Note re: Effect of wind on transit times
7 March 2012

The sum of the aircraft motion relative to the air (**V**) and the motion of the air relative to the ground (**w**) must give the motion relative to the ground (**s**). The equations below then constrain the five unknowns $\{a, b, \alpha, \gamma, \tau\}$ where a and b are the respective magnitudes of **V** and **w**, α is the angle of offset between the flight direction and the direction of motion relative to the ground or the orientation of **s**, γ is the angle between the heading (or orientation of **V**) and the wind direction, and τ is the flight time to cover the distance $|\mathbf{s}|$. The known quantities are **s**, β= angle between the ground track and the wind direction, the wind speed w_s, and the flight speed V_t. The last two equations come from the law of sines:

$$\alpha+\beta+\gamma=\pi \tag{1}$$

$$a = V_t\tau \tag{2}$$

$$b = w_s\tau \tag{3}$$

$$\frac{\sin\beta}{a} = \frac{\sin\alpha}{a} \tag{4}$$

$$\frac{\sin\gamma}{s} = \frac{\sin\beta}{a} \tag{5}$$

From (2), (3) and (4),

$$\sin\alpha = \frac{b}{a}\sin\beta = \frac{w_s}{V_t}\sin\beta$$

which gives α in terms of the known quantities $w_{s,}V_{t,}\beta$:

$$\alpha = \arcsin\left(\frac{w_s}{V_t}\sin\beta\right) \ .$$

Equation (1) then gives the angle γ. Equation (5) then gives

$$a = s\frac{\sin\beta}{\sin\gamma}$$

and so defines the time required for the flight segment:

$$\tau = \frac{a}{V_t}$$