Xpol Calibration

Technical Document

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1. CALIBRATION THEORY

1.1 Point Scattering

The transmitted power density, *S*, of a radar can be expressed as

(1) $$S_t = \frac{P_t G}{4\pi r^2}\ ,\ (\mathrm{W/m^2}),$$

where P_t is the transmitted power in (W), G is the antenna gain and r is the distance to the point target reflector in (m).
The power density captured by the reflector and re-radiated towards the radar is

(2) $$S_r = \frac{S_t \sigma}{4\pi r^2 l_t} = \frac{P_t G \sigma_{crn}}{(4\pi)^2 r^4}\ ,\ (\mathrm{W/m^2}).$$

The received power at the antenna feed is the product of the antenna effective aperture and the incident power density of the scattered field according to

(3) $$P_r = A_{eff} S_r = A_{eff} \frac{P_t G \sigma}{(4\pi)^2 r^4}\ ,$$

and since
(4) $$A_{eff} = G\lambda^2 / 4\pi\ ,$$

the received power can be written in the standard form of the radar range equation for a point target as

(5) $$P_r = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 r^4}\ ,\ (\mathrm{W}).$$

The power measured by the radar data system at the output of the radar receiver is the received power at the antenna feed, P_r and amplified by the receiver gain, G_{rec}, so the power measured by the radar data system can be expressed as

(6) $$P_{rm} = \frac{P_t G^2 G_{rec} \lambda^2 \sigma}{(4\pi)^3 r^4}.$$

By solving for σ, the expression for the maximum reflector cross section can be obtained as a function of range and the maximum measurable power ($P_{rm\_\mathbf{max}}$):

(7) $$\sigma = \frac{P_{rm\_\max} (4\pi)^3 r^4}{P_t G^2 G_{rec} \lambda^2}$$

1.2 Corner Reflector Calibration

The “Far-Field” range of an antenna is approximated as

(8) $r_{ff} = 2D^2 / \lambda$,

where D is the antenna diameter and λ is the radar wavelength. Beyond this range the antenna pattern is well established and the antenna gain is effectively independent of range, and thus if a reflector is used to calibrate the radar, then it is convenient to place it at or beyond this distance. Inside this distance the range dependence of the antenna gain has to be taken in consideration. For the Xpol, with a 1.8 m diameter antenna, this minimum range is approximately:

(9) $r_{ff} = 2(1.8)^2 / 0.03 = 216\,\boldsymbol{m}$,

We can consider using a corner reflector as a calibration target outside these far filed distances. The radar cross section of a corner reflector is given by

(10) $\sigma_{crn} = \dfrac{4\pi l^4}{3\lambda^2}\ (\mathrm{m}^2)$,

where l is the length of the inside corner of the reflector, and λ is the radar wavelength, both in units of meters.

The reflector calibration measures the overall system gain at the center of the antenna beam. The fixed radar parameters and constants can be lumped into the radar calibration constant, C_{crn}, such that

(11) $C_{crn} = \dfrac{P_t G^2 G_{rec} \lambda^2}{(4\pi)^3}$, resulting the following expression for the received power:

(12) $P_{rm} = \dfrac{C_{crn}\sigma}{r^4}$.

This calibration constant can be expressed in terms of the corner reflector measurement data according to

(13) $C_{crn} = \dfrac{P_{rm} r_{crn}^4}{\sigma_{crn}}$.

1.3 Volume Scatterering

The radar range equation for a point target can be extended to a volume target by replacing σ with the product of volume reflectivity, η (m^2/m^3) and the volume of a range cell, *V*. The range cell volume of a pulsed radar, with an approximately Gaussian shaped antenna pattern, may *be approximated as*

(14) $$V = r^2 \frac{c\tau}{2}\int_0^{\pi}\int_0^{2\pi} f^2(\theta,\phi)\sin(\theta)d\theta d\phi = r^2\frac{c\tau}{2}\pi\theta_{3dB}^2/8\ln 2,\ (m^3),$$

where,
r is the range to the center of the volume cell in meters,

(15) $$f = \exp(-\frac{4\ln(2)\theta^2}{\theta_{3dB}^2})\exp(-\frac{4\ln(2)\phi^2}{\phi_{3dB}^2})$$

$c = 3\times 10^8$ m/s, speed of light in free space,
$\tau =$ transmit pulse length (sec.), and
for the Xpol antenna half power beamwidths in elevation and azimuth may be approximated to be equal so $\theta_{3db} = \phi_{3dB}$ (radians).

The resulting expression for the measured power from a volume target is [Ref. 2, page 75, Eq. 4.16]:

(16) $$P_{Vm} = \frac{P_t G^2 G_{rec}\lambda^2}{(4\pi)^3}\frac{c\tau\pi\theta_{3dB}^2\eta}{r^2 16\ln 2} = \frac{C_{crn} c\tau\pi\theta_{3dB}^2\eta}{r^2 16\ln 2}.$$

Solving for the volume reflectivity and combining the fixed variables into a new constant, C_η,

(17) $$\eta = P_{Vm} r^2 \frac{1024\pi^2\ln 2}{P_t G^2 G_{rec}\lambda^2 c\tau\theta_{3dB}^2}.$$

So the following expression can be used to estimate the volume reflectivity at some range r from the measured power (in units of dBm):

(18) $$\eta\left(dBm^2 m^{-3}\right) = P_{Vm}\left(dBm\right) + 20\log[r(m)] + 10\log(C_n),$$

where

(19) $$C_\eta = \frac{1024\pi^2\ln 2}{P_t G^2 G_{rec}\lambda^2 c\tau\theta_{3dB}^2}$$ and here *r* is in units of meters.

A more common measure of volume reflectivity is the radar reflectivity factor, *Z*, defined as the sixth moment of the drop diameter concentration in a volume according to

(20) $$Z = \sum_{i=1}^{N_v} d_i^6,$$

where
d_i is the *i* th particle out of N_v total number of particles in the volume cell.
If the particle size is much smaller than the incident wave, so that

(21) $|\frac{n\pi d}{\lambda}| << 1$,

where
n is the complex refractive index of the particle,
then *Z* may be related to η as

(22) $\eta\left(m^2/m^3\right) = \frac{\pi^5}{\lambda^4}|K|^2\,10^{-18}\,Z(mm^6/m^3)$, so $Z = \eta\frac{\lambda^4 10^{18}}{\pi^5|K|^2}$

where

(23) $K = \frac{n^2-1}{n^2+1}$ at 9.4 GHz $|K|^2 \approx 0.94$.

By substituting Z in place of η in Eq. (17), the following expression can be obtained for the measured power from a volume scatterer:

(24) $Z = P_{Vm} r^2 \frac{\lambda^2 1024\ln 2}{P_t G^2 G_{rec} c\tau\pi^3\theta_{3dB}^2|K|^2}10^{18}$.

The radar reflectivity factor can be expressed in terms of the C_η and the measured power as

(25) $Z\left(mm^6/m^3\right) = P_{Vm} r^2 C_\eta \frac{\lambda^4}{\pi^5|K|^2}\times 10^{18}$.

We can define a new calibration constant, C_Z, by combining $C_\eta\frac{\lambda^4}{\pi^5|K|^2}\times 10^{18}$ and converting the units of *r* to km (all other lengths remain in meters):

(26) $C_Z = C_n\frac{\lambda^4}{\pi^5|K|^2}\times 10^{24}$.

(27) $C_Z = \frac{1024\ln 2\lambda^2}{P_t G^2 G_{rec} c\tau\pi^3|K|^2\theta_{3dB}^2}10^{24}$, where

(28) $dBZ\left(mm^6/m^3\right) = P_{Vm}(dBm) + 20\log[r(km)] + 10\log(C_Z)$.

2. MEASURED RESULTS

The Xpol radar has the following set of typical calibration parameters:

V: $P_t =$ 70.7 dBm -0.9 dB (waveguide loss) = 69.8 dBm,
H: $P_t =$ 70.5 dBm -0.8 dB (waveguide loss) = 69.7 dBm,

V: $G =$ 42.2 dB; from antenna test report
H: $G =$ 42.1 dB; from antenna test report

V: $G_{rec} =$ 31.0 dB – 0.9 dB (waveguide loss) = 30.1 dB
H: $G_{rec} =$ 31.6 dB – 0.8 dB (waveguide loss) = 30.8 dB

$\lambda =$ 0.032 m
$|K|^2 \approx 0.94$

V: $\theta_{3db} =$ 0.023 rad (~ 1.3 deg); from antenna test report.
V: $\theta_{3db} =$ 0.024 rad (~ 1.35 deg); from antenna test report.

Using Equation (27) and substituting the range resolution (Δr) for $\left(c\tau/2\right)$:

$$C_Z = \frac{1024 \ln 2 \lambda^2}{P_t G^2 G_{rec} c \tau \pi^3 |K|^2 \theta_{3dB}^2} 10^{24} = \frac{512 \ln 2 \lambda^2}{P_t G^2 G_{rec} \pi^3 |K|^2 \theta_{3dB}^2} 10^{24} \times \frac{1}{\Delta r},$$

V: 10log(C_Z) = 67.6 dB -10log(Δr),
H: 10log(C_Z) = 67.4 dB -10log(Δr).

The following expression is the final equation for the radar reflectivity factor, Z in units of mm^6/m^3 is

V: $\boxed{10\log(Z) = P_{Vm}(dBm) + 20\log[r(km)] - 10\log(\Delta r) + 67.6}$,
H: $\boxed{10\log(Z) = P_{Vm}(dBm) + 20\log[r(km)] - 10\log(\Delta r) + 67.4}$,

where Δr is the range resolution (30, 75, 105, 150) in units of meters.

Note that this calibration analysis is an approximation, as it assumes no atmospheric attenuation, no signal loss in the digital receiver filter, and a constant receiver gain.

3. REFERENCES

[1] Ulaby, More and Fung, 1989: “Microwave Remote Sensing”, Vol. 2, *Artech House*.
[2] Doviak and Zrnic, 1984: “Doppler Radar and Weather Observations”, *Academic Press*.