

```
<< NC`
<< NCAIgebra`
<< SDP`
<< NCSE`
<< NCSEBackwardsCompatible`
```

You are using the version of NCAIgebra which is found in:

C:\Users\ericm\NC\

You can now use "<< NCAIgebra`" to load NCAIgebra.

NCAIgebra - Version 5.0.4

Compatible with Mathematica Version 10 and above

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Helton 2002
Helton and Miller June 1991
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The program was written by the authors and by:

David Hurst, Daniel Lamm, Orlando Merino, Robert Obar,
Henry Pfister, Mike Walker, John Wavrik, Lois Yu,
J. Camino, J. Griffin, J. Ovall, T. Shaheen, John Shopple.
The beginnings of the program come from eran@slac.
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For NCAIgebra updates see:

www.github.com/NCAIgebra/NC
www.math.ucsd.edu/~ncalg

```
(*Setting up basic test variables*)
```

```
a1pent = DiagonalMatrix[{1, -1, 0, 0, 1}];
```

```
a2pent = DiagonalMatrix[{0, 0, 1, -1, 1}];
```

```
apentg2d5 = {a1pent, a2pent};
```

```
aIrred1g2d4 = {{{{2, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -2}},  
  {{1, -2, 1, -2}, {-2, -1, 0, 1}, {1, 0, 2, 3}, {-2, 1, 3, -2}}};
```

```
testXg2n2 = {{{{1, 2}, {2, 1}}, {{0, 1}, {1, -1}}};
```

```
(*Testing MakePositiveMatrix *)
```

```
M = MakeMatrixSigned[10, 3] // N;
```

```
(*The eigenvalues should have 7 positive numbers and 3 negative numbers*)
```

```
Eigenvalues[M]
```

```
{27.4178, 17.5165, 13.4005, 8.2764, 3.39504,  
 0.739394, 0.399032, -0.114303, -0.000363481, -4.50777 × 10-6}
```

```
(*Testing DirectSum*)
```

```
DirectSum[{a1pent, a2pent}] == {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

```
True
```

```
(*Testing DirectSumTuple*)
```

```
DirectSumTuple[{aIrred1g2d4, apentg2d5}] ==
```

```
{{{2, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0},  
  {0, 0, 0, -2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0},  
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1}},  
  {{1, -2, 1, -2, 0, 0, 0, 0, 0, 0}, {-2, -1, 0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 2, 3, 0, 0, 0, 0, 0, 0},  
  {-2, 1, 3, -2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}}
```

```
True
```

```
(*Testing MakeFunctionalRational *)
```

MakeFunctionalRational[3, 4]

$$\left\{ \left\{ \frac{4}{25}, \frac{91}{100}, -\frac{59}{100}, \frac{31}{50}, -\frac{43}{50}, \frac{19}{20}, -\frac{89}{50}, \frac{8}{25}, \frac{61}{100}, -\frac{9}{50}, \frac{167}{100}, \frac{13}{10}, -\frac{3}{2}, \frac{2}{25}, -\frac{3}{20}, \frac{13}{25}, \frac{157}{100}, -\frac{13}{10}, \frac{7}{25}, -\frac{33}{20}, \frac{29}{50}, -\frac{2}{25}, -\frac{137}{100}, -\frac{61}{100}, \frac{8}{25}, \frac{39}{50}, \frac{23}{20}, \frac{181}{100}, \frac{147}{100}, -\frac{71}{50} \right\}, \right. \\ \left. \frac{4}{25} X[1, 1, 1] + \frac{91}{100} X[1, 1, 2] - \frac{59}{100} X[1, 1, 3] + \frac{31}{50} X[1, 1, 4] - \frac{43}{50} X[1, 2, 2] + \frac{19}{20} X[1, 2, 3] - \frac{89}{50} X[1, 2, 4] + \frac{8}{25} X[1, 3, 3] + \frac{61}{100} X[1, 3, 4] - \frac{9}{50} X[1, 4, 4] + \frac{167}{100} X[2, 1, 1] + \frac{13}{10} X[2, 1, 2] - \frac{3}{2} X[2, 1, 3] + \frac{2}{25} X[2, 1, 4] - \frac{3}{20} X[2, 2, 2] + \frac{13}{25} X[2, 2, 3] + \frac{157}{100} X[2, 2, 4] - \frac{13}{10} X[2, 3, 3] + \frac{7}{25} X[2, 3, 4] - \frac{33}{20} X[2, 4, 4] + \frac{29}{50} X[3, 1, 1] - \frac{2}{25} X[3, 1, 2] - \frac{137}{100} X[3, 1, 3] - \frac{61}{100} X[3, 1, 4] + \frac{8}{25} X[3, 2, 2] + \frac{39}{50} X[3, 2, 3] + \frac{23}{20} X[3, 2, 4] + \frac{181}{100} X[3, 3, 3] + \frac{147}{100} X[3, 3, 4] - \frac{71}{50} X[3, 4, 4] \right\}$$

Func = MakeFunctionalRational[4, 5];

WeightVec = Func[[1]];

Functional = Func[[2]];

MakeFunctionalRational[4, 5, WeightVector → WeightVec][[2]] == Functional

True

(*Testing MakeRationalFunctional (should be the same as MakeFunctionalRational) *)

Func2 = MakeRationalFunctional[2, 3]

$$\left\{ \left\{ -\frac{81}{50}, -\frac{43}{100}, \frac{3}{2}, -\frac{1}{50}, -\frac{87}{50}, -\frac{1}{25}, -\frac{32}{25}, \frac{51}{50}, \frac{61}{100}, \frac{139}{100}, \frac{193}{100}, \frac{3}{50} \right\}, \right. \\ \left. -\frac{81}{50} X[1, 1, 1] - \frac{43}{100} X[1, 1, 2] + \frac{3}{2} X[1, 1, 3] - \frac{1}{50} X[1, 2, 2] - \frac{87}{50} X[1, 2, 3] - \frac{1}{25} X[1, 3, 3] - \frac{32}{25} X[2, 1, 1] + \frac{51}{50} X[2, 1, 2] + \frac{61}{100} X[2, 1, 3] + \frac{139}{100} X[2, 2, 2] + \frac{193}{100} X[2, 2, 3] + \frac{3}{50} X[2, 3, 3] \right\}$$

MakeRationalFunctional[4, 5, WeightVector → WeightVec][[2]] == Functional

True

(*Testing BoundedQ*)

BoundedQ[apentg2d5]

unboundedSpec = {{ {1, 0}, {0, -1}}, {{0, 1}, {1, 2}}};

BoundedQ[unboundedSpec]

True

False

(*Testing GrabFunctionalCoefs*)

```
GrabFunctionalCoefs[4, 5, Func[[2]]] == Func[[1]]
```

```
True
```

```
(*Testing ArvesonTest *)
```

```
tpoint = FindExtremePoint[aIrred1g2d4, 2, DiagnosticLevel → 0]
```

```
{{{-0.516743, -0.00345445}, {-0.00345445, -0.510631}},  
{{0.0328666, -0.014158}, {-0.014158, 0.0579145}}}
```

```
ArvesonTest[aIrred1g2d4, tpoint]
```

```
ArvesonTest[aIrred1g2d4, tpoint, NumericsAssessment → False]
```

```
ArvesonTest[aIrred1g2d4, tpoint, NumericsAssessment → False, EigGapTol → 10−10]
```

```
ArvesonTest[aIrred1g2d4, tpoint, NumericsAssessment → False, EigMagTol → 10−10]
```

```
ArvesonTest[aIrred1g2d4, tpoint, EigGapTol → 10−10]
```

```
ArvesonTest[aIrred1g2d4, tpoint, EigMagTol → 10−10]
```

```
{True, {6.7972 × 10−9, 0.228125}}
```

```
{True, {6.7972 × 10−9, 0.228125}}
```

```
{False, {6.7972 × 10−9, False,  
{2.0178, 2.00227, 1.54157, 1.53953, 0.456161, 0.442664, 3.00888 × 10−9, 4.28953 × 10−10}}}}
```

```
{False, {6.7972 × 10−9, False,  
{2.0178, 2.00227, 1.54157, 1.53953, 0.456161, 0.442664, 3.00888 × 10−9, 4.28953 × 10−10}}}}
```

```
{BadNullNumerics, {6.7972 × 10−9, BadNullNumerics}}
```

```
{BadNullNumerics, {6.7972 × 10−9, BadNullNumerics}}
```

```
(*Testing EuclideanTest *)
```

```
EuclideanTest[aIrred1g2d4, tpoint]
```

```
EuclideanTest[aIrred1g2d4, tpoint, NumericsAssessment → False]
```

```
EuclideanTest[aIrred1g2d4, tpoint, NumericsAssessment → False, EigGapTol → 10−10]
```

```
EuclideanTest[aIrred1g2d4, tpoint, EigGapTol → 10−10]
```

```
{True, {6.7972 × 10−9, 0.435687}}
```

```
{True, {6.7972 × 10−9, 0.435687}}
```

```
{False, {6.7972 × 10−9, False,  
{2.0178, 2.00227, 1.54157, 1.53953, 0.456161, 0.442664, 3.00888 × 10−9, 4.28953 × 10−10}}}}
```

```
{BadNullNumerics, {6.7972 × 10−9, BadNullNumerics}}
```

```
(*Testing FindExtremePoint*)
FindExtremePoint[aIrred1g2d4, 2]
{{{ {-0.499563, -0.0494347}, {-0.0494347, 0.413203}},
  {{0.00254905, -0.00485196}, {-0.00485196, 0.0921361}}}, { $\frac{93}{50}$ ,  $\frac{17}{50}$ ,  $-\frac{28}{25}$ ,  $\frac{89}{50}$ ,  $\frac{17}{100}$ ,  $-\frac{141}{100}$ },
  True, { $2.18134 \times 10^{-9}$ , 0.259461}, True, { $2.18134 \times 10^{-9}$ , 0.259461}, 2, 2, 2,  $9.18513 \times 10^{-16}$ }
```

```
BadWeightVec = {1};
extPt1 = FindExtremePoint[aIrred1g2d4, 2, WeightVector → WeightVec]
```

WARNING: The WeightVector Option does not have the correct format.

```
GoodWeightVec = FindExtremePoint[aIrred1g2d4, 2, DiagnosticLevel → 2][[2]]
extPt2 =
  FindExtremePoint[aIrred1g2d4, 2, WeightVector → GoodWeightVec, DiagnosticLevel → 2]
```

```
{ $\frac{179}{100}$ ,  $-\frac{119}{100}$ ,  $-\frac{33}{50}$ ,  $-\frac{11}{50}$ ,  $\frac{121}{100}$ ,  $\frac{19}{25}$ }
{{{ {-0.461468, 0.228759}, {0.228759, 0.382813}},
  {{0.0334752, -0.0463794}, {-0.0463794, -0.137697}}},
  { $\frac{179}{100}$ ,  $-\frac{119}{100}$ ,  $-\frac{33}{50}$ ,  $-\frac{11}{50}$ ,  $\frac{121}{100}$ ,  $\frac{19}{25}$ }, True, True}
```

```
(*The weight vector used to generate extPt1 should be WeightVec*)
```

```
extPt2[[2]] == GoodWeightVec
```

```
True
```

```
extPt3 = FindExtremePoint[aIrred1g2d4, 1, MyFunctional → MakeFunctionalTrace]
FindExtremePoint[aIrred1g2d4, 1,
  MyFunctional → MakeFunctionalTrace, WeightMatrix → extPt3[[2]]]
```

```
{{{ {0.0926665}}, {{-0.24403}}}, { $\frac{9}{100}$ ,  $\frac{57}{100}$ ,  $\frac{21}{100}$ ,  $-\frac{27}{50}$ },
  { $\frac{57}{100}$ ,  $\frac{53}{10}$ ,  $\frac{69}{25}$ ,  $-\frac{171}{50}$ }, { $\frac{21}{100}$ ,  $\frac{69}{25}$ ,  $\frac{117}{50}$ ,  $-\frac{47}{50}$ }, { $-\frac{27}{50}$ ,  $-\frac{171}{50}$ ,  $-\frac{47}{50}$ ,  $\frac{349}{100}$ }}, True,
  { $8.33391 \times 10^{-10}$ , 0.0997247}, True, { $8.33391 \times 10^{-10}$ , 0.0997247}, 1, 1, True, 0}
```

```
{{{ {0.0926665}}, {{-0.24403}}}, { $\frac{9}{100}$ ,  $\frac{57}{100}$ ,  $\frac{21}{100}$ ,  $-\frac{27}{50}$ },
  { $\frac{57}{100}$ ,  $\frac{53}{10}$ ,  $\frac{69}{25}$ ,  $-\frac{171}{50}$ }, { $\frac{21}{100}$ ,  $\frac{69}{25}$ ,  $\frac{117}{50}$ ,  $-\frac{47}{50}$ }, { $-\frac{27}{50}$ ,  $-\frac{171}{50}$ ,  $-\frac{47}{50}$ ,  $\frac{349}{100}$ }}, True,
  { $8.33391 \times 10^{-10}$ , 0.0997247}, True, { $8.33391 \times 10^{-10}$ , 0.0997247}, 1, 1, True, 0}
```

```
BadWeightMat = {{1}};
FindExtremePoint[aIrred1g2d4, 1,
  MyFunctional → MakeFunctionalTrace, WeightMatrix → BadWeightMat]
```

WARNING: The WeightMatrix Option does not have the correct format.

```

FindExtremePoint[aIrred1g2d4, 1, EigMagTol → 10−10]

FindExtremePoint[aIrred1g2d4, 1, NumericsAssessment → False, EigMagTol → 10−10]
{{{{-0.515736}}, {{0.0548043}}}, { $\frac{17}{10}$ ,  $-\frac{93}{100}$ },
BadNullNumerics, {8.24652 × 10−10, BadNullNumerics}, BadNullNumerics,
{8.24652 × 10−10, BadNullNumerics}, BadNullNumerics, 1, True, 0}

{{{0.418376}}, {{0.0903829}}}, { $-\frac{53}{50}$ ,  $-\frac{131}{100}$ }, False,
{8.65285 × 10−10, False, {2.1252, 1.15587, 0.718926, 6.22076 × 10−10}},
False, {8.65285 × 10−10, False, {2.1252, 1.15587, 0.718926, 6.22076 × 10−10}},
BadNullNumerics, 1, True, 0}

FindExtremePoint[aIrred1g2d4, 1, ChopCutoff → 10−5]
{{{{-0.513313}}, {{0.0162605}}}, { $\frac{117}{100}$ ,  $\frac{73}{100}$ }, True,
{6.13358 × 10−10, 0.184675}, True, {6.13358 × 10−10, 0.184675}, 1, 1, True, 0}

FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 0]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 1]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 2]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 3]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 4]

{{{0.431281}}, {{-0.15691}}}

{{{{-0.492865}}, {{-0.00668724}}}, { $\frac{91}{100}$ ,  $\frac{103}{100}$ }}

{{{{-0.480019}}, {{0.088191}}}, { $\frac{2}{5}$ ,  $-\frac{31}{50}$ }, True, True}

{{{0.263992}}, {{-0.244092}}}, { $-\frac{13}{25}$ ,  $\frac{47}{25}$ },
True, {1.57797 × 10−9, 0.23863}, True, {1.57797 × 10−9, 0.23863}}

{{{0.505703}}, {{-0.0710458}}}, { $-\frac{197}{100}$ ,  $\frac{18}{25}$ }, True,
{4.98517 × 10−10, 0.403864}, True, {4.98517 × 10−10, 0.403864}, 1, 1, True, 0}

(*Testing FindExtremeAndAnalyze*)
FindExtremeAndAnalyze[aIrred1g2d4, 2, 4, 929]

```

```

{}
0
False
False
0
False
{{}}, 0
4
0
{
  {
    {
      {
        {0.489031, -0.060212}, {-0.060212, 0.333009}},
        {
          {-0.0340076, 0.06756}, {0.06756, 0.141055}},
          {
             $-\frac{181}{100}, \frac{53}{100}, -\frac{34}{25}, \frac{3}{20}, -\frac{27}{20}, -\frac{181}{100}$ ,
            True, {8.67882 × 10-10, 0.402784}, True, {8.67882 × 10-10, 0.402784}, 2, 2,
            2, 5.83351 × 10-16},
            {
              {0.302849, 0.369109}, {0.369109, -0.215324}},
              {
                {-0.0464095, -0.0894016}, {-0.0894016, 0.0790968}},
                {
                  -1,  $-\frac{48}{25}, \frac{13}{100}, \frac{3}{4}, \frac{41}{25}, -\frac{13}{10}$ ,
                  True, {6.77585 × 10-8, 0.377911}, True, {6.77585 × 10-8, 0.377911}, 2, 2, 2, 2.6266 × 10-15},
                  {
                    {0.445946, 0.246734}, {0.246734, -0.456936}},
                    {
                      {-0.0206605, -0.0161709}, {-0.0161709, 0.0385141}},
                      {
                         $-\frac{27}{20}, -\frac{161}{100}, \frac{137}{100}, \frac{3}{50}, -\frac{3}{2}, -\frac{16}{25}$ , True, {3.29794 × 10-9, 0.364671},
                        True, {3.29794 × 10-9, 0.364671}, 2, 2, 2, 8.25905 × 10-16},
                        {
                          {0.457342, 0.00832793}, {0.00832793, 0.481399}},
                          {
                            {0.043506, -0.0472277}, {-0.0472277, -0.092922}},
                            {
                               $-\frac{89}{100}, -\frac{107}{100}, -2, -\frac{49}{100}, \frac{69}{50}, \frac{79}{50}$ ,
                              True, {9.0701 × 10-10, 0.362036}, True, {9.0701 × 10-10, 0.362036}, 2, 2, 2, 7.29152 × 10-16}},
                              {{}}, 0, False, False, 0, False, {{}}, 0, 4, 0, {}
                            }
                        }
                    }
                }
            }
          }
        }
      }
    }
  }
}

```

FindExtremeAndAnalyze[aIrred1g2d4, 2, 4, 929]

```

{}
0
False
False
0
False
{{}, 0}
4
0
{
  {
    {
      {
        {0.489031, -0.060212}, {-0.060212, 0.333009}},
        {
          {-0.0340076, 0.06756}, {0.06756, 0.141055}},
          {
             $-\frac{181}{100}, \frac{53}{100}, -\frac{34}{25}, \frac{3}{20}, -\frac{27}{20}, -\frac{181}{100}$ ,
            True, {8.67882 × 10-10, 0.402784}, True, {8.67882 × 10-10, 0.402784}, 2, 2,
            2, 5.83351 × 10-16},
            {
              {0.302849, 0.369109}, {0.369109, -0.215324}},
              {
                {-0.0464095, -0.0894016}, {-0.0894016, 0.0790968}},
                {
                  -1,  $-\frac{48}{25}, \frac{13}{100}, \frac{3}{4}, \frac{41}{25}, -\frac{13}{10}$ ,
                  True, {6.77585 × 10-8, 0.377911}, True, {6.77585 × 10-8, 0.377911}, 2, 2, 2, 2.6266 × 10-15},
                  {
                    {0.445946, 0.246734}, {0.246734, -0.456936}},
                    {
                      {-0.0206605, -0.0161709}, {-0.0161709, 0.0385141}},
                      {
                         $-\frac{27}{20}, -\frac{161}{100}, \frac{137}{100}, \frac{3}{50}, -\frac{3}{2}, -\frac{16}{25}$ , True, {3.29794 × 10-9, 0.364671},
                        True, {3.29794 × 10-9, 0.364671}, 2, 2, 2, 8.25905 × 10-16},
                        {
                          {0.457342, 0.00832793}, {0.00832793, 0.481399}},
                          {
                            {0.043506, -0.0472277}, {-0.0472277, -0.092922}},
                            {
                               $-\frac{89}{100}, -\frac{107}{100}, -2, -\frac{49}{100}, \frac{69}{50}, \frac{79}{50}$ ,
                              True, {9.0701 × 10-10, 0.362036}, True, {9.0701 × 10-10, 0.362036}, 2, 2, 2, 7.29152 × 10-16}},
                              {{}, 0, False, False, 0, False, {{}, 0}, 4, 0, {}}
                            }
                          }
                        }
                      }
                    }
                  }
                }
              }
            }
          }
        }
      }
    }
  }
}

```

test1 = FindExtremeAndAnalyze[aIrred1g2d4, 2, 2, 929, MyFunctional → MakeFunctionalTrace]


```

{}
0
False
False
0
False
{{}, 0}
2
0
{
  {
    {
      {
        {-0.487308, -0.0012309}, {-0.0012309, -0.511937}
      },
      {
        {-0.0112868, 0.00358504}, {0.00358504, 0.0604445}
      }
    },
    {
      {
 $\frac{36}{25}, \frac{3}{25}, -\frac{33}{25}, \frac{12}{5}, -\frac{18}{25}, \frac{57}{25}, \frac{3}{25}, \frac{24}{25}$ 
      },
      {
 $\frac{3}{25}, \frac{29}{20}, \frac{157}{100}, -\frac{52}{25}, -\frac{39}{50}, \frac{31}{100}, -\frac{227}{100}, \frac{38}{25}$ 
      },
      {
 $-\frac{33}{25}, \frac{157}{100}, \frac{163}{50}, -\frac{531}{100}, 0, -\frac{153}{100}, -\frac{271}{100}, \frac{29}{25}$ 
      },
      {
 $\frac{12}{5}, -\frac{52}{25}, -\frac{531}{100}, \frac{131}{10}$ 
      },
      {
 $-\frac{3}{2}, \frac{133}{100}, \frac{27}{100}, -\frac{79}{50}$ 
      },
      {
 $-\frac{18}{25}, -\frac{39}{50}, 0, -\frac{3}{2}, \frac{217}{100}, \frac{97}{100}, -\frac{3}{50}, -\frac{33}{100}$ 
      },
      {
 $\frac{57}{25}, \frac{31}{100}, -\frac{153}{100}, \frac{133}{100}, \frac{97}{100}, \frac{186}{25}, -\frac{57}{25}, \frac{151}{50}$ 
      },
      {
 $\frac{3}{25}, -\frac{227}{100}, -\frac{271}{100}, \frac{27}{100}, -\frac{3}{50}$ 
      },
      {
 $-\frac{57}{25}, \frac{1163}{100}, -\frac{38}{25}$ 
      },
      {
 $\frac{24}{25}, \frac{38}{25}, \frac{29}{25}, -\frac{79}{50}, -\frac{33}{100}, \frac{151}{50}, -\frac{38}{25}, \frac{779}{100}$ 
      }
    },
    True,
    {
 $4.94648 \times 10^{-10}$ , 0.215611
    },
    True,
    {
 $4.94648 \times 10^{-10}$ , 0.215611
    },
    2, 2, 2,  $9.00269 \times 10^{-16}$ 
  },
  {
    {
      {-0.489217, 0.00580788}, {0.00580788, -0.516079}
    },
    {
      {0.0815091, 0.00617168}, {0.00617168, 0.0529645}
    }
  },
  {
    {
      1,  $\frac{6}{5}, \frac{1}{10}, \frac{1}{5}, \frac{19}{10}, \frac{19}{10}, -\frac{1}{5}, \frac{3}{5}$ 
    },
    {
 $\frac{6}{5}, \frac{72}{25}, -\frac{42}{25}, -\frac{6}{5}, -\frac{3}{25}, \frac{18}{5}, -\frac{51}{25}, -\frac{27}{25}$ 
    },
    {
 $\frac{1}{10}, -\frac{42}{25}, \frac{121}{50}, \frac{91}{50}, \frac{59}{20}, -\frac{33}{50}, \frac{151}{100}, \frac{279}{100}$ 
    },
    {
 $\frac{1}{5}, -\frac{6}{5}, \frac{91}{50}, \frac{173}{100}, \frac{87}{25}, -\frac{17}{50}, \frac{133}{50}, \frac{117}{100}$ 
    },
    {
 $\frac{19}{10}, -\frac{3}{25}, \frac{59}{20}, \frac{87}{25}, \frac{1093}{100}, \frac{389}{100}, \frac{301}{50}, \frac{61}{50}$ 
    },
    {
 $\frac{19}{10}, \frac{18}{5}, -\frac{33}{50}, -\frac{17}{50}, \frac{389}{100}, \frac{721}{50}, -\frac{439}{100}, -\frac{91}{100}$ 
    },
    {
 $-\frac{1}{5}, -\frac{51}{25}, \frac{151}{100}, \frac{133}{50}, \frac{301}{50}, -\frac{439}{100}, \frac{23}{2}, \frac{129}{100}$ 
    },
    {
 $\frac{3}{5}, -\frac{27}{25}, \frac{279}{100}, \frac{117}{100}, \frac{61}{50}, -\frac{91}{100}, \frac{129}{100}, \frac{1327}{100}$ 
    }
  },
  True,
  {
 $7.43351 \times 10^{-10}$ , 0.234679
  },
  2, 2, 2,  $1.03257 \times 10^{-15}$ 
},
{{}, 0, False, False, 0, False, {{}, 0}, 2, 0, {}
}

```

(*Testing FindExtremeAndAnalyze *)

FindExtremeAndAnalyze[aIrred1g2d4, 2, 2, 929, MyFunctional → MakeFunctionalTrace] == test1

`{}``0``False``False``0``False``{{}, 0}``2``0``True``(*Testing MakeIrreducibleA*)``MakeIrreducibleA[3, 4, 100]`

$$\left\{ \left\{ \left\{ -5, 1, \frac{23}{10}, -\frac{7}{10} \right\}, \left\{ 1, \frac{11}{5}, \frac{3}{5}, -\frac{3}{10} \right\}, \left\{ \frac{23}{10}, \frac{3}{5}, \frac{4}{5}, \frac{14}{5} \right\}, \left\{ -\frac{7}{10}, -\frac{3}{10}, \frac{14}{5}, \frac{11}{5} \right\} \right\}, \right. \\ \left. \left\{ \left\{ \frac{12}{5}, \frac{5}{2}, \frac{11}{10}, \frac{11}{5} \right\}, \left\{ \frac{5}{2}, 1, -\frac{9}{10}, -\frac{11}{10} \right\}, \left\{ \frac{11}{10}, -\frac{9}{10}, -\frac{4}{5}, -\frac{9}{5} \right\}, \left\{ \frac{11}{5}, -\frac{11}{10}, -\frac{9}{5}, \frac{17}{5} \right\} \right\}, \right. \\ \left. \left\{ \left\{ -4, -\frac{33}{10}, \frac{13}{5}, 0 \right\}, \left\{ -\frac{33}{10}, -\frac{6}{5}, -\frac{19}{10}, -\frac{3}{2} \right\}, \left\{ \frac{13}{5}, -\frac{19}{10}, \frac{9}{5}, \frac{33}{10} \right\}, \left\{ 0, -\frac{3}{2}, \frac{33}{10}, -\frac{23}{5} \right\} \right\} \right\}$$
`(*Testing MakeIrreducibleBoundedA *)``MakeIrreducibleBoundedA[4, 3, 100]`

$$\left\{ \left\{ \left\{ -\frac{23}{5}, -\frac{6}{5}, -\frac{4}{5} \right\}, \left\{ -\frac{6}{5}, \frac{18}{5}, \frac{3}{2} \right\}, \left\{ -\frac{4}{5}, \frac{3}{2}, -\frac{2}{5} \right\} \right\}, \right. \\ \left. \left\{ \left\{ -\frac{9}{5}, \frac{33}{10}, -4 \right\}, \left\{ \frac{33}{10}, \frac{19}{5}, -\frac{3}{10} \right\}, \left\{ -4, -\frac{3}{10}, -\frac{2}{5} \right\} \right\}, \right. \\ \left. \left\{ \left\{ 1, -\frac{23}{10}, \frac{1}{10} \right\}, \left\{ -\frac{23}{10}, -\frac{1}{5}, \frac{5}{2} \right\}, \left\{ \frac{1}{10}, \frac{5}{2}, -\frac{11}{5} \right\} \right\}, \right. \\ \left. \left\{ \left\{ -1, \frac{1}{5}, \frac{41}{10} \right\}, \left\{ \frac{1}{5}, -5, \frac{1}{10} \right\}, \left\{ \frac{41}{10}, \frac{1}{10}, \frac{23}{5} \right\} \right\} \right\}$$
`(*Testing LMI*)``LA = LMI[apentg2d5, testXg2n2]`

$$\left\{ \{0, -2, 0, 0, 0, 0, 0, 0, 0, 0\}, \{-2, 0, 0, 0, 0, 0, 0, 0, 0, 0\}, \right. \\ \{0, 0, 2, 2, 0, 0, 0, 0, 0, 0\}, \{0, 0, 2, 2, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, -1, 0, 0, 0, 0\}, \\ \{0, 0, 0, 0, -1, 2, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 0, 0, 0\}, \\ \left. \{0, 0, 0, 0, 0, 0, 0, 0, 0, -3\}, \{0, 0, 0, 0, 0, 0, 0, 0, -3, 1\} \right\}$$
`(*Testing Lambda*)``IdentityMatrix[10] - Lambda[apentg2d5, testXg2n2] == LA``True`

(*Testing Tuple operations.*)

TupleNorm[apentg2d5]

2

LeftMultTuple[DiagonalMatrix[{-1, -1, 0, 1, 1}], apentg2d5]

```
{{{-1, 0, 0, 0, 0}, {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 1}},
{{0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, -1, 0}, {0, 0, 0, 0, 1}}}
```

RightMultTuple[

```
{ {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, 0, 0}}, apentg2d5]
{{ {0, 1, 0, 0, 0}, {0, 0, -1, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
{ {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, -1}, {0, 0, 0, 0, 0}}}
```

ConjTuple[DiagonalMatrix[{-1, -1, 0, 1, 1}], apentg2d5]

```
{{ {1, 0, 0, 0, 0}, {0, -1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 1}},
{ {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, -1, 0}, {0, 0, 0, 0, 1}}}
```

TupleComm[DiagonalMatrix[{-1, -1, 0, 1, 1}], apentg2d5]

```
{{ {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}},
{ {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}}}
```

TupleComm[

```
{ {1, 1, 2, 3, 4}, {4, 3, 2, 5, 1}, {1, 2, 3, 2, 1}, {3, 3, 3, 3, 3}, {5, 3, 2, 1, 2}}, apentg2d5]
{{ {0, -2, -2, -3, 0}, {8, 0, 2, 5, 2}, {1, -2, 0, 0, 1}, {3, -3, 0, 0, 3}, {0, -6, -2, -1, 0}},
{ {0, 0, 2, -3, 4}, {0, 0, 2, -5, 1}, {-1, -2, 0, -4, 0}, {3, 3, 6, 0, 6}, {-5, -3, 0, -2, 0}}}
```