

```
In[77]:= << NC`
<< NCAAlgebra`
<< SDP`
<< NCSE`
<< NCSEBackwardsCompatible`

You are using the version of NCAAlgebra which is found in:

C:\Users\ericm\NC\

You can now use "<< NCAAlgebra`" to load NCAAlgebra.
```

```
In[82]:= (*Setting up basic test variables*)

a1pent = DiagonalMatrix[{1, -1, 0, 0, 1}];
a2pent = DiagonalMatrix[{0, 0, 1, -1, 1}];
apentg2d5 = {a1pent, a2pent};

aIrred1g2d4 = {{ {2, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -2}},
  {{1, -2, 1, -2}, {-2, -1, 0, 1}, {1, 0, 2, 3}, {-2, 1, 3, -2}}};

testXg2n2 = {{ {1, 2}, {2, 1}}, {{0, 1}, {1, -1}}};
(*Testing MakePositiveMatrix *)

M = MakeMatrixSigned[10, 3] // N;

(*The eigenvalues should have 7 positive numbers and 3 negative numbers*)
Eigenvalues[M]
Out[88]= {26.0593, 16.0985, 9.61343, 8.14445, 3.05963,
  1.30188, 0.455811, -0.275575, -0.00605642, -3.17661 × 10-6}
```

```
In[89]:= (*Testing DirectSum*)

DirectSum[{a1pent, a2pent}] == {{1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}
```

Out[89]= True

```
In[90]:= (*Testing DirectSumTuple*)
DirectSumTuple[{aIrred1g2d4, apentg2d5}] ==
{{ {2, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, -1, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, -2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, -1, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1}},
  {{1, -2, 1, -2, 0, 0, 0, 0, 0, 0}, {-2, -1, 0, 1, 0, 0, 0, 0, 0, 0}, {1, 0, 2, 3, 0, 0, 0, 0, 0, 0},
  {-2, 1, 3, -2, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 1}}}
```

Out[90]= True

```
In[91]:= (*Testing MakeFunctionalRational *)
```

```
In[92]:= MakeFunctionalRational[3, 4]
```

```
Out[92]= { { - $\frac{89}{50}$ , - $\frac{147}{100}$ , - $\frac{9}{100}$ , - $\frac{24}{25}$ ,  $\frac{31}{50}$ , - $\frac{139}{100}$ ,  $\frac{123}{100}$ ,  $\frac{161}{100}$ , - $\frac{27}{100}$ ,  $\frac{173}{100}$ , - $\frac{9}{50}$ , 1, - $\frac{63}{50}$ , - $\frac{77}{100}$ ,  $\frac{51}{100}$ , - $\frac{11}{100}$ ,  
- $\frac{33}{25}$ , - $\frac{99}{100}$ , - $\frac{173}{100}$ , - $\frac{31}{25}$ ,  $\frac{47}{100}$ , - $\frac{77}{100}$ ,  $\frac{1}{2}$ ,  $\frac{169}{100}$ ,  $\frac{43}{25}$ , - $\frac{27}{20}$ , - $\frac{53}{100}$ ,  $\frac{127}{100}$ , - $\frac{23}{100}$ , - $\frac{103}{100}$  },  
- $\frac{89}{50}$  X[1, 1, 1] -  $\frac{147}{100}$  X[1, 1, 2] -  $\frac{9}{100}$  X[1, 1, 3] -  $\frac{24}{25}$  X[1, 1, 4] +  $\frac{31}{50}$  X[1, 2, 2] -  
 $\frac{139}{100}$  X[1, 2, 3] +  $\frac{123}{100}$  X[1, 2, 4] +  $\frac{161}{100}$  X[1, 3, 3] -  $\frac{27}{100}$  X[1, 3, 4] +  $\frac{173}{100}$  X[1, 4, 4] -  
 $\frac{9}{50}$  X[2, 1, 1] + X[2, 1, 2] -  $\frac{63}{50}$  X[2, 1, 3] -  $\frac{77}{100}$  X[2, 1, 4] +  $\frac{51}{100}$  X[2, 2, 2] -  
 $\frac{11}{100}$  X[2, 2, 3] -  $\frac{33}{25}$  X[2, 2, 4] -  $\frac{99}{100}$  X[2, 3, 3] -  $\frac{173}{100}$  X[2, 3, 4] -  $\frac{31}{25}$  X[2, 4, 4] +  
 $\frac{47}{100}$  X[3, 1, 1] -  $\frac{77}{100}$  X[3, 1, 2] +  $\frac{1}{2}$  X[3, 1, 3] +  $\frac{169}{100}$  X[3, 1, 4] +  $\frac{43}{25}$  X[3, 2, 2] -  
 $\frac{27}{20}$  X[3, 2, 3] -  $\frac{53}{100}$  X[3, 2, 4] +  $\frac{127}{100}$  X[3, 3, 3] -  $\frac{23}{100}$  X[3, 3, 4] -  $\frac{103}{100}$  X[3, 4, 4] }
```

```
In[93]:= Func = MakeFunctionalRational[4, 5];
```

```
WeightVec = Func[[1]];
```

```
Functional = Func[[2]];
```

```
MakeFunctionalRational[4, 5, WeightVector → WeightVec][[2]] == Functional
```

```
Out[96]= True
```

```
In[97]:= (*Testing MakeRationalFunctional (should be the same as MakeFunctionalRational) *)
```

```
Func2 = MakeRationalFunctional[2, 3]
```

```
Out[97]= { {  $\frac{41}{100}$ , - $\frac{163}{100}$ ,  $\frac{26}{25}$ ,  $\frac{193}{100}$ , - $\frac{33}{25}$ ,  $\frac{7}{5}$ ,  $\frac{44}{25}$ , - $\frac{77}{50}$ ,  $\frac{173}{100}$ ,  $\frac{12}{25}$ ,  $\frac{3}{25}$ , - $\frac{19}{25}$  },  
 $\frac{41}{100}$  X[1, 1, 1] -  $\frac{163}{100}$  X[1, 1, 2] +  $\frac{26}{25}$  X[1, 1, 3] +  $\frac{193}{100}$  X[1, 2, 2] -  $\frac{33}{25}$  X[1, 2, 3] +  $\frac{7}{5}$  X[1, 3, 3] +  
 $\frac{44}{25}$  X[2, 1, 1] -  $\frac{77}{50}$  X[2, 1, 2] +  $\frac{173}{100}$  X[2, 1, 3] +  $\frac{12}{25}$  X[2, 2, 2] +  $\frac{3}{25}$  X[2, 2, 3] -  $\frac{19}{25}$  X[2, 3, 3] }
```

```
In[98]:= MakeRationalFunctional[4, 5, WeightVector → WeightVec][[2]] == Functional
```

```
Out[98]= True
```

```
In[99]:= (*Testing BoundedQ. apentg2d5 is bounded. unboundedSpec is unbounded*)
```

```
In[100]:= BoundedQ[apentg2d5]
```

```
unboundedSpec = { {{1, 0}}, {0, -1}}, {{0, 1}}, {1, 2}}};
```

```
BoundedQ[unboundedSpec]
```

```
Out[100]= True
```

```
Out[102]= False
```

```
In[103]:= (*Testing GrabFunctionalCoefs*)
```

```

In[104]:= GrabFunctionalCoefs[4, 5, Func[[2]]] == Func[[1]]
Out[104]= True

In[105]:= (*Testing ArvesonTest *)

In[106]:= tpoint = FindExtremePoint[aIrred1g2d4, 2, DiagnosticLevel → 0]
Out[106]= {{ {0.135519, -0.26907}, {-0.26907, -0.400833} },
  { {-0.209502, 0.093237}, {0.093237, -0.0236474} }}

In[107]:= ArvesonTest[aIrred1g2d4, tpoint]
ArvesonTest[aIrred1g2d4, tpoint, NumericsAssessment → False]

ArvesonTest[aIrred1g2d4, tpoint, NumericsAssessment → False, EigGapTol → 10-10]
ArvesonTest[aIrred1g2d4, tpoint, NumericsAssessment → False, EigMagTol → 10-10]

ArvesonTest[aIrred1g2d4, tpoint, EigGapTol → 10-10]
ArvesonTest[aIrred1g2d4, tpoint, EigMagTol → 10-10]
Out[107]= {True, {8.35892 × 10-10, 0.244114}}
Out[108]= {True, {8.35892 × 10-10, 0.244114}}
Out[109]= {False, {8.35892 × 10-10, False,
  {2.22161, 2.01249, 1.52593, 1.47376, 0.461585, 0.304635, -2.54642 × 10-10, 2.14355 × 10-10}}}
Out[110]= {False, {8.35892 × 10-10, False,
  {2.22161, 2.01249, 1.52593, 1.47376, 0.461585, 0.304635, -2.54642 × 10-10, 2.14355 × 10-10}}}
Out[111]= {BadNullNumerics, {8.35892 × 10-10, BadNullNumerics}}
Out[112]= {BadNullNumerics, {8.35892 × 10-10, BadNullNumerics}}

In[113]:= (*Testing EuclideanTest *)
EuclideanTest[aIrred1g2d4, tpoint]
EuclideanTest[aIrred1g2d4, tpoint, NumericsAssessment → False]

EuclideanTest[aIrred1g2d4, tpoint, NumericsAssessment → False, EigGapTol → 10-10]

EuclideanTest[aIrred1g2d4, tpoint, EigGapTol → 10-10]
Out[113]= {True, {8.35892 × 10-10, 0.45797}}
Out[114]= {True, {8.35892 × 10-10, 0.45797}}
Out[115]= {False, {8.35892 × 10-10, False,
  {2.22161, 2.01249, 1.52593, 1.47376, 0.461585, 0.304635, -2.54642 × 10-10, 2.14355 × 10-10}}}
Out[116]= {BadNullNumerics, {8.35892 × 10-10, BadNullNumerics}}

```

```

In[117]:= (*Testing FindExtremePoint*)
FindExtremePoint[aIrred1g2d4, 2]
Out[117]= {{ {0.409107, 0.244034}, {0.244034, -0.352799}},
  {{-0.107136, -0.0158977}, {-0.0158977, -0.0575013}}, {-153/100, -6/5, 1/2, 27/25, 31/25, 31/20},
  True, {1.89329 × 10-9, 0.354982}, True, {1.89329 × 10-9, 0.354982}, 2, 2, 2, 6.32088 × 10-16}

In[118]:= BadWeightVec = {1};
extPt1 = FindExtremePoint[aIrred1g2d4, 2, WeightVector → WeightVec]
WARNING: The WeightVector Option does not have the correct format.

In[120]:= GoodWeightVec = FindExtremePoint[aIrred1g2d4, 2, DiagnosticLevel → 2] [[2]]
extPt2 =
  FindExtremePoint[aIrred1g2d4, 2, WeightVector → GoodWeightVec, DiagnosticLevel → 2]
Out[120]= {-17/50, 19/25, 3/2, 8/5, -11/20, 83/50}
Out[121]= {{ {0.211352, -0.173259}, {-0.173259, -0.459119}},
  {{-0.232953, 0.0569464}, {0.0569464, -0.0125841}}},
  {-17/50, 19/25, 3/2, 8/5, -11/20, 83/50}, True, True}

In[122]:= (*The weight vector used to generate extPt1 should be WeightVec*)

In[123]:= extPt2[[2]] == GoodWeightVec
Out[123]= True

```

```
In[124]:= extPt3 = FindExtremePoint[aIrred1g2d4, 1, MyFunctional → MakeFunctionalTrace]
FindExtremePoint[aIrred1g2d4, 1,
MyFunctional → MakeFunctionalTrace, WeightMatrix → extPt3[[2]]]
```

```
Out[124]= {{ {{{-0.420649}}, {{-0.0548663}}},
{{
 $\frac{1309209489}{2500000000}$ ,  $-\frac{509782287}{625000000}$ ,  $\frac{2035981227}{5000000000}$ ,  $-\frac{3558706599}{5000000000}$ },
{{
 $-\frac{509782287}{625000000}$ ,  $\frac{1934375029}{500000000}$ ,  $-\frac{7078797611}{2500000000}$ ,  $\frac{3275533873}{1250000000}$ },
{{
 $\frac{2035981227}{5000000000}$ ,  $\frac{7078797611}{2500000000}$ ,  $\frac{21299966683}{5000000000}$ ,  $-\frac{8964968913}{2500000000}$ },
{{
 $-\frac{3558706599}{5000000000}$ ,  $\frac{3275533873}{1250000000}$ ,  $-\frac{8964968913}{2500000000}$ ,  $\frac{3358157529}{1000000000}$ }}, True,
{5.74852 × 10-11, 0.0907989}, True, {5.74852 × 10-11, 0.0907989}, 1, 1, True, 0}
```

```
Out[125]= {{ {{{-0.420649}}, {{-0.0548663}}},
{{
 $\frac{1309209489}{2500000000}$ ,  $-\frac{509782287}{625000000}$ ,  $\frac{2035981227}{5000000000}$ ,  $-\frac{3558706599}{5000000000}$ },
{{
 $-\frac{509782287}{625000000}$ ,  $\frac{1934375029}{500000000}$ ,  $-\frac{7078797611}{2500000000}$ ,  $\frac{3275533873}{1250000000}$ },
{{
 $\frac{2035981227}{5000000000}$ ,  $\frac{7078797611}{2500000000}$ ,  $\frac{21299966683}{5000000000}$ ,  $-\frac{8964968913}{2500000000}$ },
{{
 $-\frac{3558706599}{5000000000}$ ,  $\frac{3275533873}{1250000000}$ ,  $-\frac{8964968913}{2500000000}$ ,  $\frac{3358157529}{1000000000}$ }}, True,
{5.74852 × 10-11, 0.0907989}, True, {5.74852 × 10-11, 0.0907989}, 1, 1, True, 0}
```

```
In[126]:= BadWeightMat = {{1}};
FindExtremePoint[aIrred1g2d4, 1,
MyFunctional → MakeFunctionalTrace, WeightMatrix → BadWeightMat]
```

WARNING: The WeightMatrix Option does not have the correct format.

```
In[128]:= FindExtremePoint[aIrred1g2d4, 1, EigMagTol → 10^(-10)]

FindExtremePoint[aIrred1g2d4, 1, NumericsAssessment → False, EigMagTol → 10^(-10)]
```

```
Out[128]= {{ {{{{0.344038}}, {{-0.211703}}}, {{- $\frac{79}{100}$ ,  $\frac{153}{100}$ }},
BadNullNumerics, {9.03807 × 10-9, BadNullNumerics}, BadNullNumerics,
{9.03807 × 10-9, BadNullNumerics}, BadNullNumerics, 1, True, 0}
```

```
Out[129]= {{ {{{{0.475721}}, {{-0.117252}}}, {{- $\frac{121}{100}$ ,  $\frac{111}{100}$ }}, True,
{1.42602 × 10-10, 0.345419}, True, {1.42602 × 10-10, 0.345419}, 1, 1, True, 0}
```

```
In[130]:= FindExtremePoint[aIrred1g2d4, 1, ChopCutoff → 10^(-5)]
```

```
Out[130]= {{ {{{{-0.484486}}, {{0.0852365}}}, {{ $\frac{131}{100}$ ,  $-\frac{193}{100}$ }}, True,
{3.77343 × 10-10, 0.164081}, True, {3.77343 × 10-10, 0.164081}, 1, 1, True, 0}
```

```
In[131]:= FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 0]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 1]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 2]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 3]
FindExtremePoint[aIrred1g2d4, 1, DiagnosticLevel → 4]
```

```
Out[131]= {{ {0.353095} }, { {-0.206926} } }
```

```
Out[132]= { { {0.469699} }, { {0.0427115} } }, { - $\frac{121}{100}$ , - $\frac{109}{100}$  }
```

```
Out[133]= { { { -0.508288 } }, { {0.0651978} } }, {  $\frac{121}{100}$ , - $\frac{107}{100}$  }, True, True }
```

```
Out[134]= { { {0.0111763} }, { {0.269817} } }, {  $\frac{11}{25}$ , - $\frac{193}{100}$  }, True,
{  $2.57103 \times 10^{-8}$ , 0.0901878 }, True, {  $2.57103 \times 10^{-8}$ , 0.0901878 } }
```

```
Out[135]= { { { -0.519689 } }, { {0.0442104} } }, {  $\frac{21}{20}$ , - $\frac{21}{100}$  }, True,
{  $5.65853 \times 10^{-10}$ , 0.223958 }, True, {  $5.65853 \times 10^{-10}$ , 0.223958 }, 1, 1, True, 0 }
```

```
In[136]:=
```

```
In[137]:= (*Testing FindExtremeAndAnalyze*)
FindExtremeAndAnalyze[aIrred1g2d4, 2, 4, 929]
```

```
{}
```

```
0
```

```
False
```

```
False
```

```
0
```

```
False
```

```
{{}, 0}
```

```
4
```

```
0
```

```
Out[137]= {{{{{0.489031, -0.0602111}, {-0.0602111, 0.33301}},
  {{{-0.0340095, 0.0675598}, {0.0675598, 0.141054}}}, {- $\frac{181}{100}$ ,  $\frac{53}{100}$ , - $\frac{34}{25}$ ,  $\frac{3}{20}$ , - $\frac{27}{20}$ , - $\frac{181}{100}$ },
  True, { $9.33972 \times 10^{-9}$ , 0.402784}, True, { $9.33972 \times 10^{-9}$ , 0.402784}, 2, 2, 2,  $5.83351 \times 10^{-16}$ },
  {{{{0.302852, 0.369107}, {0.369107, -0.215323}},
  {{{-0.0464088, -0.0894016}, {-0.0894016, 0.0790987}}}, {-1, - $\frac{48}{25}$ ,  $\frac{13}{100}$ ,  $\frac{3}{4}$ ,  $\frac{41}{25}$ , - $\frac{13}{10}$ },
  True, { $2.29174 \times 10^{-8}$ , 0.377899}, True, { $2.29174 \times 10^{-8}$ , 0.377899}, 2, 2, 2,  $7.75401 \times 10^{-16}$ },
  {{{{0.445945, 0.246735}, {0.246735, -0.456935}},
  {{{-0.0206601, -0.0161706}, {-0.0161706, 0.0385131}}},
  {- $\frac{27}{20}$ , - $\frac{161}{100}$ ,  $\frac{137}{100}$ ,  $\frac{3}{50}$ , - $\frac{3}{2}$ , - $\frac{16}{25}$ }, True, { $1.26034 \times 10^{-9}$ , 0.364671},
  True, { $1.26034 \times 10^{-9}$ , 0.364671}, 2, 2, 2,  $8.25906 \times 10^{-16}$ },
  {{{{0.457336, 0.00832913}, {0.00832913, 0.481396}},
  {{{0.0435104, -0.0472308}, {-0.0472308, -0.0929233}}},
  {- $\frac{89}{100}$ , - $\frac{107}{100}$ , -2, - $\frac{49}{100}$ ,  $\frac{69}{50}$ ,  $\frac{79}{50}$ }, True, { $7.76101 \times 10^{-10}$ , 0.362032},
  True, { $7.76101 \times 10^{-10}$ , 0.362032}, 2, 2, 2,  $7.29081 \times 10^{-16}$ }},
  {{{}, 0, False, False, 0, False, {{}, 0}, 4, 0, {}}}}
```

```
In[138]:= FindExtremeAndAnalyze[aIrred1g2d4, 2, 4, 929]
```

```

{}
0
False
False
0
False
{{}, 0}
4
0
Out[138]= {{{{{0.489031, -0.0602111}, {-0.0602111, 0.33301}},
  {{{-0.0340095, 0.0675598}, {0.0675598, 0.141054}}}, {- $\frac{181}{100}$ ,  $\frac{53}{100}$ , - $\frac{34}{25}$ ,  $\frac{3}{20}$ , - $\frac{27}{20}$ , - $\frac{181}{100}$ },
  True, { $9.33972 \times 10^{-9}$ , 0.402784}, True, { $9.33972 \times 10^{-9}$ , 0.402784}, 2, 2, 2,  $5.83351 \times 10^{-16}$ },
  {{{{0.302852, 0.369107}, {0.369107, -0.215323}},
    {{{-0.0464088, -0.0894016}, {-0.0894016, 0.0790987}}}, {-1, - $\frac{48}{25}$ ,  $\frac{13}{100}$ ,  $\frac{3}{4}$ ,  $\frac{41}{25}$ , - $\frac{13}{10}$ },
    True, { $2.29174 \times 10^{-8}$ , 0.377899}, True, { $2.29174 \times 10^{-8}$ , 0.377899}, 2, 2, 2,  $7.75401 \times 10^{-16}$ },
    {{{{0.445945, 0.246735}, {0.246735, -0.456935}},
      {{{-0.0206601, -0.0161706}, {-0.0161706, 0.0385131}}},
      {- $\frac{27}{20}$ , - $\frac{161}{100}$ ,  $\frac{137}{100}$ ,  $\frac{3}{50}$ , - $\frac{3}{2}$ , - $\frac{16}{25}$ }, True, { $1.26034 \times 10^{-9}$ , 0.364671},
      True, { $1.26034 \times 10^{-9}$ , 0.364671}, 2, 2, 2,  $8.25906 \times 10^{-16}$ },
      {{{{0.457336, 0.00832913}, {0.00832913, 0.481396}},
        {{{0.0435104, -0.0472308}, {-0.0472308, -0.0929233}}},
        {- $\frac{89}{100}$ , - $\frac{107}{100}$ , -2, - $\frac{49}{100}$ ,  $\frac{69}{50}$ ,  $\frac{79}{50}$ }, True, { $7.76101 \times 10^{-10}$ , 0.362032},
        True, { $7.76101 \times 10^{-10}$ , 0.362032}, 2, 2, 2,  $7.29081 \times 10^{-16}$ }}},
    {{{}, 0, False, False, 0, False, {{}, 0}, 4, 0, {}}}}
Out[139]= test1 = FindExtremeAndAnalyze[aIrred1g2d4, 2, 2, 929, MyFunctional → MakeFunctionalTrace]
{}
0
False
False
0
False
{{}, 0}
2
0
Out[139]= {{{{{-0.514262, -0.0148366}, {-0.0148366, -0.46052}},
  {{{0.0445587, 0.0200919}, {0.0200919, -0.0282194}}},

```



$$\begin{aligned}
& \left\{ \left\{ \frac{24470032041}{10000000000}, -\frac{1647353799}{10000000000}, \frac{12597383799}{5000000000}, -\frac{5346273933}{5000000000}, \right. \right. \\
& \quad \left. \frac{22807817487}{10000000000}, \frac{129366783}{5000000000}, \frac{837051579}{10000000000}, -\frac{20861527869}{10000000000} \right\}, \\
& \quad \left\{ \frac{1647353799}{10000000000}, \frac{14484514061}{10000000000}, \frac{8801934231}{5000000000}, \frac{3420230127}{5000000000}, \frac{22434676193}{10000000000}, \right. \\
& \quad \left. \frac{10673524087}{2271768551}, \frac{11909066601}{10000000000}, \frac{12597383799}{5000000000}, \frac{8801934231}{5000000000}, \right. \\
& \quad \left. \frac{5000000000}{10000000000}, \frac{10000000000}{10000000000}, \frac{10000000000}{5000000000}, -\frac{5000000000}{5000000000}, \right. \\
& \quad \frac{1157885171}{250000000}, \frac{49368509}{31250000}, \frac{25899976163}{5000000000}, \frac{3519297987}{250000000}, \frac{6423640677}{2500000000}, \frac{5815092589}{2500000000} \Big\}, \\
& \quad \left\{ \frac{5346273933}{5000000000}, \frac{3420230127}{5000000000}, \frac{49368509}{31250000}, \frac{15173783253}{10000000000}, \frac{15033946241}{10000000000}, \right. \\
& \quad \left. \frac{13343742891}{10000000000}, \frac{75219061}{156250000}, \frac{7282128701}{2000000000} \right\}, \left\{ -\frac{22807817487}{10000000000}, \frac{22434676193}{10000000000}, \right. \\
& \quad \left. \frac{25899976163}{15033946241}, \frac{72892934043}{8033514471}, \frac{50149538141}{126227849}, \frac{126227849}{62500000}, \right. \\
& \quad \left. \frac{129366783}{5000000000}, \frac{10673524087}{5000000000}, \frac{3519297987}{2500000000}, -\frac{13343742891}{10000000000}, \right. \\
& \quad \left. \frac{8033514471}{5000000000}, \frac{68883576839}{10000000000}, -\frac{5809040233}{10000000000}, -\frac{23206290531}{5000000000} \right\}, \\
& \quad \left\{ \frac{837051579}{1000000000}, \frac{2271768551}{1000000000}, \frac{6423640677}{2500000000}, \frac{75219061}{156250000}, \frac{50149538141}{10000000000}, -\frac{5809040233}{10000000000}, \right. \\
& \quad \left. \frac{97817191051}{10000000000}, \frac{1909182629}{400000000} \right\}, \left\{ -\frac{20861527869}{10000000000}, \frac{11909066601}{10000000000}, -\frac{5815092589}{2500000000}, \right. \\
& \quad \left. \frac{7282128701}{2000000000}, \frac{126227849}{62500000}, -\frac{23206290531}{5000000000}, -\frac{1909182629}{400000000}, \frac{129830467717}{10000000000} \right\}, \\
& \text{True}, \{8.28136 \times 10^{-10}, 0.224435\}, \text{True}, \{8.28136 \times 10^{-10}, 0.224435\}, \\
& 2, 2, 2, 9.19547 \times 10^{-16}\}, \\
& \{ \{ \{-0.425966, -0.123927\}, \{-0.123927, -0.329291\} \}, \\
& \quad \{ \{0.0683595, -0.082029\}, \{-0.082029, 0.13235\} \} \}, \\
& \quad \left\{ \frac{3256870761}{10000000000}, \frac{4501659789}{5000000000}, -\frac{216690993}{400000000}, \frac{2270033613}{10000000000}, \frac{5766080553}{10000000000}, \right. \\
& \quad \left. \frac{7101609291}{10000000000}, \frac{216348579}{312500000}, -\frac{9862493373}{10000000000}, \frac{4501659789}{5000000000}, \frac{2531427937}{5000000000}, \right. \\
& \quad \left. \frac{7047161149}{5000000000}, \frac{7239035441}{5000000000}, -\frac{3385600047}{5000000000}, \frac{2710488813}{5000000000}, \frac{374040823}{250000000}, -\frac{1924321113}{5000000000}, \right. \\
& \quad \left. \frac{216690993}{400000000}, \frac{7047161149}{5000000000}, \frac{33245015439}{5000000000}, \frac{7227898817}{10000000000}, \frac{10263287101}{10000000000}, \right. \\
& \quad \left. \frac{40458059317}{10000000000}, \frac{22092165043}{5000000000}, \frac{37845098439}{10000000000} \right\}, \left\{ \frac{2270033613}{10000000000}, \frac{7239035441}{5000000000}, \right. \\
& \quad \left. \frac{7227898817}{10000000000}, \frac{13133463899}{10000000000}, \frac{3561265573}{5000000000}, -\frac{256517733}{312500000}, -\frac{19165987239}{10000000000}, -\frac{365111891}{200000000}, \right. \\
& \quad \left. \frac{5766080553}{10000000000}, \frac{3385600047}{5000000000}, \frac{10263287101}{10000000000}, \frac{3561265573}{5000000000}, \right. \\
& \quad \left. \frac{13218057399}{2000000000}, \frac{47623410771}{10000000000}, -\frac{126160599}{156250000}, -\frac{68699260119}{10000000000} \right\},
\end{aligned}$$

$$\left\{ -\frac{7101609291}{10000000000}, \frac{2710488813}{5000000000}, \frac{40458059317}{10000000000}, -\frac{256517733}{312500000}, -\frac{47623410771}{10000000000}, \right.$$

$$\frac{31823147473}{5000000000}, \frac{47511986209}{10000000000}, \frac{73038527259}{10000000000}, \left\{ -\frac{216348579}{312500000}, -\frac{374040823}{250000000}, \frac{22092165043}{5000000000}, \right.$$

$$\left. -\frac{19165987239}{10000000000}, -\frac{126160599}{156250000}, \frac{47511986209}{10000000000}, \frac{10127595571}{1000000000}, \frac{33233838289}{5000000000} \right\},$$

$$\left\{ -\frac{9862493373}{10000000000}, -\frac{1924321113}{5000000000}, \frac{37845098439}{10000000000}, -\frac{365111891}{200000000}, \right.$$

$$\left. -\frac{68699260119}{10000000000}, \frac{73038527259}{10000000000}, \frac{33233838289}{5000000000}, \frac{1313918949}{1000000000} \right\}, \text{True},$$

$$\{3.90132 \times 10^{-9}, 0.186939\}, \text{True}, \{3.90132 \times 10^{-9}, 0.186939\}, 2, 2, 2, 7.22625 \times 10^{-16}\},$$

$$\{\{\}, 0, \text{False}, \text{False}, 0, \text{False}, \{\{\}, 0\}, 2, 0, \{\}\}$$

```
In[140]:= (*Testing FindExtremeAndAnalyze *)
```

```
FindExtremeAndAnalyze[aIrred1g2d4, 2, 2, 929, MyFunctional → MakeFunctionalTrace] == test1
```

```
{}
```

```
0
```

```
False
```

```
False
```

```
0
```

```
False
```

```
{{}, 0}
```

```
2
```

```
0
```

```
Out[140]= True
```

```
In[141]:= (*Testing MakeIrreducibleA*)
```

```
MakeIrreducibleA[3, 4, 100]
```

```
Out[141]= {{ {-5, 1,  $\frac{23}{10}$ ,  $-\frac{7}{10}$ }, {1,  $\frac{11}{5}$ ,  $\frac{3}{5}$ ,  $-\frac{3}{10}$ }, { $\frac{23}{10}$ ,  $\frac{3}{5}$ ,  $\frac{4}{5}$ ,  $\frac{14}{5}$ }, { $-\frac{7}{10}$ ,  $-\frac{3}{10}$ ,  $\frac{14}{5}$ ,  $\frac{11}{5}$ }},
```

```
{ { $\frac{12}{5}$ ,  $\frac{5}{2}$ ,  $\frac{11}{10}$ ,  $\frac{11}{5}$ }, { $\frac{5}{2}$ , 1,  $-\frac{9}{10}$ ,  $-\frac{11}{10}$ }, { $\frac{11}{10}$ ,  $-\frac{9}{10}$ ,  $-\frac{4}{5}$ ,  $-\frac{9}{5}$ }, { $\frac{11}{5}$ ,  $-\frac{11}{10}$ ,  $-\frac{9}{5}$ ,  $\frac{17}{5}$ }},
```

```
{ {-4,  $-\frac{33}{10}$ ,  $\frac{13}{5}$ , 0}, { $-\frac{33}{10}$ ,  $-\frac{6}{5}$ ,  $-\frac{19}{10}$ ,  $-\frac{3}{2}$ }, { $\frac{13}{5}$ ,  $-\frac{19}{10}$ ,  $\frac{9}{5}$ ,  $\frac{33}{10}$ }, {0,  $-\frac{3}{2}$ ,  $\frac{33}{10}$ ,  $-\frac{23}{5}$ }}}
```

```
In[142]:= (*Testing MakeIrreducibleBoundedA *)
```

In[143]:=

**MakeIrreducibleBoundedA[4, 3, 100]**

Out[143]=  $\left\{ \left\{ \left\{ -\frac{23}{5}, -\frac{6}{5}, -\frac{4}{5} \right\}, \left\{ -\frac{6}{5}, \frac{18}{5}, \frac{3}{2} \right\}, \left\{ -\frac{4}{5}, \frac{3}{2}, -\frac{2}{5} \right\} \right\}, \right.$   
 $\left. \left\{ \left\{ -\frac{9}{5}, \frac{33}{10}, -4 \right\}, \left\{ \frac{33}{10}, \frac{19}{5}, -\frac{3}{10} \right\}, \left\{ -4, -\frac{3}{10}, -\frac{2}{5} \right\} \right\}, \right.$   
 $\left\{ \left\{ 1, -\frac{23}{10}, \frac{1}{10} \right\}, \left\{ -\frac{23}{10}, -\frac{1}{5}, \frac{5}{2} \right\}, \left\{ \frac{1}{10}, \frac{5}{2}, -\frac{11}{5} \right\} \right\},$   
 $\left. \left\{ \left\{ -1, \frac{1}{5}, \frac{41}{10} \right\}, \left\{ \frac{1}{5}, -5, \frac{1}{10} \right\}, \left\{ \frac{41}{10}, \frac{1}{10}, \frac{23}{5} \right\} \right\} \right\}$

In[144]:= **(\*Testing LMI\*)**In[145]:= **LA = LMI[apentg2d5, testXg2n2]**

Out[145]=  $\{ \{0, -2, 0, 0, 0, 0, 0, 0, 0, 0\}, \{ -2, 0, 0, 0, 0, 0, 0, 0, 0, 0\},$   
 $\{0, 0, 2, 2, 0, 0, 0, 0, 0, 0\}, \{0, 0, 2, 2, 0, 0, 0, 0, 0, 0\}, \{0, 0, 0, 0, 1, -1, 0, 0, 0, 0\},$   
 $\{0, 0, 0, 0, -1, 2, 0, 0, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 1, 0, 0\}, \{0, 0, 0, 0, 0, 0, 1, 0, 0, 0\},$   
 $\{0, 0, 0, 0, 0, 0, 0, 0, 0, -3\}, \{0, 0, 0, 0, 0, 0, 0, 0, 0, -3, 1\} \}$

In[146]:= **(\*Testing Lambda\*)****IdentityMatrix[10] - Lambda[apentg2d5, testXg2n2] == LA**Out[146]= **True**In[147]:= **(\*Testing Tuple operations.\*)****TupleNorm[apentg2d5]**Out[147]=  $\sqrt{2}$ In[148]:= **LeftMultTuple[DiagonalMatrix[{-1, -1, 0, 1, 1}], apentg2d5]**

Out[148]=  $\{ \{ \{ -1, 0, 0, 0, 0 \}, \{ 0, 1, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 1 \} \},$   
 $\{ \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, -1, 0 \}, \{ 0, 0, 0, 0, 1 \} \} \}$

In[149]:= **RightMultTuple[****{ {0, 1, 0, 0, 0}, {0, 0, 1, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, 0, 0} }, apentg2d5]**

Out[149]=  $\{ \{ \{ 0, 1, 0, 0, 0 \}, \{ 0, 0, -1, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \} \},$   
 $\{ \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 1, 0 \}, \{ 0, 0, 0, 0, -1 \}, \{ 0, 0, 0, 0, 0 \} \} \}$

In[150]:= **ConjTuple[DiagonalMatrix[{-1, -1, 0, 1, 1}], apentg2d5]**

Out[150]=  $\{ \{ \{ 1, 0, 0, 0, 0 \}, \{ 0, -1, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 1 \} \},$   
 $\{ \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, -1, 0 \}, \{ 0, 0, 0, 0, 1 \} \} \}$

In[151]:= **TupleComm[DiagonalMatrix[{-1, -1, 0, 1, 1}], apentg2d5]**

Out[151]=  $\{ \{ \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \} \},$   
 $\{ \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \}, \{ 0, 0, 0, 0, 0 \} \} \}$

In[152]:= **TupleComm[****{ {1, 1, 2, 3, 4}, {4, 3, 2, 5, 1}, {1, 2, 3, 2, 1}, {3, 3, 3, 3, 3}, {5, 3, 2, 1, 2} }, apentg2d5]**

Out[152]=  $\{ \{ \{ 0, -2, -2, -3, 0 \}, \{ 8, 0, 2, 5, 2 \}, \{ 1, -2, 0, 0, 1 \}, \{ 3, -3, 0, 0, 3 \}, \{ 0, -6, -2, -1, 0 \} \},$   
 $\{ \{ 0, 0, 2, -3, 4 \}, \{ 0, 0, 2, -5, 1 \}, \{ -1, -2, 0, -4, 0 \}, \{ 3, 3, 6, 0, 6 \}, \{ -5, -3, 0, -2, 0 \} \} \}$