Deutsch-Jozsa_Alg

December 27, 2025

1 Deutsch Algorithm

This notebook will go over the basics of Deutsch Algorithm and how to implement it on Qiskit.

1.1 Deutsch Problem

Suppose a function $f : \{0, 1\} \rightarrow \{0, 1\}$ If the output is same for different inputs, it is a constant function If the output is different it is a balanced function

Classically, in order to evalute the function, two calls of the function is needed. But with a Quantum Machine, we could evaluate the function with one iteration of the algorithm

1.1.1 Let's start with building the function

We want a function that can do the following

x	$f_1(x)$	$f_2(x)$	$f_3(x)$	$f_4(x)$
0	0	0	1	1
1	0	1	0	1

[67]:
```
from qiskit import QuantumCircuit

def target_function(case: int):
    if case not in [1, 2, 3, 4]:
        raise ValueError("'case' must be 1, 2, 3, or 4.")

    f = QuantumCircuit(2)
    if case in [2,3]:
        f.cx(0,1)
    if case in [3,4]:
        f.x(1)
    return f
```

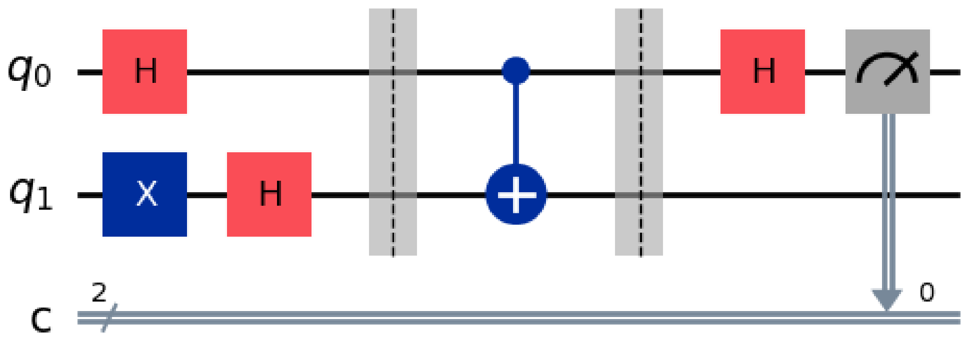
1.1.2 Now start building our Deutsch Algorithm

Initialize the qubits to $|-\rangle|+\rangle$ state, which can be expressed as $\frac{1}{2}(|0\rangle - |1\rangle)|0\rangle + \frac{1}{2}(|0\rangle - |1\rangle)|1\rangle$

Here, I am following Qiskit's qubit ordering convention, which puts the top qubit to the right and the bottom qubit to the left.

[68]:
```
qc = QuantumCircuit(2,2) # 2 qubit system, 2 classical bits to store measurement

qc.h(0)
qc.x(1)
qc.h(1)
qc.barrier()
qc.compose(target_function(2), inplace=True)
qc.barrier()
qc.h(0)
qc.measure(0,0)

qc.draw("mpl")
```

[68]:



Here, I am skipping some math. But we will acquire 0 for q_0 from a constant function and 1 for q_1 from a balanced function.

The final state can be written as

$$\begin{cases} (-1)^{f(0)}|-\rangle|0\rangle, & \text{if } f(0) \oplus f(1) = 0, \\ (-1)^{f(0)}|-\rangle|1\rangle, & \text{if } f(0) \oplus f(1) = 1. \end{cases}$$

where $\oplus$ denotes addition modulo 2 (or binary dot product)

1.2 Loading Real Backend

This codes sets up backend service from the least busy QPU (Make sure you have an IBM account)

[69]:
```
from qiskit_ibm_runtime import QiskitRuntimeService
from qiskit_ibm_runtime import SamplerV2 as Sampler

service = QiskitRuntimeService()
```

```
backend = service.least_busy(simulator=False, operational=True) # Check what␣
↪the parameters mean
print("Using backend: ",backend.name)
```

```
management.get:WARNING:2025-12-27 00:59:11,042: Loading default saved account
```

```
Using backend:  ibm_torino
```

1.3 Loading Backend Sampler

A simulator would also work well for this scale of algorithm

[70]:
```
from qiskit.primitives import BackendSamplerV2
from qiskit_aer import AerSimulator
from qiskit_aer.noise import NoiseModel

noise_model = NoiseModel.from_backend(backend)

# Defining a backend simulator with Aer
backend_sim = AerSimulator(noise_model=noise_model)
sampler_sim = BackendSamplerV2(backend=backend_sim)
```

1.4 Transpiling the Circuit

Optimizing the ciruit for real hardware topology

[71]:
```
from qiskit.transpiler.preset_passmanagers import generate_preset_pass_manager

target = backend.target # QPU we are targeting
pm = generate_preset_pass_manager(target=target, optimization_level=3)

qc_isa = pm.run(qc)
```

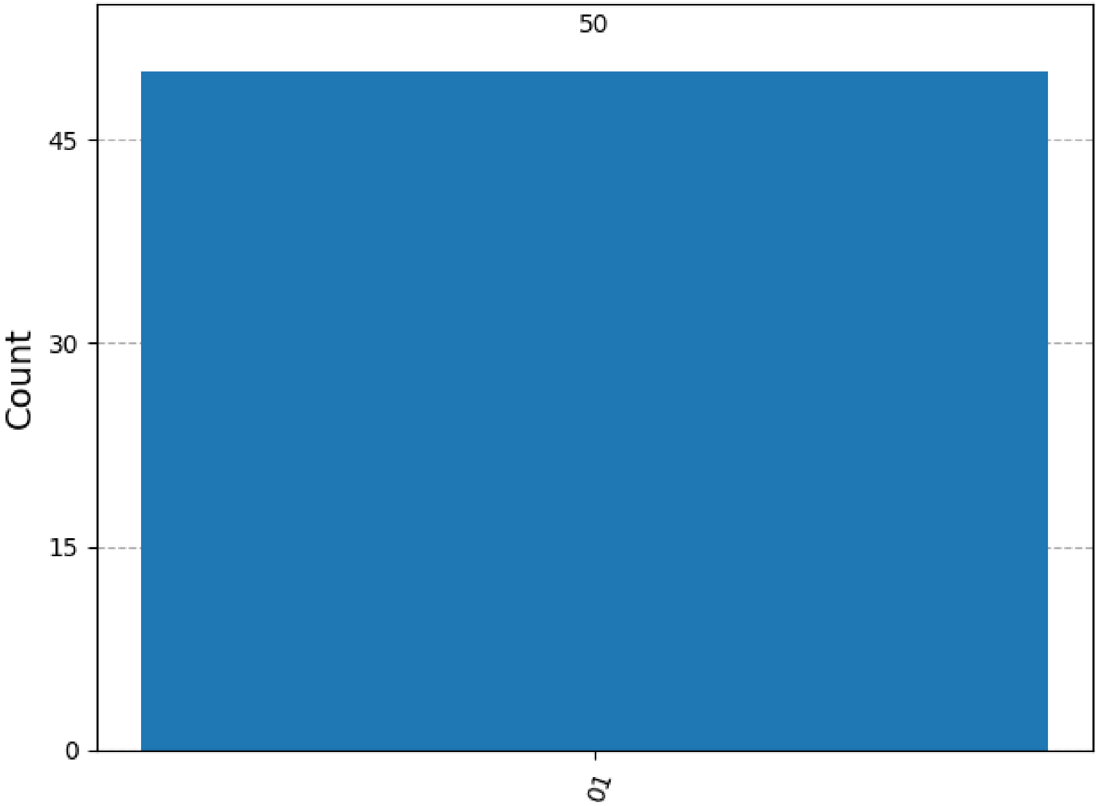
1.5 Execute

[ ]:
```
# job = sampler.run([qc_isa], shots=1) # Uncomment to run on real QPU
job = sampler_sim.run([qc_isa], shots=50)

result = job.result()
counts = result[0].data.c.get_counts()
```

1.6 Visualize Results

[ ]:
```
from qiskit.visualization import plot_histogram

plot_histogram(counts)
```

[ ]:



The function we use for this demonstration is function 2, which the result (1 on the right qubit) agrees with our calculations.