Introduction to Algorithms

Lecture 3: Unknown Array Size Problem

Yao-Chung Fan
yfan@nchu.edu.tw

Client, Implementation, Interface

Separate interface and implementation.

Ex: stack, queue, bag, priority queue, symbol table, union-find, ...

Benefits.

- Client can't know details of implementation $\Rightarrow$
  client has many implementation from which to choose.
- Implementation can't know details of client needs $\Rightarrow$
  many clients can re-use the same implementation.
- Design: creates modular, reusable libraries.
- Performance: use optimized implementation where it matters.

Client: program using operations defined in interface.

Implementation: actual code implementing operations.

Interface: description of data type, basic operations.

Problem: String Token Reserves

Given a string e.g., “Today is a good day”

Return: day good a is Today (a reverse of the given string token)

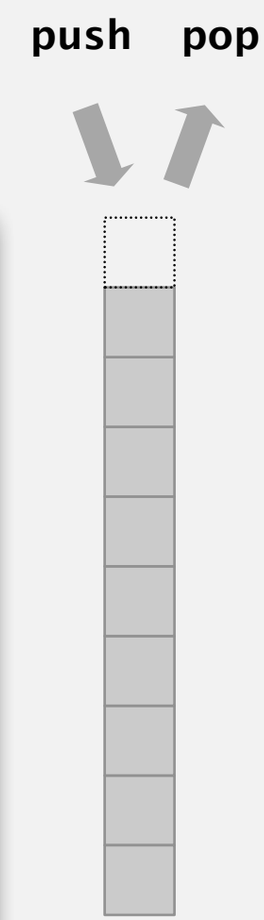
```
% more tinyTale.txt
it was the best of times ...

% java ReverseStrings < tinyTale.txt
... times of best the was it
```

How to Implement such a requirement ?

Sample client

Warmup client. Reverse sequence of strings from standard input.

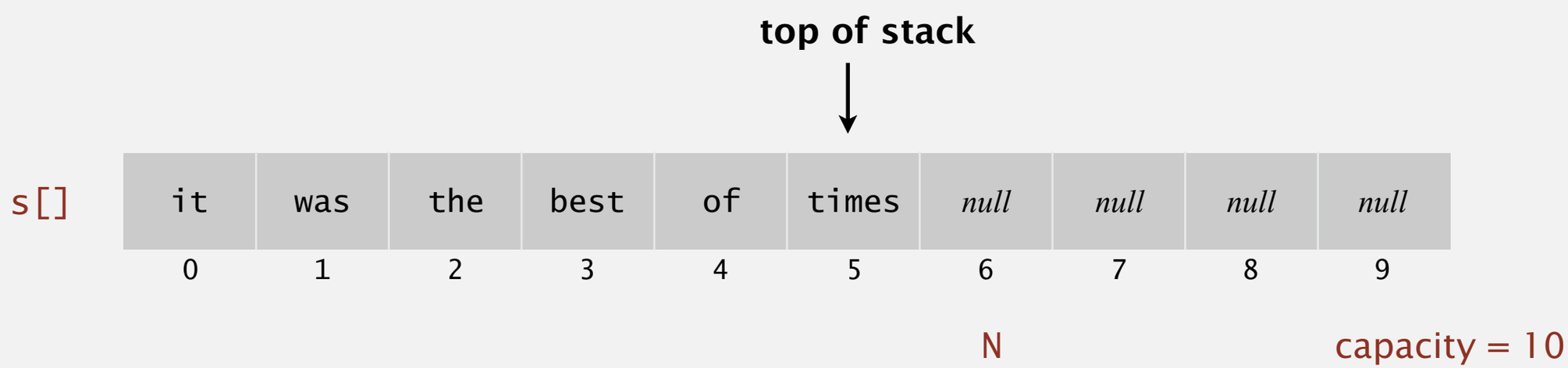
- Read string and push onto stack.
- Pop string and print.




```
public static void main(String[] args)
{
   StackOfStrings stack = new StackOfStrings();
   while (!StdIn.isEmpty())
      stack.push(StdIn.readString());
   while (!stack.isEmpty())
      StdOut.println(stack.pop());
}
```

Array implementation for String Token Reserves

- Use array `s[]` to store `N` items on stack.
- `push()`: add new item at `s[N]`.
- `pop()`: remove item from `s[N-1]`.



Array implementation for String Token Reserves

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int N = 0;

   public FixedCapacityStackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

use to index into array;
then increment N

decrement N;
then use to index into array

Defect. Stack overflows when N exceeds capacity.

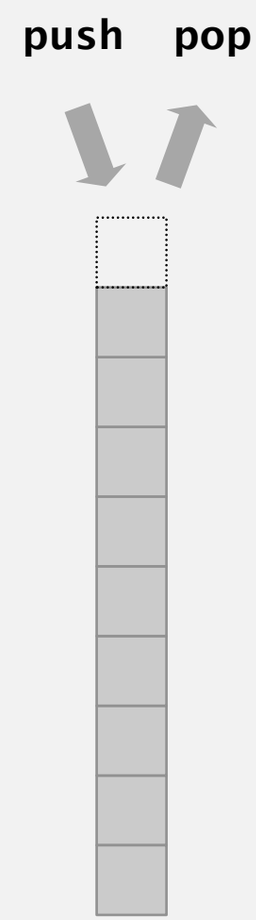
Unknown Array Size Problem

- *Linked List Implementation*
- *Resizing Arrays Implementation*

Stack API

Warmup API. Stack of strings data type.

public class StackOfStrings		
	StackOfStrings()	*create an empty stack*
void	push(String item)	*insert a new string onto stack*
String	pop()	*remove and return the string most recently added*
boolean	isEmpty()	*is the stack empty?*
int	size()	*number of strings on the stack*



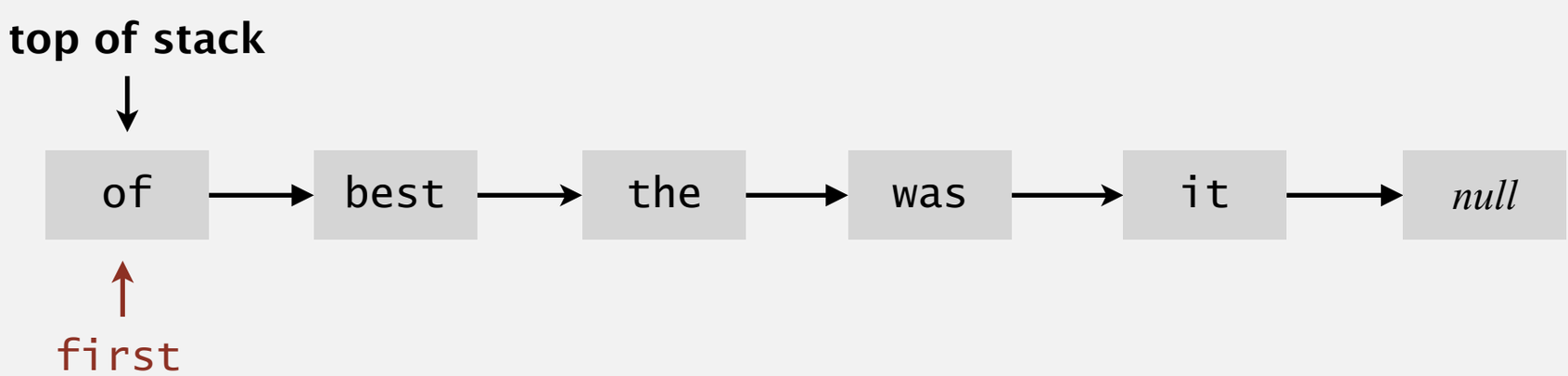
Warmup client. Reverse sequence of strings from standard input.

Stack: linked-list implementation

- Maintain pointer `first` to first node in a singly-linked list.
- Push new item before `first`.
- Pop item from `first`.

inner class

```
private class Node
{
   String item;
   Node next;
}
```



Stack: linked-list implementation in Java

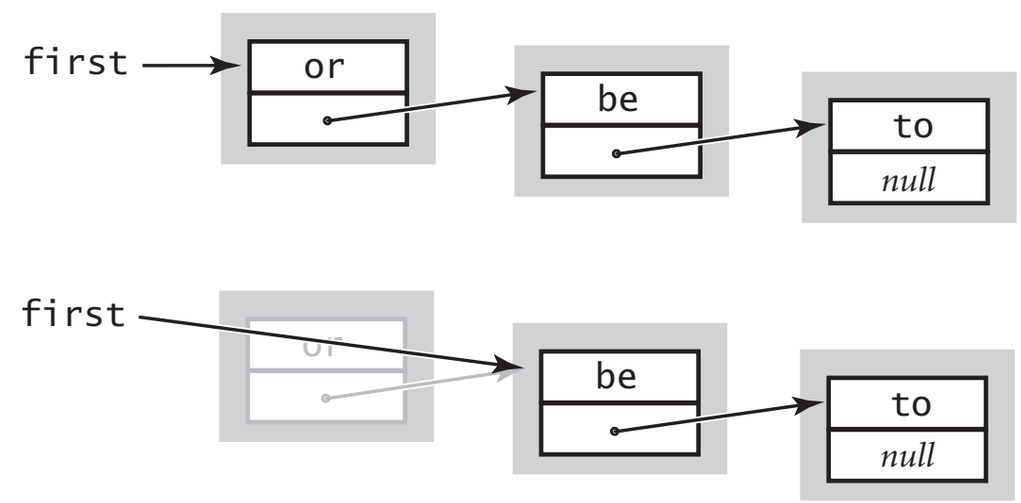
```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```

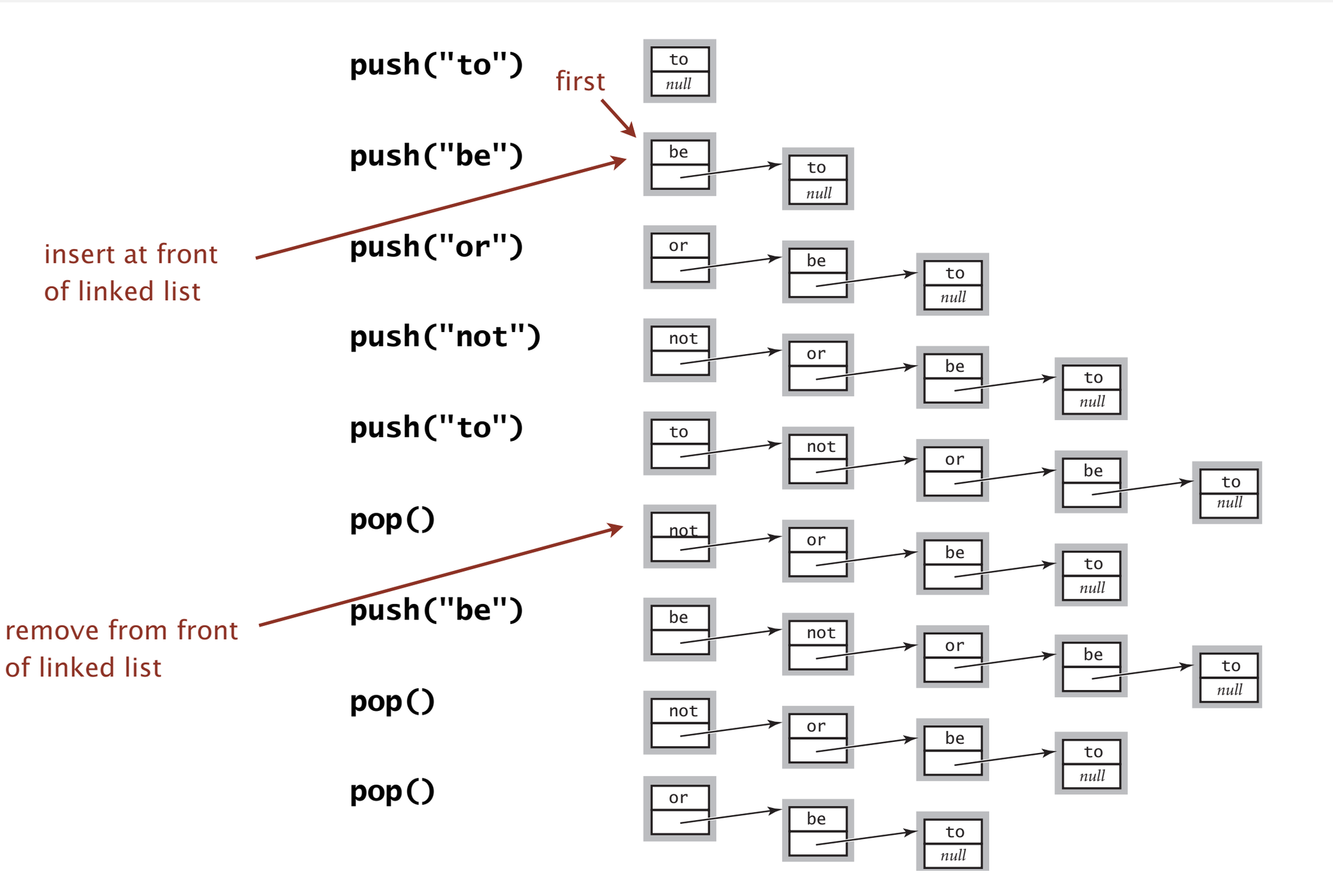


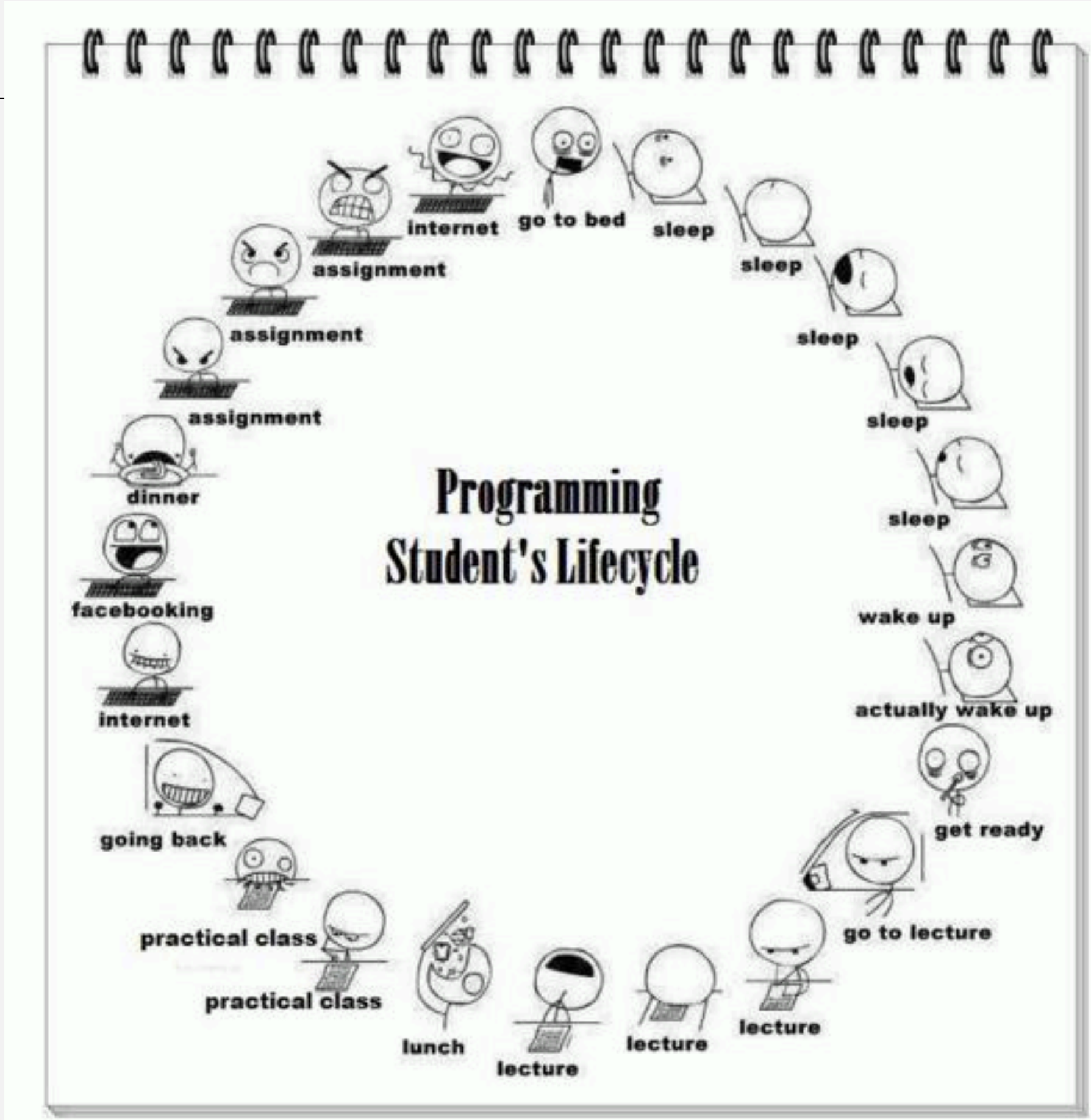
return saved item

```
return item;
```

Stack: linked-list representation

Maintain pointer to first node in a linked list; insert/remove from **front**.



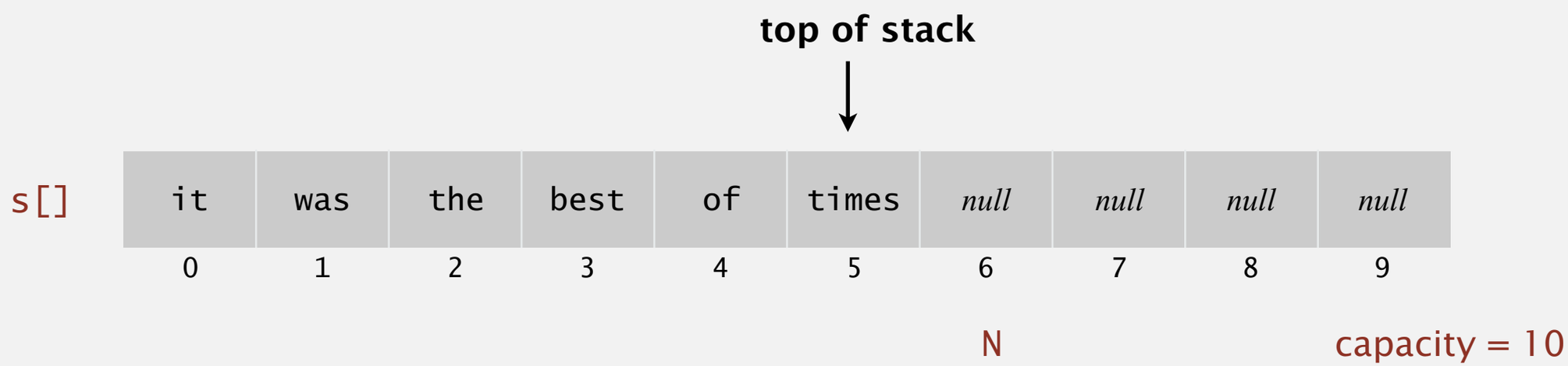

Programming
Student's Lifecycle
go to bed
sleep
sleep
sleep
sleep
sleep
wake up
actually wake up
get ready
go to lecture
lecture
lecture
lecture
lunch
practical class
practical class
going back
internet
facebooking
dinner
assignment
assignment
assignment
internet

Unknown Array Size Problem

- *Linked List Implementation*
- *resizing arrays Implementation*

Fixed-capacity stack: array implementation

- Use array `s[]` to store `N` items on stack.
- `push()`: add new item at `s[N]`.
- `pop()`: remove item from `s[N-1]`.



Fixed-capacity stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int N = 0;

   public FixedCapacityStackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

use to index into array;
then increment N

decrement N;
then use to index into array

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

Stack: resizing-array implementation

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

```
private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < N; i++)
      copy[i] = s[i];
   s = copy;
}
```

First try.

- `push()`: increase size of array `s[]` by 1.
- `pop()`: decrease size of array `s[]` by 1.

Too expensive.

infeasible for large N

- Need to copy all items to a new array, for each operation.
- Array accesses to insert first N items = $N + (2 + 4 + 6 + 8 \ldots + 2(N-1)) \sim N^2$.

1 array access per push

2(k−1) array accesses to expand to size k (ignoring cost to create new array)

Stack: resizing-array implementation

Q. How to grow array?

A. If array is full, create a new array of twice the size, and copy items.

"repeated doubling"

```
public ResizingArrayStackOfStrings()
{  s = new String[1];  }

public void push(String item)
{
   if (N == s.length) resize(2 * s.length);
   s[N++] = item;
}

private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < N; i++)
      copy[i] = s[i];
   s = copy;
}
```

Array accesses to insert first N = 2^i items. $N + (2 + 4 + 8 + \ldots + N) \sim 3N.$

1 array access per push

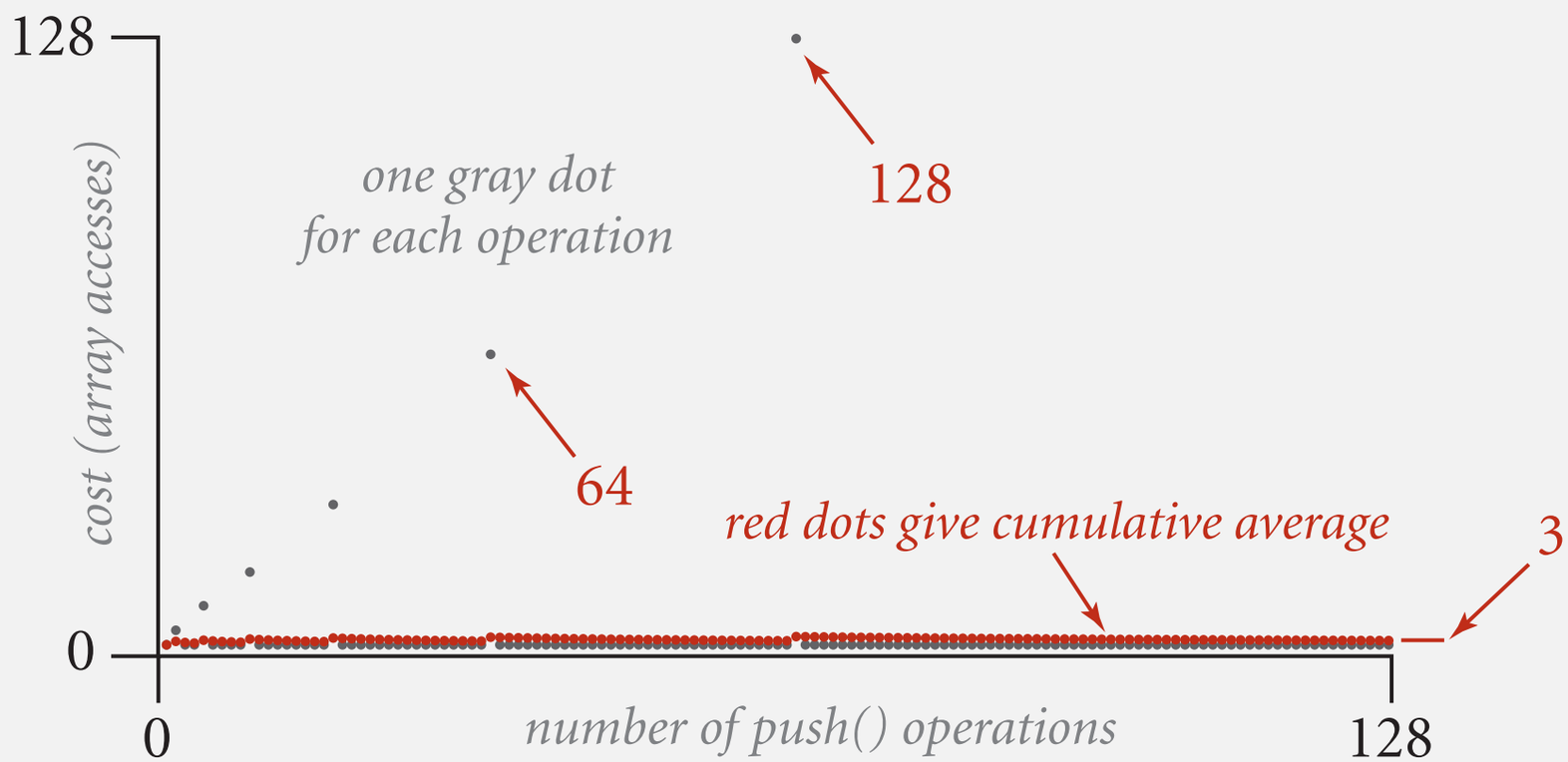
k array accesses to double to size k (ignoring cost to create new array)

Stack: amortized cost of adding to a stack

Cost of inserting first N items. $N + (2 + 4 + 8 + \ldots + N) \sim 3N.$

1 array access per push

k array accesses to double to size k (ignoring cost to create new array)



Stack: resizing-array implementation

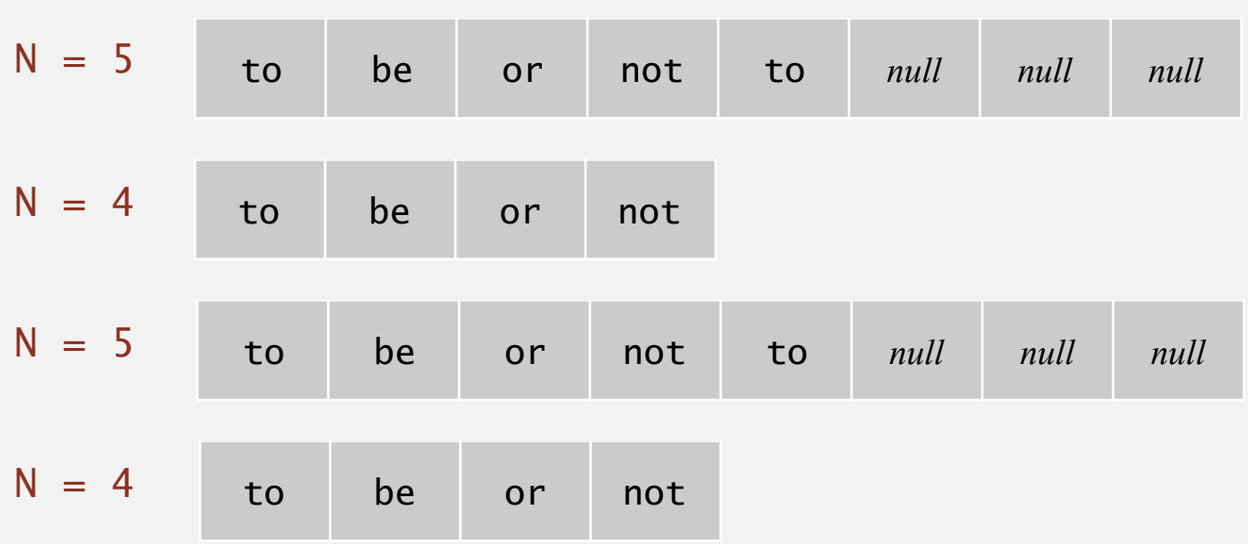
Q. How to shrink array?

First try.

- `push()`: double size of array `s[]` when array is full.
- `pop()`: halve size of array `s[]` when array is one-half full.

Too expensive in worst case.

- Consider push-pop-push-pop-... sequence when array is full.
- Each operation takes time proportional to N.



Stack: resizing-array implementation

Q. How to shrink array?

Efficient solution.

- `push()`: double size of array `s[]` when array is full.
- `pop()`: halve size of array `s[]` when array is one-quarter full.

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   if (N > 0 && N == s.length/4) resize(s.length/2);
   return item;
}
```

Invariant. Array is between 25% and 100% full.

Stack: resizing-array implementation trace

push()	pop()	N	a.length	a[] 0	1	2	3	4	5	6	7
		0	1	*null*							
to		1	1	to							
be		2	2	to	be						
or		3	4	to	be	or	*null*				
not		4	4	to	be	or	not				
to		5	8	to	be	or	not	to	*null*	*null*	*null*
-	to	4	8	to	be	or	not	*null*	*null*	*null*	*null*
be		5	8	to	be	or	not	be	*null*	*null*	*null*
-	be	4	8	to	be	or	not	*null*	*null*	*null*	*null*
-	not	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
that		4	8	to	be	or	that	*null*	*null*	*null*	*null*
-	that	3	8	to	be	or	*null*	*null*	*null*	*null*	*null*
-	or	2	4	to	be	*null*	*null*				
-	be	1	2	to	*null*						
is		2	2	to	is						

Trace of array resizing during a sequence of push() and pop() operations

Stack resizing-array implementation: performance

Amortized analysis. Starting from an empty data structure, average running time per operation over a worst-case sequence of operations.

Proposition. Starting from an empty stack, any sequence of N push and pop operations takes time proportional to N.

	best	worst	amortized
construct	1	1	1
push	1	N	1
pop	1	N	1
size	1	1	1

doubling and halving operations

order of growth of running time for resizing stack with N items

Stack implementations: resizing array vs. linked list

Tradeoffs.
Can implement a stack with either resizing array or linked list; client can use interchangeably.

Which one is better?

Stack resizing-array implementation: memory usage

Q. How much memory does a `ResizingArrayStackOfStrings` use to store N strings in the best case? Worst case?

Count only the memory owned by the stack (not the strings themselves).

Stack resizing-array implementation: memory usage

Proposition. Uses between $\sim 8\,N$ and $\sim 32\,N$ bytes to represent a stack with N items.

- $\sim 8\,N$ when full.
- $\sim 32\,N$ when one-quarter full.

```
public class ResizingArrayStackOfStrings
{
   private String[] s;   ←——  8 bytes × array size
   private int N = 0;
   …
}
```

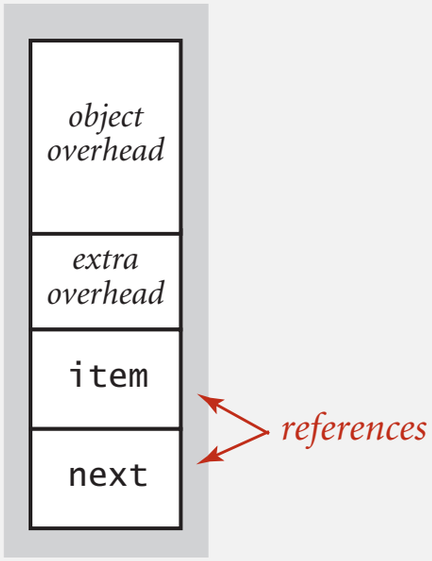
Stack: linked-list implementation performance

Proposition. Every operation takes constant time in the worst case.

Proposition. A stack with N items uses $\sim 40\,N$ bytes.

inner class

```
private class Node
{
   String item;
   Node next;
}
```



16 bytes (object overhead)

8 bytes (inner class extra overhead)

8 bytes (reference to String)

8 bytes (reference to Node)

---

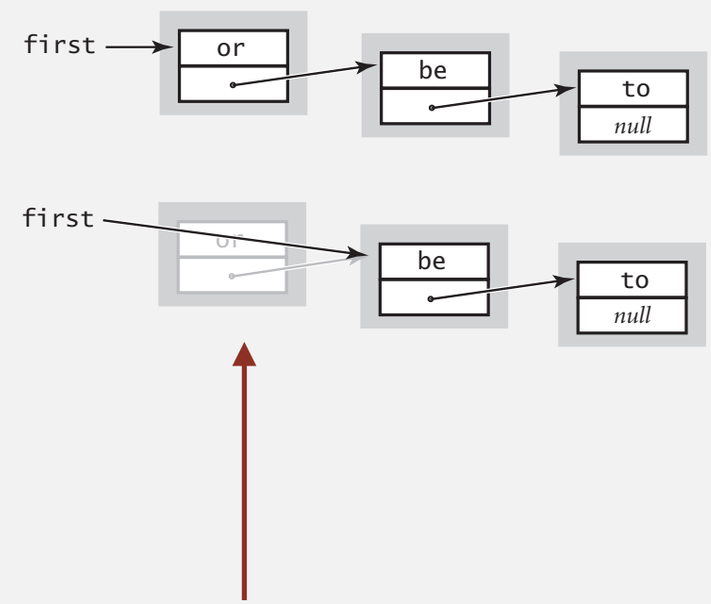
40 bytes per stack node

Something You Need to Know Object Loitering

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

delete first node

```
first = first.next;
```



Loitering. Holding a reference to an object when it is no longer needed.

Stack implementations: resizing array vs. linked list

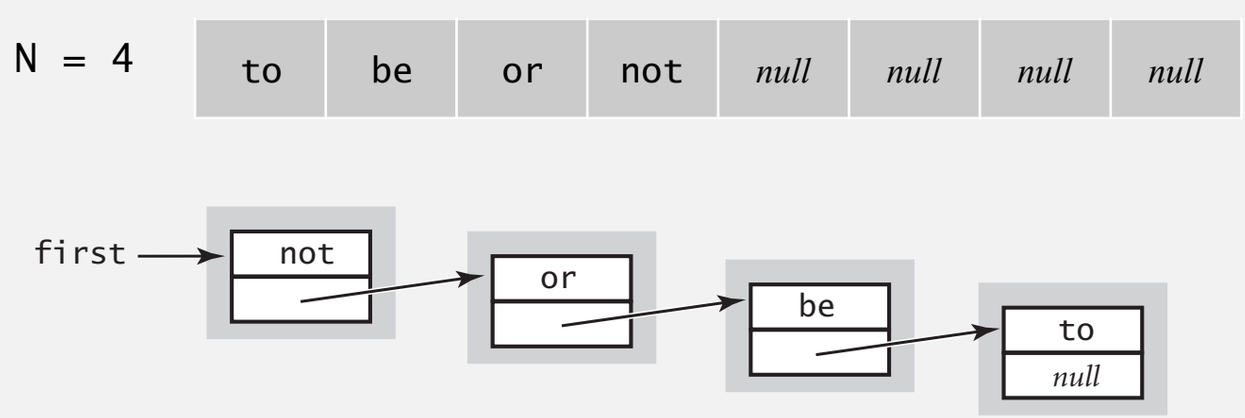
Tradeoffs. Can implement a stack with either resizing array or linked list; client can use interchangeably. Which one is better?

Linked-list implementation.

- Every operation takes constant time in the worst case.
- Uses extra time and space to deal with the links.

Resizing-array implementation.

- Every operation takes constant amortized time.
- Less wasted space.



Java collections library

List interface. `java.util.List` is API for an sequence of items.

https://docs.oracle.com/javase/7/docs/api/index.html?java/util/package-tree.html

`public interface List<Item> implements Iterable<Item>`

	`List()`	*create an empty list*
`boolean`	`isEmpty()`	*is the list empty?*
`int`	`size()`	*number of items*
`void`	`add(Item item)`	*append item to the end*
`Item`	`get(int index)`	*return item at given index*
`Item`	`remove(int index)`	*return and delete item at given index*
`boolean`	`contains(Item item)`	*does the list contain the given item?*
`Iterator<Item>`	`iterator()`	*iterator over all items in the list*
...		

?

`java.util.ArrayList` implementation

`java.util.LinkedList` implementation

Which one is better ? What is the difference ?

`java.util.ArrayList` v.s. `java.util.LinkedList`

Java collections library

Lesson. Don't use a library until you understand its API!

```
public interface List<Item> implements Iterable<Item>
```

	`List()`	*create an empty list*
`boolean`	`isEmpty()`	*is the list empty?*
`int`	`size()`	*number of items*
`void`	`add(Item item)`	*append item to the end*
`Item`	`get(int index)`	*return item at given index*
`Item`	`remove(int index)`	*return and delete item at given index*
`boolean`	`contains(Item item)`	*does the list contain the given item?*
`Iterator<Item>`	`iterator()`	*iterator over all items in the list*
	...	

Implementations. `java.util.ArrayList` uses resizing array; `java.util.LinkedList` uses linked list.