Chapter 2

2.1. The proof is as follows:

$$\begin{aligned}
(x+y)\cdot(x+z) &= xx + xz + xy + yz \\
&= x + xz + xy + yz \\
&= x(1+z+y) + yz \\
&= x \cdot 1 + yz \\
&= x + yz
\end{aligned}$$

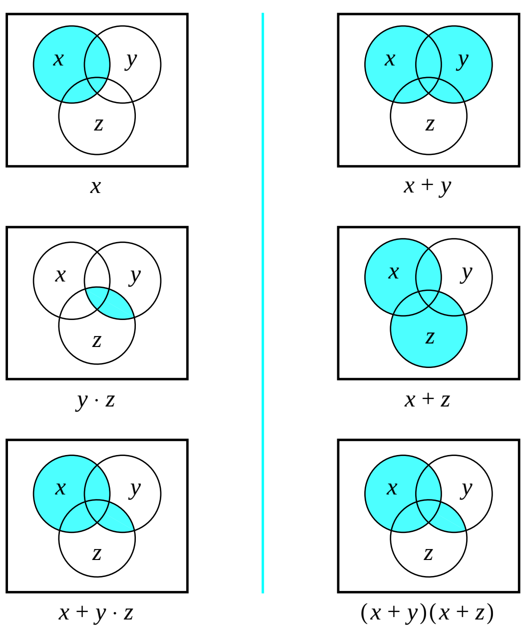
2.2. The proof is as follows:

$$\begin{aligned}
(x+y)\cdot(x+\overline{y}) &= xx + xy + x\overline{y} + y\overline{y} \\
&= x + xy + x\overline{y} + 0 \\
&= x(1+y+\overline{y}) \\
&= x \cdot 1 \\
&= x
\end{aligned}$$

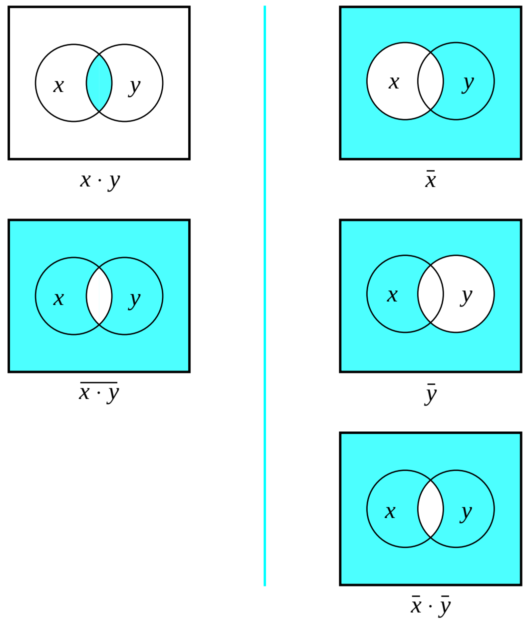
2.3. Manipulate the left hand side as follows:

$$\begin{aligned}
xy + yz + \overline{x}z &= xy + (x+\overline{x})yz + \overline{x}z \\
&= xy + xyz + \overline{x}yz + \overline{x}z \\
&= xy(1+z) + \overline{x}(y+1)z \\
&= xy \cdot 1 + \overline{x} \cdot 1 \cdot z \\
&= xy + \overline{x}z
\end{aligned}$$

2.4. Proof using Venn diagrams:

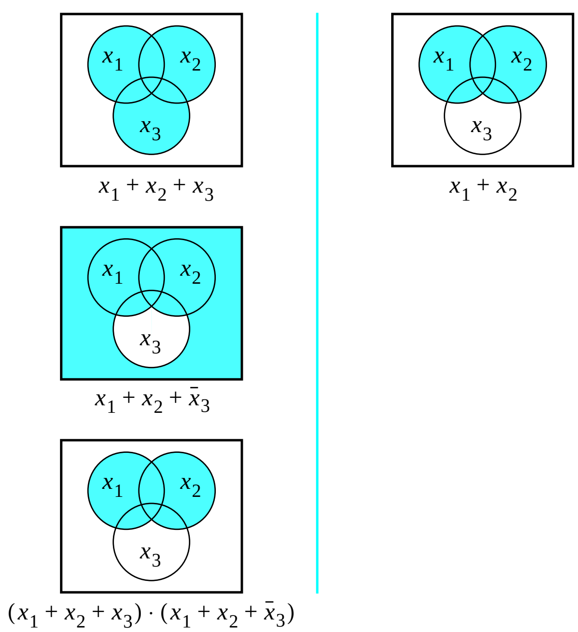


2.5. Proof of 15a using Venn diagrams:



A similar proof is constructed for 15b.

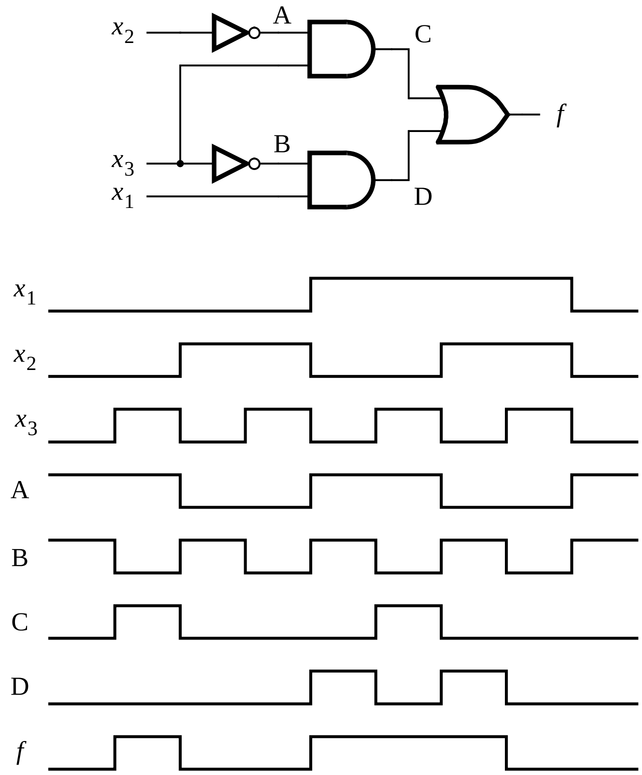
2.6. Proof using Venn diagrams:



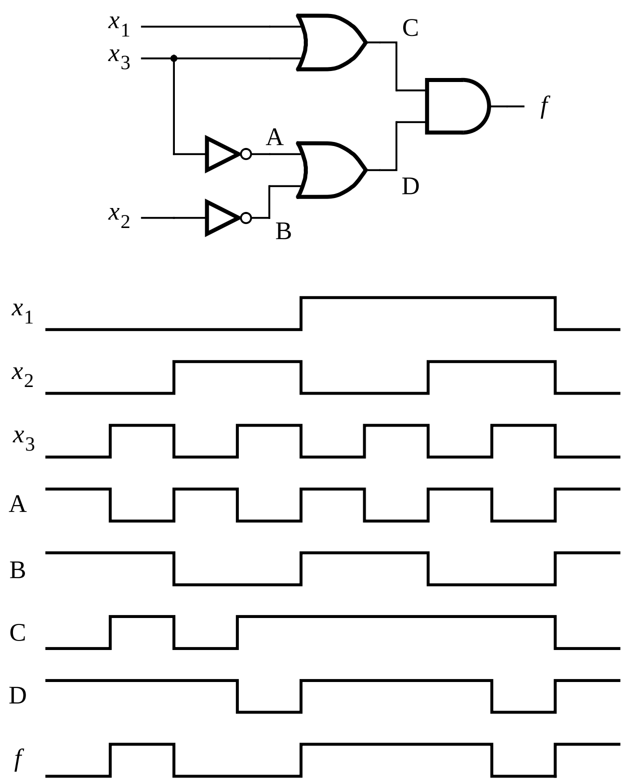
2.7. A possible approach for determining whether or not the expressions are valid is to try to manipulate the left and right sides of an expression into the same form, using the theorems and properties presented in section 2.5. While this may seem simple, it is an awkward approach, because it is not obvious what target form one should try to reach. A much simpler approach is to construct a truth table for each side of an expression. If the truth tables are identical, then the expression is valid. Using this approach, we can show that the answers are:

(a) Yes
(b) Yes
(c) No

2.8. Timing diagram of the waveforms that can be observed on all wires of the circuit:



2.9. Timing diagram of the waveforms that can be observed on all wires of the circuit:



2.10. Starting with the canonical sum-of-products for f get

$$
\begin{aligned}
f &= \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2\overline{x}_3 + \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2x_3 + x_1x_2\overline{x}_3 + x_1x_2x_3 \\
&= x_1(\overline{x}_2\overline{x}_3 + \overline{x}_2x_3 + x_2\overline{x}_3 + x_2x_3) + x_2(\overline{x}_1\overline{x}_3 + \overline{x}_1x_3 + x_1\overline{x}_3 + x_1x_3) \\
&\quad + x_3(\overline{x}_1\overline{x}_2 + \overline{x}_1x_2 + x_1\overline{x}_2 + x_1x_2) \\
&= x_1(\overline{x}_2(\overline{x}_3 + x_3) + x_2(\overline{x}_3 + x_3)) + x_2(\overline{x}_1(\overline{x}_3 + x_3) + x_1(\overline{x}_3 + x_3)) \\
&\quad + x_3(\overline{x}_1(\overline{x}_2 + x_2) + x_1(\overline{x}_2 + x_2)) \\
&= x_1(\overline{x}_2 \cdot 1 + x_2 \cdot 1) + x_2(\overline{x}_1 \cdot 1 + x_1 \cdot 1) + x_3(\overline{x}_1 \cdot 1 + x_1 \cdot 1) \\
&= x_1(\overline{x}_2 + x_2) + x_2(\overline{x}_1 + x_1) + x_3(\overline{x}_1 + x_1) \\
&= x_1 \cdot 1 + x_2 \cdot 1 + x_3 \cdot 1 \\
&= x_1 + x_2 + x_3
\end{aligned}
$$

2.11. Starting with the canonical product-of-sums for f can derive:

$$
\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + x_3) \\
&= ((x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3))((x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)) \cdot \\
&\quad ((\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3))((\overline{x}_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + x_3)) \\
&= (x_1 + x_2 + x_3\overline{x}_3)(x_1 + \overline{x}_2 + x_3\overline{x}_3) \cdot \\
&\quad (\overline{x}_1 + x_2 + x_3\overline{x}_3)(\overline{x}_1 + \overline{x}_2x_2 + x_3) \\
&= (x_1 + x_2)(x_1 + \overline{x}_2)(\overline{x}_1 + x_2)(\overline{x}_1 + x_3)
\end{aligned}
$$

$$
\begin{aligned}
&= (x_1 + x_2\overline{x}_2)(\overline{x}_1 + x_2x_3) \\
&= x_1(\overline{x}_1 + x_2x_3) \\
&= x_1\overline{x}_1 + x_1x_2x_3 \\
&= x_1x_2x_3
\end{aligned}
$$

2.12. Derivation of the minimum sum-of-products expression:

$$
\begin{aligned}
f &= x_1x_3 + x_1\overline{x}_2 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\
&= x_1(\overline{x}_2 + x_2)x_3 + x_1\overline{x}_2(\overline{x}_3 + x_3) + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\
&= x_1\overline{x}_2x_3 + x_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + \overline{x}_1x_2x_3 + \overline{x}_1\overline{x}_2\overline{x}_3 \\
&= x_1x_3 + (x_1 + \overline{x}_1)x_2x_3 + (x_1 + \overline{x}_1)\overline{x}_2\overline{x}_3 \\
&= x_1x_3 + x_2x_3 + \overline{x}_2\overline{x}_3
\end{aligned}
$$

2.13. Derivation of the minimum sum-of-products expression:

$$
\begin{aligned}
f &= x_1\overline{x}_2\overline{x}_3 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3(\overline{x}_4 + x_4) + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3\overline{x}_4 + x_1\overline{x}_2\overline{x}_3x_4 + x_1x_2x_4 + x_1\overline{x}_2x_3\overline{x}_4 \\
&= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2(\overline{x}_3 + x_3)\overline{x}_4 + x_1x_2x_4 \\
&= x_1\overline{x}_2\overline{x}_3 + x_1\overline{x}_2\overline{x}_4 + x_1x_2x_4
\end{aligned}
$$

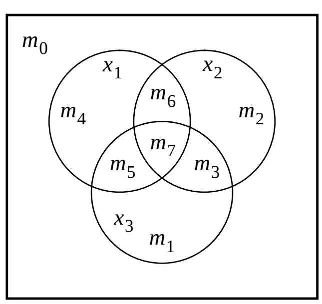
2.14. The simplest POS expression is derived as

$$
\begin{aligned}
f &= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3 + x_4) \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3 + x_4) \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)((x_1 + \overline{x}_2 + x_4)(x_3 + \overline{x}_3)) \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_4) \cdot 1 \\
&= (x_1 + x_3 + x_4)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + x_4)
\end{aligned}
$$

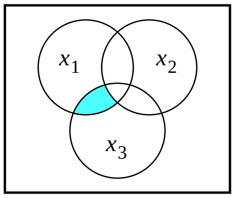
2.15. Derivation of the minimum product-of-sums expression:

$$
\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3) \\
&= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)(x_1 + (\overline{x}_2 + x_3))(\overline{x}_1 + (\overline{x}_2 + x_3)) \\
&= (x_1 + x_2)(\overline{x}_2 + x_3)
\end{aligned}
$$

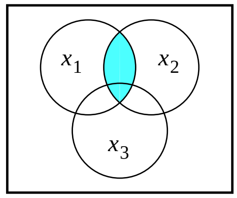
2.16. (a) Location of all minterms in a 3-variable Venn diagram:



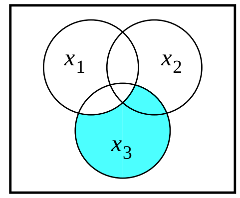
(b) For $f = x_1\overline{x}_2x_3 + x_1x_2 + \overline{x}_1x_3$ have:



$x_1 \cdot \bar{x}_2 \cdot x_3$

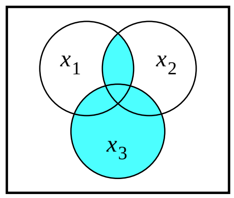


$x_1 \cdot x_2$



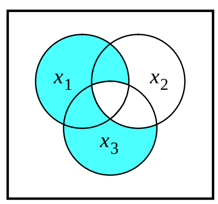
$\bar{x}_1 \cdot x_3$

Therefore, f is represented as:



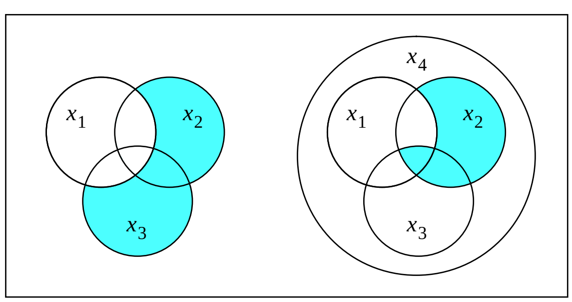
$$f = x_3 + x_1x_2$$

2.17. The function in Figure 2.18 in Venn diagram form is:



2.18. In Figure P2.1a it is possible to represent only 14 minterms. It is impossible to represent the minterms $\overline{x}_1\overline{x}_2x_3x_4$ and $x_1x_2\overline{x}_3\overline{x}_4$.
In Figure P2.1b, it is impossible to represent the minterms $x_1x_2\overline{x}_3\overline{x}_4$ and $x_1x_2x_3\overline{x}_4$.

2.19. Venn diagram for $f = \overline{x}_1\overline{x}_2x_3\overline{x}_4 + x_1x_2x_3x_4 + \overline{x}_1x_2$ is



2.20. The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3 \\ &= (\overline{x}_1 + x_1) x_2 x_3 + x_1 (\overline{x}_2 + x_2) \overline{x}_3 \\ &= x_2 x_3 + x_1 \overline{x}_3 \end{aligned}$$

2.21. The simplest SOP implementation of the function is

$$\begin{aligned} f &= \overline{x}_1 \overline{x}_2 x_3 + \overline{x}_1 x_2 x_3 + x_1 \overline{x}_2 \overline{x}_3 + x_1 x_2 \overline{x}_3 + x_1 x_2 x_3 \\ &= \overline{x}_1 (\overline{x}_2 + x_2) x_3 + x_1 (\overline{x}_2 + x_2) \overline{x}_3 + (\overline{x}_1 + x_1) x_2 x_3 \\ &= \overline{x}_1 x_3 + x_1 \overline{x}_3 + x_2 x_3 \end{aligned}$$

Another possibility is

$$f = \overline{x}_1 x_3 + x_1 \overline{x}_3 + x_1 x_2$$

2.22. The simplest POS implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= ((x_1 + x_3) + x_2)((x_1 + x_3) + \overline{x}_2)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= (x_1 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3) \end{aligned}$$

2.23. The simplest POS implementation of the function is

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)((\overline{x}_1 + x_3) + x_2)((\overline{x}_1 + x_3) + \overline{x}_2) \\ &= (x_1 + x_2)(\overline{x}_1 + \overline{x}_3) \end{aligned}$$

2.24. The simplest SOP expression for the function is

$$\begin{aligned} f &= x_1 \overline{x}_3 \overline{x}_4 + x_2 \overline{x}_3 x_4 + x_1 \overline{x}_2 \overline{x}_3 \\ &= x_1 \overline{x}_3 \overline{x}_4 + x_2 \overline{x}_3 x_4 + x_1 x_2 \overline{x}_3 + x_1 \overline{x}_2 \overline{x}_3 \\ &= x_1 \overline{x}_3 \overline{x}_4 + x_2 \overline{x}_3 x_4 + x_1 \overline{x}_3 \\ &= x_2 \overline{x}_3 x_4 + x_1 \overline{x}_3 \end{aligned}$$

2.25. The simplest SOP expression for the function is

$$
\begin{aligned}
f &= \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1 x_4 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\
&= \overline{x}_1\overline{x}_3\overline{x}_5 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1 x_4 x_5 + \overline{x}_1\overline{x}_3 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\
&= \overline{x}_1\overline{x}_3 + \overline{x}_1\overline{x}_3\overline{x}_4 + \overline{x}_1 x_4 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\
&= \overline{x}_1\overline{x}_3 + \overline{x}_1 x_4 x_5 + x_1\overline{x}_2\overline{x}_3 x_5 \\
&= \overline{x}_1\overline{x}_3 + \overline{x}_1 x_4 x_5 + \overline{x}_2\overline{x}_3 x_5
\end{aligned}
$$

2.26. The simplest POS expression for the function is

$$
\begin{aligned}
f &= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(x_1 + \overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)(x_1 + \overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3 + x_4)(\overline{x}_2 + \overline{x}_3) \\
&= (\overline{x}_1 + \overline{x}_3 + \overline{x}_4)(\overline{x}_2 + \overline{x}_3)
\end{aligned}
$$

2.27. The simplest POS expression for the function is

$$
\begin{aligned}
f &= (\overline{x}_2 + x_3 + x_5)(x_1 + \overline{x}_3 + x_5)(x_1 + x_2 + x_5)(x_1 + \overline{x}_4 + \overline{x}_5) \\
&= (\overline{x}_2 + x_3 + x_5)(x_1 + \overline{x}_3 + x_5)(x_1 + \overline{x}_2 + x_5)(x_1 + x_2 + x_5)(x_1 + \overline{x}_4 + \overline{x}_5) \\
&= (\overline{x}_2 + x_3 + x_5)(x_1 + \overline{x}_3 + x_5)(x_1 + x_5)(x_1 + \overline{x}_4 + \overline{x}_5) \\
&= (\overline{x}_2 + x_3 + x_5)(x_1 + x_5)(x_1 + x_5(\overline{x}_4 + \overline{x}_5)) \\
&= (\overline{x}_2 + x_3 + x_5)(x_1 + x_5)(x_1 + x_5\overline{x}_4) \\
&= (\overline{x}_2 + x_3 + x_5)(x_1 + x_5)(x_1 + \overline{x}_4)
\end{aligned}
$$

2.28. The lowest-cost circuit is defined by

$$
f(x_1, x_2, x_3) = x_1 x_2 + x_1 x_3 + x_2 x_3
$$

2.29. The function, *f*, of this circuit is equal to 0 when either none of the inputs or all three inputs are equal to 0; otherwise, *f* is equal to 1. Therefore, using the POS form, the desired circuit can be realized as

$$
\begin{aligned}
f(x_1, x_2, x_3) &= \Pi \mathrm{M}(0, 3) \\
&= (x_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)
\end{aligned}
$$

2.30. The circuit can be implemented as

$$
\begin{aligned}
f &= x_1 x_2 x_3 \overline{x}_4 + x_1 x_2 \overline{x}_3 x_4 + x_1 \overline{x}_2 x_3 x_4 + \overline{x}_1 x_2 x_3 x_4 + x_1 x_2 x_3 x_4 \\
&= x_1 x_2 x_3(\overline{x}_4 + x_4) + x_1 x_2(\overline{x}_3 + x_3) x_4 + x_1(\overline{x}_2 + x_2) x_3 x_4 + (\overline{x}_1 + x_1) x_2 x_3 x_4 \\
&= x_1 x_2 x_3 + x_1 x_2 x_4 + x_1 x_3 x_4 + x_2 x_3 x_4
\end{aligned}
$$

2.31. The truth table that corresponds to the timing diagram in Figure P2.3 is

x_1	x_2	x_3	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

The simplest SOP expression is $f = \overline{x}_1\overline{x}_2\overline{x}_3 + \overline{x}_1 x_2 x_3 + x_1\overline{x}_2 x_3 + x_1 x_2\overline{x}_3$.

2.32. The truth table that corresponds to the timing diagram in Figure P2.3 is

x_1	x_2	x_3	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

The simplest POS expression is $f = (x_1 + x_2 + \overline{x}_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)$.

2.33. The truth table that corresponds to the timing diagram in Figure P2.4 is

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

The simplest SOP expression is derived as follows:

$$\begin{aligned} f &= \overline{x}_1\overline{x}_2x_3 + \overline{x}_1x_2\overline{x}_3 + \overline{x}_1x_2x_3 + x_1\overline{x}_2\overline{x}_3 + x_1x_2x_3 \\ &= \overline{x}_1(\overline{x}_2 + x_2)x_3 + \overline{x}_1x_2(\overline{x}_3 + x_3) + (\overline{x}_1 + x_1)x_2x_3 + x_1\overline{x}_2\overline{x}_3 \\ &= \overline{x}_1 \cdot 1 \cdot x_3 + \overline{x}_1x_2 \cdot 1 + 1 \cdot x_2x_3 + x_1\overline{x}_2\overline{x}_3 \\ &= \overline{x}_1x_3 + \overline{x}_1x_2 + x_2x_3 + x_1\overline{x}_2\overline{x}_3 \end{aligned}$$

2.34. The truth table that corresponds to the timing diagram in Figure P2.4 is

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

The simplest POS expression is $f = (x_1 + x_2 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + x_3)$.

2.35. (a)

x_1	x_0	y_1	y_0	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

(b) The simplest POS expression is $f = (x_1 + \overline{y}_1)(\overline{x}_1 + y_1)(x_0 + \overline{y}_0)(\overline{x}_0 + y_0)$.

2.36. (a)

x_1	x_0	y_1	y_0	f
0	0	0	0	1
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

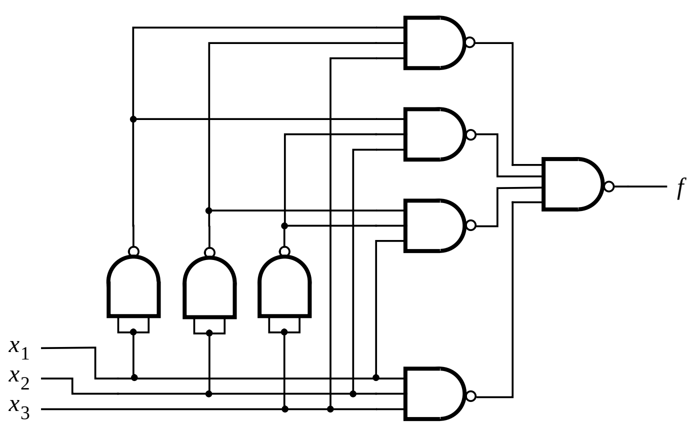
(b) The canonical SOP expression is

$$\begin{aligned} f &= \overline{x}_1\overline{x}_0\overline{y}_1\overline{y}_0 + \overline{x}_1 x_0\overline{y}_1\overline{y}_0 + \overline{x}_1 x_0\overline{y}_1 y_0 + x_1\overline{x}_0\overline{y}_1\overline{y}_0 + x_1\overline{x}_0\overline{y}_1 y_0 + x_1\overline{x}_0 y_1\overline{y}_0 \\ &\quad + x_1 x_0\overline{y}_1\overline{y}_0 + x_1 x_0\overline{y}_1 y_0 + x_1 x_0 y_1\overline{y}_0 + x_1 x_0 y_1 y_0 \end{aligned}$$

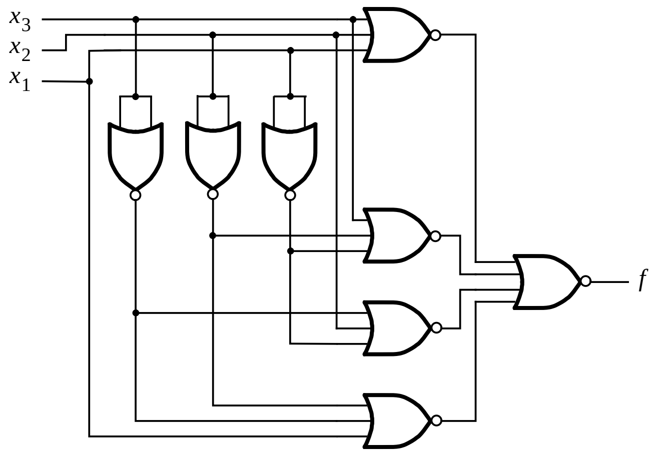
(c) The simplest SOP expression is

$$f = x_1 x_0 + \overline{y}_1\overline{y}_0 + x_1\overline{y}_0 + x_0\overline{y}_1$$

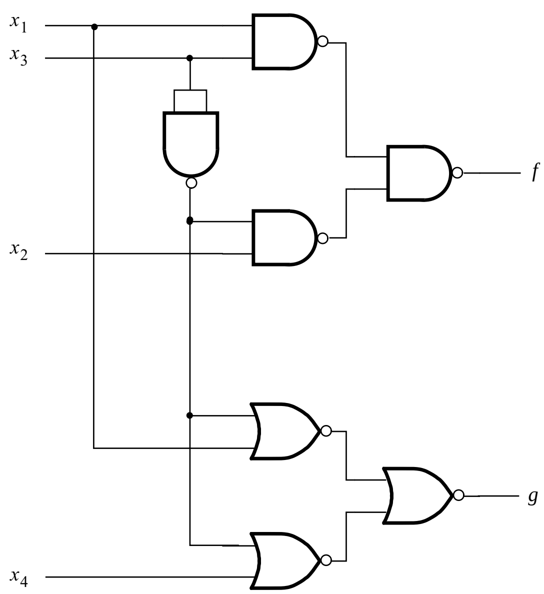
2.37. Using the ciruit in Figure 2.27a as a starting point, the function in Figure 2.24 can be implemented using NAND gates as follows:



2.38. Using the ciruit in Figure 2.27*b* as a starting point, the function in Figure 2.24 can be implemented using NOR gates as follows:



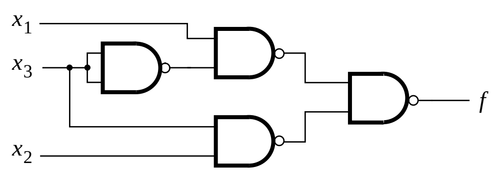
2.39. The circuit in Figure 2.33 can be implemented using NAND and NOR gates as follows:



2.40. The minimum-cost SOP expression for the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$ is

$$f = x_1\overline{x}_3 + x_2x_3$$

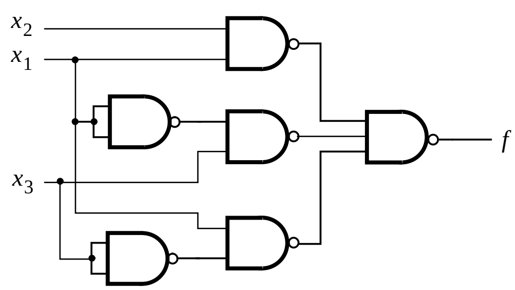
The corresponding circuit implemented using NAND gates is



2.41. A minimum-cost SOP expression for the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7)$ is

$$f = x_1x_2 + x_1\overline{x}_3 + \overline{x}_1x_3$$

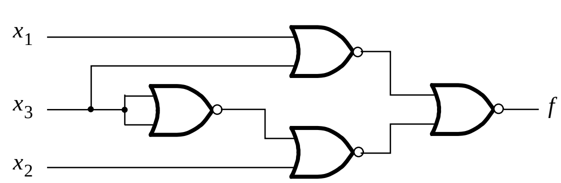
The corresponding circuit implemented using NAND gates is



2.42. The minimum-cost POS expression for the function $f(x_1, x_2, x_3) = \sum m(3, 4, 6, 7)$ is

$$f = (x_1 + x_3)(x_2 + \overline{x}_3)$$

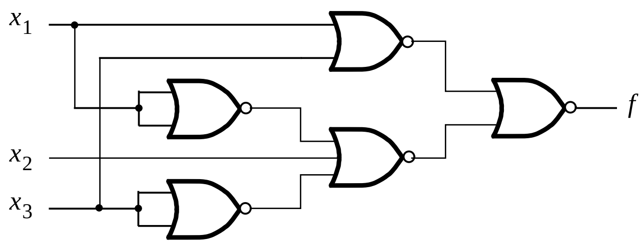
The corresponding circuit implemented using NOR gates is



2.43. The minimum-cost POS expression for the function $f(x_1, x_2, x_3) = \sum m(1, 3, 4, 6, 7)$ is

$$f = (x_1 + x_3)(\overline{x}_1 + x_2 + \overline{x}_3)$$

The corresponding circuit implemented using NOR gates is



2.44. The simplest SOP expression is derived as

$$\begin{aligned} f &= x_1\overline{x}_3 + x_1x_2 + \overline{x}_1\overline{x}_2 + \overline{x}_2x_3 \\ &= x_1\overline{x}_3 + \overline{x}_2x_3 + x_1\overline{x}_2 + x_1x_2 + \overline{x}_1\overline{x}_2 \\ &= x_1\overline{x}_3 + \overline{x}_2x_3 + x_1(\overline{x}_2 + x_2) + \overline{x}_2(x_1 + \overline{x}_1) \\ &= x_1\overline{x}_3 + \overline{x}_2x_3 + x_1 + \overline{x}_2 \\ &= x_1 + \overline{x}_2 \end{aligned}$$

2.45. The simplest SOP expression is derived as

$$\begin{aligned} f &= \overline{x}_1\overline{x}_2x_3 + x_1x_3 + x_2x_3 + x_1x_2\overline{x}_3 \\ &= \overline{(x_1 + x_2)}x_3 + (x_1 + x_2)x_3 + x_1x_2x_3 \\ &= x_3 + x_1x_2\overline{x}_3 \\ &= x_3 + x_1x_2 \end{aligned}$$

2.46. The simplest POS expression is derived as

$$\begin{aligned} \overline{f} &= \overline{x_2 + x_1x_3 + \overline{x}_1\overline{x}_3} \\ &= \overline{x}_2(\overline{x}_1 + \overline{x}_3)(x_1 + x_3) \\ &= \overline{x}_2(\overline{x}_1x_3 + \overline{x}_3x_1) \\ &= \overline{x}_2\overline{x}_1x_3 + \overline{x}_2\overline{x}_3x_1 \end{aligned}$$

$Then$

$$\begin{aligned} f &= \overline{\overline{x}_2\overline{x}_1x_3 + \overline{x}_2\overline{x}_3x_1} \\ &= \overline{\overline{x}_2\overline{x}_1x_3} \cdot \overline{\overline{x}_2\overline{x}_3x_1} \\ &= (x_2 + x_1 + \overline{x}_3)(x_2 + x_3 + \overline{x}_1) \end{aligned}$$

2.47. The simplest POS expression is derived as

$$
\begin{aligned}
f &= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(\overline{x}_1 + x_2 + x_3)(\overline{x}_1 + \overline{x}_2 + x_3)(x_1 + x_2 + \overline{x}_3 + x_4) \\
&= ((x_1 + x_2)(x_1 + \overline{x}_2)(\overline{x}_1 + x_2)(\overline{x}_1 + \overline{x}_2) + x_3)(x_1 + x_2 + \overline{x}_3 + x_4) \\
&= ((x_1 + x_2\overline{x}_2)(\overline{x}_1 + x_2\overline{x}_2) + x_3)(x_1 + x_2 + \overline{x}_3 + x_4) \\
&= ((x_1 + 0)(\overline{x}_1 + 0) + x_3)(x_1 + x_2 + \overline{x}_3 + x_4) \\
&= (x_1\overline{x}_1 + x_3)(x_1 + x_2 + \overline{x}_3 + x_4) \\
&= (0 + x_3)(x_1 + x_2 + \overline{x}_3 + x_4) \\
&= x_3(x_1 + x_2 + \overline{x}_3 + x_4) \\
&= x_3(x_1 + x_2 + x_4)
\end{aligned}
$$

2.50. The function can be specified by using the minterms as follows:

```
ENTITY problem46 IS
    PORT ( x1, x2, x3   : IN    BIT ;
           f            : OUT   BIT ) ;
END problem46 ;

ARCHITECTURE LogicFunc OF problem46 IS
BEGIN
    f <= (NOT x1 AND NOT x2 AND NOT x3) OR (NOT x1 AND NOT x2 AND x3) OR
         (NOT x1 AND x2 AND x3) OR (x1 AND NOT x2 AND NOT x3) OR
         (x1 AND NOT x2 AND x3) OR (x1 AND x2 AND NOT x3) ;
END LogicFunc ;
```

The simplest SOP expression for this function is

$$f = \overline{x}_2 + x_1\overline{x}_3 + \overline{x}_1 x_3$$

Using this expression, we can replace the statement that specifies f in the above VHDL code with the statement

```
f <= NOT x2 OR (x1 AND NOT x3) OR (NOT x1 AND x3) ;
```

Another way of specifying the function is by using the maxterms, M_2 and M_7, in which case the VHDL statement would be

```
f <= (x1 OR NOT x2 OR x3) AND (NOT x1 OR NOT x2 OR NOT x3) ;
```

2.51. The VHDL code is

```
ENTITY prob47 IS
      PORT ( x1, x2, x3, x4   : IN      STD_LOGIC ;
                f1, f2                : OUT  STD_LOGIC ) ;
END prob47 ;

ARCHITECTURE LogicFunc OF prob47 IS
BEGIN
      f1 <= (x1 AND NOT x3) OR (x2 AND NOT x3) OR
              NOT x3 AND NOT x4) OR (x1 AND x2) OR
              x1 AND NOT x4) ;
      f2 <= (x1 OR NOT x3) AND (x1 OR x2 OR NOT x4) AND
              x2 OR NOT x3 OR NOT x4) ;
END LogicFunc ;
```

2.52. The VHDL code is

```
ENTITY prob48 IS
      PORT ( x1, x2, x3, x4   : IN      STD_LOGIC ;
                f1, f2                : OUT  STD_LOGIC ) ;
END prob48 ;

ARCHITECTURE LogicFunc OF prob48 IS
BEGIN
      f1 <= ((x1 AND x3) OR (NOT x1 AND NOT x3)) OR
              ((x2 AND x4) OR (NOT x2 AND NOT x4)) ;
      f2 <= (x1 AND x2 AND NOT x3 AND NOT x4) OR
              (NOT x1 AND NOT x2 AND x3 AND x4) OR
              (x1 AND NOT x2 AND NOT x3 AND x4) OR
              (NOT x1 AND x2 AND x3 AND NOT x4);
END LogicFunc ;
```