RL DC motor

Control laws design

Dynamics of DC motor

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -ax_2 - \mathbb{F} + bu(t)$$

$$\begin{cases} x_d = \theta_{desired} \\ x_1 = \theta_{actual} \\ x_2 = \dot{\theta}_{actual} = w_{actual} \end{cases}$$

Define the tracking error dynamics function

$$c = x_d - x_1$$
$$\dot{c} = \dot{x}_d - \dot{x}_1 = \dot{x}_d - x_2$$
$$\ddot{c} = \ddot{x}_d - \ddot{x}_1 = \ddot{x}_d - \dot{x}_2 \Rightarrow \ddot{c} = \ddot{x}_d - [-ax_2 - \mathbb{F} + bu]$$

Since the bristle state in LuGre model is challenged to derive or estimate, the feedforward compensation terms approximate the friction model by using Columb friction model.
To compensate disturbances in error function, the control input can be designed as follows:

$$u(t) = v_{PID} + \alpha x_2 + \beta \ddot{\theta}_d(t) + \gamma_1 sign^+(\dot{\theta}(t)) + \gamma_2 sign^-(\dot{\theta}(t))$$

$$, where \quad v_{PID} = \frac{1}{b}\left[k_p c + k_d \dot{c} + k_i \int_0^t c(\tau)d\tau\right], \alpha = \frac{a}{b}, \beta = \frac{1}{b}, \gamma_1 = \frac{f_c^+}{b}, \gamma_2 = \frac{f_c^-}{b}$$

Observation space

Num	Observation	Min	Max	Starting value
0	Motor Angular Position θ (rad)	-Inf	Inf	0
1	Motor Angular Velocity ω (rad/s)	-Inf	Inf	0
2	Motor Angular Position Error c (rad)	-Inf	Inf	0
3	Error Backward Difference $\dot{c}$ (rad)	-Inf	Inf	0
4	Error integration $\int c\,dt$ (rad)	-Inf	Inf	0
5	Desired Motor Angular Velocity $\dot{\theta}_d$ (rad)	-Inf	Inf	0
6	Desired Motor Angular Acceleration $\ddot{\theta}_d$ (rad/s)	-Inf	Inf	0

Action space

Num	Action	Min	Max
0	k_p	0	150
1	k_i	0	60
2	k_d	0	20
3	α	0	0.519
4	β	0	0.061
5	γ_1	0	1.698
6	γ_2	0	1.622

System ID. result

Para.	Value
a	8.4645
b	32.6486
f_c^+	27.7231
f_c^-	26.4779

$$\alpha = \frac{a}{b}, \beta = \frac{1}{b}$$

$$\gamma_1 = \frac{f_c^+}{b}, \gamma_2 = \frac{f_c^-}{b}$$