

NCKU Programming Contest Training Course

Computational Geometry

2017/05/31

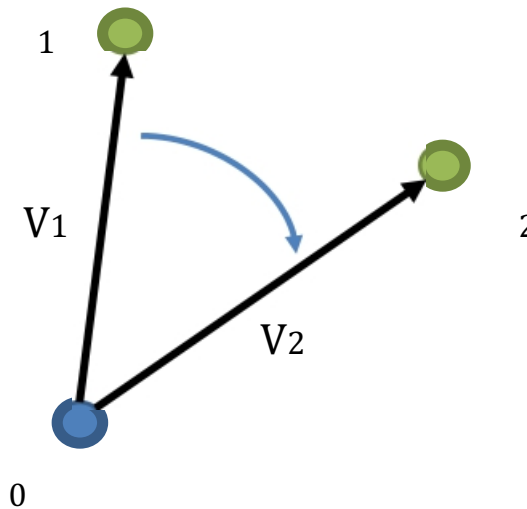
Jheng-Huang Hong

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National Cheng Kung University
Tainan, Taiwan



Line intersection

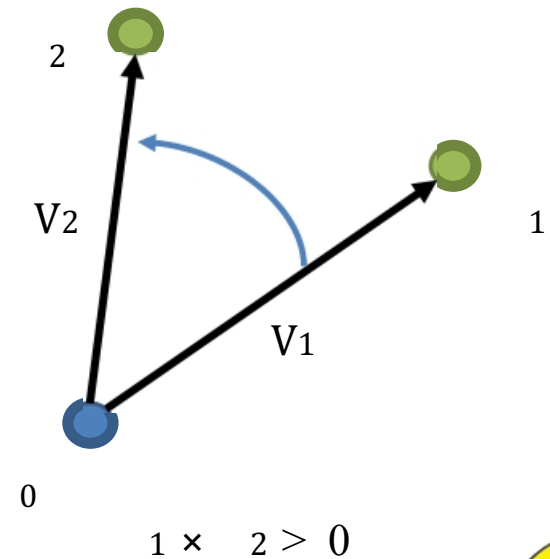
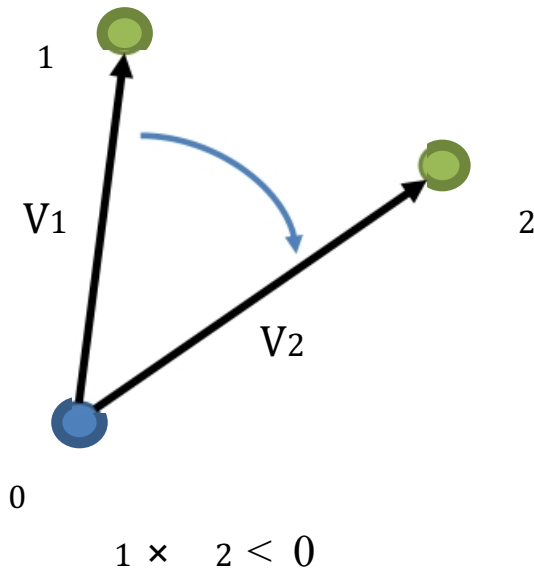
- The vector V_2 is clockwise/counterclockwise from V_1 ?



Line intersection

- Cross Product :

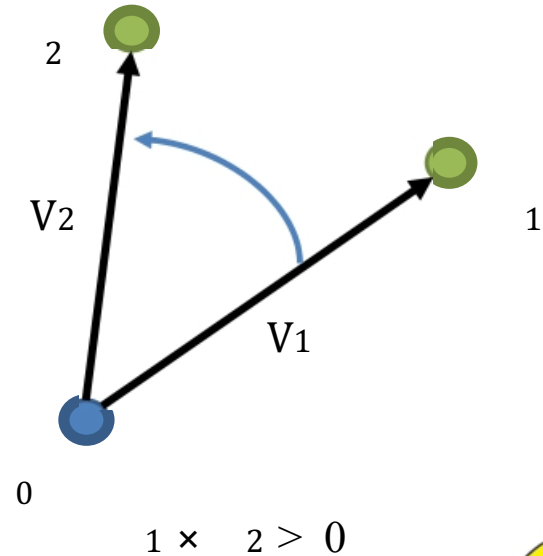
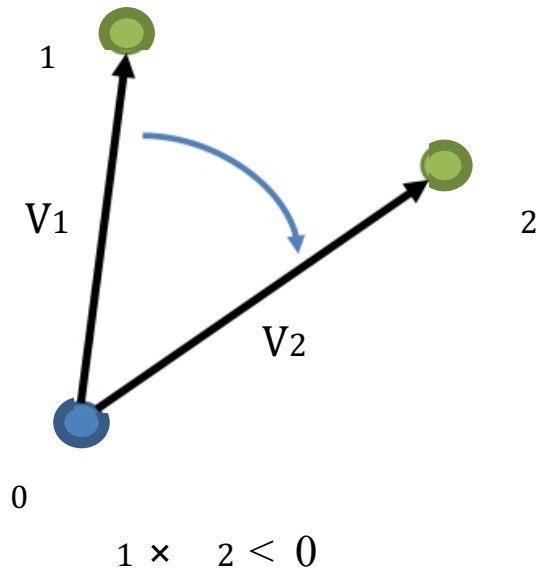
$$\mathbf{v}_1 \times \mathbf{v}_2 = \det \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 1 \cdot 2 - 2 \cdot 1 = \|\mathbf{v}_1\| \|\mathbf{v}_2\|$$



Line intersection

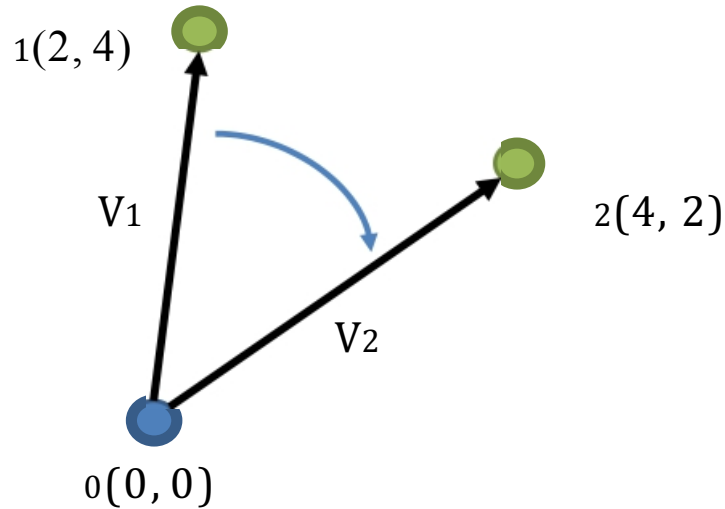
- Cross Product :

$$1 \times 2 = (1 - 0) \times (2 - 0) = (1 - 0)(2 - 0) - (2 - 0)(1 - 0)$$



Line intersection

- Cross Product :

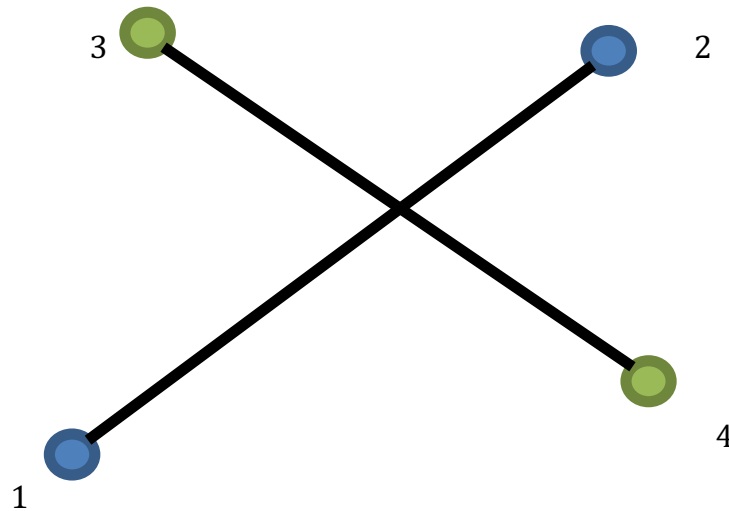


$$1 \times 2 = (1 - 0) \times (2 - 0) = (2 - 0)(2 - 0) - (4 - 0)(4 - 0) = -12$$



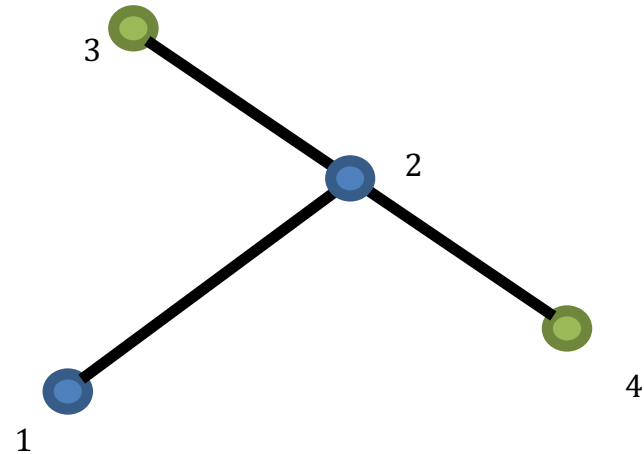
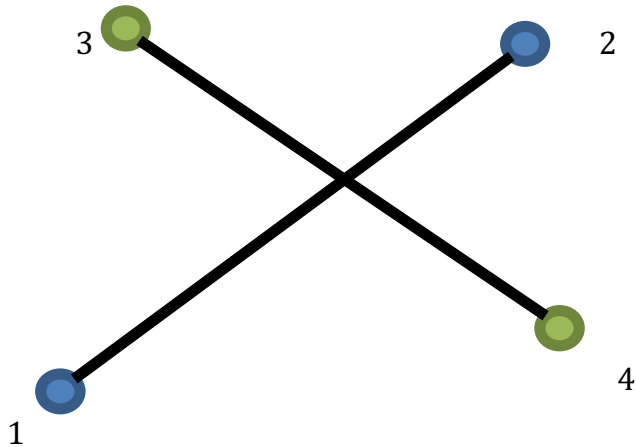
Line intersection

- Line intersection problem



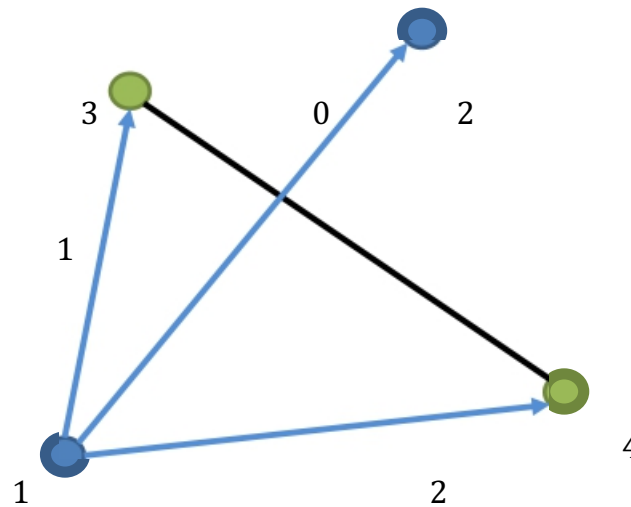
Line intersection

- Two situation



Line intersection

- Cross Product :

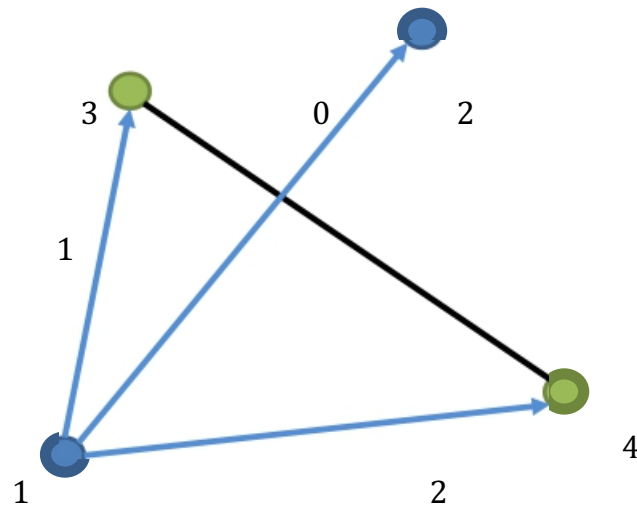


$$(0 \times 1) \cdot (0 \times 2) = ?$$



Line intersection

- Cross Product :

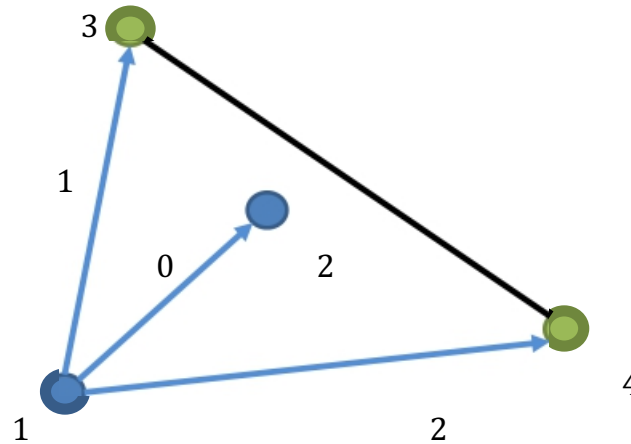


$$(0 \times 1) \cdot (0 \times 2) < 0$$



Line intersection

- Cross Product :

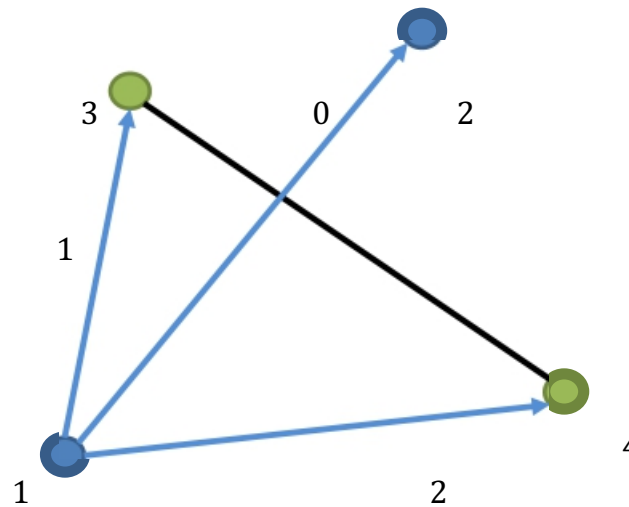


$$(0 \times 1) \cdot (0 \times 2) < 0$$



Line intersection

- Cross Product :

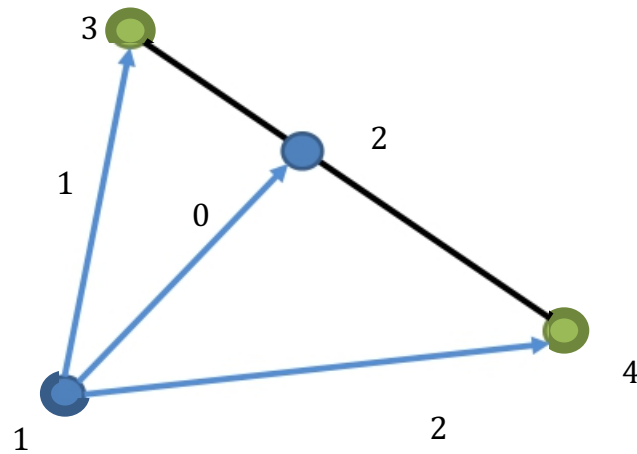


$$\frac{((x_2 - x_1) \times (y_3 - y_1)) \cdot ((x_2 - x_1) \times (y_4 - y_1))}{0} < 0 \ \&\& \ ((x_4 - x_3) \times (y_1 - y_3)) \cdot ((x_4 - x_3) \times (y_2 - y_3)) < 0$$



Line intersection

- Cross Product :

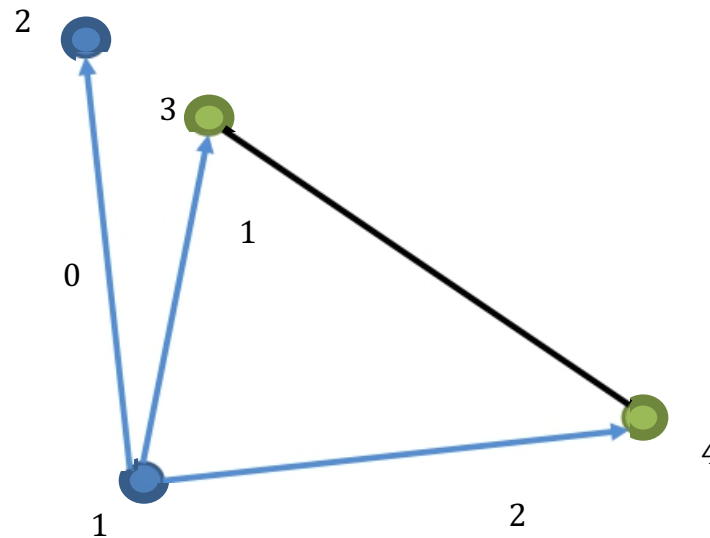


$$(4 - 3) \times (2 - 3) = 0$$



Line intersection

- Cross Product :

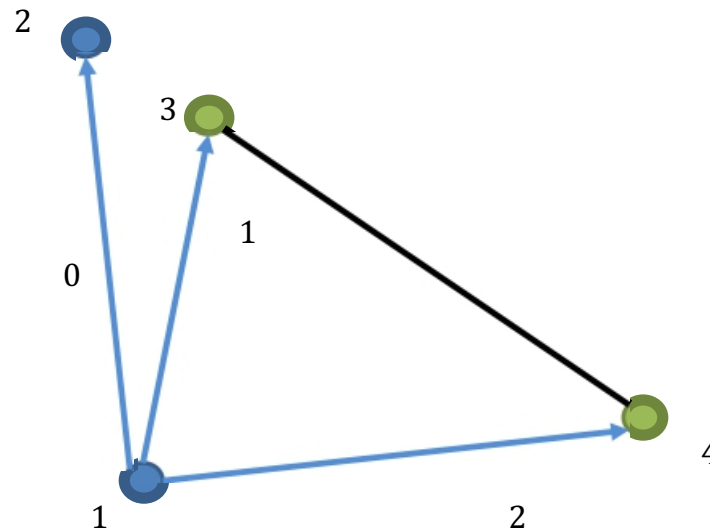


$$(4 - 3) \times (2 - 3) = 0$$



Line intersection

- Cross Product :

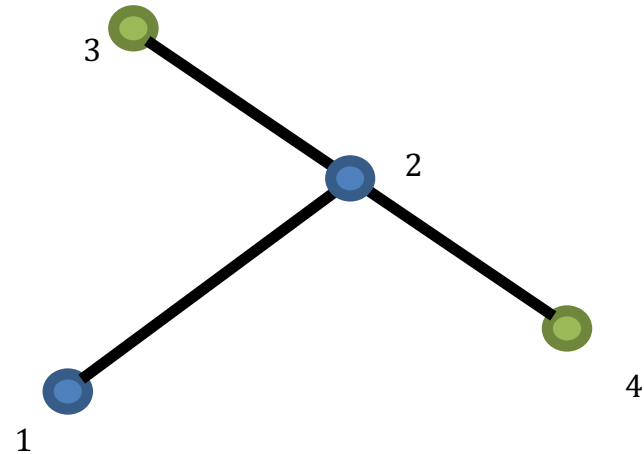
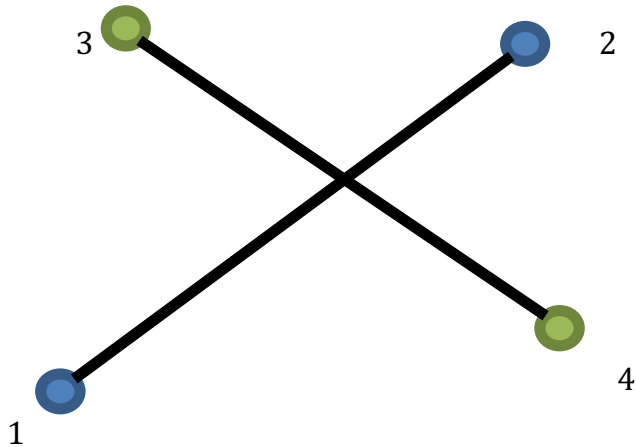


$$\min(3, 4) \leq 2 \leq \max(3, 4) \ \&\& \ \min(3, 4) \leq 2 \leq \max(3, 4)$$



Line intersection

- Two situation



Practice - 1

UVa 191 - Intersection



Convex Hull

- 中譯「凸包」或「凸殼」。在多維空間中有一群散佈各處的點，「凸包」是包覆這群點的所有外殼當中，表面積暨容積最小的一個外殼，而最小的外殼一定是凸的。
- 「凸」的定義是：圖形內任意兩點的連線不會經過圖形外部。「凸」並不是指表面呈弧狀隆起，事實上凸包是由許多平坦表面組成的。

演算法筆記 – Convex Hull



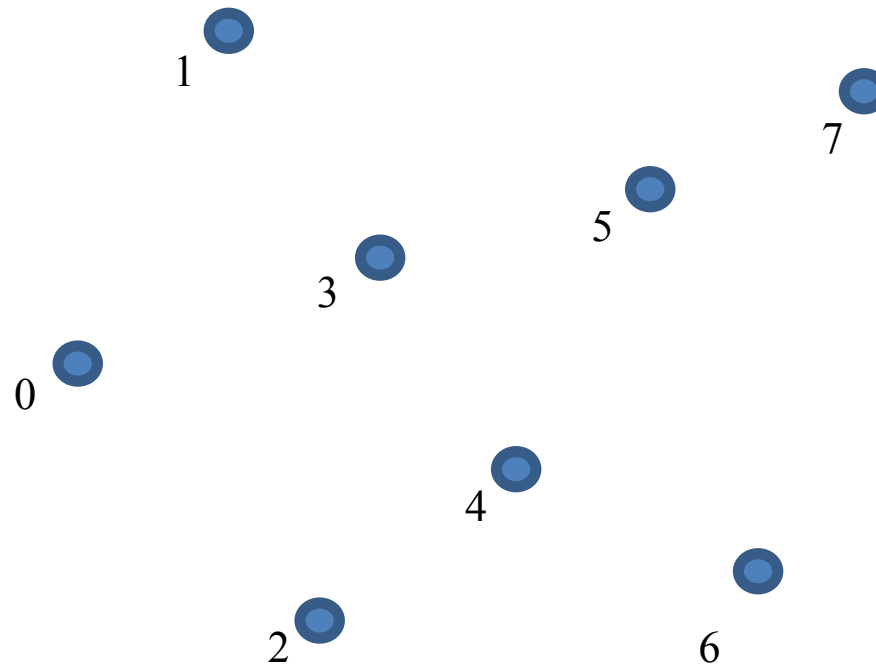
Convex Hull

- Algorithm
 - Brute Force
 - Graham-Scan
 - Andrew's Monotone Chain



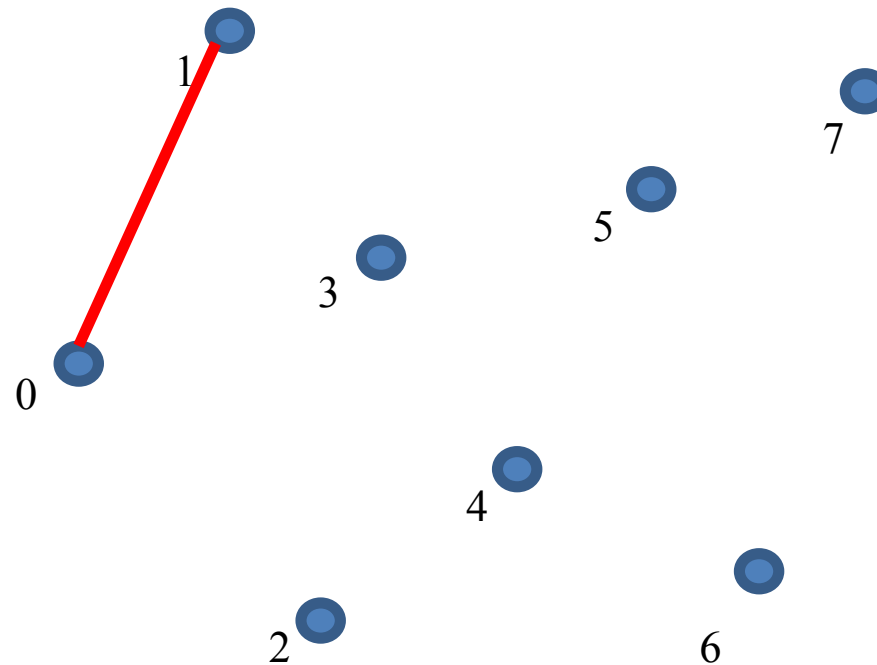
Andrew's Monotone Chain

- Step1 : Sort by x



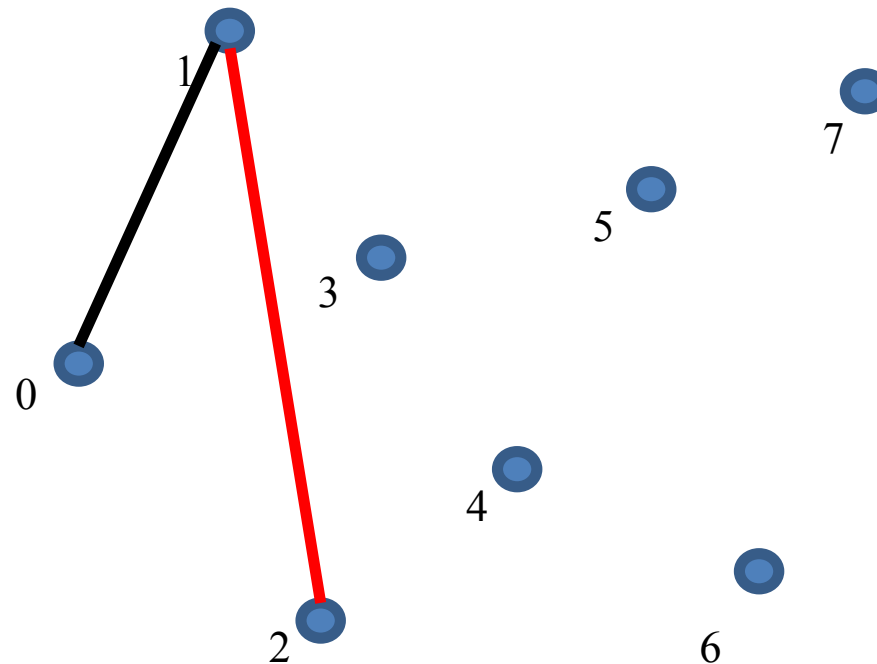
Andrew's Monotone Chain

- Step2 : Connect points



Andrew's Monotone Chain

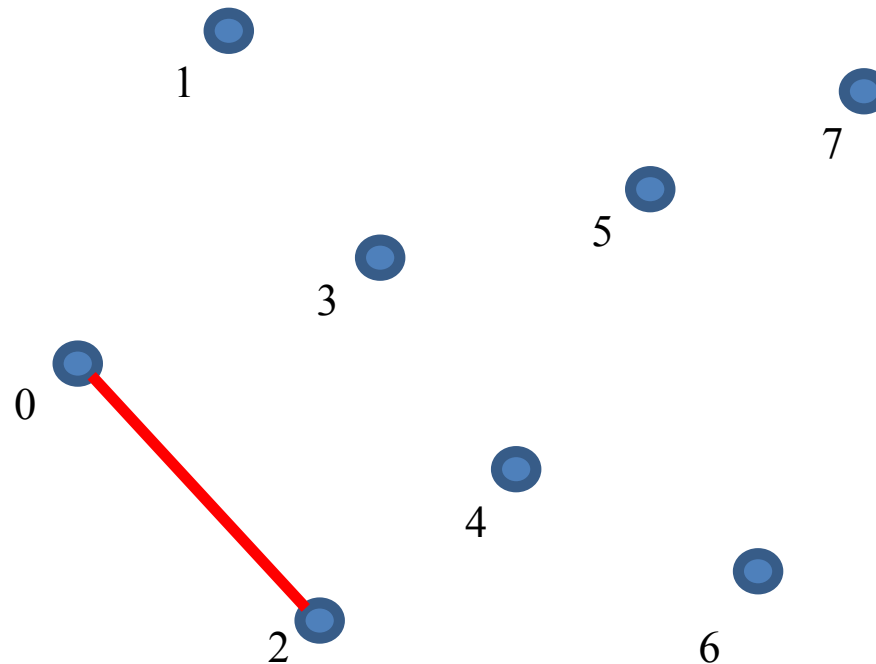
- Step2 : Connect points



Andrew's Monotone Chain

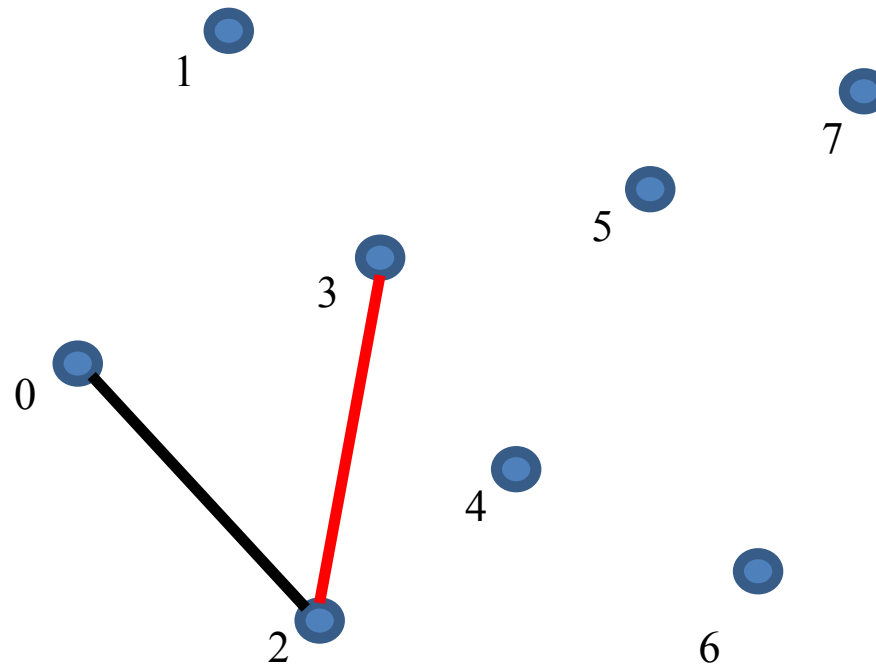
- Step3 : Use cross product

$$p_1 \times p_2 < 0$$



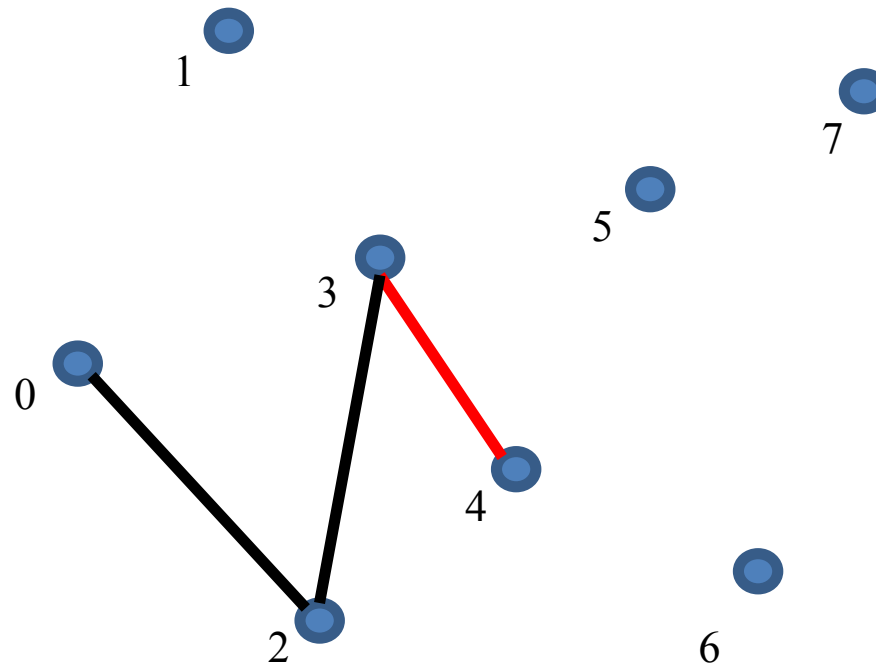
Andrew's Monotone Chain

- Step3 : Use cross product



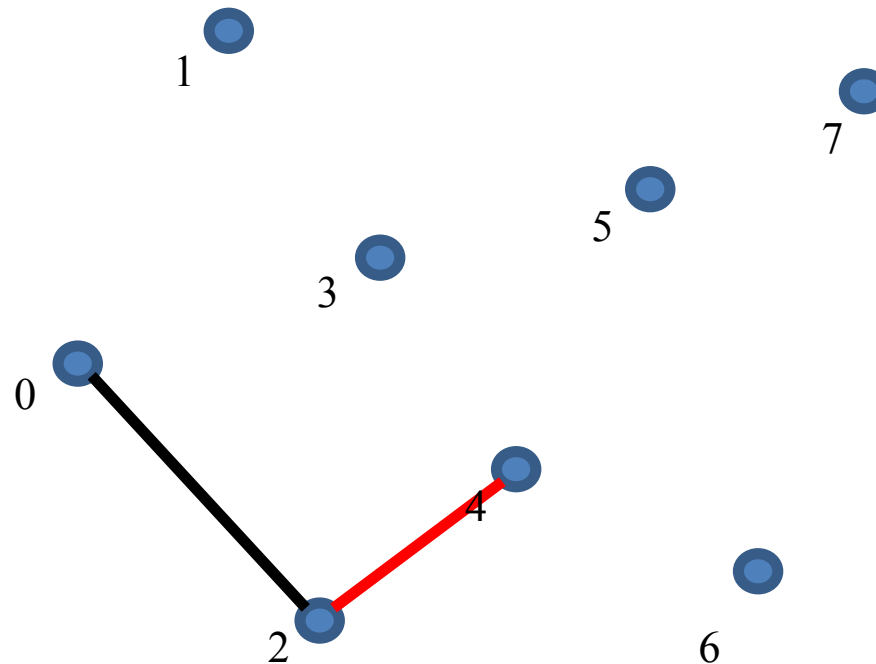
Andrew's Monotone Chain

- Step3 : Use cross product



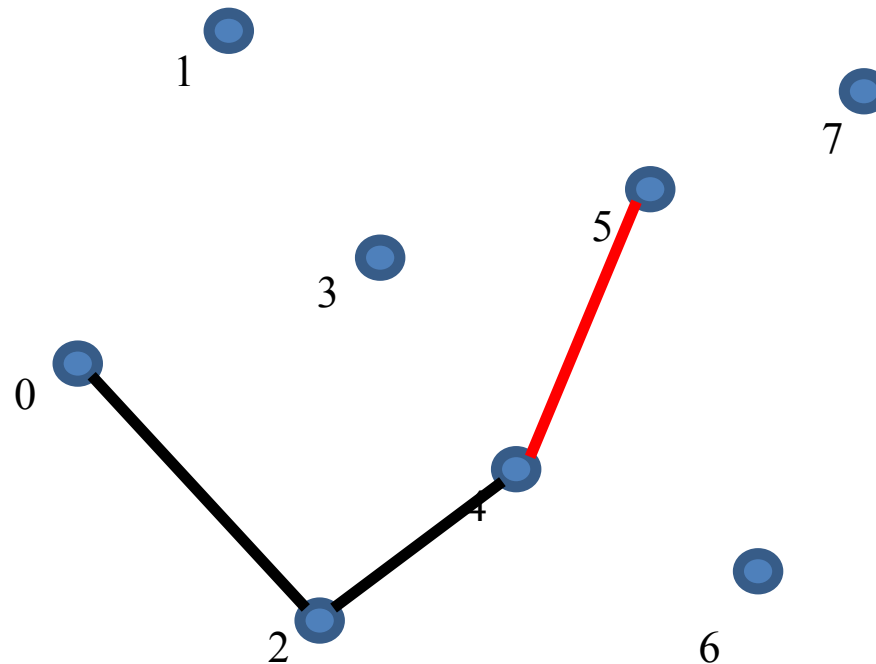
Andrew's Monotone Chain

- Step3 : Use cross product



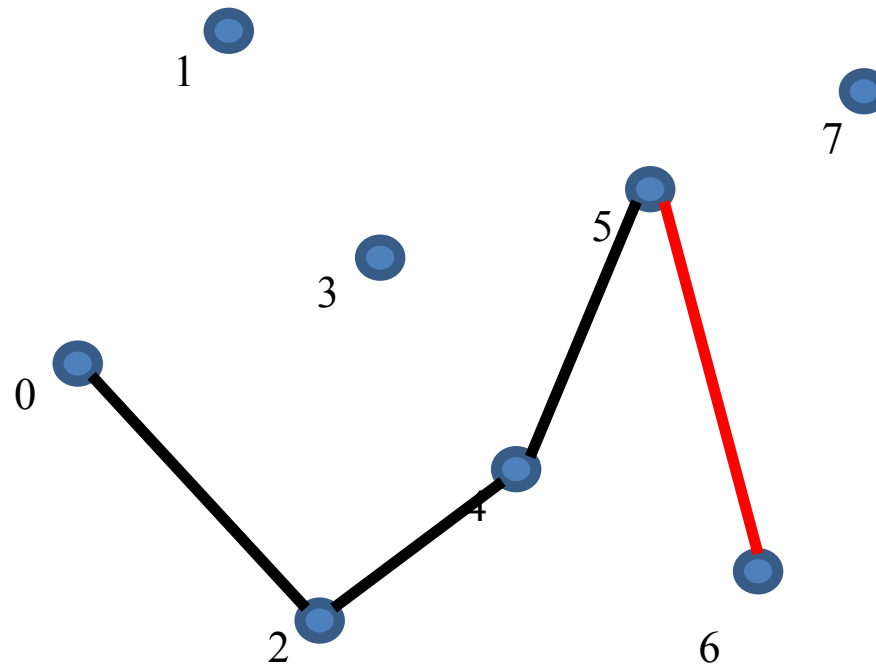
Andrew's Monotone Chain

- Step3 : Use cross product



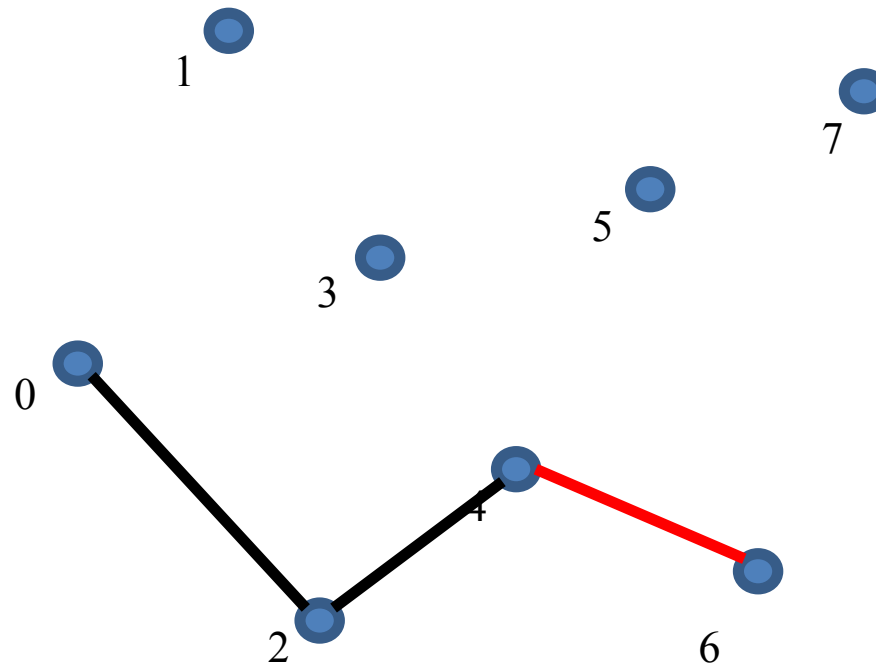
Andrew's Monotone Chain

- Step3 : Use cross product



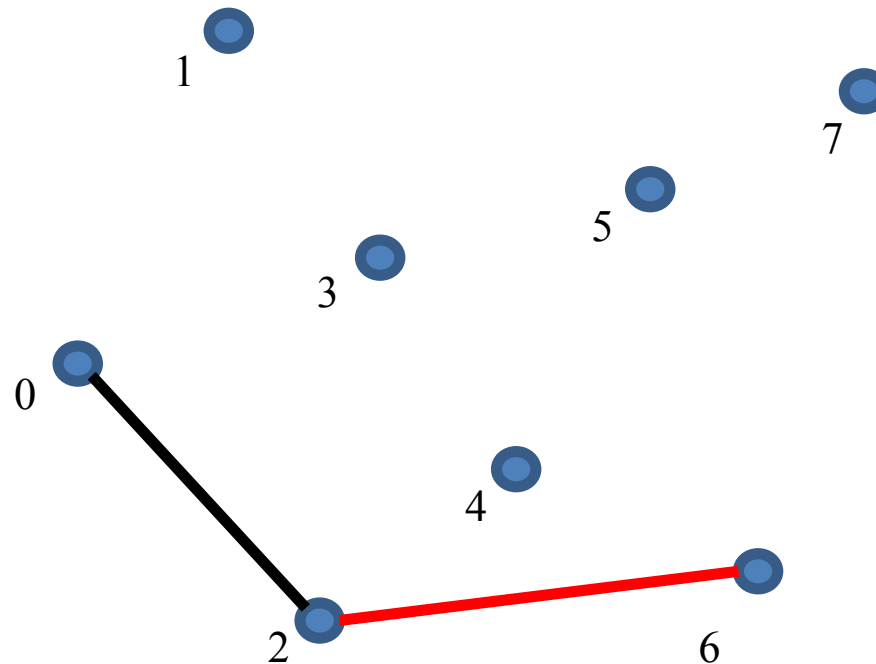
Andrew's Monotone Chain

- Step3 : Use cross product



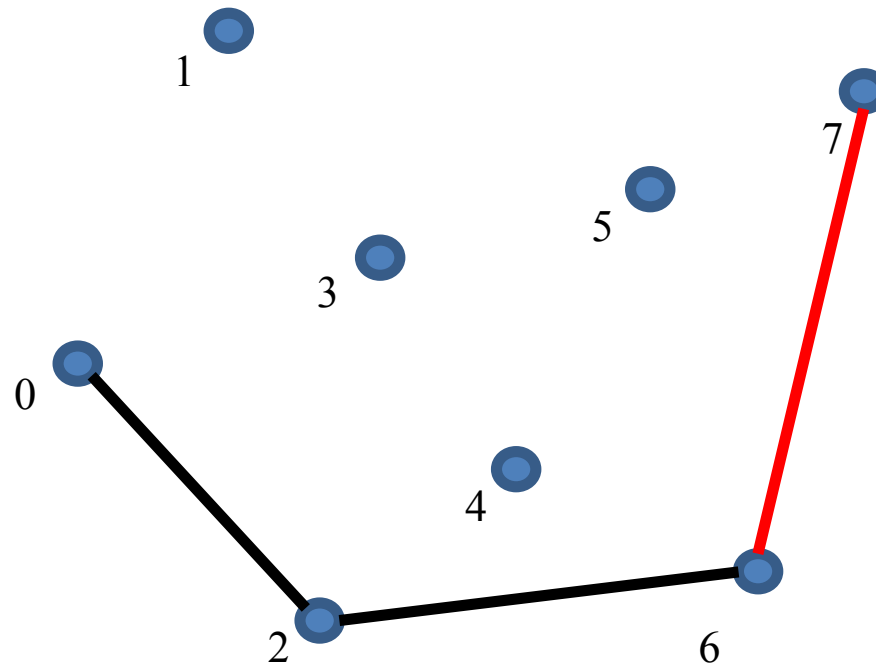
Andrew's Monotone Chain

- Step3 : Use cross product



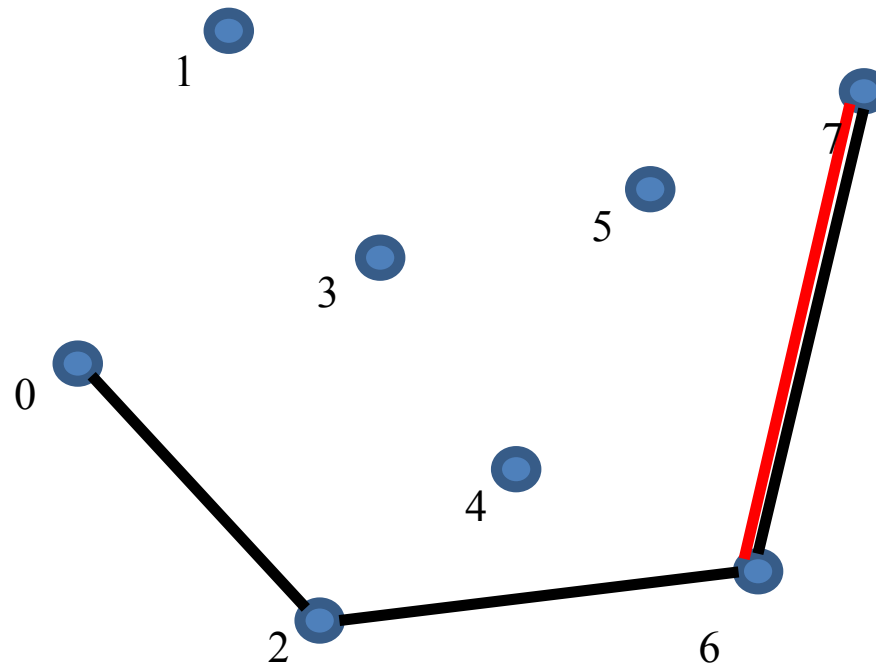
Andrew's Monotone Chain

- Step3 : Use cross product



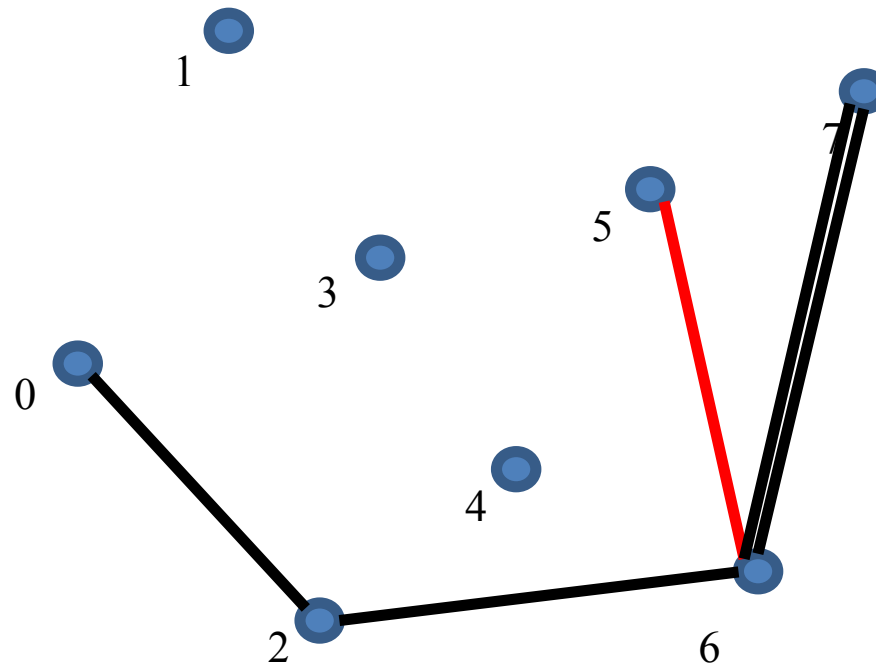
Andrew's Monotone Chain

- Step3 : Use cross product



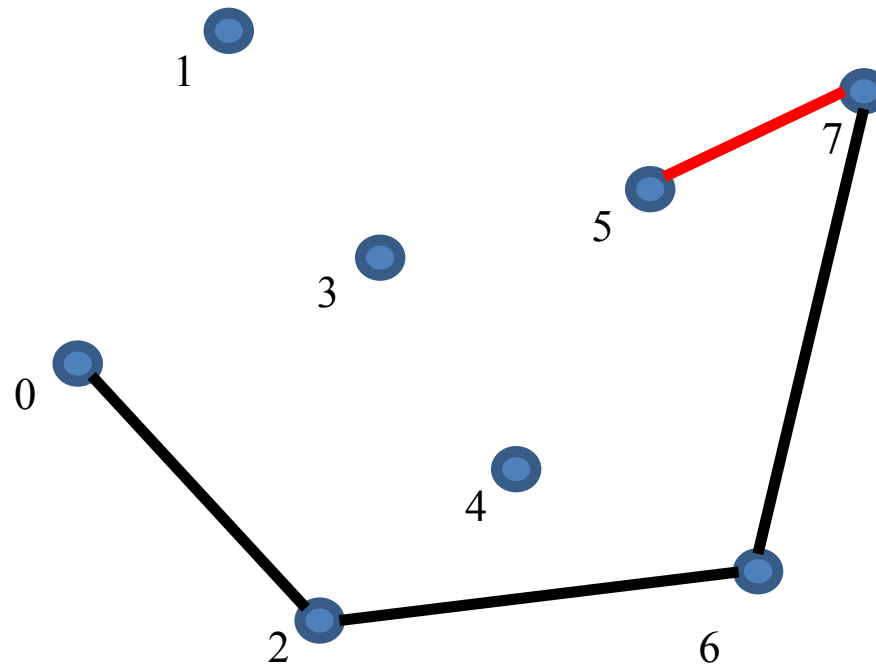
Andrew's Monotone Chain

- Step3 : Use cross product



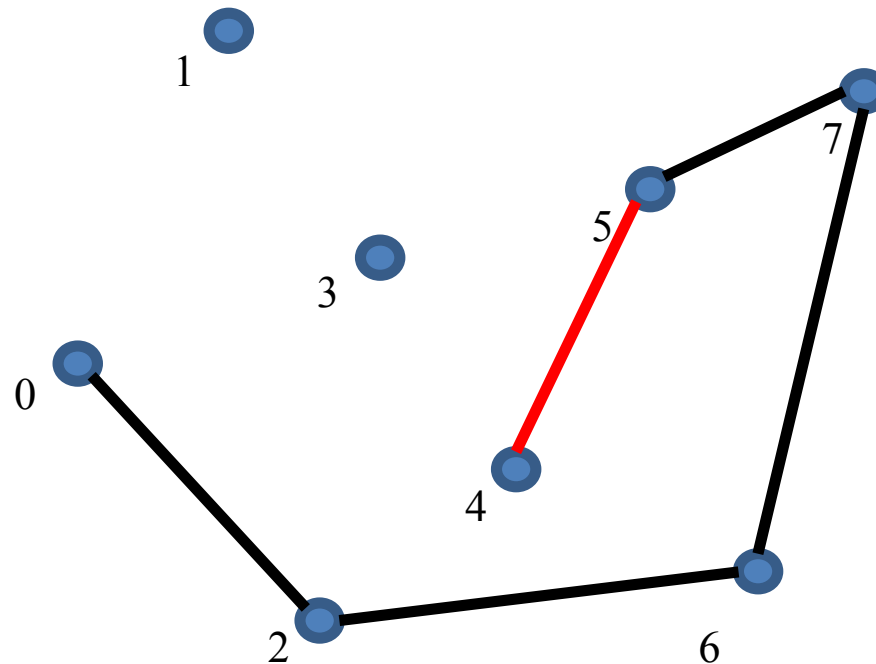
Andrew's Monotone Chain

- Step3 : Use cross product



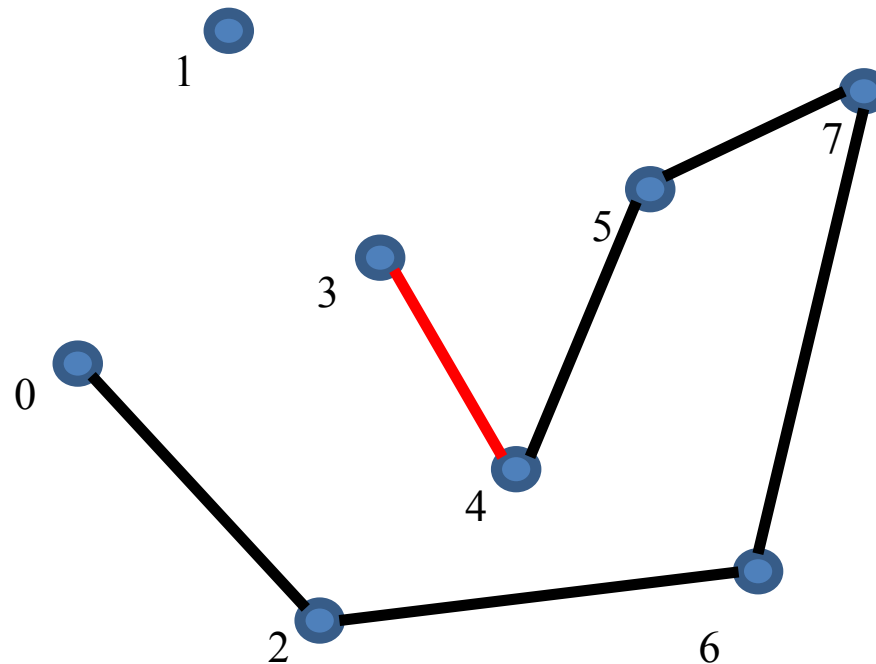
Andrew's Monotone Chain

- Step3 : Use cross product



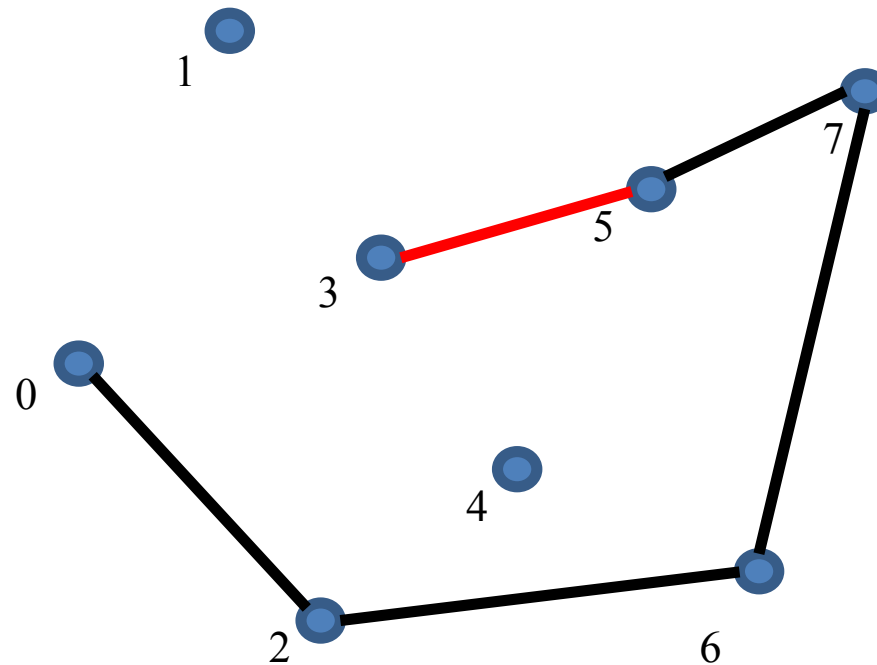
Andrew's Monotone Chain

- Step3 : Use cross product



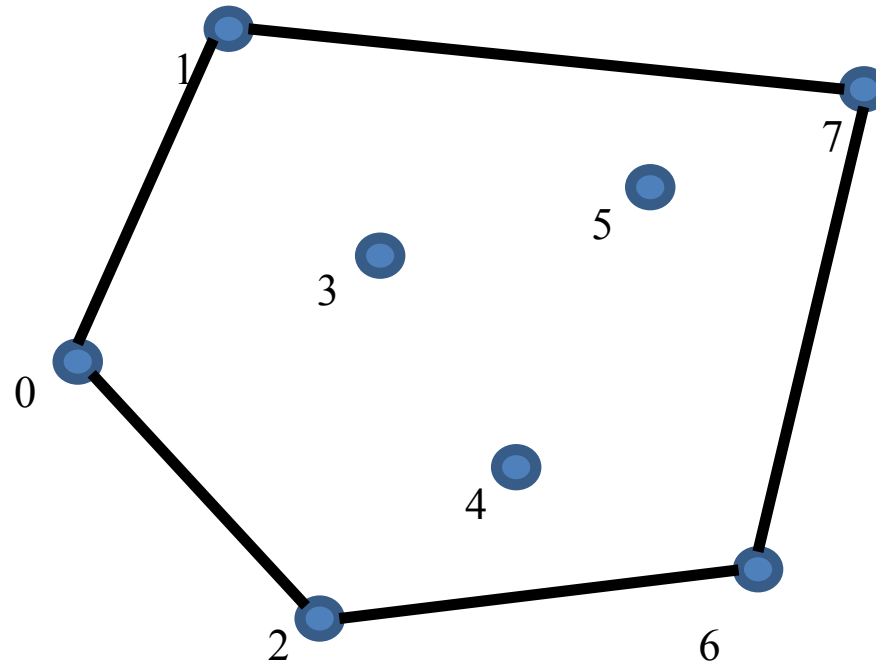
Andrew's Monotone Chain

- Step3 : Use cross product



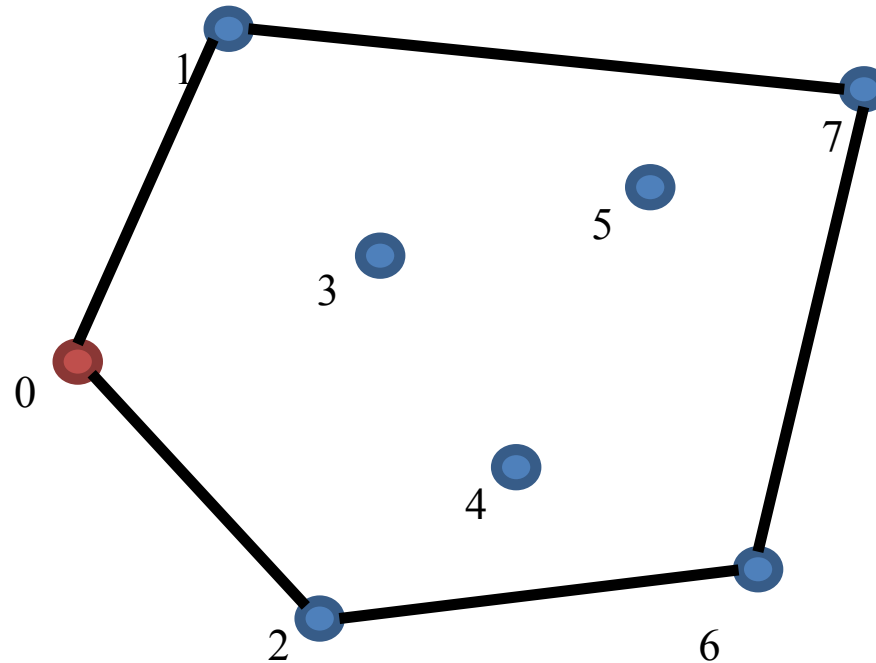
Andrew's Monotone Chain

- Step3 : Use cross product



Andrew's Monotone Chain

- Step4 : Delete starting point



Practice - 1

UVa 218 - Moth Eradication

