NCKU Programming Contest Training Course
Strong Connected Component(SCC)
2018/04/25

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made by
tommy5198 & free999 &
electorn & petermouse

Outline

- Articulation/Bridge
  (in undirected graph)

- Strongly Connected Component(SCC)
  (in directed graph)

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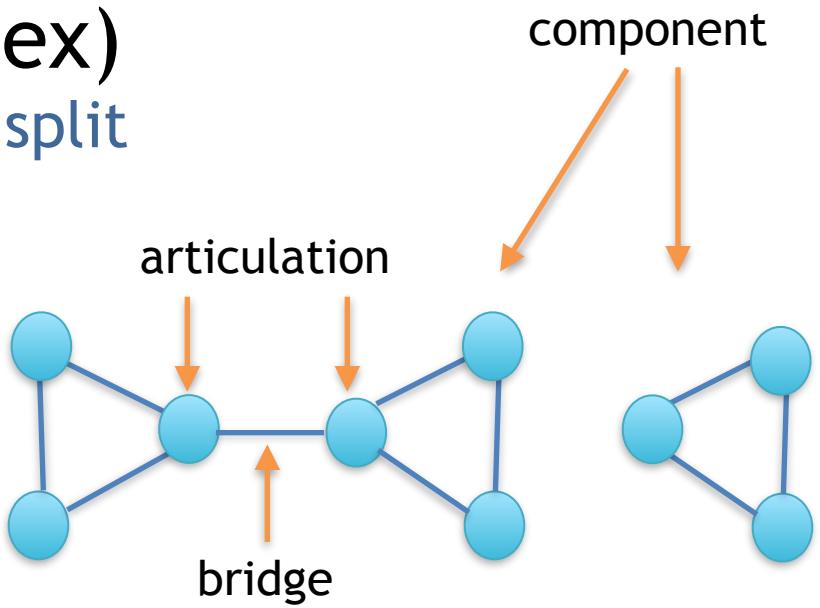
& petermouse

Graph

- connected graph/component
  connected graph/component iff all pairwise vertices exist at least one path & no more vertices can be added
- articulation(cut-vertex)
  remove articulation vertex split one component to two
- bridge(cut-edge)
  same as articulation



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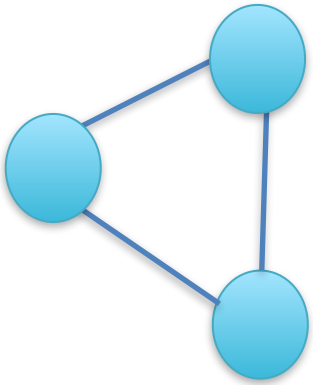
Articulation/Bridge

- Find Articulation in Graph
  - Graph become non-connected if remove a Articulation.
    V times DFS = O(V*(V+E)) —> too slow!

  - Vertex is not Articulation if can find alternative path
    —> find cycle!

  - Use DFS —> O(V+E)



Articulation/Bridge

- Concept
  - DFS traversel will construct a DFS tree
  - if vertex u's children can't back to u's ancestors
    —> u is Articulation
  - if vertex u is root and has at least 2 child
    —> u is Articulation

- Bridge?
  - two Articulation u, v have an edge —> (u, v) is Bridge!

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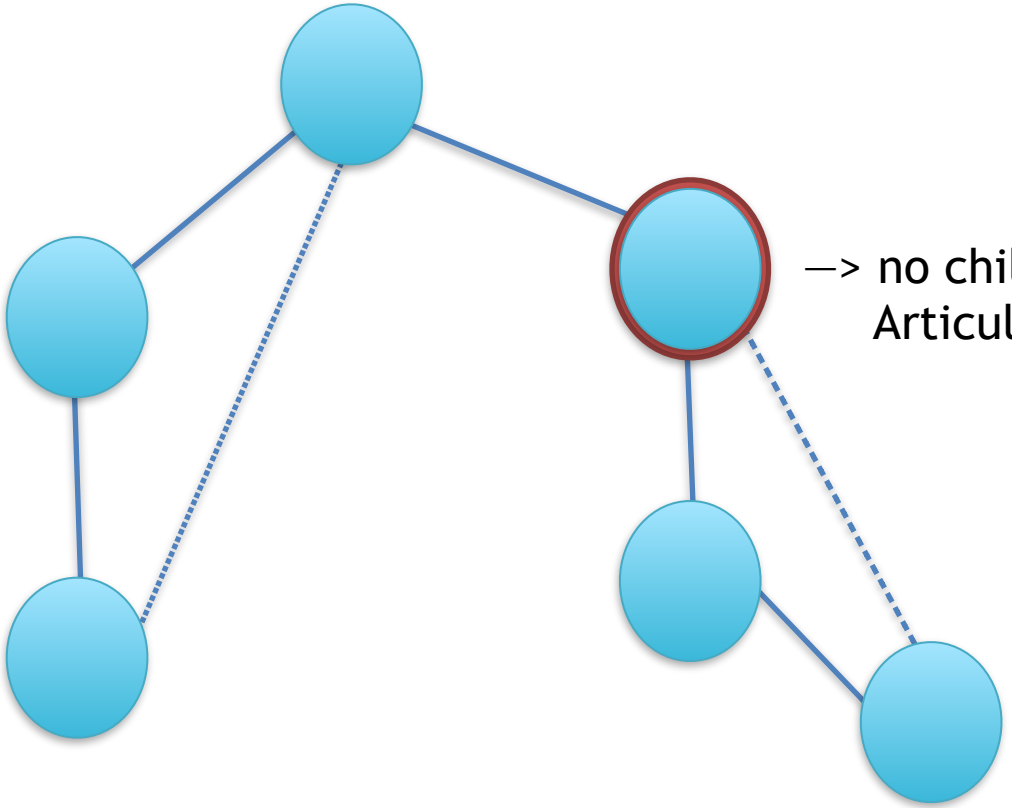
Articulation/Bridge



—> no child can back to ancestor.
Articulation!

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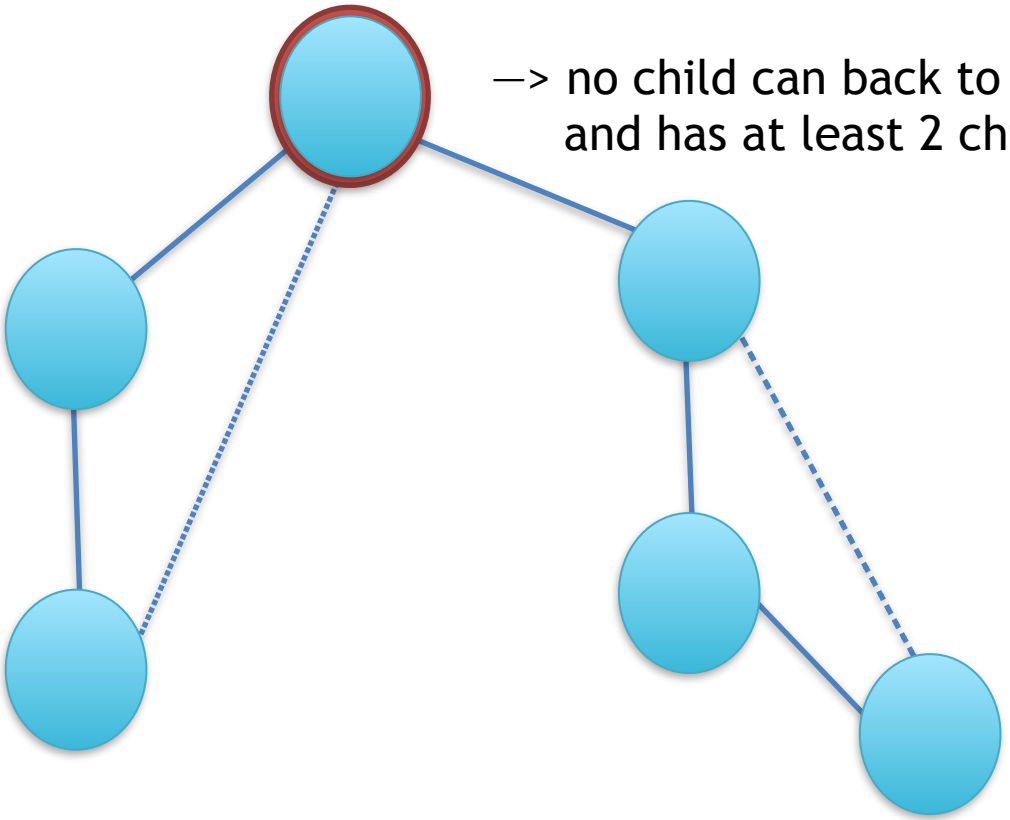
Articulation/Bridge



—> no child can back to ancestor
and has at least 2 child. Articulation!

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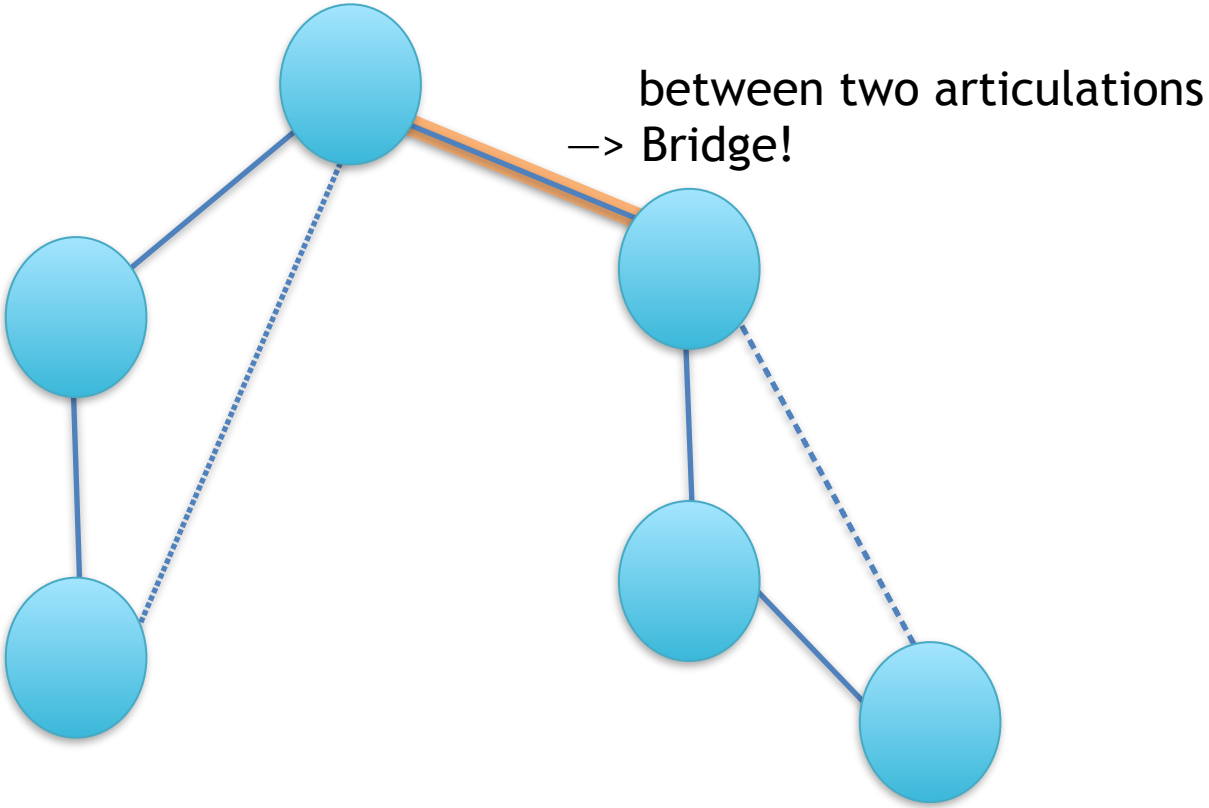
Articulation/Bridge



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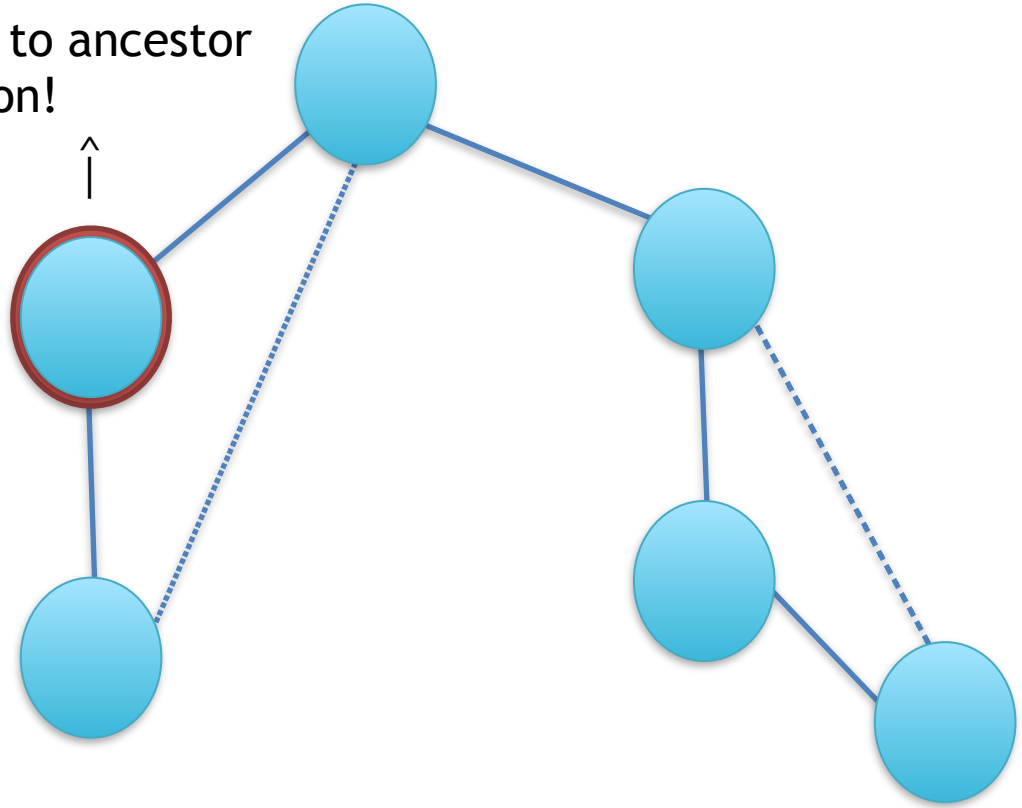
Articulation/Bridge

child can back to ancestor
NOT Articulation!



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Articulation/Bridge

- dfn[u] = DFS traversal order
  - first visit time each vertex u in DFS

- low[u] = min(dfn[u], lowest low[v])
  - if edge (u,v) exist and v is not u's parent

Articulation/Bridge

- Articulation
  - if vertex u's children can't back to u's ancestors
  —> dfn[u] <= low[v], v is u's child

  - if vertex u is root and has at least 2 child
  —> count child >= 2
- Bridge?
  - two Articulation u, v —> dfn[u] < low[v], v is u's child

made by tommy5198

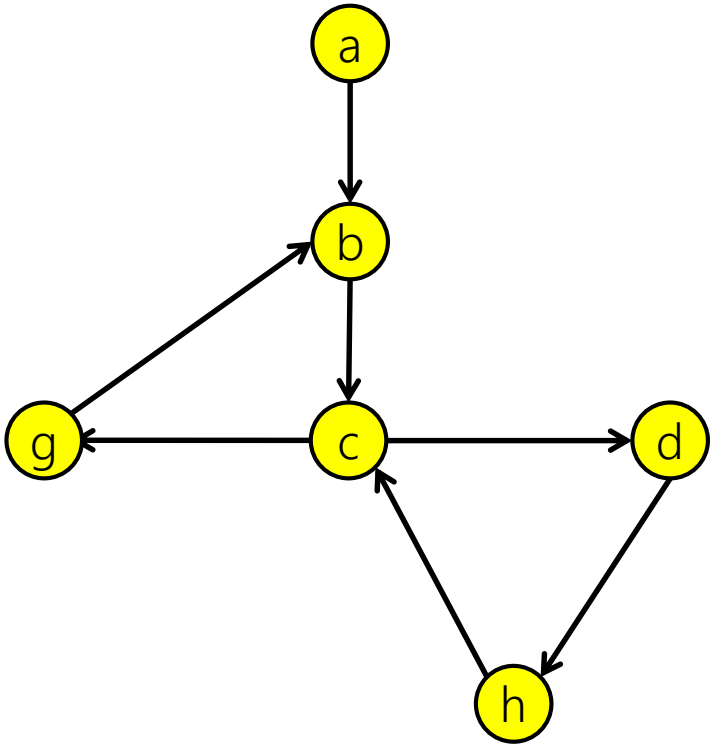
Cut vertex

- code

```
23  void DFS(int prev,int cur)
24  {
25      bool cut = false;
26      int child = 0;
27      low[cur] = dfn[cur] = ++dfsn;
28      for(int idx = adj_list[cur]; ~idx; idx = edge[idx].next) {
29          if(!dfn[edge[idx].to]) {
30              DFS(cur, edge[idx].to);
31              low[cur] = min(low[cur], low[edge[idx].to]);
32              if(low[edge[idx].to] >= dfn[cur])
33                  cut = true;
34              ++child;
35          } else if(edge[idx].to != prev)
36              low[cur] = min(low[cur], dfn[edge[idx].to]);
37      }
38      if((child > 1 || prev != -1) && cut)
39          ans++;
40  }
```

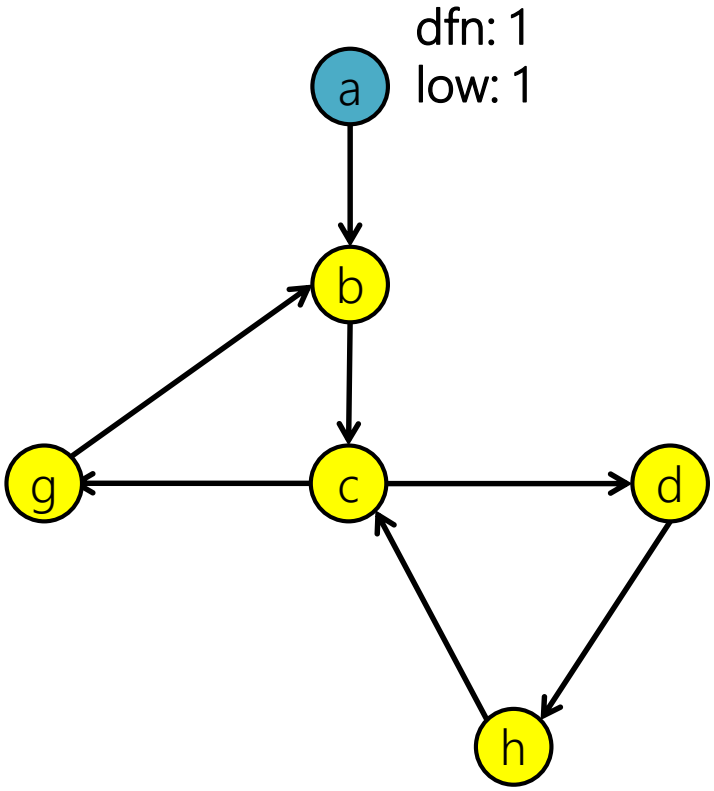

made by petermouse

Cut vertex

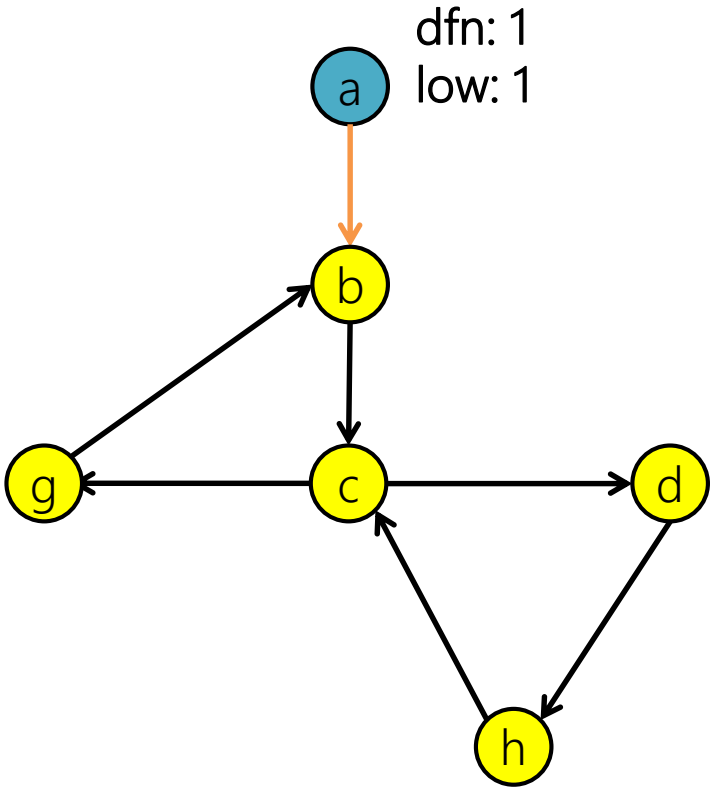


Note: This is an undirected graph

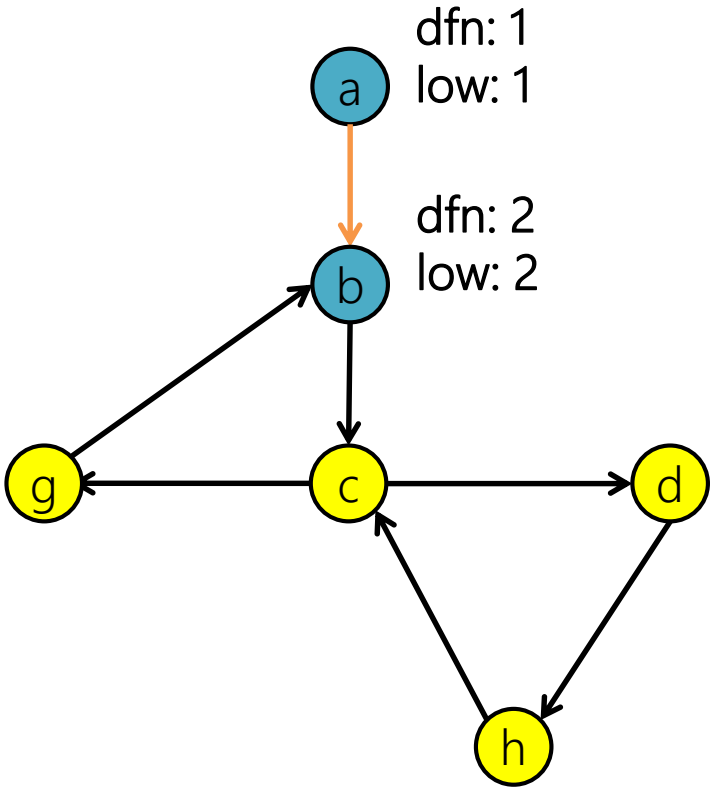
Cut vertex



Cut vertex

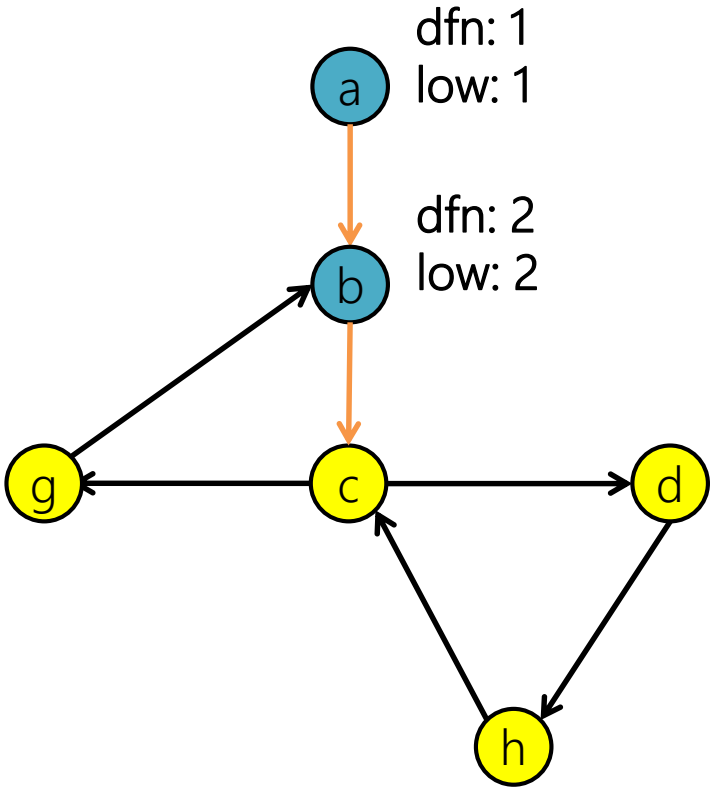


Cut vertex



made by riljian

Cut vertex



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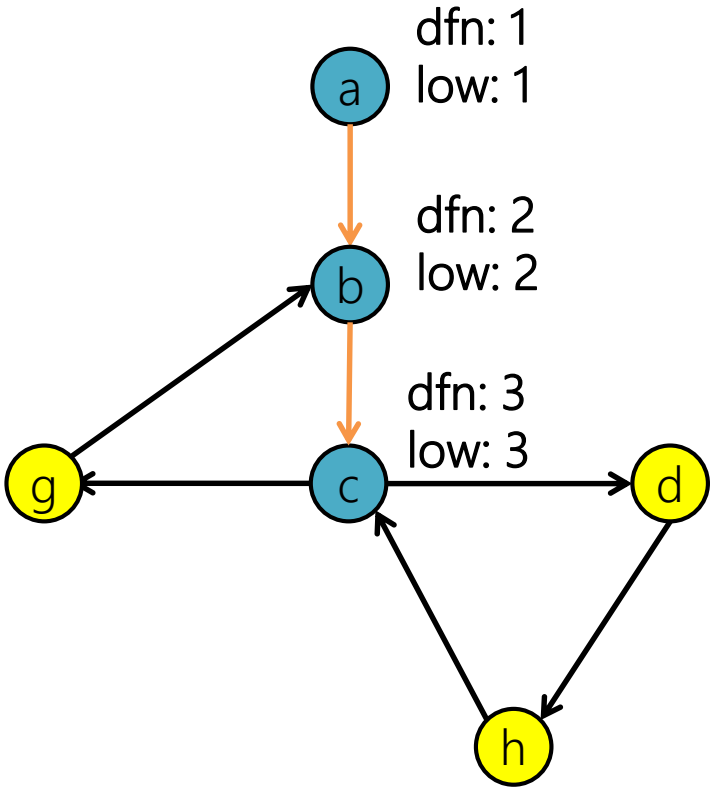
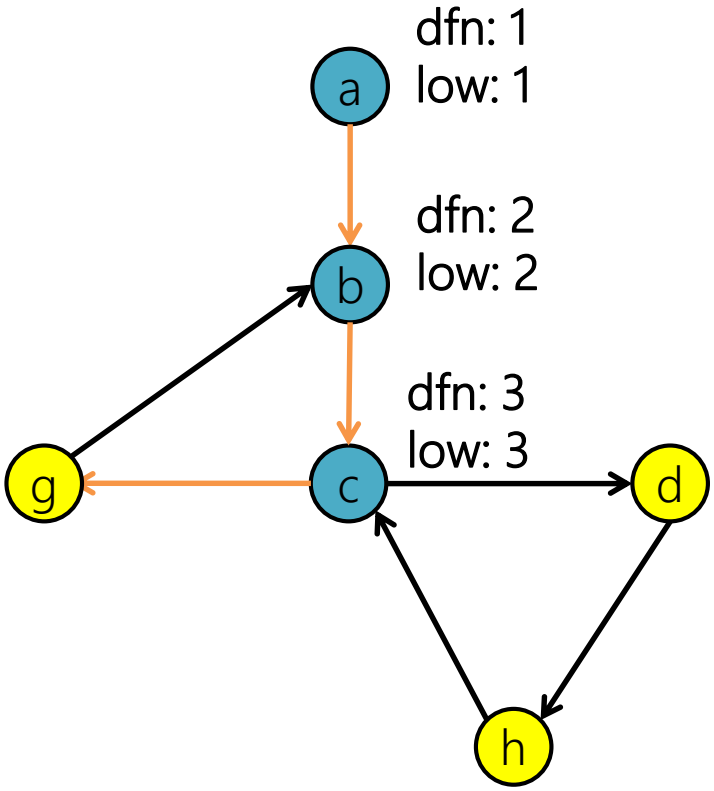
Cut vertex



Cut vertex



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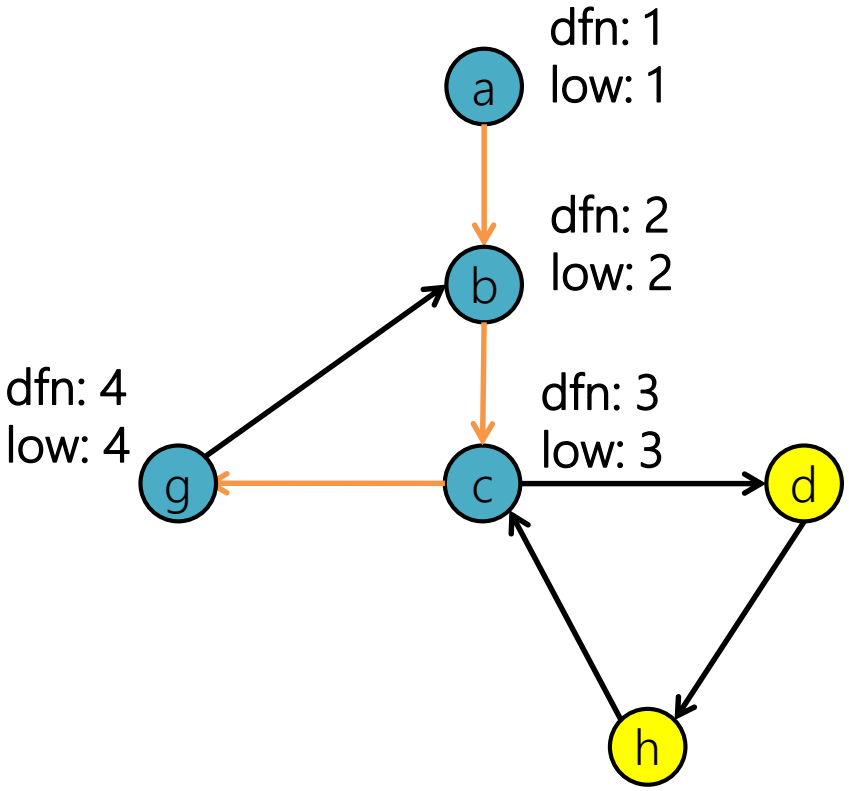
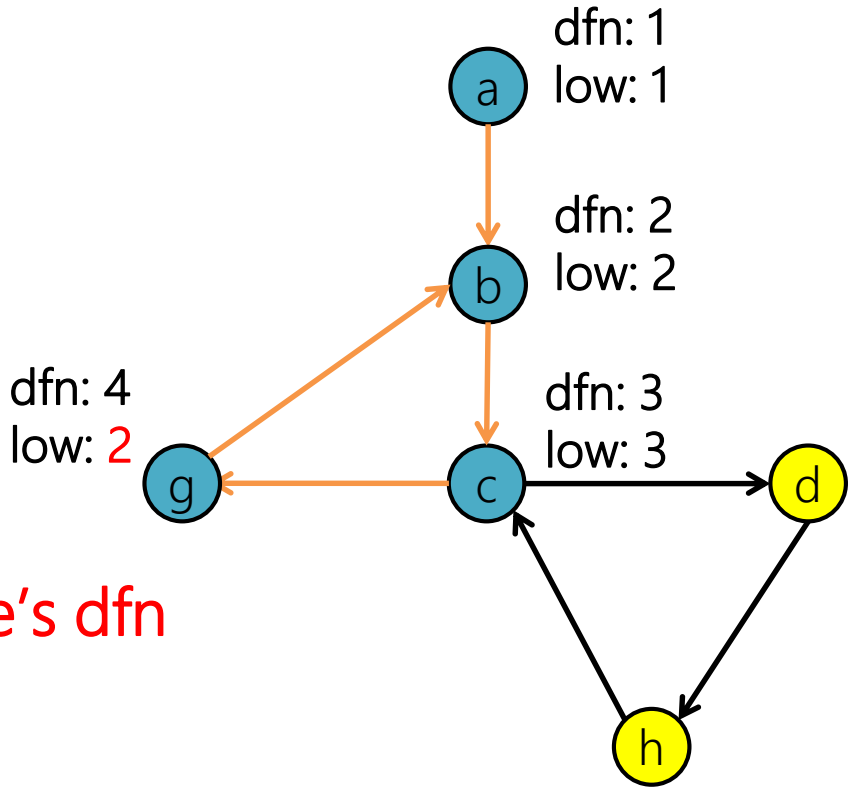
Cut vertex



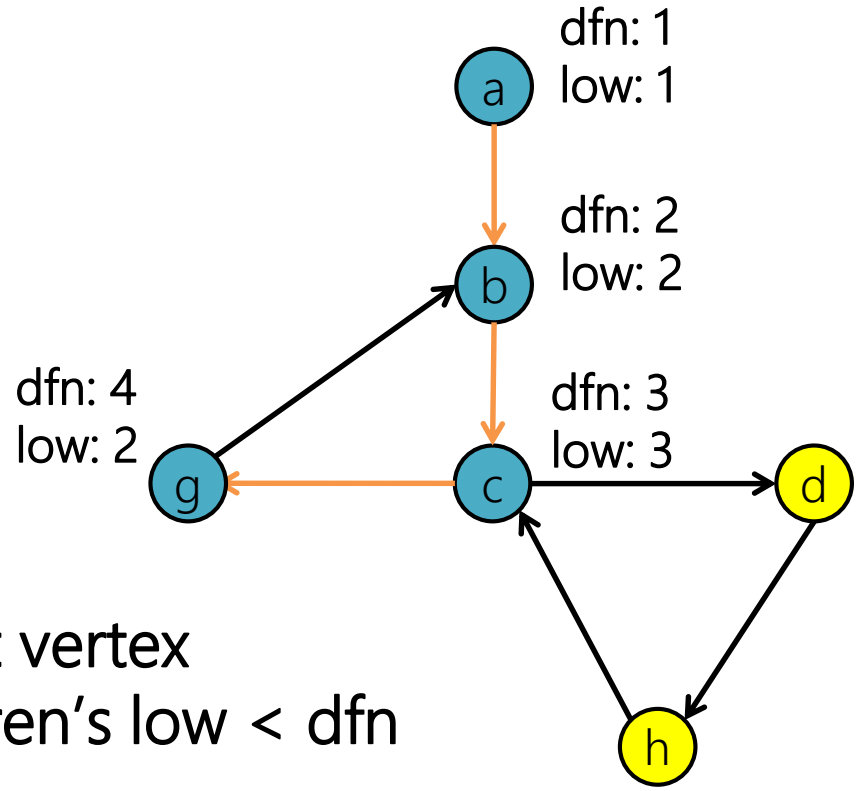
Cut vertex



child node's dfn

made by riljian

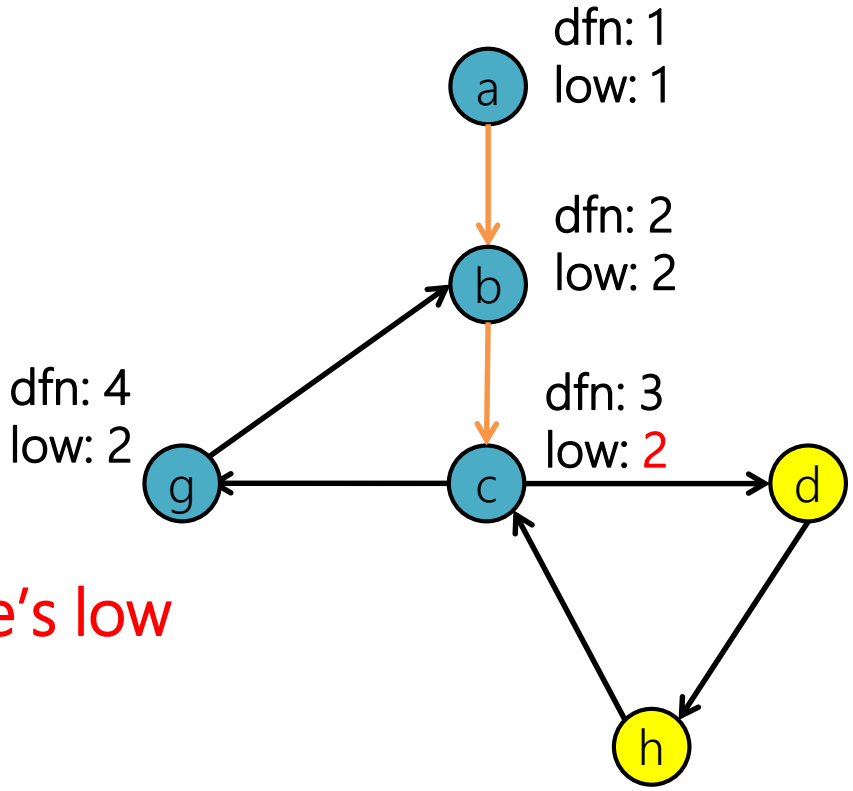
Cut vertex



g is not cut vertex
since children's low < dfn

made by riljian

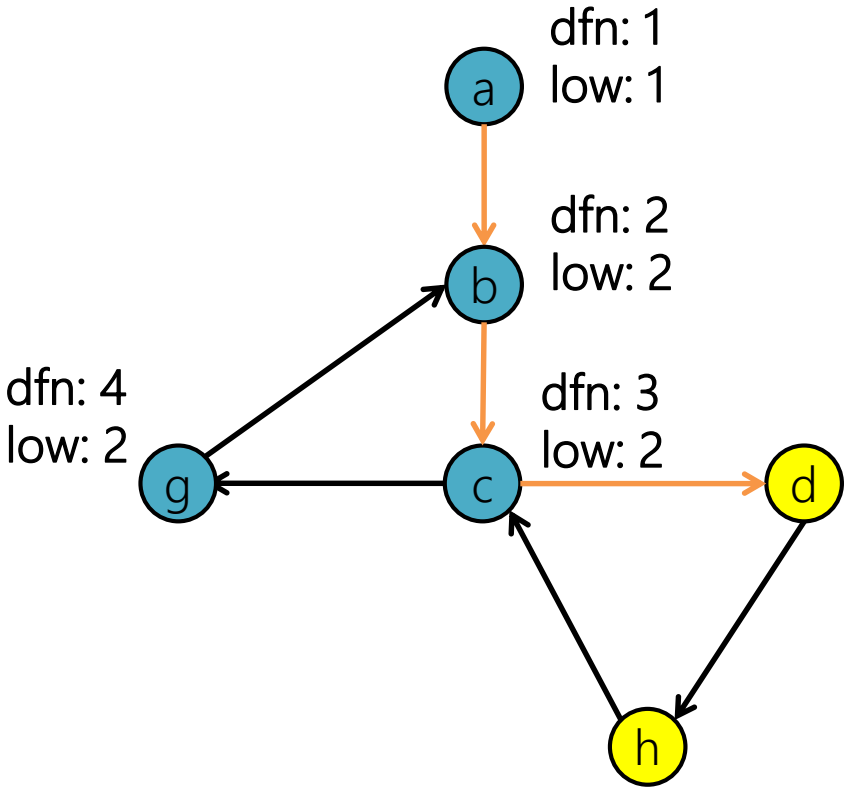
Cut vertex



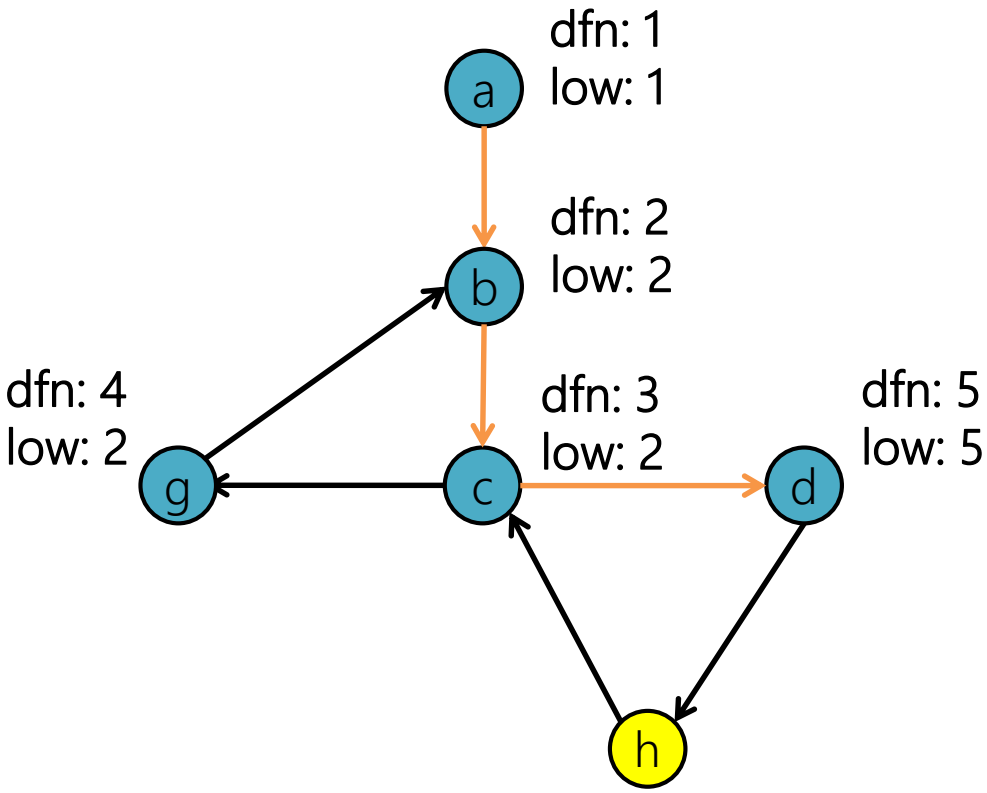
child node's low

made by riljian

Cut vertex

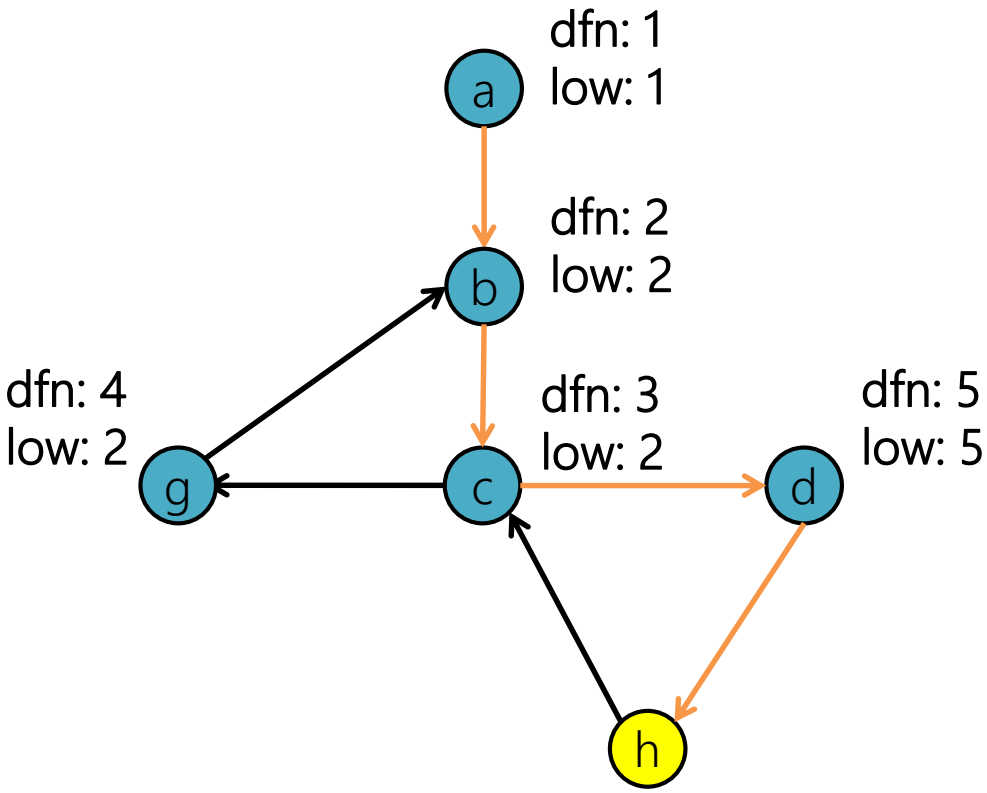


Cut vertex



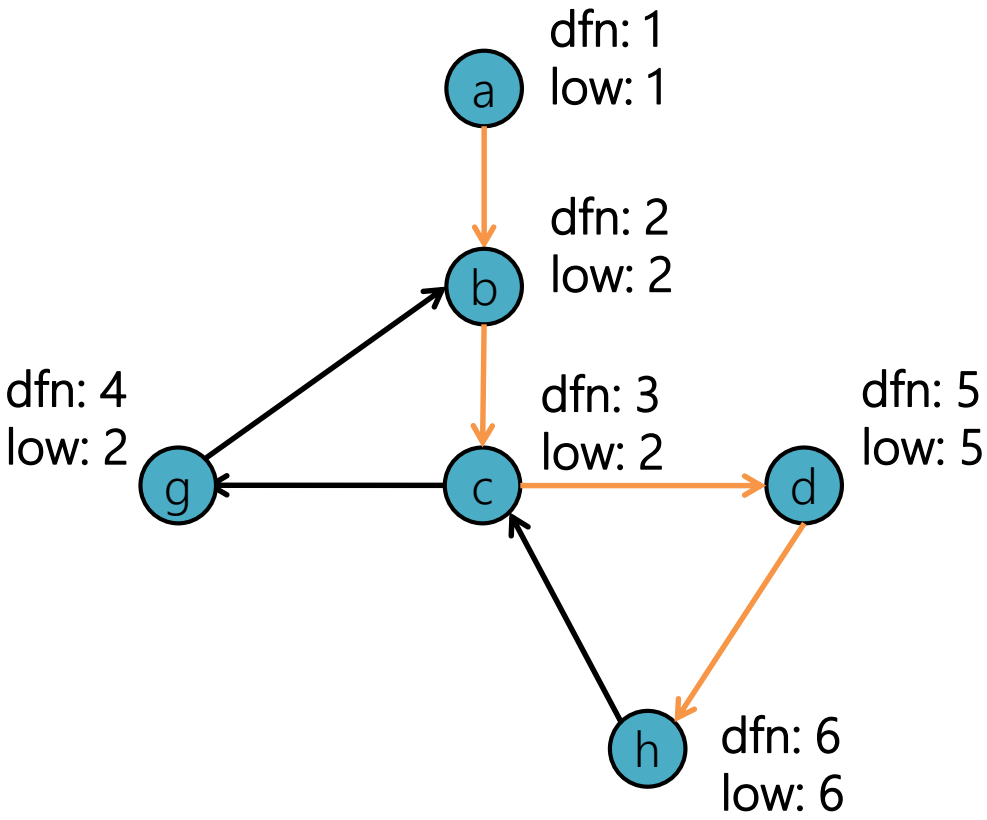
made by riljian

Cut vertex



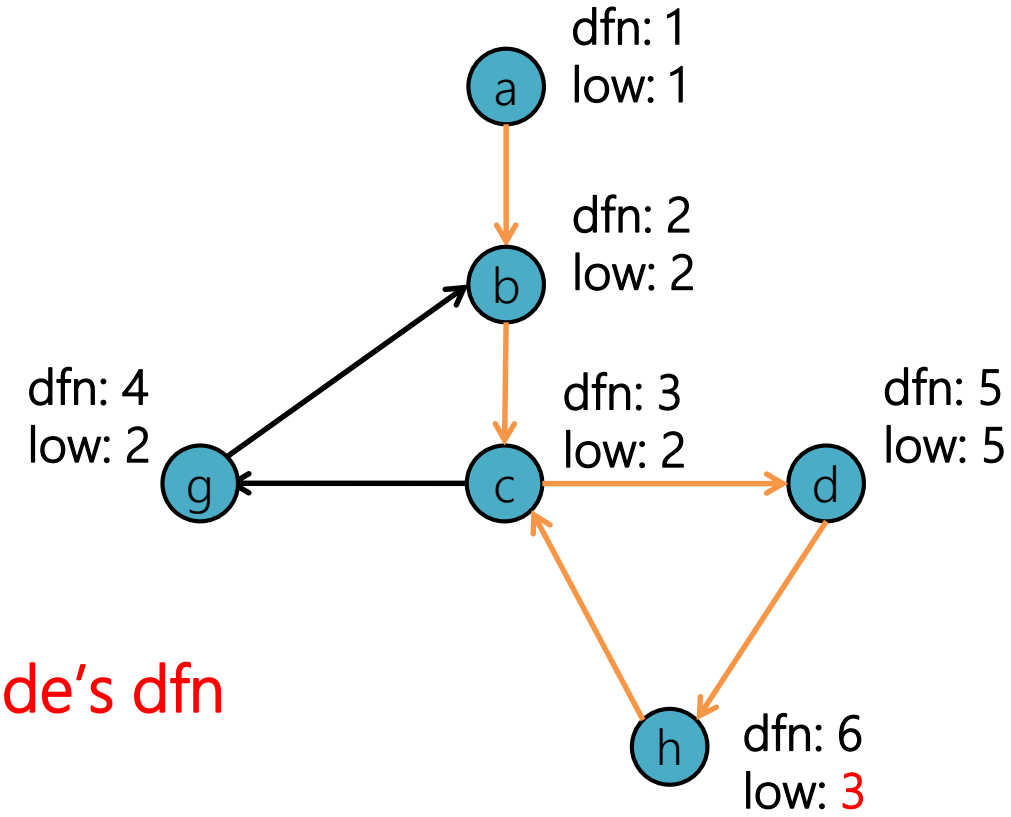
made by riljian

Cut vertex



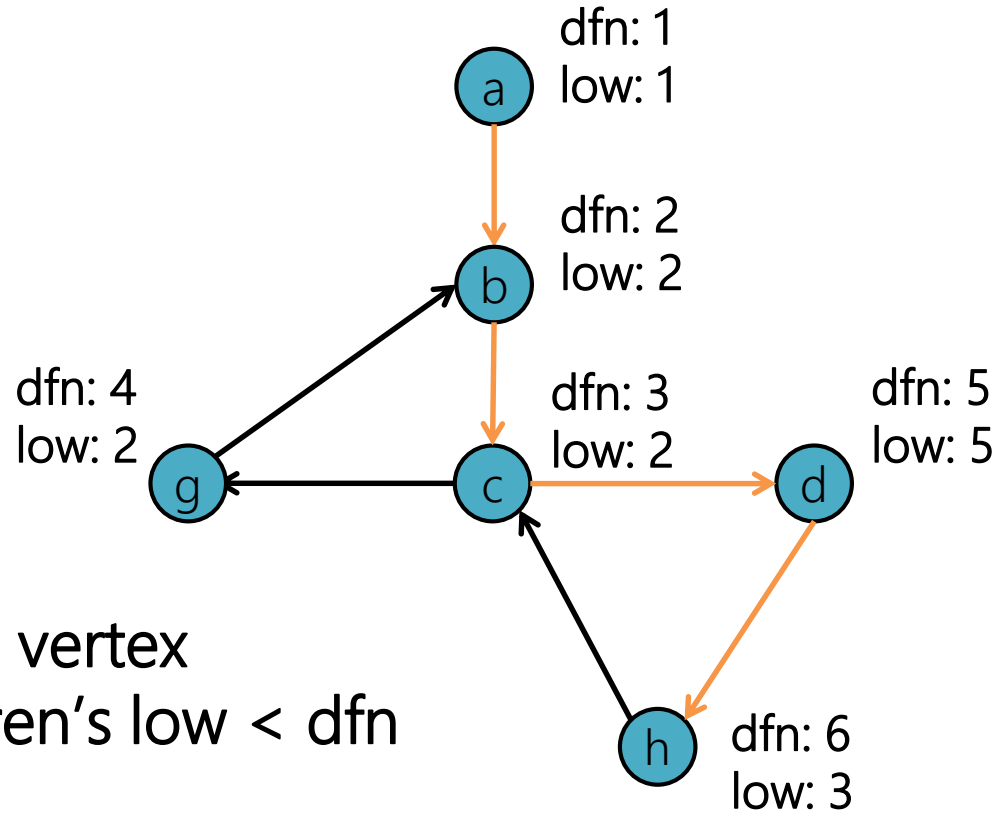
made by riljian

Cut vertex



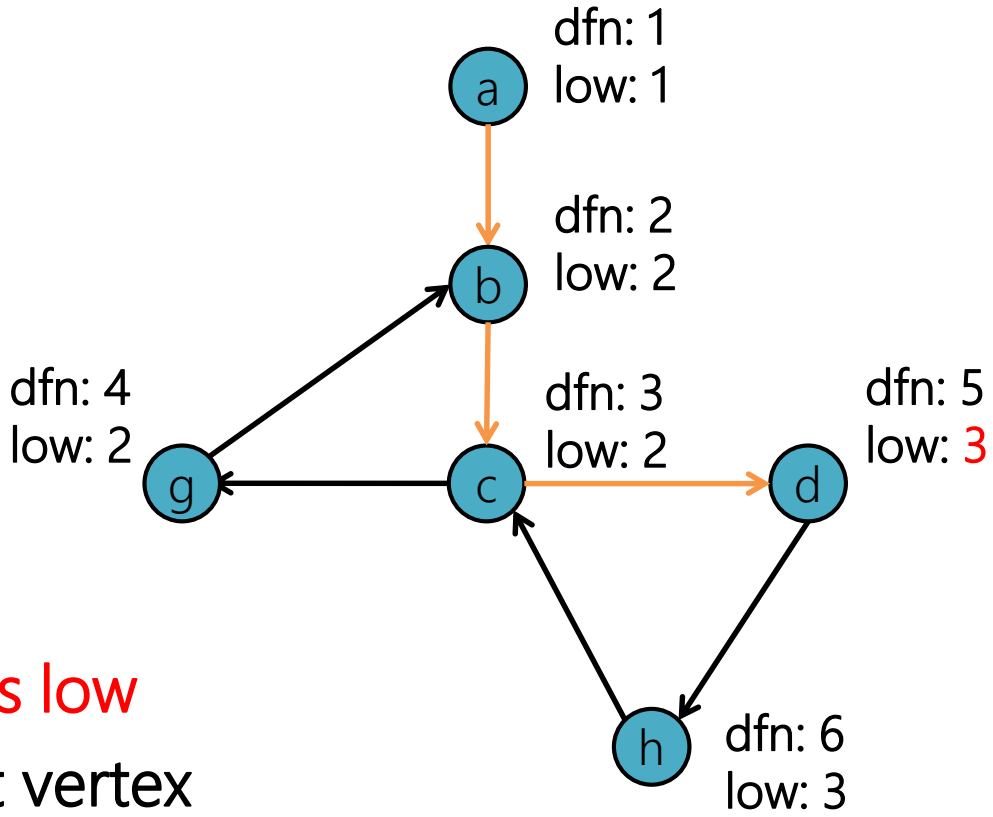
!!! child node's dfn

Cut vertex



h is not cut vertex
since children's low < dfn

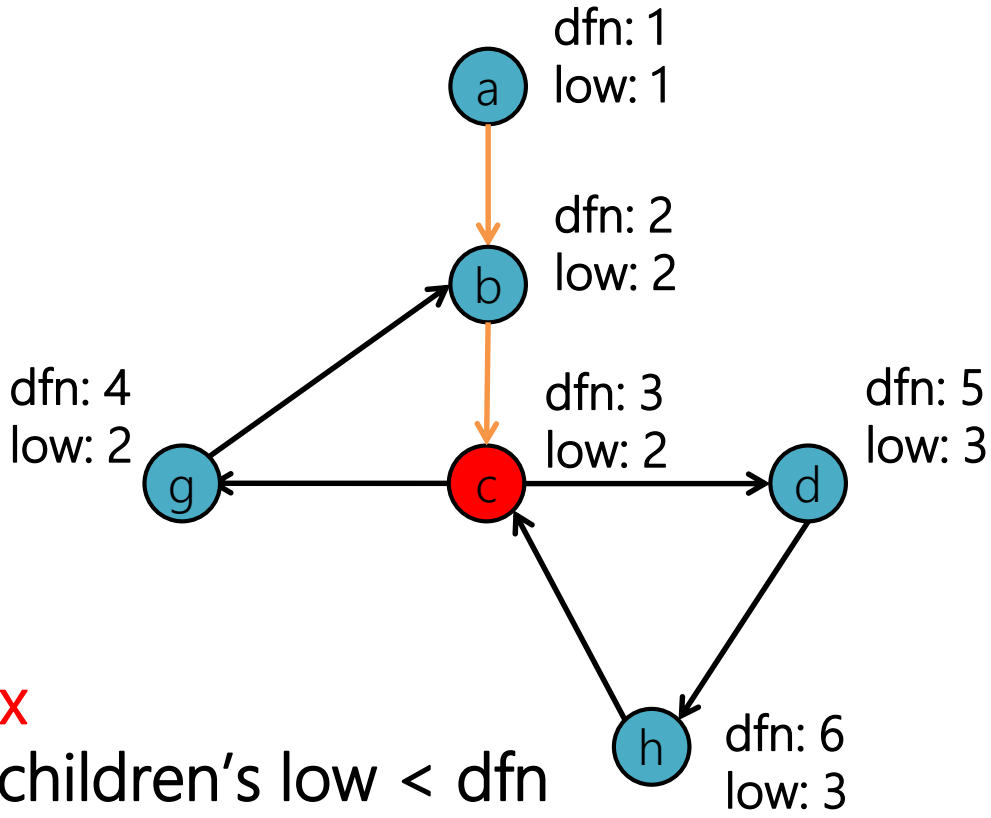
Cut vertex



child node's low

d is not cut vertex
since children's low < dfn

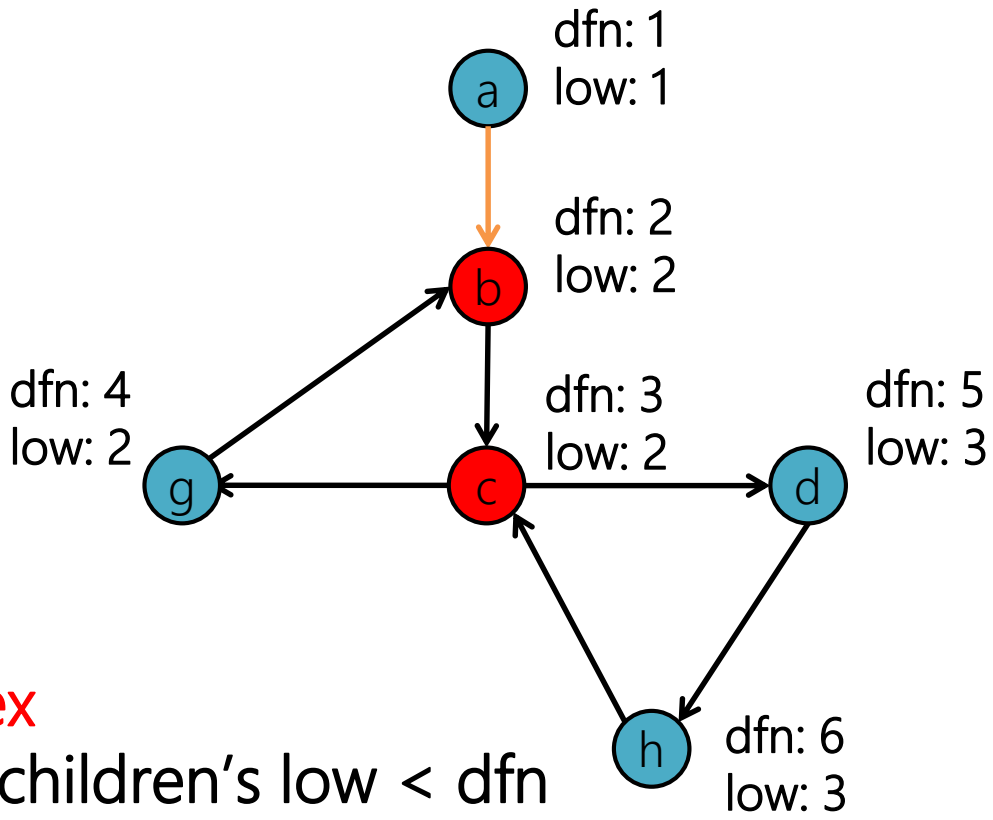
Cut vertex



c is cut vertex
since not all children's low < dfn

made by riljian

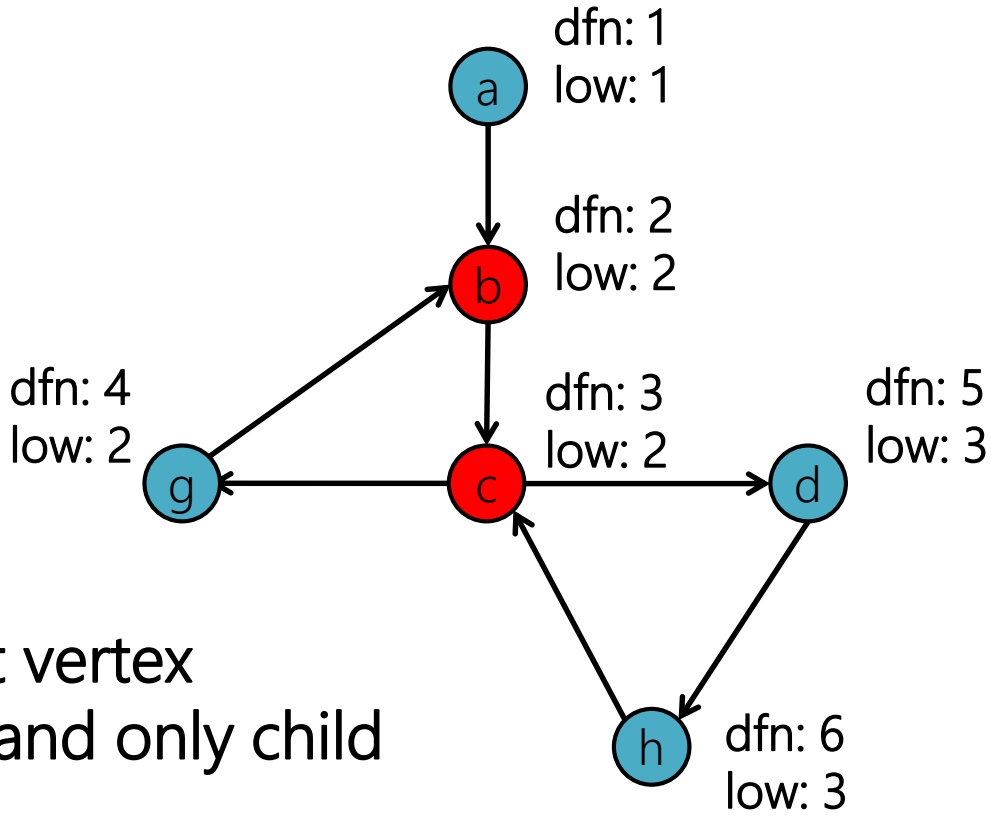
Cut vertex



d is cut vertex
since not all children's low < dfn

made by riljian

Cut vertex



a is not cut vertex
since root and only child

made by riljian

Practice

UVA - 315

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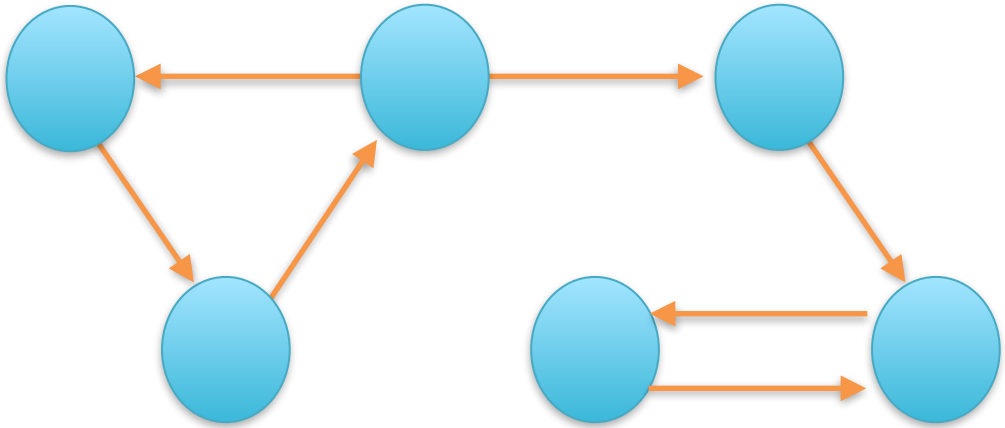
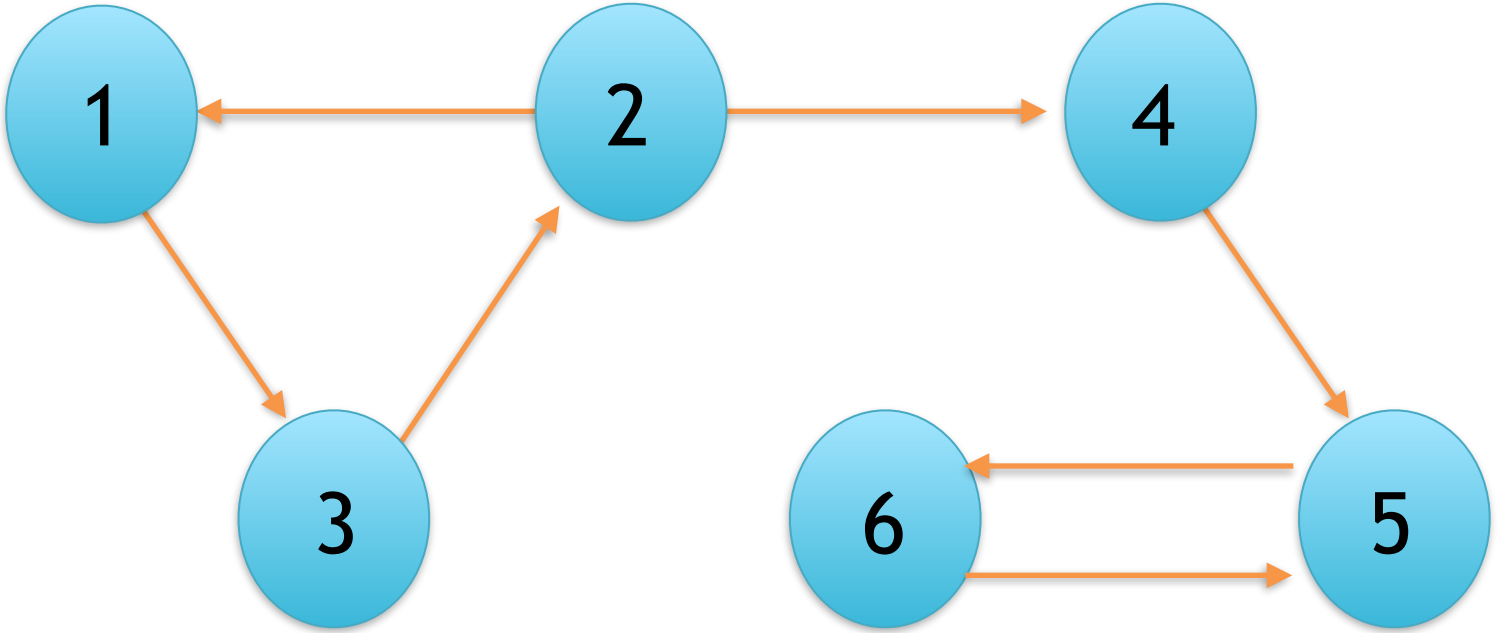
SCC

- connected component in directed graph
  - same definition in undirected graph



SCC



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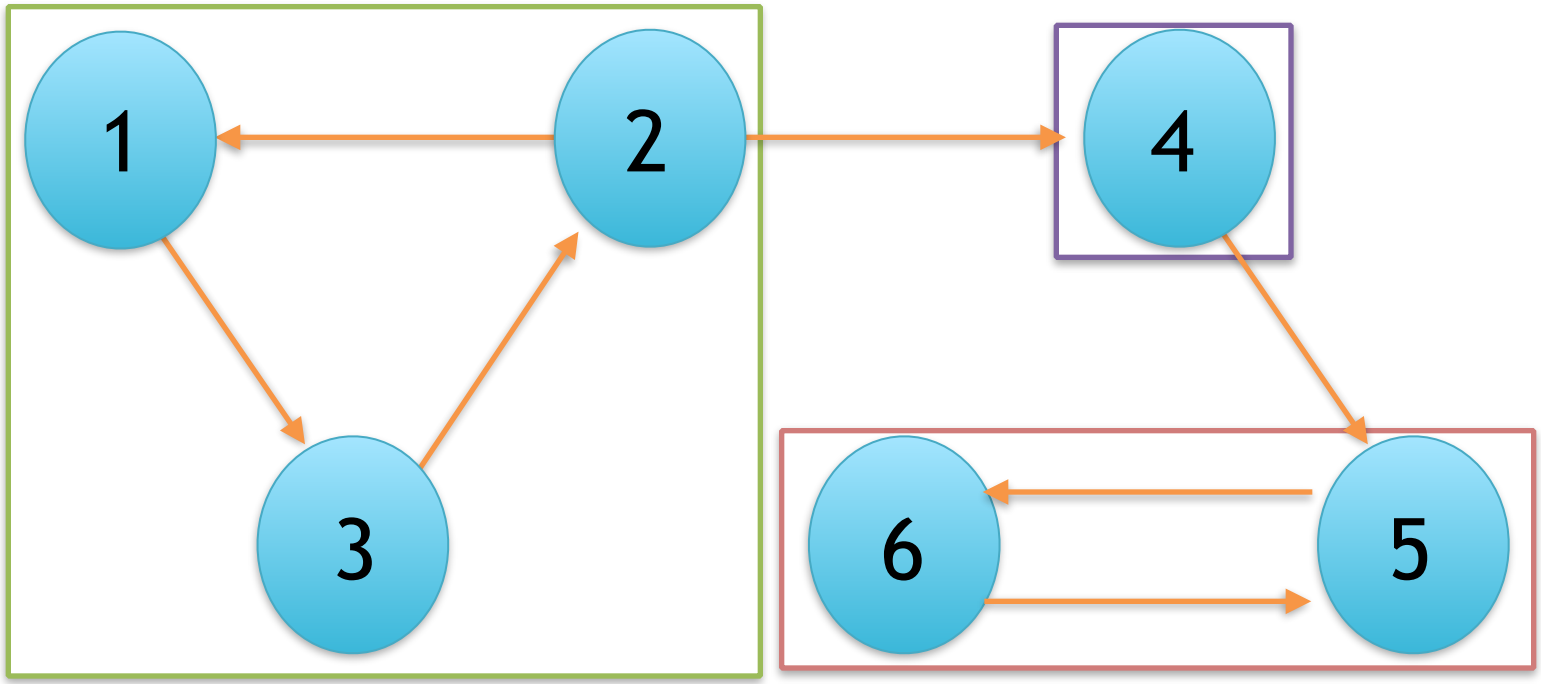
SCC



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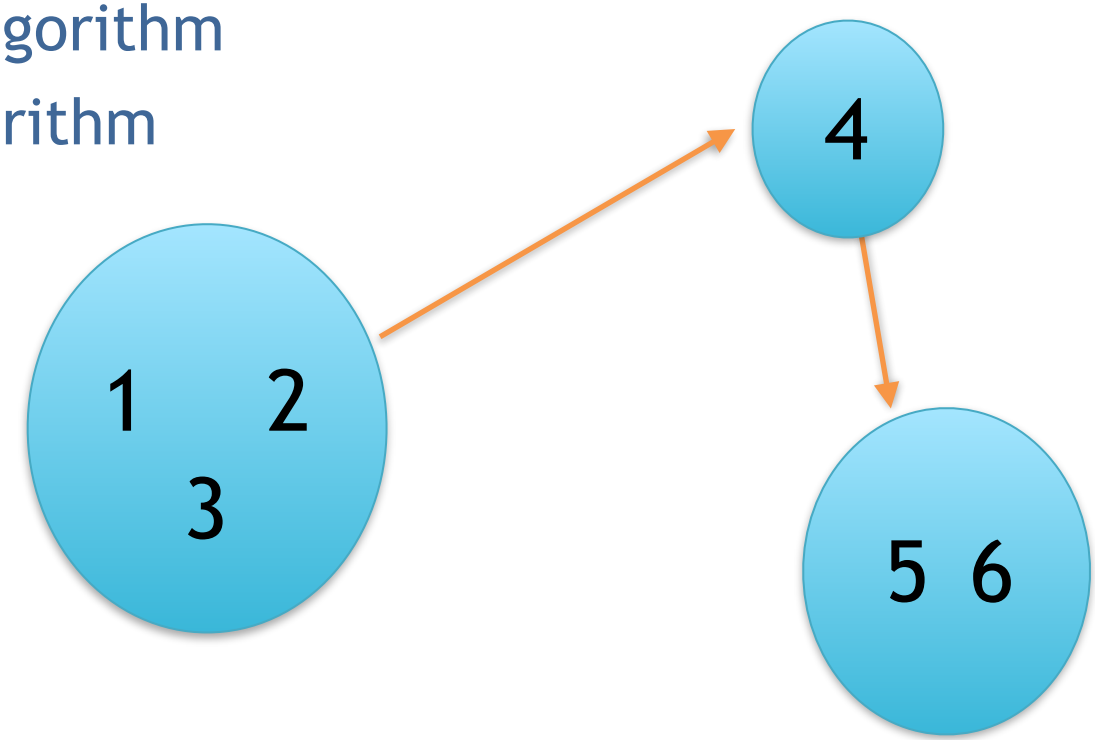
SCC

find all SCCs, contract all cycles —> DAG (directed acyclic graph)

- Kosaraju's Algorithm
- Tarjan's Algorithm



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SCC

- Kosaraju's algorithm

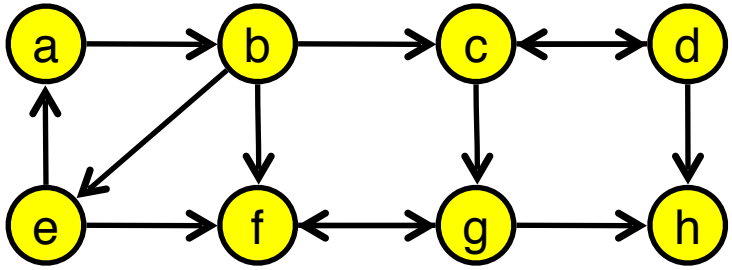
STRONGLY-CONNECTED-COMPONENTS(G)

1. Call DFS(G) to compute finishing time for each vertex.
2. Compute transpose of G i.e., G^T.
3. Call DFS(G^T) but this time consider the vertices in order of decreasing finish time.
4. Out the vertices of each tree in DFS-forest.

twice DFS —> total complexity: O(V+E)

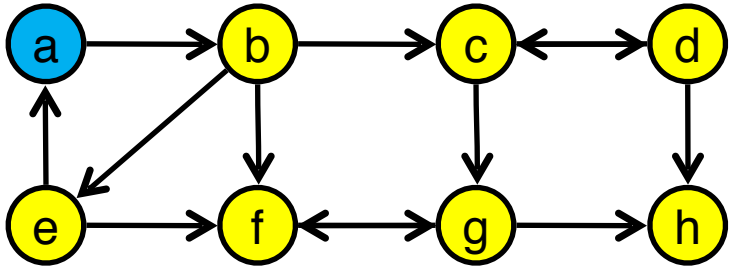
SCC

- Algorithm



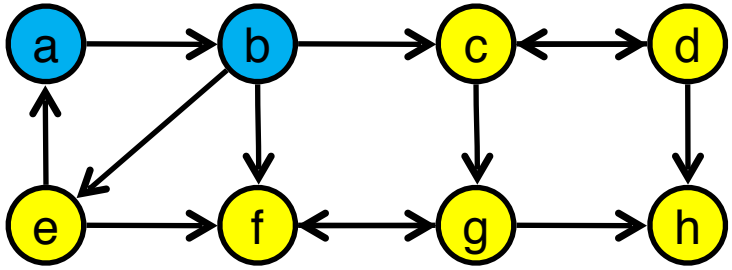
SCC

- Algorithm



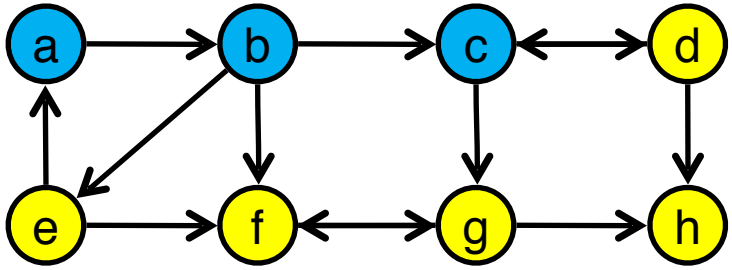
SCC

- Algorithm



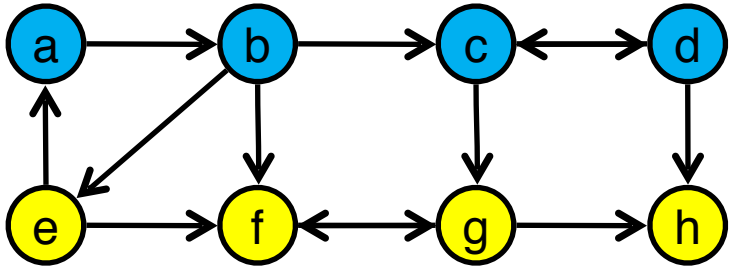
SCC

- Algorithm



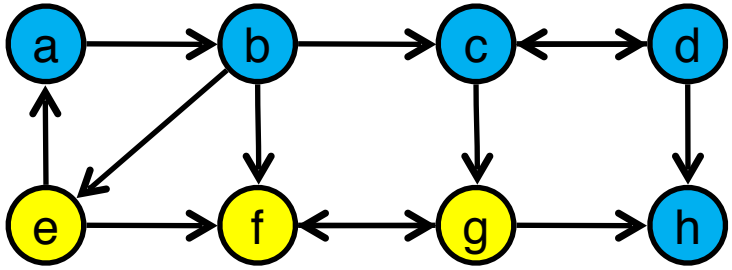
SCC

- Algorithm



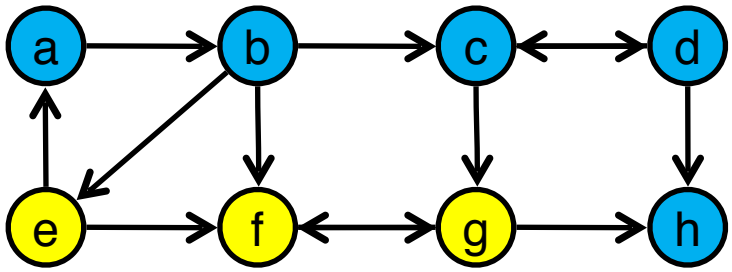
SCC

- Algorithm



SCC

- Algorithm

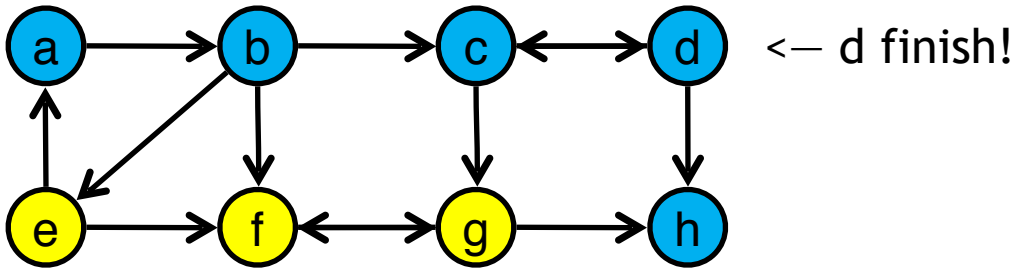


<— h finish!

h									

SCC

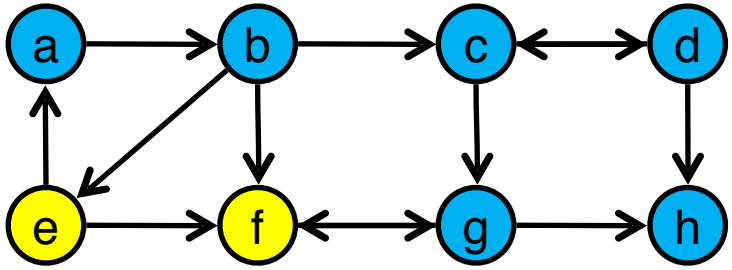
- Algorithm



h	d								

SCC

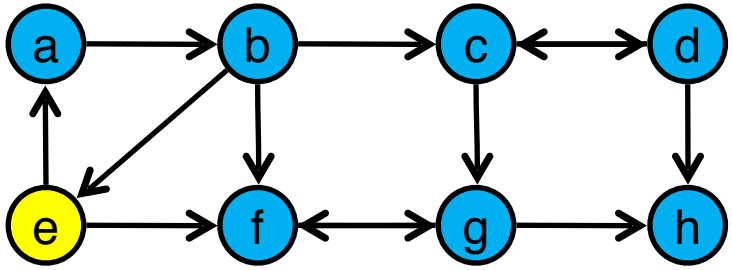
- Algorithm



h	d								

SCC

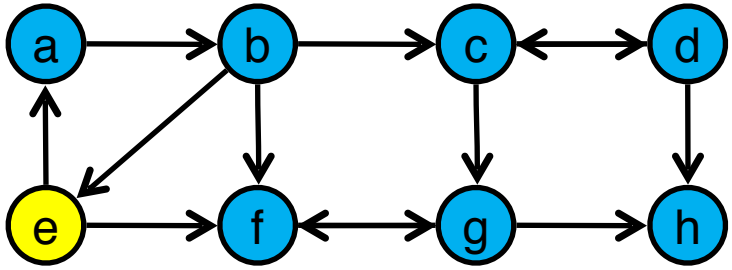
- Algorithm



h	d								

SCC

- Algorithm

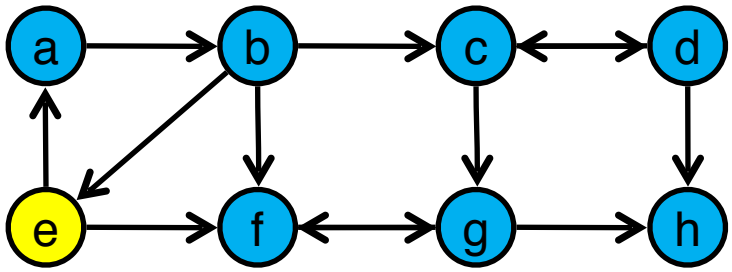


^ f finish!

h	d	f							

SCC

- Algorithm

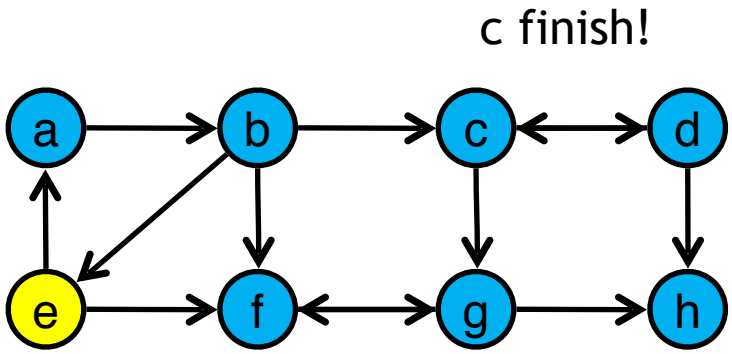


^ g finish!

h	d	f	g						

SCC

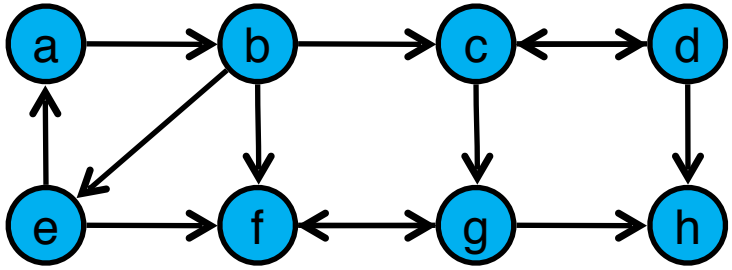
- Algorithm



h	d	f	g	c					

SCC

- Algorithm



h	d	f	g	c					

SCC

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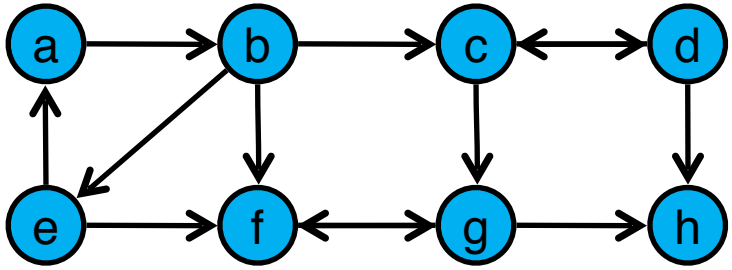


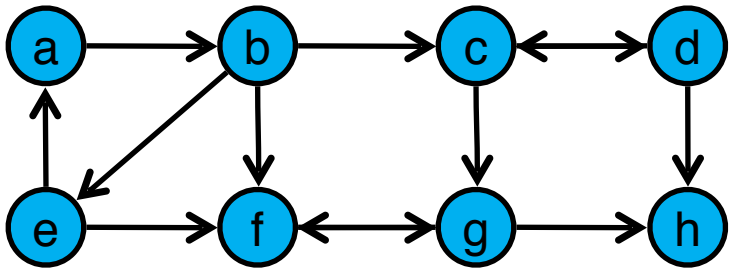
^ e finish!

h	d	f	g	c	e				

SCC

- Algorithm

b finish!

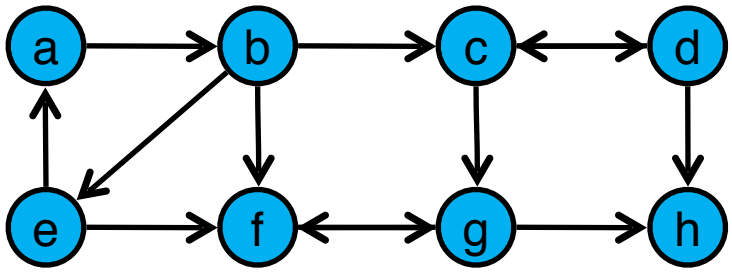


h	d	f	g	c	e	b			

SCC

- Algorithm

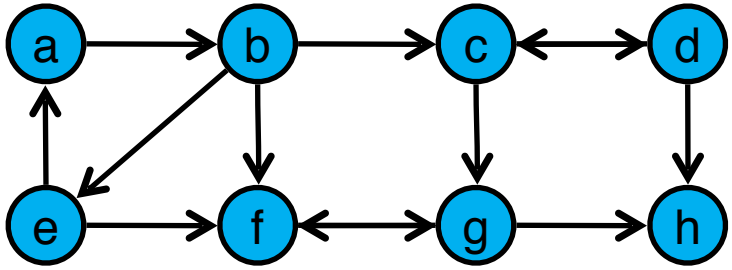
a finish!



h	d	f	g	c	e	b	a		

SCC

- Algorithm
  - Reverse the graph



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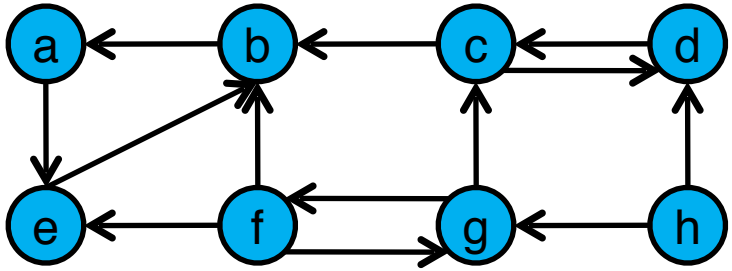
SCC

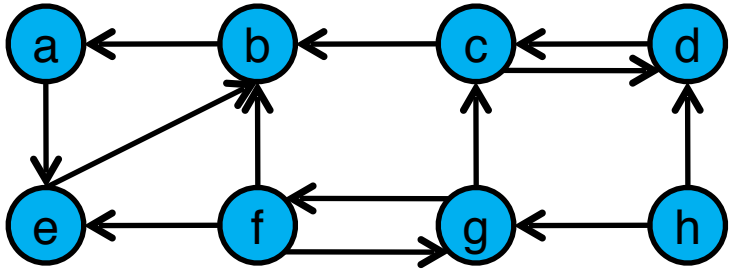
- Algorithm
  - Reverse the graph



h	d	f	g	c	e	b	a		

SCC

- Algorithm
  - Reverse the graph
  - Re-search by the ending time



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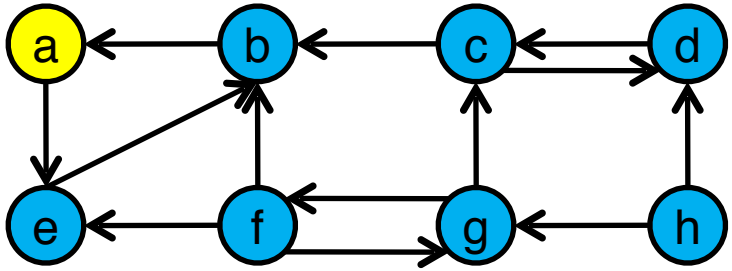
SCC

- Algorithm
  - Reverse the graph
  - Re-search by the ending time



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SCC

- Algorithm
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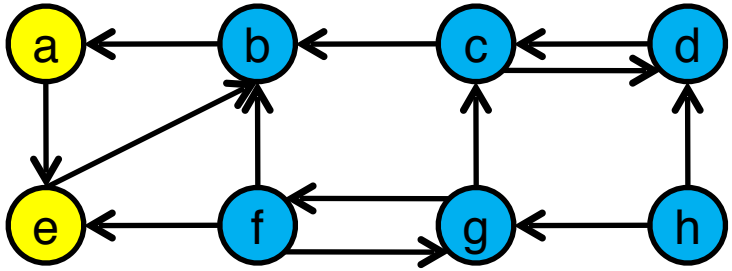

h	d	f	g	c	e	b	a		

SCC

- Algorithm
  - Reverse the graph
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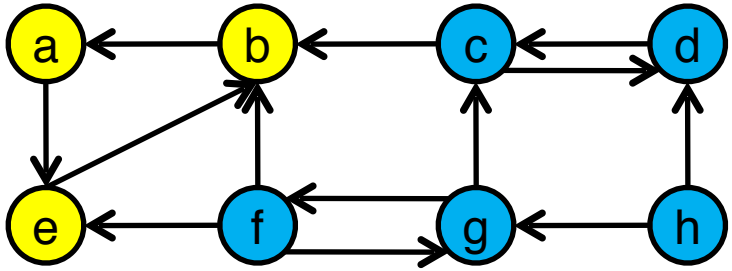

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SCC

- Algorithm
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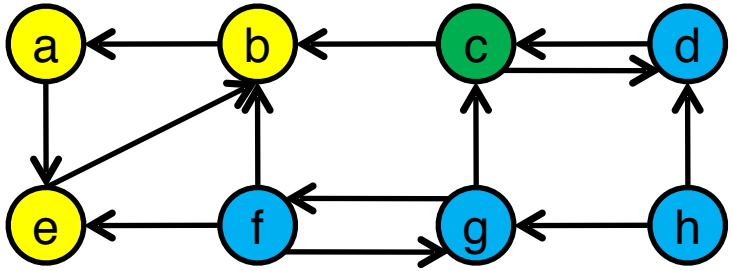

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- Algorithm
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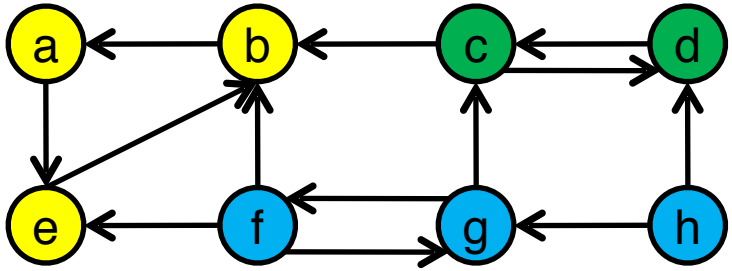

h	d	f	g	c	e	b	a		

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- Algorithm
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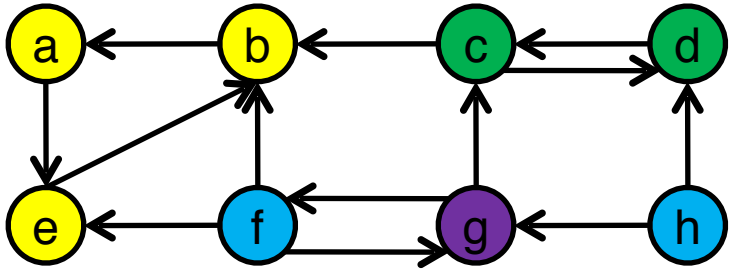

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- Algorithm
  - Reverse the graph
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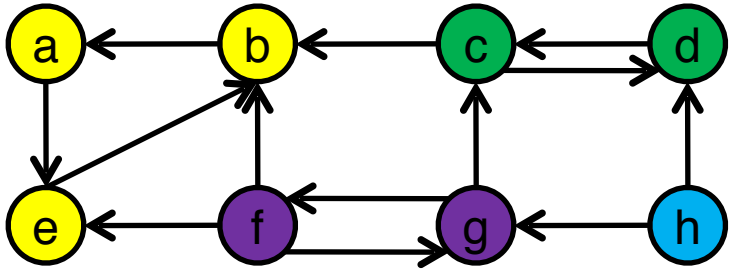

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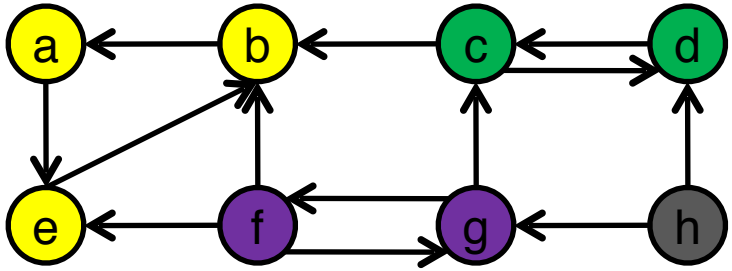
- Algorithm
  - Reverse the graph
  - Re-search by the ending time



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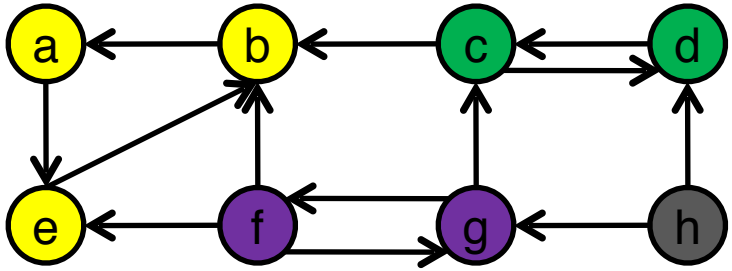
- Algorithm
  - Reverse the graph
  - Re-search by the ending time

4 components



SCC

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```
42 void DFS(int v) {
43     int top;
44     dfn[v] = low[v] = ++dfn_cnt;
45     stk.push(v);
46     in_stk[v] = true;
47     for (int idx = adj_list[v]; ~idx; idx = edge[idx].next) {
48         if (!dfn[edge[idx].to]) {
49             DFS(edge[idx].to);
50             low[v] = min(low[v], low[edge[idx].to]);
51         } else if (in_stk[edge[idx].to]) {
52             low[v] = min(low[v], dfn[edge[idx].to]);
53         }
54     }
55
56     if (dfn[v] == low[v]) {
57         do {
58             top = stk.top();
59             stk.pop();
60             in_stk[top] = false;
61         } while (top != v);
62         ++ans;
63     }
64 }
```


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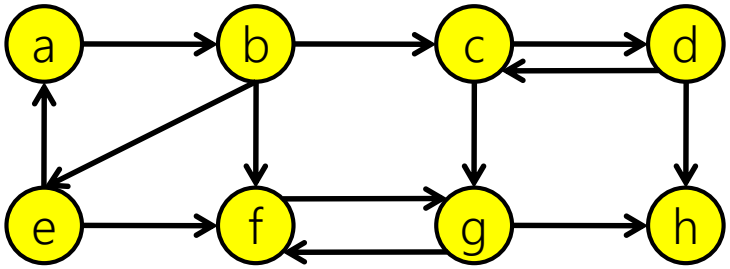

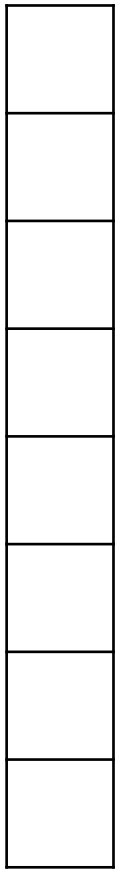

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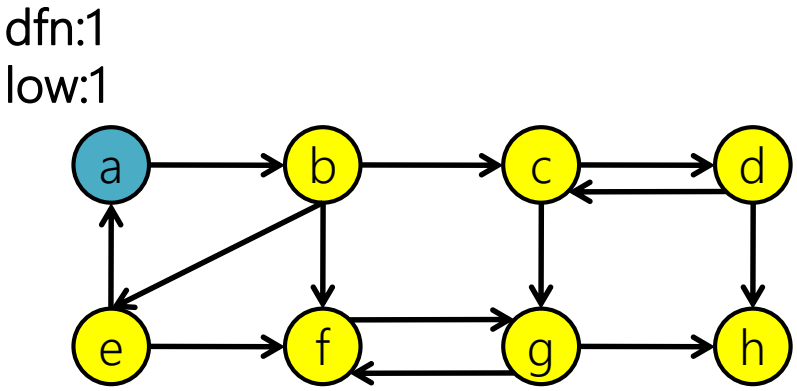

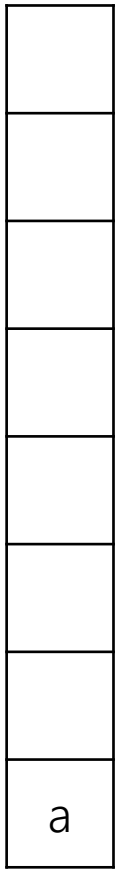

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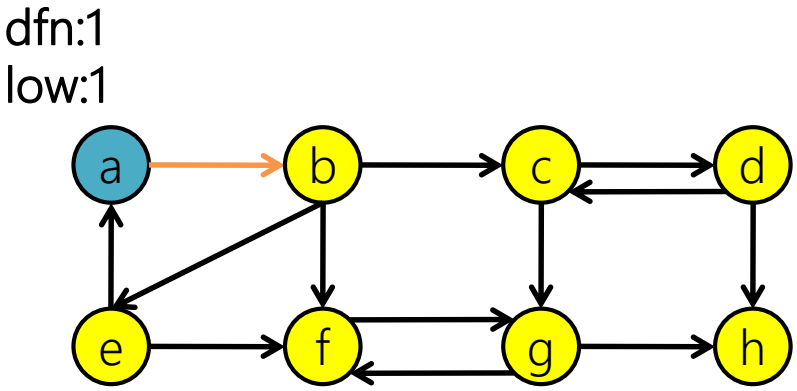

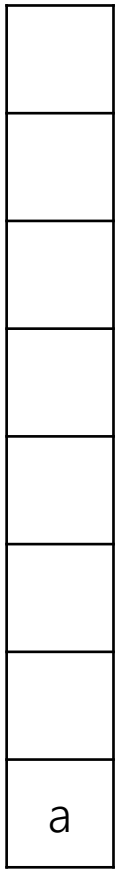

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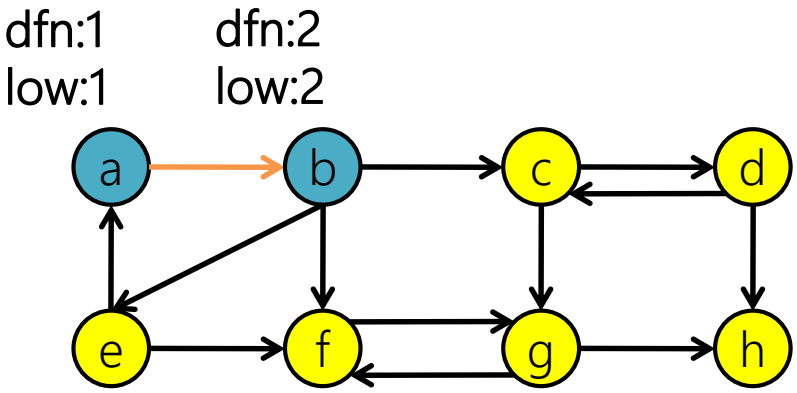

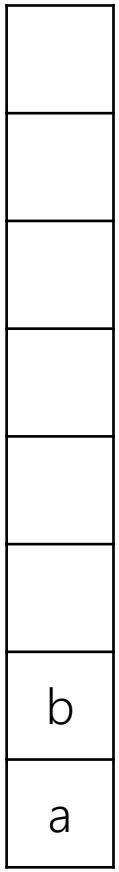

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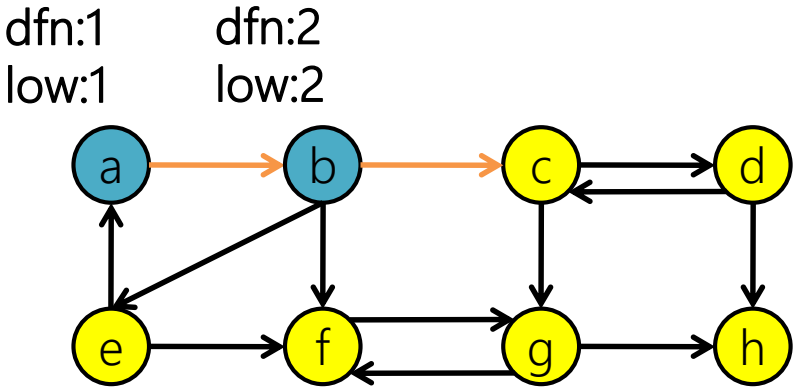

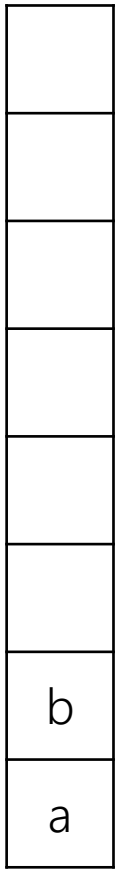

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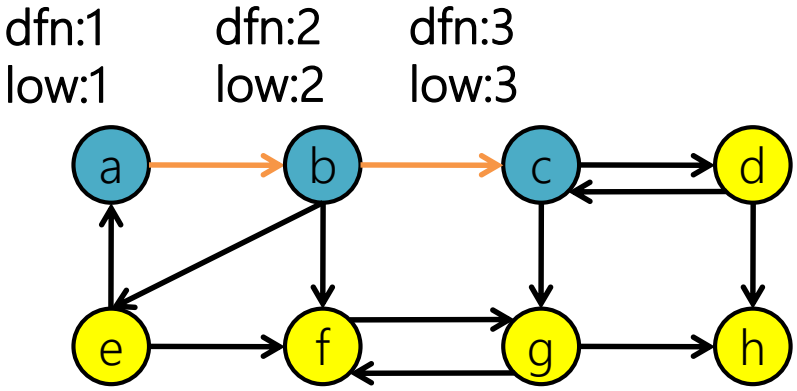

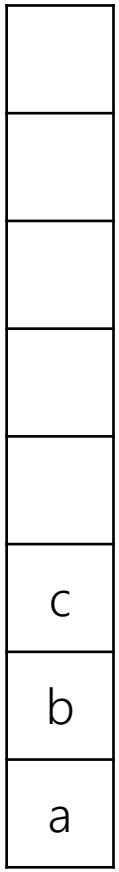

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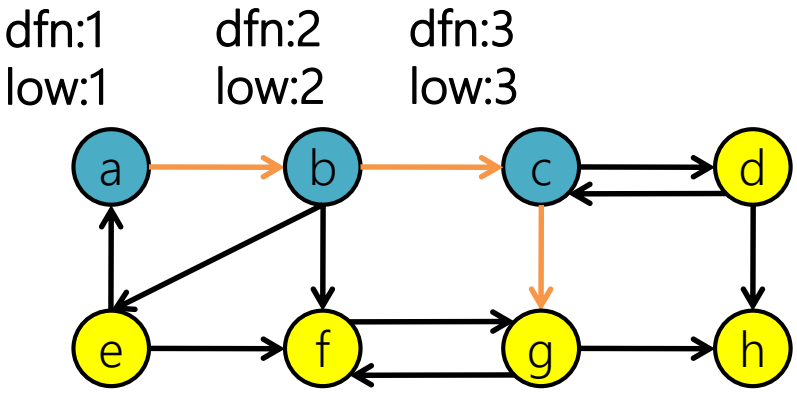

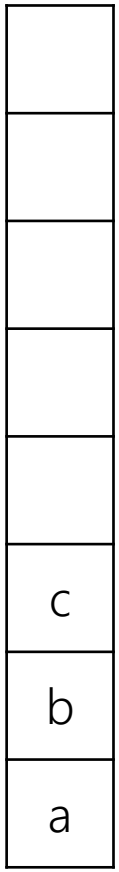

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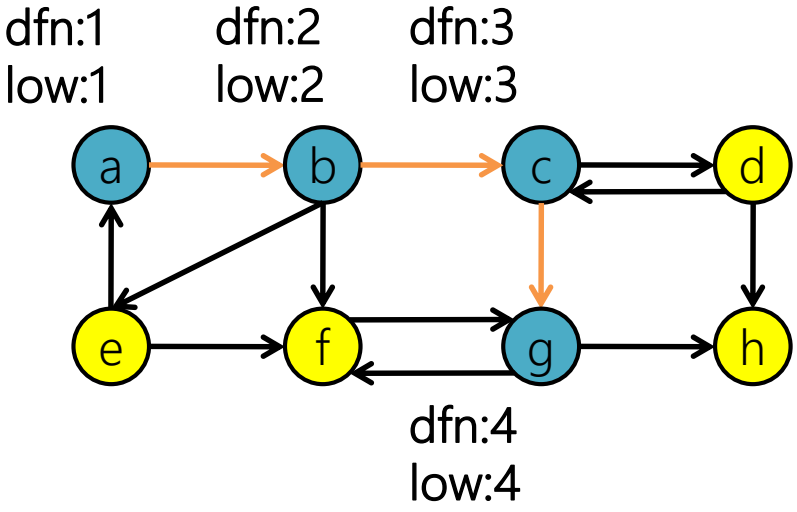

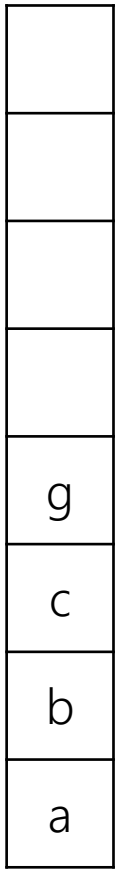

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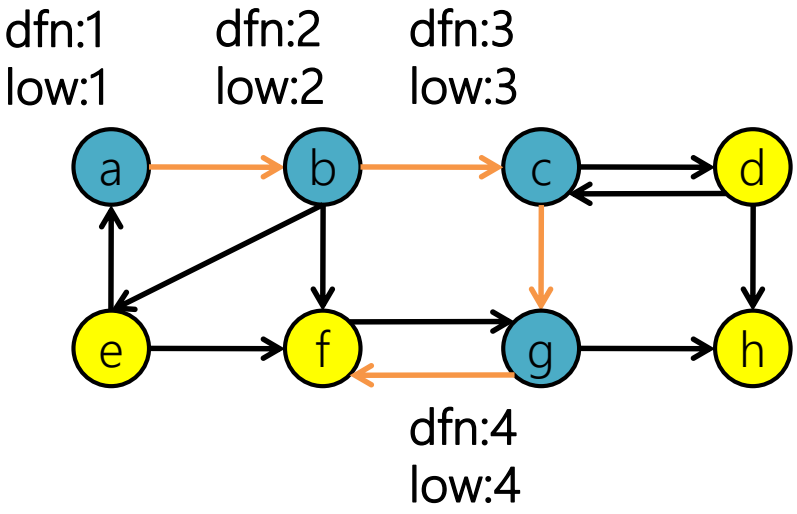

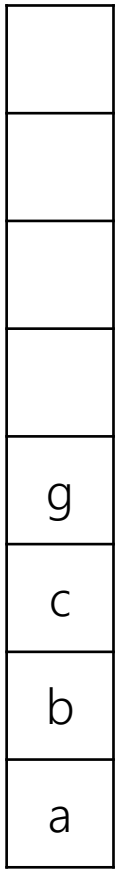

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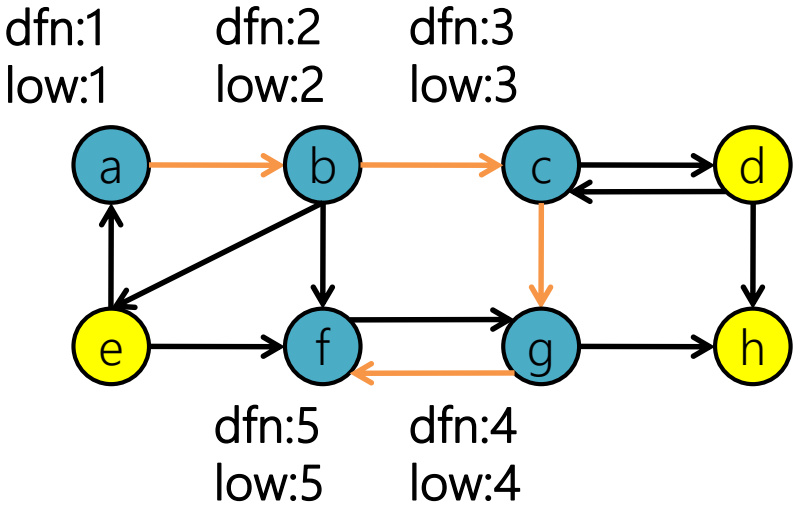

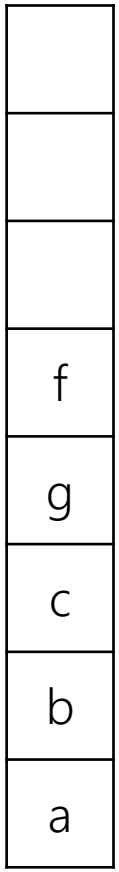

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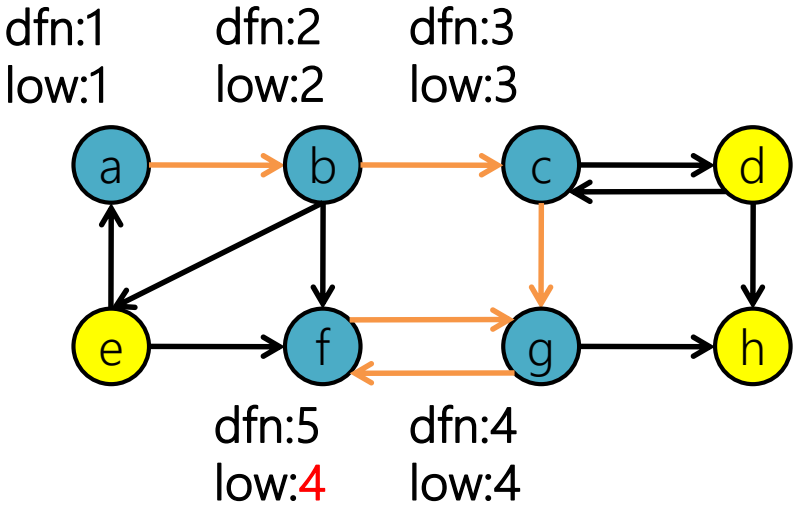

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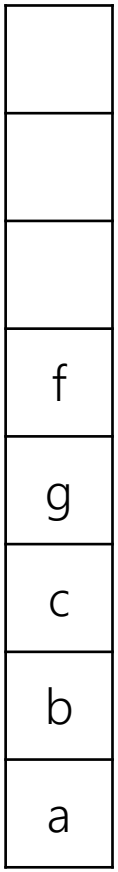

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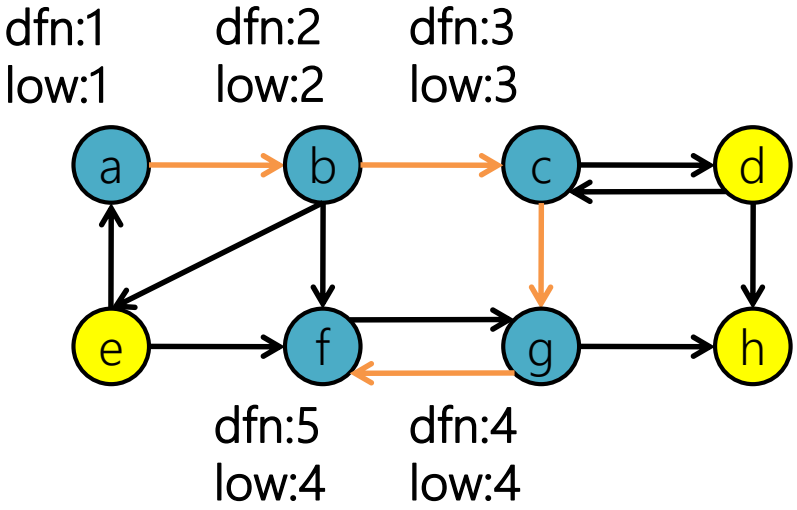
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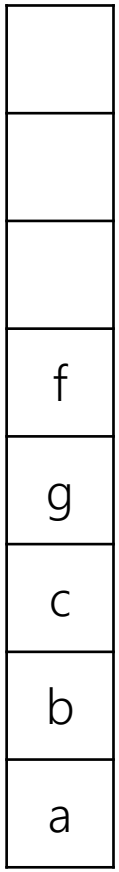
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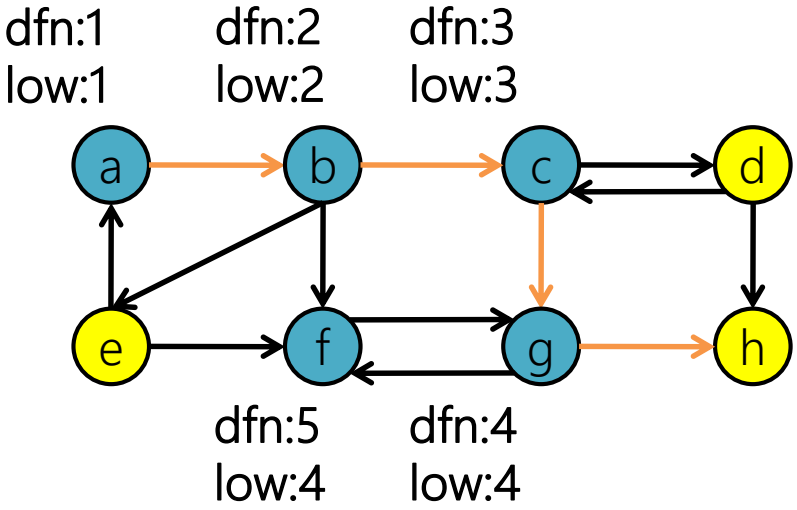
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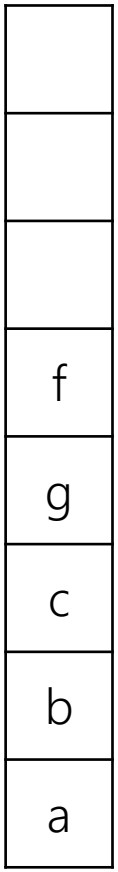
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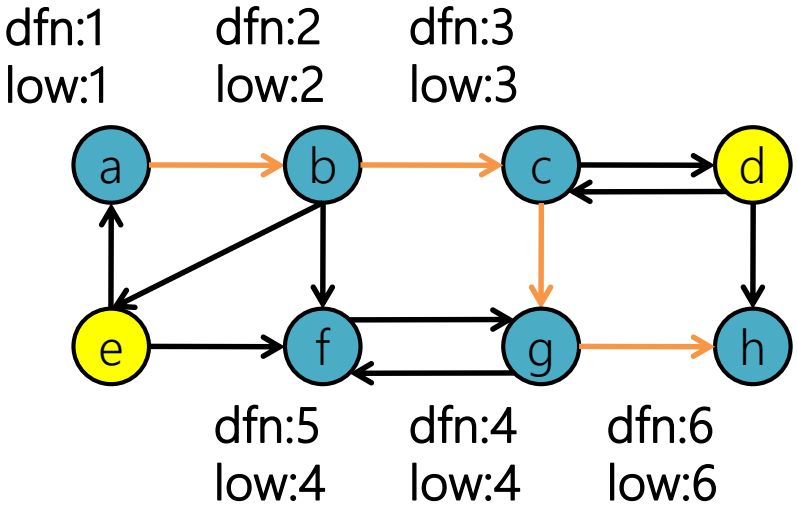

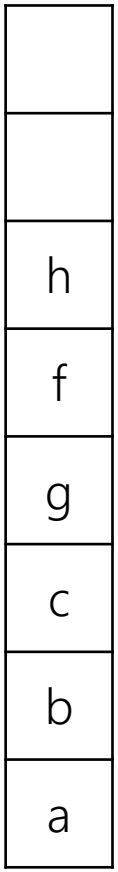

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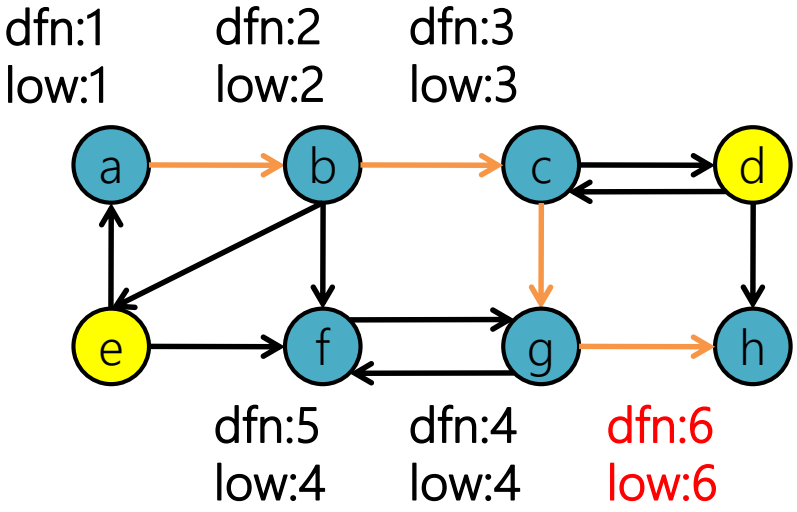

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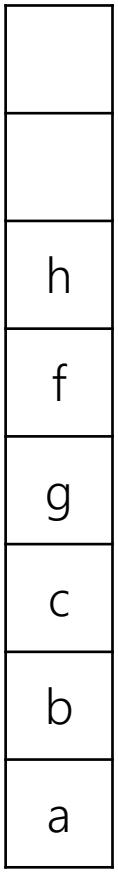

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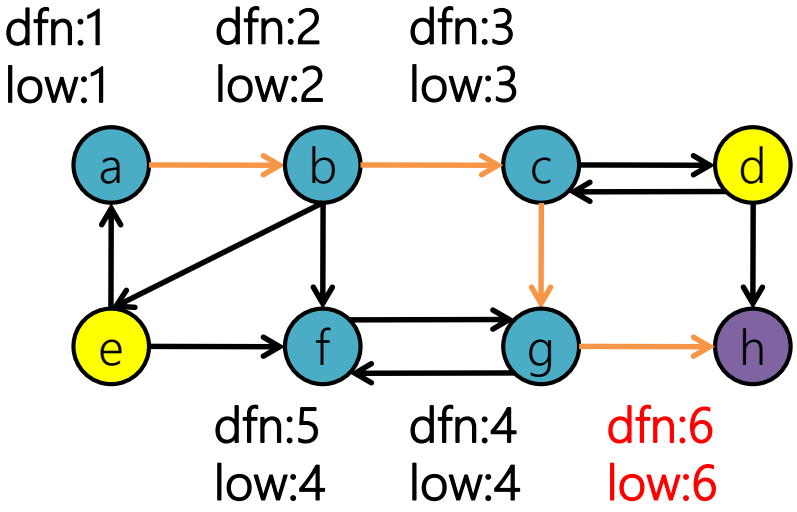

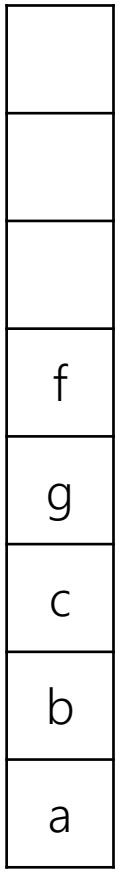

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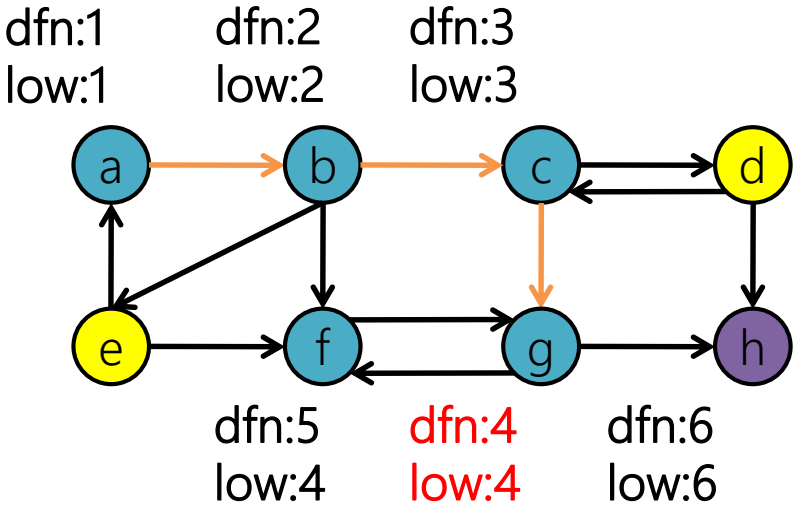

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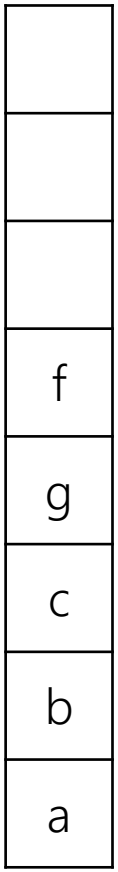

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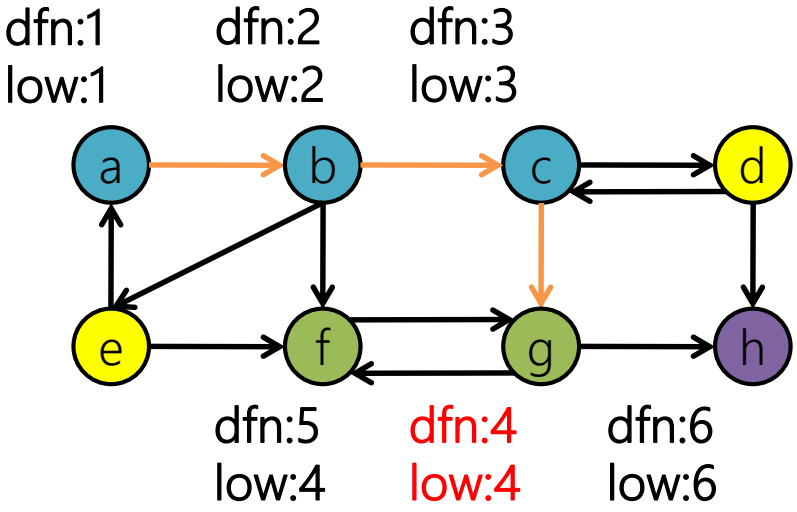

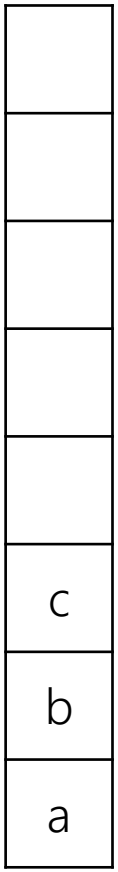

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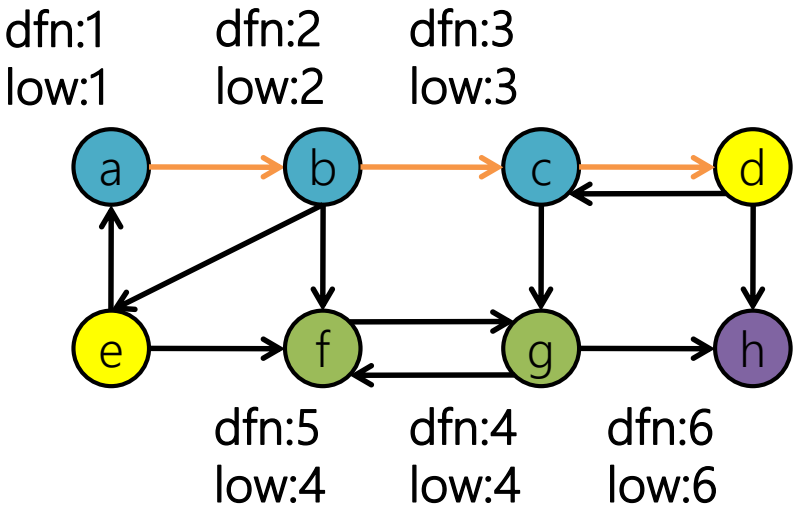

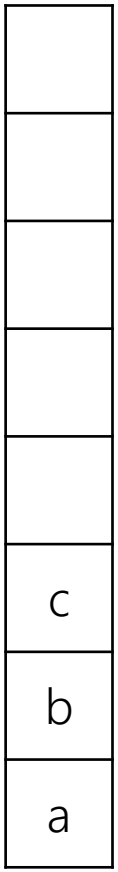

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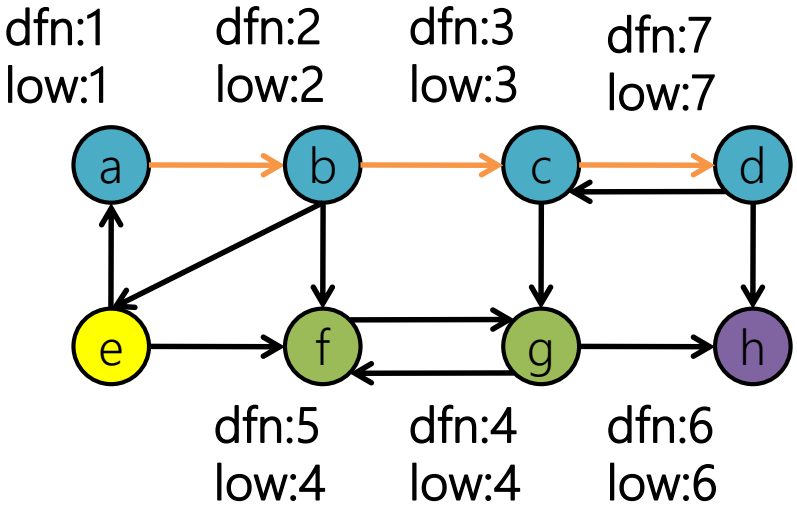

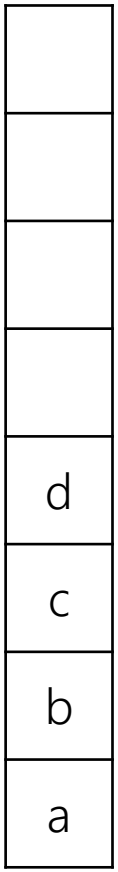

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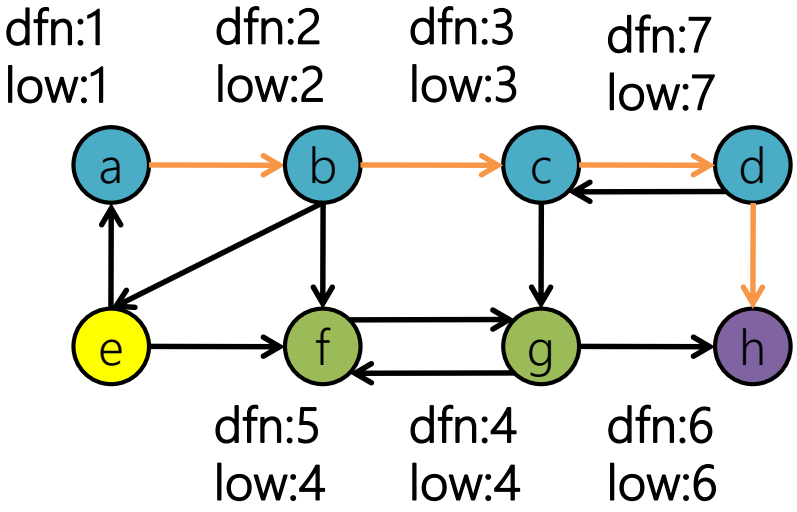

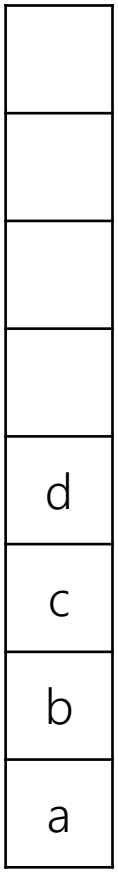

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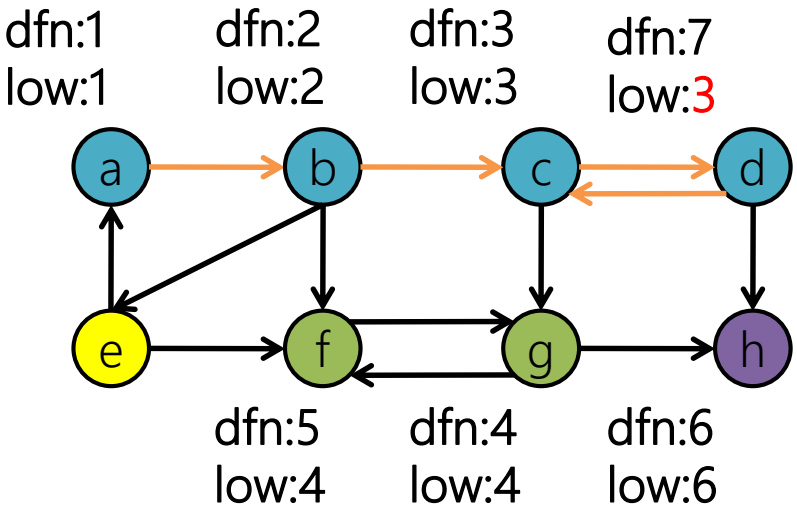

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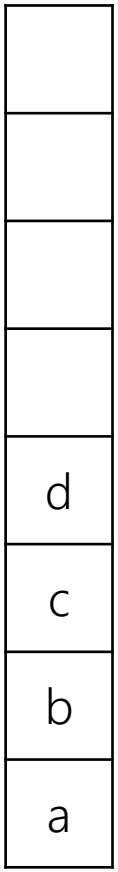

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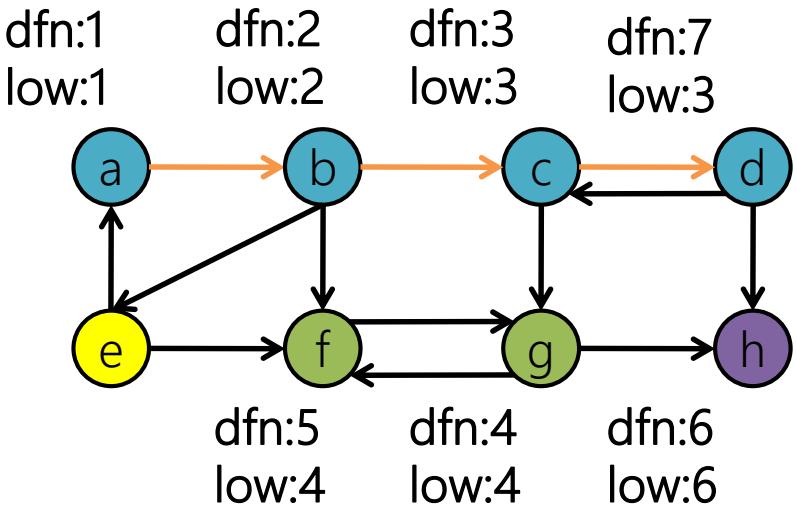

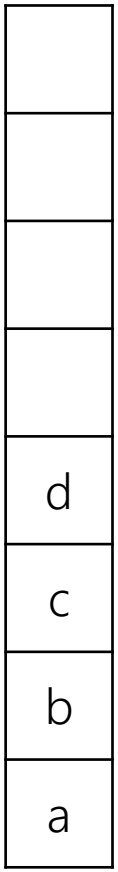

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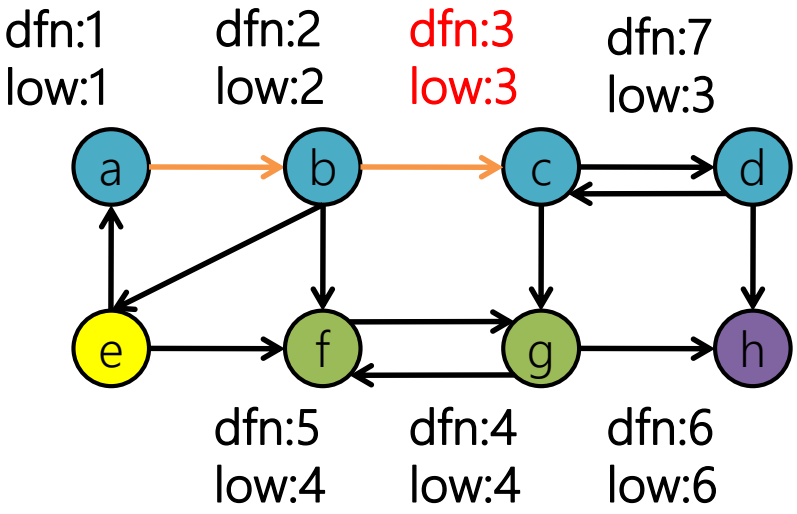

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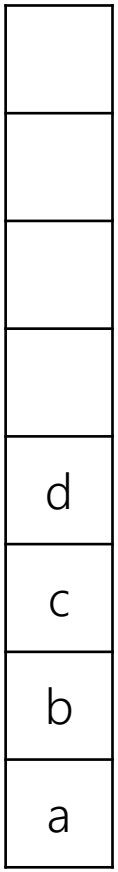

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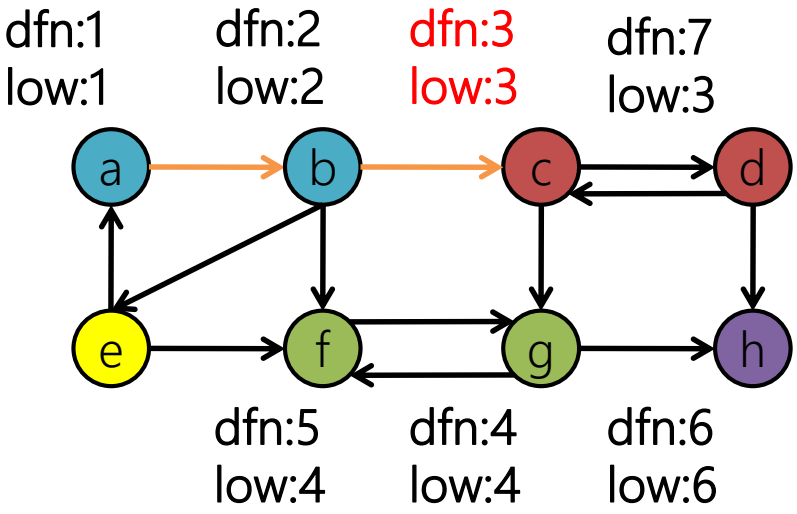

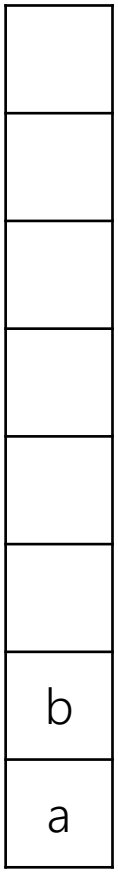

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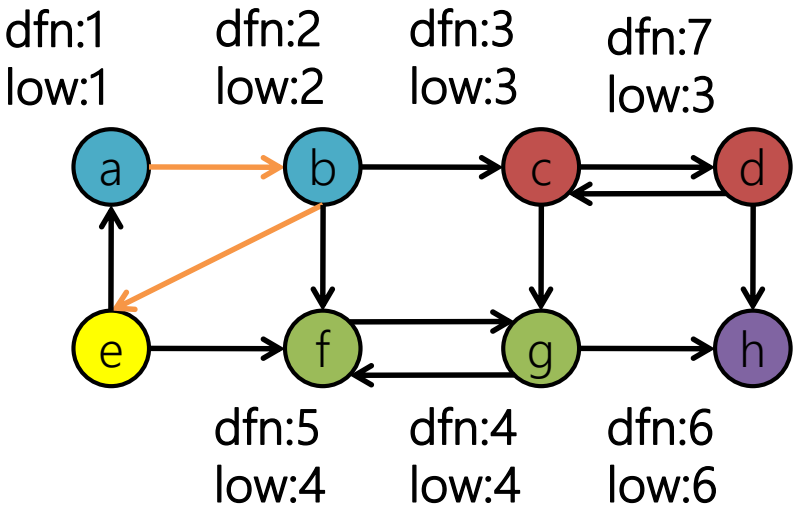

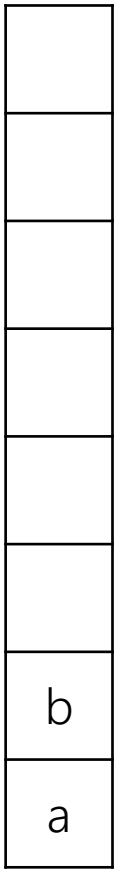

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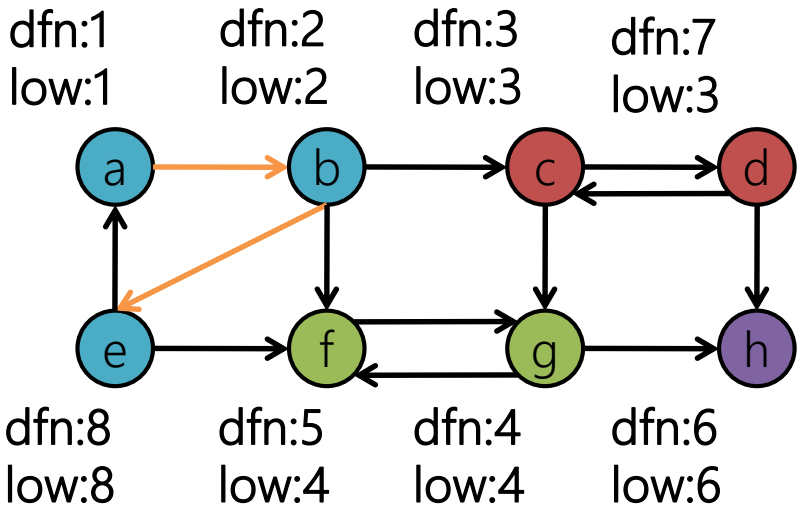

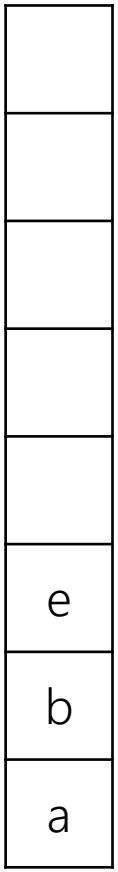

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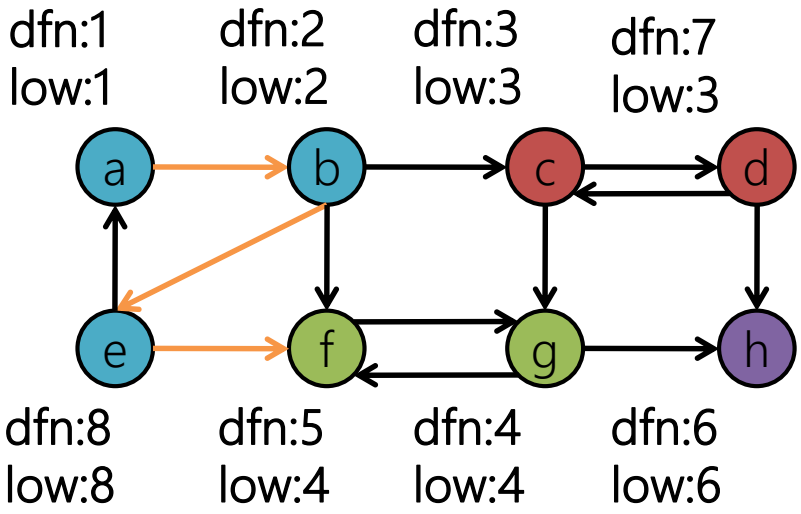

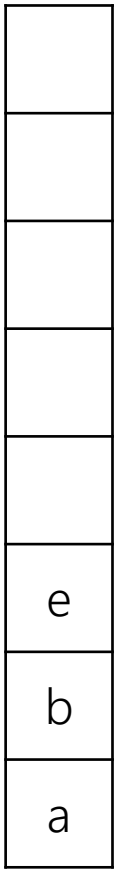

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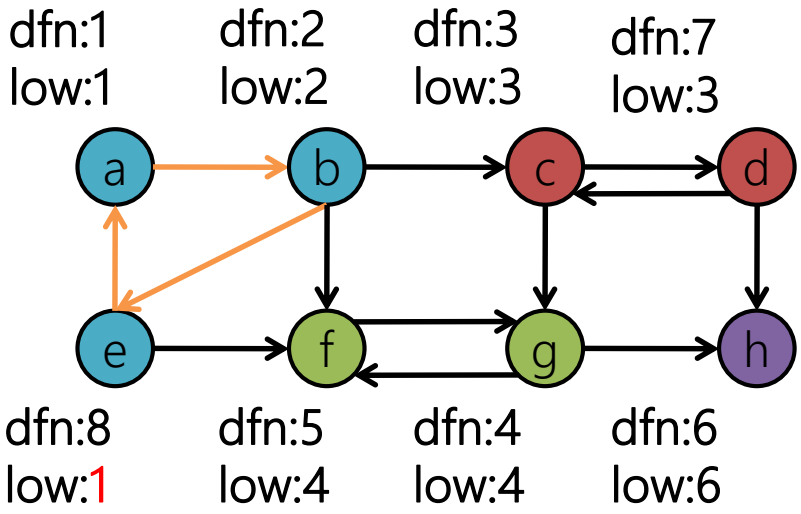

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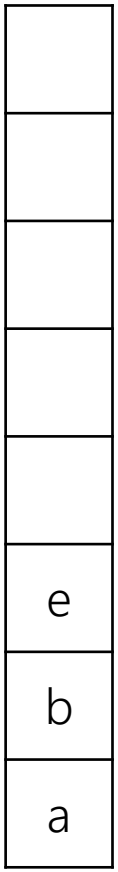

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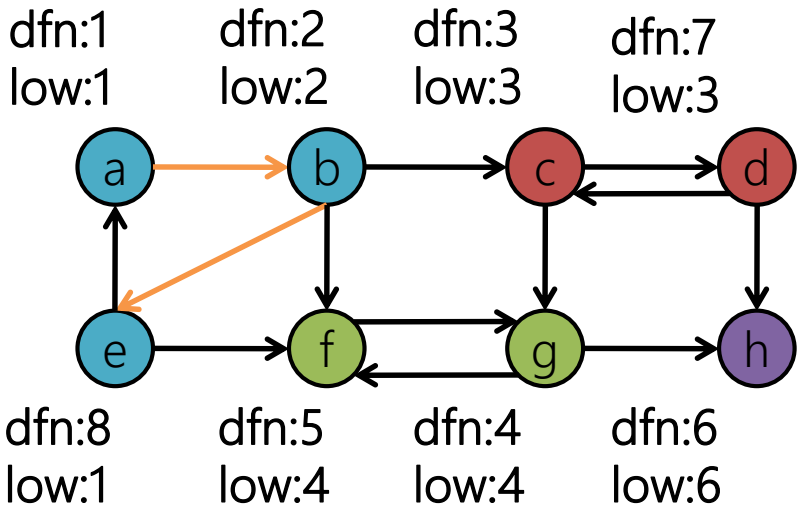

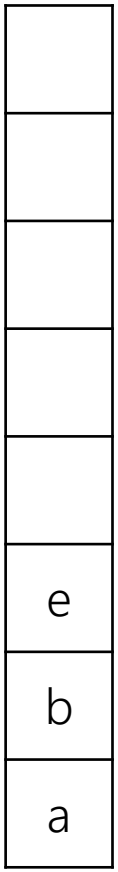

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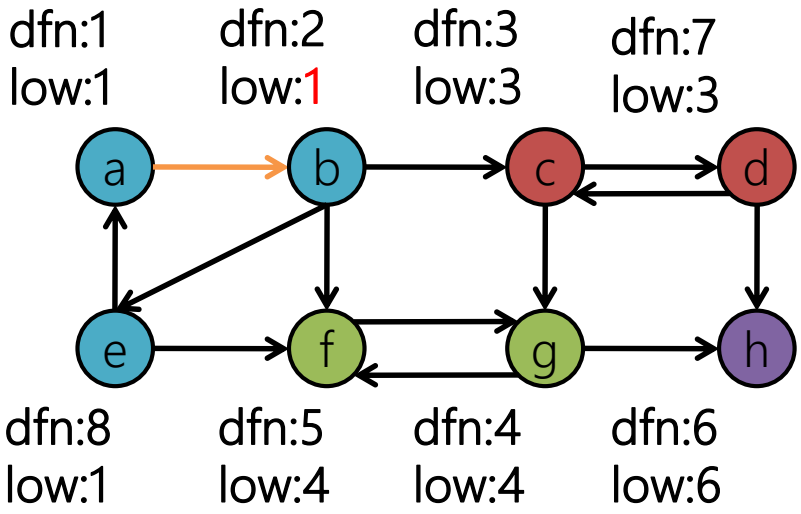

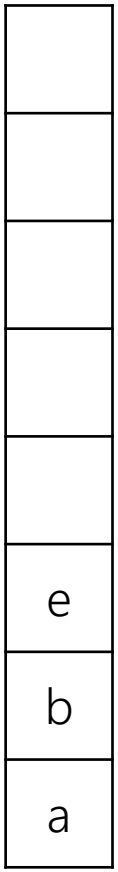

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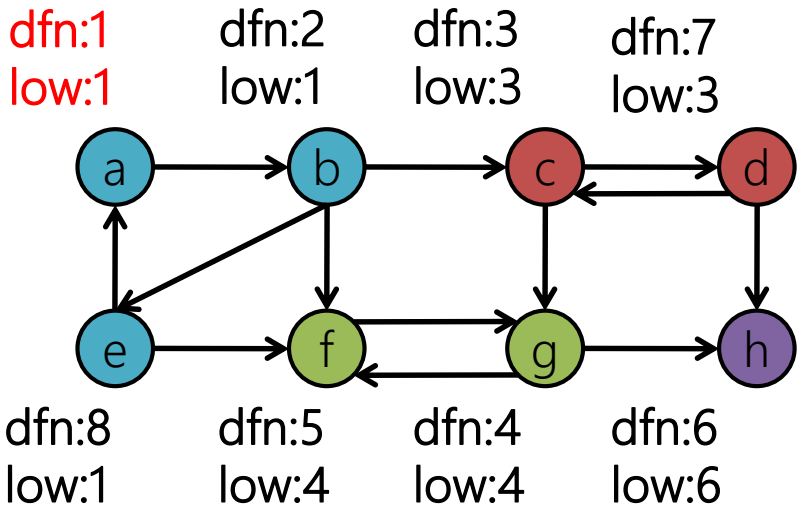

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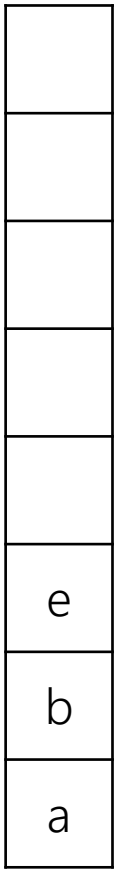

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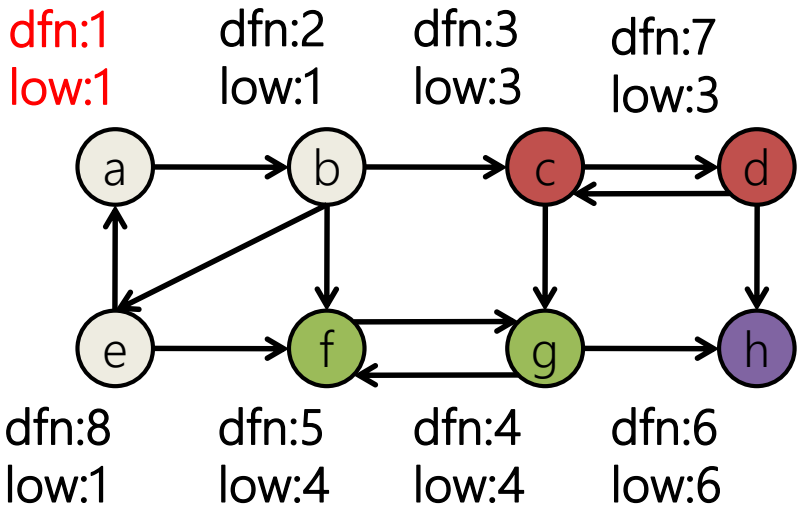

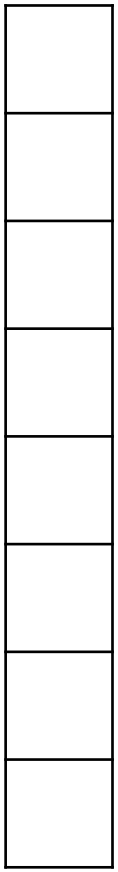

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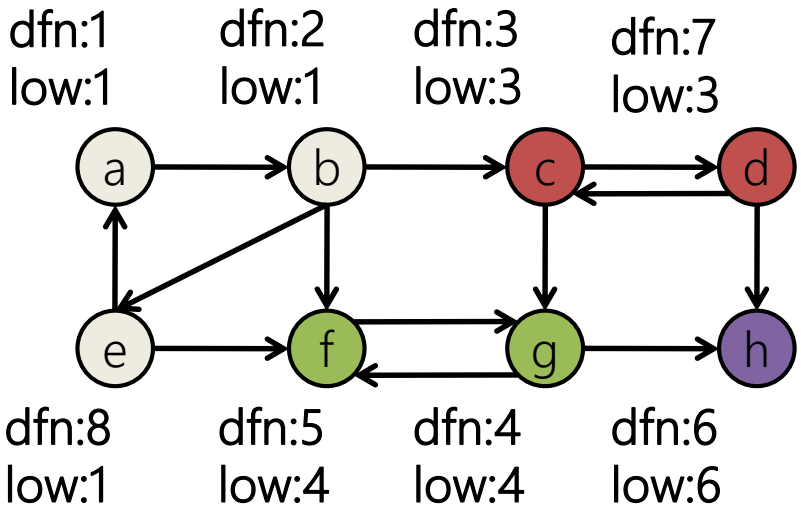

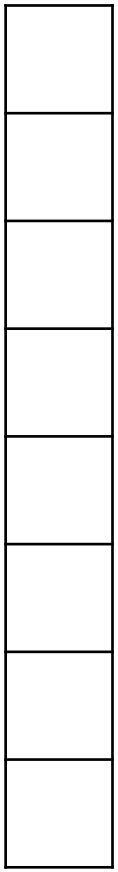

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