Nonlinear F-16 Model Description

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Nomenclature

State variables:

V_T	-	Total velocity, m/s
α	-	angle of attack, rad
β	-	angle of sideslip, rad
ϕ	-	roll angle, rad
θ	-	pitch angle, rad
ψ	-	yaw angle, rad
q_0, q_1, q_2, q_3	-	quaternion components
p	-	body-axis roll rate, rad/s
q	-	body-axis pitch rate, rad/s
r	-	body-axis yaw rate, rad/s
x_E	-	x-position w.r.t. earth, m
y_E	-	y-position w.r.t. earth, m
z_E	-	z-position w.r.t. earth, m
pow	-	power setting, %

Control variables:

δ_{th}	-	throttle setting, (0-1)
δ_e	-	elevator deflection, rad
δ_a	-	aileron deflection, rad
δ_r	-	rudder deflection, rad

Parameters:

δ_{LEF}	-	leading edge flap deflection, rad
ρ	-	air density, kg/m^3
b	-	reference wing span, m
$\bar{c}$	-	mean aerodynamic chord, m
C_{l_T}	-	total rolling moment coefficient
C_{m_T}	-	total pitching moment coefficient
C_{n_T}	-	total yawing moment coefficient
C_{X_T}	-	total axial force coefficient
C_{Y_T}	-	total lateral force coefficient
C_{Z_T}	-	total normal force coefficient
F_T	-	total engine thrust, N
g	-	gravitational constant, m/s^2
h_E	-	engine angular momentum, kg.m^2/s
I_x	-	roll moment of inertia, kg.m^2

I_y	-	pitch moment of inertia, kg.m^2
I_z	-	yaw moment of inertia, kg.m^2
I_{xz}	-	product moment of inertia, kg.m^2
I_{xy}	-	product moment of inertia, kg.m^2
I_{yz}	-	product moment of inertia, kg.m^2
$\bar{L}$	-	rolling moment, N.m
$\bar{M}$	-	pitching moment, N.m
$\bar{N}$	-	yawing moment, N.m
m	-	total aircraft mass, kg
M	-	Mach number
p_s	-	static pressure, Pa
$\bar{q}$	-	dynamic pressure, Pa
S	-	reference wing area, m^2
u	-	velocity in x-axis direction, m/s
T	-	temperature, K
v	-	velocity in y-axis direction, m/s
w	-	velocity in z-axis direction, m/s
x_{cg}	-	center of gravity location, m
x_{cg_r}	-	reference c.g. location, m
$\bar{X}$	-	axial force component, N
$\bar{Y}$	-	lateral force component, N
$\bar{Z}$	-	normal force component, N

1 Aircraft Dynamics

In this section the nonlinear dynamical model of the F-16 aircraft is derived. Mass and geometric data are given in Table 2. This derivation is based on [Blakelock, 1991], [Cook, 1997] and [Lewis and Stevens, 1992]. The course notes [Mulder et al., 2000] are also recommended for a very detailed flight dynamics analysis.

1.1 Reference Frames

Before describing the equations of motion of an aircraft, some frame of reference is needed to describe the motion in. The most commonly used reference frames are the earth-fixed reference frame F_E and the body-fixed reference frame F_B, see Figure 1. Both reference frames are right-handed and orthogonal. In the earth-fixed reference frame the Z_E-axis points to the center of the earth, the X_E-axis points in some arbitrary direction, e.g. the north, and the Y_E-axis is perpendicular to the X_E-axis. This frame is useful for describing the position and orientation of the aircraft. In the body-fixed reference frame, the origin is at the aircraft center of gravity, while the X_B-axis points forward through the nose, the Y_B-axis through the starboard (right) wing and the Z_B-axis downwards.

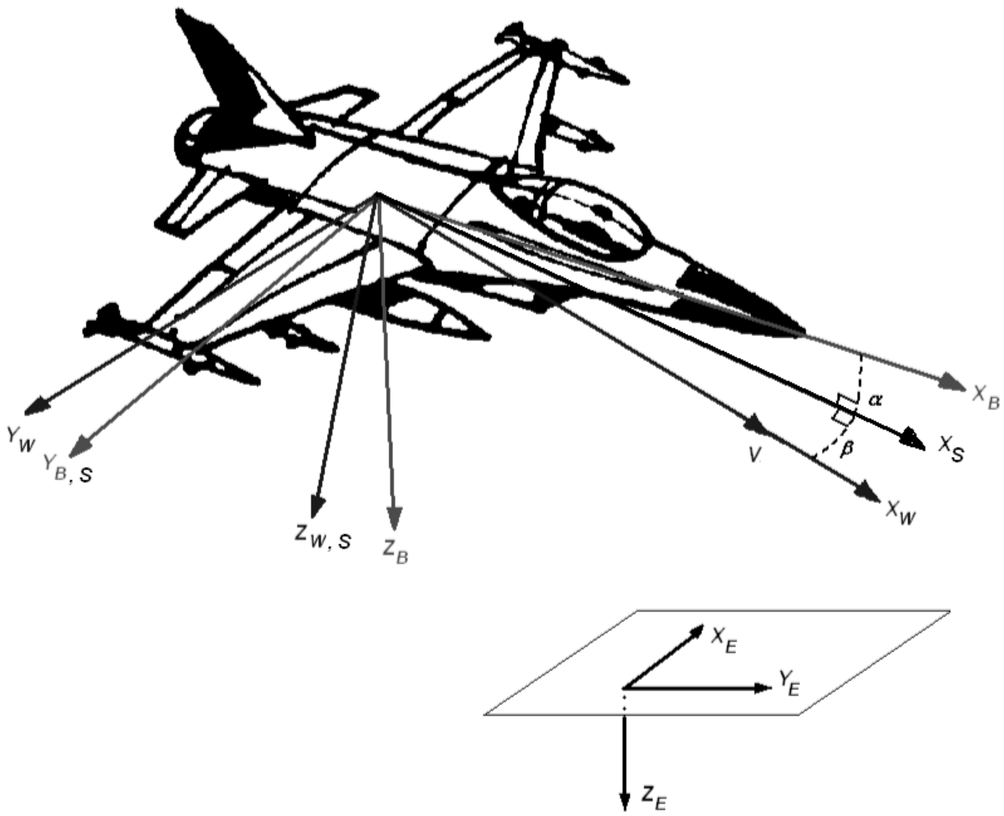

Figure 1: Aircraft reference frames: earth-fixed reference frame F_E, body-fixed reference frame F_B, stability-axes reference frame F_S and wind-axes reference frame F_W

Two other frames are indicated in Figure 1, these are the stability-axes reference frame F_S

and the wind-axes reference frameF_W. The stability-axes reference frame is obtained from the body-fixed reference frame by a left-handed rotation through angle of attack α, and is used for analyzing the effect of perturbations from steady-state flight. The wind-axes reference frame is obtained from the stability-axes reference frame by a rotation around the z-axis through sideslip angle β. The lift, drag and side forces are defined naturally in this reference frame. Using this reference frame can be convenient when describing the equations of motion.
The transformation from a vector in the body-fixed reference frame to the stability-axes reference frame can be described by a rotation over angle of attack α

$$\mathbf{T}_s = T_{s/b}\mathbf{T}_b = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \mathbf{T}_b \tag{1}$$

and the transformation from a vector in the body-fixed reference frame to the wind-axes reference frame can be described by a rotation over side slip angle β, followed by a rotation over angle of attack α

$$\mathbf{T}_w = T_{w/b}\mathbf{T}_b \tag{2}$$

where

$$\begin{aligned} T_{w/b} &= \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} T_{s/b} && (3) \\ &= \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}. && (4) \end{aligned}$$

1.2 Aircraft Variables

A number of assumptions has to be made, before proceeding with the derivation of the equations of motion:

1. *The aircraft is a rigid-body*, which means that any two points on or within the airframe remain fixed with respect to each other. This assumption is quite valid for fighter aircraft.

2. *The earth is flat and non-rotating and regarded as an inertial reference.* This assumption is valid when dealing with control design of aircraft, but not when analyzing inertial guidance systems.

3. *The mass is constant during the time interval over which the motion is considered*, the fuel consumption is neglected during this time-interval. This assumption is necessary to apply Newton's motion laws.

4. *The mass distribution of the aircraft is symmetric relative to the X_BOZ_B-plane*, this implies that the products of inertia I_{yz} and I_{xy} are equal to zero. This assumption is valid for most aircraft.

Under the above assumptions the motion of the aircraft has 6 degrees of freedom (rotation and translation in 3 dimensions). The aircraft dynamics can be described by its position, orientation, velocity and angular velocity over time. $\mathbf{p_E} = (x_E, y_E, z_E)^T$ is the position vector expressed in an earth-fixed coordinate system. $\mathbf{V}$ is the velocity vector given by $\mathbf{V} = (u, v, w)^T$, where u is the longitudinal velocity, v the lateral velocity and w the normal velocity. The orientation vector is given by $\mathbf{\Phi} = (\phi, \theta, \psi)^T$, where ϕ is the roll angle, θ the pitch angle and ψ the yaw angle, and the angular velocity vector is given by $\boldsymbol{\omega} = (p, q, r)^T$, where p,q and r are the roll, pitch and yaw angular velocities. Various components of the aircraft motions are illustrated in Figure 2.

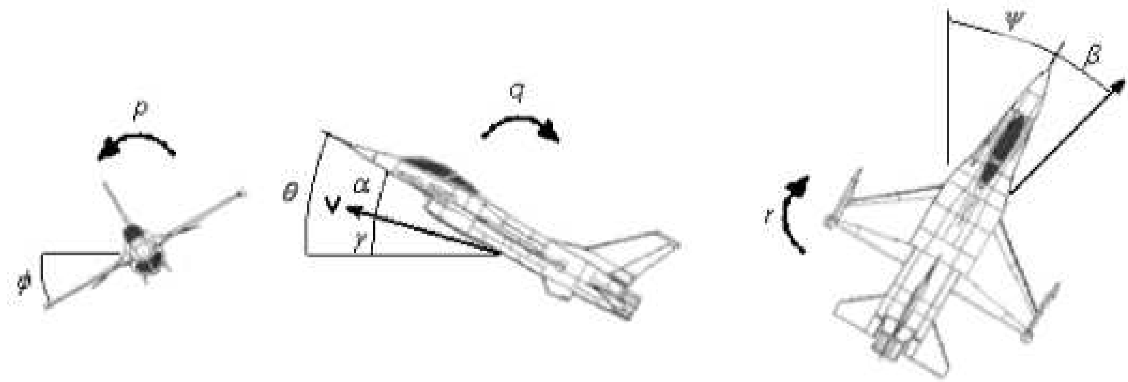

Figure 2: Aircraft orientation angles ϕ, θ and ψ, aerodynamic angles α and β, and the angular rates p, q and r. All angles are positive in the figure.

The relation between the attitude vector $\mathbf{\Phi}$ and the angular velocity vector $\boldsymbol{\omega}$ is given by [Mulder et al., 2000]

$$\dot{\mathbf{\Phi}} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{bmatrix} \boldsymbol{\omega} \tag{5}$$

Defining V_T as the total velocity and using Figure 2, we find the following relations:

$$\begin{aligned} V_T &= \sqrt{u^2 + v^2 + w^2} \\ \alpha &= \arctan\frac{w}{u} \\ \beta &= \arcsin\frac{v}{V_T} \end{aligned} \tag{6}$$

Furthermore, when $\beta = \phi = 0$, the flight path angle γ can be defined as

$$\gamma = \theta - \alpha \tag{7}$$

1.3 Equations of Motion for a Rigid Body Aircraft

The equations of motion for the aircraft can be derived from Newton's Second Law of motion, which states that the summation of all external forces acting on a body must be equal to the time rate of change of its momentum, and the summation of the external moments acting on a body must be equal to the time rate of change of its angular momentum. In the inertial, Earth-fixed reference frame F_E, Newton's Second Law can be expressed by two vector equations, see e.g. page 48 of [Mulder et al., 2000]

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{V})\Big]_E \tag{8a}$$

$$\mathbf{M} = \frac{d\mathbf{H}}{dt}\Big]_E \tag{8b}$$

where $\mathbf{F}$ represents the sum of all externally applied forces, m is the mass of the aircraft, $\mathbf{V}$ is the velocity vector, $\mathbf{M}$ represents the sum of all applied torques and $\mathbf{H}$ is the angular momentum.

Force equation

First, we will further evaluate the force equation (8a) and for that it is necessary to obtain an expression for the time rate of change of the velocity vector with respect to earth. This process is complicated by the fact that the velocity vector may be rotating while it is changing in magnitude. Using the equation of Coriolis in appendix A of [Blakelock, 1991] we obtain

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{V})\Big]_B + \boldsymbol{\omega} \times m\mathbf{V} \tag{9}$$

where $\boldsymbol{\omega}$ is the total angular velocity of the aircraft with respect to the earth (inertial reference frame). Expressing the vectors as the sum of their components with respect to the body-fixed reference frame F_B gives

$$\mathbf{V} = \mathbf{i}u + \mathbf{j}v + \mathbf{k}w \tag{10a}$$

$$\boldsymbol{\omega} = \mathbf{i}p + \mathbf{j}q + \mathbf{k}r \tag{10b}$$

where $\mathbf{i}$,$\mathbf{j}$ and $\mathbf{k}$ are unit vectors along the aircraft's X_B, Y_B and Z_B axes, respectively. Expanding (9) using (10) results in

$$\begin{aligned} F_x &= m(\dot{u} + qw - rv) \\ F_y &= m(\dot{v} + ru - pw) \\ F_z &= m(\dot{w} + pv - qu) \end{aligned} \tag{11}$$

where the external forces F_x, F_y and F_z depend on the weight vector $\mathbf{W}$, the aerodynamic force vector $\mathbf{R}$ and the thrust vector $\mathbf{E}$. We assume the thrust produced by the engine, F_T, acts parallel to the aircraft's X_B-axis, which makes the thrust vector $\mathbf{E}$ equal to

$$\begin{aligned} E_x &= F_T \\ E_y &= 0 \\ E_z &= 0 \end{aligned} \tag{12}$$

The components of $\mathbf{W}$ and $\mathbf{R}$ along the body-axes are

$$
\begin{aligned}
W_x &= -mg\sin\theta \\
W_y &= mg\sin\phi\cos\theta \\
W_z &= mg\cos\phi\cos\theta
\end{aligned}
\tag{13}
$$

and

$$
\begin{aligned}
R_x &= \bar{X} \\
R_y &= \bar{Y} \\
R_z &= \bar{Z}
\end{aligned}
\tag{14}
$$

where g is the gravity constant. The size of the aerodynamic forces $\bar{X}$,$\bar{Y}$ and $\bar{Z}$ is determined by the amount of air diverted by the aircraft in different directions. For a full discussion on the different aerodynamic forces, see [Mulder et al., 2000] or [Lewis and Stevens, 1992]. The amount of air diverted by the aircraft mainly depends on the following factors:

- the speed V_T (or Mach number M) and density of the airflow ρ,
- the geometry of the aircraft: wing area S, wing span b and mean aerodynamic chord $\bar{c}$,
- the orientation of the aircraft relative to the airflow: angle of attack α and side slip angle β,
- the control surface deflections δ,
- the angular rates p, q, r,

There are other variables such as the time derivatives of the aerodynamic angles that also play a role, but these effects are less prominent, since we assumed that the aircraft is a rigid body. This motivates the standard way of modeling the aerodynamic force:

$$
\begin{aligned}
\bar{X} &= \bar{q}SC_{X_T}(\alpha, \beta, p, q, r, \delta, ...) \\
\bar{Y} &= \bar{q}SC_{Y_T}(\alpha, \beta, p, q, r, \delta, ...) \\
\bar{Z} &= \bar{q}SC_{Z_T}(\alpha, \beta, p, q, r, \delta, ...)
\end{aligned}
\tag{15}
$$

where $\bar{q} = \frac{1}{2}\rho V_T^2$ is the aerodynamic pressure. The coefficients C_{X_T}, C_{Y_T} and C_{Z_T} are usually obtained from wind tunnel data and flight tests.

Combining equations (13) and (14) and the thrust components (12) with (11), results in the complete body-axes force equation:

$$
\begin{aligned}
\bar{X} + F_T - mg\sin\theta &= m(\dot{u} + qw - rv) \\
\bar{Y} + mg\sin\phi\cos\theta &= m(\dot{v} + ru - pw) \\
\bar{Z} + mg\cos\phi\sin\theta &= m(\dot{w} + pv - qu)
\end{aligned}
\tag{16}
$$

Moment equation

Now, we will return to equation (8b) to obtain the equations of angular motion. The time rate of change of **H** is required and since **H** can change in magnitude and direction, (8b) can be written as

$$\mathbf{M} = \frac{d\mathbf{H}}{dt}\Big]_B + \boldsymbol{\omega} \times \mathbf{H} \tag{17}$$

In the body-fixed reference frame, under assumptions 1 and 3, we can express the angular momentum **H** as

$$\mathbf{H} = I\boldsymbol{\omega} \tag{18}$$

where, under assumption 4, the inertia matrix is defined as

$$I = \begin{bmatrix} I_x & 0 & -I_{xz} \\ 0 & I_y & 0 \\ -I_{xz} & 0 & I_z \end{bmatrix} \tag{19}$$

Expanding (17) using (18) results in

$$\begin{aligned} M_x &= \dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - pqI_{xz} \\ M_y &= \dot{q}I_y + pq(I_x - I_z) + (p^2 - r^2)I_{xz} \\ M_z &= \dot{r}I_z - \dot{p}I_{xz} + pq(I_y - I_x) + qrI_{xz}. \end{aligned} \tag{20}$$

The external moments M_x, M_y and M_z are those due to aerodynamics and thrust. As a result the aerodynamic moments are

$$\begin{aligned} M_x &= \bar{L} + L_T \\ M_y &= \bar{M} + M_T \\ M_z &= \bar{N} + N_T \end{aligned} \tag{21}$$

where $\bar{L}$,$\bar{M}$ and $\bar{N}$ are the aerodynamic moments and L_T, M_T and N_T are the moments due to thrust. The aerodynamic moments can be expressed in a similar way as the aerodynamic forces in (15)

$$\begin{aligned} \bar{L} &= \bar{q}SbC_{l_T}(\alpha, \beta, p, q, r, \delta, ...) \\ \bar{M} &= \bar{q}S\bar{c}C_{m_T}(\alpha, \beta, p, q, r, \delta, ...) \\ \bar{N} &= \bar{q}SbC_{n_T}(\alpha, \beta, p, q, r, \delta, ...) \end{aligned} \tag{22}$$

Assuming that the engine is mounted so that the thrust point lies in the xz-plane of the body-fixed reference frame with an offset from the center of gravity by z_T along the z-axis, moments due to thrust are defined as

$$\begin{aligned} L_T &= 0 \\ M_T &= F_T z_T \\ N_T &= 0 \end{aligned} \tag{23}$$

Combining (20) and (21), the complete body-axis moment equation is formed as

$$
\begin{aligned}
\bar{L} + L_T &= \dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - pqI_{xz} \\
\bar{M} + M_T &= \dot{q}I_y + pq(I_x - I_z) + (p^2 - r^2)I_{xz} \\
\bar{N} + N_T &= \dot{r}I_z - \dot{p}I_{xz} + pq(I_y - I_x) + qrI_{xz}.
\end{aligned}
\tag{24}
$$

1.4 Gathering the Equations of Motion

The equations of motion derived in the previous sections are now written as a system of 12 scalar first order differential equations.

$$\dot{u} = rv - qw - g\sin\theta + \frac{1}{m}(\bar{X} + F_T) \tag{25a}$$
$$\dot{v} = pw - ru + g\sin\phi\cos\theta + \frac{1}{m}\bar{Y} \tag{25b}$$
$$\dot{w} = qu - pv + g\cos\phi\cos\theta + \frac{1}{m}\bar{Z} \tag{25c}$$

$$\dot{p} = (c_1 r + c_2 p)q + c_3\bar{L} + c_4(\bar{N} + h_E q) \tag{26a}$$
$$\dot{q} = c_5 pr - c_6(p^2 - r^2) + c_7(\bar{M} + F_T z_T - h_E r) \tag{26b}$$
$$\dot{r} = (c_8 p - c_2 r)q + c_4\bar{L} + c_9(\bar{N} + h_E q) \tag{26c}$$

$$\dot{\phi} = p + \tan\theta(q\sin\phi + r\cos\phi) \tag{27a}$$
$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{27b}$$
$$\dot{\psi} = \frac{q\sin\phi + r\cos\phi}{\cos\theta} \tag{27c}$$

$$\dot{x}_E = u\cos\psi\cos\theta + v(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi) \tag{28a}$$
$$+ w(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) \tag{28b}$$
$$\dot{y}_E = u\sin\psi\cos\theta + v(\sin\psi\sin\theta\sin\phi + \cos\psi\cos\phi) \tag{28c}$$
$$+ w(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) \tag{28d}$$
$$\dot{z}_E = -u\sin\theta + v\cos\theta\sin\phi + w\cos\theta\cos\phi \tag{28e}$$

where the moment of inertia components are given by [Lewis and Stevens, 1992]

$$
\begin{array}{rcc}
\Gamma c_1 = (I_y - I_z)I_z - I_{xz}^2 & \Gamma c_4 = I_{xz} & c_7 = \dfrac{1}{I_y} \\
\Gamma c_2 = (I_x - I_y + I_z)I_{xz} & c_5 = \frac{I_z - I_x}{I_y} & \Gamma c_8 = I_x(I_x - I_y) + I_{xz}^2 \\
\Gamma c_3 = I_z & c_6 = \frac{I_{xz}}{I_y} & \Gamma c_9 = I_x
\end{array}
$$

with $\Gamma = I_x I_z - I_{xz}^2$.

1.5 Equations of Motion Using Quaternion Orientation

The derived equations of motion make use of Euler angle approach for the orientation model. The disadvantage of the Euler angle method is that the differential equations for $\dot{p}$ and $\dot{r}$ become singular when pitch angle θ passes through $\pm\frac{\pi}{2}$. To avoid these singularities quaternions are used for the aircraft orientation presentation. A detailed explanation about quaternions and their properties can be found in [Lewis and Stevens, 1992]. With the quaternions presentation we now have a aircraft system representation existing of 13 scalar first order differential equations:

$$\dot{u} = rv - qw + \frac{1}{m}\left(\bar{X} + F_T\right) + 2(q_1q_3 - q_0q_2)g \tag{29a}$$

$$\dot{v} = pw - ru + \frac{1}{m}\bar{Y} + 2(q_2q_3 + q_0q_1)g \tag{29b}$$

$$\dot{w} = qu - pv + \frac{1}{m}\bar{Z} + (q_0^2 - q_1^2 - q_2^2 + q_3^2)g \tag{29c}$$

$$\dot{p} = (c_1r + c_2p)q + c_3\bar{L} + c_4(\bar{N} + h_Eq) \tag{30a}$$

$$\dot{q} = c_5pr - c_6(p^2 - r^2) + c_7(\bar{M} + F_Tz_T - h_Er) \tag{30b}$$

$$\dot{r} = (c_8p - c_2r)q + c_4\bar{L} + c_9(\bar{N} + h_Eq) \tag{30c}$$

$$\begin{bmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{bmatrix}\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{31}$$

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{bmatrix} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{32}$$

where

$$\begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \pm\begin{bmatrix} \cos\phi/2\cos\theta/2\cos\psi/2 + \sin\phi/2\sin\theta/2\sin\psi/2 \\ \sin\phi/2\cos\theta/2\cos\psi/2 - \cos\phi/2\sin\theta/2\sin\psi/2 \\ \cos\phi/2\sin\theta/2\cos\psi/2 + \sin\phi/2\cos\theta/2\sin\psi/2 \\ \cos\phi/2\cos\theta/2\sin\psi/2 - \sin\phi/2\sin\theta/2\cos\psi/2 \end{bmatrix}$$

1.6 Wind-Axes Reference Frame

It is more convenient to express the force equations in the wind-axes reference frame, especially for control design. This will mean the following transformations:

$$\begin{array}{rcl} V_T & = & \sqrt{u^2 + v^2 + w^2} \\ \alpha & = & \arctan\frac{w}{u} \\ \beta & = & \arcsin\frac{v}{V_T} \end{array} \quad \Leftrightarrow \quad \begin{array}{rcl} u & = & V_T\cos\alpha\cos\beta \\ v & = & V_T\sin\beta \\ w & = & V_T\sin\alpha\cos\beta. \end{array} \tag{33}$$

Taking the derivative of α, β and V_T results in

$$\begin{aligned}
\dot{V}_T &= \frac{u\dot{u} + v\dot{v} + w\dot{w}}{V_T} \\
\dot{\alpha} &= \frac{u\dot{w} - w\dot{u}}{u^2 + w^2} \\
\dot{\beta} &= \frac{\dot{v}V_T - v\dot{V}_T}{V_T^2 \cos\beta}.
\end{aligned} \tag{34}$$

Substituting (25) and (33) in (34) and neglecting some small terms gives the force equation in the wind-axes reference frame [Lewis and Stevens, 1992]

$$\begin{aligned}
\dot{V}_T &= \frac{1}{m}(-D + F_T \cos\alpha\cos\beta + mg_1) \\
\dot{\alpha} &= q - (p\cos\alpha + r\sin\alpha)\tan\beta + \frac{1}{mV_T\cos\beta}(-L - F_T\sin\alpha + mg_3) \\
\dot{\beta} &= p\sin\alpha - r\cos\alpha + \frac{1}{mV_T}(Y - F_T\cos\alpha\sin\beta + mg_2)
\end{aligned} \tag{35}$$

where the drag force D, the side force Y and the lift force L are defined as

$$\begin{aligned}
D &= -\bar{X}\cos\alpha\cos\beta - \bar{Y}\sin\beta - \bar{Z}\sin\alpha\cos\beta \\
Y &= -\bar{X}\cos\alpha\sin\beta + \bar{Y}\cos\beta - \bar{Z}\sin\alpha\sin\beta \\
L &= \bar{X}\sin\alpha - \bar{Z}\cos\alpha
\end{aligned} \tag{36}$$

and the gravity components g_1, g_2 and g_3 as [Lewis and Stevens, 1992]

$$\begin{aligned}
g_1 &= g(-\cos\alpha\cos\beta\sin\theta + \sin\beta\sin\phi\cos\theta + \sin\alpha\cos\beta\cos\phi\cos\theta) \\
g_2 &= g(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta) \\
g_3 &= g(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta).
\end{aligned} \tag{37}$$

1.7 ISA Atmospheric Model

For the atmospheric data an approximation of the International Standard Atmosphere (ISA) is used [Mulder et al., 2000].

$$\begin{aligned}
T &= T_0 - 0.0065h \\
\rho &= \rho_0 e^{-\frac{g}{287.05T}h} \\
a &= \sqrt{1.4 \times 287.05T}
\end{aligned}$$

where $T_0 = 288.15$ is the temperature at sea level and $\rho_0 = 1.225$ is the air density at sea level. This atmospheric model is only valid in the troposphere ($h < 11000$ m). Given the aircraft's altitude (h in meters) it returns the current temperature (T in Kelvin), the current air density (ρ in kg/m^3) and the speed of sound (a in m/s).

2 Control Variables

The F-16 model allows for control over thrust, elevator, ailerons, rudder, and the thrust vector in horizontal and vertical direction. The thrust is measured in Newtons. All deflections are defined positive in the conventional way, i.e. positive thrust causes an increase in acceleration along the body x-axis, a positive elevator deflection results in a decrease in pitch rate, a positive aileron deflection gives a decrease in roll rate and a positive rudder deflection decreases the yaw rate. The engine nozzle deflections are defined positive downwards and to the right. The F-16 also has a leading edge flap, which helps to fly the aircraft at high angles of attack. The deflection of the leading edge flap δ_{LEF} is not controlled directly by the pilot, but is governed by the following transfer function dependent on angle of attack α and static and dynamic pressures:

$$\delta_{LEF} = 1.38\frac{2s + 7.25}{s + 7.25}\alpha - 9.05\frac{\bar{q}}{p_s} + 1.45$$

The control surfaces of the F-16 are driven by servo-controlled actuators to produce the deflections commanded by the flight control system, u, which are the true control variables. The actuators of the control surfaces are modeled as a first-order low-pass filters with certain gain and saturation limits in range and deflection rate. These limits can be found in Table 1. The gains of the actuators are 1/0.136 for the leading edge flap and 1/0.0495 for the other control surfaces.

The maximum values and units for all control variables are given in Table 1

Table 1: The control input units and maximum values

Control	units	MIN.	MAX.	rate limit
Elevator	deg	-25	25	± 60 deg/s
Ailerons	deg	-21.5	21.5	± 80 deg/s
Rudder	deg	-30	30	± 120 deg/s
Leading edge flap	deg	0	25	± 25 deg/s

3 Engine Model

The F-16 aircraft is powered by an afterburning turbofan jet engine, which is modeled taking into account throttle gearing and engine power level lag. The thrust response is modeled with a first-order lag, and the lag time constant is a function of the actual engine power level and the commanded power. The commanded power level to the throttle position is a linear relationship apart from a change of slope when the military power level is reached at 0.77 throttle setting [Nguyen et al., 1979]:

$$P_c(\delta_{th}) = \begin{cases} 64.94\delta_{th} & if \quad \delta_{th} \leq 0.77 \\ 217.38\delta_{th} - 117.38 & if \quad \delta_{th} > 0.77 \end{cases}. \tag{38}$$

Note that throttle position is limited to the range $0 \leq \delta_{th} \leq 1$. As mentioned before the engine power level dynamic response is modeled using a first-order lag. The actual power level derivative P_a is given by [Nguyen et al., 1979]:

$$\dot{P}_a = \frac{1}{\tau_{eng}}(P_c - P_a) \tag{39}$$

where

$$P_c = \begin{cases} P_c & if \quad P_c \geq 50 \text{ and } P_a \geq 50 \\ 60 & if \quad P_c \leq 50 \text{ and } P_a < 50 \\ 40 & if \quad P_c < 50 \text{ and } P_a \geq 50 \\ P_c & if \quad P_c < 50 \text{ and } P_a < 50 \end{cases} \tag{40}$$

$$\frac{1}{\tau_{eng}} = \begin{cases} 5.0 & if \quad P_c \geq 50 \text{ and } P_a \geq 50 \\ \frac{1}{\tau_{eng}}^* & if \quad P_c \leq 50 \text{ and } P_a < 50 \\ 5.0 & if \quad P_c < 50 \text{ and } P_a \geq 50 \\ \frac{1}{\tau_{eng}}^* & if \quad P_c < 50 \text{ and } P_a < 50 \end{cases} \tag{41}$$

$$\frac{1}{\tau_{eng}}^* = \begin{cases} 1.0 & if \quad (P_c - P_a) \leq 25 \\ 0.1 & if \quad (P_c - P_a) \geq 50 \\ 1.9 - 0.036(P_c - P_a) & if \quad 25 < (P_c - P_a) < 50 \end{cases} . \tag{42}$$

The engine thrust data is in tabular form as a function of actual power level, altitude, and Mach number over the ranges 0 m $\leq h \leq$ 15240 m and $0 \leq M \leq 1$, for idle, military, and maximum power settings [Nguyen et al., 1979]. The thrust is computed as follows:

$$F_T = \begin{cases} T_{idle} + (T_{mil} - T_{idle})(\frac{P_a}{50}) & if \quad P_a < 50 \\ T_{mil} + (T_{max} - T_{mil})(\frac{P_a - 50}{50}) & if \quad P_a \geq 50 \end{cases} . \tag{43}$$

The engine angular momentum h_E is assumed to act along the aircraft X body-axis with a constant value of 216.9 kg.m^2/s.

4 Aerodynamic Models

Two different aerodynamic models of the F-16 aircraft are available [Russel, 2003]. The aerodynamic data of the most accurate model, referred to as the high-fidelity model, have been derived from low-speed static and dynamic (force oscillation) wind-tunnel tests conducted with sub-scale models of the F-16 in wind-tunnel facilities at the NASA Ames and Langley Research Centers [Nguyen et al., 1979]. The aerodynamic data in [Nguyen et al., 1979] are given in tabular form and are valid for the following flight envelope:

- $-20 \leq \alpha \leq 90$ degrees;
- $-30 \leq \beta \leq 30$ degrees;
- $0.1 \leq M \leq 0.6$.

In the Appendix of [Lewis and Stevens, 1992] also contain aerodynamic data for an F-16, this model is referred to as the low-fidelity model. The low-fidelity model does not use the leading edge flap. Furthermore, the longitudinal and lateral motions are much more decoupled. These aerodynamic data are valid for the following flight envelope:

- $-10 \leq \alpha \leq 45$ degrees;
- $-30 \leq \beta \leq 30$ degrees;
- $0.1 \leq M \leq 0.6$.

The next section gives the aerodynamic force and moment equations valid for both aerodynamic models. Note, however, that for the low-fidelity model a lot of the coefficients are equal to zero.

5 Aerodynamic Coefficients

A few additional assumptions have been made for the F-16 model:

1. Horizontal tail has only symmetric deflection. Although differential tail deflection is possible for the F-16 (and indeed desirable at flight conditions with high dynamic pressure), no data was available for its implementation.
2. Speed brakes were not used.

Total coefficient equations have been used to sum the various aerodynamic contributions to a given force or moment coefficient as follows.

For the X-axis force coefficient C_{X_T}:

$$C_{X_T} = C_X(\alpha, \beta, \delta_e) + \delta C_{X_{LEF}}\Big(1 - \frac{\delta_{LEF}}{25}\Big) + \frac{q\bar{c}}{2V_T}\Big[C_{X_q}(\alpha) + \delta C_{X_{q_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big]$$

where

$$\delta C_{X_{LEF}} = C_{X_{LEF}}(\alpha, \beta) - C_X(\alpha, \beta, \delta_e = 0^o)$$

For the Y-axis force coefficient C_{Y_T}:

$$\begin{aligned} C_{Y_T} &= C_Y(\alpha, \beta) + \delta C_{Y_{LEF}}\Big(1 - \frac{\delta_{LEF}}{25}\Big) + \Big[\delta C_{Y_{\delta_a}} + \delta C_{Y_{\delta_{a_{LEF}}}}\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big]\Big(\frac{\delta_a}{20}\Big) + \delta C_{Y_{\delta_r}}\Big(\frac{\delta_r}{30}\Big) \\ &+ \frac{rb}{2V_T}\Big[C_{Y_r}(\alpha) + \delta C_{Y_{r_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] + \frac{pb}{2V_T}\Big[C_{Y_p}(\alpha) + \delta C_{Y_{p_{LEF}}}(\alpha)\Big(1 - \frac{\delta_{LEF}}{25}\Big)\Big] \end{aligned}$$

where

$$\begin{aligned} \delta C_{Y_{LEF}} &= C_{Y_{LEF}}(\alpha, \beta) - C_Y(\alpha, \beta) \\ \delta C_{Y_{\delta_a}} &= C_{Y_{\delta_a}}(\alpha, \beta) - C_Y(\alpha, \beta) \\ \delta C_{Y_{\delta_{a_{LEF}}}} &= C_{Y_{\delta_{a_{LEF}}}}(\alpha, \beta) - C_{Y_{LEF}}(\alpha, \beta) - \delta C_{Y_{\delta_a}} \\ \delta C_{Y_{\delta_r}} &= C_{Y_{\delta_r}}(\alpha, \beta) - C_Y(\alpha, \beta) \end{aligned}$$

For the Z-axis force coefficient C_{Z_T}:

$$C_{Z_T} = C_Z(\alpha,\beta,\delta_e) + \delta C_{Z_{LEF}}\Big(1-\frac{\delta_{LEF}}{25}\Big) + \frac{q\bar{c}}{2V_T}\Big[C_{Z_q}(\alpha) + \delta C_{Z_{q_{LEF}}}(\alpha)\Big(1-\frac{\delta_{LEF}}{25}\Big)\Big]$$

where

$$\delta C_{Z_{LEF}} = C_{Z_{LEF}}(\alpha,\beta) - C_Z(\alpha,\beta,\delta_e = 0^o)$$

For the rolling-moment coefficient C_{l_T}:

$$\begin{aligned} C_{l_T} &= C_l(\alpha,\beta,\delta_e) + \delta C_{l_{LEF}}\Big(1-\frac{\delta_{LEF}}{25}\Big) + \Big[\delta C_{l_{\delta_a}} + \delta C_{l_{\delta_{a_{LEF}}}}\Big(1-\frac{\delta_{LEF}}{25}\Big)\Big]\Big(\frac{\delta_a}{20}\Big) \\ &+ \delta C_{l_{\delta_r}}\Big(\frac{\delta_r}{30}\Big) + \frac{rb}{2V_T}\Big[C_{l_r}(\alpha) + \delta C_{l_{r_{LEF}}}(\alpha)\Big(1-\frac{\delta_{LEF}}{25}\Big)\Big] \\ &+ \frac{pb}{2V_T}\Big[C_{l_p}(\alpha) + \delta C_{l_{p_{LEF}}}(\alpha)\Big(1-\frac{\delta_{LEF}}{25}\Big)\Big] + \delta C_{l_\beta}(\alpha)\beta \end{aligned}$$

where

$$\begin{aligned} \delta C_{l_{LEF}} &= C_{l_{LEF}}(\alpha,\beta) - C_l(\alpha,\beta,\delta_e = 0^o) \\ \delta C_{l_{\delta_a}} &= C_{l_{\delta_a}}(\alpha,\beta) - C_l(\alpha,\beta,\delta_e = 0^o) \\ \delta C_{l_{\delta_{a_{LEF}}}} &= C_{l_{\delta_{a_{LEF}}}}(\alpha,\beta) - C_{l_{LEF}}(\alpha,\beta) - \delta C_{l_{\delta_a}} \\ \delta C_{l_{\delta_r}} &= C_{l_{\delta_r}}(\alpha,\beta) - C_l(\alpha,\beta,\delta_e = 0^o) \end{aligned}$$

For the pitching-moment coefficient C_{m_T}:

$$\begin{aligned} C_{m_T} &= C_m(\alpha,\beta,\delta_e) + C_{Z_T}\Big[x_{cg_r} - x_{cg}\Big] + \delta C_{m_{LEF}}\Big(1-\frac{\delta_{LEF}}{25}\Big) \\ &+ \frac{q\bar{c}}{2V_T}\Big[C_{m_q}(\alpha) + \delta C_{m_{q_{LEF}}}(\alpha)\Big]\Big(1-\frac{\delta_{LEF}}{25}\Big) + \delta C_m(\alpha) + \delta C_{m_{ds}}(\alpha,\delta_e) \end{aligned}$$

where

$$\delta C_{m_{LEF}} = C_{m_{LEF}}(\alpha,\beta) - C_m(\alpha,\beta,\delta_e = 0^o)$$

For the yawing-moment coefficient C_{n_T}:

$$\begin{aligned} C_{n_T} &= C_n(\alpha,\beta,\delta_e) + \delta C_{n_{LEF}}\Big(1-\frac{\delta_{LEF}}{25}\Big) + \Big[\delta C_{n_{\delta_a}} + \delta C_{n_{\delta_{a_{LEF}}}}\Big(1-\frac{\delta_{LEF}}{25}\Big)\Big]\Big(\frac{\delta_a}{20}\Big) \\ &+ \delta C_{n_{\delta_r}}\Big(\frac{\delta_r}{30}\Big) + \frac{rb}{2V_T}\Big[C_{n_r}(\alpha) + \delta C_{n_{r_{LEF}}}(\alpha)\Big(1-\frac{\delta_{LEF}}{25}\Big)\Big] \\ &+ \frac{pb}{2V_T}\Big[C_{n_p}(\alpha) + \delta C_{n_{p_{LEF}}}(\alpha)\Big(1-\frac{\delta_{LEF}}{25}\Big)\Big] + \delta C_{n_\beta}(\alpha)\beta - C_{Y_T}\Big[x_{cg_r} - x_{cg}\Big]\frac{\bar{c}}{b} \end{aligned}$$

where

$$
\begin{aligned}
\delta C_{n_{LEF}} &= C_{n_{LEF}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_e = 0^o) \\
\delta C_{n_{\delta_a}} &= C_{n_{\delta_a}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_e = 0^o) \\
\delta C_{n_{\delta_{a_{LEF}}}} &= C_{n_{\delta_{a_{LEF}}}}(\alpha, \beta) - C_{n_{LEF}}(\alpha, \beta) - \delta C_{n_{\delta_a}} \\
\delta C_{n_{\delta_r}} &= C_{n_{\delta_r}}(\alpha, \beta) - C_n(\alpha, \beta, \delta_e = 0^o)
\end{aligned}
$$

6 MATLAB/Simulink script files

Descriptions are included inside the files. Most files are based on or copied from the F-16 models by R. S. Russel [Russel, 2003] and Y. Huo. The S-functions `F16_dyn.c` and `F16_dynam.c` both contain the aircraft dynamics, they only differ in the use of quaternions vs. Euler angles. The `runF16model.m` script file allows the user to trim the aircraft for steady-state flight conditions.

```
runF16model.m
tgear.m
trim_fun.m

F16_dyn.c
F16_dynam.c
lofi_f16_aerodata.c
hifi_f16_aerodata.c
engine_model.c
mexndinterp.c
ISA_atmos.c
```

Appendix

Table 2: F-16 mass and geometric data

Parameter	Symbol	Value
aircraft mass (kg)	m	9295.44
reference wing span (m)	b	9.144
reference wing area (m^2)	S	27.87
mean aerodynamic chord (m)	$\bar{c}$	3.45
roll moment of inertia (kg.m^2)	I_x	12874.8
pitch moment of inertia (kg.m^2)	I_y	75673.6
yaw moment of inertia (kg.m^2)	I_z	85552.1
product moment of inertia (kg.m^2)	I_{xz}	1331.4
product moment of inertia (kg.m^2)	I_{xy}	0.0
product moment of inertia (kg.m^2)	I_{yz}	0.0
c.g. location (m)	x_{cg}	$0.3\bar{c}$
reference c.g. location (m)	x_{cg_r}	$0.35\bar{c}$
engine angular momentum (kg.m^2/s)	h_E	216.9

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