Experiments on Aristotle's Thesis:

Towards an experimental philosophy of conditionals

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Abstract

Two experiments ($N_1 = 141$, $N_2 = 40$) investigate two versions of Aristotle's Thesis for the first time. Aristotle's Thesis is a negated conditional, which consists of one propositional variable with a negation either in the antecedent (version 1) or in the consequent (version 2). This task allows to infer if people interpret indicative conditionals as material conditionals or as conditional events. In the first experiment I investigate between-participants the two versions of Aristotle's Thesis crossed with abstract versus concrete task material. The modal response for all four groups is consistent with the conditional event and inconsistent with the material conditional interpretation. This observation is replicated in the second experiment. Moreover, the second experiment rules out scope ambiguities of the negation of conditionals. Both experiments provide new evidence against the material conditional interpretation of conditionals and support the conditional event interpretation. Finally, I discuss implications for modeling indicative conditionals and the relevance of this work for experimental philosophy.

Keywords: experimental philosophy; conditionals; probability; negation

1 Introduction

Experimental philosophers investigated people's intuitions already on a wide variety of philosophical topics, including causation, consciousness, cross-cultural intuitions, epistemology, morality, free will, and intentional action (see, e.g., Phillips, 2011; Knobe and Nichols, 2008; Feltz, 2009). Conditionals, however, have not been discussed by experimental philosophers yet. This paper aims to extend the domain of experimental philosophy to conditionals. After a brief review of philosophical and psychological intuitions on probabilistic interpretation of indicative conditionals, I report two new experiments on reasoning about conditionals to clarify the interpretation and negation of indicative conditionals.

There is a long tradition of psychological investigations of conditionals. Standard tasks include Wason's selection task, truth table tasks, and conditional elimination tasks (like modus ponens; see, e.g., Evans, Newstead, and Byrne, 1993). The propositional calculus was taken for granted as *the* rationality norm: rational inferences are consistent with the laws of logic and indicative conditionals (of the form "If A, then B") should be interpreted as material conditionals (denoted by $A \supset B$). People's inferences, however, diverged from the rationality postulates of classical logic.

Philosophers argued for probabilistic interpretations of indicative conditionals by relating conditionals to conditional probability, $P(B|A)$ (e.g., Adams, 1975; Bennett, 2003; Douven, 2008; Ramsey, 1978). The argument of the conditional probability function is the conditional event $B|A$. The conditional event cannot be captured within the framework of classical logic. Contrary to the material conditional ($A \supset B$) and the conjunction ($A \wedge B$), the conditional event cannot be expressed by any Boolean function: the conditional event is *void*, if the antecedent (A) is false (see Table 1).

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. . . . *insert Table 1 about here* . . . .

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There are striking *a priori* reasons, why indicative conditionals are not material ones. As an example, imagine that Jones "is about to be dealt a five card poker hand from a shuffled deck of 52 cards" (Adams, 2005, p. 1). Someone asserts:

If Jones's first card is an ace, then Jones's second card is an ace.

How sure can you be, that the sentence in the box holds? If you interpret the sentence as a conditional event and assign a conditional probability, you obtain a very low probability value[1], which is intuitively plausible,

$$P(\text{Jones's second card is an ace} \,|\, \text{Jones's first card is an ace}) = \frac{3}{51} \simeq 0.06\,.$$

If you interpret the sentence in the box as a material conditional, you obtain in this scenario an intuitively implausible high probability value[2],

$$P(\text{Jones's first card is an ace} \supset \text{Jones's second card is an ace}) = \frac{205}{221} \simeq 0.93\,.$$

Some of the arguments in favor of the conditional event interpretation have been confirmed empirically. Contrary to the material conditional, the conditional event interpretation avoids the paradoxes of the material conditional and people do not endorse these paradoxes (Pfeifer and Kleiter, 2011). Premise strengthening and contraposition do not hold under the conditional event interpretation and people do not endorse these argument forms (Pfeifer and Kleiter, 2010).

In recent years, probabilistic rationality norms emerged in the psychology of reasoning to better deal with the defeasibility and uncertainty and to match closer everyday inference (e.g., Evans and Over, 2004; Oaksford and Chater, 2009; Pfeifer and Kleiter, 2005b, 2009). Within these probabilistic approaches, a new hypothesis concerning the interpretation of conditionals emerged: people interpret indicative conditionals (If A, then B) as conditional events ($B|A$). Recent studies (e.g., Fugard, Pfeifer, and Mayerhofer, 2011; Fugard, Pfeifer, Mayerhofer, and Kleiter, in press; Oberauer, 2006; Over, Hadjichristidis, Evans, Handley, and Sloman, 2007) provide strong empirical evidence for this hypothesis.

The present paper further investigates the conditional event hypothesis. Specifically, it investigates two versions of Aristotle's Thesis for the first time. Aristotle's Thesis is a negated conditional, which consists of one propositional variable with a negation either in the antecedent (version 1) or in the consequent (version 2):

(AT #1) $\neg(\neg A \to A)$

(AT #2) $\neg(A \to \neg A)$

"$\neg$" denotes negation.[3] "$A \to B$" denotes the indicative conditional *If A, then B*, where the semantics of $\to$ is not specified. AT #1 and AT #2 are intuitively plausible. Consider an instance in natural language:

It is not the case that: If I do not win the lottery, then I win the lottery.

Likewise, it is plausible to assert

It is not the case that: If I win the lottery, then I do not win the lottery.

In classical logic, where "$\rightarrow$" is interpreted as a material conditional, (AT #1) and (AT #2) are not theorems. Both formulas are contingent ("$\equiv$" denotes equivalence):

$$\neg(\neg A \supset A) \quad \equiv \quad \neg A \wedge \neg A \quad \equiv \quad \neg A$$

$$\neg(A \supset \neg A) \quad \equiv \quad A \wedge A \quad \equiv \quad A$$

In this paper, I propose an interpretation that justifies the rationality of high beliefs in AT #1 and AT #2. Specifically, I formalize AT in terms of coherence based probability logic (Pfeifer and Kleiter, 2006a, 2009).

In the psychology of reasoning three interpretations of indicative conditionals (If A, then B) are currently debated: the conditional event interpretation ($P(B|A)$), the material conditional interpretation ($P(A \supset B)$), and the conjunction interpretation ($P(A \wedge B)$). The conjunction interpretation is discussed in the theory of mental models (Johnson-Laird and Byrne, 2002) and the suppositional theory of conditional reasoning (Evans and Over, 2004). Both theories predict, roughly speaking, that if people process conditionals superficially, then they use the conjunction interpretation.

The coherence approach to probability goes back to De Finetti (1980, 1974) and more recent work includes, e.g., Walley (1991); Lad (1996); Biazzo and Gilio (2000); Coletti and Scozzafava (2002). Coherence is in the tradition of subjective probability theory in which probabilities are conceived as *degrees of belief*. Degrees of belief are coherent descriptions of incomplete knowledge states. One key feature is that the coherence approach defines the probability function on an *arbitrary* family of conditional events. Therefore, it does not require a complete algebra as in the standard approach to probability. Conditional probability, $P(B|A)$, is a *primitive* notion. The probability value is assigned *directly* to the conditional event, $B|A$, as a whole (and not by definition via the fraction of the joint and the marginal probability, $P(A \wedge B)/P(A)$). Therefore, the probability axioms are formulated for conditional probabilities in the framework of coherence and not for absolute probabilities (as it is done in the standard approach to probability).

Coherence based probability logic defines the consequence relation as a *deductive* one. The probabilistic inference problem consists of how to transmit the probabilities of the premises to the probability of the conclusion. Usually, the coherent probability of the conclusion is constrained by a lower and an upper probability. The coherent conclusion of the modus ponens, for example, is in the interval $xy \leq P(B) \leq$

$xy+1-x$, where the two premises are $P(A)=x$ and $P(B|A)=y$, respectively.[4] As only two probabilities are given, the coherent probability of the conclusion is *imprecise*. If $P(B|\neg A)=z$ is added to the premise set of the probabilistic modus ponens, then the resulting argument form allows for inferring a *precise* probability value of the conclusion: $P(B)=xy+(1-x)z$.

Coherence based probability logic (Pfeifer and Kleiter, 2006a, 2009) has received strong empirical support in a series of experiments on the rules of the nonmonotonic System P (Pfeifer and Kleiter, 2003, 2005a, 2006b), the paradoxes of the material conditional (Pfeifer and Kleiter, 2011), the conditional syllogisms (Pfeifer and Kleiter, 2007), and on how people interpret conditionals (Fugard et al., in press, 2011).

Table 2 lists the probability logical predictions according to the different interpretations of indicative conditionals. The conditional event and the conjunction interpretation predict that people should hold a strong belief in both versions of AT: the probability value 1 is the only coherent assessment. The material conditional interpretation predicts that people cannot tell whether AT holds: any (point or interval) value from zero to one is coherent. Experiment 1 investigates these predictions empirically.

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.... *insert Table 2 about here* ....

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2 Experiment 1

2.1 Method and design

The sample consists of 141 psychology students (110 females and 31 males). The median age of the sample is 21 (1st Qu. = 20, 3rd Qu. =23). 91% of the participants were in their third semester.

The data were collected in a lecture hall during an introductory course on cognitive neuroscience. One week before the experiment, the students learned classical truthtables, including the material conditional and the concepts of logical truth, logical falsehood and logical contingency. At the very beginning of the unit, in which the experiment took place, the truthtables as well as related concepts were repeated. Then the lecture continued with an unrelated topic. In the middle of the lecture unit, the experiment started.

Four versions of the task material were distributed in such a way that the seating-distance between the participants of each condition was maximized. The four conditions consisted in an abstract and in a concrete version of AT #1 and AT #2.

The abstract version of AT #1 was formulated as follows:

> The letter "*A*" denotes a sentence, like "It is raining".
>
> There are sentences, where you can infer only on the basis of their logical form, whether they are guaranteed to be false or guaranteed to be true. For example:
>
> - "*A* and not-*A*" is guaranteed to be false.
> - "*A* or not-*A*" is guaranteed to be true.
>
> There are sentences, where you cannot infer only on the basis of their logical form, whether they are true or false. The sentence "*A*" ("It is raining."), for example, can be true but it can just as well be false: this depends upon whether it is actually raining.

In this part of the instruction the concepts "logical truth", "logical falsehood", and "logical contingency" are explained once again to the participants, without mentioning these technical terms explicitly. To make clear, that the task concerns the natural language version of AT as a whole and to avoid ambiguities of the scope of the question, AT was put into a box:

> Evaluate the following sentence (please tick exactly one alternative):
>
> $\boxed{\text{It is not the case, that: If not-}A\text{, then }A.}$
>
> | | |
> |---|---|
> | The sentence in the box is guaranteed to be false | ☐ |
> | The sentence in the box is guaranteed to be true | ☐ |
> | One cannot infer whether the sentence is true or false | ☐ |

The abstract version of AT #2 was identical to the abstract version of AT #1 except for the sentence in the box: the conditional "If not-*A*, then *A*" was replaced by "If *A*, then not-*A*".

The concrete version of AT was formulated by replacing the abstract letters by concrete objects. A further difference to the abstract version was the use of an implicit negation in the conditional. Negations are hard to process in general. Moreover, concrete task material is easier to process than abstract material.

Thus, the concrete versions were hypothesized to be easier to process for the participants than the abstract versions.

In the concrete versions of AT, the participants were asked to imagine that there is either a dog or a cat behind a door, but not both. As in the abstract version of the task, the concepts “logical truth”, “logical falsehood”, and “logical contingency” were introduced informally. Then, the participants were asked to

> Evaluate the following sentence (please tick exactly one alternative):
>
> It is not the case, that: If there is a cat behind the door, then there is a dog behind the door.

The response-format was the same as in the abstract versions. The other concrete version of AT differed only in one respect: the words “cat” and “dog” changed their positions in the conditional in the box.

Strictly speaking, there is no difference between AT #1 and AT #2 in the concrete version of the task. The vignette makes clear that “cat” means “*not*-dog” and “dog” means “*not*-cat”.

In all versions of the task, the task was presented together with the instructions on one page. This helps to minimize working memory demands.

2.2 Results and discussion

In all four conditions of the experiment the participants were asked to rank on a scale how clear and comprehensible the task was to them. Furthermore, the participants evaluated the confidence in the correctness of their solution and the task difficulty. Figure 1 presents the results of the participants’ task evaluations. The mean rating of the task comprehensibility is close to “very clear”, which indicates that the participants were not swamped by processing the conditional and the two negations. Moreover, the mean subjective confidence in the correctness of the participants’ responses and the task difficulty were in the middle regions of the respective scales. This suggests that the participants did not opt out of the task. If a task is obscure or if it is perceived to be too easy or too hard, then there is a danger that the participants do not engage themselves properly in the task, and—in the worst case—opt out of doing the task. In sum, the results suggest that the task and the vignette stories are not obscure, and perceived as being neither too easy nor too difficult.

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There are also no statistically significant differences between the responses in AT #1 and AT #2. Therefore, the data of AT #1 and AT #2 are pooled. Figure 2 summarizes the main results of Experiment 1. The modal response is consistent with the conditional event interpretation and inconsistent with the material conditional interpretation of conditionals. There is a slightly higher proportion of participants in the concrete than in the abstract condition who hold a strong belief in AT. However, this difference is statistically not significant.

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. . . . *insert Figure 2 about here* . . . .

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A minority of the participants expressed a strong belief that the sentence in the box is false. Exploration questions[5] after the experiment revealed that some participants reasoned about the conditional only. These participants overlooked that the conditional is negated. If this negation is ignored, these ratings are perfectly rational under the conditional event interpretation: coherence requires $P(A|\neg A) = 0$ and $P(\neg A|A) = 0$.

One key result of Experiment 1 is that the conditional event interpretation predicts the modal response of the participants. Another key result indicates that the literal formalization of AT by the material conditional interpretation does not predict the modal response of the participants. The data suggest that people do not interpret AT #1 as $\neg(\neg A \supset A)$ and that they do not interpret AT #2 as $\neg(A \supset \neg A)$. Thus, AT provides a watershed to experimentally differentiate between the material conditional and the conditional event interpretations of indicative conditionals. However, as noted above, AT alone does not distinguish between the conditional event and the conjunction interpretation: both predict that AT is guaranteed to be true. Experiment 2 will address this issue.

Proponents of the material conditional interpretation may argue that the results of Experiment 1 provide evidence against the wide scope reading of the negation of conditionals,

$$\neg \underbrace{(A \to \neg A)}_{\text{wide scope}} .$$

One may argue that people interpret indicative conditionals as material conditionals and material conditionals are negated by negating the consequent,

$$(A \to \neg \underbrace{\neg A)}_{\text{narrow scope}} \quad .$$

Consequently, AT # 1 reduces to $\neg A \supset \neg A$ and AT # 2 reduces to $A \supset A$, which leads to the same predictions as given by the conditional event interpretation.[6]

Experiment 2 is designed to (i) clarify this scope ambiguity, (ii) differentiate between the conjunction and the conditional event interpretation of indicative conditionals, and to (iii) improve the experimental conditions.

3 Experiment 2

3.1 Method and design

Forty students (20 females and 20 males) of the University of Salzburg were tested individually in experimental rooms of the Psychology Department. Psychology students and students with a formal background were not included in the sample. The participants received 5 € for participation. Between participant explicit ($n_1 = 20$) versus implicit negation ($n_2 = 20$) conditions were varied and each participant had to solve 12 tasks (see Table 3). All tasks were concrete.

As in Experiment 1, the first part of the instruction explains the concepts "logical truth", "logical falsehood", and "logical contingency" without mentioning technical terms. In the explicit negation condition, the participants were asked to imagine the following situation:

> Hans expects to be visited by Thea and Ida. He is sitting in his room. Suddenly someone knocks at the door. Hans is absolutely certain, that either Thea or Ida is knocking.
>
> Evaluate the following sentence (please tick exactly one alternative):

It is **not** the case, that: **If** Ida knocks, **then** Ida **does not** knock.

The response format was identical to the one used in Experiment 1. The logical structure of the sentence in the box was formatted in boldface to reduce the probability that participants overlook the negation in front of the conditional. Since the aim of the experiment is to investigate intuitions about the *degrees of belief* in (negated) conditionals an epistemic component (“Hans is absolutely certain . . . ”) is added in Experiment 2. In the implicit versions of AT, “Ida **does not** knock” was replaced by “Thea knocks”.

The vignette stories were adapted to other argument forms to differentiate between the conditional event and the conjunction interpretation. Table 3 lists the order of the task items in both conditions. Items #1 and #3 are designed to replicate the findings of AT #2 and AT #1, respectively, of Experiment 1. Item #2 may be called “negated reflexivity”. It differentiates between the narrow and the wide scope reading of the negation of the material conditional. Item #4 (“reflexivity”) differentiates between the conditional event interpretation and the conjunction interpretation of indicative conditionals. Items # 5 and # 6 are control items.

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. . . . *insert Table 3 about here* . . . .

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The items #7–#10 are adapted from Pfeifer and Kleiter (2011). They serve (i) to replicate findings and (ii) to provide further possibilities to differentiate among the material conditional, conditional event, and conjunction interpretation. Items #7–#10 correspond to a version of the probabilistic truth table task, where the participants are instructed to imagine a pack of 120 cards. On each card, there is either a circle or a square, either in red or in blue. The pack consists of 40 red circle cards, 40 red square cards, 20 blue circle cards and 20 blue square cards. The pack is shuffled and then one card is randomly chosen. One cannot see what is printed on this card. The task consists in evaluating four conditionals on a scale with the labels “does not hold for sure” and “holds for sure”. The four conditionals and the probability logical predictions are contained in Figure 3. The participants’ interpretation can be inferred from their degree of belief in the conditional. Item #11 and #12 correspond to two paradoxes of the material conditional.

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3.2 Results and discussion

There is no statistically significant difference between the two between-participant conditions. Therefore, the subsequent analysis is conducted on the pooled data. Experiment 2 replicates the results of Experiment 1: the data are consistent with the conditional event interpretation and inconsistent with the material conditional interpretation of indicative conditionals (see Table 3). Moreover, negated reflexivity (item #2) rules out the narrow scope reading of the material conditional. The conjunction interpretation is ruled out by reflexivity (item #4), the paradox of the material conditional (item #12), and the results of the probabilistic truth table task (items #7–#10).

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. . . . *insert Table 4 about here* . . . .

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Scoring the data in the six tasks reveals that the mean consistency of the responses with the conditional event was the highest one out of the four interpretations (see Table 4). This reflects again, that the conditional event is the best predictor for the conditional event responses.

4 Concluding remarks

Mental probability logic (Pfeifer and Kleiter, 2005b, 2009), the suppositional theory of conditional reasoning (Evans and Over, 2004), and the probabilistic approach by Oaksford and Chater (2009) are examples of recent psychological theories that argue for the conditional event interpretation of indicative conditionals. This study is in line with this research and provides new evidence for the conditional event interpretation.

This paper investigates Aristotle's Thesis, reflexivity and negated reflexivity for the first time empirically. Moreover, the second experiment resolves scope ambiguities of the negation of conditionals. Neither people without training in logic (Experiment 2) nor people who just learned the truthtables and the material conditional (Experiment 1) interpret conditionals as material conditionals. The modal response pattern in all tasks corresponds to the conditional event interpretation of conditionals.

The truth functions of the material conditional and the conjunction correspond to the truth conditions of the explicit and the implicit mental models, respectively, of basic conditionals. According to the theory of mental models the core meaning of indicative conditionals is the material conditional (Johnson-Laird and Byrne, 2002). The present data do not support this approach.

The tasks on Aristotle's theses differ in several respects to previous studies on conditional reasoning. First, the argument form is an inference from the empty premise set. In the abstract version, the belief in the negated conditional may be established by reasoning about a sentence in the box (i.e., the conclusion) only. In the concrete versions, the information communicated in the instructions before the box does not belong to the premise set. This information explains the relationship between the cat and the dog in this scenario. Thus, the logical form corresponds to an inference from the empty premise set as well. Second, Aristotle's thesis consists of only one propositional variable (A). However, the logical form is complex, since it is composed of two negations and one conditional. Wason's selection task, the tasks related to conditional elimination inferences (like the suppression tasks, modus ponens, modus tollens, etc.), and the variants of truth table tasks are not inferences from the empty premise set and usually involve two propositional variables.

The narrow and the wide scope readings of the negation of a conditional *If A, then B* are well defined for material conditionals ($A \supset \neg B$ and $\neg(A \supset B)$, respectively). The negation of a conditional event $B|A$, however, is well defined for the narrow scope reading only ($\neg B|A$). One might propose that negating a conditional event means that one is completely uncertain about $B|A$. Using imprecise probabilities, this could mean that the $P(B|A)$ is probabilistically uninformative, i.e., $0 \leq P(B|A) \leq 1$ is coherent. Assigning the unit interval expresses a situation of complete ignorance about $B|A$. However, the present data do not support this hypothesis: for almost-all participants Aristotle's thesis is probabilistically informative.

The data of both experiments show that the acceptability/assertability conditions of $A \to B$ are consistent with $P(B|A)$ but inconsistent with $P(A \supset B)$. Some philosophers (e.g., Lewis, 1976; Grice, 1975) claim that conditionals are truth functional and that $A \to B$ is acceptable/assertable *iff* (i) $P(A \supset B)$ is high, and (ii) $P(A \supset B|A)$ is high (and close to $P(A \supset B)$). On the first sight, this could be a way to save the material conditional interpretation of indicative conditionals. This is not the case: Jackson (1987, p. 31) notes that condition (i) and (ii) imply that $P(B|A)$ is high.

Connexive logicians investigate a branch of non-classical logic where a standard logical vocabulary is used but certain non-theorems of classical logic like AT #1 and AT #2 are theorems (McCall, 1966; Angell, 2002). An implication that satisfies Aristotle's theses and the Boethius' theses ($(A \to B) \to \neg(A \to \neg B)$ and $(A \to \neg B) \to \neg(A \to B)$) and where $\to$ cannot be understood as a biconditional (i.e., $(A \to B) \to (B \to A)$ is not a theorem) is called a *connexive implication* (Wansing, 2010).

Aristotle's thesis provides an important empirical watershed between the conditional event and the material conditional interpretation of indicative conditionals. Other empirically interesting argument forms that allow for investigating different interpretations of conditionals include the paradoxes of the material conditional (Pfeifer and Kleiter, 2011) and (non)monotonic argument forms (Pfeifer and Kleiter, 2005a, 2009, 2010). The results point from different angles to the same direction: people's intuitions on the meaning of conditionals converge on the conditional event interpretation.

The present paper provides a new formalization of Aristotle's thesis in probability logical terms. Its main empirical result is that coherent conditional probabilities are natural building blocks for modeling indicative conditionals. I am convinced that "armchair philosophy" and careful experimental work can fruitfully interact. On the one hand, formal philosophy provides tools to make psychological hypotheses precise: without a proper formalism many fruitful hypotheses cannot even be formulated. On the other hand, experimental studies can empirically validate philosophical theories. Empirical investigations provide important external quality criteria for logical theories which are beyond the purely formal ones (like soundness or completeness). The present study illustrates how the domain of experimental philosophy is extended to conditionals.

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Notes

[1]Of all 52 cards only four ones are aces. If the first card is an ace, three aces are left for drawing a second ace out of the 51 remaining cards. Thus, $P(\text{Jones's second card is an ace} \,|\, \text{Jones's first card is an ace}) = 3/51 \simeq 0.06$.

[2]Let "A" denote the antecedent and "B" denote the consequent of the conditional. $P(A \supset B) = 1 - P(A \wedge \neg B) = 1 - \left(\frac{4}{52} \times \frac{48}{51}\right) = \frac{205}{221} \simeq 0.93$.

[3]The name "Aristotle's Thesis" was coined by McCall (1966). Aristotle wrote in his *Prior Anlaytics* "…if B is not great, B itself is great. But this is impossible." (quoted after Lukasiewicz, 1957, p. 50).

[4]The law of total probability states that $P(B) = P(B|A) \times P(A) + P(B|\neg A) \times P(\neg A)$. As $P(\neg A) = 1 - P(A)$, the only unknown value is $P(B|\neg A)$. If $P(B|\neg A) = 0$, then $P(B) = P(B|A) \times P(A)$ (which is the tightest coherent lower probability bound of the conclusion). If $P(B|\neg A) = 1$, then $P(B) = P(B|A) \times P(A) + 1 - P(A)$ (which is the tightest coherent upper probability bound of the conclusion).

[5]The participants were asked informally in the lecture hall how they understood the task material and how they solved the task.

[6]I thank Igor Douven for this point.

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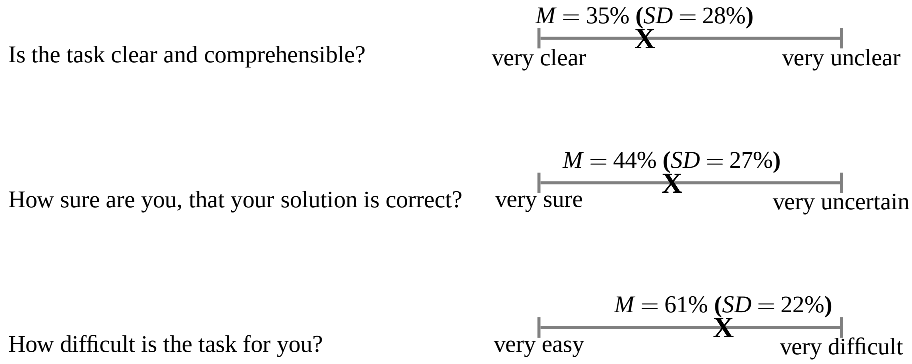

Figure 1: Mean ratings of the task comprehensibility, confidence in correctness, and difficulty (Experiment 1, $N_1 = 141$).

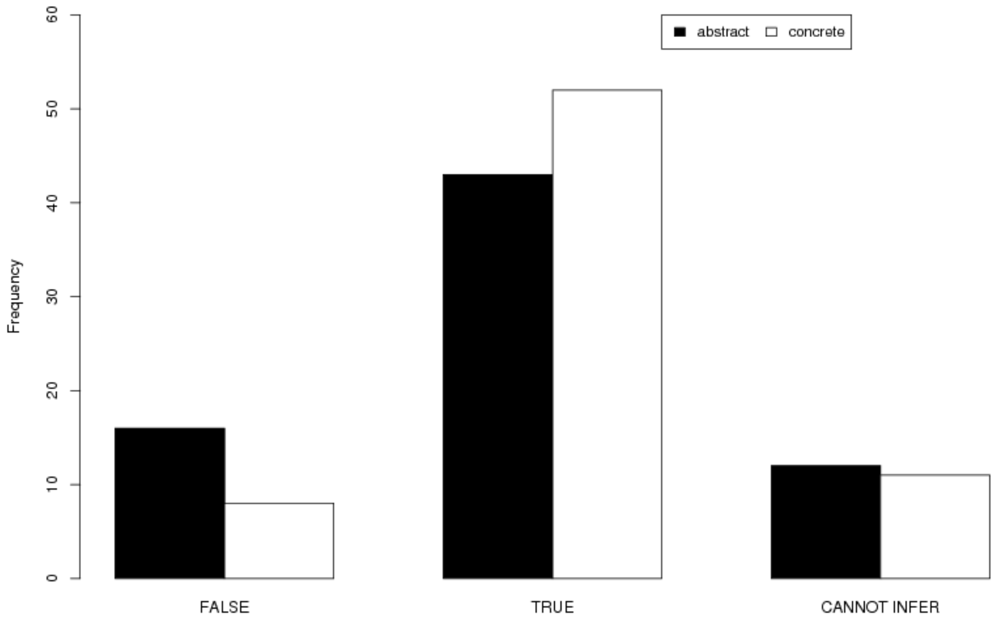

Figure 2: Response frequencies in Experiment 1 (pooled data of AT #1 and #2; $N_1 = 141$).

		Material conditional	*Conjunction*	*Conditional event*
A	B	$A \supset B$	$A \wedge B$	$B \vert A$
true	*true*	*true*	*true*	*true*
true	*false*	*false*	*false*	*false*
false	*true*	*true*	*false*	void
false	*false*	*true*	*false*	void

Table 1: The three most prominent psychological predictions of the interpretation of indicative conditionals. They differ only in the last two lines, where the antecedent (A) is false. The conditional event is *partially* truth-functional, as it is *void*, if the antecedence is false.

AT	$P(\cdot \supset \cdot)$	$P(\cdot \wedge \cdot)$	$P(\cdot\|\cdot)$
$\neg(\neg A \rightarrow A)$	$0 \leq P(\neg A) \leq 1$	$P(\neg(\neg A \wedge A)) = 1$	$P(\neg A\|\neg A) = 1$
$\neg(A \rightarrow \neg A)$	$0 \leq P(A) \leq 1$	$P(\neg(A \supset \neg A)) = 1$	$P(\neg\neg A\|A) = 1$

Table 2: Probability logical interpretations of the two versions of Aristotle's Thesis (AT) in terms of the material conditional ($P(\cdot \supset \cdot)$), the conjunction ($P(\cdot \wedge \cdot)$), and the conditional event ($P(\cdot|\cdot)$).

Item	Argument form	Prediction				Responses in percent		
			WS	NS				
		$\cdot\vert\cdot$	$\cdot\supset\cdot$	$\cdot\supset\cdot$	$\cdot\wedge\cdot$	T	F	CT
#1	$\neg(A\rightarrow\neg A)$	T	CT	T	T	**78**	18	5
#2	$\neg(A\rightarrow A)$	F	F	CT	CT	10	**88**	2
#3	$\neg(\neg A\rightarrow A)$	T	CT	T	T	**80**	13	8
#4	$A\rightarrow A$	T	T	T	CT	**93**	3	5
#5	$A\rightarrow B$	CT	CT	CT	CT	0	13	**88**
#6	$\neg(A\rightarrow B)$	CT	CT	CT	CT	20	3	**78**
#11	from B infer $A\rightarrow B$	U	H		U	40	0	**60**
#12	from B infer $A\rightarrow\neg B$	U	H		L	5	30	**65**

Table 3: Results ($N_2 = 40$) of Experiment 2. WS=wide and NS=narrow scope reading of negated material conditionals, CT=can't tell, T=true, F=false, U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability. Items #7–#10 are part of the probabilistic truth table task (see text and Figure 3). Conditional event is the best predictor (**bold**).

Interpretation		Mean	SD
	Scope		
$\cdot\vert\cdot$		4.6	1.2
$\cdot\supset\cdot$	narrow	3.0	0.9
$\cdot\wedge\cdot$		2.6	0.9
$\cdot\supset\cdot$	wide	2.4	0.8

Table 4: Scoring of the items #1–#4 and items #11–#12 ($N_2 = 40$, 6 tasks, min $= 0$, max $= 6$). The conditional event obtains the highest score.

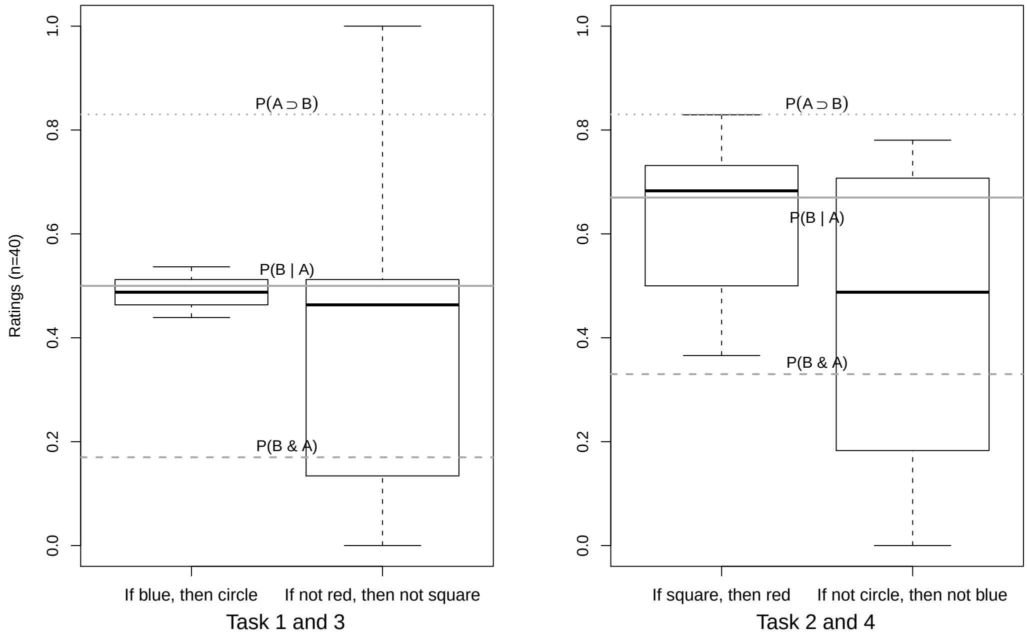

Figure 3: Results of the probabilistic truth table task (Experiment 2, $N_2 = 40$). The boxes contain 50% of the responses, the thick line indicates the median. The whiskers indicate 1.5 $\times$ the interquartile range. Normative predictions are printed in gray. Conditional probability is the best predictor.