Identity, Quantification, and Number

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1.

When I was a student, I was taught that there was an intimate connection between quantification, identity, and number.

First quantification and number. To say that there is something of a certain sort, I was taught--to 'quantify over' things of that sort--is to say that there is at least one thing of that sort. I was taught that these expressions are synonymous:

1. There is an F.
2. Something is F.
3. There is at least one thing that is F.

For that matter, I was taught that the above expressions are synonymous with this one:

4. The number of Fs is one or more.

This last expression is interesting, for it implies that there is a number--a unique number--of Fs, whatever F may be. This suggests that the following schema holds generally:

5. If anything is F, then there is some number of Fs.

Take any predicate you like--'tree', 'rock', 'thing within a mile of here', or even 'thing' generally: if anything satisfies that predicate, there is always some number of things satisfying it (at a given time, perhaps).

This claim is problematic. How many bald men are in the village of Little Piddling? Suppose ten men in Little Piddlng are definitely bald, five more are borderline cases, and the rest are definitely not bald. What is the number of bald men in the village? It's tempting to say that there is no number of them. At any rate no number is a true answer to the question, How many bald men are there in Little Piddling? (Whether you agree depends on your view of vaguenss. On the epistemic theory, some number between ten and fifteen is the number of bald men in Little Piddling, but we can never know which it is. Supervaluationists will say that, although there is a number of bald men in the village, no number is that number.) But even if this is right, we can still say something about how many bald men there are in the village: there are at least ten and no more than fifteen. The question of how many there are has an answer, though perhaps not a precise one.

5 is problematic for another reason too: there might be things that are

too many for there to be any number of them. Suppose there is a set of all the natural numbers, and that every set has a power set (the set of all its subsets). (These claims are widely held.) The power set of any set has more members than that set has. It follows from this that for any number, transfinite cardinals included, there are more sets than that. But even if this is right, we can still say <u>something</u> about how many sets there are: there are more than any number can capture. The question of how many sets there are has an answer, even though the answer is not a number.

So 5 is probably too strong. Perhaps we can replace it with something like this:

6. If anything is F, then it is possible to say, at least roughly, how many Fs there are.

(This doesn't mean that it is possible to <u>know</u> how many.) And we could state 4 more accurately as 'There are one or more Fs'.

As for the connection between identity and number, I was taught that for this and that to be identical is for them to be one, and that for them to be distinct is for them to be two:

7. x=y iff x and y are one;
8. ~x=y iff x and y are two.

It is because of this connection between identity and number, I was taught, that we call the relation here expressed by the '=' sign 'numerical identity'.

2.

Because I was taught these things at an impressionable age, perhaps, they seem to me very obvious. But Lowe says I was taught wrongly. He denies that 1-4 are synonymous. To say that there is something of a certain sort, on his view, is to say nothing at all about the number of things of that sort, or about how many there are. There can be something of a certain sort without the number of things of that sort being one or more. There can even be something of a certain sort without there being at least <u>one</u> thing of that sort.

Lowe also denies 7 and 8. He says there might be something, x, that is identical with something, y, even though x and y are not one--not one of anything. And there might be something, x, and something else, y, not identical with x, even though x and y are not two (not two of anything). (In fact Lowe claims that 7 and 8 hold neither left to right nor right to left: x and y might be one without being identical, or two without being non-identical. As he puts it, things might have 'determinate countability but not determinate identity'. I won't discuss this claim here.)

How can this be? How can there be something without there being at least one such thing? How can things be numerically identical without being one, or numerically different without being two?

Lowe's answer is that some things are uncountable. Not uncountable in the sense that they are too many to count--in the way that the real numbers are uncountable because there are too many of them to be matched up one to one with the natural numbers. Nor is Lowe's claim merely that some things have no precise number, like the bald men of Little Piddling. He says there are things that are uncountable in the sense of having no quantity at all. We can't even begin to count them, in the way that we can at least begin to count the real numbers. Nor can we say approximately how many there are, as we can with the bald men. There are things, Lowe says, such that "it makes no sense even to inquire how many there are" (<u>Possibility of Metaphysics</u> 74). We can't say that there is one of them. We can't say that there is more than one. We can't say anything at all about how many there are.

The claim that there are things of a certain kind but we cannot meaningfully speak of how many may sound ungrammatical. The word 'things' is plural, the thought goes, and plurality implies more than one; so it is a mere grammatical fact that there couldn't be <u>things</u> of a certain kind without there being more than one thing of that kind; and to say that there is more than one thing of that kind is to say something about how many there are. Lowe concedes that the claim 'there are uncountable things' is paradoxical. But we might be able to put the thought unparadoxically by saying simply that there is <u>something</u> of a certain kind, but no number, even approximately, of things of that kind. Still, a number of things that Lowe says, and that I shall say in trying to expound his view, are paradoxical, and I don't know how to put them unparadoxically.

3.

Lowe's clearest example of uncountable things, or uncountable something, is arbitrary masses of homogeneous stuff. (He also says that tropes and facts are uncountable. I won't discuss tropes or facts.) Inconveniently, 'mass' is a count noun, and the claim that arbitrary masses are uncountable is paradoxical once again, and I don't know how to make it unparadoxical. But never mind.

As far as we know there is no homogeneous stuff: for every actual stuff, there is a smallest possible portion of it. You can't have a portion of gold smaller than a single atom. Nor, it seems, can you have a portion of gold that isn't composed of gold atoms. Since there is a number of gold atoms (borderline cases aside), it therefore seems to follow that there is a number of masses of gold: either it is zero (if there are no masses of gold at all--something many reputable philosophers have believed) or it is the same as the number of non-empty subsets of the set of gold atoms--2^n -1, if the number of gold atoms is n.

But never mind actual stuffs. Lowe thinks there <u>could</u> be homogeneous stuff, with no atoms: stuff, any mass of which is infinitely divisible into smaller masses of the same stuff. I don't share Lowe's confident belief that homogeneous stuff is possible. I have no idea whether it is. But I am prepared to give Lowe the benefit of the doubt. Let us imagine that there is some of what Lewis called atomless gunk. Lowe says that in this case we can ask how much gunk there is--how many kilos or cubic meters of it, say--but not how many masses of it there are. Nor would there be any answer, even an imprecise one such as 'at least one', to the question how many.

I don't know whether Lowe needs to appeal to homogeneous stuff to make his point. (Lowe is unsure as well: 72.) If there could be uncountable masses of gunk, then there might well be uncountable masses of actual stuff too. The smallest actual particles--electrons, quarks, and so on--appear to take up space (they're not merely point-sized). And you might suppose that whatever takes up space is composed of arbitrary parts: if a material object occupies a certain region of space, every part of that region is occupied by a part of the object (this seems to be Lowe's view). If you hold this view, and you accept universal composition, you will conclude that there actually are arbitrary masses of matter. There might not be arbitrary masses of any particular sort of matter--of gold, for instance--but there will be arbitrary masses of something. And if arbitrary masses of gunk are uncountable, arbitrary masses of actual matter ought to be uncountable too.

But let us set this point aside and return to our imaginary gunk. How does Lowe's claim that masses of gunk would be uncountable conflict with the connections between quantification, identity, and number that I was taught? Well, Lowe says there might be gunk, and yet there would not be at least one thing that was gunk. If there were at least one thing that was gunk, then we could say something about how many masses of gunk there are, namely that there is at least one, contrary to Lowe's claim that masses of gunk would be uncountable. According to Lowe we cannot say that there is at least one mass of gunk. Thus, to say that there is gunk, or that something is gunk, is not to say that there is at least one thing that is gunk, or at least one of anything. That goes against the connection I was taught between quantification and number.

Now suppose there is gunk in England and also gunk in France. In that case the gunk in England is not the gunk in France. It is numerically different gunk. (That seems obvious.) But although something is the gunk in England and something else is the gunk in France, Lowe says, there are not therefore at least two masses of gunk, or two things that are gunk. Lowe expresses this by saying that masses have determinate identity but not determinate countability (72): for any masses x and y, either x is y or x is not y; but we cannot say either that x and y are one or that x and y are two. That goes against the connection I was taught between identity and number.

4.

Why does Lowe think that arbitrary masses of gunk are uncountable? He says it is because they have arbitrary boundaries (74). A mass needn't contrast in any way with its surroundings. This means that any given mass of gunk is infinitely divisible into smaller masses of gunk. Why this should show or even suggest that arbitrary masses are uncountable is unclear to me. However, since my time is brief, I won't discuss this issue.

Whatever the arguments for it, Lowe's claim seems to me fascinating but wrong. In fact it seems to me so obviously wrong that it's hard to know how to object to it. It's like objecting to the claim that contradictions can be true. Any argument for the claim that contradictions can never be true is bound to have premises that are less clearly true than their conclusion. It will end up making the claim that contradictions can be true look like an interesting hypothesis: there are some inconclusive objections to it, but of course there are objections to any philosophical claim. Objections to the claim that there can be uncountable objects run the same risk. But I will make an attempt anyway.

First, Lowe's claim contradicts Frege's account of number, or cardinality. For there to be exactly two giraffes, Frege said, is for there to be a giraffe, and another giraffe, different from the first, and such that any giraffe is identical with one or the other of them:

> There are exactly two giraffes <u>iff</u> (Ex) x is a giraffe & (Ey) y is a giraffe & ~x=y & (z) if z is a giraffe, then either z=x or z=y.

According to Lowe this may be correct, but only because giraffes are countable objects; it isn't true for masses of gunk. It is a metaphysical truth and not a logical truth. We can't generalize it to yield an acount of what it is for there to be two of something in general. So what is it for there to be two of something? The best Lowe can say is something like this:

> There are exactly two Fs <u>iff</u> (Ex) x is an F, and (Ey) y is an F, and ~x=y, and (z) if z is an F, then either z=x or z=y, and Fs are countable objects.

This means that we cannot give an account of number or cardinality in purely logical terms. The idea of there being two of something is not a logical notion but a metaphysical one. This is a consequence that few philosophers of mathematics will welcome.

5.

Here is another objection. Lowe says that, although we cannot ask how many masses of gunk there are, we can ask how many <u>pieces</u> of gunk there are (73). A piece of gunk is roughly a mass of gunk that is connected, as opposed to disconnected or scattered--you can draw a line between any parts of it that never goes outside the mass--and whose boundaries are not arbitrary.[1] A piece of gunk contrasts with its surroundings: it is entirely

[1] Lowe's definition is different: he says a piece of stuff is a mass of stuff that is connected and which is not a proper part of any larger connected

surrounded by non-gunk, or else its parts are bonded to one another more strongly than they are bonded to things that are not parts of it. (I don't know why Lowe insists that countable pieces be connected; if there could be non-arbitrary masses of gunk at all, there could be non-arbitrary disconnected masses--think of archipelagos or galaxies. But never mind.) Suppose I have a cubical piece of gunk on my desk, holding down some tedious committee papers. Then Lowe will concede that there is <u>one</u> piece of gunk there. And if I have another piece of gunk on a shelf, modelled into a likeness of Margaret Thatcher, then I have <u>two</u> pieces of gunk. The idea that I might have a cubical piece of gunk on my desk and a Thatcher-shaped piece of gunk on my shelf without having at least two pieces of gunk would be even harder to understand than the idea that there might be uncountable things.

So non-arbitrary pieces of gunk are countable. And pieces of gunk are masses of gunk: a piece is a special sort of mass, one that is connected and has non-arbitrary boundaries. This means that <u>some</u> masses of gunk--those that are pieces--are countable after all. Suppose I have two pieces of gunk in my office. Then I have at least two masses of gunk in my office. It follows that there are at least two masses of gunk. And in that case we can say something about how many masses of gunk there are: there are at least two. We can at least begin to count the masses, even if we can never finish the job. And if ther are at least two masses, surely we can ask how many there are.

I suppose Lowe will reply that if masses of gunk are countable, that is only because a special subclass of them--the non-arbitrary pieces--are countable. Still, <u>arbitrary</u> masses are uncountable, and so there are uncountable things.

This does not yet solve the problem, though. It seems possible for something to be a piece of gunk at one time and a mere arbitrary mass at another time. If we break a piece of gunk into two pieces, it seems that each of the resulting smaller pieces was previously a mere arbitrary part of the original piece. An arbitrary mass of gunk (if it is connected) can come to be a piece of gunk by having the surrounding gunk detached from it. Likewise, it seems that a piece of gunk can cease to be a piece and revert to being a mere arbitrary mass, for instance by being fused together with another piece.

Suppose there have been ten pieces of gunk in my office over the course of the past year, and they have all now ceased to be pieces, but still exist as arbitrary masses. Now if there have been ten masses of gunk, and none of them has ceased to exist or ceased to be a mass of gunk, doesn't it follow that there are still ten masses of gunk? And if each of those ten masses is now an arbitrary mass of gunk, are there not now at least ten arbitrary

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mass of stuff of the same kind (PM 73). As far as I can see, many arbitrary masses of gunk could qualify as pieces by this account. So I assume it isn't what he meant to say.

masses of gunk? It seems so. A thing's countability could hardly be a mere temporary feature of it: a thing couldn't be countable at one time and not countable at another. Could it? But then we can say something about how many arbitrary masses of gunk there are.

For that matter, I should have thought that a thing's countability or non-countability could not be a mere accidental or contingent feature of it: whatever is countable (or non-countable) must necessarily be countable (or non-countable) if it exists at all. If x and y are two, I should have thought, then they are necessarily two, and if they are not two, then they are necessarily not two. There could no more be contingent countability or non-countability than there could be contingent identity or non-identity. But then if pieces are masses, every arbitrary mass could have been a piece. Any arbitrary mass of gunk could be made into a piece of gunk by having its parts pushed together and the surrounding gunk removed. There are no arbitrary masses of gunk that it would be absolutely impossible to make into pieces of gunk: no arbitrary mass is essentially arbitrary. Thus, every mass is possibly countable; and if nothing could be contingently countable, it follows that every mass is in fact countable.

6.

Lowe might reply by denying that an arbitrary mass of gunk could come to be a piece of gunk, or that a piece could be made into an arbitrary mass. More generally, he might say that nothing that is in fact a piece could have been an arbitrary mass or vice versa: every piece is essentially a piece, and every arbitrary mass is essentially an arbitrary mass. Arbitrary and non-arbitrary masses of gunk are two different metaphysical kinds.

What happens, then, when we take a piece of gunk and break it into two pieces? We seem to end up with two non-arbitrary masses. If there are any masses of gunk, surely there are two masses here. Did those masses not exist before we broke up the original piece? The suggestion is that they didn't. They came into being when we broke up the original piece. The process of breaking a piece of gunk in two creates two new masses of gunk. I suppose it also destroys something. It's tempting to say that it destroys two arbitrary masses, namely those that were made of the same gunk before the breaking as the two resulting pieces are afterwards. Lowe can't say that. But perhaps he can say that there is an arbitrary mass of gunk, and there is an arbitrary mass of gunk different from it, and each of them necessarily ceases to exist when the gunk surrounding it is removed as the piece they are parts of is broken. Likewise, Lowe might say that if we take two smaller pieces of gunk and fuse them to make a larger piece, the two smaller pieces don't become arbitrary masses, but cease to exist. The arbitrary mass whose boundaries end up the same as one of the original pieces was not once a piece, but comes into being in the course of the fusing. And Lowe might say that it is not possible for a given piece of gunk to have had other gunk fused to it (for then it would have been an uncountable arbitrary mass); nor is it possible for a given arbitrary mass of gunk to have been surrounded by non-gunk (for then it would have been a countable piece).

Someone might say this; but it's pretty unattractive. If we break a piece of gunk into its right and left halves (or if we do something we would loosely describe in these words), we want to say that the gunk in its left half still exists, and the gunk in its right half still exists. We have merely separated one from the other. We haven't destroyed or created any gunk. The gunk we began with is all still there, only reconfigured. The gunk that makes up each resulting piece existed before we broke the original piece in two. But if we have the same gunk, how could we not have the same mass of gunk?

Lowe might say that the arbitrary masses we begin with all still exist after we break our piece in two; but rather than becoming pieces, some of them merely come to <u>coincide</u> with pieces. Breaking our original piece in two creates two new <u>pieces</u> of gunk, but it doesn't create any new <u>mass</u> of gunk. The mass coinciding with each new piece existed beforehand. This would imply that pieces of gunk aren't masses at all. Pieces and masses are different metaphysical categories. Pieces are countable, masses are not.

But this doesn't help. On this story, each piece of gunk coincides, after the breaking, with a mass of gunk: exactly one mass. And in that case we can after all say something about how many masses there are: there is at least one. Moreover, if there are two pieces of gunk, and each coincides with exactly one mass of gunk, surely it follows that there are two masses of gunk. And if each of those masses was once an arbitrary mass (with arbitrary boundaries), then there were once two arbitrary masses. So some arbitrary masses are countable. And since any arbitrary mass could come to coincide with a piece, all arbitrary masses are potentially countable.

7.

I have argued that there are two good reasons to think that there could not be uncountable masses, and I have confessed that I cannot see any reason to suppose that there could be such things. I have said nothing about Lowe's claim that tropes and facts are uncountable. What if Lowe is wrong and there couldn't be uncountable things? That would support the traditional connections between quantification, identity, and number that I was taught as a student. (At least it would support the left-to-right versions of 7 and 8.) It would also support 6, the claim that if there is anything of a given kind--any kind at all--it is possible to say how many things of that kind there are.

This contradicts another of Lowe's claims (and a claim that Lowe is not alone in making), namely that we cannot ask how many <u>things</u> there are--'thing' in the sense of the most general count noun, so that everything is a thing. Since 'thing' is a predicate, 6 implies that we can <u>say</u> how many things there are; and if we can say it, we can ask it. There may not be a definite <u>number</u> of things, either owing to vagueness or because there are too many things to be numbered; but there is an answer to the question of how many there are.