

# Containment Relations in Anatomical Ontologies

Maureen Donnelly, Ph.D.

IFOMIS, Saarland University, Germany

**Abstract** Containment relations are needed in anatomy for describing the locations of anatomical individuals. My lungs are contained in my thoracic cavity. They are also, in a somewhat different way, contained in my chest. Note that in the first case my lungs are not part of the container. Thus, a parthood relation alone does not enable us to fully describe containment relations among anatomical entities.

Ontologies such as the FMA and GALEN use containment relations to link anatomical classes. To provide the ontologies with clear semantics and consistent reasoning strategies, it is necessary to precisely determine the logical properties of their containment relations. In this paper, I define different types of containment relations in a formal theory and show that the logical properties of these relations differ significantly. The formal relations are used to partially analyze and highlight differences between the FMA's and GALEN's containment relations.

## 1. Introduction

Both the Foundational Model of Anatomy (FMA) [1] and GALEN [2] use containment relations to link pairs of anatomical classes (where an *anatomical class* is understood as a kind or type which has anatomical individuals as instances). For example, the FMA asserts: *Heart contained\_in Middle Mediastinal Space*.<sup>1</sup> GALEN asserts: *Heart isContainedIn Mediastinum*, *Larynx isContainedIn Neck*, *Pleural Space isContainedIn Pleural Membrane*, and *Tooth isContainedIn Tooth Socket*. These assertions imply that individual instances of the appropriate classes (e.g. my larynx, my neck, and so on) stand in certain containment relations [3]. However, different instance-level containment relations seem to underlie the class-level containment relations used in these assertions. My heart is contained in my middle mediastinal space in the way that a table is contained in the interior of a room -- my heart is not itself part of the space, but it occupies part of the space. By contrast, my larynx is part of my neck. The space within a pleural membrane is entirely surrounded by, but is not part of and does not occupy part of, the pleural membrane. By contrast, only a part of the tooth (the root) is surrounded by the socket.

The purpose of this paper is to clearly distinguish different types of instance-level containment relations

in the context of a formal spatial theory. I will show that containment relations roughly corresponding to those of the FMA and GALEN assertions above have different logical properties. Thus, an explicit distinction between different containment relations is needed not only for disambiguating the containment assertions, but also for implementing consistent automated reasoning within or across ontologies.

The outline of this paper is as follows. In §2, I present a formal theory, Parthood and Containment Theory (PCT), in which five containment relations are defined. In §3, I discuss the logical properties of these relations. In §4, I use the relations of PCT to compare the very different containment relations used in the FMA and GALEN. Though none of the PCT's relations exactly matches the containment relations used in the FMA or GALEN, PCT is an important first step toward an adequate formal treatment of these relations.

## 2. Parthood and Containment Theory

PCT is a time-independent theory which can be used to describe static relations among individuals during a fixed time-frame. An important project for further work is to incorporate time and change into PCT.

### 2.1 Parthood Relations

The parthood relation (symbolized as “P”) holds between individuals,  $x$  and  $y$ , when  $x$  is part of  $y$ . For example, my left atrium is part of my heart. PCT has three parthood axioms:

(P1)  $Pxx$  (every object is part of itself)

(P2)  $Pxy \ \& \ Pyx \rightarrow x = y$  (if  $x$  is part of  $y$  and  $y$  is part of  $x$ , then  $x$  and  $y$  are identical)

(P3)  $Pxy \ \& \ Pyz \rightarrow Pxz$  (if  $x$  is part of  $y$  and  $y$  is part of  $z$ , then  $x$  is part of  $z$ )

Additional relations can be defined in terms of parthood.

**Proper Parthood:**  $x$  is a *proper part* of  $y$  if  $x$  is any part of  $y$  other than  $y$  itself. Symbolically:

$$PPxy =: Pxy \ \& \ x \neq y.$$

For example, my hand is a proper part of my body.

**Overlap:**  $x$  and  $y$  *overlap* if there is some object,  $z$ , that is part of both  $x$  and  $y$ . Symbolically:

$$Oxy =: \exists z (Pzx \ \& \ Pzy).$$

My bony pelvis and my vertebral column overlap: my sacrum and my coccyx are part of both.

### 2.2 The Region Function

For introducing containment relations which are distinct from parthood relations, PCT needs additional vocabulary. A region is a part of the fixed background space in which an organism is located. The region function (symbolized “ $r$ ”) maps

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<sup>1</sup> Throughout this paper, I use italics and initial capitals for the names of classes.

individual  $x$  to the unique spatial region  $r(x)$  at which  $x$  is exactly located.

PCT's axioms for the region function are:

(R1)  $PPxy \rightarrow PPr(x)r(y)$  (if  $x$  is a proper part of  $y$ , then  $x$ 's region is proper part of  $y$ 's region)

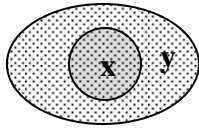
(R2)  $r(r(x)) = r(x)$  ( $x$ 's spatial region is its own spatial region)

I use the region function to introduce the first type of containment relation.

**Region Containment:**  $x$  is *r-contained* in  $y$  if  $x$ 's region is part of  $y$ 's region. Symbolically:

$$CNT-IN_rxy =: Pr(x)r(y).$$

See Figure 1. For example, my heart is r-contained in my middle mediastinal space. It is also r-contained in my thoracic cavity, my chest, and my body.



**Figure 1:  $x$  is r-contained in  $y$**

Since r-containment is defined in terms of the relation between  $x$ 's region and  $y$ 's region, it depends only on  $x$ 's and  $y$ 's exact locations and not on whether  $x$  and  $y$  stand in parthood relations. My heart is not part of middle mediastinal space or my thoracic cavity. On the other hand, it follows from axiom (R1) that: if  $x$  is part of  $y$ , then  $x$  is r-contained in  $y$ . Thus, my heart is r-contained in my chest, my larynx is r-contained in my neck, and so on.

### 2.3 The Convex Hull Function

With only parthood relations and the region function, we cannot introduce the type of containment relation that holds between my pleural space and my pleural membrane. The region of my pleural space is not part of the region of my pleural membrane.

In these cases, the containee lies within a region which is somewhat bigger than the container, called its convex hull. A *convex* region is one which includes any line segment connecting any of its parts. For example, the region occupied by a solid ball is convex. Regions occupied by a drinking glass or my pleural membrane are not convex. The *convex hull* of an individual  $x$  is the smallest convex region of which  $x$ 's region is part. For example, the convex hull of my pleural membrane extends over both the pleural membrane and the space inside the pleural membrane. See also [4].

I add to PCT a convex hull function (symbolized "ch") which maps each individual to its convex hull.

PCT has three axioms for the convex hull function:

(CH1)  $Pr(x)ch(x)$  ( $x$ 's region is part of  $x$ 's convex hull)

(CH2)  $CNT-IN_rxy \rightarrow Pch(x)ch(y)$

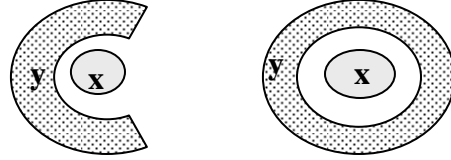
(if  $x$  is r-contained in  $y$ , then  $x$ 's convex hull is part of  $y$ 's convex hull)

(CH3)  $ch(ch(x)) = ch(x)$  ( $x$ 's convex hull is its own convex hull)

**Surround Containment:**  $x$  is *s-contained* in  $y$  if  $x$ 's region is part of  $y$ 's convex hull and  $x$ 's region does not overlap  $y$ 's region. Symbolically:

$$CNT-IN_sxy =: Pr(x)ch(y) \ \& \ \sim Or(x)r(y)$$

See Figure 2. For example, my pleural space is s-contained in my pleural membrane and the cavity of my stomach is s-contained in the wall of my stomach.



**Figure 2: Two possibilities for  $x$  is s-contained in  $y$**

Notice that, because r-containment requires that containee and container are located at overlapping regions and s-containment requires that containee and container are located at non-overlapping regions, r-containment and s-containment are mutually exclusive. A bolus of food is r-contained (but not s-contained) in my stomach cavity. It is s-contained (but not r-contained) in the wall of my stomach.

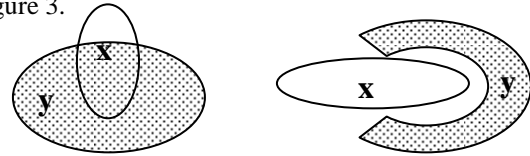
We can define a very general containment relation which includes both r-containment and s-containment.

**General Containment:**  $x$  is *g-contained* in  $y$  if  $x$ 's region is part of  $y$ 's convex hull. Symbolically:

$$CNT-IN_gxy =: Pr(x)ch(y).$$

See Figures 1 and 2: all examples of r-containment or s-containment are also examples of g-containment. For example, a bolus of food in my stomach is g-contained in both my stomach cavity and the wall of my stomach.

However, even g-containment is not broad enough to include the relation between a tooth and its socket. The region of the tooth merely overlaps, but is not part of, the convex hull of the socket. For such cases, we need a partial containment relation. See figure 3.



**Figure 3: Two possibilities for  $x$  is partially contained in  $y$**

**Partial Containment**  $x$  is *partially contained* in  $y$  if  $x$ 's region overlaps  $y$ 's convex hull. Symbolically:

$$P-CNT-INxy =: Or(x)ch(y).$$

### 2.4 Material and Immaterial Individuals

The containment relation used by the FMA depends on a distinction between material and immaterial individuals. To better approximate this containment relation, I introduce into PCT a predicate *material*

(M) where  $x$  is considered material as long as  $x$  is has some material part. For example, my stomach and heart are material.  $x$  is *immaterial* (IM) if and only if  $x$  is not material. For example, my stomach cavity and the cavity of my left atrium are immaterial.

Axioms for the material predicate include the following:

(M1)  $Mx \ \& \ Pxy \rightarrow My$  (if  $y$  has a material part, then  $y$  is material)

**Material-Region Containment:**  $x$  is *mr-contained* in  $y$  if  $x$  is material,  $y$  is immaterial, and  $x$ 's region is part of  $y$ 's region. Symbolically:

$CNT-IN_{mr}xy =: Mx \ \& \ IMy \ \& \ Pr(x)r(y)$ .

For example, my heart is mr-contained in my middle mediastinal space. But my heart is NOT mr-contained in material individuals, such as my chest.

### 3. Logical Properties of Containment Relations

The various containment relations introduced in the preceding section have significantly different logical properties. I limit the discussion in this section to properties which are most relevant to the current treatment of spatial relations the FMA and GALEN.

#### 3.1 Transitivity

Both the FMA and GALEN implement transitivity reasoning on some spatial relations. A binary relation  $R$  is *transitive* if whenever  $x$  stands in  $R$  to  $y$  and  $y$  stands in  $R$  to  $z$ ,  $x$  must stand in  $R$  to  $z$ . For example, axiom (P3) of §2.1 says that the parthood relation (P) is transitive.

The following three of PCT's containment relations are transitive:  $CNT-IN_r$ ,  $CNT-IN_g$ , and  $CNT-IN_{mr}$ . For example, given that my heart is r-contained in my middle mediastinal space and my middle mediastinal space is r-contained in my thoracic cavity, my heart must also be r-contained in my thoracic cavity. Also, given that my pleural space is g-contained in my pleural membrane and my pleural membrane is g-contained in chest, my pleural space must also be g-contained in my chest.

Note that there is an important difference in the transitivity of mr-containment and the transitivity of r-containment and g-containment. As the examples in the previous paragraph show, we can use the transitivity of r-containment and g-containment to generate further assertions. We cannot, however, use transitivity reasoning on mr-containment to generate further assertions. This is because it is NEVER the case that  $x$  is mr-contained in  $y$  and  $y$  is mr-contained in  $z$ , since these assertions could both hold only if  $y$  were both material and immaterial. Thus, e.g., my heart is mr-contained in my middle mediastinal space, but my middle mediastinal space, as an immaterial entity, cannot be mr-contained in any individual.

The remaining PCT containment relations –  $CNT-IN_s$  and  $P-CNT-IN$  – are NOT transitive. For

example, even though my tooth is partially contained in its socket, a filling or dental instrument may be partially contained in my tooth without also being partially contained in the tooth socket.

### 3.2 Interaction between Containment Relations and the Proper Parthood Relation

Both the FMA and GALEN make extensive use of class-level versions of the proper parthood relation (or of more specialized versions of this relation) [1, 2, 5]. For this reason, it is of interest to see how each of PCT's containment relations interacts with the proper parthood relation.

Some of PCT's containment relations hold between  $x$  and  $y$  whenever  $x$  is a proper part of  $y$ . In particular, if  $x$  is proper part of  $y$ , then  $x$  is r-contained in  $y$ ,  $x$  is g-contained in  $y$ , and  $x$  is partially contained in  $y$ .

By contrast, the remaining PCT containment relations exclude parthood: if  $x$  is a proper part of  $y$ , then  $x$  cannot be either s-contained or mr-contained in  $y$ .

Compositional reasoning is another important aspect of the interaction between proper parthood and containment relations. In some cases, we can make an inference about the containment relation holding between  $x$  and  $z$  from a conjunction of the form  $PPxy \ \& \ C^*yz$  where  $C^*$  is one of PCT's containment relations. Table 1 shows the strongest assertion concerning the containment relation between  $x$  and  $z$  that can be inferred from conjunctions of this form. (A similar table can be constructed for conjunctions of the form  $C^*xy \ \& \ PPyz$ .)

	$x$ is a proper part of $y$
$CNT-IN_r yz$	$CNT-IN_r xz$
$CNT-IN_s yz$	$CNT-IN_s xz$
$CNT-IN_g yz$	$CNT-IN_g xz$
$CNT-IN_{mr} yz$	$CNT-IN_r xz$
$P-CNT-IN yz$	

**Table 1: Inferences from:  $x$  is a proper part of  $y$  and  $y$  is contained in  $z$**

The blank cell in the last row indicates that NO assertion describing a containment relation between  $x$  and  $z$  can be inferred from the information in the row and column headings. Given that the T1 segment of my esophagus is a proper part of my esophagus and my esophagus is partially contained in my abdominal cavity, it does NOT follow that the T1 segment of my esophagus stands in any containment relation to my abdominal cavity. On the other hand, given that my heart is r-contained in my middle mediastinal space (row 1) and my left atrium is a proper part of my heart (column 1), it follows that my left atrium is also r-contained in my middle mediastinal space.

In some cases, the inference concerning the relation of  $x$  to  $z$  is stated in terms of a weaker containment relation than that of the premises. From

x is a proper part of y and y is mr-contained in z, we can infer that x is r-contained in z but NOT that x is mr-contained in z. To see this, note that my stomach cavity is a proper part of my stomach and my stomach is mr-contained in my abdominal cavity, but my stomach cavity cannot be mr-contained in my abdominal cavity since my stomach cavity is immaterial. In these cases, an ontology which includes only the stronger relation (e.g. mr-containment, but not r-containment) will not be able to provide any inferred assertion about the relation of x to z unless further information is given concerning the materiality or immateriality of the relevant individuals.

#### 4. Containment Relations in the FMA and GALEN

As mentioned in §1, the FMA and GALEN use relations which hold among classes rather than individual instances of these classes. Thus, the containment relations of the FMA and GALEN cannot be identical to any of the PCT relations (which hold only among individuals). However, if properly defined, each class-level containment relation **C** should correspond to some one instance-level containment relation **C** in the sense that: class A stands in relation **C** to class B only if relevant instances of A stand in relation **C** to relevant instances of B [3, 6, 7]. For example, given the GALEN assertions: *Larynx isContainedIn Neck*, *Pleural Space isContainedIn Pleural Membrane*, *Tooth isContainedIn Tooth Socket*, and so on, there should be some one instance-level containment relation **C** (possibly, but not necessarily, a PCT relation) such that **C** holds between my larynx and my neck, between my pleural space and my pleural membrane, between each of my teeth and their sockets, and so on.

As is shown in [7], several different class-level containment relations might correspond in this way to any one instance-level containment relation. Distinguishing between these different types of class-level relations is important but involves complications which go well beyond the scope of this paper. Thus, I focus here only on the question of what instance-level containment relation might correspond either to the FMA's or GALEN's containment relations.

The FMA has one containment relation, *contained\_in*.<sup>2</sup> The FMA's developers state that A *contained\_in* B holds only when A is a subclass of either *Body Substance* or *Anatomical Structure* (both subclasses of *Material Physical Anatomical Entity*) and B is a subclass of *Anatomical Space* (a

subclass of *Immaterial Physical Anatomical Entity*) [1]. The FMA's *contained\_in* assertions bear this out. Besides *Heart contained\_in Middle Mediastinal Space*, the FMA asserts, for example: *Liver contained\_in Abdominal Cavity*, *Urinary Bladder contained\_in Pelvic Cavity*, and *Urine contained\_in Lumen of Urinary Bladder*. It, thus, appears as though PCT's CNT-IN<sub>mr</sub> could be the instance-level relation corresponding to the FMA's *contained\_in*. Clearly, my heart is mr-contained in my middle mediastinal space, my liver is mr-contained in my abdominal cavity, and so on.

If *contained\_in* is a class version of CNT-IN<sub>mr</sub>, then *contained\_in* should be irreflexive and asymmetric (§3.1), and A *contained\_in* B should never hold when A *part\_of* B holds (§3.2) (where *part\_of* is the FMA's most general class-level proper parthood relation). We should also expect that, although *contained\_in* is transitive, transitivity reasoning on *contained\_in* cannot be used to generate further assertions (§3.1). Since the FMA does not include a class-level version of the more general relation CNT-IN<sub>r</sub>, compositional reasoning over *contained\_in* and *part\_of* should not generate additional containment assertions unless it is combined with further information on whether the instances of the relevant classes are material or immaterial (§3.2).

That the FMA's use of *contained\_in* does not violate any of the above restrictions may be taken as further evidence that CNT-IN<sub>mr</sub> is intended as its underlying instance-level containment relation. However, *contained\_in* assertions are often missing from the FMA where the CNT-IN<sub>mr</sub> relation does hold among instances of the relevant classes. For example, although each person's heart is mr-contained in both her middle mediastinal space and her thoracic cavity, only *Heart contained\_in Middle Mediastinal Space* is asserted in the FMA. Also, although each person's urine is mr-contained in both the lumen of his urinary bladder and his pelvic cavity, only *Urine contained\_in Lumen of Urinary Bladder* is asserted in the FMA. The explanation for these missing assertions could be that input of containment information into the FMA has not yet been completed. On the other hand, it may be that the FMA intends *contained\_in* as a class version of a specialized sub-relation of CNT-IN<sub>mr</sub>. Such a relation would hold between a material individual and only one particular of its mr-containers. If this is the intention, then the exact interpretation the FMA's containment relation, detailing the conditions that the special mr-container must satisfy, needs to be worked out. In this case, the logical properties of the FMA's *contained\_in* should differ slightly from those of a class-level version of CNT-IN<sub>mr</sub>.

<sup>2</sup> The FMA and GALEN also both have inverse containment relations. I do not consider these in this paper.

GALEN has one general containment relation, *isContainedIn*, which is divided into several sub-relations. For the most part, I ignore the distinctions between these sub-relations and focus only on *isContainedIn*. However, it is important to note that the two immediate sub-relations of *isContainedIn* are *isPartitivelyContainedIn* and *isNonPartitivelyContainedIn*. Thus, unlike the FMA's *contained\_in*, GALEN's *isContainedIn* is a class version of a relation such as *CNT-IN<sub>r</sub>*, *CNT-IN<sub>g</sub>*, or *P-CNT-IN* (but not *CNT-IN<sub>s</sub>* or *CNT-IN<sub>mr</sub>*) which is compatible with parthood.

An investigation of GALEN's *contained\_in* assertions shows that the underlying containment relation is not limited to *CNT-IN<sub>r</sub>*. As seen above, GALEN asserts *Pleural Space isContainedIn Pleural Membrane* and *Tooth isContainedIn Tooth Socket*. But a person's pleural space is not r-contained in her pleural membrane and a tooth is not r-contained in its socket. The general containment relation *CNT-IN<sub>g</sub>* fits most of GALEN's containment assertions including *Pleural Space isContainedIn Pleural Membrane* (a person's pleural space is g-contained in her pleural membrane), *Larynx isContainedIn Neck* (a person's larynx is g-contained in his neck), and so on. But *Tooth isContainedIn Tooth Socket* corresponds not to g-containment, but to partial containment. Thus, unless assertions such as *Tooth isContainedIn Tooth Socket* are eliminated, only the weak partial containment relation is compatible with all of GALEN's containment assertions.

However, GALEN's automated reasoning over *isContainedIn* is much too strong for a class-level partial containment relation. Among other things, GALEN implements unrestricted transitivity reasoning on *isContainedIn*. But since *P-CNT-IN* is not transitive (§3.1), a class-level version of *P-CNT-IN* should not be transitive.

Also, GALEN lacks *isContainedIn* assertions in many cases where the *P-CNT-IN* relation (or the stronger *CNT-IN<sub>g</sub>* relation) holds among instances of the relevant classes. For example, the left side of a person's heart is g-contained (and thus also partially contained) in her heart, but *Left Side of Heart isContainedIn Heart* is not asserted in GALEN. Such missing containment assertions are not just a matter of incomplete input. The fact that GALEN divides its primary class-level parthood relation (*isDivisionOf*) into sub-relations which exclude *isPartitivelyContainedIn* shows that GALEN does not consider that every case of parthood is also a case of containment [2]. Thus, GALEN seems to intend *isContainedIn* as a more specialized version of either *P-CNT-IN* or (if assertions such as *Tooth isContainedIn Tooth Socket* are eliminated) *CNT-*

*IN<sub>g</sub>*. The intended relation should be compatible with parthood in some cases (my larynx and neck) but not in others. However, the exact interpretation and logical properties of GALEN's intended containment relation are not clear and remain to be worked out.

## 5. Conclusions

An examination of the FMA's and GALEN's assertions in terms of PCT leaves open the question of whether either ontology's primary containment relation is a class-version of any of the PCT relations. What is clear, however, is that the FMA's and GALEN's containment relations function quite differently with the FMA's *contained\_in* being closer to a class-level version of *CNT-IN<sub>mr</sub>* and GALEN's *isContainedIn* being closer to a class-level version of *CNT-IN<sub>g</sub>*. Thus, precise semantics for each ontology's containment relations are crucial for comparing the spatial information embodied in the two ontologies. One important project for further work is collaboration with the developers of the FMA or GALEN which will result in precise semantics for their containment relations. Until these issues are settled, it remains unclear exactly how to understand and make use of the containment information included in the ontologies.

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