**Steps towards a definition of ‘rate’**

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**Some relevant background on process profiles**  
There are certain structural dimensions of objects – e.g. mass, temperature, volume – which involve qualities that objects have to have essentially. These qualities can vary, but they must have some value. (Compare the anatomists idea of a *Bauplan*.)  
  
There are, I want to claim, analogous structural dimensions of processes, tentatively called 'profiles'. Thus processes of a given sort – e.g. the functioning of a heart – might have a motion profile, an auditory profile, a blood output profile, and so on. Quantitative values are associated directly with process profiles, with the process as a whole only in a secondary sense.

While the motion and blood output and auditory profiles would necessarily have some value for any functioning of the heart as a pump. This is clear if we define an auditory profile as follows:

An auditory profile is a part of a process in an independent continuant that consists in changes in sound quality where (1) the process in question is of a type typical members of which involve changes in sound quality which typically are of a similar type, (2) ‘changes in sound quality’ may involve also periods of silence or of unvarying sound quality.

Examples of processes with no auditory profile by the above include:

Doing mental arithmetic, reading, bacteria entering your bloodstream, hydrogen atoms moving through deep space …  
(To deal with the bacteria case, which I suppose may involve micro-sounds, we may need to involve a granularity factor – thus sounds below a certain threshold will be ignored)

**The BFO background**

Basic Axioms:

1. There are four kinds of occurrents:

processes (connected and scattered), which are always extended in time

process boundaries (of instantaneous extent), both internal (when a threshold is crossed) and external (beginnings and endings)

temporal regions (points and intervals)

spatiotemporal regions

2. Processes are changes; they have participants, which are the independent continuants which change. Whenever a quality of an independent continuant changes then (trivially) this independent continuant changes also.

3. There are projection relations from any entity to a corresponding region (e.g. from process to spatiotemporal region; from process to temporal region; from process boundary to temporal instant).

4. Instantiation relations for occurrents are never time indexed.

Continuants can change their type from one type to the next (e.g. fetus becomes an embryo becomes an infant …); occurrents can never change their type from one time to the next, because occurrents can never change, because occurrents *are* changes.

5. Multiple inheritance is to be avoided.

6. Processes can be arbitrarily summed and divided. In particular, we can identify sub-processes which are fiat segments projecting onto temporal intervals – e.g. your heart-beating from 4pm to 5pm today; the 4th year of your life.

**Towards a definition of ‘rate’**

Question: How, on the basis of the above, and of the BFO framework, do we do justice to rates of change, which seem intuitively to be attributes of processes in some ways analogous to BFO:qualities?

To answer this question we consider the specific case of beating processes, as for example of a heart.

How do we do justice to the fact that, intuitively, a process of heart beating can change its rate from one time to the next.

Answer: There are two sub-processes (by 5.), one spanning the first time, the other spanning the second time, and the rate of one is different from the rate of the other.

What follows is an attempt to make this work, given that there are beating processes which change continuously (so that we have to make sense of a beating process boundary having a certain rate *at an instant*)

p instanceOf beating-at-n-bpm =def.

**Case 1** (see Figure 1): p is regular, and extends across at least one cycle

there is some decomposition of p into similar process parts p = sum of p1, …, pn, for n ≥ 1, which is such that each of the pi projects onto a time interval of time 1/n

Figure 1

**Case 2** (see figure 2): the rate of beating of a process is varying, but for a certain interval t1 (from 1.5 to 2 seconds), which is less than the extent of the relevant cycle, it is beating at a rate of n bpm.

Here it is as if at the relevant time the process could be extended to Case 1. We need to say something like:

during this interval the beating process is similar to a 0.5 second long segment of a process that is otherwise similar and is beating at n bpm according to our definition. Perhaps like this:

p instance of partial\_interval\_beating-at-n bpm =def. there is some time interval t, p projects onto t, there is some process q such that q instance\_of beating-at-n bpm and p is similar to a segment of q that is of length equal to that of t

Figure 2

**Case 3** (see Figure 3): there might be an instantaneous process (process boundary) p1 to which we can assign a rate of n bpm. To this end we need to use the idea of limits; however close an interval around n we choose, we can find an interval around t1 in which the beating process is arbitrarily similar to a process that is an instance of partial\_interval\_beating-at-n bpm.

Thus suppose that the heart is beating as in Figure 3 and that its rate of beating is decreasing continuously between t1 and t2. We want to say that at the mid-point the heart is beating at 64 bpm. Yet by our definitions above at no time in the given interval do we have an instance of beating-at-64-bpm in either Case 1 or Case. Remember that we cannot *make* time-indexed instantiation assertions concerning processes at all.

We first define

two beating processes are δ-similar =def. the difference between their rates (defined under either case 2 or case 3) is less than or equal to δ bpm.

We then define

p instance of instantaneous\_beating-at-n bpm =def. p is a process boundary (instantaneous process part) in the interior of some process p1 and given any δ we can find some process p2 such that p interior part of p and p2 part of p1 & p1 δ-similar to some process that is an instance of partial\_interval\_beating-at-n bpm

Figure 3

**Case 4: Acceleration (see Figure 4)**

**p instance of beating\_at\_a\_rate\_increasing\_by n bpm2**

(i.e. in every minute the rate rises by n bpm)

=def. for any instantaneous process boundary p1 in the interior of p, if p1 instance of instaneous\_beating-at-m1 bpm & p1 projects onto t1, then for any δ there is some instantaneous process boundary p2 such that p2 instance of instaneous\_beating-at-m2 & p2 projects onto t2 and p1 δ-similar to some process that is an instance of partial\_interval\_beating-at-n bpm & the difference between nand

is less than δ

Figure 4

**The master argument for profiles now reads as follows:**

‘Similar’ in all of the above has to mean: have a similar beating-motion profile

For suppose that each of the pi involves e.g. the making of a noise of a quite different sort, or the flashing of a color of a quite different sort, or any other kind of change.

Footnote:

The idea, illustrated by Figures 1-3, is to compute the rate of change as the [limiting value](http://en.wikipedia.org/wiki/Limit_of_a_function) of the [ratio of the differences](http://en.wikipedia.org/wiki/Difference_quotient) Δ*y* / Δ*x* as Δ*x* becomes infinitely small.

In [Leibniz's notation](http://en.wikipedia.org/wiki/Leibniz%27s_notation), such an [infinitesimal](http://en.wikipedia.org/wiki/Infinitesimal) change in *x* is denoted by *dx*, and the derivative of *y* with respect to *x* is written

 \frac{dy}{dx} \,\!