Applied Bayesian Analysis : NCSU ST 540

Homework 5

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let Y_i be the number of concussions from team $i = 1, ..., 32$. The model is

$$Y_i|\lambda_i \sim Poisson(\lambda_i)$$

and the prior is

$$\lambda_i|\theta \sim Gamma(1, \theta)$$

where

$$\theta \sim Gamma(0.1, 0.1)$$

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(1) Derive the full conditional distribution of λ_1

Assuming independence of Y_i we can write the full joint distribution as

$$P(Y_1, ..., Y_n, \lambda_1, ...\lambda_n, \theta) \propto \prod_{i=1}^{n} P(Y_i|\lambda_i)P(\lambda_i|\theta)P(\theta)$$

Now

$$P(\lambda_1|Y_1, ..., Y_n, \lambda_2, ...\lambda_n, \theta) \propto P(Y_1|\lambda_1)P(\lambda_1|\theta)P(\theta) \quad \prod_{i=2}^{n} P(Y_i|\lambda_i)P(\lambda_i|\theta)$$

Putting the expressions for the densities in here and dropping the product terms on the right hand side unrelated to λ_1 we have that

$$P(\lambda_1|Y_1, ..., Y_n, \lambda_2, ...\lambda_n, \theta) \propto \frac{\lambda_1^{y_1}}{y_1!}e^{-\lambda_1} \quad \theta e^{-\theta\lambda_1} \quad \frac{(0.1)^{0.1}}{\Gamma(0.1)}\theta^{0.1-1}e^{-0.1\,\theta} \propto \lambda_1^{(y_1+1)-1}e^{-\lambda_1(1+\theta)}$$

The last experssion we recognise as the kernel of a $Gamma(y_1 + 1, 1 + \theta)$ distribution.

(1) Derive the full conditional distribution of θ
(2) Write Gibbs sampling code to draw samples from the joint distribution of $(\lambda_1, ..., \lambda_32, \theta)$.
(3) Show trace plots of the samples for λ_1 and θ.
(4) Plot the estimated posterior mean of the λ_i versus Y_i and comment on whether the code is returning reasonable estimates. Turn in your solution on one piece of paper. Also, turn in MCMC code on a second piece of paper stapled to the solution.

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df <- read.csv("ConcussionsByTeamAndYear.csv",
    header = TRUE)
```