

Applied Bayesian Analysis : NCSU ST 540

Homework 7

Bruce Campbell

In this assignment we perform Bayesian linear regression for the microbiome data on the course website

<https://www4.stat.ncsu.edu/~reich/ABA/assignments/homes.RData>

Let Y_i be the precipitation for observation i and X_{ij} equal one if OTU j is present in sample i .

First, extract the 50 OTU with the largest absolute correlation between X_{ij} and Y_i . Then fit a Bayesian linear regression model precipitation as the response and with these 50 covariates (and an intercept term) using two priors:

- (1) Uninformative normal priors: $\beta_j \sim Normal(0, 100^2)$
- (2) Hierarchical normal priors: $\beta_j | \tau \sim Normal(0, \tau^2)$ where $\tau^2 \sim InvGamma(0 : 01, 0 : 01)$
- (3) Bayesian LASSO: $\beta_j | \tau^2 \sim DE(0, \tau^2)$ where $\tau^2 \sim InvGamma(0 : 01, 0 : 01)$

Compare convergence and the posterior distribution of the regression coefficients under these three priors. In particular, are the same OTU's significant in all three fits?

Load data and select 50 most ocrrelated OUT variables.

```
library(rjags)
library(coda)
library(modeest)
load("homes.RData")

X <- OTU != 0
Y <- homes$MeanAnnualPrecipitation

C_xy <- cor(X, Y)

top <- function(x, n) {
  tail(order(x), n)
}

indices <- top(C_xy, 50)

X <- X[, indices]

top.corr <- C_xy[indices]
```

```
# Y <- scale(Y) X <- scale(X)
```

```
DEBUG <- FALSE
if (DEBUG) {
  nSamples <- 10000
  n.chains <- 1
} else {
  nSamples <- 10000
  n.chains <- 1
}
```

It's not specified what the prior variance is for $E[Y_j|X_j]$. We will assume $Y|\beta \sim N(y \cdot \beta, \sigma^2)$ where $\sigma^2 \sim InvGamma(0.1, 0.1)$

```
n <- nrow(X)

sigma.beta <- 100
inv.gamma.param <- 0.01
p <- ncol(X)

model_string.normal_uniformative <- "model{
  # Likelihood
  for(i in 1:n){
    Y[i] ~ dnorm(mu[i],inv.var)
    mu[i] <- intercept +inprod(X[i,],beta[])
  }

  # Prior for beta
  for(j in 1:p){
    beta[j] ~ dnorm(0,1/sigma.beta^2)
  }
  intercept ~ dnorm(0,1/sigma.beta^2)

  # Prior for the inverse variance
  inv.var ~ dgamma(inv.gamma.param, inv.gamma.param)
  sigma <- 1/sqrt(inv.var)
}"

model.normal_uniformative <- jags.model(textConnection(model_string.normal_uniformative), data

## Compiling model graph
## Resolving undeclared variables
## Allocating nodes
## Graph information:
## Observed stochastic nodes: 1133
```

```

##      Unobserved stochastic nodes: 52
##      Total graph size: 61098
##
## Initializing model
update(model.normal_uninformative, nSamples, progress.bar="none"); # Burnin
samp.coeff.normal_uninformative <- coda.samples(model.normal_uninformative, variable.names=c("int"

```

(2) Assess convergence of the samplers

We sample from our model after burn in. Not all of the plots are not presented we assesed convergence by; - viewing the time sereies for the intercept and each of the predictors. For this we utilized the coda package. - ran multiple chains and viewed evaluated the autocorrelation plots. - calculated the posterior means for the intercept and the β_{aj} - utilized the mlv funtions in the modeest to calculate the MAP estimated of the posterior modes - compared the 95% prediction intervals for the intercepts against the p-values from the logistic regression maximum likelihood model - Gelman plots are optionally produced whem the nuymber of MCMC chains is greater than one.

Some of the code is run conditionally through the DEBUG flag.

```
summary(samp.coeff.normal_uninformative)
```

```

##
## Iterations = 10001:30000
## Thinning interval = 1
## Number of chains = 1
## Sample size per chain = 20000
##
## 1. Empirical mean and standard deviation for each variable,
##     plus standard error of the mean:
##
##           Mean        SD Naive SE Time-series SE
## beta[1]   -2.7523  1.955  0.01383      0.04386
## beta[2]   -3.2156  1.872  0.01324      0.03941
## beta[3]   -0.2299  1.727  0.01221      0.03081
## beta[4]    1.6352  1.855  0.01311      0.04566
## beta[5]   -1.9435  1.928  0.01364      0.03871
## beta[6]   -4.2102  1.777  0.01257      0.03309
## beta[7]    1.1921  1.587  0.01122      0.03188
## beta[8]   -0.9285  1.504  0.01064      0.02602
## beta[9]   -1.8372  1.632  0.01154      0.03184
## beta[10]   0.1512  1.590  0.01124      0.02909
## beta[11]  -3.4235  1.794  0.01269      0.04170
## beta[12]  -0.2794  1.460  0.01033      0.02084
## beta[13]   2.2910  1.439  0.01017      0.01819
## beta[14]  -1.4831  1.522  0.01077      0.02364
## beta[15]   0.2902  1.818  0.01285      0.03914
## beta[16]   2.0028  1.596  0.01128      0.03355

```

```

## beta[17] -1.9156 1.643 0.01162      0.02565
## beta[18] -2.3804 1.798 0.01272      0.03165
## beta[19] -4.3252 1.884 0.01332      0.05108
## beta[20] 2.6645 1.832 0.01296      0.03606
## beta[21] -0.9085 1.904 0.01346      0.04224
## beta[22] -1.8324 1.822 0.01288      0.03901
## beta[23] 1.1196 1.704 0.01205      0.03501
## beta[24] 3.0988 1.533 0.01084      0.02474
## beta[25] 0.5196 1.603 0.01134      0.02974
## beta[26] 2.6608 1.867 0.01320      0.04053
## beta[27] 1.6665 1.951 0.01380      0.04670
## beta[28] 5.2513 1.910 0.01351      0.04569
## beta[29] -0.1789 1.598 0.01130      0.02487
## beta[30] -1.2692 1.744 0.01234      0.03332
## beta[31] 0.4905 1.904 0.01346      0.04282
## beta[32] 1.4549 1.913 0.01353      0.04143
## beta[33] 2.5071 1.639 0.01159      0.03761
## beta[34] 2.5273 2.031 0.01436      0.05152
## beta[35] 3.7270 1.899 0.01343      0.04172
## beta[36] 4.1416 1.467 0.01037      0.02833
## beta[37] 3.9701 1.891 0.01337      0.04166
## beta[38] 2.0902 1.777 0.01256      0.03251
## beta[39] 2.0913 1.897 0.01341      0.03906
## beta[40] 5.5296 1.911 0.01352      0.04606
## beta[41] 2.1965 1.843 0.01303      0.04120
## beta[42] 1.2081 1.715 0.01213      0.03805
## beta[43] 1.1020 1.792 0.01267      0.03908
## beta[44] 5.4614 1.854 0.01311      0.04421
## beta[45] 6.2316 1.690 0.01195      0.03263
## beta[46] 2.6092 1.594 0.01127      0.02618
## beta[47] 1.5390 1.690 0.01195      0.03096
## beta[48] 4.9301 1.589 0.01123      0.02441
## beta[49] 6.9068 1.743 0.01233      0.03737
## beta[50] 13.8127 26.946 0.19054     12.64367
## intercept 40.3121 26.941 0.19051     12.69317
##
## 2. Quantiles for each variable:
##
##          2.5%    25%   50%   75%  97.5%
## beta[1] -6.49082 -4.08341 -2.7834 -1.45957 1.1927
## beta[2] -6.86762 -4.48320 -3.2169 -1.95179 0.4794
## beta[3] -3.59432 -1.39256 -0.2200  0.93940 3.1694
## beta[4] -1.99029  0.38543  1.6252  2.88274 5.2837
## beta[5] -5.72492 -3.24711 -1.9467 -0.64659 1.8146
## beta[6] -7.68534 -5.41768 -4.1861 -3.00007 -0.7560
## beta[7] -1.90469  0.11955  1.1730  2.27191 4.3210
## beta[8] -3.87052 -1.93800 -0.9277  0.08928 2.0489
## beta[9] -5.02797 -2.93617 -1.8468 -0.73671 1.3725

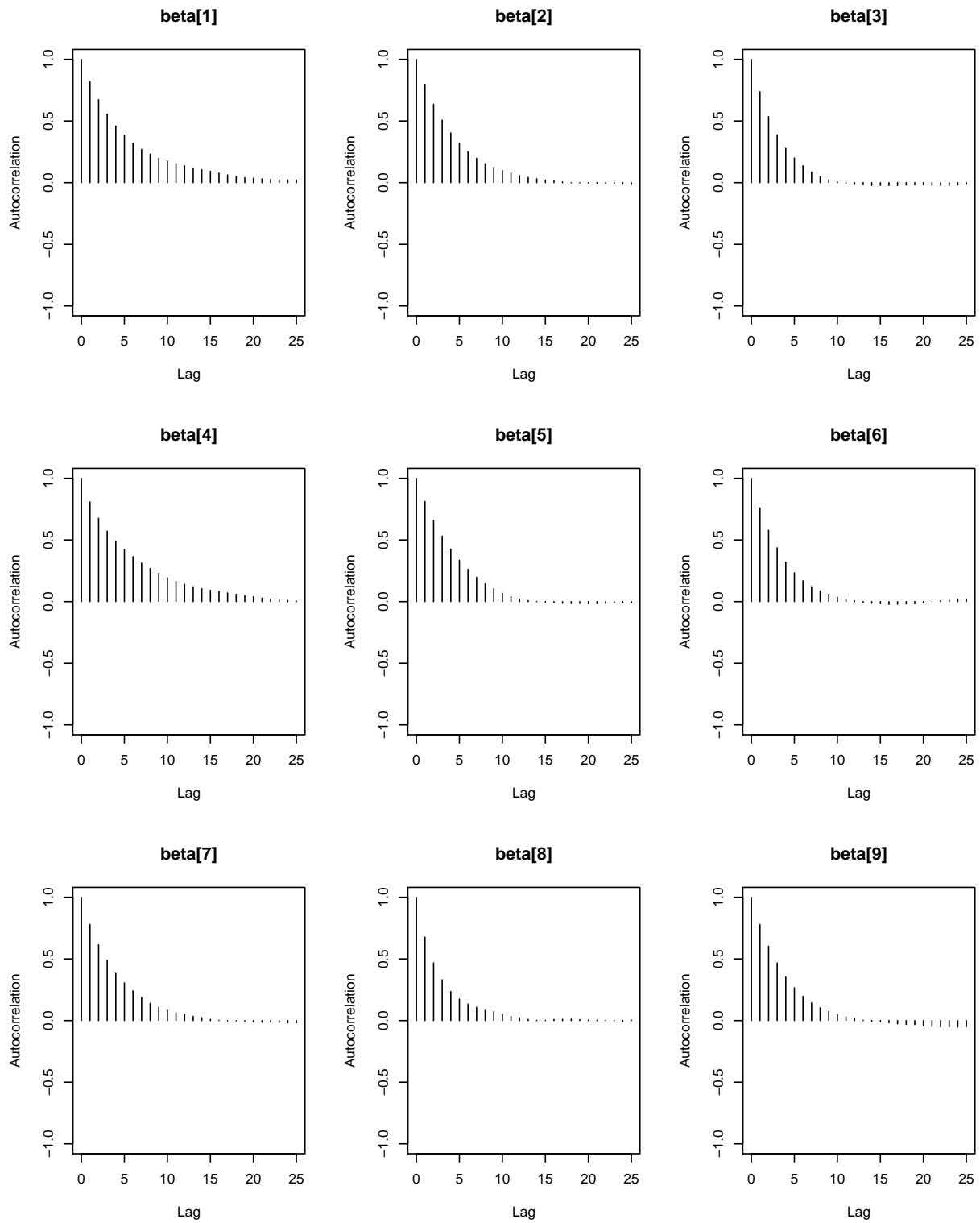
```

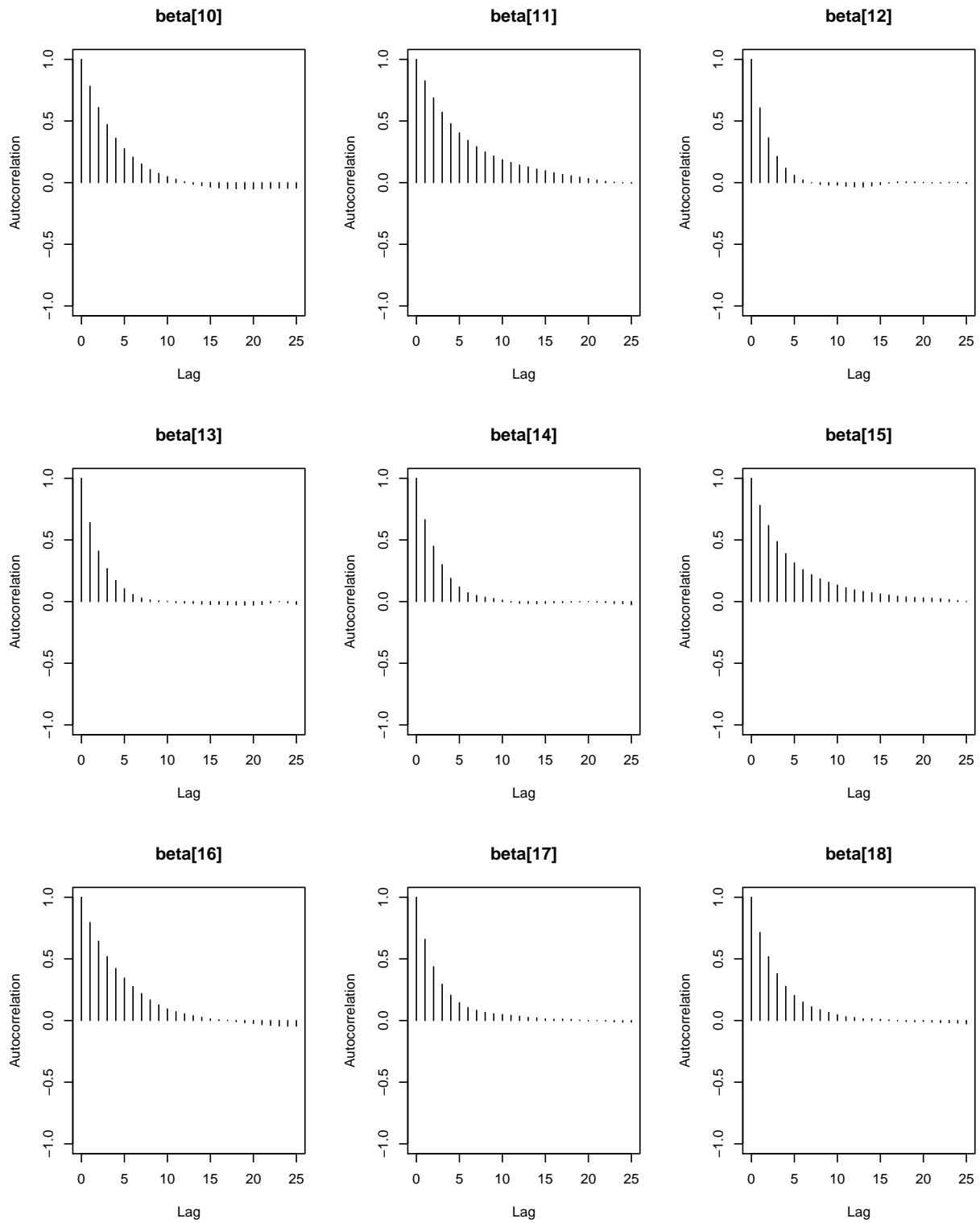
```

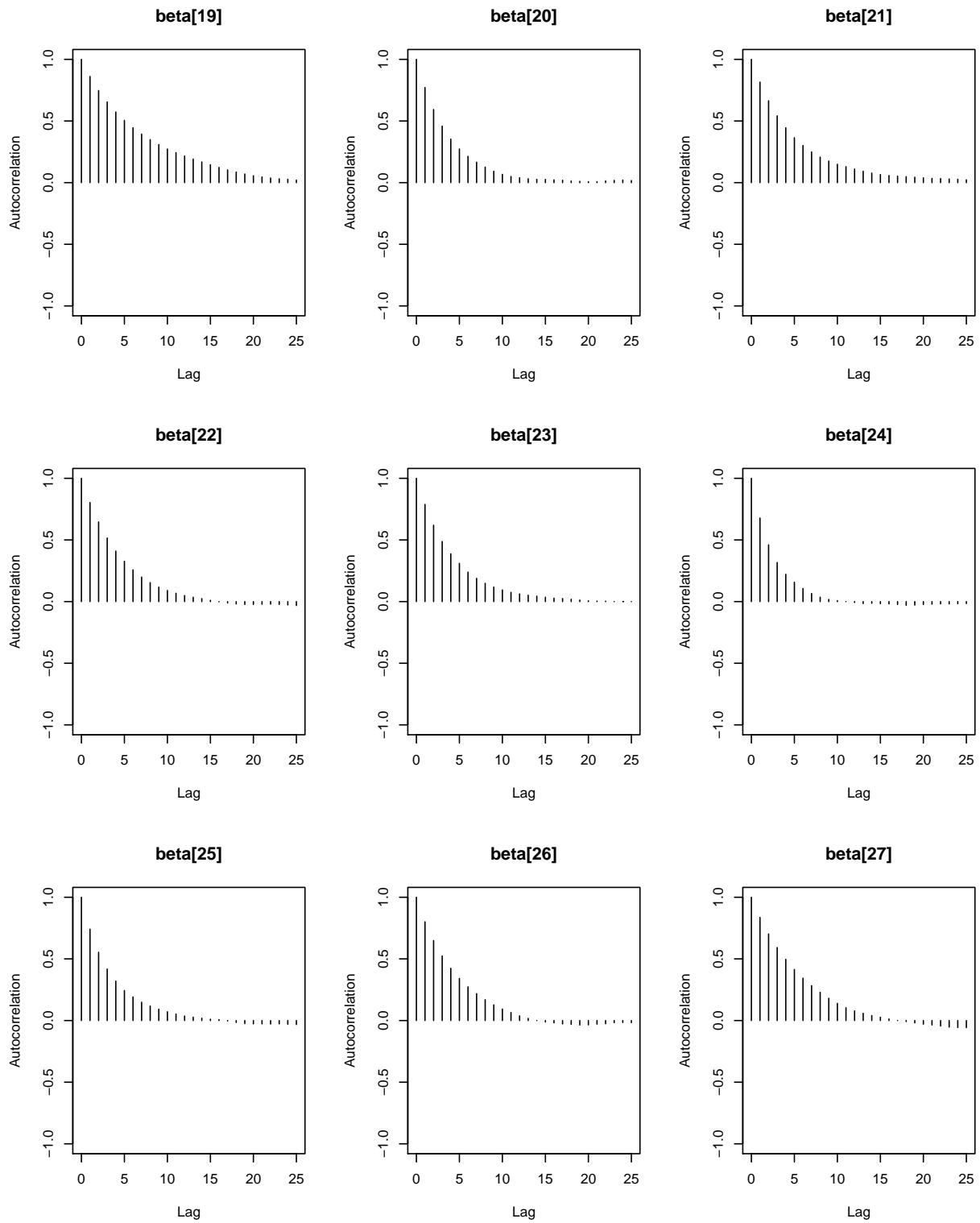
## beta[10] -2.91898 -0.93415 0.1603 1.24229 3.2253
## beta[11] -6.89034 -4.65585 -3.4135 -2.21350 0.1253
## beta[12] -3.13993 -1.27074 -0.2718 0.69701 2.5888
## beta[13] -0.51702 1.31100 2.2948 3.25315 5.1301
## beta[14] -4.44010 -2.52333 -1.4756 -0.44983 1.4917
## beta[15] -3.27065 -0.92471 0.3085 1.50862 3.8682
## beta[16] -1.11014 0.92435 2.0036 3.07201 5.1275
## beta[17] -5.15138 -3.01363 -1.9207 -0.79995 1.2861
## beta[18] -5.90033 -3.59626 -2.3766 -1.18039 1.1062
## beta[19] -8.01524 -5.58682 -4.3163 -3.06524 -0.5929
## beta[20] -0.89197 1.42790 2.6647 3.90671 6.2476
## beta[21] -4.66815 -2.18296 -0.8925 0.37881 2.8481
## beta[22] -5.36357 -3.05770 -1.8446 -0.62290 1.8217
## beta[23] -2.21205 -0.01518 1.1065 2.27433 4.4949
## beta[24] 0.08798 2.04666 3.1042 4.14266 6.1050
## beta[25] -2.59757 -0.56860 0.5109 1.59169 3.7009
## beta[26] -1.05162 1.40357 2.6816 3.93300 6.2497
## beta[27] -2.10850 0.35295 1.6620 2.98298 5.5130
## beta[28] 1.54398 3.96849 5.2293 6.54068 9.0551
## beta[29] -3.31833 -1.26416 -0.1827 0.90865 2.9162
## beta[30] -4.63081 -2.44753 -1.2915 -0.12176 2.2146
## beta[31] -3.21447 -0.81205 0.4946 1.75500 4.2351
## beta[32] -2.37108 0.17934 1.4789 2.73939 5.1325
## beta[33] -0.71782 1.40221 2.5294 3.62523 5.6866
## beta[34] -1.52921 1.17411 2.5354 3.88075 6.5176
## beta[35] -0.04275 2.46682 3.7130 5.00951 7.4492
## beta[36] 1.27948 3.14586 4.1579 5.13688 7.0012
## beta[37] 0.28132 2.68417 3.9570 5.21972 7.7051
## beta[38] -1.40301 0.88869 2.0847 3.27908 5.5707
## beta[39] -1.70137 0.81913 2.1150 3.35042 5.7957
## beta[40] 1.76589 4.22263 5.5237 6.82990 9.2682
## beta[41] -1.44059 0.95093 2.2025 3.43053 5.8131
## beta[42] -2.20404 0.05125 1.2318 2.37594 4.5392
## beta[43] -2.40861 -0.10507 1.1141 2.31406 4.5742
## beta[44] 1.83849 4.21025 5.4452 6.70632 9.1344
## beta[45] 2.90954 5.10195 6.2244 7.37260 9.5573
## beta[46] -0.53689 1.54431 2.6060 3.69282 5.7256
## beta[47] -1.76435 0.39036 1.5516 2.67120 4.8673
## beta[48] 1.78513 3.86119 4.9285 5.99231 8.0380
## beta[49] 3.50227 5.72546 6.9180 8.07546 10.3180
## beta[50] -45.29463 -4.00377 12.9136 30.71330 65.5905
## intercept -11.47476 23.11187 41.1536 58.12242 99.6424

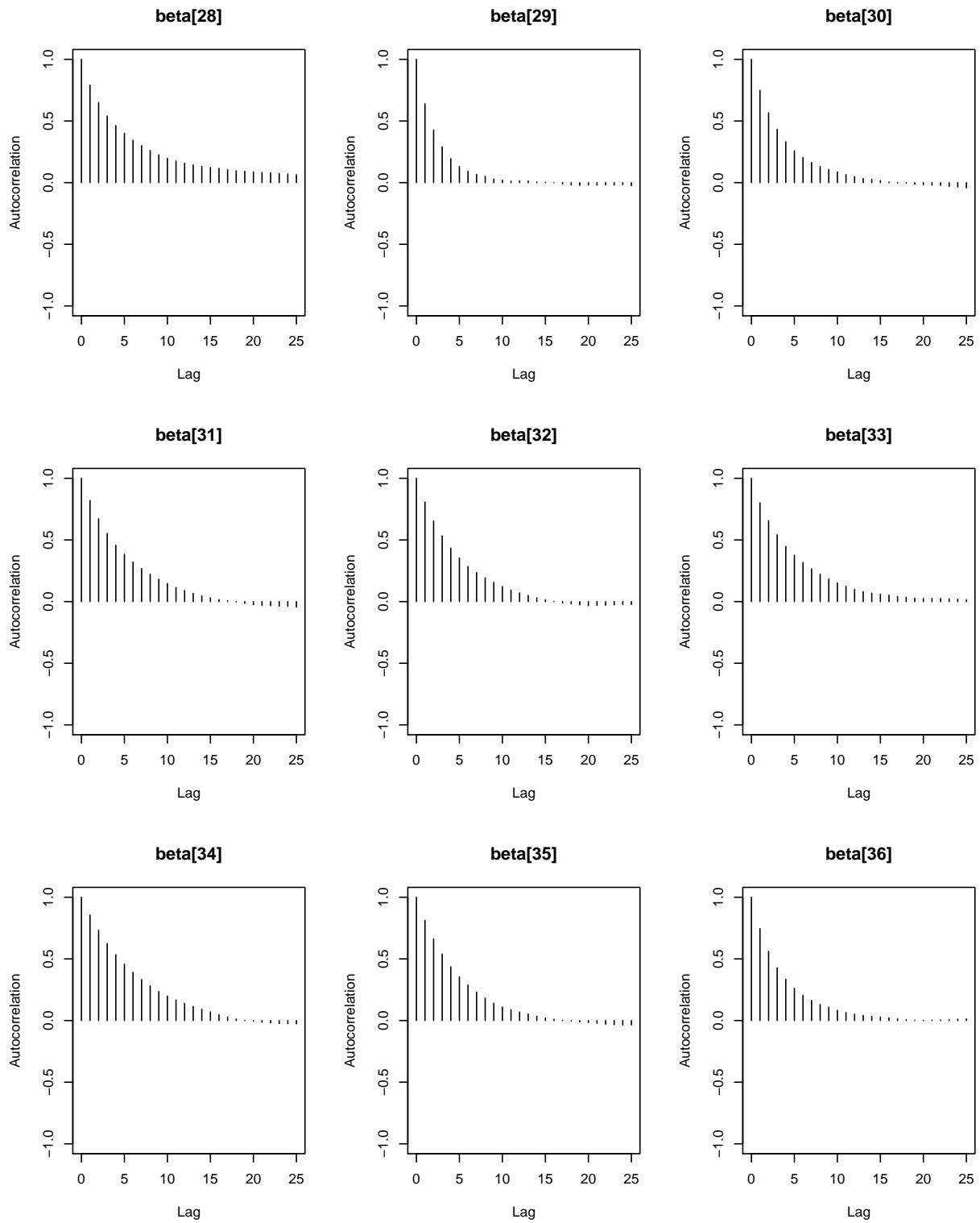
autocorr.plot(samp.coeff.normal_uninformative)

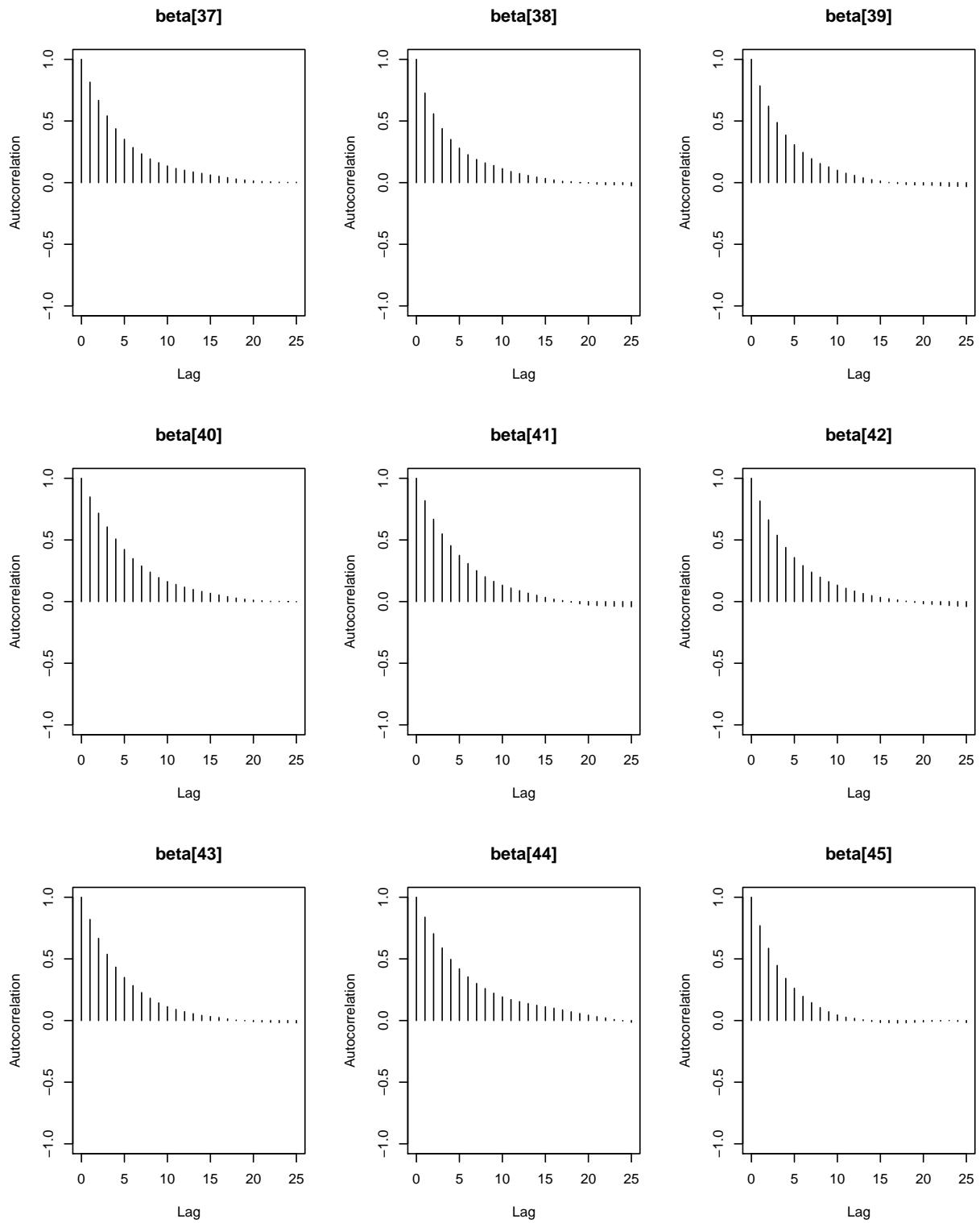
```

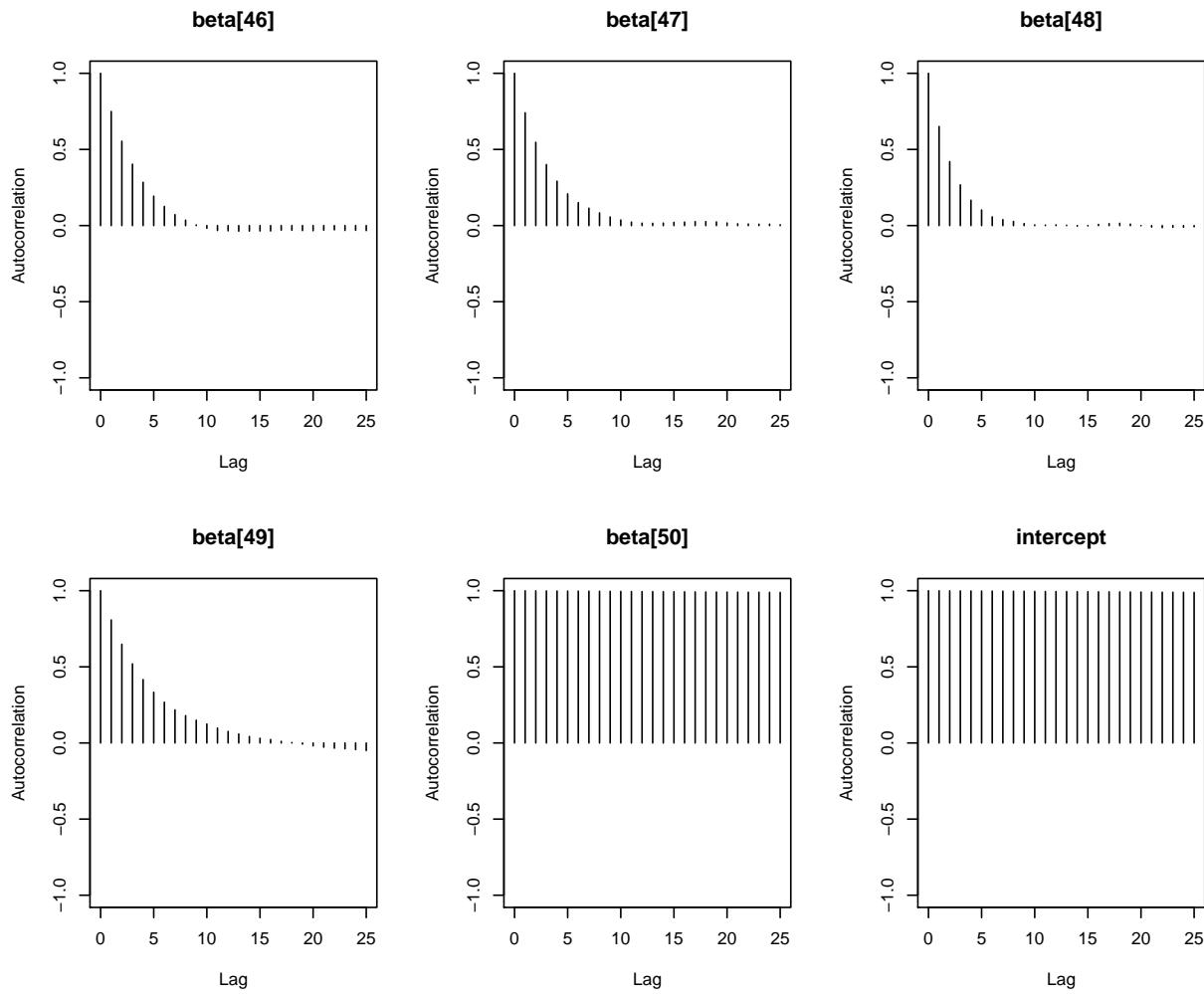






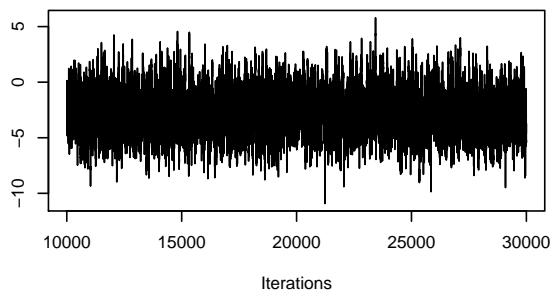




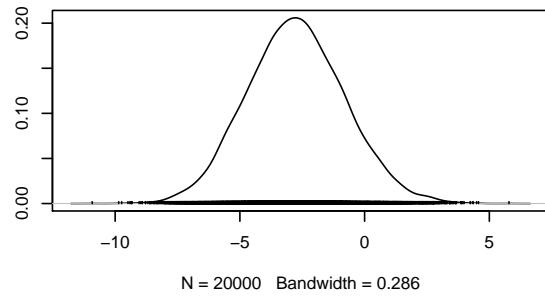


```
plot(samp.coeff.normal_uninformative)
```

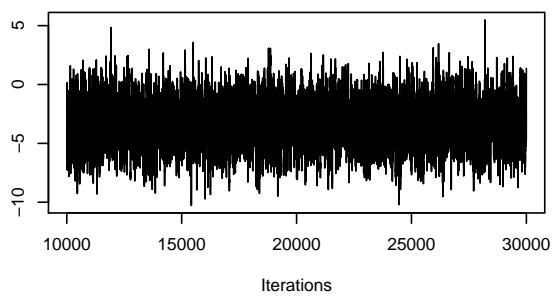
Trace of beta[1]



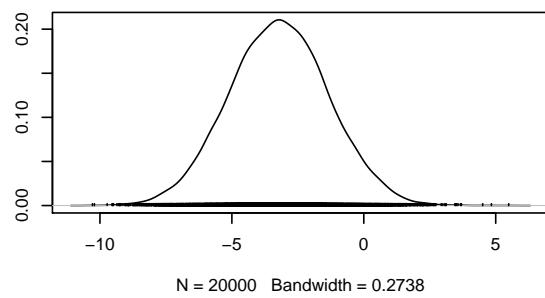
Density of beta[1]



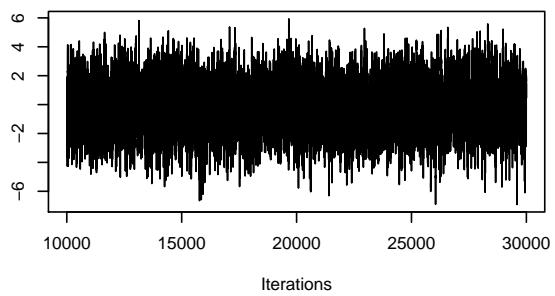
Trace of beta[2]



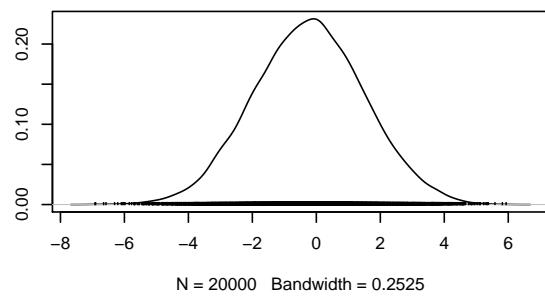
Density of beta[2]



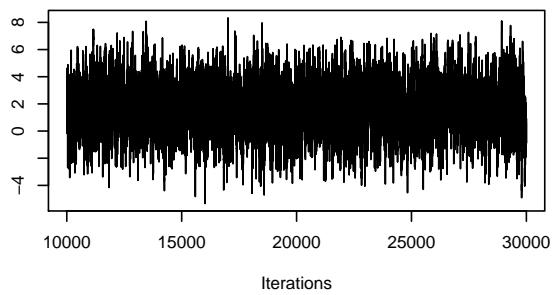
Trace of beta[3]



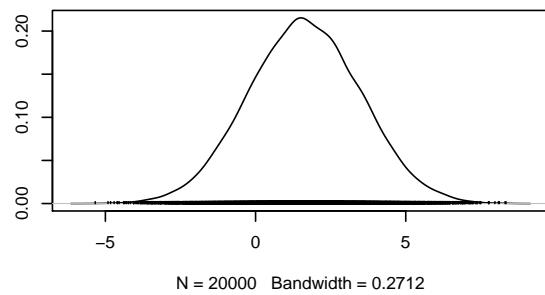
Density of beta[3]



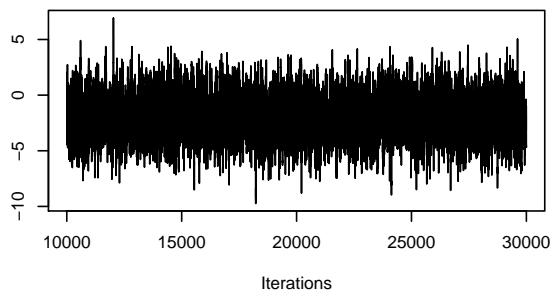
Trace of beta[4]



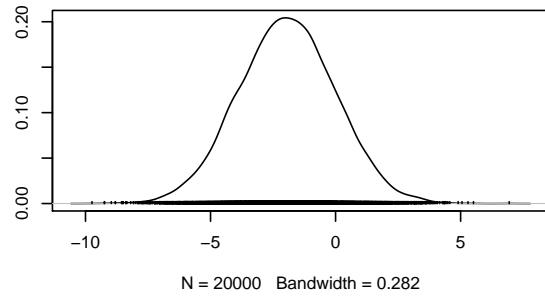
Density of beta[4]



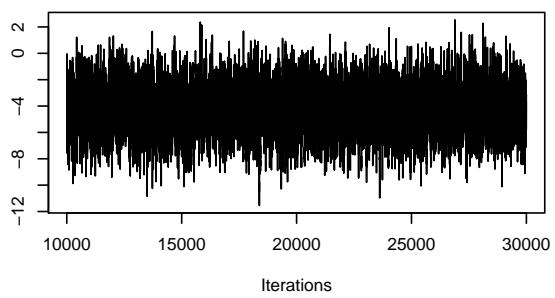
Trace of beta[5]



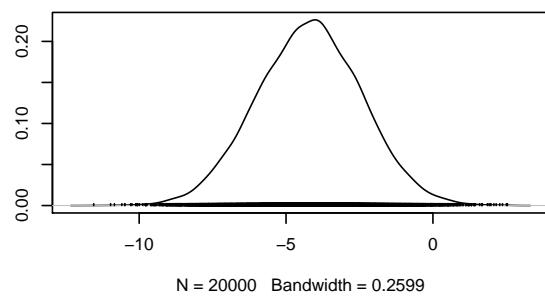
Density of beta[5]



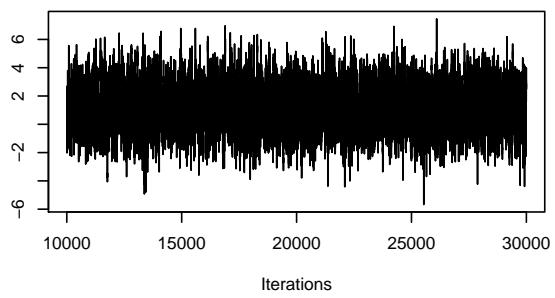
Trace of beta[6]



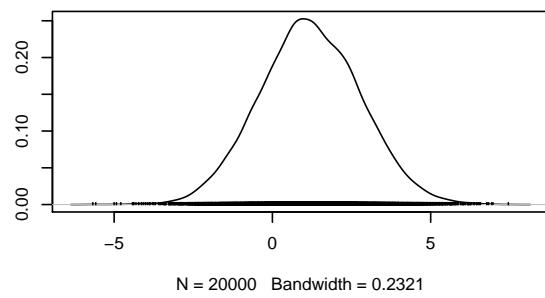
Density of beta[6]



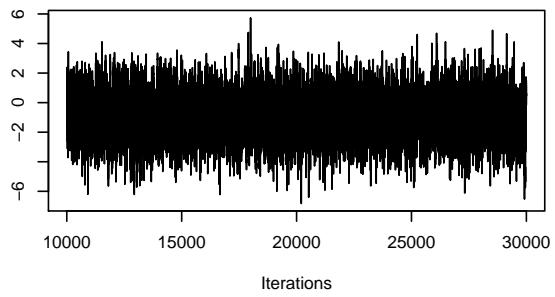
Trace of beta[7]



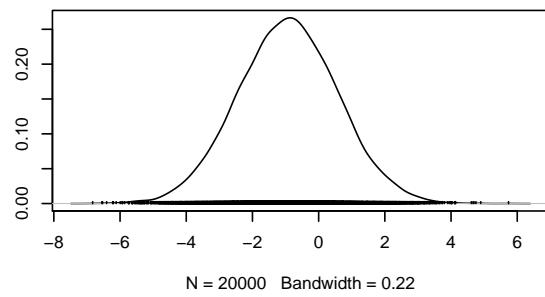
Density of beta[7]



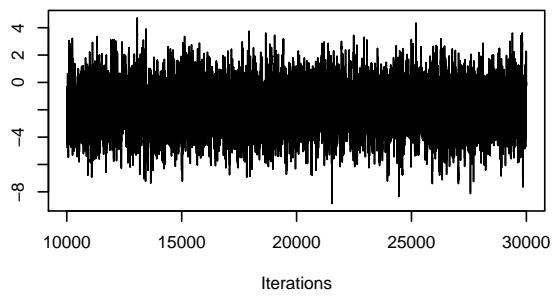
Trace of beta[8]



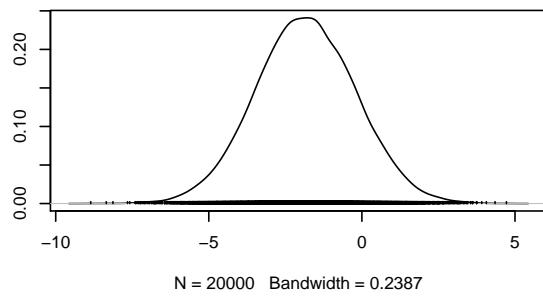
Density of beta[8]



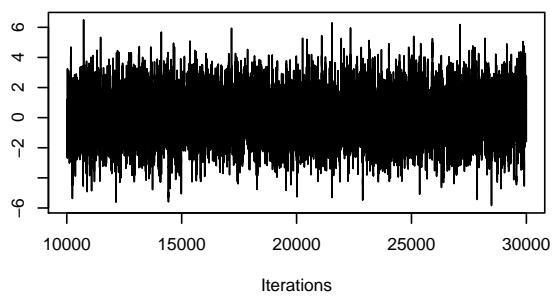
Trace of beta[9]



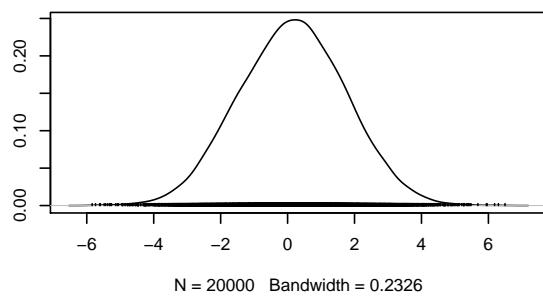
Density of beta[9]



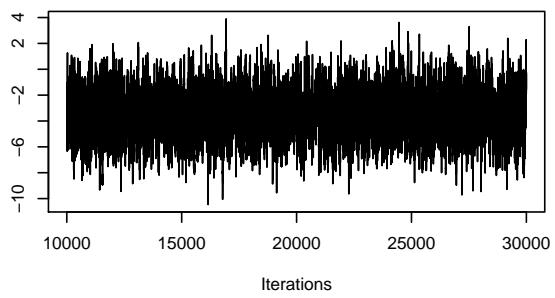
Trace of beta[10]



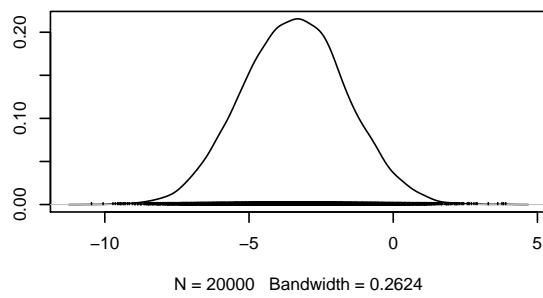
Density of beta[10]



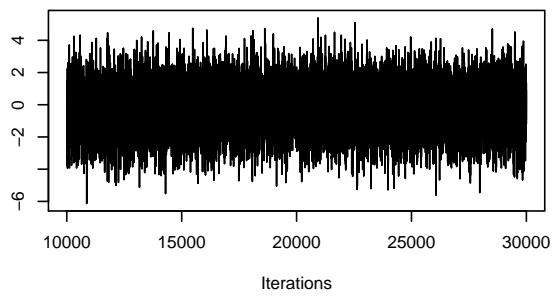
Trace of beta[11]



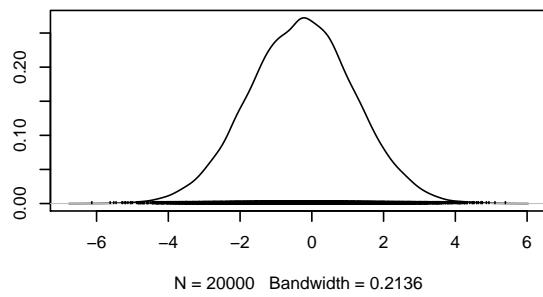
Density of beta[11]



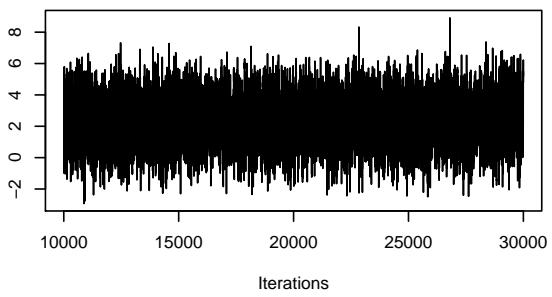
Trace of beta[12]



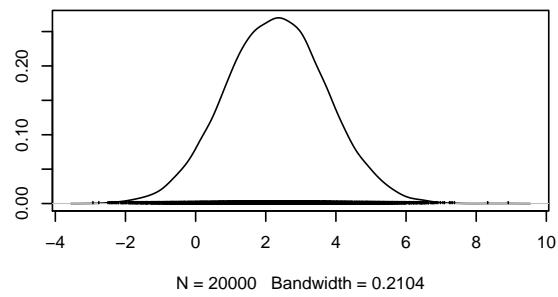
Density of beta[12]



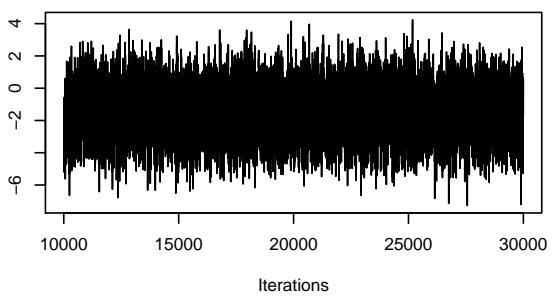
Trace of beta[13]



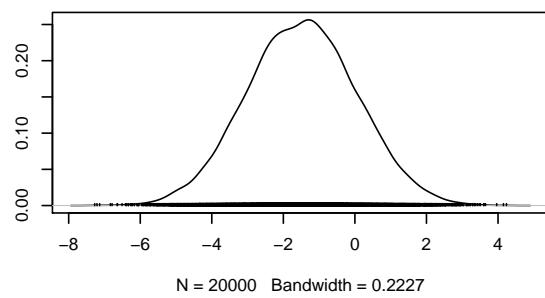
Density of beta[13]



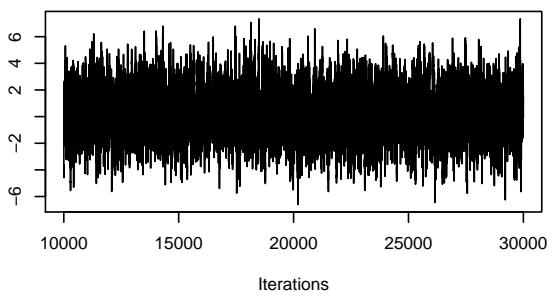
Trace of beta[14]



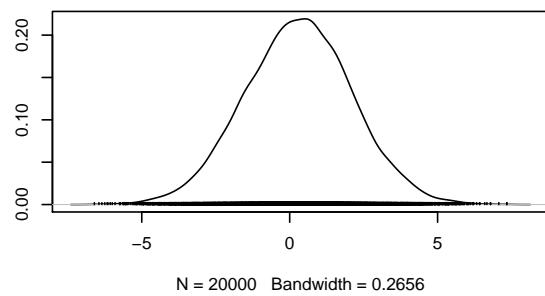
Density of beta[14]



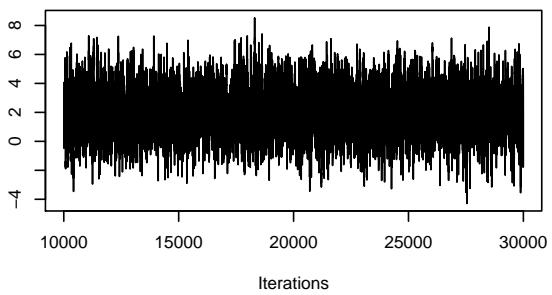
Trace of beta[15]



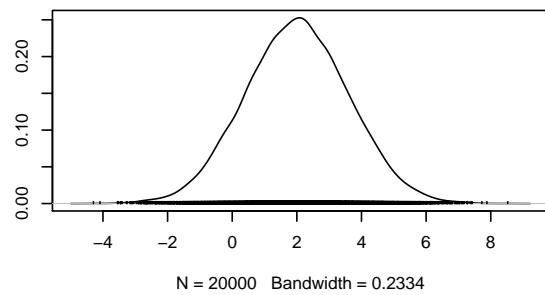
Density of beta[15]



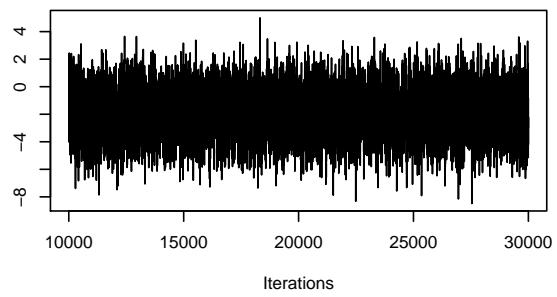
Trace of beta[16]



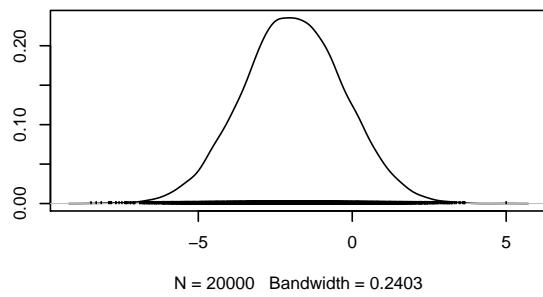
Density of beta[16]



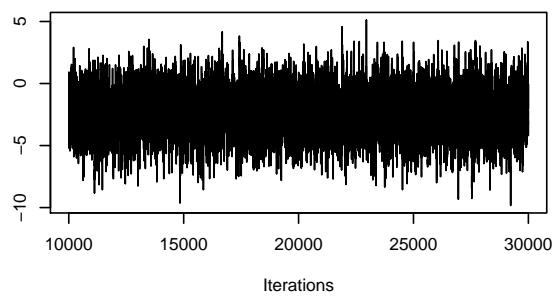
Trace of beta[17]



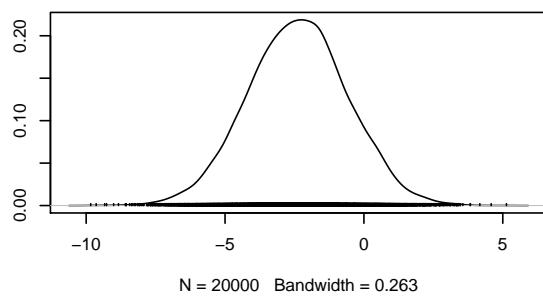
Density of beta[17]



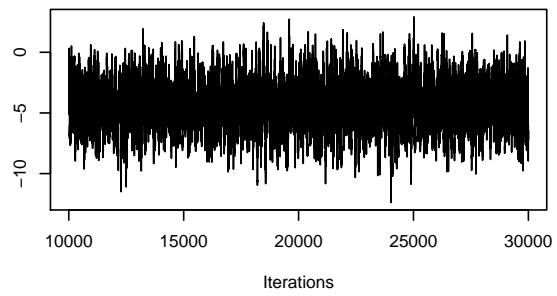
Trace of beta[18]



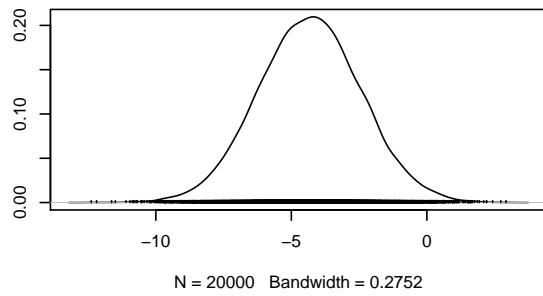
Density of beta[18]



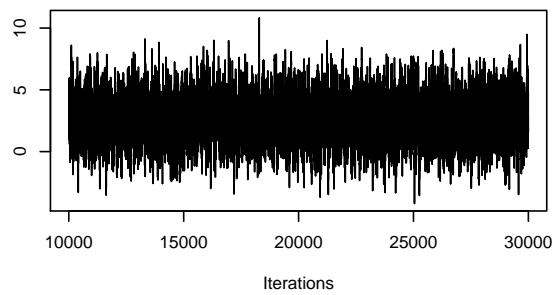
Trace of beta[19]



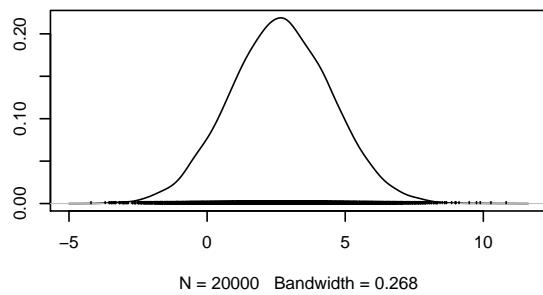
Density of beta[19]



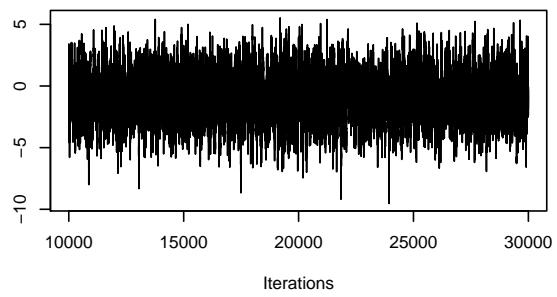
Trace of beta[20]



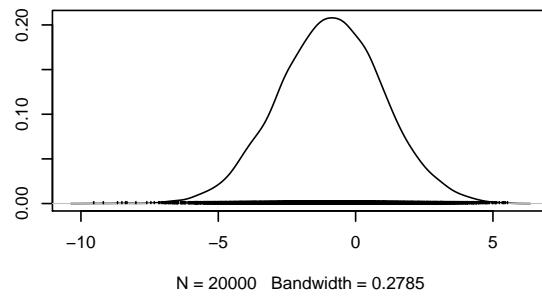
Density of beta[20]



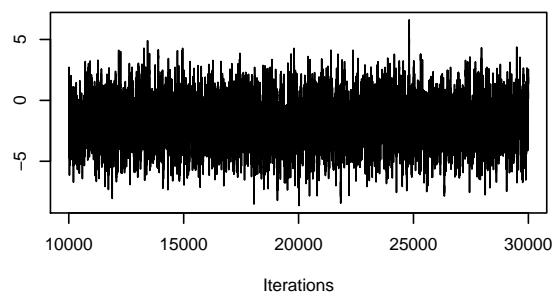
Trace of beta[21]



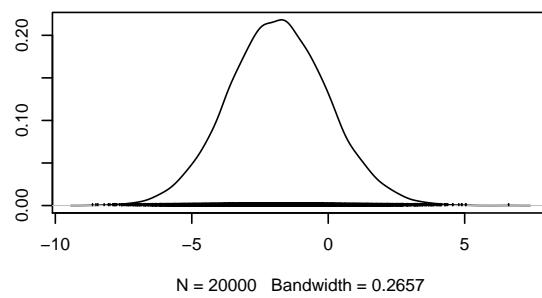
Density of beta[21]



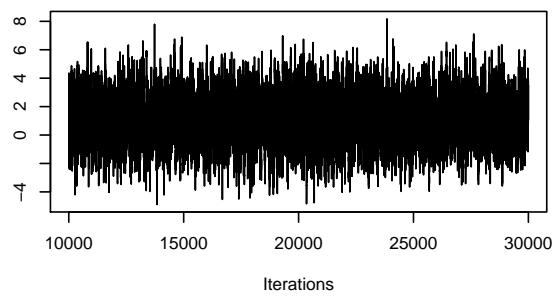
Trace of beta[22]



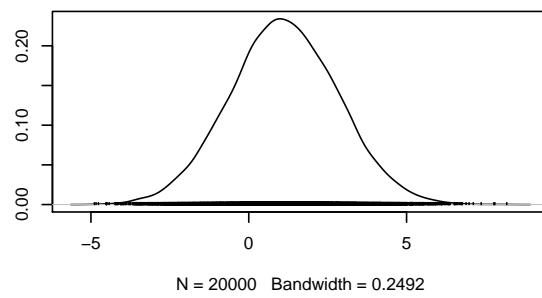
Density of beta[22]



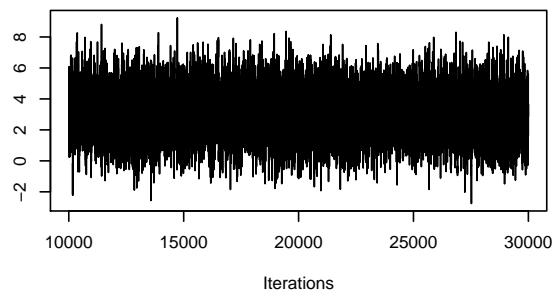
Trace of beta[23]



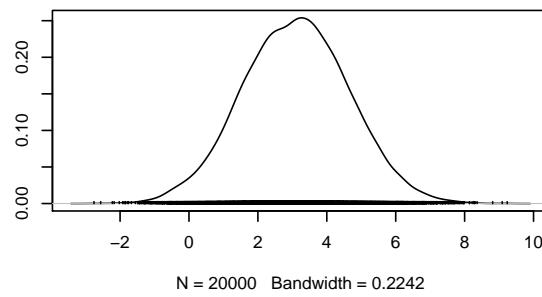
Density of beta[23]



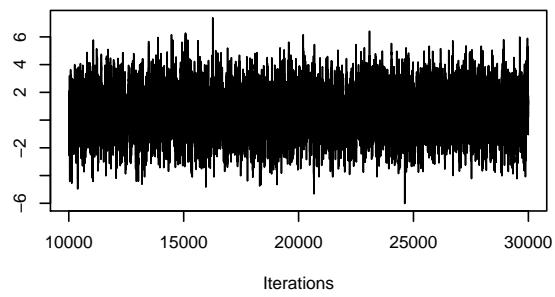
Trace of beta[24]



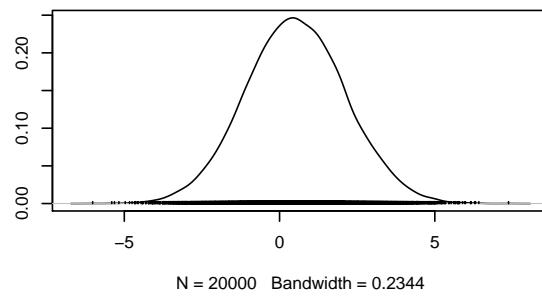
Density of beta[24]



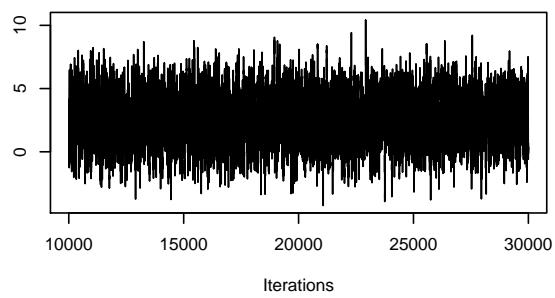
Trace of beta[25]



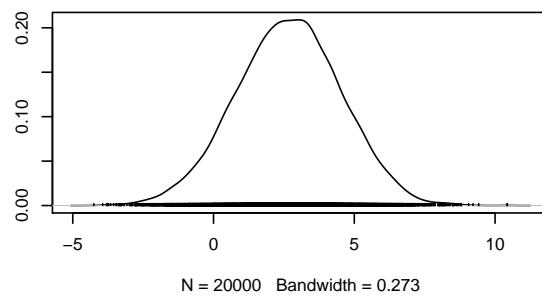
Density of beta[25]



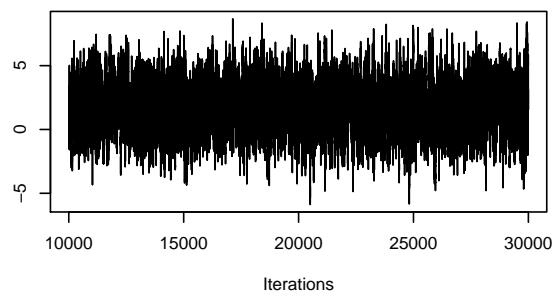
Trace of beta[26]



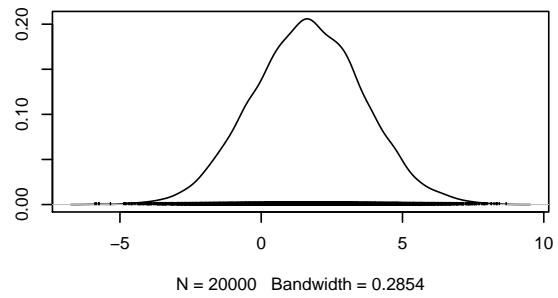
Density of beta[26]



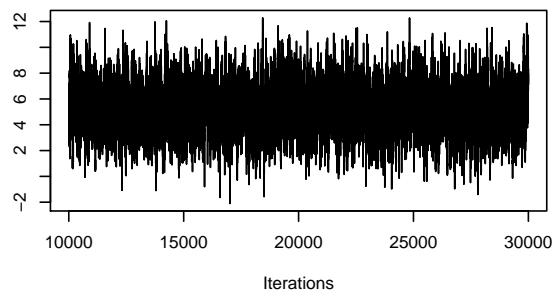
Trace of beta[27]



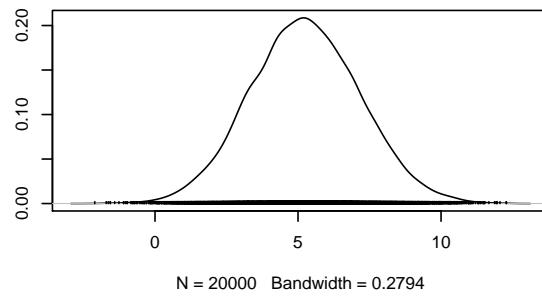
Density of beta[27]



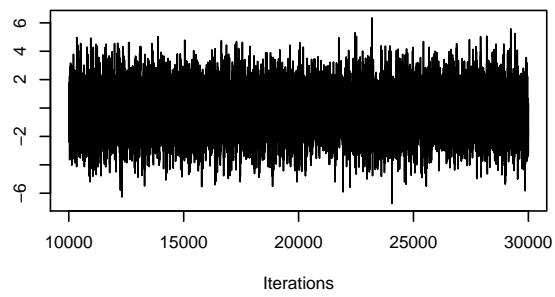
Trace of beta[28]



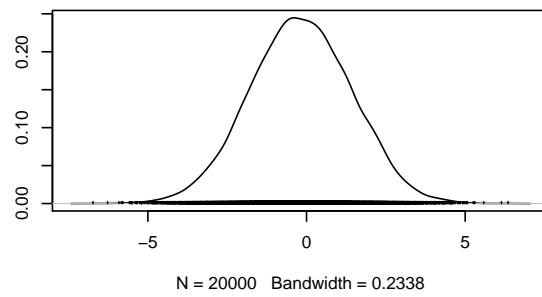
Density of beta[28]



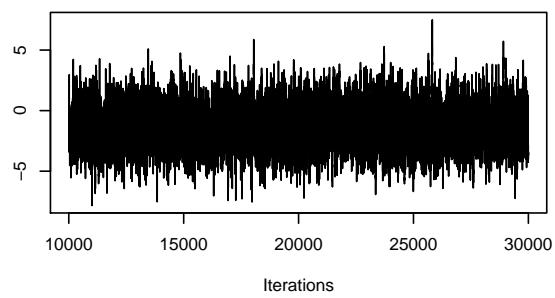
Trace of beta[29]



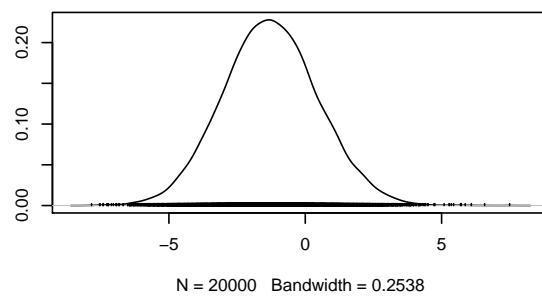
Density of beta[29]



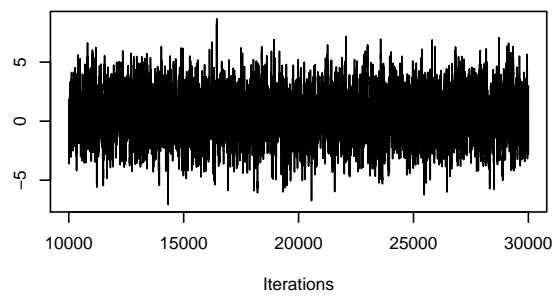
Trace of beta[30]



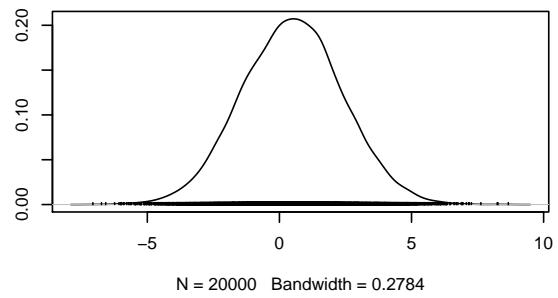
Density of beta[30]



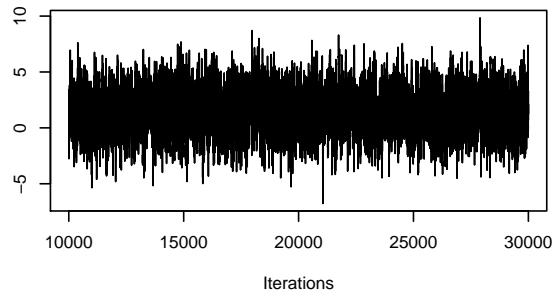
Trace of beta[31]



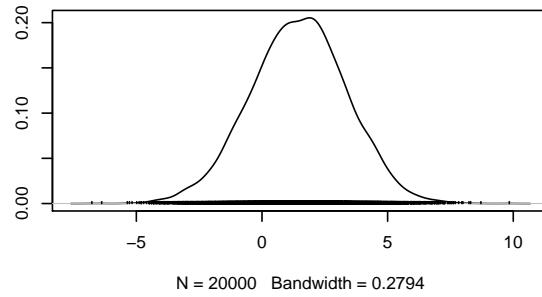
Density of beta[31]



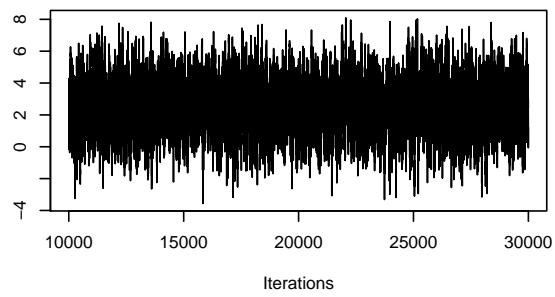
Trace of beta[32]



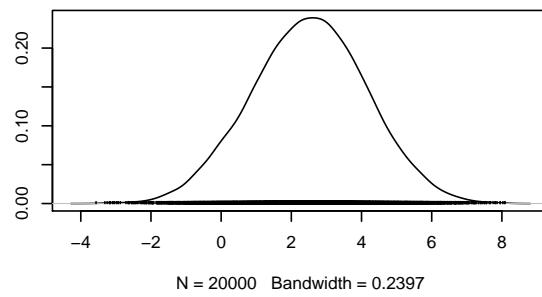
Density of beta[32]



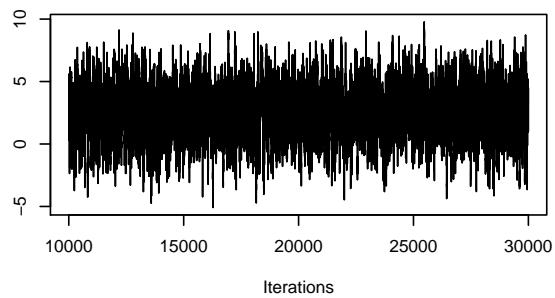
Trace of beta[33]



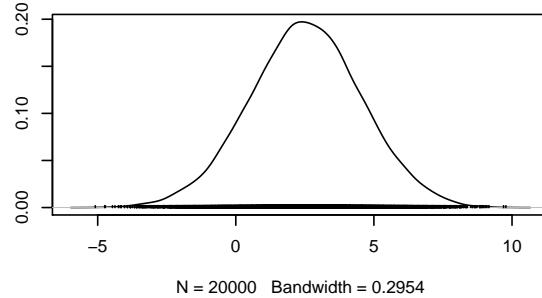
Density of beta[33]



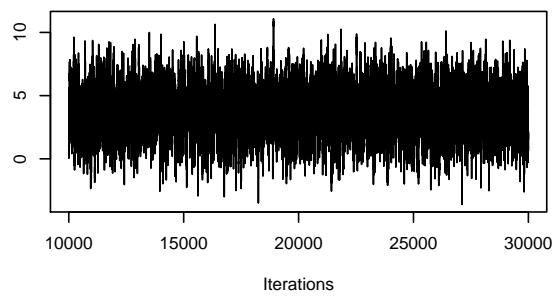
Trace of beta[34]



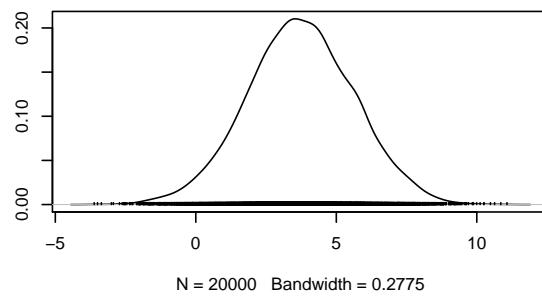
Density of beta[34]



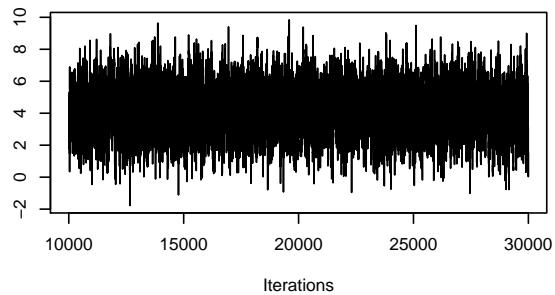
Trace of beta[35]



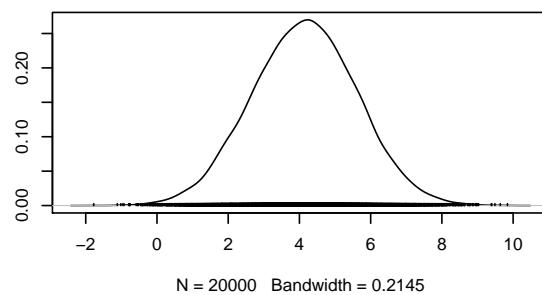
Density of beta[35]



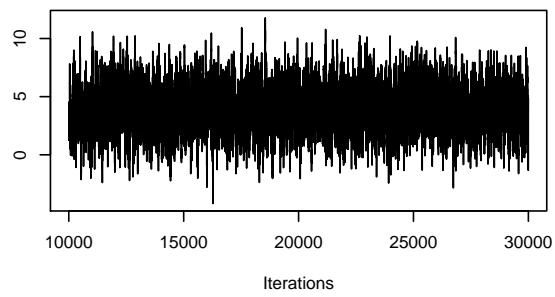
Trace of beta[36]



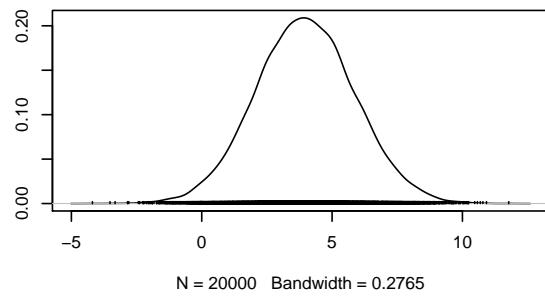
Density of beta[36]



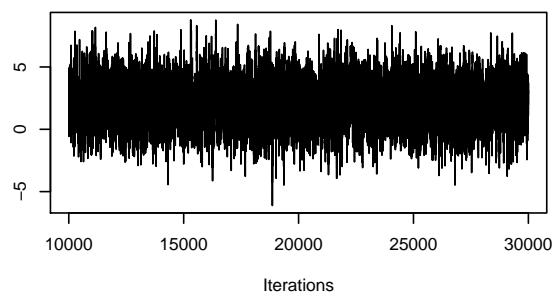
Trace of beta[37]



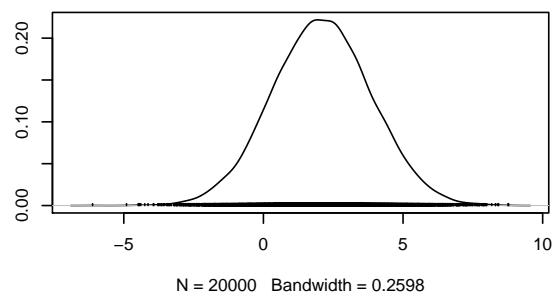
Density of beta[37]



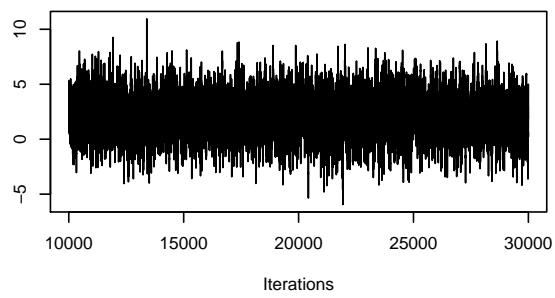
Trace of beta[38]



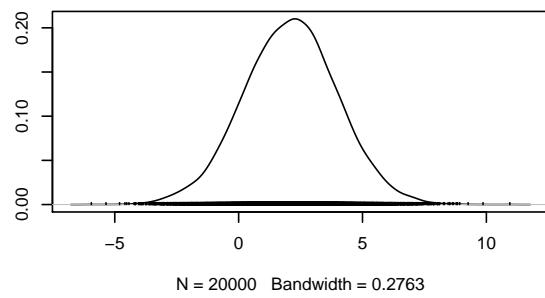
Density of beta[38]



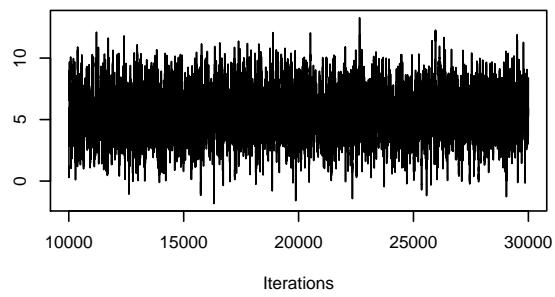
Trace of beta[39]



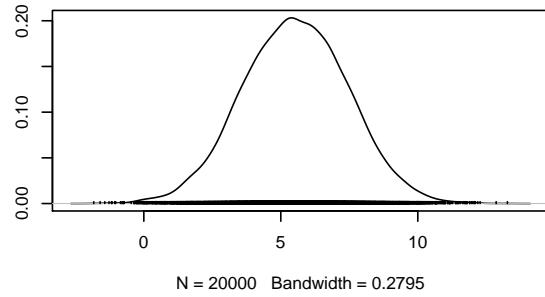
Density of beta[39]



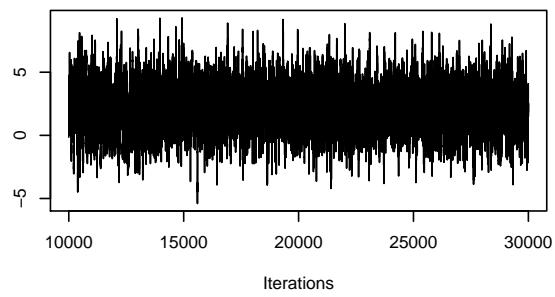
Trace of beta[40]



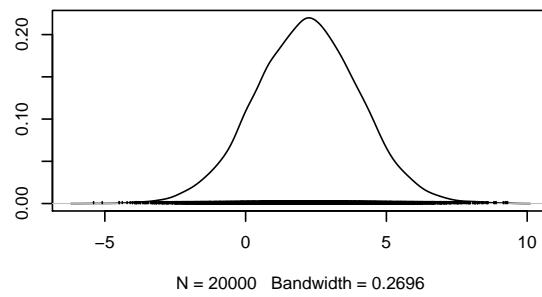
Density of beta[40]



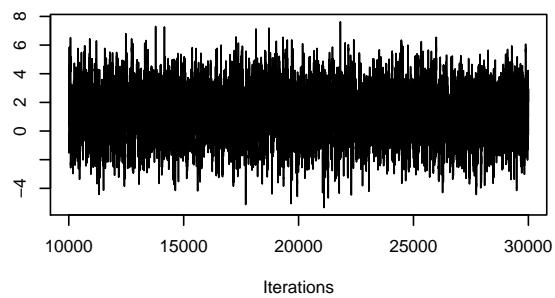
Trace of beta[41]



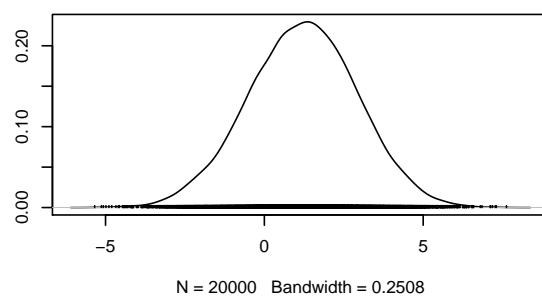
Density of beta[41]



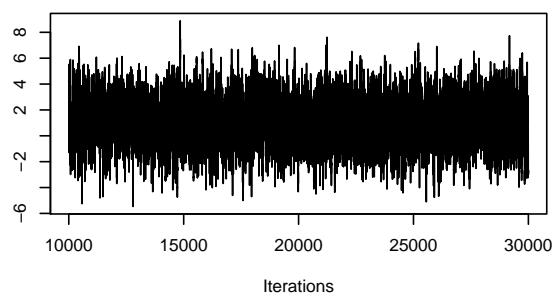
Trace of beta[42]



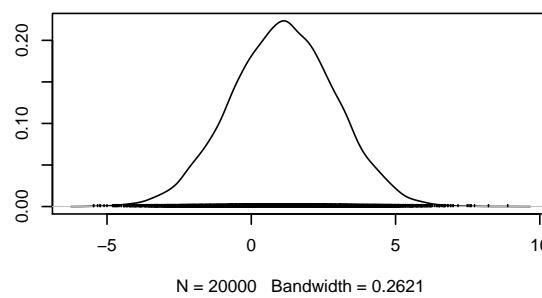
Density of beta[42]



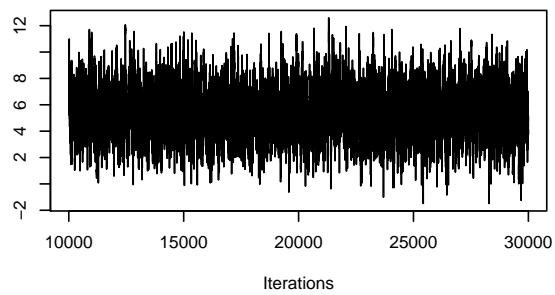
Trace of beta[43]



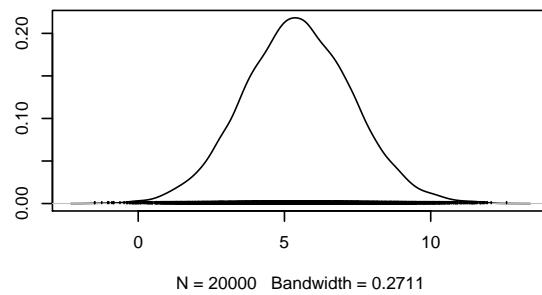
Density of beta[43]



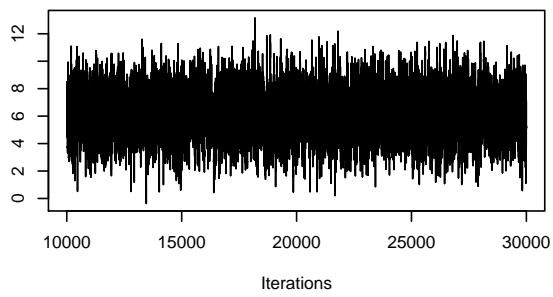
Trace of beta[44]



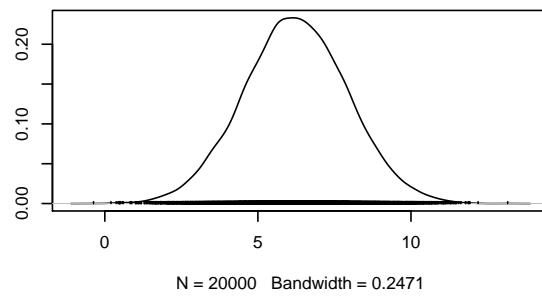
Density of beta[44]



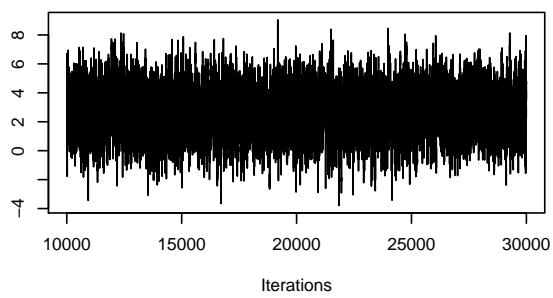
Trace of beta[45]



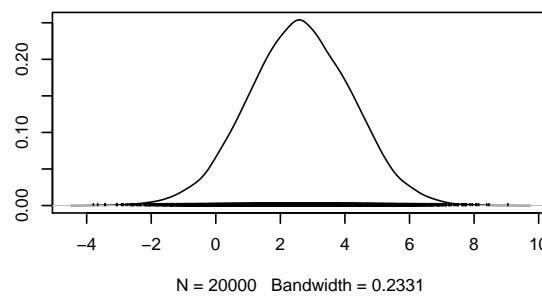
Density of beta[45]



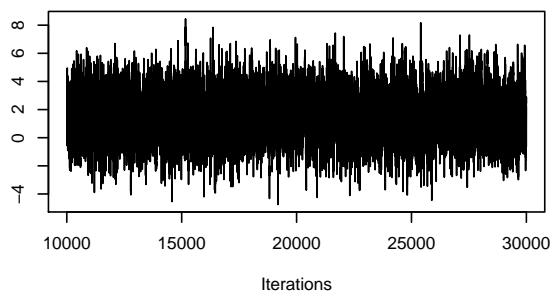
Trace of beta[46]



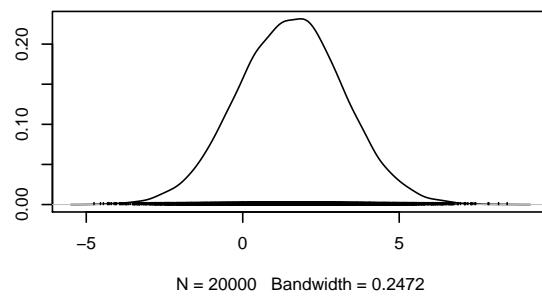
Density of beta[46]



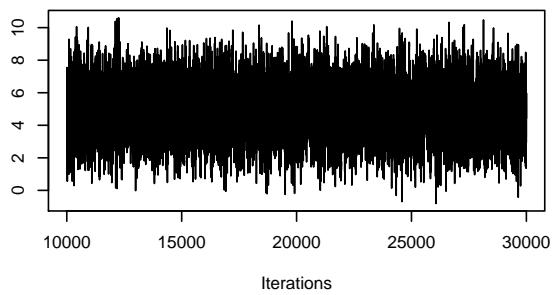
Trace of beta[47]



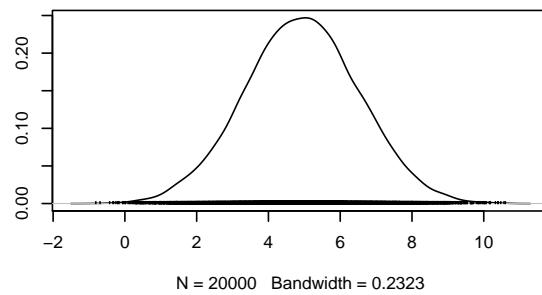
Density of beta[47]

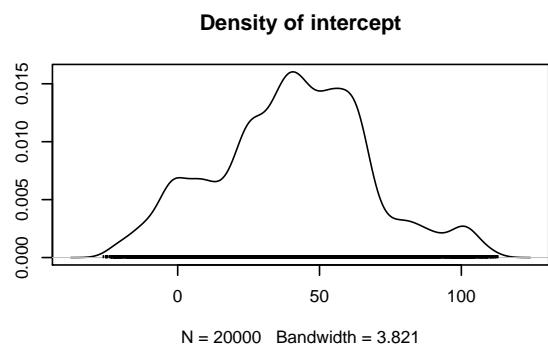
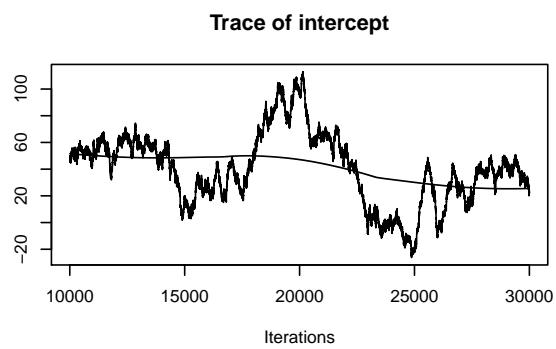
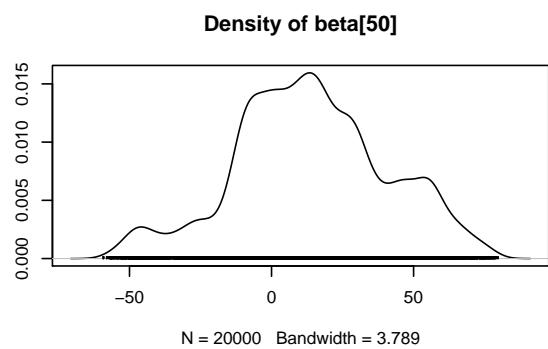
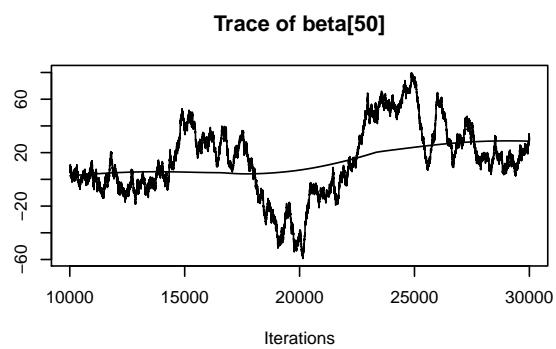
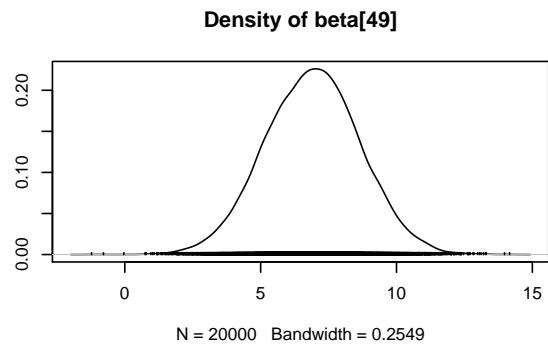
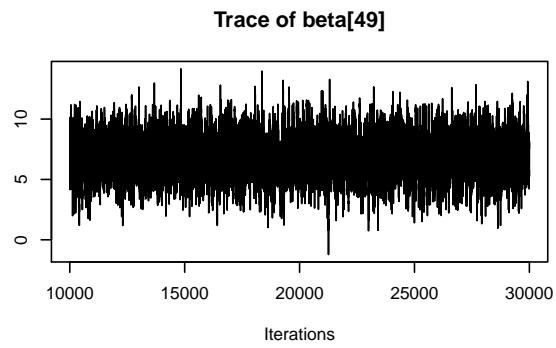


Trace of beta[48]



Density of beta[48]





```
if (DEBUG) {
  # Sample again and estimate posterior
  # means and MAP posterior modes.
  samp.coeff.normal_uninformative.jags <- jags.samples(model.normal_uninformative,
```

```

variable.names = c("intercept",
                 "beta"), n.iter = nSamples,
                 progress.bar = "none")
posterior_means.normal_uninformative <- lapply(samp.coeff.normal_uninformative.jags,
                                               apply, 1, "mean")
pander(posterior_means.normal_uninformative,
       caption = "posterior means second sample")

posterior_modes.normal_uninformative <- lapply(samp.coeff.normal_uninformative.jags,
                                                apply, 1, "mlv")
posterior_modes.normal_uninformative

if (n.chains > 1) {
  gelman.plot(samp.coeff)
}
}

```

Hierarchical Normal Priors

$$\beta_j | \tau \sim \text{Normal}(0, \tau^2) \text{ where } \tau^2 \sim \text{InvGamma}(0 : 01, 0 : 01)$$

```

beta.inv.gamma.param <- 0.01
variance.inv.gamma.param <- 0.01
p <- ncol(X)

model_string.normal_hierarchical <- "model{
  # Likelihood
  for(i in 1:n){
    Y[i] ~ dnorm(mu[i],inv.var)
    mu[i] <- intercept +inprod(X[i,],beta[])
  }

  # Prior for beta
  for(j in 1:p){
    beta[j] ~ dnorm(0,beta.inv.gamma.param)
  }
  intercept ~ dnorm(0,beta.inv.gamma.param)

  # Prior for the inverse variance
  inv.var ~ dgamma(variance.inv.gamma.param, variance.inv.gamma.param)
  sigma <- 1/sqrt(inv.var)

  #Beta Prior for the inverse variance
  inv.var.beta ~ dgamma(beta.inv.gamma.param, beta.inv.gamma.param)
}""

model.normal_hierarchical <- jags.model(textConnection(model_string.normal_hierarchical), data

```

```

## Compiling model graph
##   Resolving undeclared variables
##   Allocating nodes
## Graph information:
##   Observed stochastic nodes: 1133
##   Unobserved stochastic nodes: 53
##   Total graph size: 61096
##
## Initializing model

update(model.normal_hierarchical, nSamples, progress.bar="none"); # Burnin

samp.coeff.normal_hierarchical <- coda.samples(model.normal_hierarchical,variable.names=c("inte
summary(samp.coeff.normal_hierarchical)

##
## Iterations = 10001:30000
## Thinning interval = 1
## Number of chains = 1
## Sample size per chain = 20000
##
## 1. Empirical mean and standard deviation for each variable,
##    plus standard error of the mean:
##
##           Mean      SD Naive SE Time-series SE
## beta[1] -2.6930 1.914  0.01354      0.04219
## beta[2] -3.0459 1.769  0.01251      0.03427
## beta[3] -0.1123 1.693  0.01197      0.02928
## beta[4]  1.8991 1.841  0.01302      0.04173
## beta[5] -1.8671 1.876  0.01326      0.03959
## beta[6] -4.0411 1.776  0.01256      0.03307
## beta[7]  1.2469 1.613  0.01140      0.03270
## beta[8] -0.8527 1.485  0.01050      0.02601
## beta[9] -1.7819 1.580  0.01117      0.02824
## beta[10] 0.1569 1.569  0.01110      0.03119
## beta[11] -3.1844 1.755  0.01241      0.03909
## beta[12] -0.2575 1.481  0.01048      0.02080
## beta[13]  2.2405 1.439  0.01017      0.02136
## beta[14] -1.3930 1.498  0.01059      0.02339
## beta[15]  0.2886 1.756  0.01241      0.03664
## beta[16]  2.0388 1.583  0.01119      0.03398
## beta[17] -1.8412 1.614  0.01141      0.02563
## beta[18] -2.3790 1.750  0.01238      0.03023
## beta[19] -3.9672 1.825  0.01291      0.04646
## beta[20]  2.4884 1.847  0.01306      0.03675
## beta[21] -0.8515 1.863  0.01317      0.04022
## beta[22] -1.6541 1.804  0.01275      0.03564

```

```

## beta[23] 1.1912 1.688 0.01193      0.03399
## beta[24] 3.0088 1.519 0.01074      0.02610
## beta[25] 0.4992 1.583 0.01119      0.02460
## beta[26] 2.6441 1.809 0.01279      0.03432
## beta[27] 1.6156 1.898 0.01342      0.03830
## beta[28] 5.0133 1.849 0.01307      0.03943
## beta[29] -0.2167 1.585 0.01121      0.02472
## beta[30] -1.2406 1.716 0.01213      0.03298
## beta[31] 0.4697 1.891 0.01337      0.04199
## beta[32] 1.4520 1.864 0.01318      0.04016
## beta[33] 2.5464 1.583 0.01120      0.03357
## beta[34] 2.4351 1.984 0.01403      0.04929
## beta[35] 3.5981 1.899 0.01342      0.04100
## beta[36] 4.0984 1.480 0.01047      0.02888
## beta[37] 3.8093 1.810 0.01280      0.03902
## beta[38] 2.1241 1.746 0.01235      0.02935
## beta[39] 1.9723 1.809 0.01279      0.03630
## beta[40] 5.3939 1.891 0.01337      0.04463
## beta[41] 2.1824 1.805 0.01276      0.03850
## beta[42] 1.2841 1.673 0.01183      0.03661
## beta[43] 1.0590 1.730 0.01224      0.03588
## beta[44] 5.3669 1.762 0.01246      0.04001
## beta[45] 6.1662 1.656 0.01171      0.03151
## beta[46] 2.6032 1.552 0.01098      0.02353
## beta[47] 1.5206 1.649 0.01166      0.02952
## beta[48] 4.8025 1.546 0.01093      0.02220
## beta[49] 6.7147 1.720 0.01216      0.03699
## beta[50] 26.6699 7.502 0.05305      0.98453
## intercept 27.1040 7.512 0.05312      1.00254
##
## 2. Quantiles for each variable:
##
##          2.5%    25%    50%    75%   97.5%
## beta[1] -6.43933 -3.98218 -2.7016 -1.41296 1.1126
## beta[2] -6.46725 -4.23755 -3.0417 -1.84423 0.4125
## beta[3] -3.39907 -1.24513 -0.1218 1.03165 3.2374
## beta[4] -1.60228  0.63198  1.8629  3.13361 5.5587
## beta[5] -5.50950 -3.15291 -1.8895 -0.58345 1.7901
## beta[6] -7.46993 -5.25288 -4.0481 -2.82654 -0.5488
## beta[7] -1.93700  0.17721  1.2459  2.34002 4.3908
## beta[8] -3.71494 -1.86797 -0.8598  0.14357 2.0615
## beta[9] -4.85447 -2.85187 -1.7832 -0.72303 1.3297
## beta[10] -2.93493 -0.89017  0.1521  1.20252 3.2852
## beta[11] -6.65287 -4.37116 -3.1652 -2.00309 0.2200
## beta[12] -3.19282 -1.24274 -0.2693  0.73023 2.6930
## beta[13] -0.58731  1.27766  2.2264  3.19339 5.1094
## beta[14] -4.30461 -2.41927 -1.3946 -0.37952 1.5388
## beta[15] -3.10794 -0.89580  0.2760  1.45646 3.7679

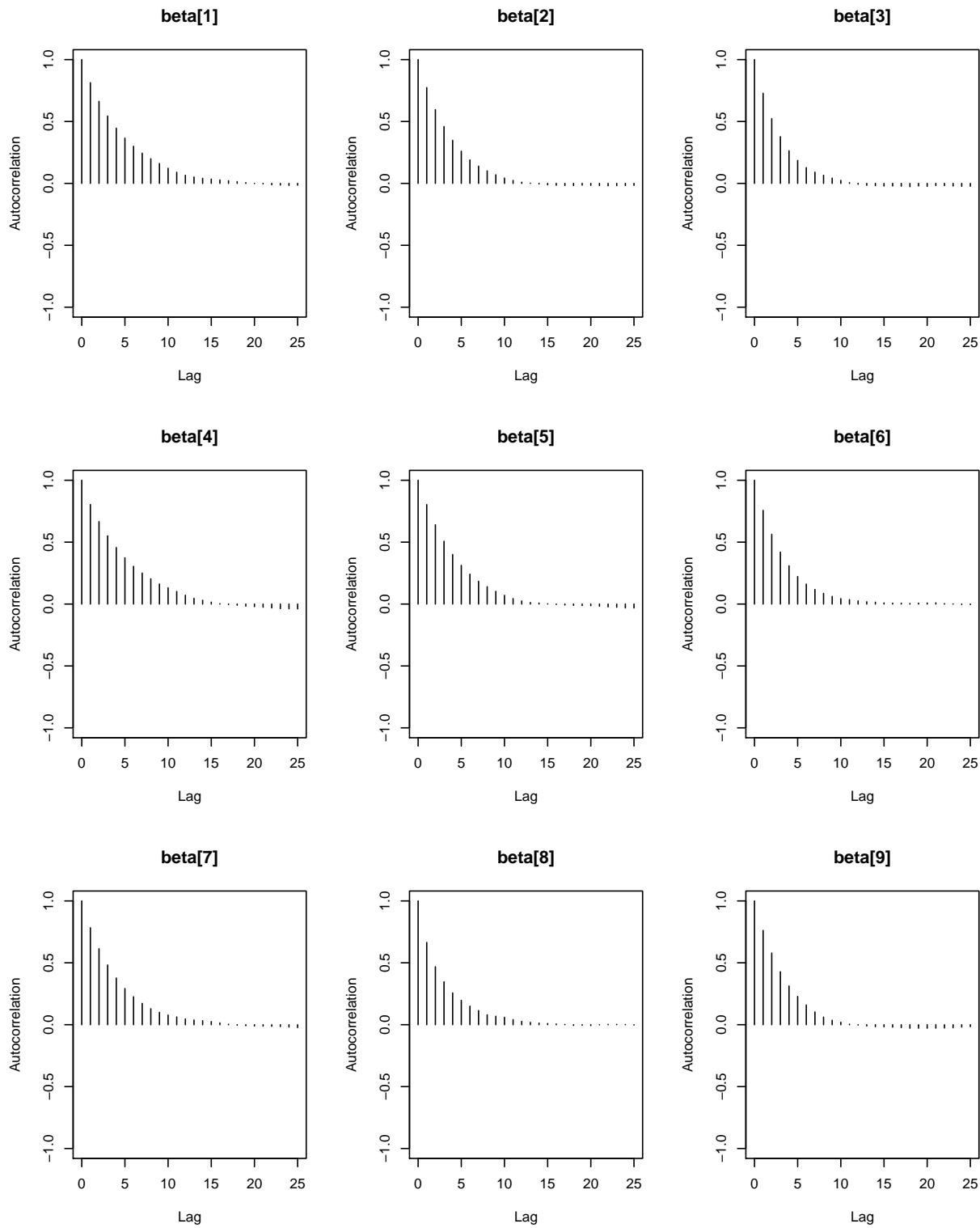
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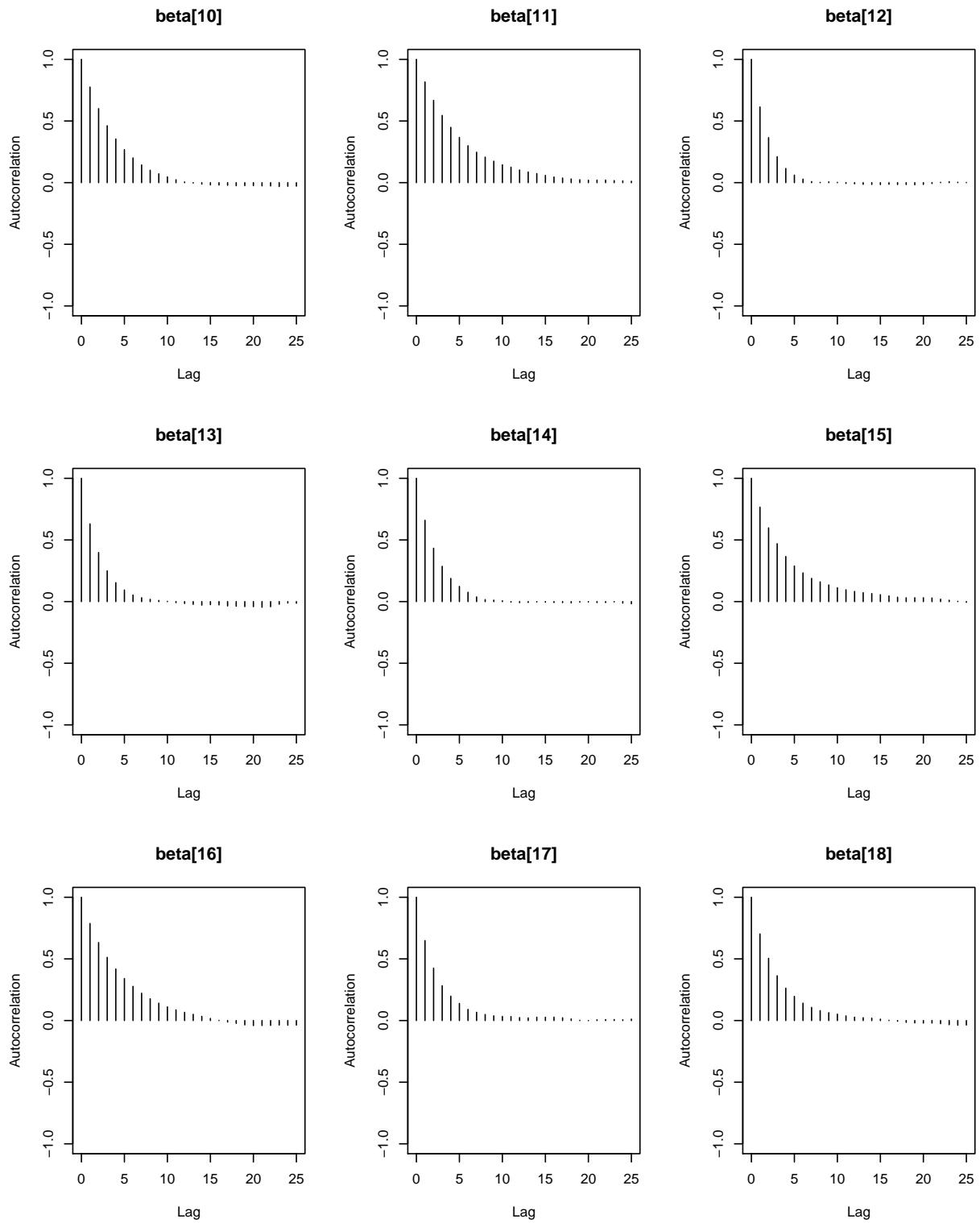
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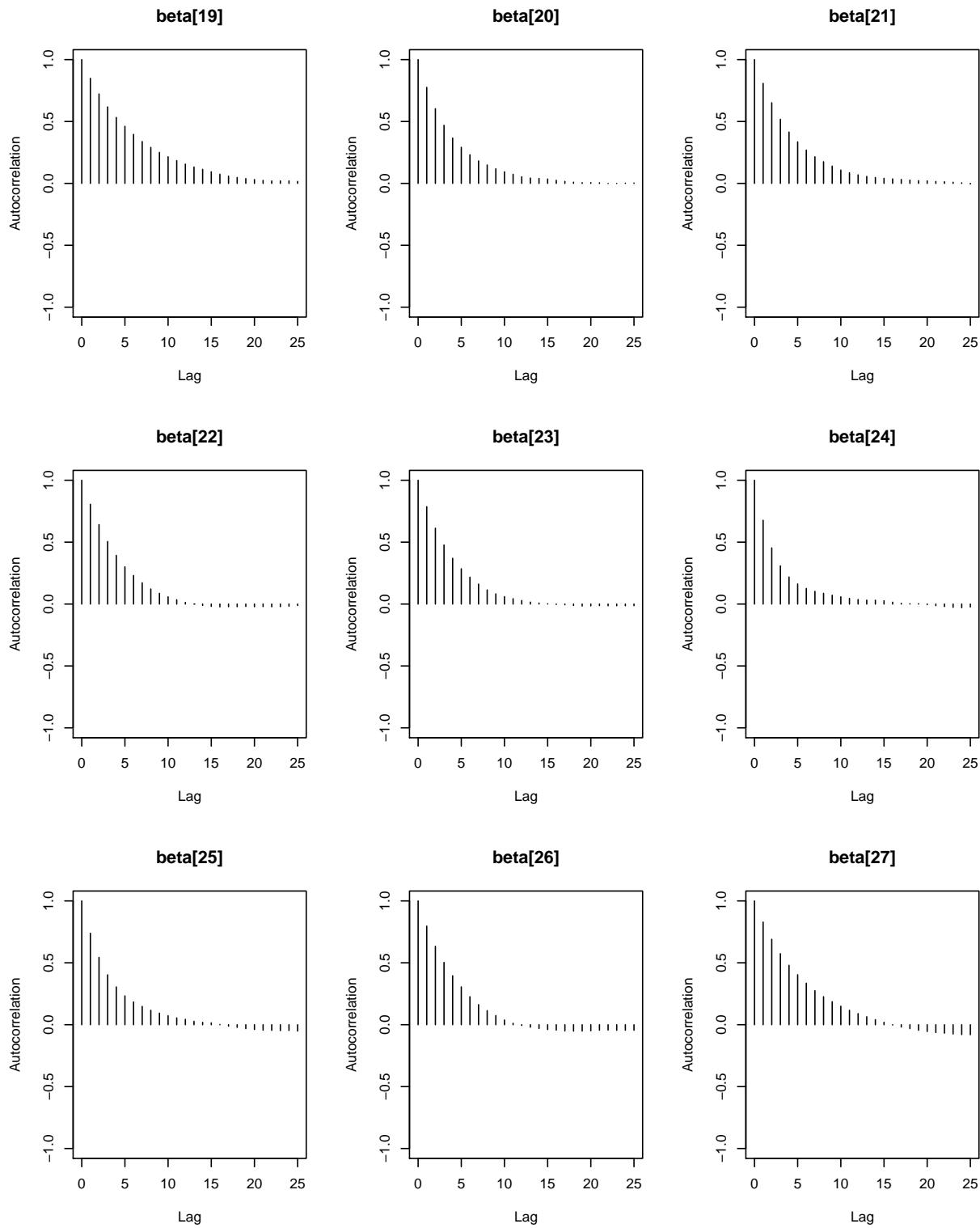
## beta[16] -1.06640 0.96054 2.0269 3.10558 5.1598
## beta[17] -5.00135 -2.94406 -1.8421 -0.74704 1.3245
## beta[18] -5.79456 -3.56431 -2.3903 -1.18956 1.0721
## beta[19] -7.50161 -5.20237 -3.9766 -2.74032 -0.3442
## beta[20] -1.05560 1.23214 2.4437 3.74330 6.1764
## beta[21] -4.51313 -2.10483 -0.8488 0.40394 2.7942
## beta[22] -5.16085 -2.86782 -1.6519 -0.46073 1.9328
## beta[23] -2.19158 0.07755 1.2013 2.31378 4.4951
## beta[24] 0.01369 1.98476 3.0181 4.04984 5.9307
## beta[25] -2.57059 -0.56844 0.4944 1.56995 3.6035
## beta[26] -0.86827 1.40122 2.6308 3.86983 6.1692
## beta[27] -2.11375 0.32154 1.6183 2.87877 5.3119
## beta[28] 1.37565 3.76748 5.0275 6.26972 8.5799
## beta[29] -3.29721 -1.30067 -0.2137 0.86222 2.8837
## beta[30] -4.57059 -2.40518 -1.2449 -0.08176 2.1351
## beta[31] -3.27980 -0.82408 0.4942 1.75516 4.1739
## beta[32] -2.23084 0.22535 1.4445 2.68902 5.0993
## beta[33] -0.52237 1.47624 2.5386 3.60506 5.6827
## beta[34] -1.50694 1.10961 2.4398 3.78777 6.2585
## beta[35] -0.14815 2.32657 3.6059 4.88098 7.2997
## beta[36] 1.16593 3.10819 4.1052 5.10819 6.9838
## beta[37] 0.31791 2.57439 3.7931 5.04752 7.3309
## beta[38] -1.28535 0.93514 2.1184 3.29640 5.5555
## beta[39] -1.64659 0.76600 1.9852 3.20493 5.4697
## beta[40] 1.73283 4.12255 5.3521 6.66255 9.1056
## beta[41] -1.42727 0.99726 2.2066 3.41949 5.6739
## beta[42] -2.03034 0.13719 1.3041 2.42304 4.5297
## beta[43] -2.32923 -0.11935 1.0617 2.24482 4.4162
## beta[44] 1.90793 4.15824 5.3769 6.57118 8.7732
## beta[45] 2.88477 5.06166 6.1696 7.29277 9.4138
## beta[46] -0.40318 1.55247 2.5956 3.65223 5.6978
## beta[47] -1.71181 0.40838 1.5228 2.64105 4.7508
## beta[48] 1.76491 3.75529 4.8020 5.83160 7.8229
## beta[49] 3.35993 5.53486 6.7149 7.89084 10.0722
## beta[50] 11.54760 22.08529 26.5411 31.41196 42.1472
## intercept 11.41302 22.32350 27.2343 31.76781 42.1716

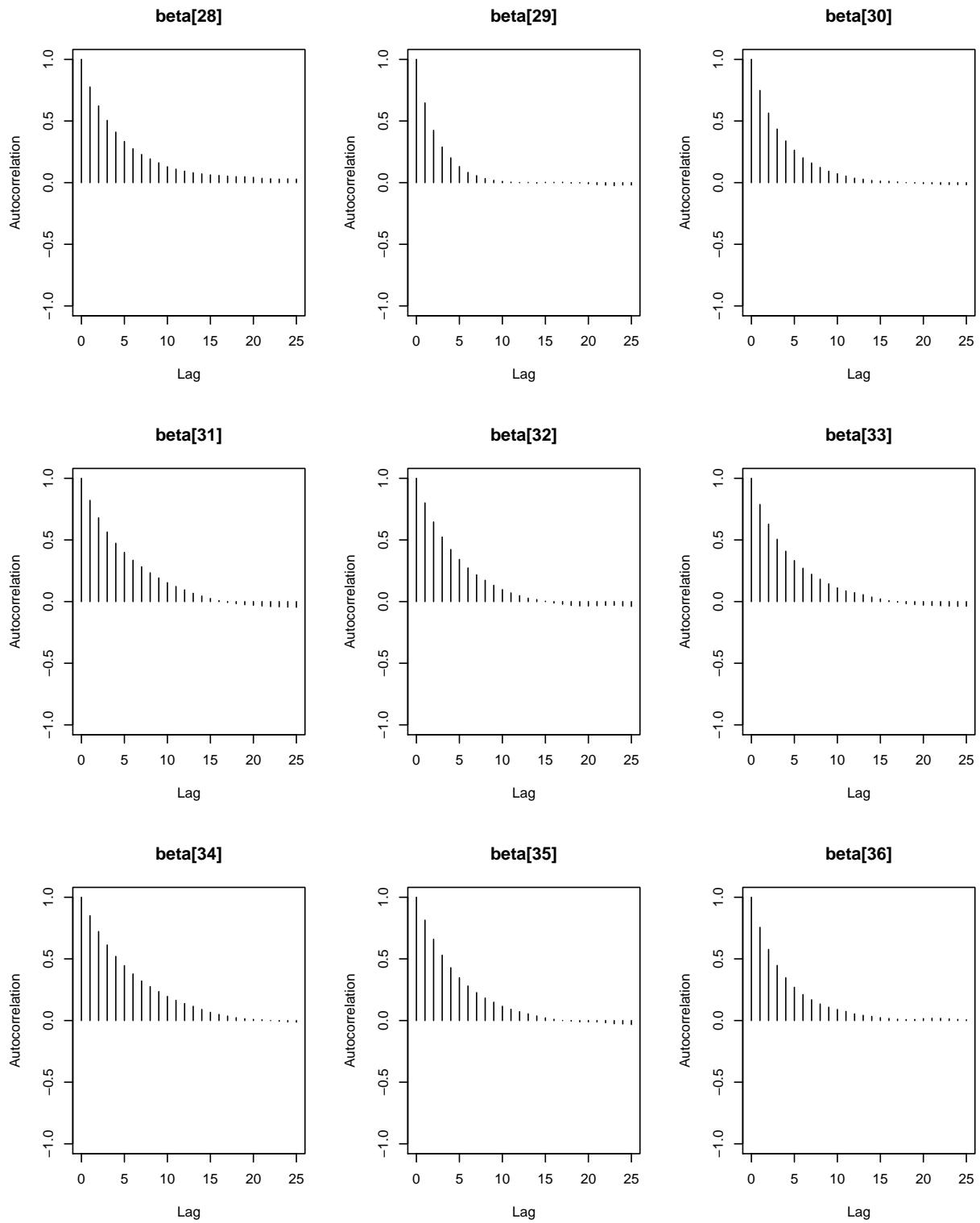
autocorr.plot(samp.coeff.normal_hierarchical)

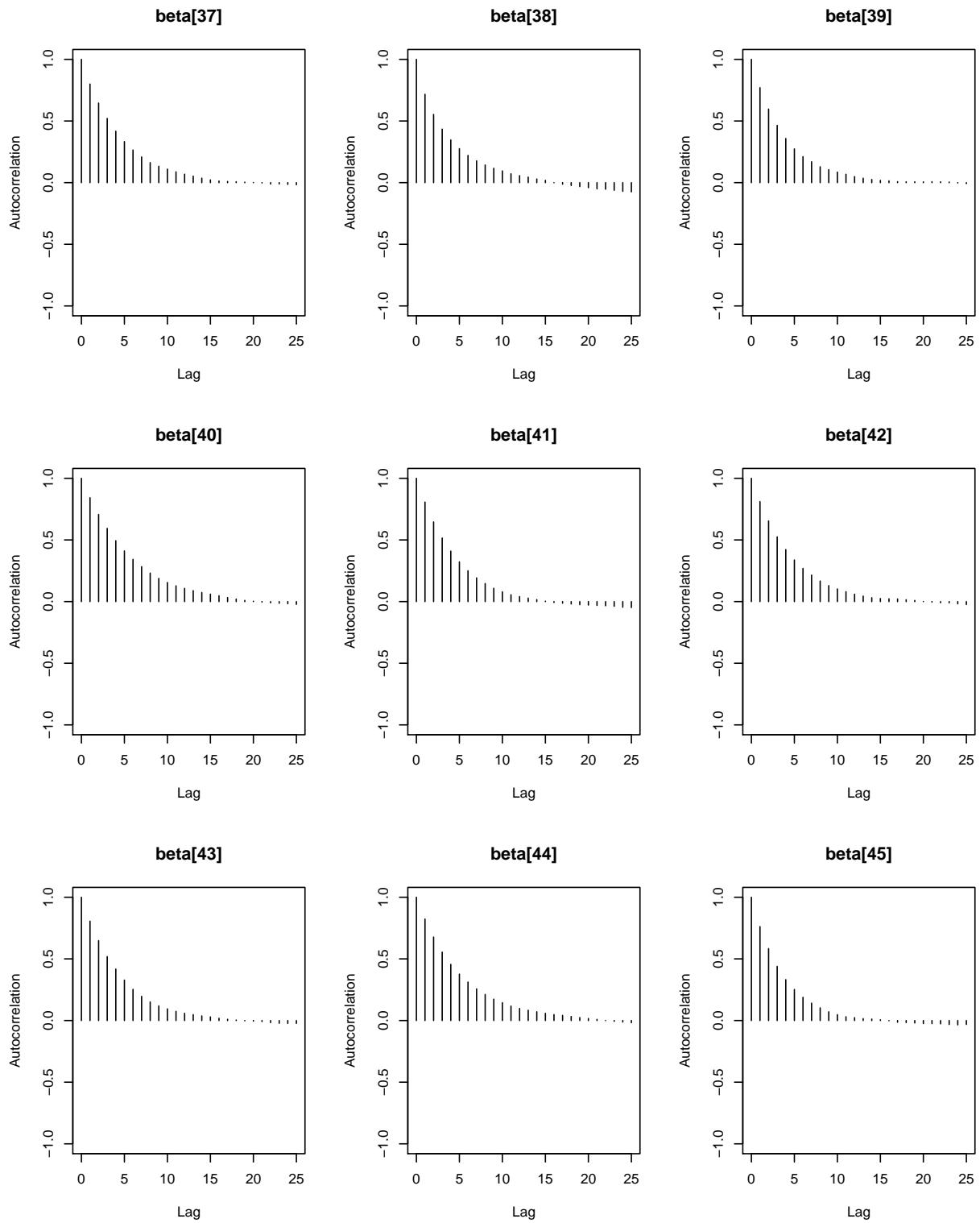
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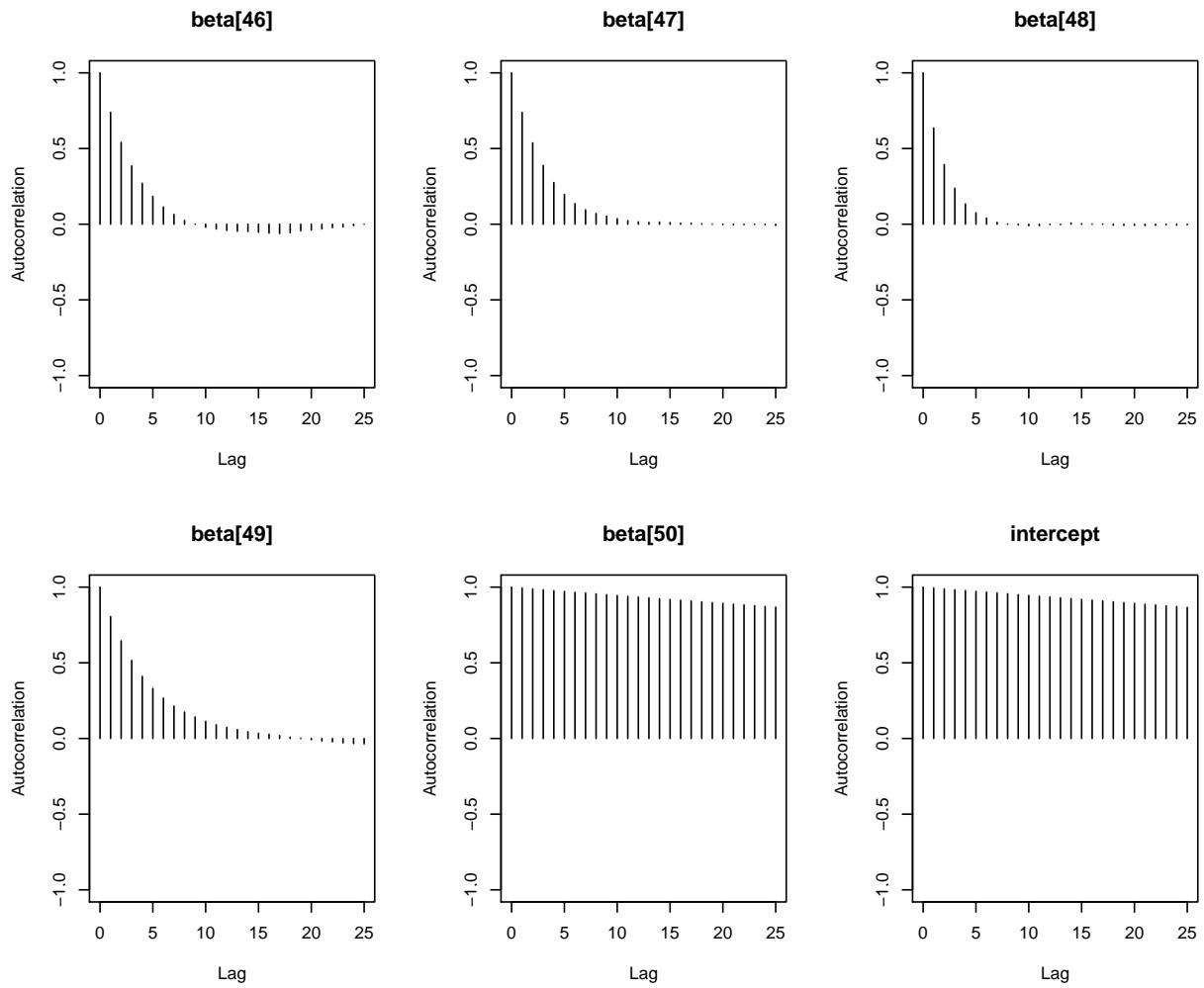




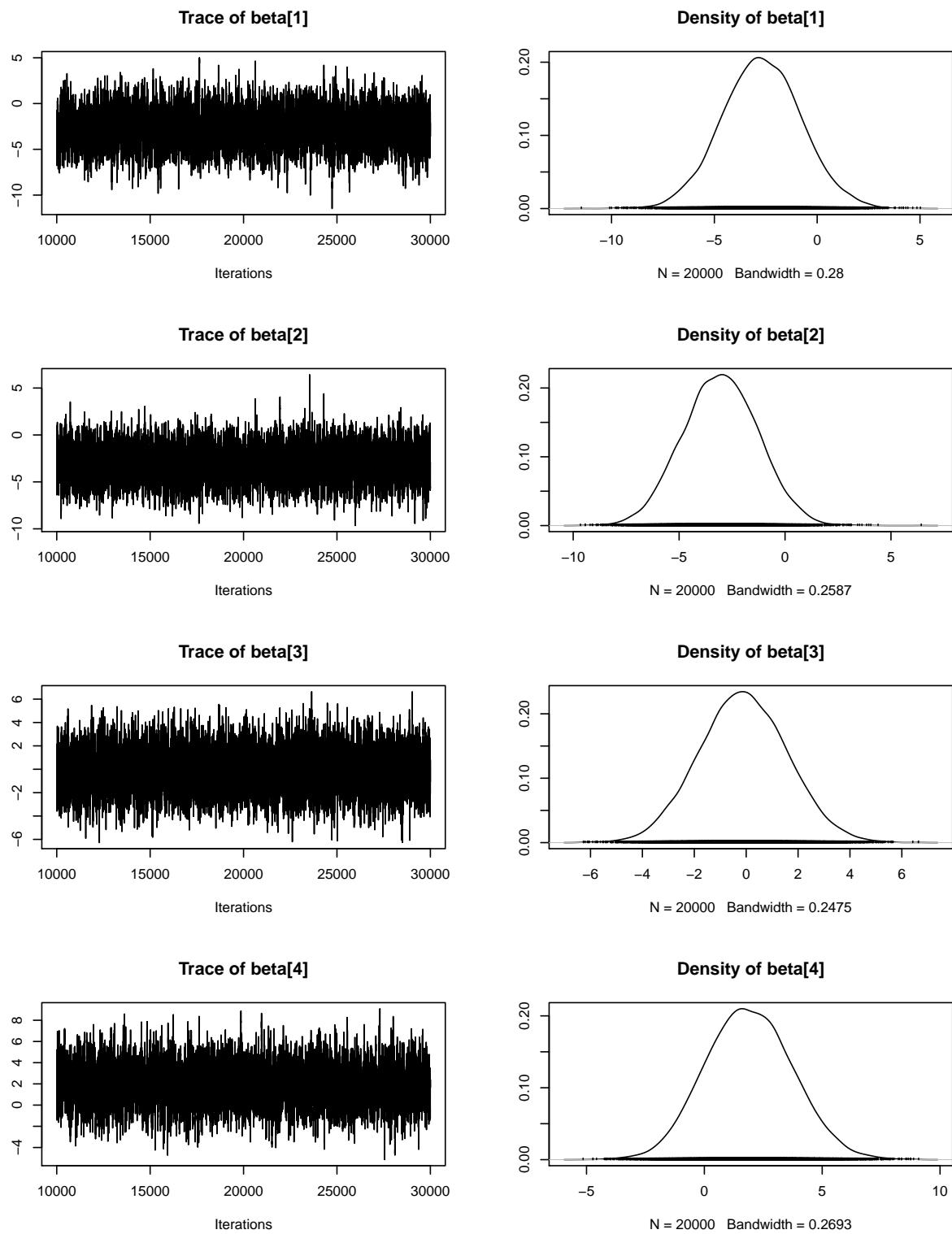


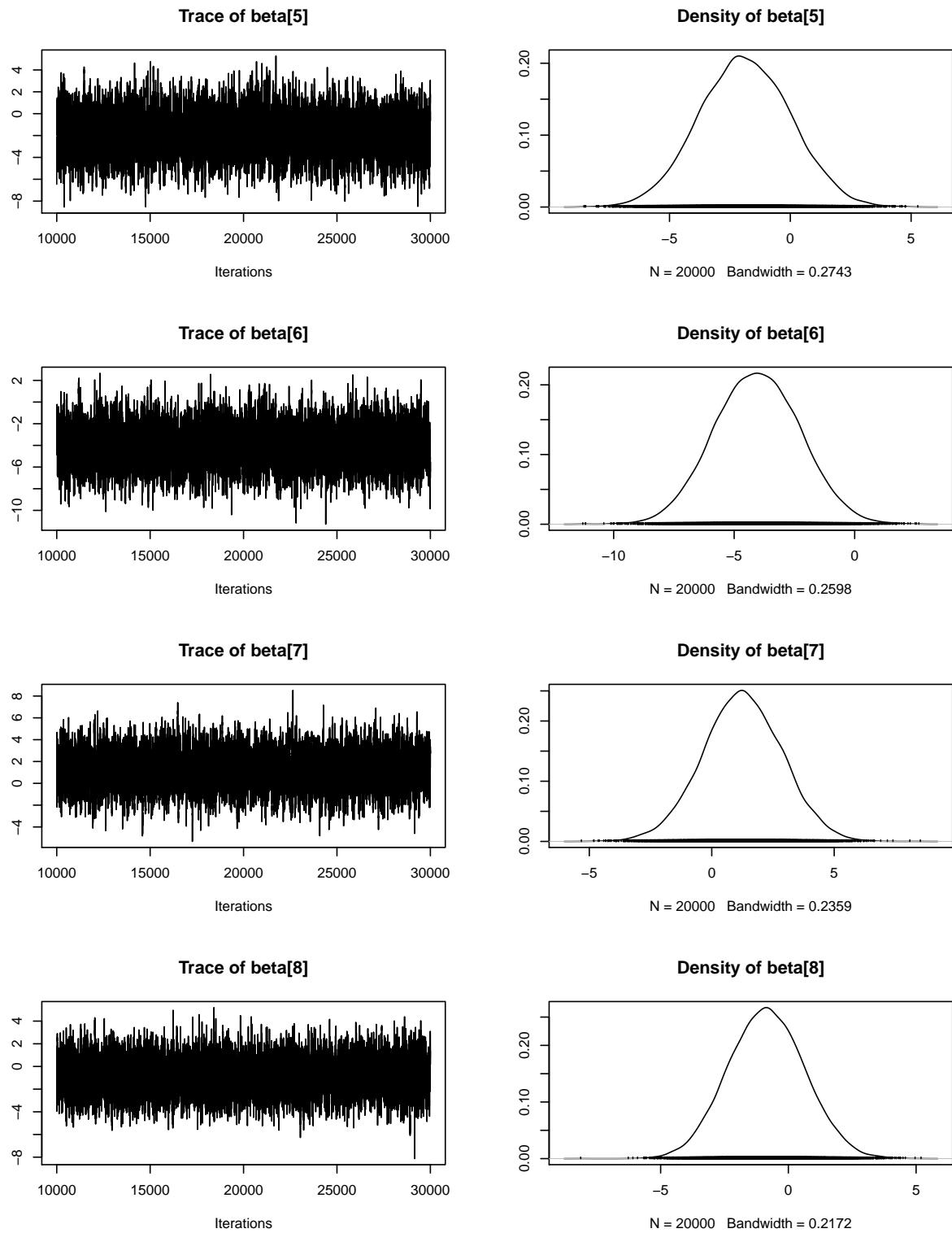




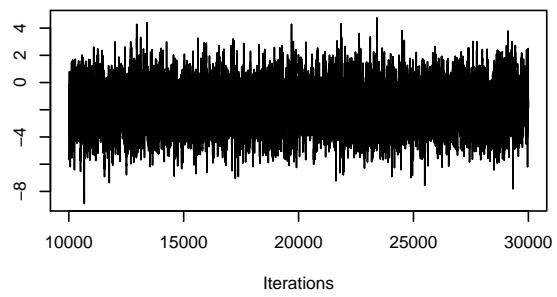


```
plot(samp.coeff.normal_hierarchical)
```

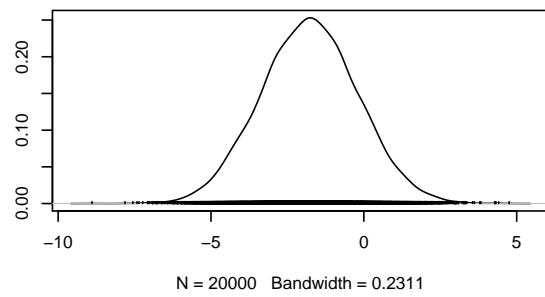




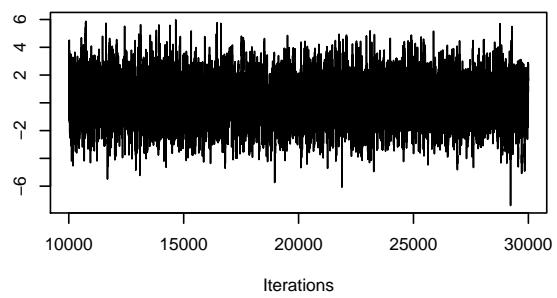
Trace of beta[9]



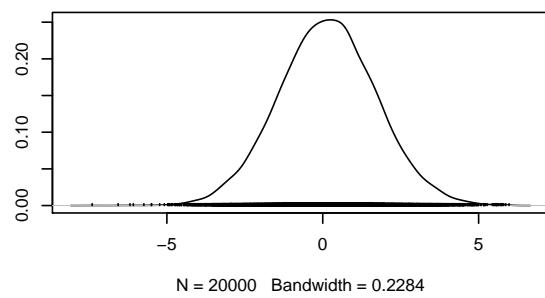
Density of beta[9]



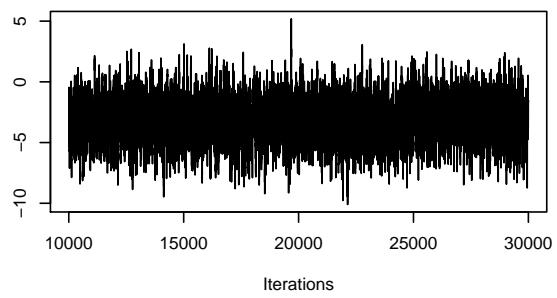
Trace of beta[10]



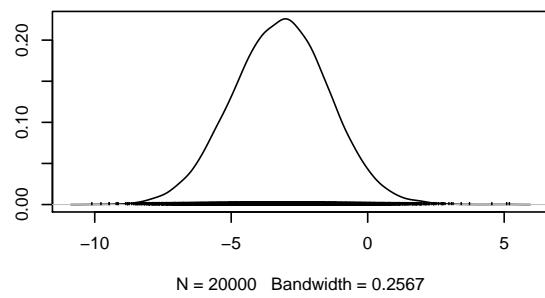
Density of beta[10]



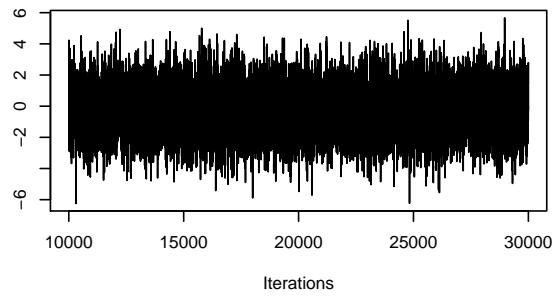
Trace of beta[11]



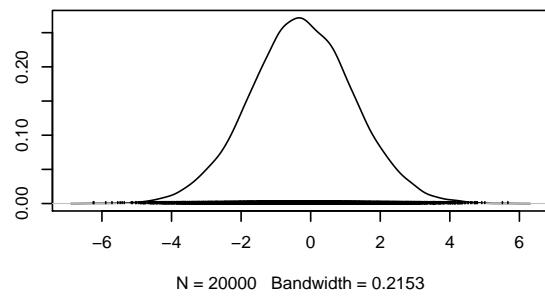
Density of beta[11]



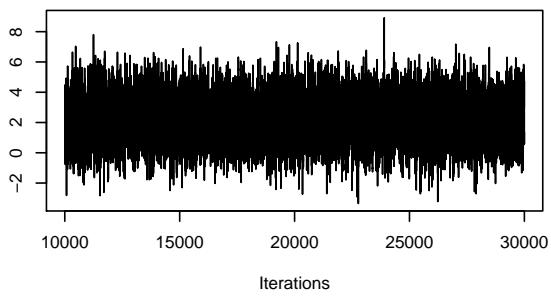
Trace of beta[12]



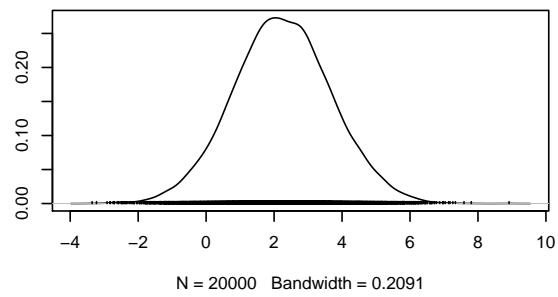
Density of beta[12]



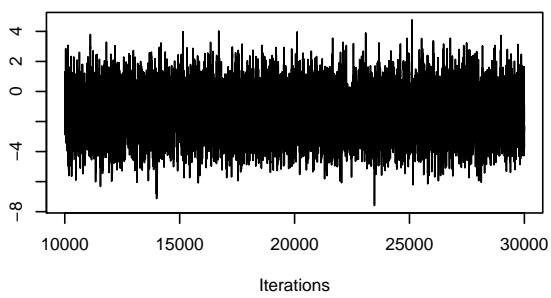
Trace of beta[13]



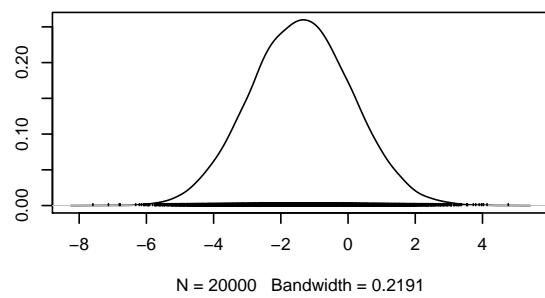
Density of beta[13]



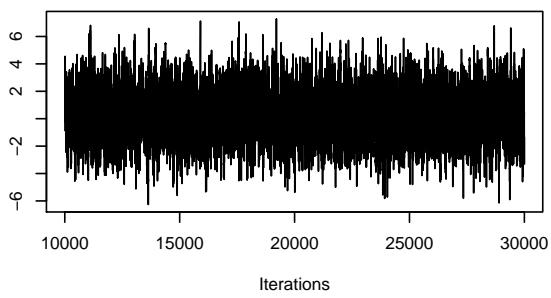
Trace of beta[14]



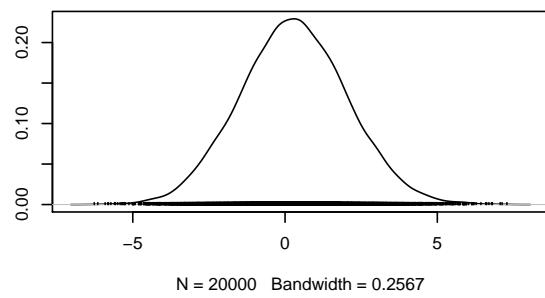
Density of beta[14]



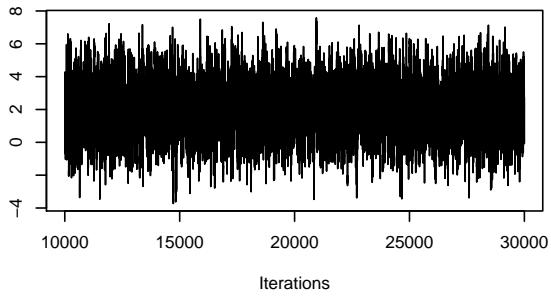
Trace of beta[15]



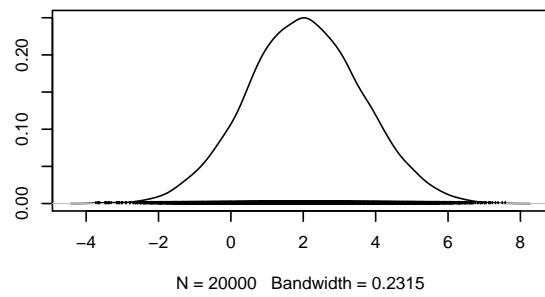
Density of beta[15]



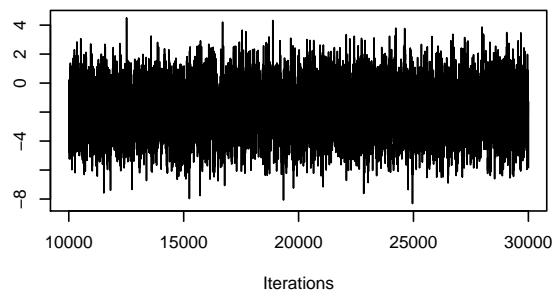
Trace of beta[16]



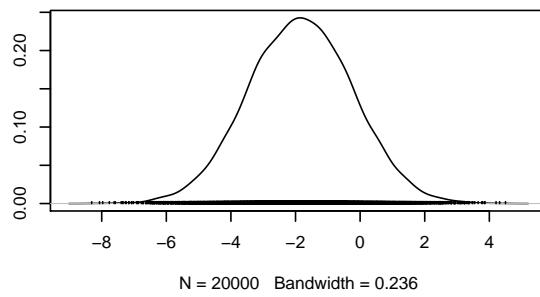
Density of beta[16]



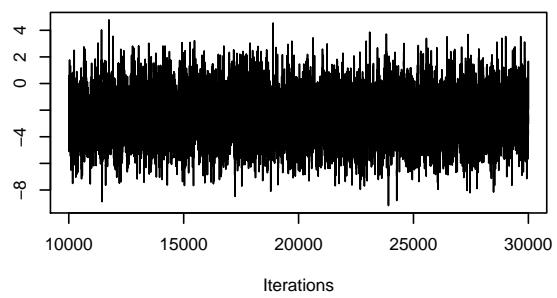
Trace of beta[17]



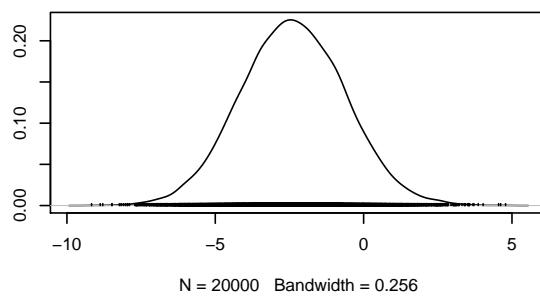
Density of beta[17]



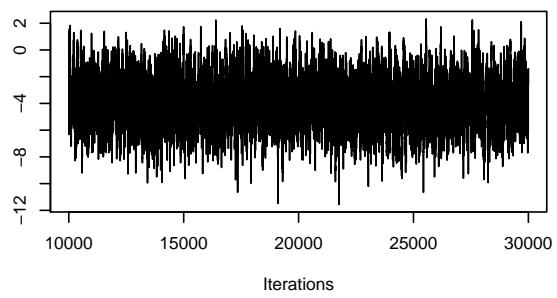
Trace of beta[18]



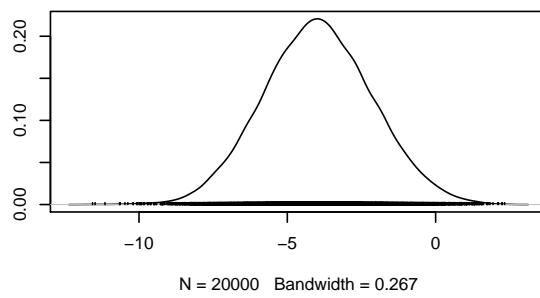
Density of beta[18]



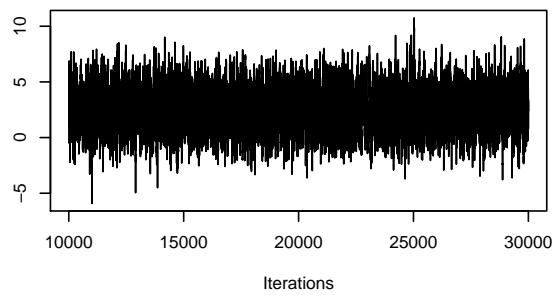
Trace of beta[19]



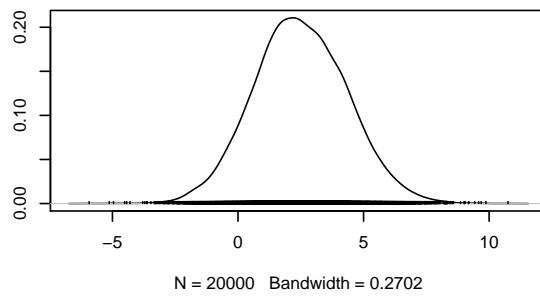
Density of beta[19]



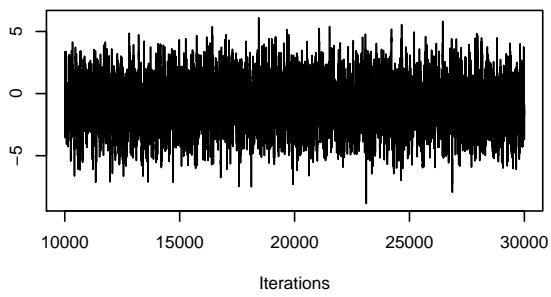
Trace of beta[20]



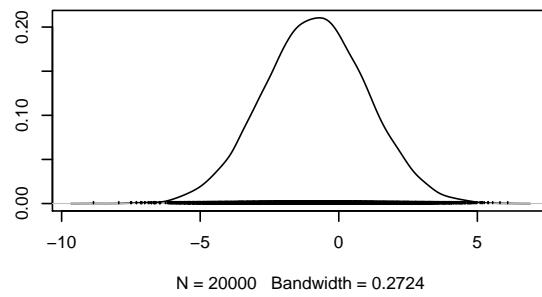
Density of beta[20]



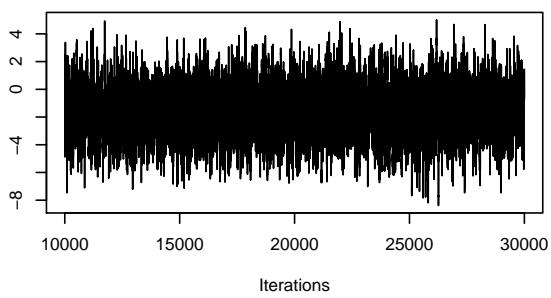
Trace of beta[21]



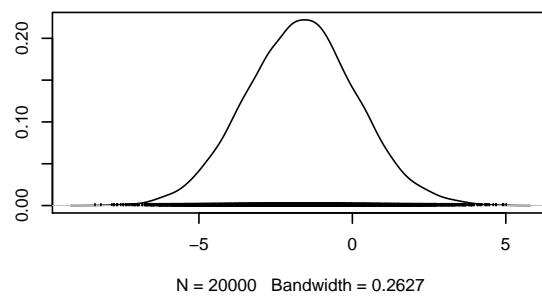
Density of beta[21]



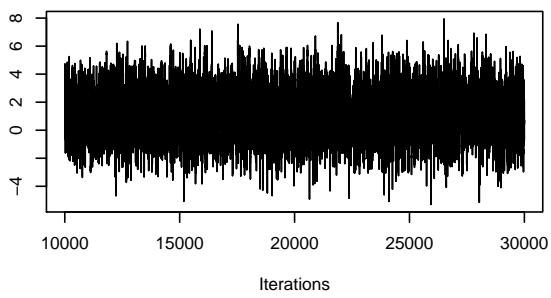
Trace of beta[22]



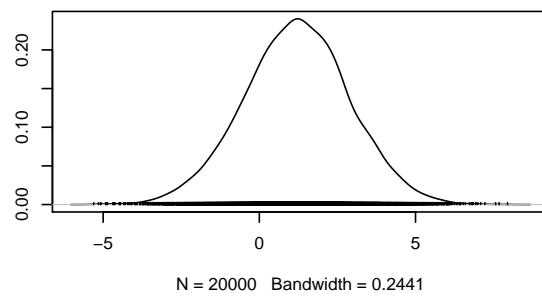
Density of beta[22]



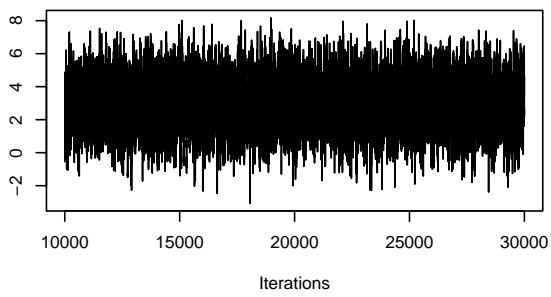
Trace of beta[23]



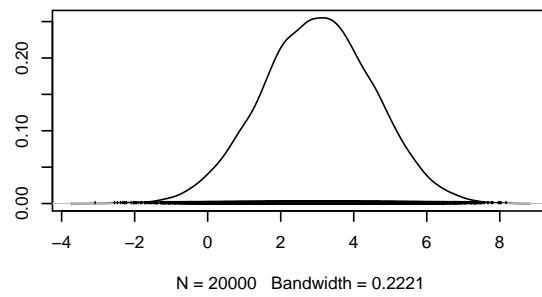
Density of beta[23]



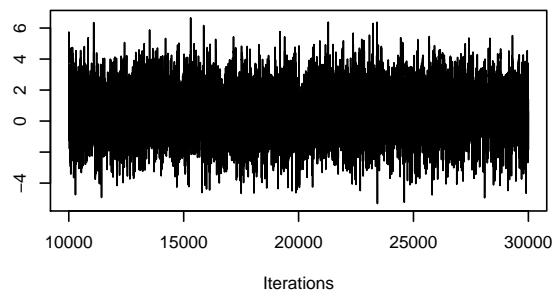
Trace of beta[24]



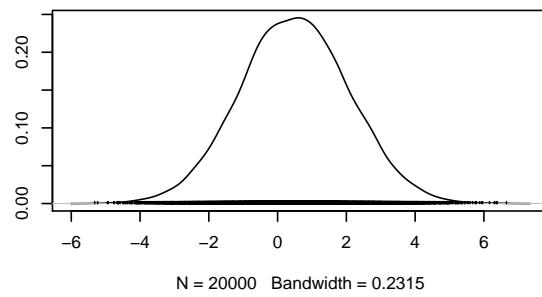
Density of beta[24]



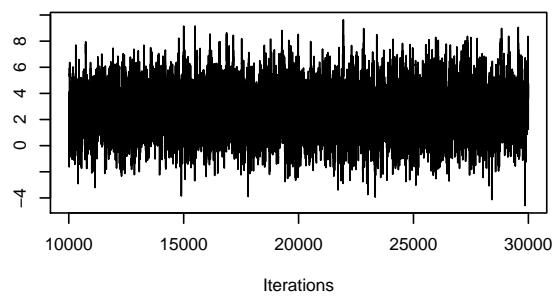
Trace of beta[25]



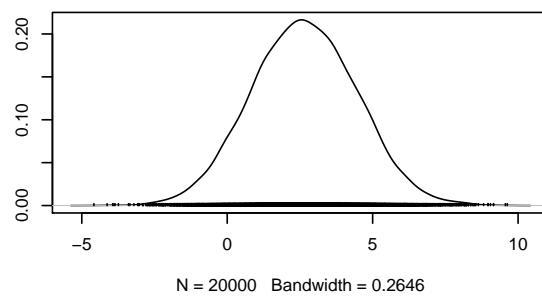
Density of beta[25]



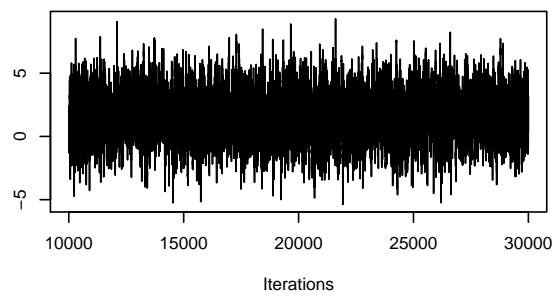
Trace of beta[26]



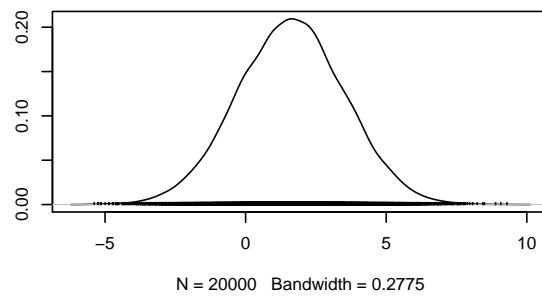
Density of beta[26]



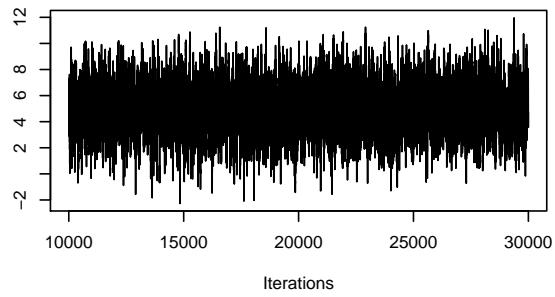
Trace of beta[27]



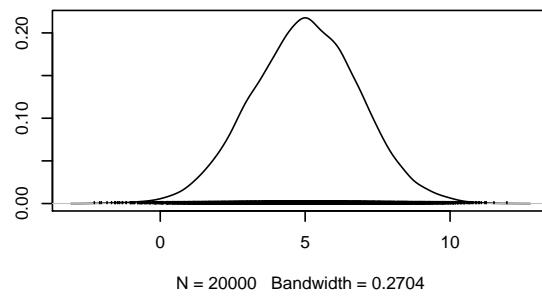
Density of beta[27]



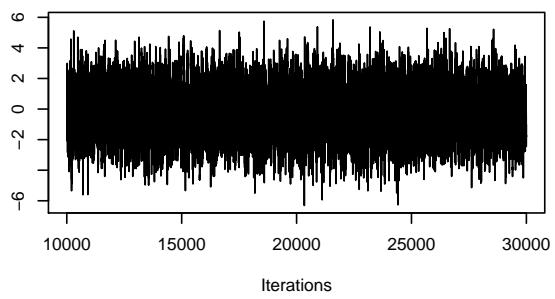
Trace of beta[28]



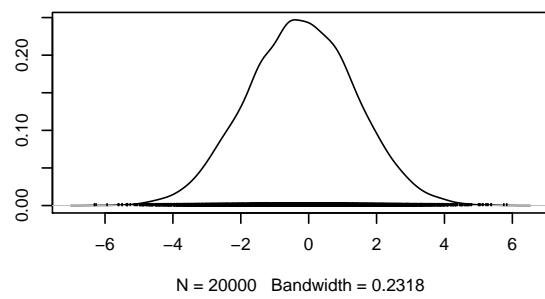
Density of beta[28]



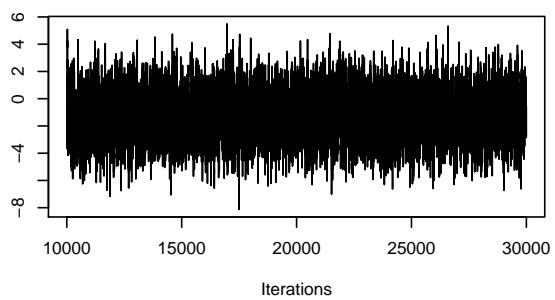
Trace of beta[29]



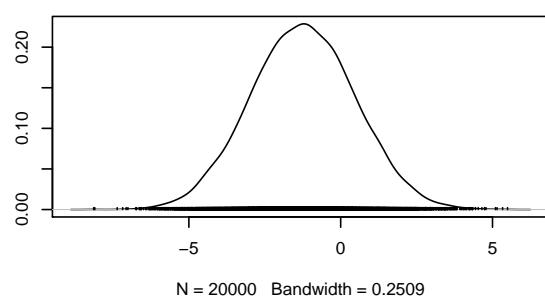
Density of beta[29]



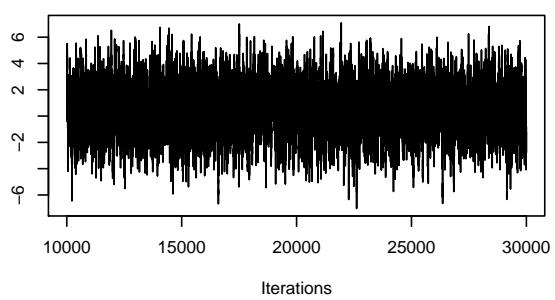
Trace of beta[30]



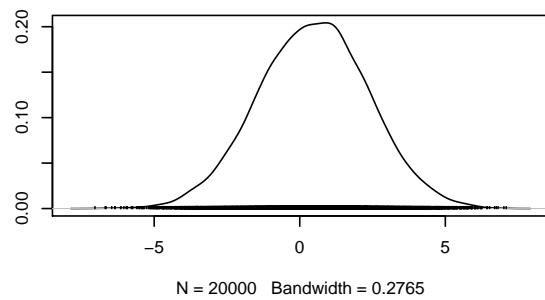
Density of beta[30]



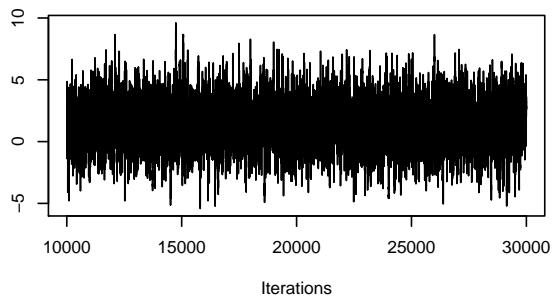
Trace of beta[31]



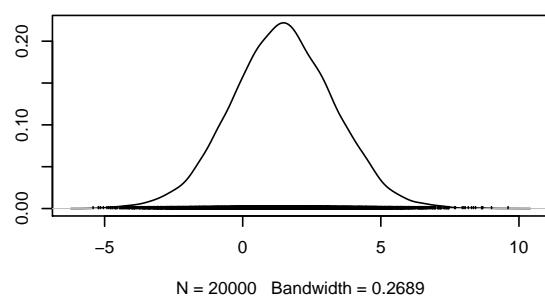
Density of beta[31]



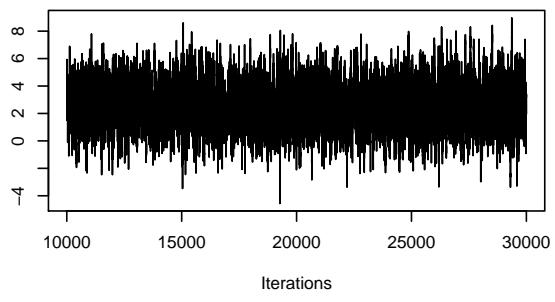
Trace of beta[32]



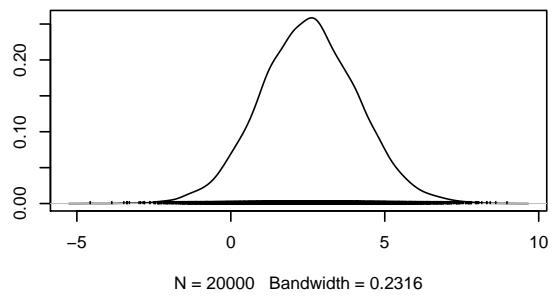
Density of beta[32]



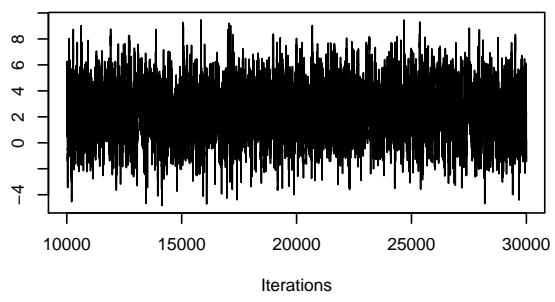
Trace of beta[33]



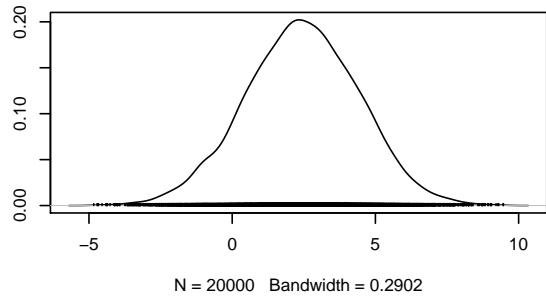
Density of beta[33]



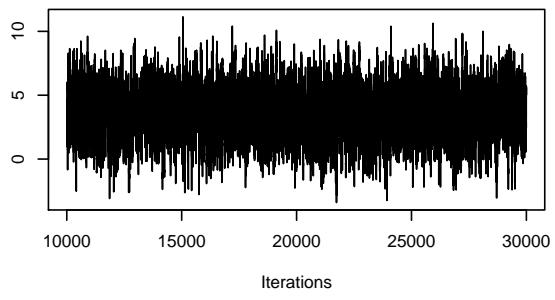
Trace of beta[34]



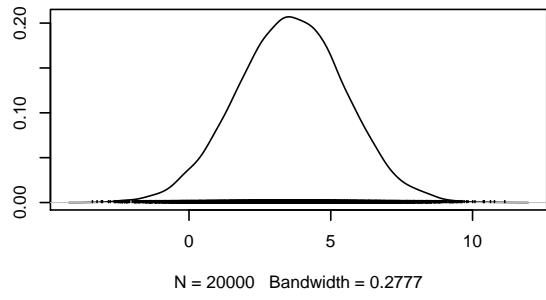
Density of beta[34]



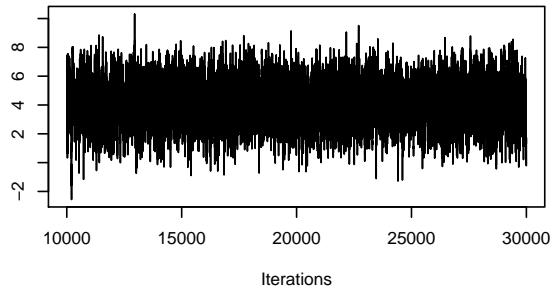
Trace of beta[35]



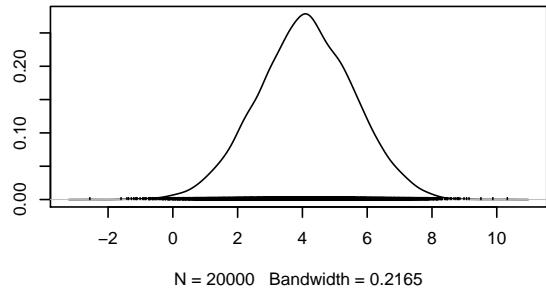
Density of beta[35]



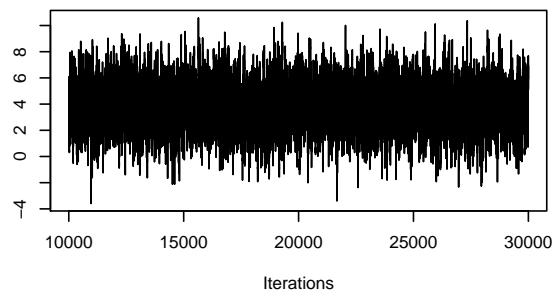
Trace of beta[36]



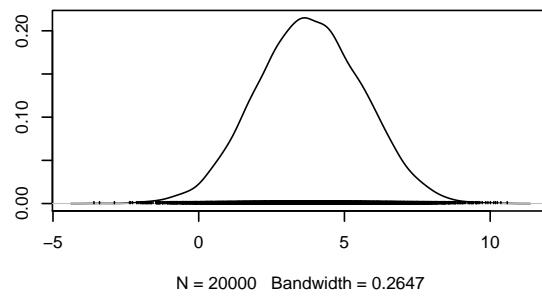
Density of beta[36]



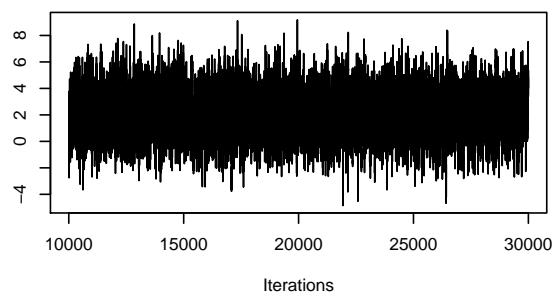
Trace of beta[37]



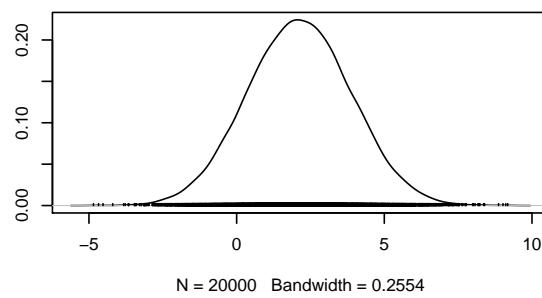
Density of beta[37]



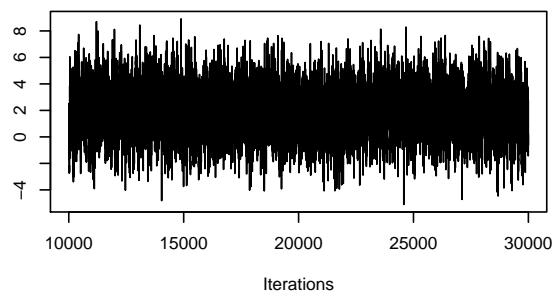
Trace of beta[38]



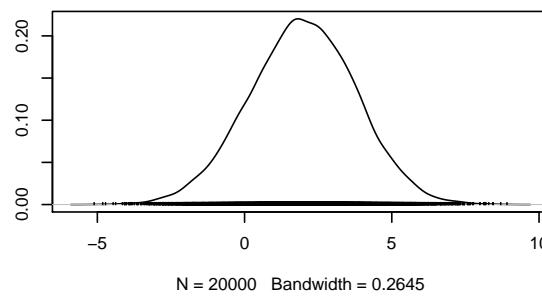
Density of beta[38]



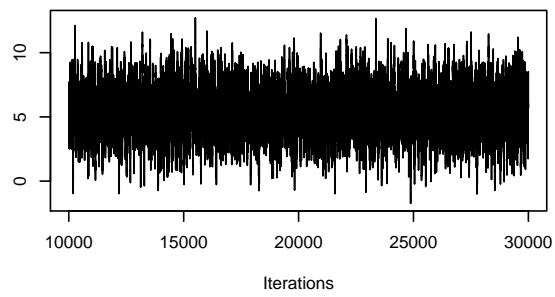
Trace of beta[39]



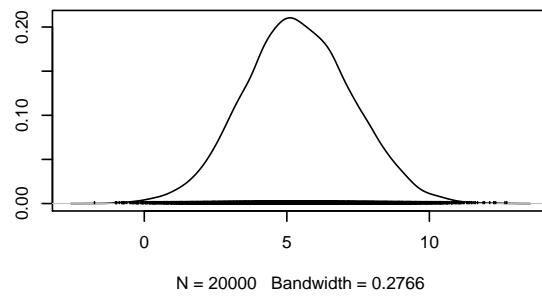
Density of beta[39]



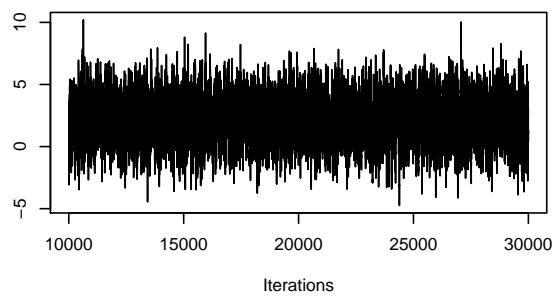
Trace of beta[40]



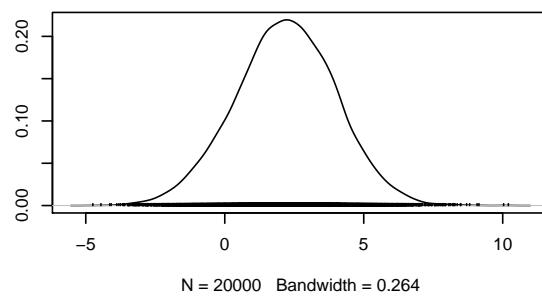
Density of beta[40]



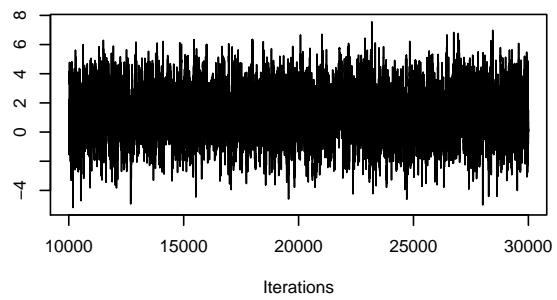
Trace of beta[41]



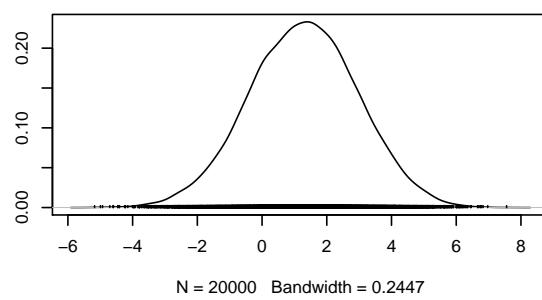
Density of beta[41]



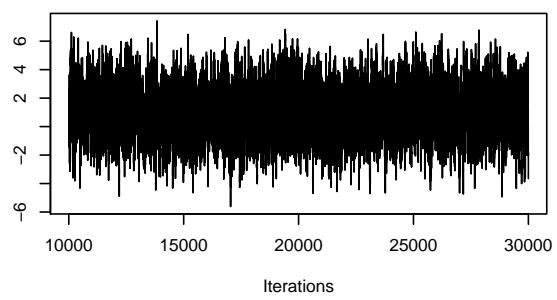
Trace of beta[42]



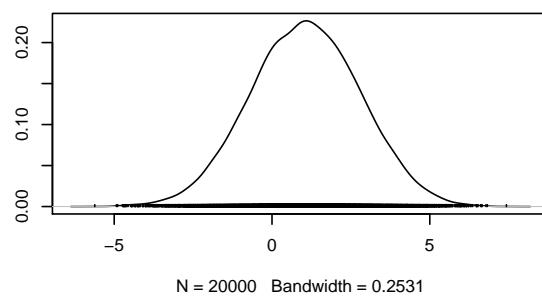
Density of beta[42]



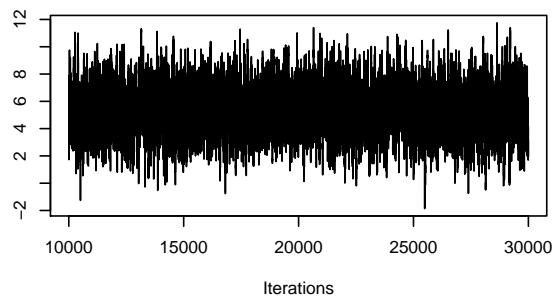
Trace of beta[43]



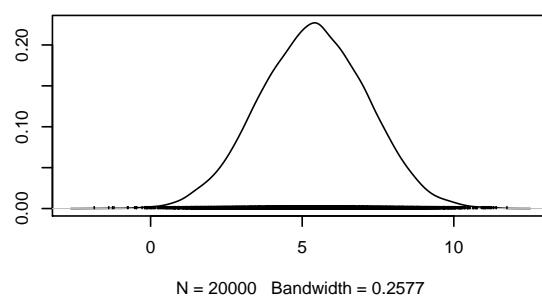
Density of beta[43]



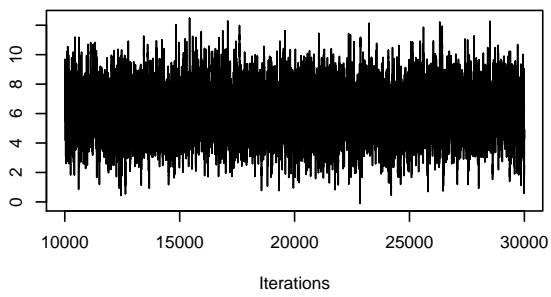
Trace of beta[44]



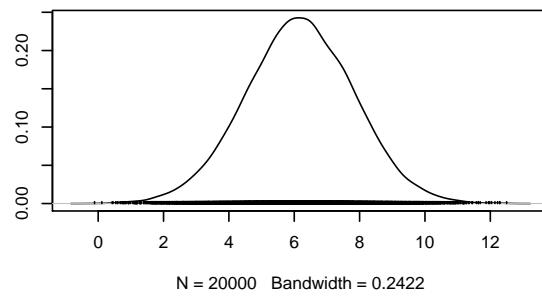
Density of beta[44]



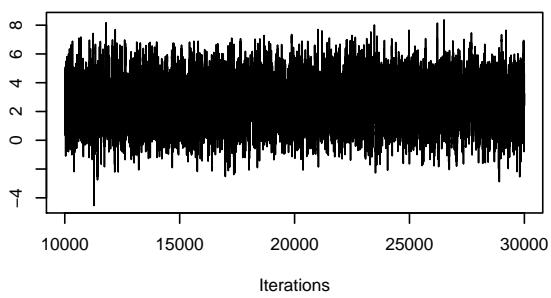
Trace of beta[45]



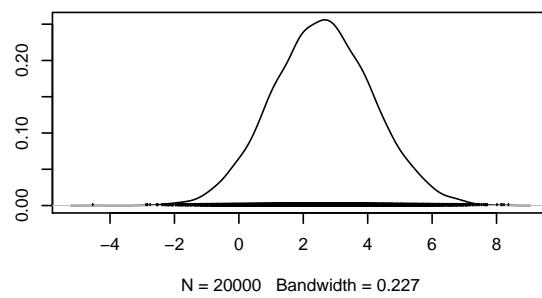
Density of beta[45]



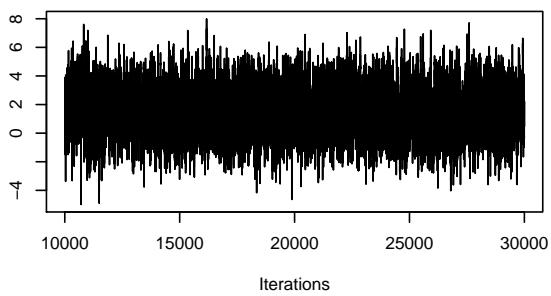
Trace of beta[46]



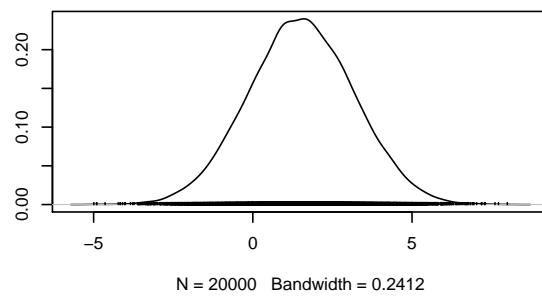
Density of beta[46]



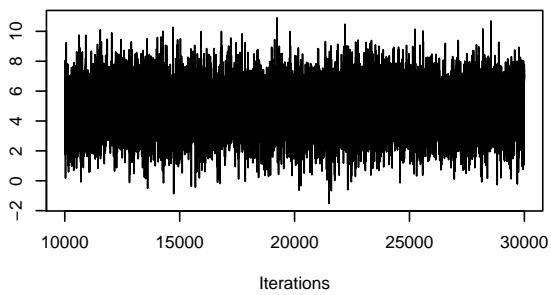
Trace of beta[47]



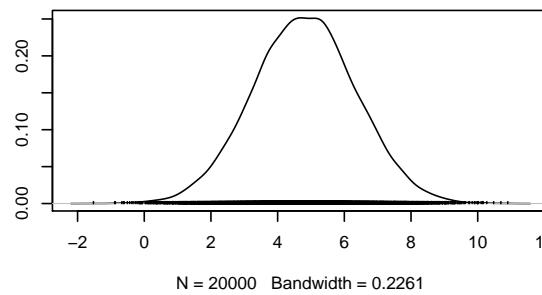
Density of beta[47]

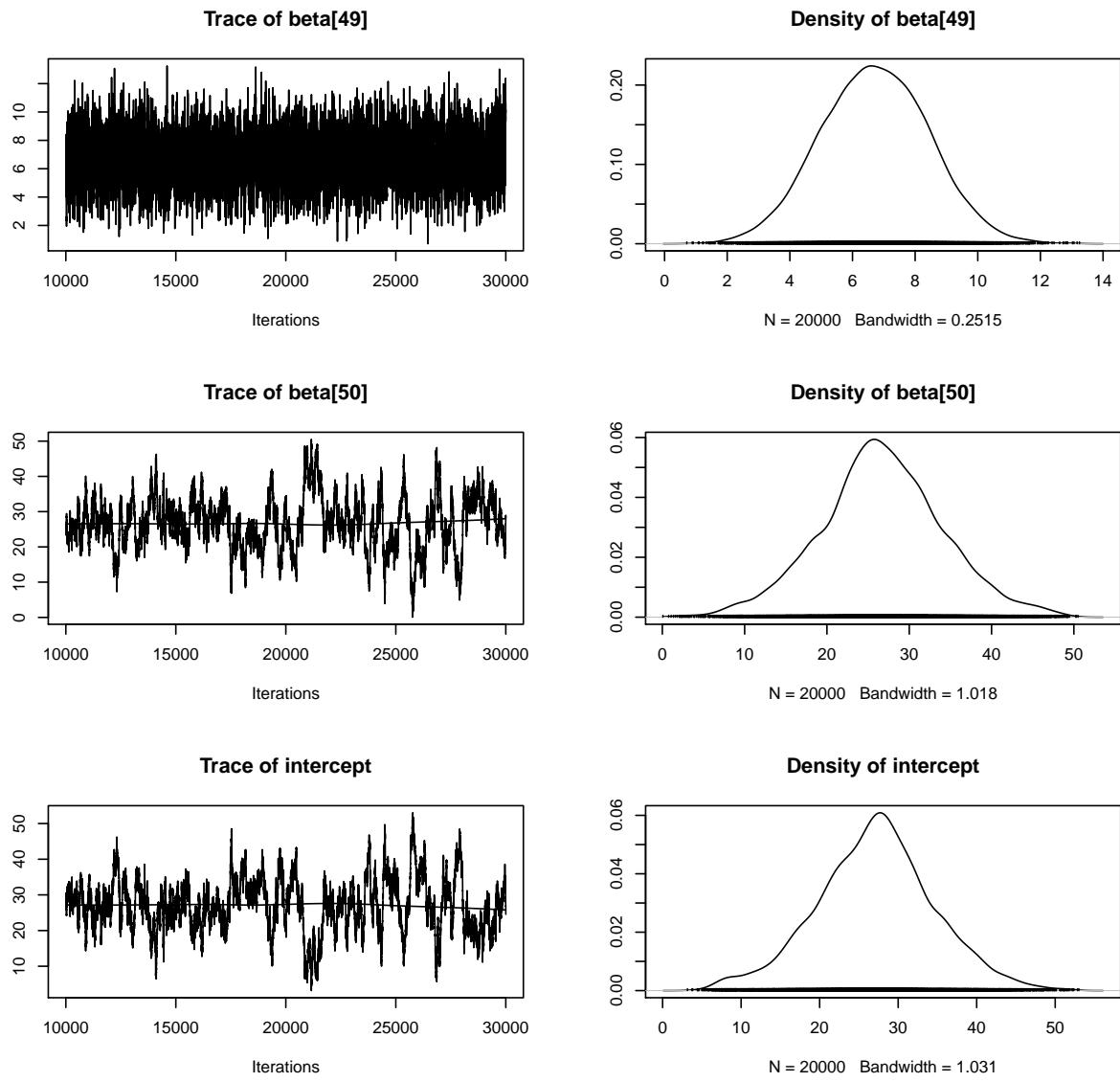


Trace of beta[48]



Density of beta[48]





```

if (DEBUG)
{
  #Sample again and estimate posterior means and MAP posterior modes.
  samp.coeff.normal_hierarchical.jags <- jags.samples(model.normal_hierarchical, variable.names =
  
```

```

posterior_means.normal_hierarchical <- lapply(samp.coeff.normal_hierarchical.jags, apply, 1,
pander(posterior_means.normal_hierarchical, caption = "posterior means second sample")
posterior_modes.normal_hierarchical <- lapply(samp.coeff.normal_hierarchical.jags, apply, 1,
posterior_modes.normal_hierarchical
if(n.chains>1)
{
  gelman.plot(samp.coeff)
}
}

```

BLASSO

```

beta.inv.gamma.param  <- 0.01
variance.inv.gamma.param  <- 0.01
p <- ncol(X)

model_string.normal_blasso <- "model{
  # Likelihood
  for(i in 1:n){
    Y[i] ~ dnorm(mu[i],inv.var)
    mu[i] <- intercept +inprod(X[i,],beta[])
  }

  # Prior for beta
  for(j in 1:p){
    beta[j] ~ ddexp(0,beta.inv.gamma.param)
  }
  intercept ~ ddexp(0,beta.inv.gamma.param)

  # Prior for the inverse variance
  inv.var ~ dgamma(variance.inv.gamma.param, variance.inv.gamma.param)
  sigma <- 1/sqrt(inv.var)

  #Beta Prior for the inverse variance
  inv.var.beta ~ dgamma(beta.inv.gamma.param, beta.inv.gamma.param)
}""

model.normal_blasso <- jags.model(textConnection(model_string.normal_blasso), data = list(Y=Y, X=X))

## Compiling model graph
## Resolving undeclared variables
## Allocating nodes
## Graph information:
##   Observed stochastic nodes: 1133
##   Unobserved stochastic nodes: 53
##   Total graph size: 61096

```

```

##  

## Initializing model  

update(model.normal_blasso, nSamples, progress.bar="none"); # Burnin  

samp.coeff.normal_blasso <- coda.samples(model.normal_blasso,variable.names=c("intercept","beta1","beta2","beta3","beta4","beta5","beta6","beta7","beta8","beta9","beta10","beta11","beta12","beta13","beta14","beta15","beta16","beta17","beta18","beta19","beta20","beta21","beta22","beta23","beta24","beta25","beta26","beta27","beta28","beta29"))  

summary(samp.coeff.normal_blasso)  

##  

## Iterations = 11001:31000  

## Thinning interval = 1  

## Number of chains = 1  

## Sample size per chain = 20000  

##  

## 1. Empirical mean and standard deviation for each variable,  

##     plus standard error of the mean:  

##  

##           Mean        SD Naive SE Time-series SE  

## beta[1]    -2.8570  1.936  0.01369      0.04969  

## beta[2]    -3.1713  1.862  0.01317      0.04431  

## beta[3]    -0.1253  1.700  0.01202      0.03493  

## beta[4]     1.6846  1.825  0.01290      0.04961  

## beta[5]    -1.9118  1.906  0.01348      0.04740  

## beta[6]    -4.0785  1.773  0.01253      0.03575  

## beta[7]     1.2132  1.606  0.01136      0.03752  

## beta[8]    -0.9197  1.508  0.01066      0.02916  

## beta[9]    -1.8215  1.641  0.01160      0.03729  

## beta[10]   0.1088  1.595  0.01128      0.03829  

## beta[11]   -3.3060  1.751  0.01238      0.04617  

## beta[12]   -0.2584  1.453  0.01027      0.02254  

## beta[13]   2.2494  1.441  0.01019      0.02523  

## beta[14]   -1.4598  1.491  0.01054      0.02649  

## beta[15]   0.2789  1.777  0.01256      0.04088  

## beta[16]   1.9975  1.588  0.01123      0.03938  

## beta[17]   -1.8904  1.598  0.01130      0.02825  

## beta[18]   -2.3552  1.762  0.01246      0.03407  

## beta[19]   -4.3446  1.852  0.01310      0.05796  

## beta[20]   2.5947  1.839  0.01300      0.04212  

## beta[21]   -0.8744  1.911  0.01351      0.04857  

## beta[22]   -1.7169  1.798  0.01271      0.04419  

## beta[23]   1.0781  1.707  0.01207      0.04127  

## beta[24]   3.1590  1.488  0.01052      0.02704  

## beta[25]   0.4449  1.554  0.01099      0.03317  

## beta[26]   2.6785  1.865  0.01319      0.04608  

## beta[27]   1.6776  1.921  0.01359      0.05234  

## beta[28]   5.1731  1.851  0.01309      0.04919  

## beta[29]   -0.1739  1.627  0.01151      0.03022

```

```

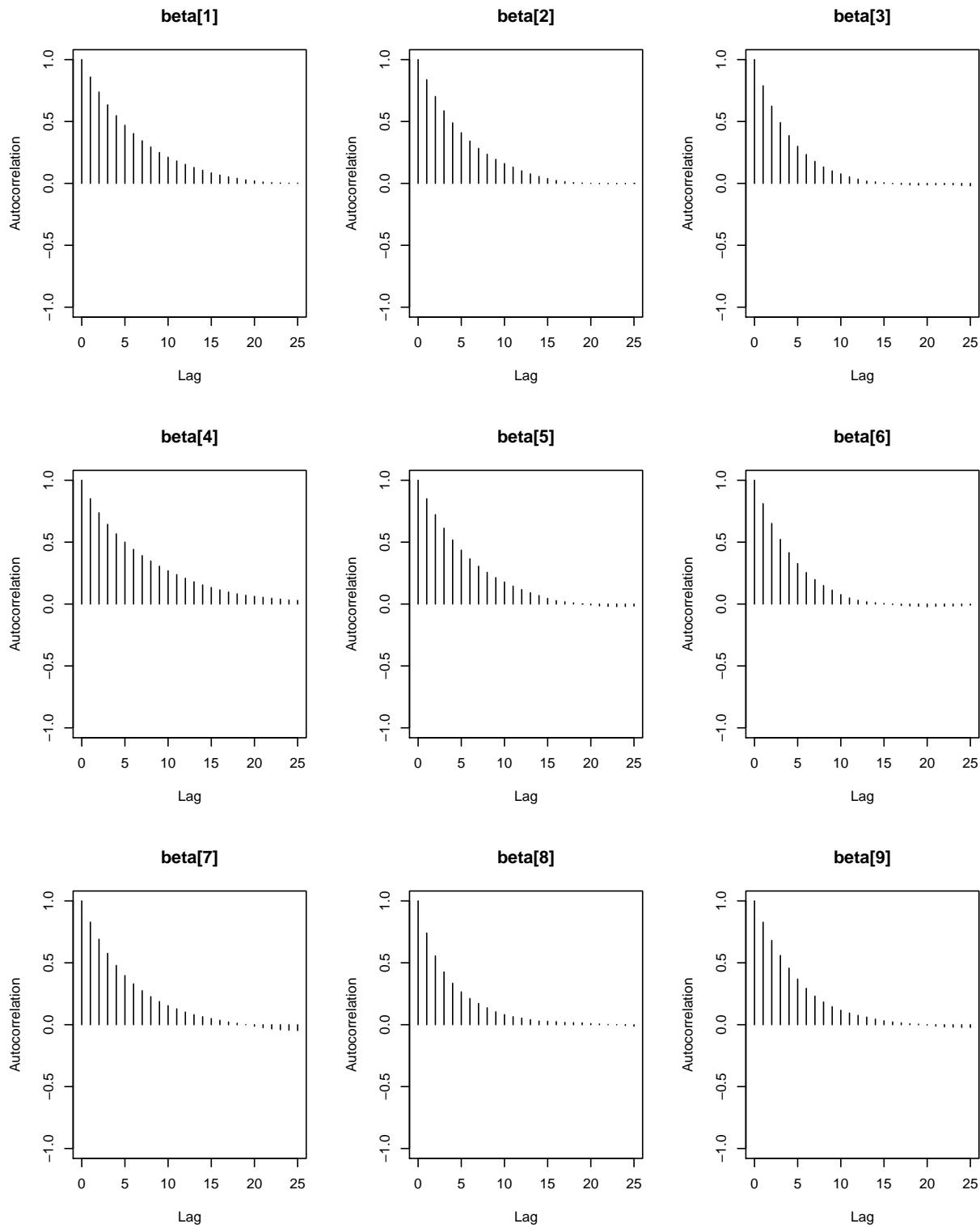
## beta[30] -1.2081 1.744 0.01233 0.03885
## beta[31] 0.5299 1.927 0.01363 0.05117
## beta[32] 1.4224 1.891 0.01337 0.04804
## beta[33] 2.4484 1.651 0.01168 0.04213
## beta[34] 2.5154 2.061 0.01458 0.06137
## beta[35] 3.6652 1.921 0.01359 0.04922
## beta[36] 4.1223 1.504 0.01063 0.03550
## beta[37] 3.9403 1.960 0.01386 0.05324
## beta[38] 2.1320 1.818 0.01285 0.03616
## beta[39] 1.9891 1.855 0.01311 0.04338
## beta[40] 5.5253 1.947 0.01377 0.05698
## beta[41] 2.2019 1.877 0.01327 0.04955
## beta[42] 1.1995 1.668 0.01180 0.04146
## beta[43] 1.0559 1.791 0.01267 0.04747
## beta[44] 5.4024 1.797 0.01271 0.05075
## beta[45] 6.1490 1.707 0.01207 0.03972
## beta[46] 2.6098 1.590 0.01124 0.03391
## beta[47] 1.5681 1.663 0.01176 0.03526
## beta[48] 4.8896 1.597 0.01129 0.02871
## beta[49] 6.8514 1.746 0.01235 0.04353
## beta[50] 70.3874 16.487 0.11658 5.68435
## intercept -16.1902 16.447 0.11630 5.76787
##
## 2. Quantiles for each variable:
##
##          2.5%      25%      50%      75%    97.5%
## beta[1] -6.67893 -4.13534 -2.8684 -1.53155 0.9416
## beta[2] -6.85232 -4.41609 -3.1636 -1.93216 0.4896
## beta[3] -3.48182 -1.25977 -0.1341  1.00154 3.1783
## beta[4] -1.84992  0.45393  1.6871  2.95453 5.2068
## beta[5] -5.68522 -3.19316 -1.9085 -0.62746 1.7993
## beta[6] -7.60259 -5.24070 -4.0648 -2.88933 -0.6385
## beta[7] -1.94150  0.14364  1.2009  2.30334 4.3687
## beta[8] -3.90509 -1.91662 -0.9157  0.08037 2.0380
## beta[9] -5.11755 -2.91912 -1.8146 -0.70066 1.3259
## beta[10] -3.08593 -0.94454  0.1106  1.17918 3.2230
## beta[11] -6.71126 -4.47181 -3.3256 -2.14538 0.1815
## beta[12] -3.13991 -1.22940 -0.2620  0.72455 2.5803
## beta[13] -0.59071  1.28412  2.2543  3.22614 5.0672
## beta[14] -4.38066 -2.46731 -1.4511 -0.43995 1.4155
## beta[15] -3.18560 -0.90124  0.2591  1.47012 3.7982
## beta[16] -1.07987  0.90117  1.9812  3.07039 5.1650
## beta[17] -5.00442 -2.98532 -1.9006 -0.80518 1.2617
## beta[18] -5.85335 -3.52713 -2.3653 -1.15403 1.0528
## beta[19] -7.93061 -5.61795 -4.3612 -3.09788 -0.6513
## beta[20] -0.93518  1.32038  2.5592  3.84218 6.2343
## beta[21] -4.60308 -2.16631 -0.8514  0.42820 2.7983
## beta[22] -5.20435 -2.92214 -1.7513 -0.51146 1.9052

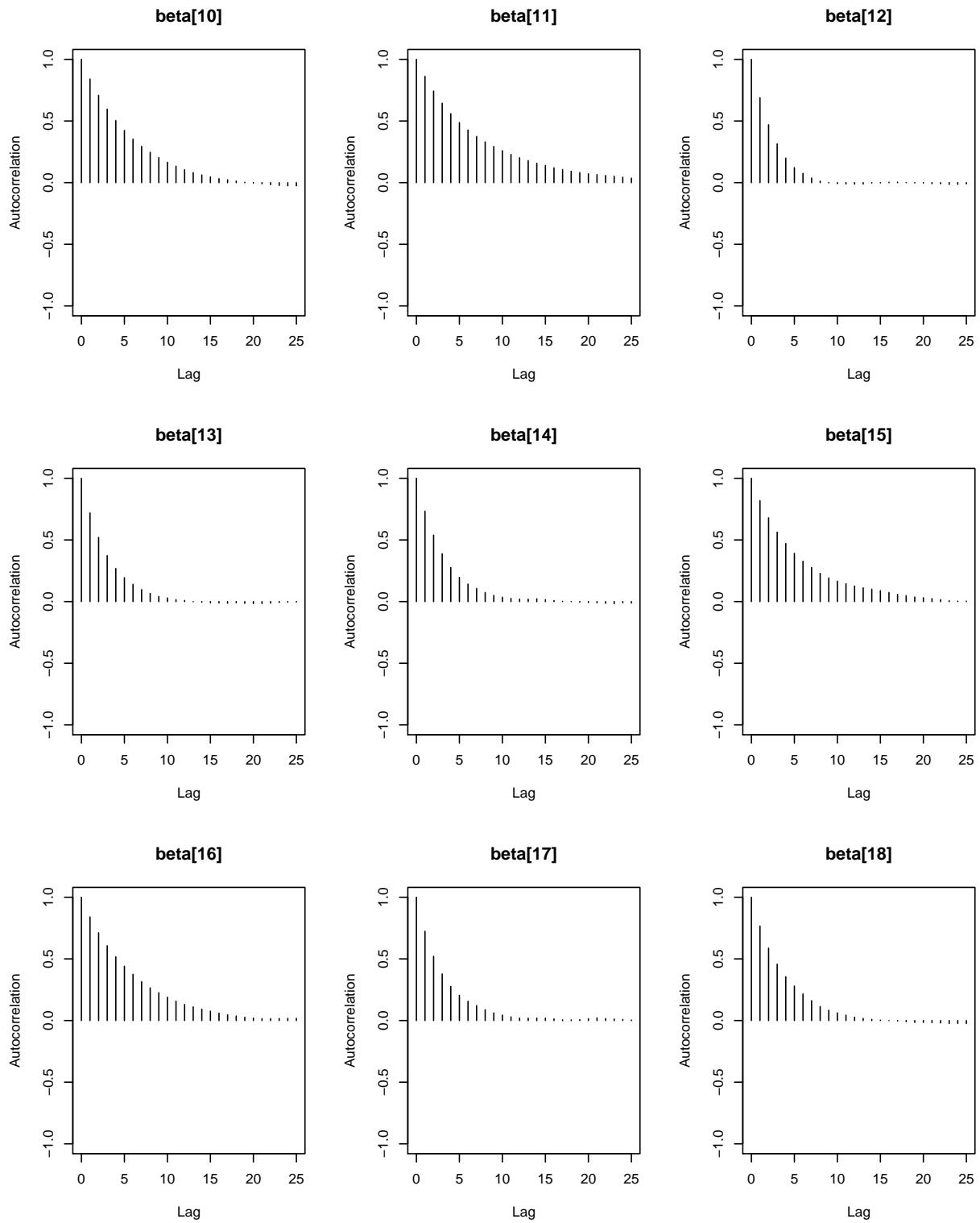
```

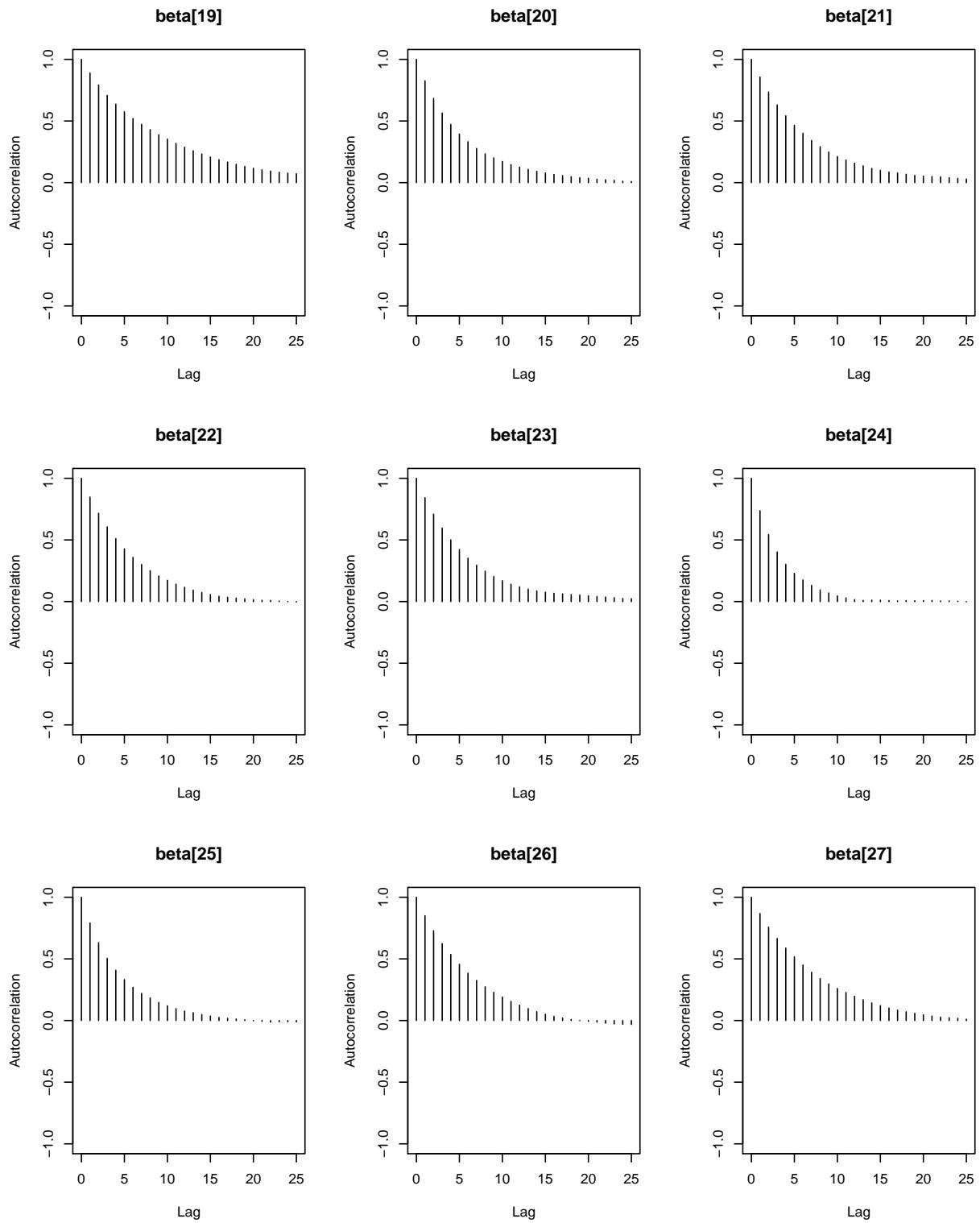
```

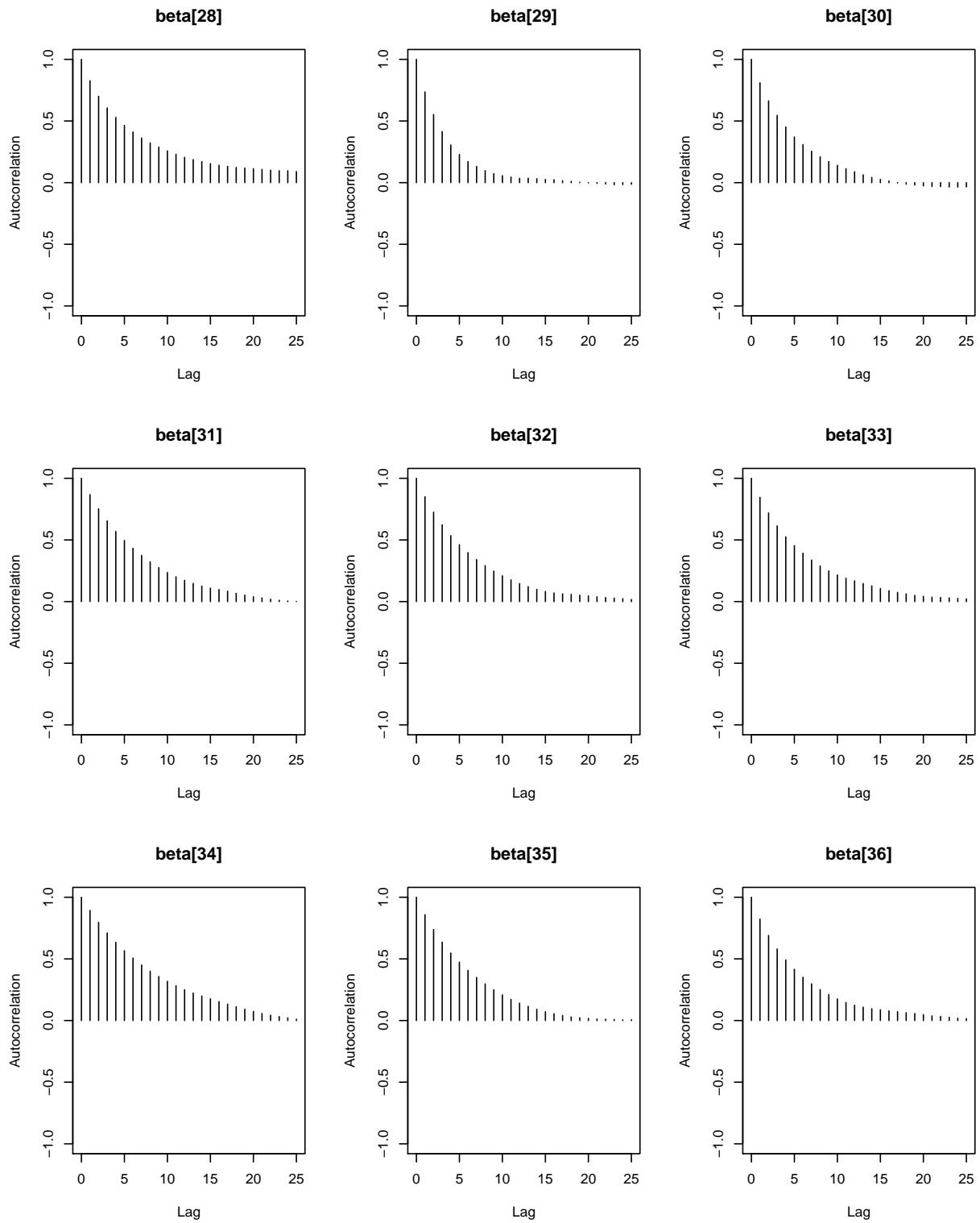
## beta[23] -2.29167 -0.05579 1.0660 2.20991 4.4594
## beta[24] 0.24232 2.15761 3.1585 4.17753 6.0556
## beta[25] -2.56887 -0.58800 0.4331 1.46682 3.5454
## beta[26] -0.91970 1.41911 2.6595 3.93249 6.4130
## beta[27] -2.08532 0.38918 1.6864 2.98276 5.4492
## beta[28] 1.58040 3.91102 5.1656 6.41719 8.8079
## beta[29] -3.37381 -1.27716 -0.1697 0.94170 2.9816
## beta[30] -4.67928 -2.37294 -1.1937 -0.04280 2.1642
## beta[31] -3.26594 -0.75627 0.5230 1.83739 4.3221
## beta[32] -2.33431 0.16391 1.4061 2.68616 5.1166
## beta[33] -0.78649 1.33548 2.4529 3.56392 5.6728
## beta[34] -1.54044 1.14041 2.5021 3.89599 6.6220
## beta[35] -0.11327 2.38745 3.6503 4.96296 7.4469
## beta[36] 1.17153 3.11479 4.1113 5.13582 7.0715
## beta[37] 0.02815 2.62126 3.9497 5.28808 7.7422
## beta[38] -1.43244 0.90152 2.1273 3.36429 5.7001
## beta[39] -1.68088 0.74528 1.9858 3.25896 5.6242
## beta[40] 1.69392 4.22449 5.5051 6.83692 9.3246
## beta[41] -1.48294 0.94127 2.1772 3.47896 5.8771
## beta[42] -2.11480 0.08219 1.2312 2.34421 4.3772
## beta[43] -2.39157 -0.16046 1.0355 2.24651 4.6490
## beta[44] 1.89033 4.19322 5.4197 6.61221 8.8732
## beta[45] 2.80566 5.01086 6.1435 7.28792 9.5525
## beta[46] -0.50053 1.52709 2.5956 3.67918 5.7428
## beta[47] -1.64149 0.43481 1.5668 2.70222 4.8178
## beta[48] 1.74971 3.82689 4.8877 5.96600 8.0629
## beta[49] 3.44534 5.63772 6.8762 8.06384 10.2304
## beta[50] 39.45128 58.76070 69.9229 82.11177 100.0735
## intercept -46.08954 -27.81224 -15.6930 -4.67371 14.7842
autocorr.plot(samp.coeff.normal_blasso)

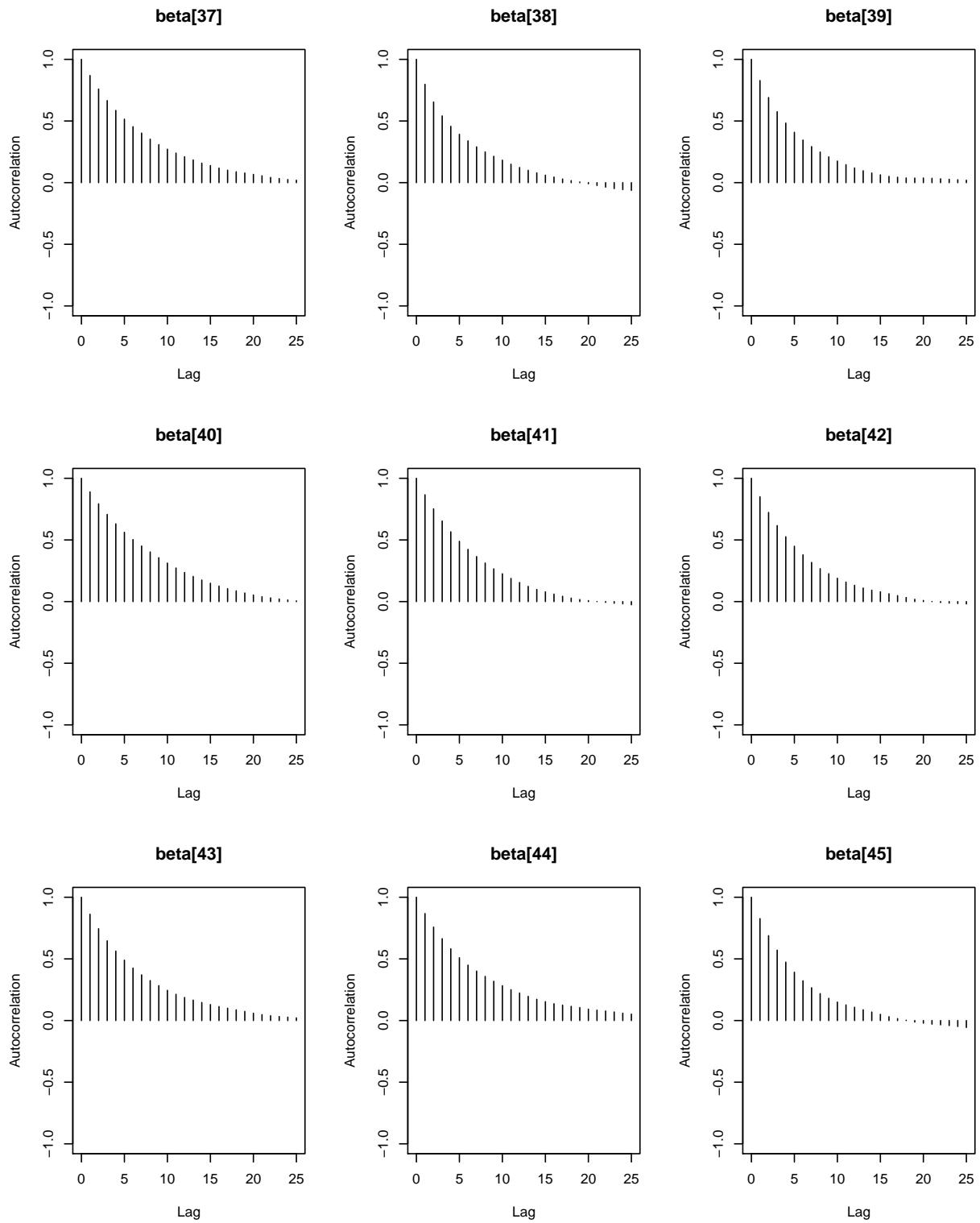
```

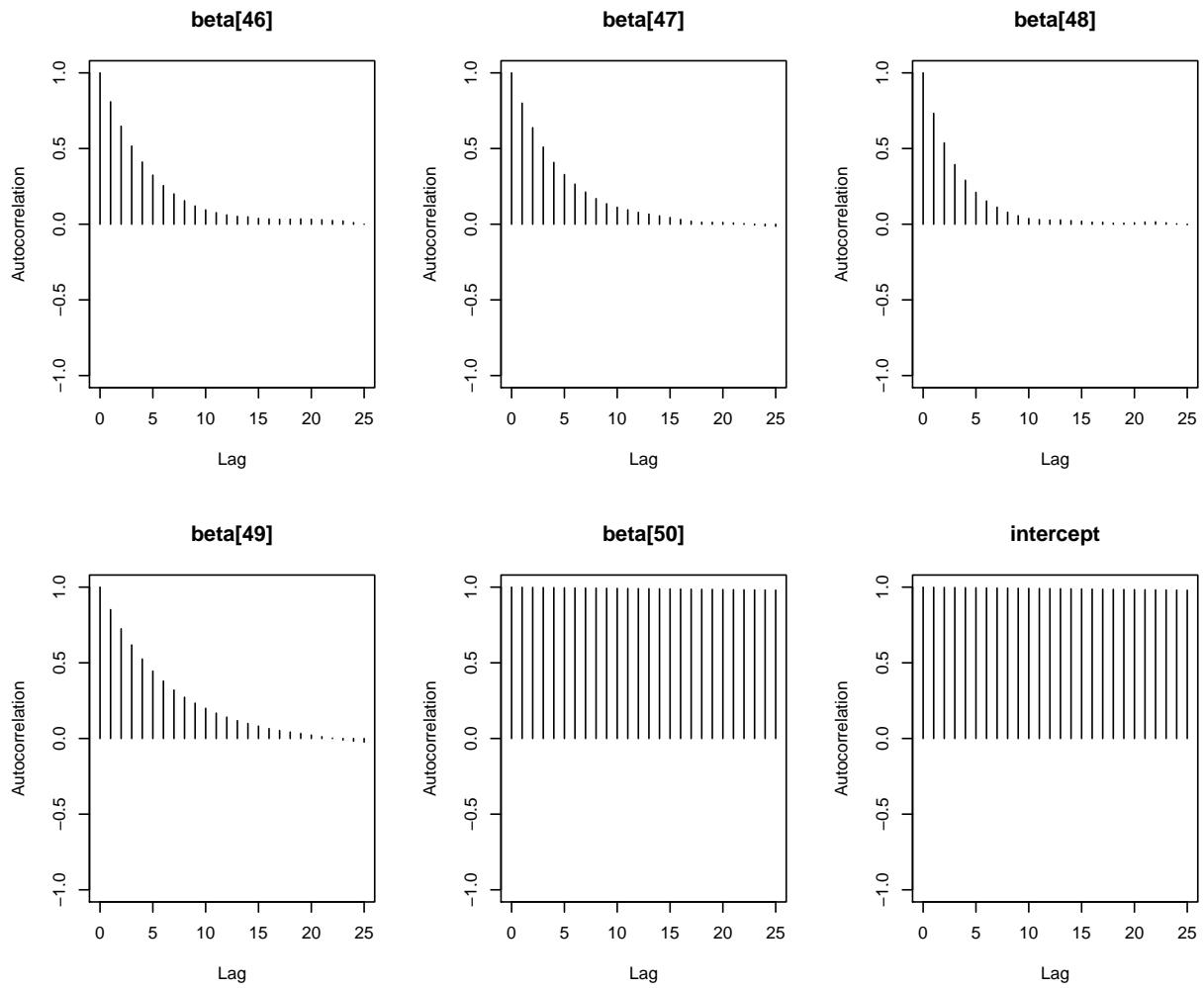




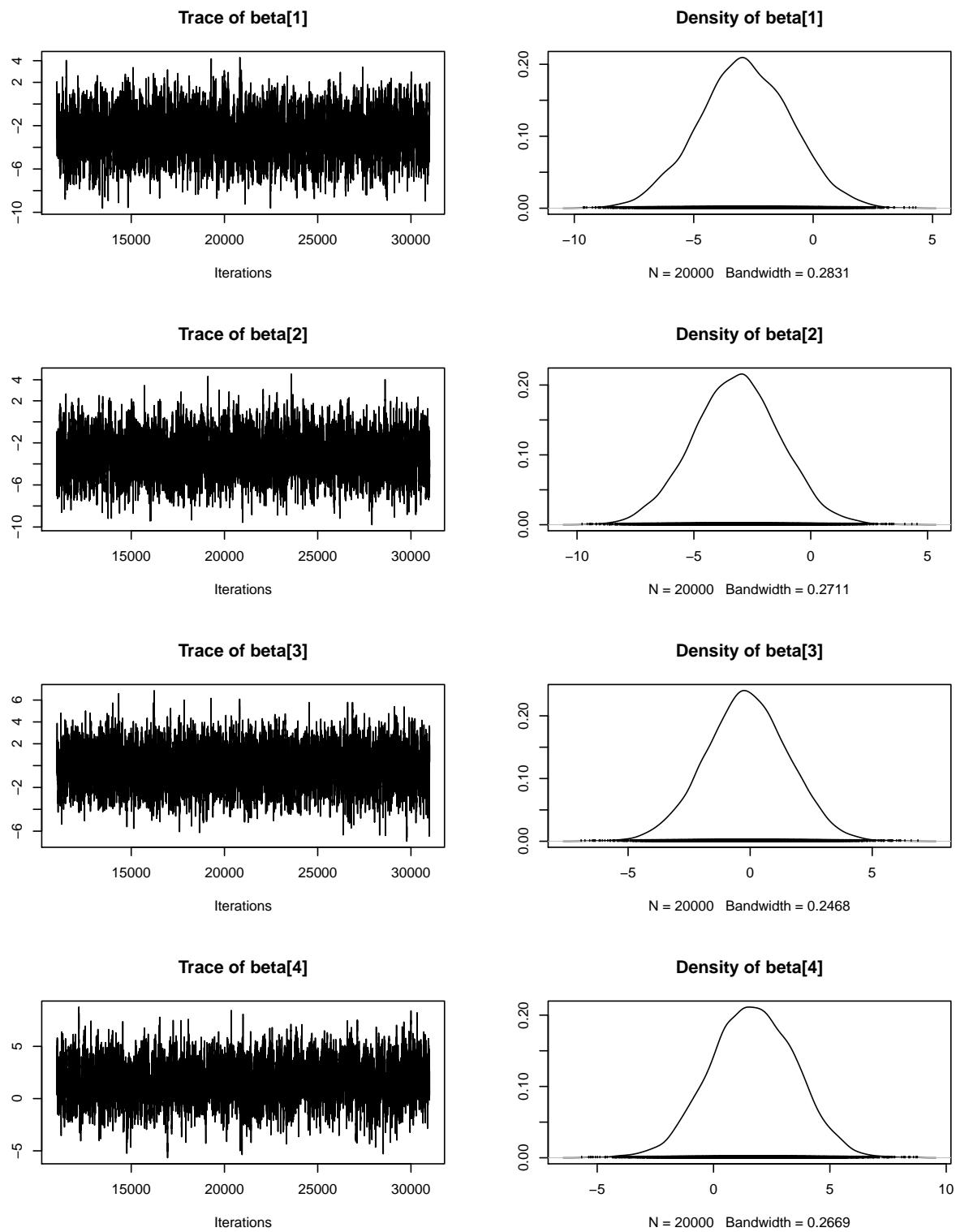




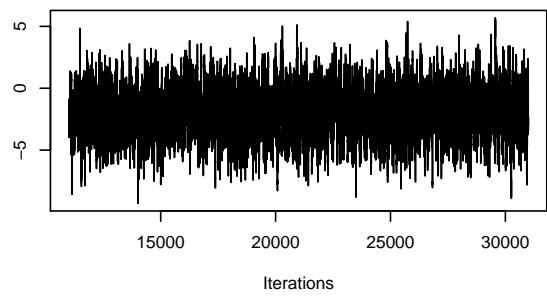




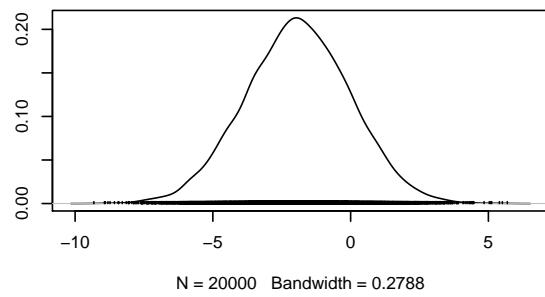
```
plot(samp.coeff.normal_blasso)
```



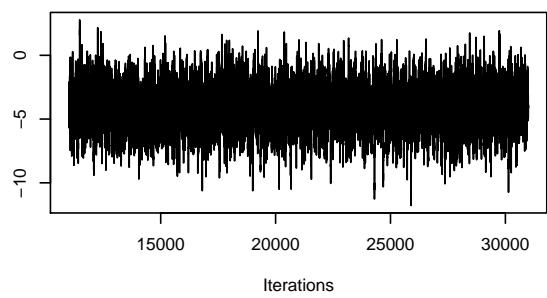
Trace of beta[5]



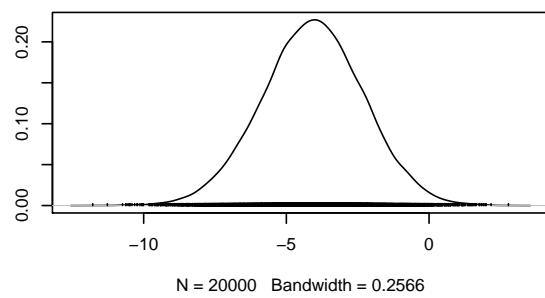
Density of beta[5]



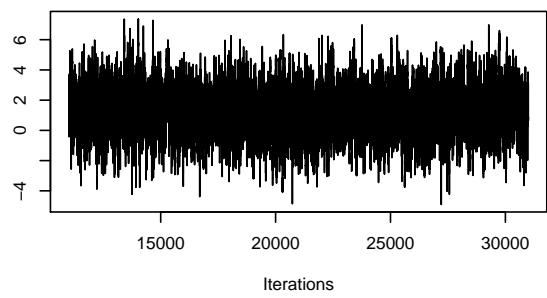
Trace of beta[6]



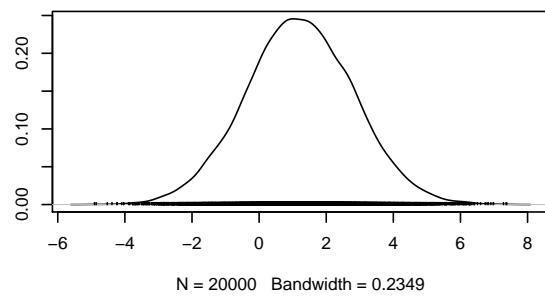
Density of beta[6]



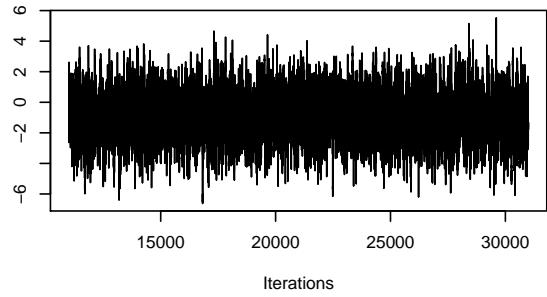
Trace of beta[7]



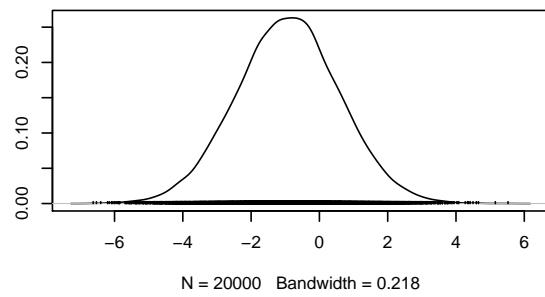
Density of beta[7]



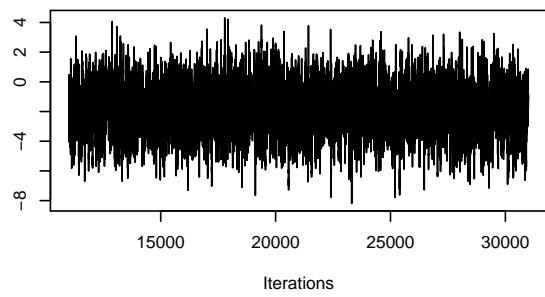
Trace of beta[8]



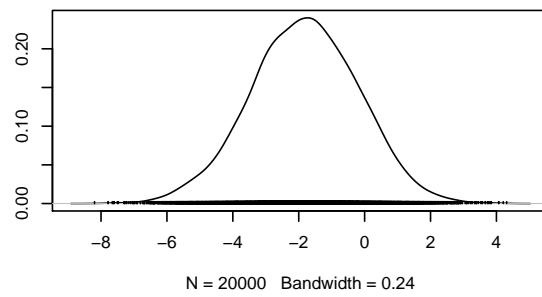
Density of beta[8]



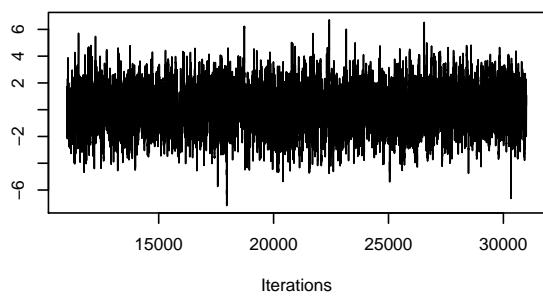
Trace of beta[9]



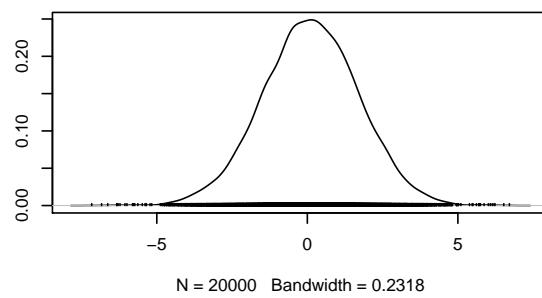
Density of beta[9]



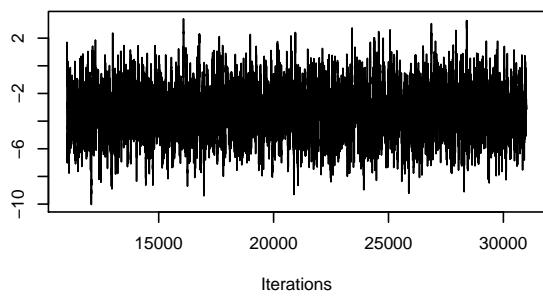
Trace of beta[10]



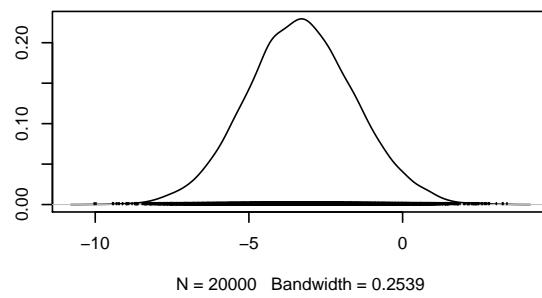
Density of beta[10]



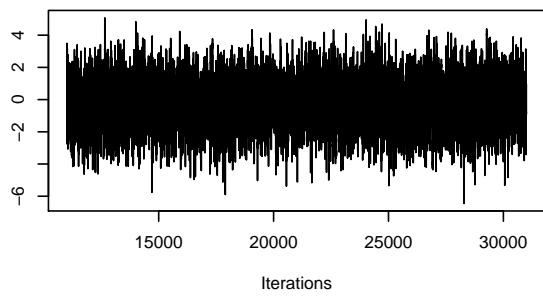
Trace of beta[11]



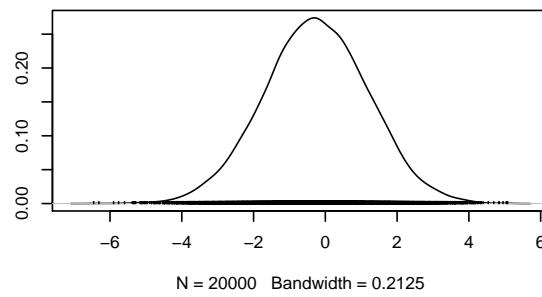
Density of beta[11]



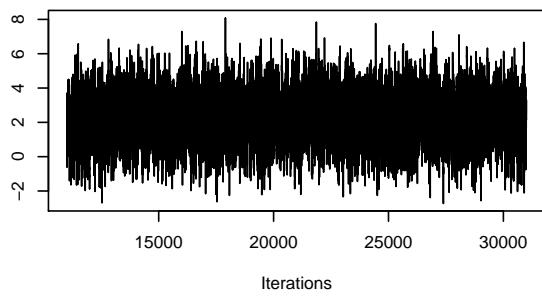
Trace of beta[12]



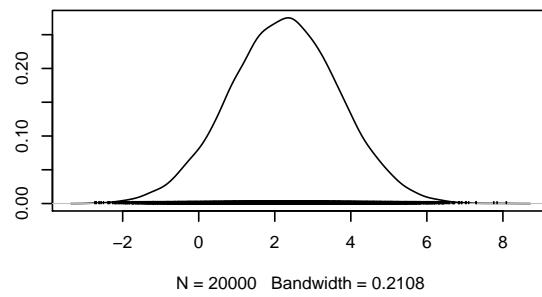
Density of beta[12]



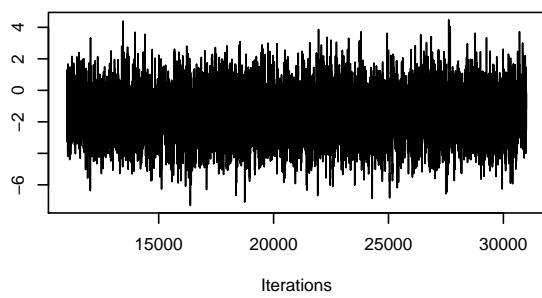
Trace of beta[13]



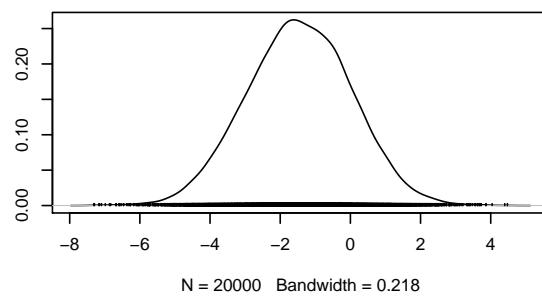
Density of beta[13]



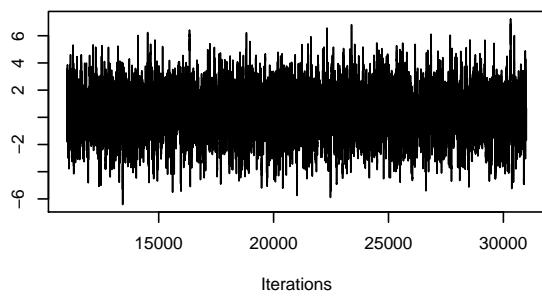
Trace of beta[14]



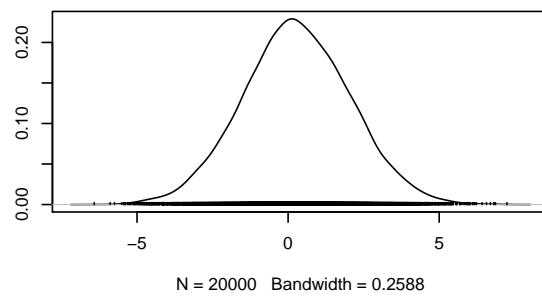
Density of beta[14]



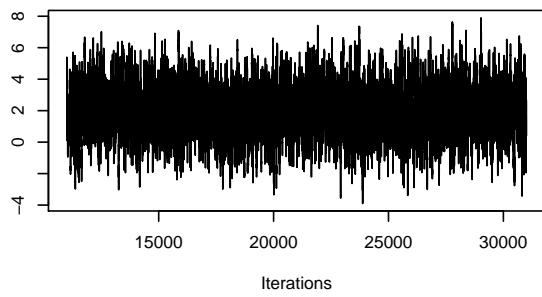
Trace of beta[15]



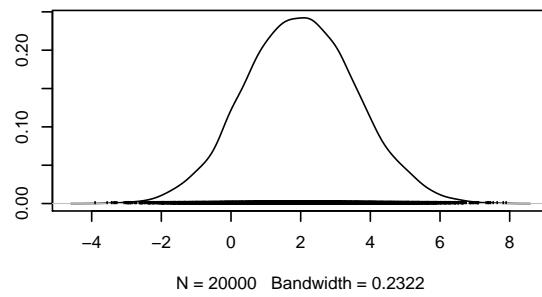
Density of beta[15]



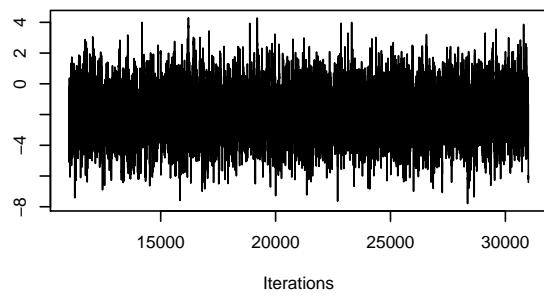
Trace of beta[16]



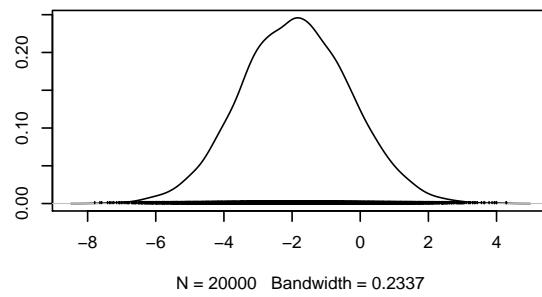
Density of beta[16]



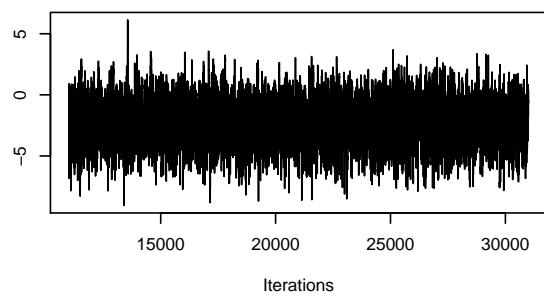
Trace of beta[17]



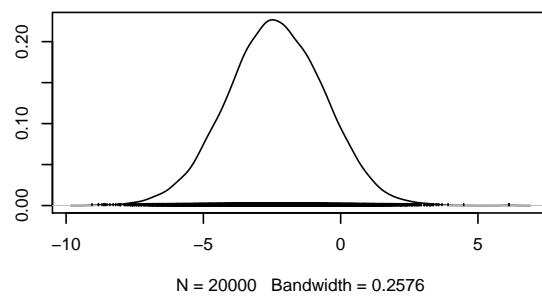
Density of beta[17]



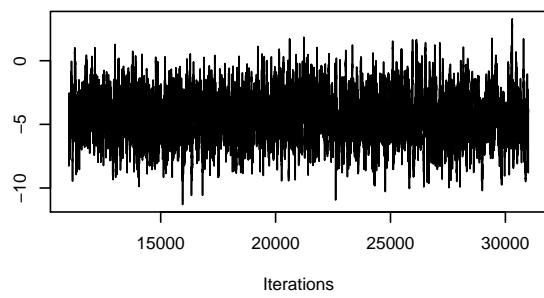
Trace of beta[18]



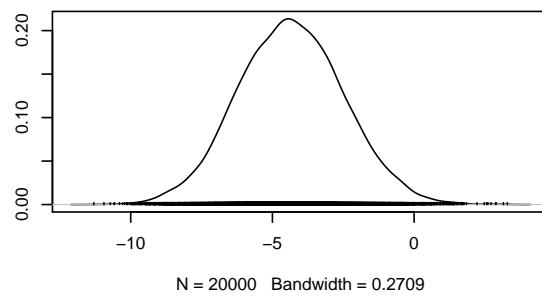
Density of beta[18]



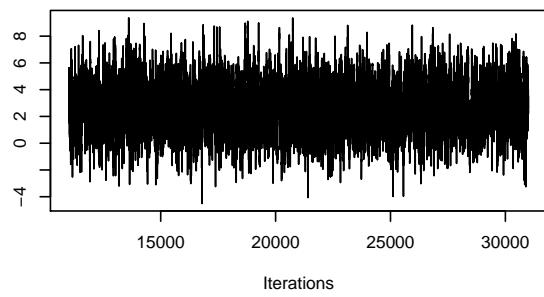
Trace of beta[19]



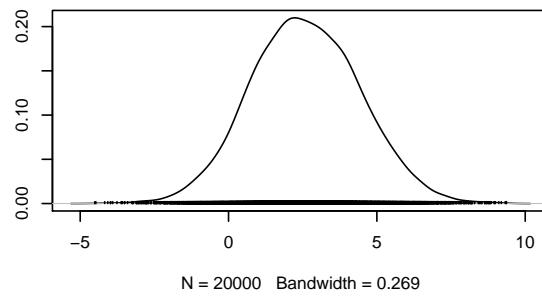
Density of beta[19]



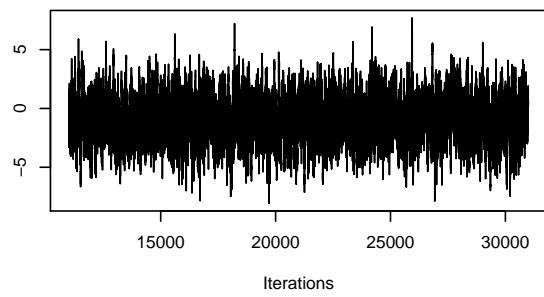
Trace of beta[20]



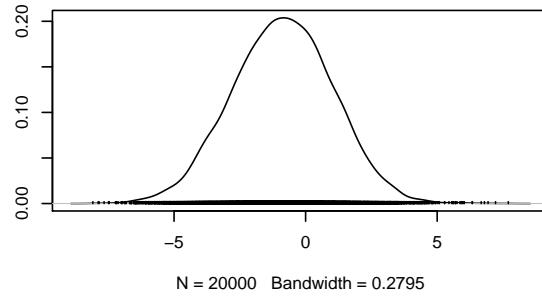
Density of beta[20]



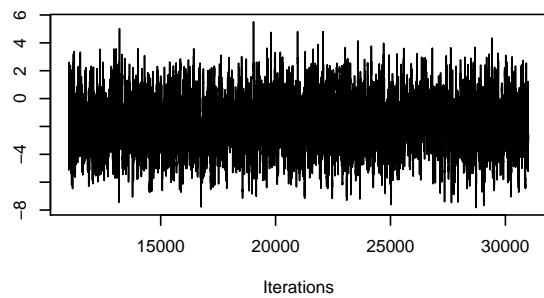
Trace of beta[21]



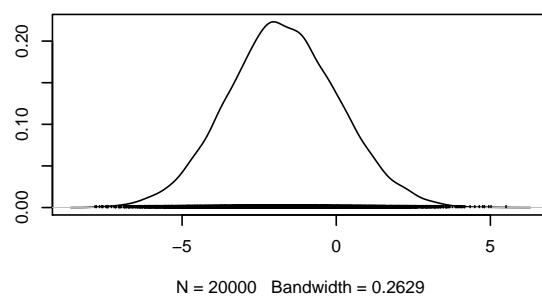
Density of beta[21]



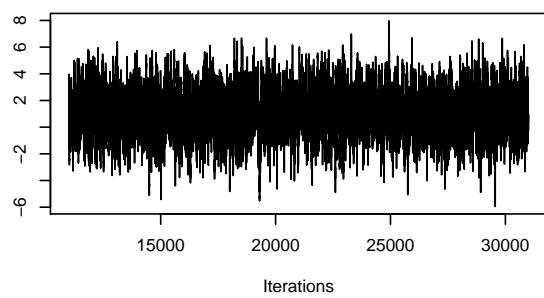
Trace of beta[22]



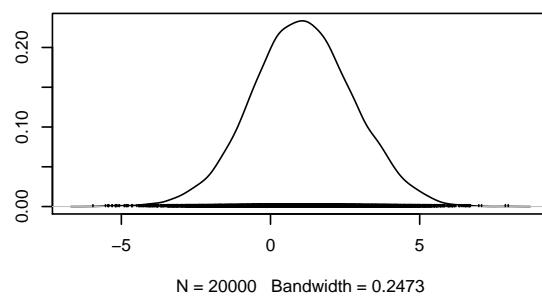
Density of beta[22]



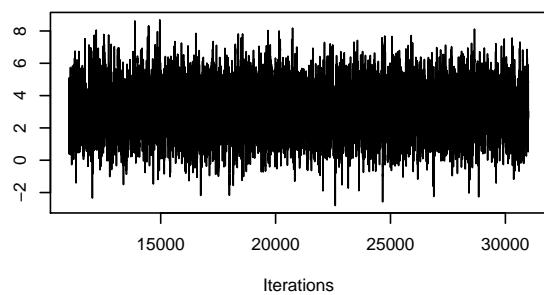
Trace of beta[23]



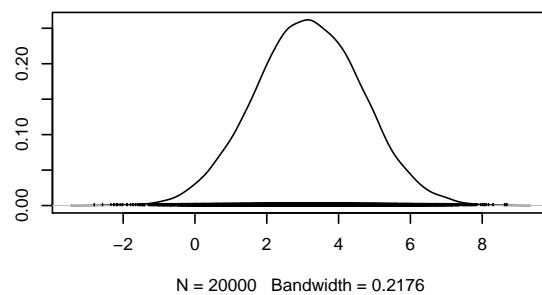
Density of beta[23]



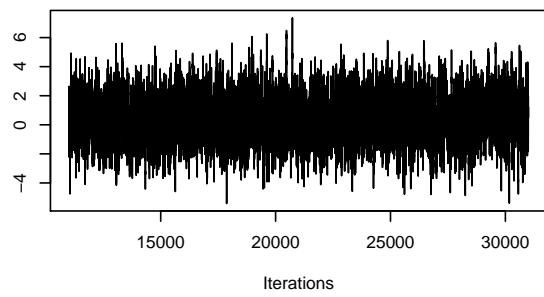
Trace of beta[24]



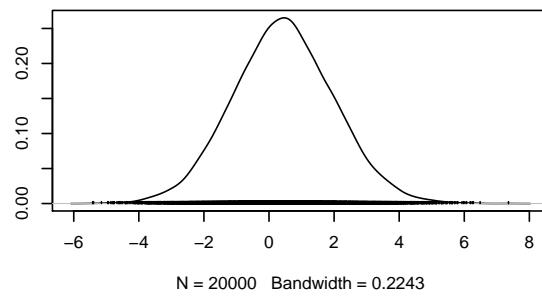
Density of beta[24]



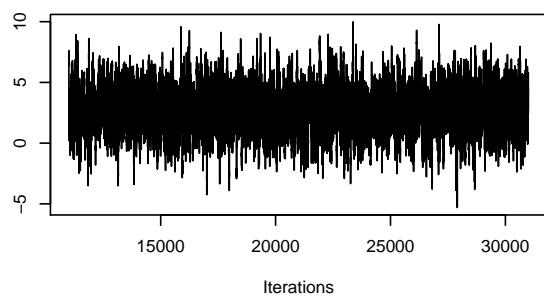
Trace of beta[25]



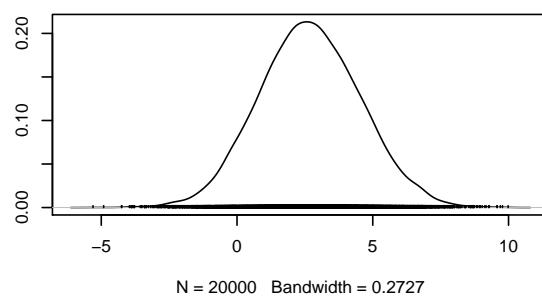
Density of beta[25]



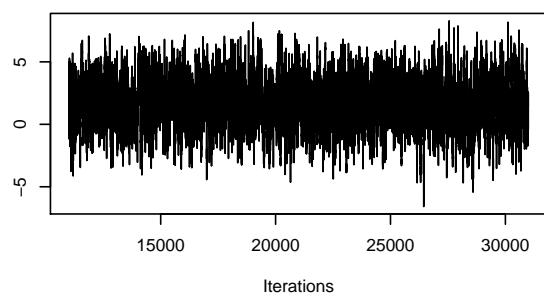
Trace of beta[26]



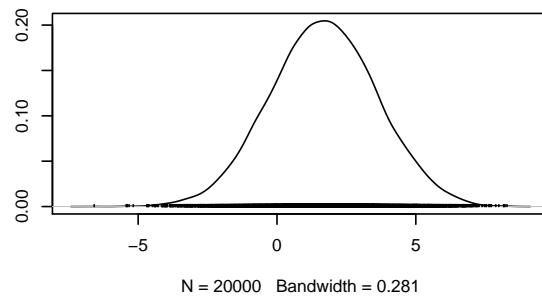
Density of beta[26]



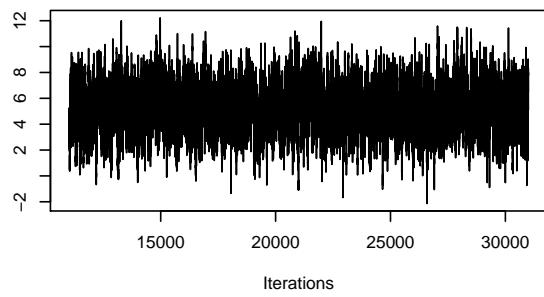
Trace of beta[27]



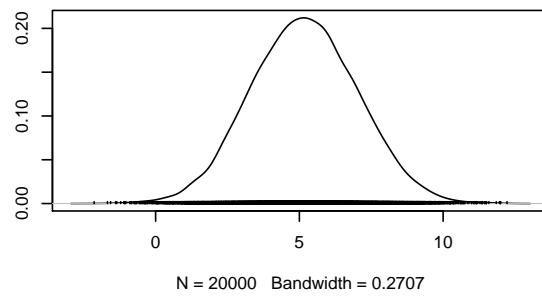
Density of beta[27]



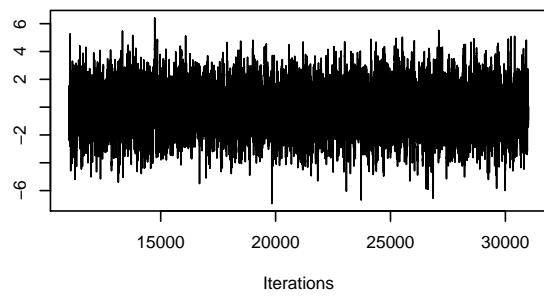
Trace of beta[28]



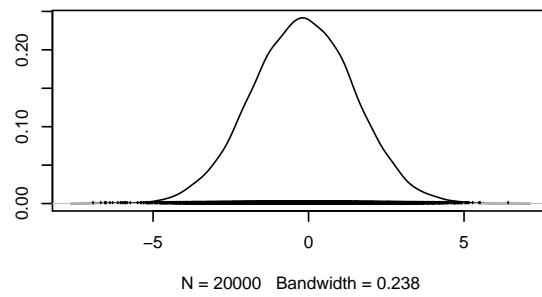
Density of beta[28]



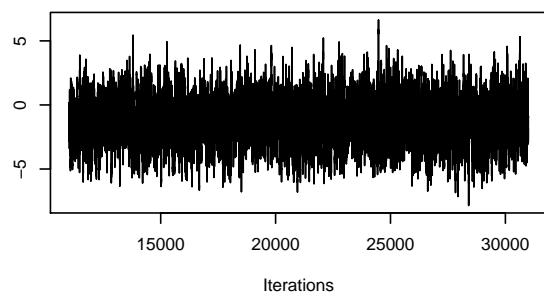
Trace of beta[29]



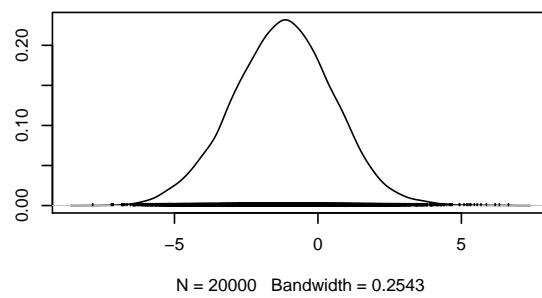
Density of beta[29]



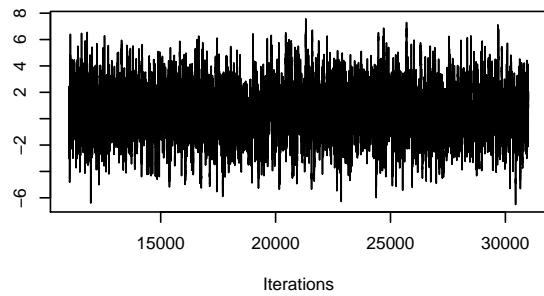
Trace of beta[30]



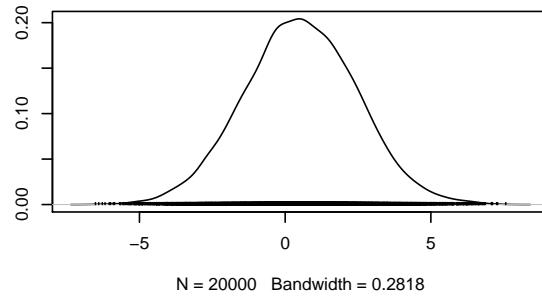
Density of beta[30]



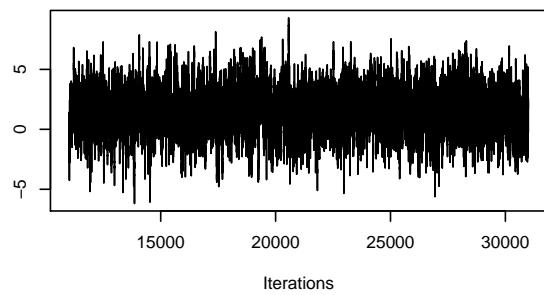
Trace of beta[31]



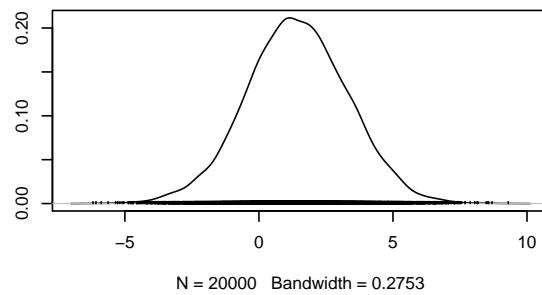
Density of beta[31]



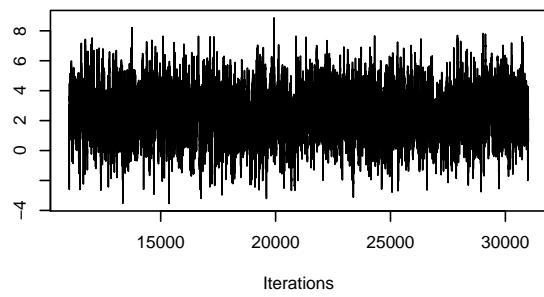
Trace of beta[32]



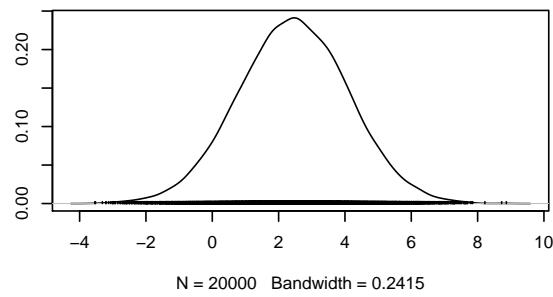
Density of beta[32]



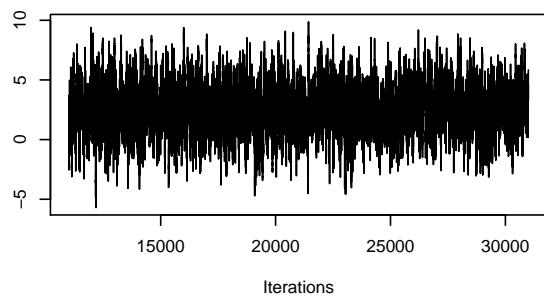
Trace of beta[33]



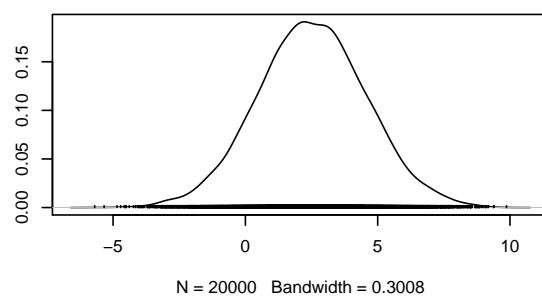
Density of beta[33]



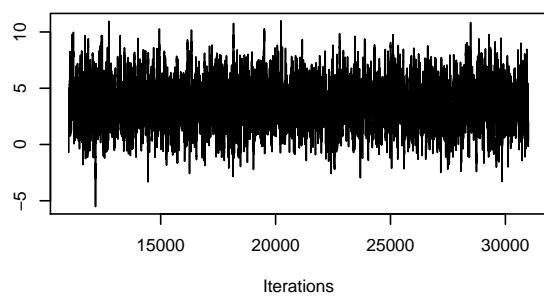
Trace of beta[34]



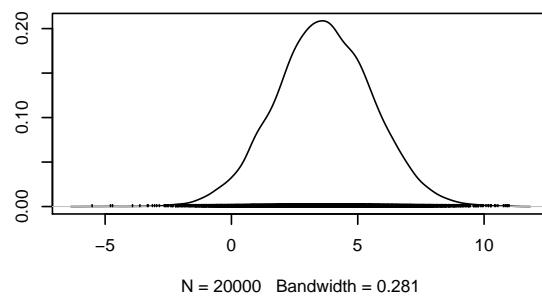
Density of beta[34]



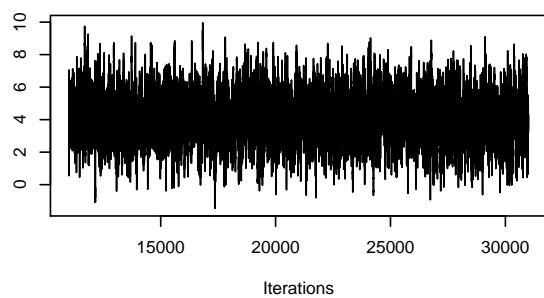
Trace of beta[35]



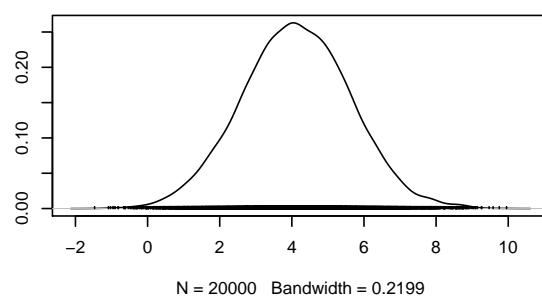
Density of beta[35]



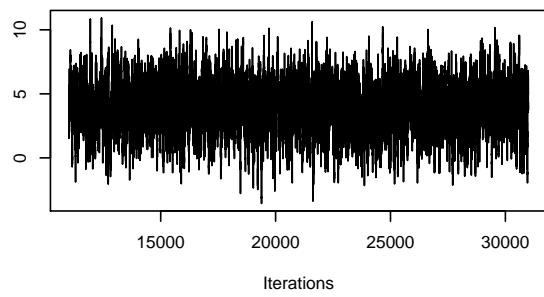
Trace of beta[36]



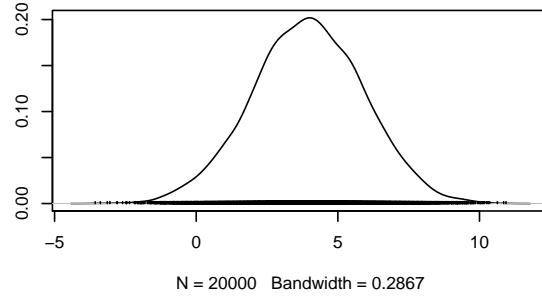
Density of beta[36]



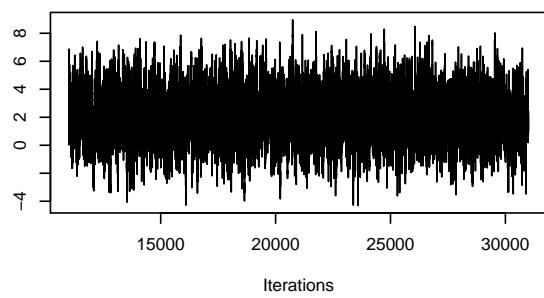
Trace of beta[37]



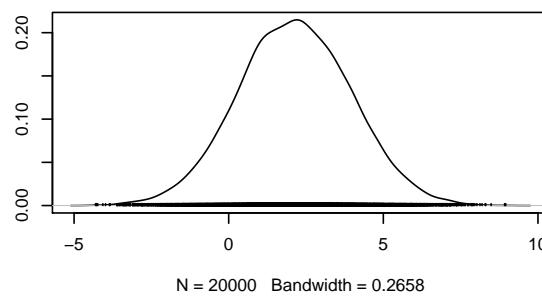
Density of beta[37]



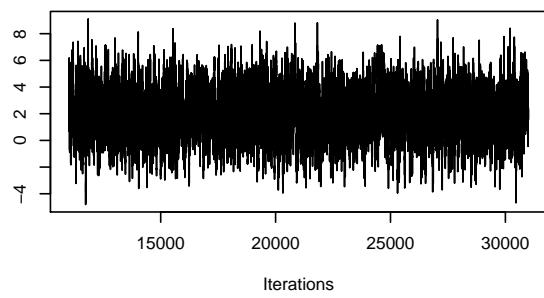
Trace of beta[38]



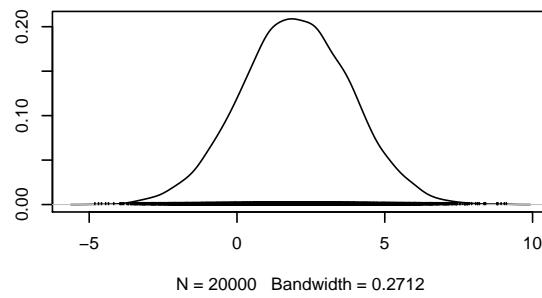
Density of beta[38]



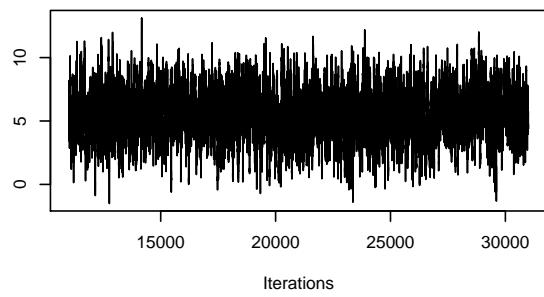
Trace of beta[39]



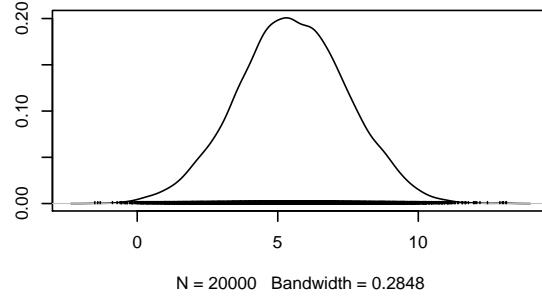
Density of beta[39]



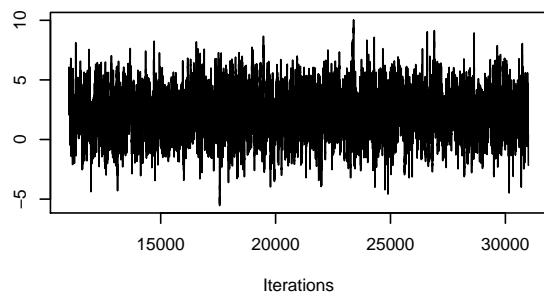
Trace of beta[40]



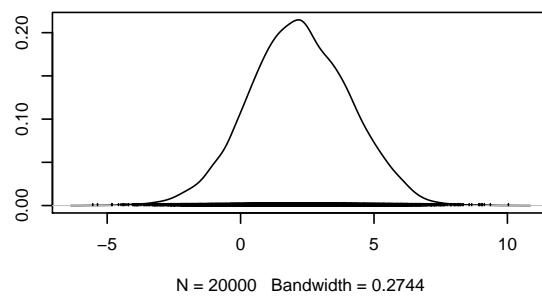
Density of beta[40]



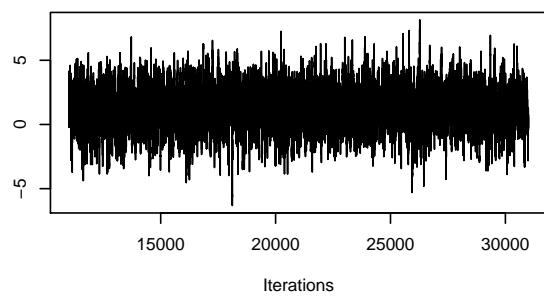
Trace of beta[41]



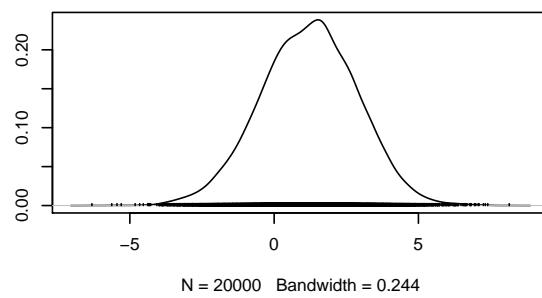
Density of beta[41]



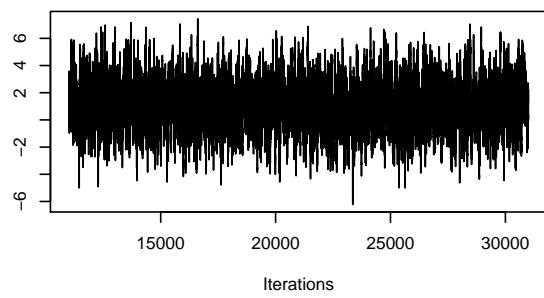
Trace of beta[42]



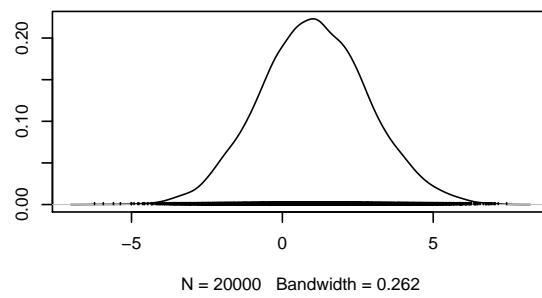
Density of beta[42]



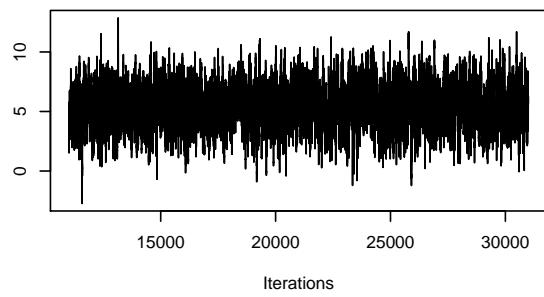
Trace of beta[43]



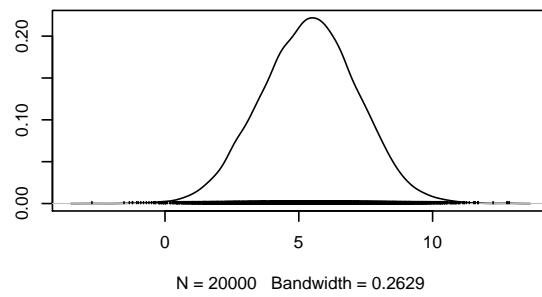
Density of beta[43]



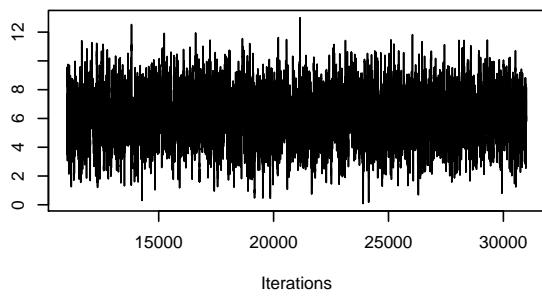
Trace of beta[44]



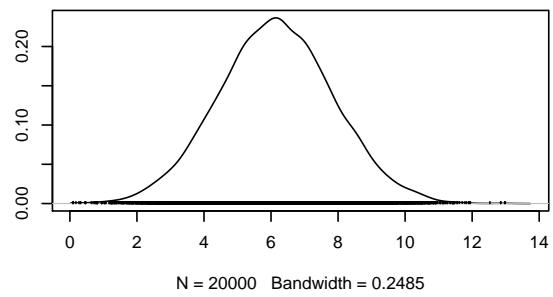
Density of beta[44]



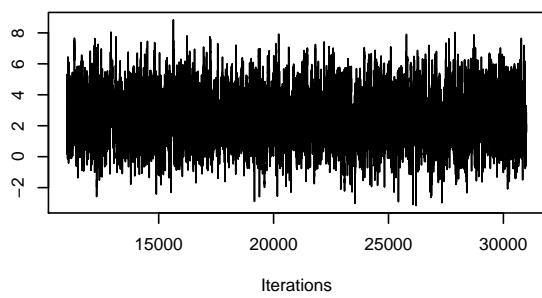
Trace of beta[45]



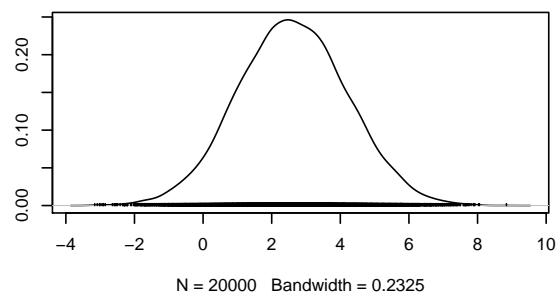
Density of beta[45]



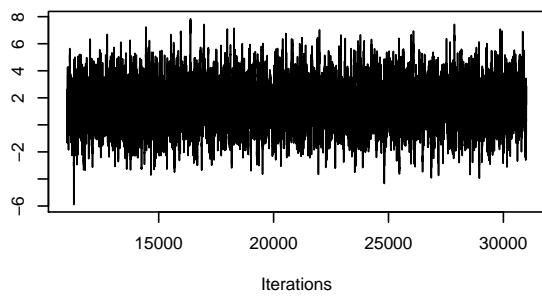
Trace of beta[46]



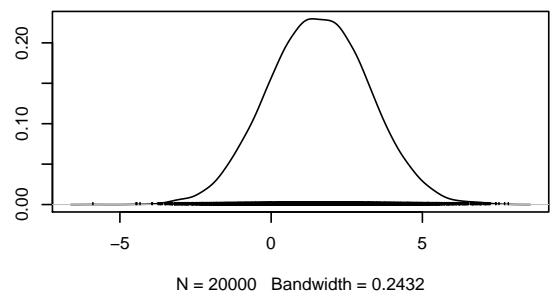
Density of beta[46]



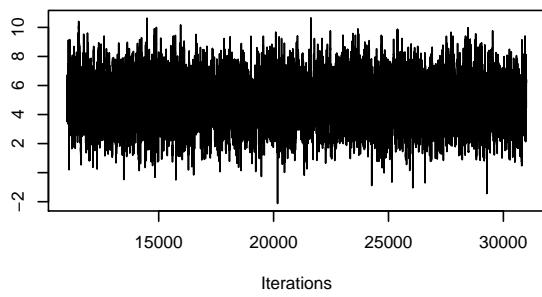
Trace of beta[47]



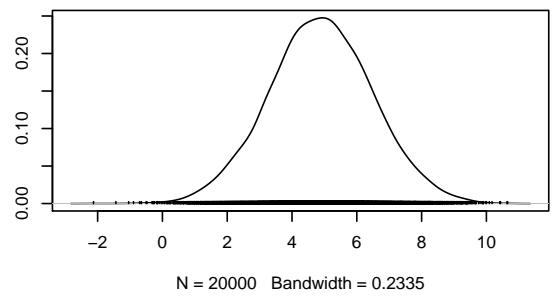
Density of beta[47]

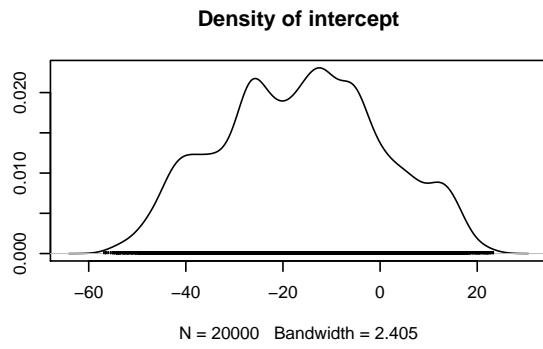
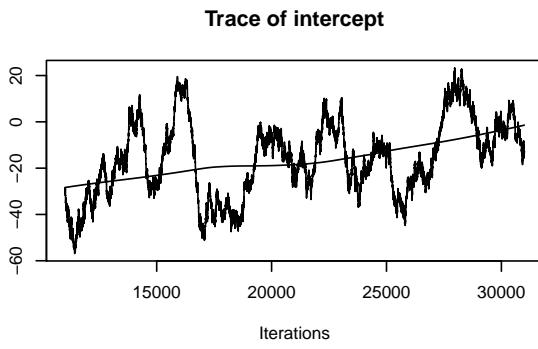
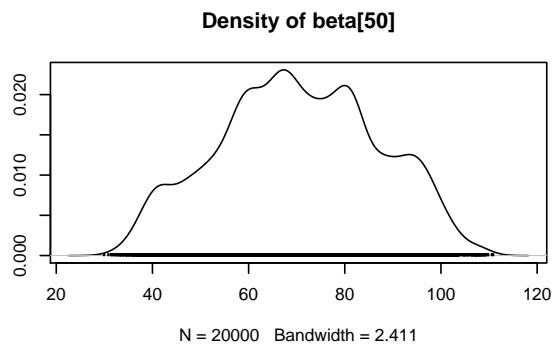
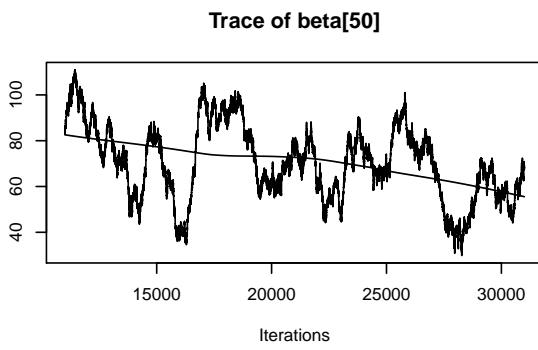
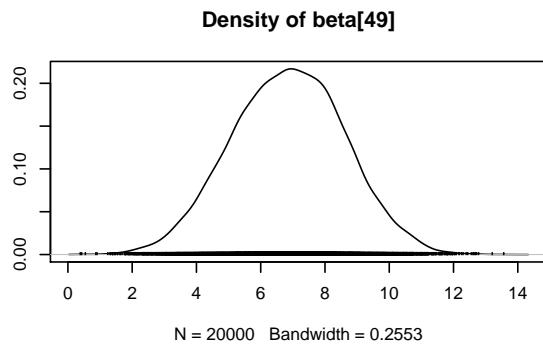
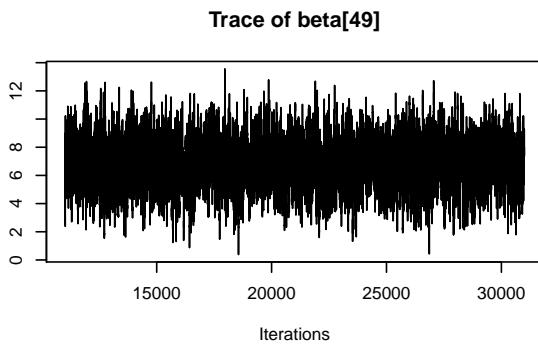


Trace of beta[48]



Density of beta[48]





```

if (DEBUG)
{
  #Sample again and estimate posterior means and MAP posterior modes.
  samp.coeff.normal_blasso.jags <- jags.samples(model.normal_blasso, variable.names = c("intercept",
  "beta[49]", "beta[50]"))
}
```

```
posterior_means.normal_blasso <- lapply(samp.coeff.normal_blasso.jags, apply, 1, "mean")
pander(posterior_means.normal_blasso, caption = "posterior means second sample")
posterior_modes.normal_blasso <- lapply(samp.coeff.normal_blasso.jags, apply, 1, "mlv")
posterior_modes.normal_blasso
if(n.chains>1)
{
  gelman.plot(samp.coeff)
}
}
```