Applied Bayesian Analysis : NCSU ST 540

Homework 1

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Bayesian analysis of Clutch Shots

In this problem we're asked to analyze success rate of NBA player clutch shots. We're given an overall success rate and data for clutch successes and attempts. Let i denote player i θ_i be our clutch success proportion, p_i the player's overall success proportion N_i the number of clutch attempts and Y_i the number of clutch successes. We will then model the likelihood with a $Binomial(N, \theta)$ distribution and for mathematical tractability we'll model the prior with a $Beta(\alpha, \beta)$ distribution.

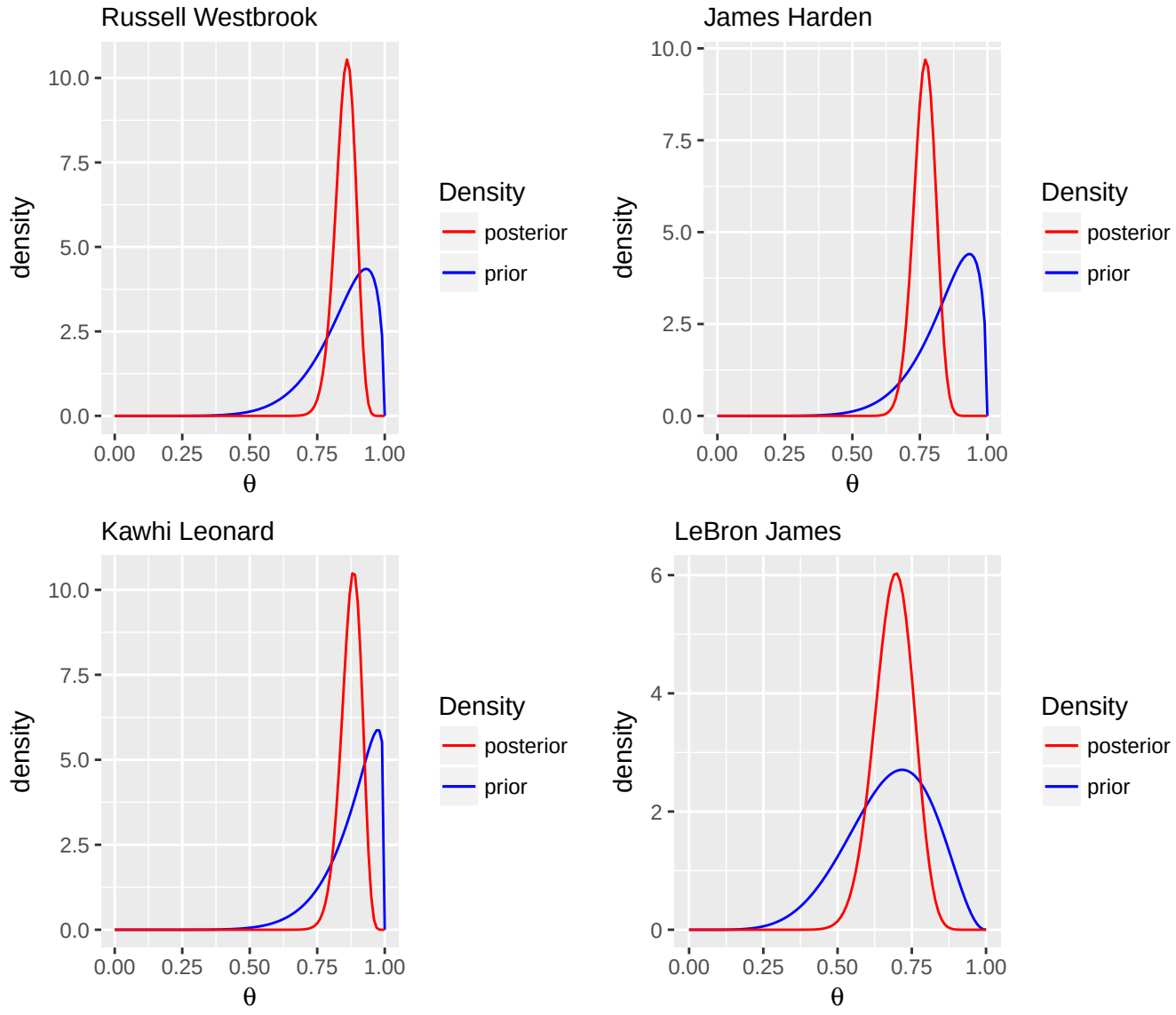
We'll use the overall proportion p_i to inform our prior. The text book does a great job explaining how the parameters of $Beta(\alpha, \beta)$ can be related to the mean $\mu = \frac{\alpha}{\alpha+\beta}$ and spread $\kappa = \alpha + \beta$ We'll choose (α, β) so the prior mean is p_i. Now the spread κ can be thought of as the minimum number of trials required for us to consider updating our belief in μ. 10 seems like the minimum number of samples we'd need to even begin to contemplate updating our beliefs and we'd like not to bias our analysis by choosing too large a number. Also the data available may play a role in helping us design our prior. The number of attempts $N_i \in [16, 95]$ so we'd not want to go above this range. It's worth contemplating if one should set $\kappa = N$. For now we will use $\kappa = 10$ and revisit that choice when we look at the sensitivity of our analysis to the prior.

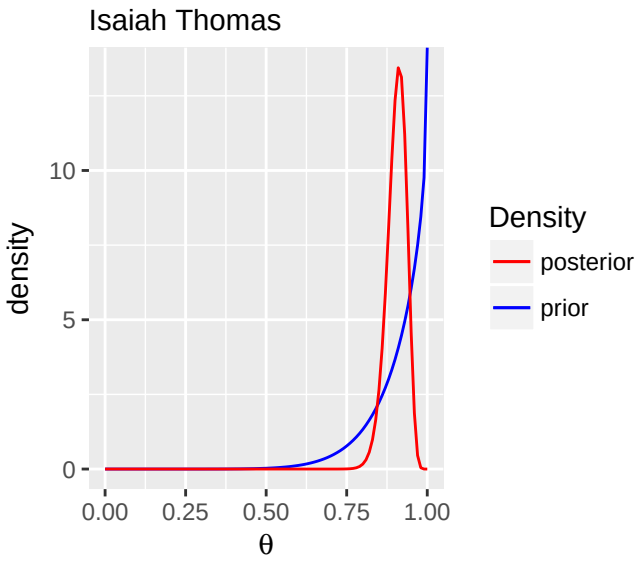
Plots of the posterior and prior used.

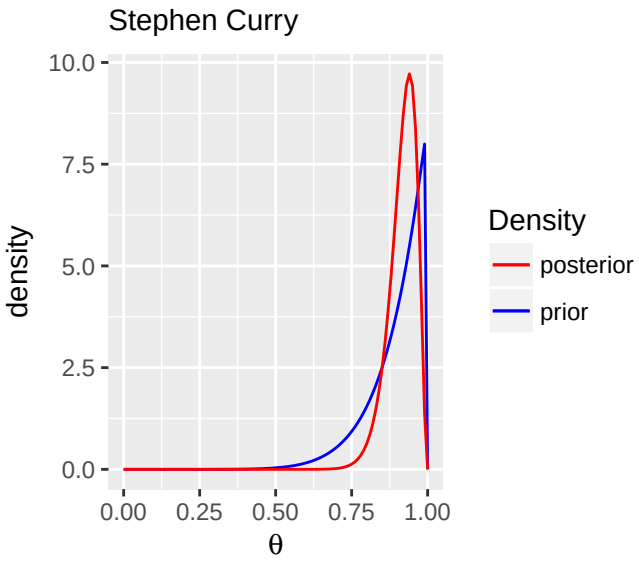
```
# We load the data from a file here.
library(readr)
library(gridExtra)
NBA_Data <- read_csv("NBA_Data.csv", col_types = cols(ClutchAttempts = col_number(),
    ClutchMakes = col_number()))
NBA_Data$clutchProportion <- NBA_Data$ClutchMakes/NBA_Data$ClutchAttempts
# pander(NBA_Data)

# We'll plot the posteriors on a single
# page after our calculations. This list
# stores the plots for us to use later.
plots <- c()

# We'll store the posterior mean and mode
# for summarizing the poseterior in a
# table.
NBA_Data[, "posteriorMean"] <- NA
NBA_Data[, "posteriorMode"] <- NA
```

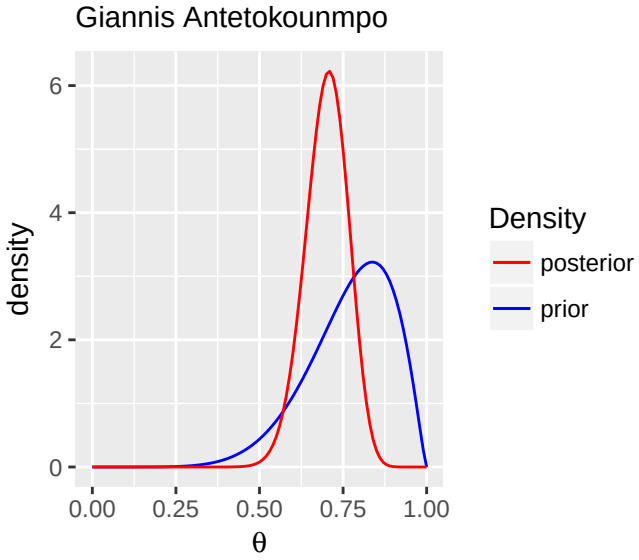
```r
# We also store the posterior parameters
# for calculating some probabilities
NBA_Data[, "posteriorA"] <- NA
NBA_Data[, "posteriorB"] <- NA


# Calculate posterior for each player
# give the prior described above.
for (i in 1:nrow(NBA_Data)) {
    # Calculate the prior parameters
    mean <- NBA_Data[i, ]$proportion
    kappa <- 10
    a = mean * kappa
    b = (1 - mean) * kappa

    if (a < 1 || b < 1)
        warning("Prior parameter warning")
    # Extract the N and Y from the data to
    # form the likelihood
    N <- NBA_Data[i, ]$ClutchAttempts
    Y <- NBA_Data[i, ]$ClutchMakes

    titleString <- NBA_Data[i, ]$PlayerName

    # Parameters for the posterior
    # distribution
    posteriorA <- Y + a
    posteriorB <- N - Y + b

    NBA_Data[i, ]$posteriorA <- posteriorA
    NBA_Data[i, ]$posteriorB <- posteriorB

    # Calculate posterior mean and variance
    posteriorMean <- posteriorA/(posteriorA +
        posteriorB)
    posteriorMode <- (posteriorA - 1)/(posteriorA +
        posteriorB - 2)
    posteriorVariance <- posteriorA * posteriorB/((posteriorA +
        posteriorB)^2 * (posteriorA + posteriorB +
        1))
    posteriorSD <- sqrt(posteriorVariance)

    # We use ggplot2's stat_function instead
    # of curve
    p1 <- ggplot(data.frame(x = c(0, 1)),
        aes(x)) + stat_function(fun = function(x) dbeta(x,
        shape1 = a, shape2 = b), aes(colour = "prior")) +
```

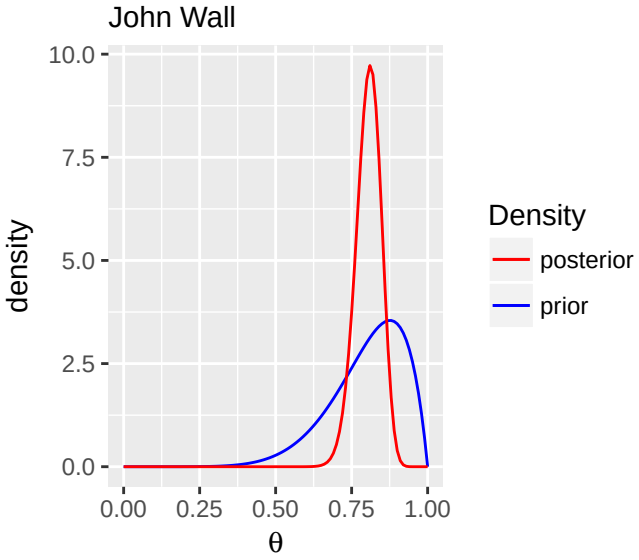
```
        ylab("density") + xlab(TeX("$\\theta$"))
    p2 <- p1 + stat_function(fun = function(x) dbeta(x,
        shape1 = posteriorA, shape2 = posteriorB),
        aes(colour = "posterior")) + ggtitle(titleString) +
        scale_colour_manual("Density", values = c("red",
            "blue")) + theme(plot.title = element_text(size = 11))
    print(p2)
    plots[[i]] <- p2
    p2 <- NULL

    NBA_Data[i, ]$posteriorMean <- posteriorMean
    NBA_Data[i, ]$posteriorMode <- posteriorMode

}
```

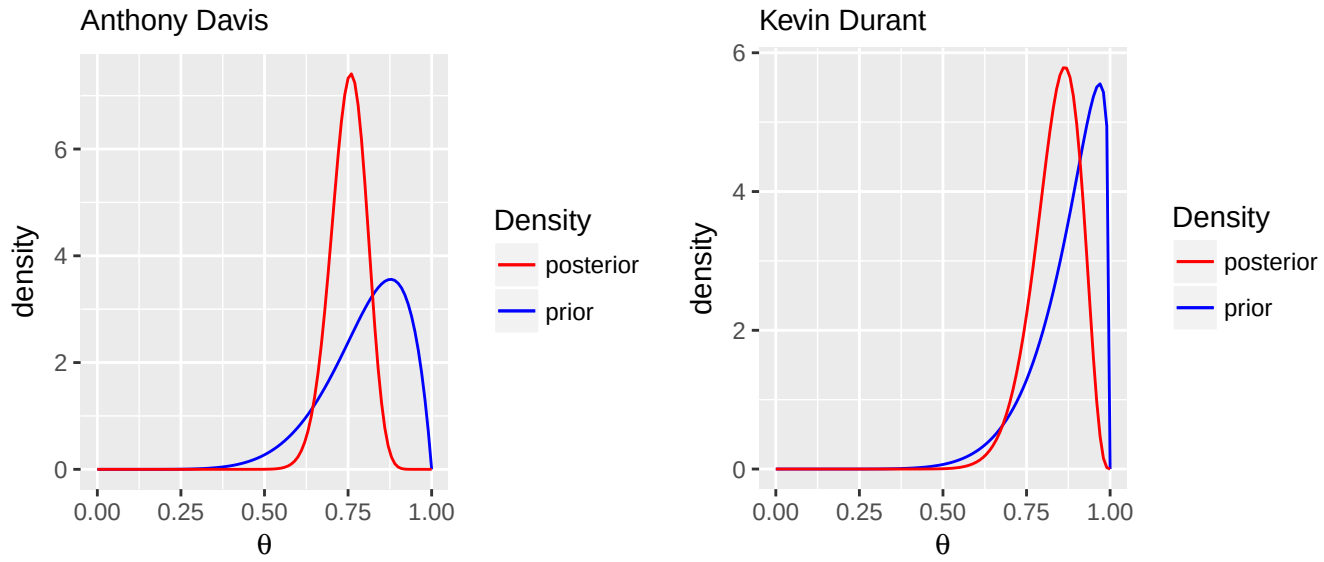



Isaiah Thomas
density
θ
Density
posterior
prior


Stephen Curry
density
θ
Density
posterior
prior


Giannis Antetokounmpo
density
θ
Density
posterior
prior


John Wall
density
θ
Density
posterior
prior




```
# MADDENING! I tried to plot them all on
# one ggplot page The titles differ but
# the plots are all the first plot.
# grid.arrange(grobs = plots, ncol = 2)
```

Summary of Posterior distributions

```
pander(data.frame(names = NBA_Data$PlayerName,
    posterior.mean = NBA_Data$posteriorMean,
    posterior.mode = NBA_Data$posteriorMode))
```

names	posterior.mean	posterior.mode
Russell Westbrook	0.8524	0.8608
James Harden	0.7664	0.7716
Kawhi Leonard	0.874	0.8845
LeBron James	0.6886	0.6966
Isaiah Thomas	0.9042	0.9131
Stephen Curry	0.9161	0.9406
Giannis Antetokounmpo	0.7	0.7082
John Wall	0.8045	0.8112
Anthony Davis	0.7503	0.7584
Kevin Durant	0.8365	0.8646

Testing the hypothesis that the cluster proportion is less than the overall proportion

Here we calculate $P(\theta|Y < p_i)$ and report the probability. We convert this to a hypothesis test at $alpha = 0.1$

```
NBA_Data[, "posteriorProbTest"] <- NA
NBA_Data[, "testResult"] <- NA
for (i in 1:nrow(NBA_Data)) {
    a <- NBA_Data[i, ]$posteriorA
    b <- NBA_Data[i, ]$posteriorB
    p <- NBA_Data[i, ]$proportion

    NBA_Data[i, ]$posteriorProbTest <- pbeta(p,
        a, b)

    NBA_Data[i, ]$testResult <- NBA_Data[i,
        ]$posteriorProbTest >= 0.1

}

pander(data.frame(name = NBA_Data$PlayerName,
    posterior.probability = NBA_Data$posteriorProbTest,
    test.result = NBA_Data$testResult))
```

name	posterior.probability	test.result
Russell Westbrook	0.3975	TRUE
James Harden	0.9827	TRUE
Kawhi Leonard	0.5272	TRUE
LeBron James	0.3989	TRUE
Isaiah Thomas	0.5255	TRUE
Stephen Curry	0.2972	TRUE
Giannis Antetokounmpo	0.8653	TRUE
John Wall	0.4457	TRUE
Anthony Davis	0.8301	TRUE
Kevin Durant	0.671	TRUE

Sensitivity Analysis

Here we calculate the sensitivity of the results to the choice of prior. We redo the hypothesis test using the completely uninformative prior $Beta(1, 1)$ first and extend to priors with smaller κ if necessary.

```
NBA_Data[, "uninformativePosteriorProbTest"] <- NA
NBA_Data[, "uninformativeTestResult"] <- NA

for (i in 1:nrow(NBA_Data)) {
    a = 1
    b = 1

    N <- NBA_Data[i, ]$ClutchAttempts
    Y <- NBA_Data[i, ]$ClutchMakes
```

```
    p <- NBA_Data[i, ]$proportion

    # Parameters for the posterior
    # distribution
    posteriorA <- Y + a
    posteriorB <- N - Y + b

    NBA_Data[i, ]$uninformativePosteriorProbTest <- pbeta(p,
        posteriorA, posteriorB)

    NBA_Data[i, ]$uninformativeTestResult <- NBA_Data[i,
        ]$uninformativePosteriorProbTest >=
        0.1

}

pander(data.frame(name = NBA_Data$PlayerName,
    posterior.probability = NBA_Data$uninformativePosteriorProbTest,
    test.result = NBA_Data$uninformativeTestResult))
```

name	posterior.probability	test.result
Russell Westbrook	0.4793	TRUE
James Harden	0.991	TRUE
Kawhi Leonard	0.6402	TRUE
LeBron James	0.4354	TRUE
Isaiah Thomas	0.6446	TRUE
Stephen Curry	0.4707	TRUE
Giannis Antetokounmpo	0.9167	TRUE
John Wall	0.5092	TRUE
Anthony Davis	0.8867	TRUE
Kevin Durant	0.8457	TRUE

```
# plot(NBA_Data$posteriorProbTest-NBA_Data$uninformativePosteriorProbTest)
```

We see that the test results are the same and looking at the probabilities they are not much different. We claim that this test is *not* sensitive to the choice of prior.