

Ray Tracing

Ray casting

From: Appel '68; Gold & Nagel '68

Problem

Polygons undersample the displayed image

Images don't have shadows, reflections, refractions

Basic idea

So sample once for every image pixel

General approach

For each screen pixel

Find the ray from the eye through the pixel

For each object in the scene

If the ray intersects the object, and is closest yet

Record intersection and object

Find color for closest intersection

This simple algorithm is a visible surface alg, called ray casting

Complexity

Worst case: object x pixel

A ray/object intersection for every object and pixel combo

Example intersections

Nice for mathematical surfaces, e.g. spheres

Take the parametric eq of line, plug into sphere equation

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

Simplify: $x^2 - 2ax + a^2 \dots$

Plug in: $(x_0 + t \, dx)^2 + \dots$

Result is quadratic in t, find roots

If no roots, no intersection

If one root, ray grazes sphere

If two, ray pierces sphere

smallest t closest

Find location by plugging in intersection value of t

Find normal by drawing line to center sphere from intersect

Harder case is polys

Use similar approach to solve for point on poly plane

Result is a constant ratio

If denominator is zero, ray/plane are parallel

Can use barycentric coordinates to test for containment in triangle

These coords are based on a set of points, rather than vectors

For triangles, we have:

Three triangle vertices A, B, C

Three weights a_1, a_2, a_3

Constrained to sum to one

Basic equation

$$P = a_1 \cdot A + a_2 \cdot B + a_3 \cdot C$$

With this system

Can address any pt on triangle's plane

If any a_i is > 1 or < 0 , pt outside triangle

If one a_i is 0, on triangle edge

If two a_i are 0, on vertex

Convert projected point to barycentric cords

Leverage constraint:

$$a_3 = 1 - a_1 - a_2$$

$$P = a_1 \cdot A + a_2 \cdot B + C \cdot (1 - a_1 - a_2)$$

$$P = (A - C) \cdot a_1 + (B - C) \cdot a_2 + C$$

Need two equations, for two unknowns

Choose two coords, e.g.

If 2 0's in poly normal, must choose those coords

$$P_x = (A_x - C_x) \cdot a_1 + (B_x - C_x) \cdot a_2 + C_x$$

$$P_y = (A_y - C_y) \cdot a_1 + (B_y - C_y) \cdot a_2 + C_y$$

Intersection if

a_1, a_2 and a_3 are > 0 and < 1

Optimizations

Speedups

- Can use bounding volumes

 - E.g. intersect with a box, if hit the box, intersect with the object

- Can use object hierarchies (in model space) to increase speed

 - E.g. Octrees -- each box contains eight smaller ones

 - Check smaller ones only if hit larger one

- Can use image space hierarchies (in screen) to increase speed

 - E.g. Beam tracing, make pixels bigger

 - E.g. Quadtrees

Quality improvements (Whitted '80)

- Adaptive supersampling

- Sample at corner of pixels

- If differences of four corners at a pixel are not too large

 - Use the average

- Otherwise, subsample with new smaller pixels and corners

Illumination extensions

Shadows

- For each light source

 - Fire an extra ray from intersection to light source

 - If intersection with object, in shadow, and ignore spec/diff terms

Reflections

- Fire a reflected ray along R

- Add a new term: ... + K_s I_r (once for r, g, b)

 - Where I_r is the intensity of the light found at reflected intersection

- Reflect until new light is minimal, or just X times

Refractions

- Fire an additional ray through the object, according to Snell's law

- Need info about the indices of refraction for the material

- Add a new term: ... + K_t I_t (for each r, g, b)

 - Where I_t is the intensity of the light found at the transmission intersection

- Snell's law:

 - Image

 - Normal N

 - Ray I strikes surface

 - Ray T is refracted vector

 - Theta_I, angle of incidence, between I and N

 - Theta_T, angle of refraction, between N and T

 - $\sin(\theta_I) / \sin(\theta_T) = \mu_T / \mu_I$

 - μ's are indices of refraction

 - Depends on material, wavelength, temperature

 - Let μ_R equal *inverse* of either side of term (μ's faster)

 - The vector of transmission T:

 - $\underline{T} = \sin(\theta_T) \underline{M} - \cos(\theta_T) \underline{N}$

 - where M is tangent to surface, projection of T on surface

 - With Snell's law plugged in, after various additional algebra/trig:

 - $\underline{T} = -\mu_R \underline{I} + \underline{N} (\mu_R (\underline{N} \cdot \underline{I}) - \sqrt{1 - \mu_R^2 (1 - (\underline{N} \cdot \underline{I})^2)})$

Total internal reflection

- When angle of refraction is larger than 90 degrees, light is in fact reflected

- E.g. you press your hand against glass, view from other side

 - If viewing angle is right, see only part of hand on glass

- This happens when sqrt above is imaginary

Frameless ray tracing

Frames

- Most moving imagery is framed

 - E.g. movies, TV

- A series of complete images, or frames

 - Each represents a complete spatial sample at one instant

 - Every pixel represents the same instant

Double buffering

- Problem:

 - In interactive rendering, when render to frame

 - Screen shows part of old, part of new image

Solution

- Use two frame buffers
- One for currently displayed image
- One for image being rendered
- Swap them when new frame is ready

Frameless

- Frameless ray tracing discards this notion
- Renders pixels as soon as calculated
- Stochastically chooses next pixel to render
- End effect is a fuzzy look -- objects moving fast are blurry

Critique:

- +: More continuous sampling in time
- +: Great reduction in delay
 - Double buffering makes you wait for:
 - Rendering time of displayed frame
 - Rendering time of next frame
- +: intuitive motion blur
- : things are blurry spatially
- : things are blurry temporally – lots of times on one screen

Critique

- +: shadows, spec reflection, refraction
- +: higher quality image
- : slower: intersections, depth
- : no diffuse interreflection
- : area, anisotropic light sources are hard
- : no soft shadows
- : no caustics, diffraction