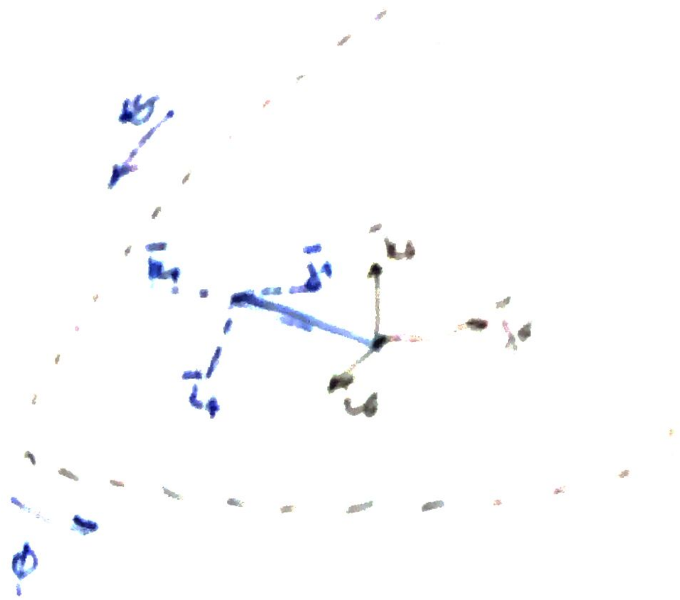
## three Dim Analysis ##



$\bar{i}_T$ points down $\bar{j}_T$ points to the right

Let, kite velocity in the $T$ frame be

$$\{\bar{v}_k\}_T = v_{kx}\bar{i}_T + v_{ky}\bar{j}_T$$

we know, from Loyd.

$$^T[C]^O = [R_y(\theta)][R_z(\phi)]$$

$$= \begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix}\begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$^O[C]^T = \left({}^T[C]^O\right)^{Tr}$$

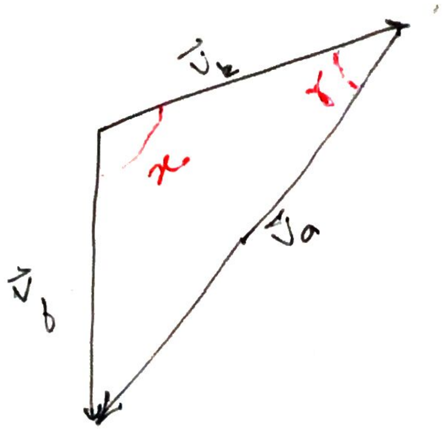
Rotate $\bar{v}_k$ in the $O$ frame.

$$\{\bar{v}_k\}_O = {}^O[C]^T\{\bar{v}_k\}_T$$
$$= (c\phi c\theta v_{kx} - s\phi v_{ky})\bar{i}_O$$
$$+ (c\theta s\phi v_{kx} + c\phi v_{ky})\bar{j}_O$$
$$- s\theta v_{kx}\bar{k}_O$$

Now, we know that flow is only along $\vec{i}_0$

$$\vec{v}_f = v_f \vec{i}_0$$

Vector diagram




from 2D analysis, we know that the angle between $\vec{v}_k$ and $\vec{v}_a$ is $\gamma = \operatorname{atan}(1/L/D)$

Angle between $\vec{v}_f$ and $\vec{v}_k$

$$x = \cos^{-1}\left(\frac{\vec{v}_f \cdot \vec{v}_k}{\|\vec{v}_f\| \, \|v_k\|}\right)$$

At this point, let's assume kite is only flying along $\vec{i}_T$ or $\vec{j}_T$ at a time.

If flying along $\vec{i}_T$, $v_{ky} = 0$

$$x_1 = \cos^{-1}\left(\frac{v_f \cdot v_{kx} \left(\cos(\theta)\cos(\phi)\right)}{v_f \cdot v_{kx}}\right)$$

$$x_1 = \cos^{-1}\left(\cos\theta \cos\phi\right)$$

If flying along $\vec{j}_T$, $v_{kx} = 0$

$$x_2 = \cos^{-1}\left(\frac{v_f \cdot v_{ky}\left(-\sin(\phi)\right)}{v_f \cdot v_{ky}}\right) = \cos^{-1}\left(\sin\phi\right)$$

Lastly, apply sine rule

$$\frac{\| v_b \|}{\sin \gamma} = \frac{\| v_k \|}{\sin(\pi - \gamma - x)}$$

$$\| v_k \| = \frac{\| v_b \| \sin(\pi - \gamma - x)}{\sin \gamma}.$$