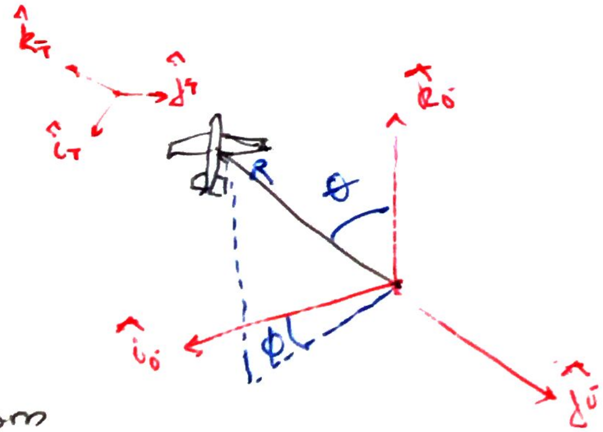
θ - Zenith angle

ϕ - Azimuth angle

R - Radius.



Rotation matrix to go from $\bar{O}$ to $\bar{T}$

$$^{\bar{T}}[C]^{\bar{O}} = [R_y(\theta)][R_z(\phi)]$$

$$= \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Position vector in $\bar{T}$ frame

$$\{\bar{r}_{cm}\}_{\bar{T}} = R\hat{k}_{\bar{T}}$$

Inertial velocity in $\bar{T}$ frame

$$\{^{O}\bar{v}_{cm}\}_{\bar{T}} = R\dot{\theta}\,\hat{i}_{\bar{T}} + \dot{\phi}R\sin(\theta)\,\hat{j}_{\bar{T}} + \dot{R}\,\hat{k}_{\bar{T}}$$

Note: Same as Loyd for $\dot{R} = 0$.

Flow is assumed to be along $\hat{i}_{\bar{O}}$

$$\{\bar{v}_f\}_{\bar{T}} = V_x\left(\cos\theta\cos\phi\,\hat{i}_{\bar{T}} - \sin(\phi)\,\hat{j}_{\bar{T}} + \cos(\phi)\sin(\theta)\,\hat{k}_{\bar{T}}\right)$$

Same as Loyd

Kite velocity through the flow

$$\{\bar{v}_A\}_{\bar{T}} = \{^{O}\bar{v}_{cm}\}_{\bar{T}} - \{\bar{v}_f\}_{\bar{T}}$$

Unit vector along $\{\bar{v}_A\}_{\bar{T}}$

$$\hat{u}_a = \frac{\{\bar{v}_A\}_{\bar{T}}}{\|\bar{v}_A\|}$$

Drag force

$$\bar{D}_R = -\frac{1}{2}\rho\left(C_L(C_L/C_D)\right)A\,\|\bar{V}_A\|^2\,\hat{u}_a$$

Vector normal to $\bar{V}_A$ and $\hat{k}_T$

$$\bar{N} = \hat{k}_T \times \bar{u}_a$$

Vector normal to $\vec{V}_A$ and $\bar{N}$ (also the longitudinal plane

$$\bar{M} = \bar{u}_a \times \bar{N}$$

Combined lift direction (as per Loyd)

$$\vec{u} = \bar{m} + \tan(\text{roll angle})\,\bar{N}$$

Lift vector

$$\bar{L} = \frac{1}{2}\rho C_L A\,\|V_A\|^2\,\frac{\bar{u}}{\|\bar{u}\|}$$

Net upward force.

$$\{\vec{F}_B\}_T = {}^{T}[C]^{O}\left(mg(BF-1)\vec{k}_O\right)$$

Inertial acceleration

$$\{\bar{a}_{cm}\}_T = \left[R\ddot{\theta} + 2\dot{R}\dot{\theta} - R\dot{\phi}^2\cos(\theta)\sin(\theta)\right]\vec{i}_T$$
$$+ \left[R\ddot{\phi}\sin(\theta) + 2\dot{\phi}\dot{\theta}R\cos(\theta) + 2\dot{\phi}\dot{R}\sin(\theta)\right]\vec{j}_T$$
$$+ \left(-\dot{\phi}^2 R\sin^2(\theta) - R\dot{\theta}^2\right)\vec{k}_T$$

$\boxed{\text{Also same as Loyd if } \dot{R} = 0}$

Total external force

$$\vec{F}_T = \bar{D}_R + \bar{L} + \vec{F}_B + T$$

Determine tether tension & angular accelerations

$$\vec{T} = -\vec{F}_T \cdot \hat{R}_{\vec{T}} - m(\dot{\phi}^2 R \sin^2\theta + R\dot{\theta}^2)$$

$$\ddot{\phi} = \frac{\vec{F}_T \cdot \vec{j}_T - 2mR\dot{\phi}\dot{\theta}\cos\theta - 2m\dot{\phi}\dot{R}\sin\theta}{2mR\sin\theta}$$

$$\ddot{\theta} = \frac{\vec{F}_T \cdot \vec{i}_T - 2m\dot{R}\dot{\theta} + mR\dot{\phi}^2\cos(\theta)\sin(\theta)}{2mR}$$