Euler Parameters (aka Euler-Rodrigues Parameters, often called quaternions because they can be viewed as the coefficients of Hamilton’s definition of a quaternion).

Euler’s rotation theorem states that the rotation of a body in three-dimensional space is fully defined by an axis and an angle of rotation about that axis. Consider two frames and We choose coincident origins to simplify mental visualizations, but in general the two frames may have undefined origins as Euler’s theorem involves only rotation and not translation.

Assume a unit vector is the instant axis of rotation and is the axis-angle. Then the Euler axis formula is:

Define the Euler parameters as:

Now we have:

The Euler parameters may be converted to/from Euler angles by comparing rotation matrices constructed with each. For example, consider the rotation matrix built using a 3-2-1 body-fixed rotation sequence about angles and , respectively.

Equating terms yields several options for computing the Euler angles. One option is:

If we differentiate the transpose we can employ to get:

One **important note** for implementation is that . You must renormalize the Euler parameters regularly. I typically renormalize every time step unless I really can’t afford the overhead or the frequencies are really high.

# Euler parameters from values

Therefore (see previous page):

Also:

Then:

Collect terms

Add to both sides

Rearrange

Substitute

Square root

Finally

Similarly

And the same approach leads to:

Likewise

And

Finally

And