

$$J_{j+1} = L_x x_{j+1} + L_u u_{j+1} + e_{j+1}^T Q_e e_{j+1} + \delta u_j^T Q_u \delta u_j + \delta x_j^T Q_x \delta x_j \quad (1)$$

$$= L_x (x_j + G_j u_{j+1} - G_j u_j) + L_u u_{j+1} + (r - x_{j+1})^T Q_e (r - x_{j+1}) + (u_{j+1} - u_j)^T Q_u (u_{j+1} - u_j) + (x_{j+1} - x_j)^T Q_x (x_{j+1} - x_j) \quad (2)$$

$$= L_x (x_j + G_j u_{j+1} - G_j u_j) + L_u u_{j+1} + (r - (x_j + G_j u_{j+1} - G_j u_j))^T Q_e (r - (x_j + G_j u_{j+1} - G_j u_j)) + (u_{j+1} - u_j)^T Q_u (u_{j+1} - u_j) + ((x_j + G_j u_{j+1} - G_j u_j) - x_j)^T Q_x ((x_j + G_j u_{j+1} - G_j u_j) - x_j) \quad (3)$$

$$= L_x (x_j + G_j u_{j+1} - G_j u_j) + L_u u_{j+1} + (r - x_j - G_j u_{j+1} + G_j u_j)^T Q_e (r - x_j - G_j u_{j+1} + G_j u_j) + (u_{j+1} - u_j)^T Q_u (u_{j+1} - u_j) + (u_{j+1} - u_j)^T G_j^T Q_x G_j (u_{j+1} - u_j) \quad (4)$$

$$= L_x (x_j + G_j u_{j+1} - G_j u_j) + L_u u_{j+1} + (r - x_j - G_j u_{j+1} + G_j u_j)^T Q_e (r - x_j - G_j u_{j+1} + G_j u_j) + (u_{j+1} - u_j)^T (Q_u + G_j^T Q_x G_j) (u_{j+1} - u_j) \quad (5)$$

$$(6)$$

$$\frac{dj_{j+1}}{du_{j+1}} = L_x G_j + L_u - 2(r - x_j - G_j u_{j+1} + G_j u_j)^T Q_e G_j + 2(u_{j+1} - u_j)^T (Q_u + G_j^T Q_x G_j) \quad (7)$$

$$= L_x G_j + L_u - 2r^T Q_e G_j + 2x_j^T Q_e G_j + 2(G_j u_{j+1})^T Q_e G_j - 2(G_j u_j)^T Q_e G_j + 2u_{j+1}^T \hat{Q}_u - 2u_j^T \hat{Q}_u \quad (8)$$

$$= L_x G_j + L_u - 2r^T Q_e G_j + 2x_j^T Q_e G_j + 2u_{j+1}^T G_j^T Q_e G_j - 2u_j^T G_j^T Q_e G_j + 2u_{j+1}^T \hat{Q}_u - 2u_j^T \hat{Q}_u \quad (9)$$

$$= L_x G_j + L_u - 2r^T Q_e G_j + 2x_j^T Q_e G_j + 2u_{j+1}^T G_j^T Q_e G_j + 2u_{j+1}^T \hat{Q}_u - 2u_j^T G_j^T Q_e G_j - 2u_j^T \hat{Q}_u \quad (10)$$

$$= L_x G_j + L_u - 2e_j^T Q_e G_j + 2u_{j+1}^T G_j^T Q_e G_j + 2u_{j+1}^T \hat{Q}_u - 2u_j^T G_j^T Q_e G_j - 2u_j^T \hat{Q}_u \quad (11)$$

$$= L_x G_j + L_u - 2e_j^T Q_e G_j + 2u_{j+1}^T (G_j^T Q_e G_j + \hat{Q}_u) - 2u_j^T (G_j^T Q_e G_j + \hat{Q}_u) \quad (12)$$

$$= L_x G_j + L_u - 2e_j^T Q_e G_j + 2u_{j+1}^T (G_j^T (Q_e + Q_x) G_j + Q_u) - 2u_j^T (G_j^T (Q_e + Q_x) G_j + Q_u) \quad (13)$$

$$(14)$$

$$\vec{0} = L_x G_j + L_u - 2e_j^T Q_e G_j + 2u_{j+1}^T (G_j^T (Q_e + Q_x) G_j + Q_u) - 2u_j^T (G_j^T (Q_e + Q_x) G_j + Q_u) \quad (15)$$

$$u_{j+1}^T (G_j^T (Q_e + Q_x) G_j + Q_u) = e_j^T Q_e G_j + u_j^T (G_j^T (Q_e + Q_x) G_j + Q_u) - \frac{1}{2} (L_x G_j + L_u) \quad (16)$$

$$(G_j^T (Q_e + Q_x) G_j + Q_u) u_{j+1} = G_j^T Q_e e_j + (G_j^T (Q_e + Q_x) G_j + Q_u) u_j - \frac{1}{2} (G_j^T L_x^T + L_u^T) \quad (17)$$

$$u_{j+1} = F_o G_j^T Q_e e_j + u_j - \frac{1}{2} F_o (G_j^T L_x^T + L_u^T) \quad (18)$$

$$u_{j+1} = u_j + F_e e_j + F_c \quad (19)$$

$$(20)$$

$$F_o \triangleq (G_j^T (Q_e + Q_x) G_j + Q_u)^{-1} \quad (21)$$

$$F_c \triangleq -\frac{1}{2} F_o (G_j^T L_x^T + L_u^T) \quad (22)$$

$$F_e \triangleq F_o G_j^T Q_e \quad (23)$$

$$\hat{Q}_u \triangleq Q_u + G_j^T Q_x G_j \quad (24)$$