$$J_{j+1} = L_x x_{j+1} + L_u u_{j+1} + e_{j+1}^T Q_e e_{j+1} + \delta u_j^T Q_u \delta u_j \tag{1}$$

$$= L_x \left(x_j + G_j u_{j+1} - G_j u_j \right) + L_u u_{j+1} + \left(r - x_{j+1} \right)^T Q_e \left(r - x_{j+1} \right) + \left(u_{j+1} - u_j \right)^T Q_u \left(u_{j+1} - u_j \right) \tag{2}$$

$$= L_x \left(x_j + G_j u_{j+1} - G_j u_j \right) + L_u u_{j+1} + \left(r - \left(x_j + G_j u_{j+1} - G_j u_j \right) \right)^T Q_e \left(r - \left(x_j + G_j u_{j+1} - G_j u_j \right) \right) + \left(u_{j+1} - u_j \right)^T Q_u \left(u_{j+1} - u_j \right) \tag{3}$$

$$= L_x \left(x_j + G_j u_{j+1} - G_j u_j \right) + L_u u_{j+1} + \left(r - x_j - G_j u_{j+1} + G_j u_j \right)^T Q_e \left(r - x_j - G_j u_{j+1} + G_j u_j \right) + \left(u_{j+1} - u_j \right)^T Q_u \left(u_{j+1} - u_j \right) \tag{4}$$

$$\tag{5}$$

$$\frac{dj_{j+1}}{du_{j+1}} = L_x G_j + L_u - 2 \left(r - x_j - G_j u_{j+1} + G_j u_j \right)^T Q_e G_j + 2 \left(u_{j+1} - u_j \right)^T Q_u \tag{6}$$

$$= L_x G_j + L_u - 2 r^T Q_e G_j + 2 x_j^T Q_e G_j + 2 (G_j u_{j+1})^T Q_e G_j - 2 (G_j u_j)^T Q_e G_j + 2 u_{j+1}^T Q_u - 2 u_j^T Q_u \tag{7}$$

$$= L_x G_j + L_u - 2 r^T Q_e G_j + 2 x_j^T Q_e G_j + 2 u_{j+1}^T G_j^T Q_e G_j - 2 u_j^T G_j^T Q_e G_j + 2 u_{j+1}^T Q_u - 2 u_j^T Q_u \tag{8}$$

$$= L_x G_j + L_u - 2 r^T Q_e G_j + 2 x_j^T Q_e G_j + 2 u_{j+1}^T G_j^T Q_e G_j + 2 u_{j+1}^T Q_u - 2 u_j^T G_j^T Q_e G_j - 2 u_j^T Q_u \tag{9}$$

$$= L_x G_j + L_u - 2 e_j^T Q_e G_j + 2 u_{j+1}^T G_j^T Q_e G_j + 2 u_{j+1}^T Q_u - 2 u_j^T G_j^T Q_e G_j - 2 u_j^T Q_u \tag{10}$$

$$= L_x G_j + L_u - 2 e_j^T Q_e G_j + 2 u_{j+1}^T \left(G_j^T Q_e G_j + Q_u \right) - 2 u_j^T \left(G_j^T Q_e G_j + Q_u \right) \tag{11}$$

$$\tag{12}$$

$$\vec{0} = L_x G_j + L_u - 2 e_j^T Q_e G_j + 2 u_{j+1}^T \left(G_j^T Q_e G_j + Q_u \right) - 2 u_j^T \left(G_j^T Q_e G_j + Q_u \right) \tag{13}$$

$$u_{j+1}^T \left(G_j^T Q_e G_j + Q_u \right) = e_j^T Q_e G_j + u_j^T \left(G_j^T Q_e G_j + Q_u \right) - \frac{1}{2} \left(L_x G_j + L_u \right) \tag{14}$$

$$\left(G_j^T Q_e G_j + Q_u \right) u_{j+1} = G_j^T Q_e e_j + \left(G_j^T Q_e G_j + Q_u \right) u_j - \frac{1}{2} \left(G_j^T L_x^T + L_u^T \right) \tag{15}$$

$$u_{j+1} = F_o G_j^T Q_e e_j + u_j - \frac{1}{2} F_o \left(G_j^T L_x^T + L_u^T \right) \tag{16}$$

$$u_{j+1} = u_j + F_e e_j + F_c \tag{17}$$

$$F_o \triangleq \left(G_j^T Q_e G_j + Q_u \right)^{-1} \tag{18}$$

$$F_c \triangleq -\frac{1}{2} F_o \left(G_j^T L_x^T + L_u^T \right) \tag{19}$$

$$F_e \triangleq F_o G_j^T Q_e \tag{20}$$