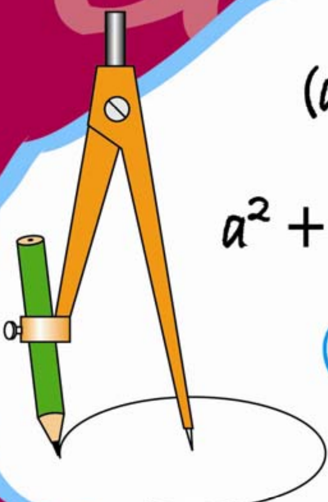
গণিত

সপ্তম শ্রেণি



$$(a+b)^2 = a^2 + 2ab + b^2$$

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

জাতীয় শিক্ষাক্রম ও পাঠ্যপুস্তক বোর্ড, ঢাকা

RvZxq wk¶vµg I cvV¨c¯—K †evW KZK 2013 wk¶vel †_‡K
mßg †kwYi cvV¨c¯—Ki‡c wbawiZ

MwYZ

mßg †kwY

iPbv
mv‡jn gwZb
W. Agj nvj`vi
W.Agj¨ P›` gÊj
†kL KZeDwÏb
nwg`v evb †eMg
G.†K.Gg knx`jvn
†gvt kvnRvnvb wmivR

m¤úv`bv
W. †gvt Ave`j gwZb
W. Avãm Qvgv`

RvZxq wk¶vµg I cvV¨c¯—K †evW, XvKv

RvZxq wk¶vµg I cvV¨c¯—K †evW

69-70 gwZwSj ewYwR¨K GjvKv, XvKv-1000
KZK cKwkZ|

[cKvkK KZK me¯^Z msiw¶Z]
cix¶vgjK ms¯‹iY

c_g cKvk : †m‡Þ¤^i, 2012

cvV¨c¯—K cYq‡b mgš^qK
†gvt bwmi DwÏb

Kw¤úDUvi K‡¤úvR
Kvjvi MwdK

c”Q`
m`kb evQvi
mRvDj Av‡e`xb

wPÎv¼b
†gvt Kwei †nv‡mb

wWRvBb
RvZxq wk¶vµg I cvV¨c¯—K †evW

miKvi KZK webvg‡j¨ weZi‡Yi Rb¨

---

g`‡Y :

cm½-K_v

wk¶v RvZxq Rxe‡bi m‡ev©ZgLx Dbœq‡bi cekZ| Avi `"Z cwieZ©bkxj we‡k¦i  P¨v‡jÄ †gvKv‡ejv K‡i evsjv‡`k‡K Dbœqb I mg„×i w`‡K wb‡q hvIqvi Rb¨ c‡qvRb mwk¶Z Rbkw³| fvlv Av‡›`vjb I gyw³hy‡×i †PZbvq †`k Movi Rb¨ wk¶v_©xi AšÍwbwnZ †gav I m¤¢vebvi cwicY weKv‡k mvnvh¨ Kiv gva¨wgK wk¶vi Ab¨Zg j¶¨| GQvov cv_wgK ¯—‡i AwR©Z wk¶vi †gšwjK Ávb I `¶Zv m¤cmwiZ I mmsnZ Kivi gva¨‡g D"PZi wk¶vi †hvM¨ K‡i †ZvjvI G ¯—‡ii wk¶vi D‡Ïk¨| Ávb∫R©‡bi GB cwµqvi wfZi w`‡q wk¶v_©x‡K †`‡ki A_©‰bwZK, mvgvwRK, mvs¯‹…wZK I cwi‡ekMZ cUf~wgi †cw¶‡Z `¶ I †hvM¨ bvMwiK K‡i †ZvjvI gva¨wgK wk¶vi Ab¨Zg we‡eP¨ welq|

RvZxq wk¶vbxwZ-2010 Gi j¶¨ I D‡Ïk¨‡K mvg‡b †i‡L cwigvwR©Z n‡q‡Q gva¨wgK ¯—‡ii wk¶vµg| cwigvwR©Z GB wk¶vµ‡g RvZxq Av`k, j¶¨, D‡Ïk¨ I mgKvjxb Pvwn`vi cwZdjb NUv‡bv n‡q‡Q, †mB mv‡_ wk¶v_©x‡`i eqm, †gav I MnY ¶gZv Abhvqx wkLbdj wbaviY Kiv n‡q‡Q| GQvov wk¶v_©xi ˆbwZK I gvbweK gj¨‡eva †_‡K ïi" K‡i BwZnvm I HwZn¨ †PZbv, gnvb gyw³hy‡×i †PZbv, wkí-mvwnZ¨-ms¯‹…wZ‡eva, †`k‡cg‡eva, cKw…Z-†PZbv Ges ag©-eY©-†MvÎ I bvix-cy"l wbwe©‡k‡l mevi cwZ mgghv©`v‡eva RvMÖZ Kivi †Póv Kiv n‡q‡Q| GKwU weÁvbgb¯‹ RvwZ MV‡bi Rb¨ Rxe‡bi cwZwU †¶‡Î weÁv‡bi ¯^ZtùZ© c‡qvM I wWwRUvj evsjv‡`‡ki icKí-2021 Gi j¶¨ ev¯—evq‡b wk¶v_©x‡`i m¶g K‡i †Zvjvi †Póv Kiv n‡q‡Q|

bZb GB wk¶vµ‡gi Av‡jv‡K cYxZ n‡q‡Q gva¨wgK ¯—‡ii cvq mKj cvV¨c¯—K| D³ cvV¨c¯—K cYq‡b wk¶v_©x‡`i mvg_©¨, ceYZv I ce AwfÁZv‡K ¸i‡Zi m‡½ we‡ePbv Kiv n‡q‡Q| cvV¨c¯—K¸‡jvi welq wbev©Pb I Dc¯vc‡bi †¶‡Î wk¶v_©xi mRbkxj cwZfvi weKvk mva‡bi w`‡K we‡kl f¦v‡e ¸iZ †`Iqv n‡q‡Q| cwZwU Aa¨v‡qi ïi‡Z wkLbdj h³ K‡i wk¶v_©xi AwR©Ze¨ Áv‡bi BwOZ c`vb Kiv n‡q‡Q Ges weweaPÎ KvR, mRbkxj ck I Ab¨vb¨ ck ms‡hvRb K‡i gj¨vqb‡K mRbkxj Kiv n‡q‡Q|

GKwesk kZ‡Ki GB h‡M Ávb-weÁv‡bi weKv‡k MwY‡Zi fwgKv AZxe ¸iZcY©| ïa ZvB bq, e¨w³MZ Rxeb †_‡K ïi K‡i cwievwiK I mvgvwRK Rxe‡bi MwY‡Zi c‡qvM A‡bK †e‡o‡Q| GB me welq we‡ePbvq †i‡L wbggva¨wgK chv‡q bZb MwYwZK welq wk¶v_x Dc‡hvMx I Avb›``vqK K‡i †Zvjvi Rb¨ MwYZ‡K mnR I m›`i f¦v‡e Dc¯vcb Kiv n‡q‡Q Ges †ek wKQ bZb MwYwZK wel‡q AšÍf³ Kiv n‡q‡Q|

GKwesk kZ‡Ki A½xKvi I cZ¨q‡K mvg‡b †i‡L cwigvwR©Z wk¶vµ‡gi Av‡jv‡K cvV¨c¯—KwU iwPZ n‡q‡Q| Kv‡RB cvV¨c¯—KwUi AviI mg„×mva‡bi Rb¨ †h‡Kv‡bv MVbgjK I hyw³m½Z civgk ¸i‡Zi m‡½ we‡ewPZ n‡e| cvV¨c¯—K cYq‡bi wecj Kgh‡Ái g‡a¨ AwZ ¯^í mg‡qi g‡a¨ c¯—KwU iwPZ n‡q‡Q| d‡j wKQ fjÎwU †_‡K †h‡Z cv‡i| cieZx ms¯‹iY¸‡jv‡Z cvV¨c¯—KwU‡K AviI m›`i, †kvfb I ÎwUg³ Kivi †Póv Ae¨vnZ _vK‡e| evbv‡bi †¶‡Î AbmZ n‡q‡Q evsjv GKv‡Wgx KZK cYxZ evbvbixwZ|

cvV¨c¯—KwU iPbv, m¤úv`bv, wPÎv¼b, bgbv ckœ` cYqb I cKvkbvi Kv‡R hviv AvšÍwiKf¦v‡e †gav I kg w`‡q‡Qb Zv‡`i ab¨ev` Ávcb KiwQ| cvV¨c¯—KwU wk¶v_©x‡`i Avbw›`Z cvV I cZ¨vkZ `¶Zv ARb wbwðZ Ki‡e e‡j Avkv Kwi|

c‡dmi †gvt †gv¯—dv KvgvjDwÏb
†Pqvig¨vb
RvZxq wk¶vµg I cvV¨c¯—K †evW©, XvKv

mwPcÎ

c_g Aa¨vq

gj` I Agj` msL¨v

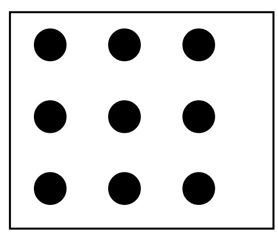
ˆewPÎ¨gq cKwZi GB ˆewPÎ¨ Avgiv MYbv I msL¨vi mvnv‡h¨ Dcjwä Kwi| ceeZx †kwY‡Z Avgiv ¯vfwweK msL¨v, cYmsL¨v I fMvsk m¤ú‡K aviYv †c‡qwQ hv gj` msL¨v wn‡m‡e cwiwPZ| G msL¨v¸‡jv‡K `BwU cYmsL¨vi Abcv‡Z cKvk Kiv hvq| msL¨vRM‡Z wKQ msL¨v i‡q‡Q †h¸‡jv `BwU cYmsL¨vi Abcv‡Z cKvk Kiv hvq bv| G¸‡jv Agj` msL¨v bv‡g cwiwPZ| G Aa¨v‡q Avgiv Agj` msL¨vi mv‡_ cwiwPZ n‡q G‡`i c‡qvM m¤ú‡K Av‡jvPbv Kie|

Aa¨vq †k‡l wk¶v_xiv–

- gj` I Agj` msL¨v kbv³ Ki‡Z cvi‡e|
- msL¨v‡iLvq gj` I Agj` msL¨vi Ae¯vb †`Lv‡Z cvi‡e|
- msL¨vi eM I eMgj e¨vL¨v Ki‡Z cvi‡e|
- Drcv`K I fvM cwµqvi gva¨‡g eMgj wbYq Ki‡Z cvi‡e|
- msL¨vi eMgj c×wZ¸‡jv c‡qvM K‡i ev¯e Rxe‡b mgm¨vi mgvavb Ki‡Z cvi‡e|

1·2 eM I eMgj

eM GKwU AvqZ, hvi evû¸‡jv ci¯úi mgvb| e‡Mi evûi ˆ`N¨ ÔKÕ GKK n‡j eM‡¶‡Îi †¶Îdj n‡e K × K eM GKK ev $K^2$ eM GKK| wecixZfv‡e, eM‡¶‡Îi †¶Îdj $K^2$ eM GKK n‡j, Gi cwZwU evûi ˆ`N¨ n‡e ÔKÕ GKK|



wP‡Î, 9wU gv‡ej‡K eMvKv‡i mvRv‡bv n‡q‡Q| mgvb `i‡Z cwZwU mwi‡Z 3wU K‡i 3wU mwi‡Z gv‡ej mvRv‡bv Av‡Q Ges †gvU gv‡e‡ji msL¨v $3 \times 3 = 3^2 = 9$| GLv‡b, c‡Z¨K mwi‡Z gv‡e‡ji msL¨v Ges mwii msL¨v mgvb| ZvB wPÎwU eMvKwZi n‡q‡Q| d‡j 3 Gi eM 9 Ges 9 Gi eMgj 3|

∴ †Kv‡bv msL¨v‡K †mB msL¨v Øviv ¸Y Ki‡j †h ¸Ydj cvIqv hvq Zv H msL¨vi eM Ges msL¨vwU ¸Yd‡ji eMgj|

wb‡Pi mviwYwU j¶ Kwi :

e‡Mi evûi ˆ`N© (wg.)	e‡Mi †¶Îdj (wg^2)
1	$1\times1=1=1^2$
2	$2\times2=4=2^2$
3	$3\times3=9=3^2$
5	$5\times5=25=5^2$
7	$7\times7=49=7^2$
a	$a\times a=a^2$

1, 4, 9, 25, 49 msL¨v¸‡jvi ˆewkó¨ n‡jv †h, G¸‡jv †Kv‡bv cYmsL¨v I Gi wb‡Ri ¸Ydj wn‡m‡e cKvk Kiv hvq| 1, 4, 9, 25, 49 G ai‡bi msL¨v eMmsL¨v|

mvaviYfv‡e GKwU ¯vfvweK msL¨v m, hw` Ab¨ GKwU ¯vfvweK msL¨v n Gi eM n^2 AvKv‡i cKvk Kiv hvq Z‡e m eMmsL¨v| G msL¨v¸‡jv‡K cYeMmsL¨v ejv nq|

cYeMmsL¨vi eMgj GKwU ¯vfvweK msL¨v|
†hgb : 21 Gi eM 21^2 ev 441 GKwU cYeMmsL¨v Ges 441 Gi eMgj 21 GKwU ¯vfvweK msL¨v|

eMmsL¨vi ag

wb‡Pi mviwY‡Z 1 †_‡K 20 msL¨vi eM †jLv n‡q‡Q|

msL¨v	eMmsL¨v	msL¨v	eMmsL¨v	msL¨v	eMmsL¨v	msL¨v	eMmsL¨v
1	1	11	121	6	36	16	256
2	4	12	144	7	49	17	289
3	9	13	169	8	64	18	324
4	16	14	196	9	81	19	361
5	25	15	225	10	100	20	400

mviwYf³ eMmsL¨v¸‡jvi GK‡Ki N‡ii A¼¸‡jv fv‡jvfv‡e ch‡e¶Y Kwi| j¶ Kwi †h, G msL¨v¸‡jvi GKK ¯vbxq A¼ 0, 1, 4, 5, 6 ev 9| †Kv‡bv eMmsL¨vi GKK ¯v‡b 2, 3, 7, ev 8 A¼wU †bB|

KvR :

1| GKwU msL¨vi GKK ¯vbxq 0, 1, 4, 5, 6, 9 n‡jB wK msL¨wU eMmsL¨v n‡e?

2| wb‡Pi msL¨v¸‡jvi †Kvb¸‡jv cYeM msL¨v? wbYq Ki|

2062, 1057, 23453, 33333, 1068

3| cvPwU msL¨v †jL hvi GKK ¯v‡bi A¼ †`‡LB Zv eMmsL¨v bq e‡j wm×vš— †bIqv hvq|

Gevi mvivY †_‡K GKK ¯’v‡b 1 i‡q‡Q Ggb eMmsL¨v wbB|

eMmsL¨v	msL¨v
1	1
81	9
121	11
361	19

GKK ¯’vbxq A¼ 1 ev 9 n‡j, Gi eMmsL¨vi GKK ¯’vbxq A¼ 1 n‡e

GKBfv‡e

eMmsL¨v	msL¨v
9	3
49	7
169	13

msL¨vi GKK ¯’vbxq A¼ 3 ev 7 n‡j Gi eMmsL¨vi GKK ¯’v‡b 9 n‡e

Ges

eMmsL¨v	msL¨v
16	4
36	6
196	14
256	16

GKK ¯’vbxq A¼ 4 ev 6 n‡j, Gi eMmsL¨vi GKK ¯’v‡b 6 _vK‡e

KvR :

1| mvivY †_‡K eMmsL¨vi GKK ¯’v‡b 4 i‡q‡Q Giƒc msL¨vi Rb¨ wbqg ˆZwi Ki|

2| wb‡Pi msL¨v¸‡jvi eMmsL¨vi GKK ¯’vbxq A¼wU KZ n‡e?

1273, 1426, 13645, 9876474, 99580

wb‡P eMgj mn K‡qKwU cY eMmsL¨vi ZvwjKv †`Iqv nj :

eMmsL¨v	eMgj	eMmsL¨v	eMgj	eMmsL¨v	eMgj
1	1	64	8	225	15
4	2	81	9	256	16
9	3	100	10	289	17
16	4	121	11	324	18
25	5	144	12	361	19
36	6	169	13	400	20
49	7	196	14	441	21

eMg‡ji wPý

eMgj cKv‡ki Rb¨ `BwU cZxKwPý e¨eüZ nq| 25 Gi eMgj †evSv‡Z †jLv nq $\sqrt{25}$ ev $(25)^{\frac{1}{2}}$ |
Avgiv Rwb, $5\times 5 = 25$, Kv‡RB 25 Gi eMgj 5|

KvR : K‡qKwU msL¨v wb‡q cY eMmsL¨vi ZwjKv ˆZwi Ki|

¸Ybxq‡Ki mvnv‡h¨ eMgj wbYq :

$$\begin{array}{r|l} 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline & 2 \end{array}$$

Avgiv Rwb, $16 = 4 \times 4 = 4^2$

$\therefore$ 16 Gi eMgj 4

$\therefore$ 16 †K †gŠwjK ¸Ybxq‡K we‡kølY K‡i cvB

$16 = 2 \times 2 \times 2 \times 2 = (2 \times 2) \times (2 \times 2)$

cwZ †Rvov †_‡K GKwU K‡i ¸YbxqK wb‡q cvB $2 \times 2 = 4$

$\therefore$ 16 Gi eMgj = $\sqrt{16} = 4$

Aevi, $36 = 6 \times 6 = 6^2$

$\therefore$ 36 Gi eMgj 6

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline & 3 \end{array}$$

$\therefore$ 36 †K †gŠwjK ¸Ybxq‡K we‡kølY K‡i cvB,

$36 = 2 \times 2 \times 3 \times 3 = (2 \times 2) \times (3 \times 3)$

cwZ †Rvov †_‡K GKwU K‡i ¸YbxqK wb‡q cvB $2 \times 3 = 6$

36 Gi eMgj = $\sqrt{36} = 6$

j¶ Kwi : ¸Ybxq‡Ki mvnv‡h¨ †Kv‡bv cY eMmsL¨vi eMgj wbYq Kivi mgq –

(1) c_‡g c`Ë msL¨wU‡K †gŠwjK ¸Ybxq‡K we‡kølY Ki‡Z n‡e|
(2) cwZ †Rvov GKB ¸YbxqK‡K GKmv‡_ cvkvcvwk wjL‡Z n‡e|
(3) cwZ †Rvov GK RvZxq ¸Ybxq‡Ki cwie‡Z GKwU ¸YbxqK wb‡q wjL‡Z n‡e|
(4) cvß ¸YbxqK¸‡jvi avivewnK ¸Ydj n‡e wb‡Yq eMgj|

D`vniY 1| 3136 Gi eMgj wbYq Ki|

mgvavb :

$$\begin{array}{r|l} 2 & 3136 \\ \hline 2 & 1568 \\ \hline 2 & 784 \\ \hline 2 & 392 \\ \hline 2 & 196 \\ \hline 2 & 98 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

GLv‡b, 3136 $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$

$= (2 \times 2) \times (2 \times 2) \times (2 \times 2) \times (7 \times 7)$

$\therefore$ 3136 Gi eMgj = $\sqrt{3136} = 2 \times 2 \times 2 \times 7 = 56$

KvR : ¸Ybxq‡Ki mvnv‡h¨ 1024 Ges 1849 Gi eMgj wbYq Ki|

1·3 fv‡Mi mvnv‡h¨ eMgj wbYq

GKwU D`vniY w`‡q fv‡Mi mvnv‡h¨ eMgj wbY‡qi c×wZ †`Lv‡bv n‡jv :

D`vniY 2| fv‡Mi mvnv‡h¨ 2304 Gi eMgj wbYq Ki :

mgvavb :

(1) 2304 msL¨wU wjwL : 23 04

(2) Wvbw`K †_‡K `BwU K‡i A¼ wb‡q †Rvov Kwi| c‡Z¨K †Rvovi Dci †iLwPý w`B : $\overline{23}\ \overline{04}$

(3) fv‡Mi mgq †hgb Lvov `vM †` Iqv nq, Wvbcv‡k Z`|c GKwU Lvov `vM w`B : $\overline{23}\ \overline{04}$ |

(4) c_g †RvowU 23| Gi ceeZx eMmsL¨wU 16, hvi eMgj $\sqrt{16}$ ev 4 ; Lvov `v‡Mi Wvbcv‡k 4 wjwL| GLb 23 Gi wVK wb‡P 16 wjwL :

$\overline{23}\ \overline{04}$ | 4
16

(5) GLb 23 †_‡K 16 we‡qvM Kwi :

$\overline{23}\ \overline{04}$ | 4
16
7

(6) we‡qvMdj 7 Gi Wv‡b cieZx †Rvov 04 emvB| 704 Gi evgw`‡K Lvov `vM (fv‡Mi wPý) w`B :

$\overline{23}\ \overline{04}$ | 4
16
| 7 04

(7) fvMd‡ji N‡ii msL¨v 4 Gi wØ¸Y 4 × 2 ev 8 wb‡Pi Lvov `v‡Mi evgcv‡k emvB| 8 Ges Lvov `v‡Mi g‡a¨ GKwU A¼ emv‡bvi g‡Zv ¯’vb ivwL :

$\overline{23}\ \overline{04}$ | 4
16
8 | 7 04

(8) GLb GKwU GK A‡¼i msL¨v L‡R †ei Kwi hv‡K 8 Gi
Wvbcv‡k emvm‡q cvß msL¨v‡K H msL¨wU Øviv ¸Y K‡i
704 Gi mgvb ev Aba¶ 704 cvIqv hvq|
G‡¶‡Î 8 n‡e| 8 msL¨wU fvMd‡j I
4 Gi Wvbcv‡k emvB|

```
      _____ ____
      23   04 | 48
      16      |
 88 | 7 04
    | 7 04
    |------
        0
```

(9) fvMd‡ji ¯’v‡b cvIqv †Mj 48| GwUB wb‡Yq eMgj|

$\therefore \sqrt{2304} = 48$

`óe¨ : fv‡Mi mvnv‡h¨ eMgj wbYq Kivi mgq msL¨vi Wvb w`K †_‡K †Rvo evav‡Z wM‡q †kl A‡¼i †Rvo bv _vK‡j G‡K †Rvov Qvov MY¨ Ki‡Z n‡e|

D`vniY 3| fv‡Mi mvnv‡h¨ 31684 Gi eMgj wbYq Ki|

mgvavb :

```
         _____ ____
      3  16   84 | 178
      1          |
 27 | 216
    | 189
     -----------
 348 | 2784
     | 2784
     |------
         0
```

$\therefore$ 31684 Gi eMgj = $\sqrt{31684} = 178$

wb‡Yq eMgj 178|

KvR : fv‡Mi mvnv‡h¨ 1444 Ges 10404 Gi eMgj wbYq Ki|

eMmsL¨v I eMgj m¤ú‡Ü D‡jøL¨ welq :

(1) †Kv‡bv msL¨vi cwZ †Rvov †gŠwjK Drcv`‡Ki Rb¨ H msL¨vi eMg‡j GKwU K‡i ¸YbxqK wb‡Z nq|

(2) †h msL¨vi me Wvbw`‡Ki A¼ A_vr GKK ¯’vbxq A¼ 2 ev 3 ev 7 ev 8 Zv cYeM bq|

(3) †h msL¨vi †k‡l we‡Rvo msL¨K kb¨ _v‡K, H msL¨v cYeM bq|

(4) GKK ¯’vbxq A¼ 1 ev 4 ev 5 ev 6 ev 9 n‡j, H msL¨v cYeM n‡Z cv‡i| †hgb : 81, 64, 25, 36, 49 BZ¨vw` eMmsL¨v|

(5) Aveui msL¨vi Wvbw`‡K †RvomsL¨K kb¨ _vK‡j H msL¨v cYeM n‡Z cv‡i| †hgb : 100, 4900 BZ¨vw` eMmsL¨v |

(6) †Kv‡bv msL¨vi GKK ¯’vbxq A¼ †_‡K ïi“ K‡i evgw`‡K GK A¼ cici hZwU †dvUv †`Iqv hvq, Gi eMg‡ji msL¨wU ZZ A¼wewkó|

†hgb, $\sqrt{81}$ = 9 (GK A¼wewkó, GLv‡b †dvUvi msL¨v 1 KviY, $8\overset{\circ}{1}$)

$\sqrt{100}$ = 10 (`B A¼wewkó, GLv‡b †dvUvi msL¨v 2 KviY, $\overset{\circ}{1}0\overset{0}{0}$)

$\sqrt{47089}$ = 217 (wZb A¼wewkó, GLv‡b †dvUvi msL¨v 3 KviY, $\overset{\circ}{4}7\overset{0}{0}8\overset{0}{9}$)

KvR :	1| 529, 3925, 5041 Ges 4489 msL¨v¸‡jvi eMgj msL¨vi GKK ¯’vbxq A¼ wbYq Ki|
	2| 3136, 1234321 Ges 52900 msL¨v¸‡jvi eMgj KZ A¼wewkó Zv wbYq Ki|

D`vniY 4| 8655 †_‡K †Kvb ¶z`Zg msL¨v we‡qvM Ki‡j we‡qvMdj GKwU cYmsL¨v n‡e?

mgvavb :

$$
\begin{array}{r|l}
 & \overline{86}\,\overline{55} \;\big|\; 93 \\
 & 81 \\
\hline
183 & 5\,55 \\
 & 5\,49 \\
\hline
 & 6
\end{array}
$$

GLv‡b, 8655 Gi eMgj fv‡Mi mvnv‡h¨ wbYq Ki‡Z wM‡q 6 Aewkó _v‡K|
mZivs c`Ë msL¨v †_‡K 6 ev` w`‡j cvß msL¨wU cY eMmsL¨v n‡e|
wb‡Yq ¶z`Zg msL¨v 6

D`vniY 5| 651201 Gi mv‡_ †Kvb ¶z`Zg msL¨v †hvM Ki‡j †hvMdj GKwU cY eMmsL¨v n‡e?

mgvavb :

$$
\begin{array}{r|l}
 & \overline{65}\,\overline{12}\,\overline{01} \;\big|\; 806 \\
 & 64 \\
\hline
1606 & 1\,12\,01 \\
 & 96\,36 \\
\hline
 & 15\,65
\end{array}
$$

†h‡nZz msL¨wUi eMgj wbYq Kivi mgq fvM‡kl 1565 Av‡Q| Kv‡RB c`Ë msL¨wU cY eMmsL¨v bq| 651201 Gi mv‡_ †Kv‡bv GKwU ¶z`Zg msL¨v †hvM Ki‡j †hvMdj cYeM n‡e Ges ZLb Gi eMgj n‡e
806 + 1 = 807
807 Gi eM = 807 × 807 = 651249
wb‡Yq ¶z`Zg msL¨wU = 651249 – 651201
= 48

Abkxjbx 1·1

1| ¸Ybxq‡Ki mvnv‡h¨ eMgj wbYq Ki :
(K) 169 (L) 529 (M) 1521 (N) 11025

2| fv‡Mi mvnv‡h¨ eMgj wbYq Ki :
(K) 225 (L) 961 (M) 3969 (N) 10404

3| wb‡Pi msL¨v¸‡jv‡K †Kvb ¶`Zg msL¨v Øviv ¸Y Ki‡j ¸Ydj cYeM msL¨v n‡e?
(K) 147 (L) 384 (M) 1470 (N) 23805

4| wb‡Pi msL¨v¸‡jv‡K †Kvb ¶`Zg msL¨v Øviv fvM Ki‡j fvMdj cYeM msL¨v n‡e?
(K) 972 (L) 4056 (M) 21952

5| 4639 †_‡K †Kvb ¶`Zg msL¨v we‡qvM Ki‡j we‡qvMdj GKwU cY eMmsL¨v n‡e?

6| 5605 Gi mv‡_ †Kvb ¶`Zg msL¨v †hvM Ki‡j †hvMdj GKwU cY eMmsL¨v n‡e?

1·4 `kwgK fMvs‡ki eMgj wbYq

cYmsL¨v ev ALÐ msL¨vi eMgj fv‡Mi mvnv‡h¨ †hfv‡e wbYq Kiv n‡q‡Q, `kwgK fMvs‡ki eMgjI †mB wbq‡gB wbYq Kiv nq| `kwgK fMvs‡ki `BwU Ask _v‡K| `kwgK we›`i evgw`‡Ki Ask‡K ALÐ ev cY Ask Ges `kwgK we›`i Wvbcv‡ki Ask‡K `kwgK Ask ejv nq|

eMgj Kivi wbqg

(1) ALÐ As‡k GKK †_‡K µgvš‡q evgw`‡K cwZ `B A‡¼i Dci `vM w`‡Z nq|
(2) `kwgK As‡k `kwgK we›`i Wvbcv‡ki A¼ †_‡K ïi“ K‡i Wvbw`‡K µgvš‡q †Rvovq †Rvovq `vM w`‡Z nq| Gi‡c hw` †`Lv hvq me‡k‡l gvÎ GKwU A¼ ewK Av‡Q, Z‡e Zvi c‡i GKwU kb¨ ewm‡q `B A‡¼i Dci `vM w`‡Z nq|
(3) mvaviY wbq‡g eMgj wbY‡qi cwµqvq ALÐ As‡ki KvR †kl K‡i `kwgK we›`i c‡ii c_g `BwU A¼ bvgv‡bvi Av‡MB eMg‡j `kwgK we›` w`‡Z nq|
(4) `kwgK we›`i GK †Rvov k‡b¨i Rb¨ eMg‡j `kwgK we›`i ci GKwU kb¨ w`‡Z nq|

D`vniY 1| 26.5225 Gi eMgj wbYq Ki|

mgvavb :

$$
\begin{array}{r|l}
 & \overline{26}\cdot\overline{52}\,\overline{25} \;\big|\; 5\cdot15 \\
 & 25 \\ \hline
101 & 1\,52 \\
 & 1\,01 \\ \hline
1025 & 51\,25 \\
 & 51\,25 \\ \hline
 & 0
\end{array}
$$

wb‡Yq eMgj = 5·15

D`vniY 2| 0·002916 Gi eMgj wbYq Ki|

mgvavb :

$$
\begin{array}{r|l}
 & \overline{0}\cdot\overline{00}\,\overline{29}\,\overline{16} \;\big|\; 0\cdot054 \\
 & 25 \\ \hline
104 & 416 \\
 & 416 \\ \hline
 & 0
\end{array}
$$

wb‡Yq eMgj = 0·054

Avmb gv‡b eMgj wbYq

D`vniY 3| 9·253 Gi eMgj wZb `kwgK ¯vb chš— wbYq Ki|

mgvavb :

$$
\begin{array}{r|l}
 & \overline{9}\cdot\overline{25}\,\overline{30}\,\overline{00}\,\overline{00} \;\big|\; 3\cdot0418 \\
 & 9 \\ \hline
604 & 25\,30 \\
 & 24\,16 \\ \hline
6081 & 1\,14\,00 \\
 & 60\,81 \\ \hline
60828 & 53\,19\,00 \\
 & 48\,66\,24 \\ \hline
 & 4\,52\,76
\end{array}
$$

wb‡Yq eMgj = 3·042 (cvq)

`óe¨ : Dc‡ii eMg‡j `kwg‡Ki ci PZ_© A¼wU 8 nIqvq ZZxq A¼wUi mv‡_ 1 †hvM K‡i wb‡Yq eMg‡ji (wZb `kwgK ¯vb chš—) Avmb gvb nj 3·042|

Avmb gvb †ei Kivi wbqg

(1) `B `kwgK ¯vb chš— eMgj wbYq Ki‡Z n‡j, wZb `kwgK ¯vb chš— eMgj wbYq Ki‡Z n‡e|

(2) wZb `kwgK ¯vb chš— eMgj wbYq Ki‡Z n‡j, msL¨vi `kwgK we›`yi ci Kgc‡¶ 6wU A¼ wb‡Z nq| `iKvi n‡j Wvbw`‡Ki †kl A‡¼i ci c‡qvRbg‡Zv kb¨ emv‡Z nq| G‡Z msL¨vi gv‡bi cwieZb nq bv|

(3) eMg‡j hZ `kwgK ¯vb chš— wbYq Ki‡Z n‡e Gi c‡ii A¼wU 0, 1, 2, 3 ev 4 n‡j c‡ei A‡¼i mv‡_ 1 †hvM n‡e bv|

(4) eMg‡j hZ `kwgK ¯'vb chš— wbYq Ki‡Z n‡e Gi c‡ii A¼wU 5, 6, 7, 8 ev 9 n‡j c‡ei A‡¼i mv‡_ 1 †hvM n‡e|

KvR : 1| 50·6944 Gi eMgj wbYq Ki|
2| 7·12 Gi eMgj `B `kwgK ¯'vb chš— wbYq Ki|

1·5 cYeM fMvsk

$\frac{50}{32}$ †K jwNô AvKv‡i wj‡L cvB $\frac{25}{16}$

GLv‡b, $\frac{25}{16}$ fMvs‡ki je 25 GKwU cY eMmsL¨v Ges ni 16 GKwU cY eMmsL¨v| mZivs $\frac{25}{16}$ GKwU cYeM fMvsk|

∴ †Kv‡bv fMvs‡ki je I ni cY eMmsL¨v ev fMvsk‡K jwNô AvKv‡i cwiYZ Ki‡j hw` Zvi je I ni cY eMmsL¨v nq, Z‡e H fMvsk‡K cYeM fMvsk ejv nq|

1·6 fMvs‡ki eMgj

fMvs‡ki j‡ei eMgj‡K n‡ii eMgj Øviv fvM Ki‡j fMvs‡ki eMgj cvIqv hvq| ni hw` cY eMmsL¨v bv nq, Z‡e ¸Yb Øviv G‡K cYeM K‡i wb‡Z nq|

D`vniY 4| $\frac{64}{81}$ Gi eMgj wbYq Ki|

mgvavb : fMvskwUi je 64 Gi eMgj = $\sqrt{64}$ = 8

Ges ni 81 Gi eMgj = $\sqrt{81}$ = 9

∴ $\frac{64}{81}$ Gi eMgj = $\sqrt{\frac{64}{81}} = \frac{8}{9}$

wb‡Yq eMgj = $\frac{8}{9}$

D`vniY 5| $52\frac{9}{16}$ Gi eMgj wbYq Ki|

mgvavb : $52\frac{9}{16}$ Gi eMgj = $\sqrt{52\frac{9}{16}} = \sqrt{\frac{841}{16}} = \frac{29}{4} = 7\frac{1}{4}$

∴ $52\frac{9}{16}$ Gi eMgj = $7\frac{1}{4}$

D`vniY 6| $2\frac{8}{15}$ Gi eMgj wZb `kwgK ¯vb chš wbYq Ki|

mgvavb : $2\frac{8}{15}$ Gi eMgj

$$=\sqrt{2\frac{8}{15}} = \sqrt{\frac{38}{15}} = \sqrt{\frac{38\times15}{15\times15}}$$

$$=\sqrt{\frac{570}{225}} = \frac{23\cdot8747}{15} = 1\cdot5916 \text{ (cvq)}$$

$\therefore$ wZb `kwgK ¯vb chš eMgj = 1·592 (cvq)

KvR : 1| $27\frac{46}{49}$ Gi eMgj wbYq Ki|
2| $1\frac{4}{5}$ Gi eMgj `B `kwgK ¯vb chš wbYq Ki|

1·7 gj` I Agj` msL¨v

1,2,3,4, ... BZ¨w` ¯vfweK msL¨v| msL¨v¸‡jv‡K fMvsk AvKv‡i wbgiƒ‡c †jLv hvq|

$1 = \frac{1}{1}$, $2 = \frac{2}{1}$, $3 = \frac{3\times2}{2} = \frac{6}{2}$, ... BZ¨w`|

Avevi, 0·1, 1·5, 2·03, ... BZ¨w` `kwgK msL¨v|

GLv‡b, $0\cdot1 = \frac{1}{10}$, $1\cdot5 = \frac{15}{10}$, $2\cdot03 = \frac{203}{100}$ hv msL¨v¸‡jvi fMvsk AvKvi|

Avevi, $0 = \frac{0}{1}$, GKwU fMvsk msL¨v|

Dc‡ii ewYZ msL¨v¸‡jv gj` msL¨v|

AZGe, kb¨, mKj ¯vfweK msL¨v I fMvsk msL¨v gj` msL¨v|

Agj` msL¨v : $\sqrt{2} = 1\cdot4142135$............ msL¨vi `kwg‡Ki c‡i A¼ msL¨v wbw`ó bq| d‡j fMvsk AvKv‡i †jLv hvq bv| Abiƒ‡c $\sqrt{3},\sqrt{5},\sqrt{6},$........... BZ¨w` msL¨v¸‡jv‡K fMvsk AvKv‡i cKvk Kiv hvq bv| G¸‡jv Agj` msL¨v|

j¶ Kwi : $\sqrt{2},\sqrt{3},\sqrt{5},\sqrt{6},$...........BZ¨w` Agj` msL¨v Ges 2,3,5,6, ... BZ¨w` cY eMmsL¨v bq| mZivs cY eMmsL¨v bq Giƒc msL¨vi eMgj Agj` msL¨v|

D`vniY 7| $0\cdot12, \sqrt{25}, \sqrt{72}, \sqrt{\frac{4}{9}}, \frac{\sqrt{49}}{7}$ msL¨v¸‡jv †_‡K Agj` msL¨v evQvB Ki|

mgvavb : GLv‡b, $0\cdot12=\frac{12}{100}=\frac{3}{25}$; hv GKwU fMvsk msL¨v

$\sqrt{25}=\sqrt{5^2}=5$, hv GKwU ¯vfweK msL¨v

$\sqrt{72}=\sqrt{2\times36}=\sqrt{2\times6^2}=6\sqrt{2}$; hv fMvsk AvKv‡i †jLv hvq bv|

Ges $\frac{\sqrt{49}}{7}=\frac{\sqrt{7^2}}{7}=\frac{7}{7}=1$; hv GKwU ¯vfweK msL¨v|

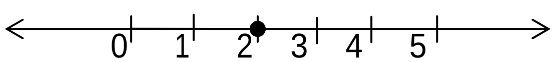
$\therefore$ $0\cdot12, \sqrt{25}, \frac{\sqrt{49}}{7}$ gj` msL¨v Ges $\sqrt{72}$ Agj` msL¨v|

KvR : $1\frac{1}{2}, \sqrt{\frac{4}{25}}, \sqrt{\frac{27}{16}}, 1\cdot0563, \sqrt{32}, \sqrt{121}$ msL¨v¸‡jv †_‡K gj` I Agj` msL¨v †ei Ki|

1·8 gj` I Agj` msL¨v‡K msL¨v‡iLvq cKvk

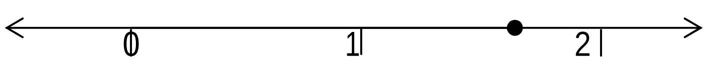
gj` msL¨vi msL¨v‡iLv

wb‡Pi msL¨v‡iLwU j¶ Ki :



Dc‡ii msL¨v‡iLwU‡Z Mvp wPýZ AskwU 2 wb‡`k K‡i|

Avevi,



Dc‡ii msL¨v‡iLwU‡Z Mvp wPýZ AskwUi Ae¯vb 1 I 2 gv‡S| Mvp wPýZ AskUK 4 fv‡Mi 3 Ask|

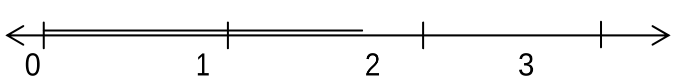
mZivs wPýZ AskwU $1+\frac{3}{4}$ ev $1\frac{3}{4}$ wb‡`k K‡i|

Agj` msL¨vi msL¨v‡iLv :

$\sqrt{3}$ GKwU Agj` msL¨v †hLv‡b, $\sqrt{3}=1\cdot732\text{.........}=1\cdot7$ (cvq)|

Gevi msL¨v‡iLvq 1 I 2 Gi gv‡Si Ask‡K mgvb 10 As‡k fvM K‡i mßg AskwU Mvp Kwi hv cvq 1.7 Z_v

$\sqrt{3}$ wb‡`k K‡i|



AZGe Mvp wPýZ AskwU $\sqrt{3}$ Gi msL¨v‡iLv|

KvR :

1| 3, $\frac{3}{2}$, 1.455 Ges $\sqrt{5}$ msL¨v¸‡jv msL¨v‡iLvq †`LvI|

D`vniY 8| †Kv‡bv evMv‡b 1296wU AvgMvQ Av‡Q| evMv‡bi ^`N¨ I c‡¯'i Dfq w`‡Ki c‡Z¨K mvwi‡Z mgvb msL¨K AvgMvQ _vK‡j c‡Z¨K mvwi‡Z Mv‡Qi msL¨v wbYq Ki|

mgvavb : evMv‡bi ^`N¨ I c‡¯'i Dfq w`‡Ki c‡Z¨K mvwi‡Z mgvb msL¨K AvgMvQ Av‡Q|

$\therefore$ c‡Z¨K mvwi‡Z AvgMv‡Qi msL¨v n‡e 1296 Gi eMgj|

GLb,

$$\begin{array}{r|l|l} & \overline{12}\,\overline{96} & 36 \\ & 9 & \\ \hline 66 & 3\,96 & \\ & 3\,96 & \\ \hline & 0 & \end{array}$$

wb‡Yq AvgMv‡Qi msL¨v 36 wU|

D`vniY 9| GKwU ¯'vDU `j‡K 9, 10, Ges 12 mvwi‡Z mvRv‡bv hvq| Avevi Zv‡`i eMvKv‡i I mvRv‡bv hvq| H ¯'vDU `‡j Kgc‡¶ KZRb ¯'vDU i‡q‡Q|

mgvavb : ¯'vDU `j‡K 9, 10 Ges 12 mvwi‡Z mvRv‡bv hvq| d‡j ¯'vDU Gi msL¨v 9, 10 Ges 12 Øviv wefvR¨| Gi/c ¶`Zg msL¨v n‡e 9, 10 Ges 12 Gi j.mv.¸.|

GLv‡b,

$$\begin{array}{r|l} 2 & 9,\ 10,\ 12 \\ \hline 3 & 9,\ 5,\ 6 \\ \hline & 3,\ 5,\ 2 \end{array}$$

$\therefore$ 9, 10 Ges 12 Gi j.mv.¸. $= 2 \times 2 \times 3 \times 3 \times 5 = (2 \times 2) \times (3 \times 3) \times 5$

cvß j.mv.¸. $(2 \times 2) \times (3 \times 3) \times 5$ †K eMvKv‡i mvRv‡bv hvq bv|

$(2 \times 2) \times (3 \times 3) \times 5$ †K eMmsL¨v Ki‡Z n‡j Kgc‡¶ 5 Øviv ¸Y Ki‡Z n‡e|

$\therefore$ 9, 10 Ges 12 mvwi‡Z Ges eMvKv‡i mvRv‡bvi Rb¨ ¯'vDU Gi msL¨v c‡qvRb

$(2 \times 2) \times (3 \times 3) \times (5 \times 5) = 900$

wb‡Yq ¯'vDU Gi msL¨v 900|

Abkxjbx 1·2

1| $\frac{289}{361}$ Gi eMgj KZ?

(K) $\frac{13}{19}$ (L) $\frac{17}{19}$ (M) $\frac{19}{13}$ (N) $\frac{19}{17}$

2| 1·1025 Gi eMgj KZ?

(K) 1·5 (L) 1·005 (M) 1·05 (N) 0·05

wb‡P Z_¨ †_‡K 3–5 bs c‡ki DËi `vI :

3| `BwU µwgK msL¨vi e‡Mi Aši 25|

(1) GKwU msL¨v 12 n‡j AciwU KZ?

(K) 5 (L) 9 (M) 11 (N) 13

(2) msL¨v `BwUi eM Kx Kx?

(K) 144, 169 (L) 121, 144 (M) 169, 196 (N) 196, 225

(3) `BwU msL¨vi g‡a¨ †KvbwUi eM †_‡K 25 we‡qvM Ki‡j we‡qvMdj GKwU cY eMmsL¨v n‡e?

(K) eowU (L) †QvUwU (M) DfqwU (N) GKwU I bv

4| wb‡Pi Z_¨¸‡jv j¶ Ki :

i. 0·0001 Gi eMgj 0·01

ii. $\frac{16}{225}$ GKwU cYeM fMvsk

iii. $\sqrt{3}$ Gi gvb cvq 2 Gi mgvb

Dc‡ii Z‡_¨i Av‡jv‡K wb‡Pi †KvbwU mwVK?

(K) i I ii (L) ii I iii (M) i I iii (N) i, ii I iii

5| GKRb KlK evMvb Kivi Rb¨ 595wU PvivMvQ wK‡b Av‡bb| c‡Z¨KwU PvivMv‡Qi gj¨ 12 UvKv|

(K) PvivMvQ¸‡jv wKb‡Z Zvi KZ LiP n‡q‡Q?

(L) evMv‡b c‡Z¨K mwi‡Z mgvb msL¨K MvQ jvMv‡bvi ci KqwU PvivMvQ Aewkó _vK‡e?

(M) Li‡Pi UvKvi msL¨v I PvivMv‡Qi msL¨vi we‡qvMd‡ji mv‡_ †Kvb ¶`Zg msL¨v †hvM Ki‡j †hvMdj GKwU cY eMmsL¨v n‡e?

6| eMgj wbYq Ki :

(K) 0·36 (L) 2·25 (M) 0·0049 (N) 641·1024

(O) 0·000576 (P) 144·841225

7| `B `kwgK ¯’vb chš— eMgj wbYq Ki :

(K) 7 (L) 23·24 (M) 0·036

8| wb‡Pi fMvsk¸‡jvi eMgj wbYq Ki :

(K) $\frac{1}{64}$ (L) $\frac{49}{121}$ (M) $11\frac{97}{144}$ (N) $32\frac{241}{324}$

9| wZb `kwgK ¯’vb chš— eMgj wbYq Ki|

(K) $\frac{6}{7}$ (L) $2\frac{5}{6}$ (M) $7\frac{9}{13}$

10| 56728 Rb ˆmb¨ †_‡K Kgc‡¶ KZRb ˆmb¨ mwi‡q ivL‡j ev Zv‡`i mv‡_ Kgc‡¶ Avi KZRb ˆmb¨ †hvM w`‡j ˆmb¨`j‡K eMvKv‡i mvRv‡bv hv‡e?

11| †Kv‡bv we`¨vj‡qi 2704 Rb wk¶v_x©‡K cvZ¨wnK mgv‡ek Kivi Rb¨ eMvKv‡i mvRv‡bv n‡jv| c‡Z¨K mwi‡Z wk¶v_x©i msL¨v wbYq Ki|

12| GKwU mgevq mwgwZi hZRb m`m¨ wQj c‡Z¨‡K ZZ 20 UvKv K‡i Pv`v †`qvq †gvU 20480 UvKv n‡jv | H mwgwZi m`m¨msL¨v wbYq Ki|

13| †Kv‡bv evMv‡b 1800 wU PvivMvQ eMvKv‡i jvMv‡Z wM‡q 36wU MvQ †ewk n‡jv| c‡Z¨K mwi‡Z PvivMv‡Qi msL¨v wbYq Ki|

14| †Kvb ¶`Zg cY© eMmsL¨v 9, 15 Ges 25 Øviv wefvR¨?

15| GKwU avb¨‡¶‡Zi avb KvU‡Z kwgK †bIqv n‡jv| c‡Z¨K kwg‡Ki ˆ`wbK gRywi Zv‡`i msL¨vi 10 ¸Y| ˆ`wbK †gvU gRywi 6250 UvKv n‡j kwg‡Ki msL¨v †ei Ki|

16| `BwU µwgK msL¨vi e‡Mi Aš—i 37 n‡j, msL¨v `BwU wbYq Ki|

17| Ggb `BwU ¶`Zg µwgK msL¨v wbYq Ki hv‡`i e‡Mi Aš—i GKwU cY© eMmsL¨v|

18| GKwU ˆmb¨`j‡K 5,6,9 mwi‡Z mvRv‡bv hvq, wKš‘ eMvKv‡i mvRv‡bv hvq bv|

(K) 6 Gi ¸YbxqK¸‡jv †ei Ki|

(L) ˆmb¨msL¨v‡K †Kvb ¶`Zg msL¨v Øviv ¸Y Ki‡j ˆmb¨msL¨v‡K eMvKv‡i mvRv‡bv hv‡e?

(M) H `‡j Kgc‡¶ KZRb ˆmb¨ †hvM w`‡j ˆmb¨`j‡K eMvKv‡i mvRv‡bv hv‡e?

wØZxq Aa¨vq

mgvbcvZ I jvf-¶wZ

Avgiv ˆ`bw›`b Rxe‡b A‡bK mgm¨vi m¤§Lxb nB Ges G mKj mgm¨v AbcvZ I mgvbcv‡Zi aviYv I e¨vL¨v e¨envi K‡i mn‡R mgvavb Ki‡Z cwi| ZvB AbcvZ I mgvbcvZ m¤‡Ü aviYv _vKv I c‡qv‡Mi `¶Zv ARb Kiv wk¶v_x©‡`i Rb¨ Avek¨Kxq| Abi~c fv‡e Avgv‡`i ˆ`bw›`b Rxe‡b A‡bKLwb RvqMv R‡o Av‡Q †jb‡`b, hvi mv‡_ RwoZ jvf-¶wZ| G †cwi¶‡Z jvf-¶wZ m¤‡Ü wk¶v_x©i cwi®‹vi Ávb _vKv Acwinvh| ZvB G Aa¨v‡q AbcvZ-mgvbcvZ I jvf-¶wZ welqK welqe¯‘ we¯—wiZ fv‡e Dc¯’vcb Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_x©iv –

- eûiwk I avivewnK AbcvZ e¨vL¨v Ki‡Z cvi‡e|
- mgvbcv‡Zi aviYv e¨vL¨v Ki‡Z cvi‡e|
- mgvbcvZ ms¯µvš— mgm¨v mgvavb Ki‡Z cvi‡e|
- HwKK I AbcvZ e¨envi K‡i ev¯—e Rxe‡b mgq I KvR, bj I †PŠev”Pv, mgq I `iZ Ges †bŠKv I †¯ªvZ welqK mgm¨v mgvavb Ki‡Z cvi‡e|
- jvf-¶wZ Kx Zv e¨vL¨v Ki‡Z cvi‡e|
- jvf-¶wZ ms¯µvš— mgm¨vi mgvavb Ki‡Z cvi‡e|
- Ki, f¨vU, Kwgkb I g`welbgq ms¯µvš— ˆ`bw›`b Rxe‡bi mgm¨v mgvavb Ki‡Z cvi‡e|

2·1 eûiwkK AbcvZ I avivewnK AbcvZ

eûiwkK AbcvZ : g‡b Kwi, GKwU ev‡·i ˆ`N¨, c¯' I D”PZv h_vµ‡g 8 †m.wg., 5 †m.wg. I 6 †m.wg.

ˆ`N¨, c¯' I D”PZvi AbcvZ = 8 : 5 : 6

ms‡¶‡c, ˆ`N¨ : c¯' : D”PZv = 8 : 5 : 6

GLv‡b wZbwU iwki AbcvZ Dc¯’vcb Kiv n‡q‡Q| Gi~c wZb ev Z‡ZwaK iwki AbcvZ‡K eûiwkK AbcvZ e‡j|

avivewnK AbcvZ : g‡b Kwi, cÎ I wcZvi eq‡mi AbcvZ = 15 : 41

Ges wcZv I `v`vi eq‡mi AbcvZ = 41 : 65

`BwU AbcvZ‡K GKÎ K‡i cvB, c‡Îi eqm : wcZvi eqm : `v`vi eqm = 15 : 41 : 65 | G ai‡bi AbcvZ‡K avivewnK AbcvZ e‡j| GLv‡b j¶Yxq †h, c_g Abcv‡Zi DËi iwk I wØZxq Abcv‡Zi ce iwk mgvb| c_g Abcv‡Zi DËi iwk I wØZxq Abcv‡Zi ce iwk mgvb bv n‡j Zv‡`i‡K mgvb K‡i avivewnK AbcvZ †ei Ki‡Z nq|

`BwU AbcvZ‡K avivewnK Abcv‡Z i~cvš‡ii Rb¨ c_g Abcv‡Zi DËi iwk Øviv wØZxq Abcv‡Zi Dfq iwk‡K ¸Y Ki‡Z n‡e Ges wØZxq Abcv‡Zi ce iwk Øviv c_g Abcv‡Zi Dfq iwk‡K ¸Y Ki‡Z n‡e|

D`vniY 1| 7 : 5 Ges 8 : 9 `BwU AbcvZ| G‡`i‡K avivewnK Abcv‡Z cKvk Ki |

mgvavb : 1g AbcvZ $= 7 : 5$

$$= \frac{7}{5}$$

$$= \frac{7 \times 8}{5 \times 8} = \frac{56}{40}$$

$$= 56 : 40$$

2q AbcvZ $= 8 : 9$

$$= \frac{8}{9}$$

$$= \frac{8 \times 5}{9 \times 5} = \frac{40}{45}$$

$$= 40 : 45$$

weKí mgvavb :

1g AbcvZ $= 7 : 5 = 7 \times 8 : 5 \times 8$
$= 56 : 40$

2q AbcvZ $= 8 : 9 = 8 \times 5 : 9 \times 5$
$= 40 : 45$

∴ AbcvZ `BwUi avivewnK AbcvZ 56 : 40 : 45

KvR :
wb‡Pi AbcvZ¸‡jv‡K avivewnK Abcv‡Z cKvk Ki :
1| 12 : 17 Ges 5 : 12
2| 23 : 11 Ges 7 : 13
3| 19 : 25 Ges 9 : 17

2·2 mgvbcvZ

g‡b Kwi, †mvnvM †Kv‡bv †`vKvb †_‡K 10 UvKv w`‡q GKwU wPc‡mi c¨v‡KU Ges 25 UvKv w`‡q 1 †KwR jeY wKb‡jv| GLv‡b jeY I wPcm Gi `v‡gi AbcvZ = 25 : 10 ev 5 : 2|
Aevi, †mvnvM‡`i †kwY‡Z wk¶v_©xi msL¨v 70| G‡`i g‡a¨ QvÎ 50 Rb Ges QvÎx 20 Rb| GLv‡b QvÎ I QvÎxmsL¨vi AbcvZ = 50 : 20 ev 5 : 2| Dfq‡¶‡Î AbcvZ `BwU mgvb |

AZGi , Avgiv ej‡Z cwi, 25 : 10 = 50 : 20| GB Abcv‡Z 4wU iwk Av‡Q|
Gi g‡a¨ 1g iwk 25, 2q iwk 10, 3q iwk 50 Ges 4_© iwk 20 wn‡m‡e we‡ePbv Ki‡j Avgiv wjL‡Z cwi, 1g iwk : 2q iwk = 3q iwk : 4_© iwk|

PviwU iwki 1g I 2q iwki AbcvZ Ges 3q I 4_© iwki AbcvZ ci¯úi mgvb n‡j, iwk PviwU GKwU mgvbcvZ ˆZwi K‡i| mgvbcv‡Zi c‡Z¨K iwk‡K mgvbcvZx e‡j|

mgvbcv‡Zi 1g I 2q iwk mgRvZxq Ges 3q I 4_ iwk mgRvZxq n‡Z cv‡i|
A_vr 4 wU iwk mgRvZxq nIqvi c‡qvRb †bB| c‡Z¨K Abcv‡Zi iwk `BwU mgRvZxq n‡jB mgvbcvZ ˆZwi nq|

mgvbcv‡Zi 1g I 4_ iwk‡K cvšÍxq iwk Ges 2q I 3q iwk‡K ga¨ iwk e‡j| mgvbcv‡Z Ô=Õ wP‡ýi cwie‡Z Ô::Õ wPý I e¨envi Kiv nq| AZGe Avgiv wjL‡Z cvwi, 25 : 10 :: 50 : 20 |
Aevi, 1g iwk : 2q iwk = 3q iwk : 4_ iwk

$$\text{ev, } \frac{\text{1g iwk}}{\text{2q iwk}} = \frac{\text{3q iwk}}{\text{4\_ iwk}} \quad \text{ev, 1g iwk} \times \text{4\_ iwk} = \text{2q iwk} \times \text{3q iwk}$$

j¶ Kwi, mgvbcv‡Z hw` 2q iwk I 3q iwk mgvb nq, Z‡e 1g iwk $\times$ 4_ iwk = (2q iwk)^2

- mgvbcv‡Zi 1g I 4_ iwk‡K cvšÍxq iwk e‡j |
- mgvbcv‡Zi 2q I 3q iwk‡K ga¨ iwk e‡j |

D`vniY 2| 3, 6,7 Gi 4_ mgvbcvZx wbYq Ki|

mgvavb : GLv‡b 1g iwk 3, 2q iwk 6, 3q iwk 7

Avgiv Rwb, 1g iwk $\times$ 4_ iwk = 2q iwk $\times$ 3q iwk

3 $\times$ 4_ iwk = 6 $\times$ 7

$$\text{ev, 4\_ iwk} = \frac{{}^{2}\cancel{6} \times 7}{\cancel{3}_1} \quad \text{ev, } 14$$

wb‡Yq 4_ mgvbcwZK 14

D`vniY 3| 8, 7 Ges 14 Gi 3q iwk wbYq Ki|

mgvavb : GLv‡b 1g iwk 8, 2q iwk 7 Ges 4_ iwk 14

Avgiv Rwb, 1g iwk $\times$ 4_ iwk = 2q iwk$\times$3q iwk

ev, 8 $\times$ 14 = 7 $\times$ 3q iwk

$$\therefore \text{ 3q iwk} = \frac{8 \times \cancel{14}^{2}}{\cancel{7}_1}$$

$$= 16$$

KvR :

wb‡Pi Lwj Ni ciY Ki

(K) $\boxed{}$: 9 :: 16 : 8

(L) 9 : 18 :: 25 : $\boxed{}$

µwgK mgvbcvZ

g‡b Kwi, 5 UvKv, 10 UvKv I 20 UvKv GB wZbwU iwk Øviv 5 : 10 Ges 10 : 20 GB `BwU AbcvZ †bIqv n‡jv| GLv‡b, 5 : 10 :: 10 : 20| G ai‡bi mgvbcvZ‡K µwgK mgvbcvZ e‡j| 5 UvKv, 10 UvKv I 20 UvKv‡K µwgK mgvbcvZx e‡j|

wZbwU iwki 1g I 2q iwki AbcvZ Ges 2q I 3q iwki AbcvZ ci¯úi mgvb n‡j, mgvbcvZwU‡K µwgK mgvbcvZ e‡j| iwk wZbwU‡K µwgK mgvbcvZx e‡j| K : L :: L : M mgvbcvZwUi wZbwU iwk K, L, M µwgK mgvbcvZx n‡j, $\frac{K}{L} = \frac{L}{M}$ ev $K \times M = (L)^2$ n‡e| A_vr, 1g I 3q iwki ¸Ydj wØZxq iwki e‡Mi mgvb|

j¶ Kwi :
- 2q iwk‡K 1g I 3q iwki ga¨ mgvbcvZx ev ga¨ iwk e‡j|
- µwgK mgvbcv‡Zi wZbwU iwkB mgRvZxq|

D`vniY 4| GKwU µwgK mgvbcv‡Zi 1g I 3q iwk h_vµ‡g 4 I 16 n‡j, ga¨ mgvbcvZx I µwgK mgvbcvZ wbYq Ki|

mgvavb : Avgiv Rvwb, 1g iwk × 3q iwk = (2q iwk)^2

GLv‡b, 1g iwk = 4 Ges 3q iwk = 16

∴ $4 \times 16 = (\text{ga¨ iwk})^2$

∴ $(\text{ga¨ iwk})^2 = 64$

∴ ga¨ iwk = $\sqrt{64} = 8$

wb‡Yq µwgK mgvbcvZ 4 : 8 :: 8 : 16 Ges wb‡Yq ga¨ mgvbcvZx 8

^ÎiwkK

Avgiv Rvwb, 1g iwk × 4_ iwk = 2q iwk × 3q iwk

g‡b Kwi, 1g, 2q I 3q iwk h_vµ‡g 9, 18, 20|

Z‡e, $9 \times$ 4_ iwk $= 18 \times 20$

∴ 4_ iwk = $\frac{\overset{2}{\cancel{18}} \times 20}{\underset{1}{\cancel{9}}} = 40$

∴ 4_ iwk = 40

Gfv‡e mgvbcv‡Zi wZbwU iwk Rvbv _vK‡j 4_ iwk wbYq Kiv hvq| GB 4_ iwk wbYq Kivi c×wZ‡K ^ÎiwkK e‡j|

D`vniY 5| 5wU LvZvi `vg 200 UvKv n‡j, 7wU LvZvi `vg KZ?

mgvavb : GLv‡b LvZvi msL¨v evo‡j `vgI evo‡e|

A_vr, LvZvi msL¨vi AbcvZ = LvZvi `v‡gi AbcvZ

5 : 7 = 200 UvKv : 7wU LvZvi `vg

ev, $\frac{5}{7} = \frac{200 \text{ UvKv}}{7\text{wU LvZvi `vg}}$

ev, 7wU LvZvi `vg = $\frac{7\times \cancel{200}^{40} \text{ UvKv}}{\cancel{5}_{1}}$ = 280 UvKv|

D`vniY 6| 12 Rb †jvK GKwU KvR 9 w`‡b Ki‡Z cv‡i| GKB nv‡i KvR Ki‡j 18 R‡b KvRwU KZ w`‡b Ki‡Z cvi‡e?

mgvavb : j¶ Kwi, †jvKmsL¨v evo‡j mgq Kg jvM‡e, Avevi †jvKmsL¨v Kg‡j mgq †ewk jvM‡e|

†jvKmsL¨vi mij AbcvZ mg‡qi e¨¯Í Abcv‡Zi mgvb n‡e|

12 : 18 = wb‡Yq mgq : 9 w`b

ev, $\frac{\cancel{12}^{2}}{\cancel{18}_{3}} = \frac{\text{wb‡Yq mgq}}{9 \text{ w`b}}$

ev, wb‡Yq mgq = $\frac{2\times\cancel{9}^{3}}{\cancel{3}_{1}}$ w`b = 6 w`b

mgvbcvwZK fvM

g‡b Kwi, 500 UvKv 3 : 2 Abcv‡Z eÈb Ki‡Z n‡e|

GLv‡b 3 : 2 Abcv‡Zi ceiwk I DËi iwki †hvMdj = 3+2 = 5

$\therefore$ 1g fvM = 500 UvKvi $\frac{3}{5}$ Ask = 300 UvKv

Ges 2q fvM = 500 UvKvi $\frac{2}{5}$ Ask = 200 UvKv|

AZGe, GKwU As‡ki cwigvY = c`Ë iwk $\times \frac{\text{H As‡ki AvbcwZK msL¨v}}{\text{Abcv‡Zi ce I DËi iwki †hvMdj}}$

Gfv‡e Dc‡ii c×wZ‡Z GKwU iwk‡K wewfbfv‡M wef³ Kiv hvq|

GKwU c`Ë iwk‡K GKwaK wbw`©ó msL¨vi Abcv‡Z wef³ Kiv‡K mgvbcvwZK fvM e‡j|

D`vniY 7| 20 wgUvi Kvco‡K wZb fvB‡evb AwgZ, mwgZ I ˆPwZi g‡a¨ 5 : 3 : 2 Abcv‡Z fvM Ki‡j c‡Z¨‡Ki Kvc‡oi cwigvY KZ ?

mgvavb : Kvc‡oi cwigvY = 20 wgUvi

c`Ë AbcvZ = 5 : 3 : 2

Abcv‡Zi msL¨v¸‡jvi †hvMdj = 5+3+2 = 10

$\therefore$ Awg‡Zi Ask = 20 wgUv‡ii $\frac{5}{10}$ Ask = 10 wgUvi

mwg‡Zi Ask = 20 wgUv‡ii $\frac{3}{10}$ Ask = 6 wgUvi

Ges ˆPwZi Ask = 20 wgUv‡ii $\frac{2}{10}$ Ask = 4 wgUvi

AwgZ, mwgZ I ˆPwZi Kvc‡oi cwigvY h_vµ‡g 10 wgUvi, 6 wgUvi I 4 wgUvi|

KvR :

1| K : L = 4 : 5, L : M = 7 : 9 n‡j, K : L : M wbYq Ki|

2| 4800 UvKv Av‡qkv, wd‡ivRv I Lw`Rvi g‡a¨ 4 : 3 : 1 Abcv‡Z fvM K‡i w`‡j †K KZ UvKv cv‡e ?

3| wZbRb Qv‡Îi g‡a¨ 570 UvKv Zv‡`i eq‡mi Abcv‡Z fvM K‡i †`Iqv n‡jv| Zv‡`i eqm h_vµ‡g 10, 13 I 15 eQi n‡j, †K KZ UvKv cv‡e?

D`vniY 8| cwbi I Zc‡bi Av‡qi AbcvZ 4 : 3| Zcb I iwe‡bi Av‡qi AbcvZ 5 : 4| cwb‡ii Avq 120 UvKv n‡j, iwe‡bi Avq KZ?

mgvavb : cwbi I Zc‡bi Av‡qi AbcvZ 4 : 3 $= \frac{4}{3} = \frac{4\times 5}{3\times 5} = \frac{20}{15} = 20 : 15$

Zcb I iwe‡bi Av‡qi AbcvZ $\frac{5}{4} = \frac{5\times 3}{4\times 3} = \frac{15}{12} = 15 : 12$

cwb‡ii Avq : Zc‡bi Avq : iwe‡bi Avq = 20 : 15 : 12

$\therefore$ cwb‡ii Avq : iwe‡bi Avq = 20 : 12

ev, $\frac{\text{cwb‡ii Avq}}{\text{iwe‡bi Avq}} = \frac{20}{12}$

ev, iwe‡bi Avq = $\frac{\text{cwb‡ii Avq} \times 12}{20}$ UvKv

$= \frac{\cancel{120}^{6}\times 12}{\cancel{20}_{1}}$ UvKv ev 72 UvKv|

$\therefore$ iwe‡bi Avq 72 UvKv

Abkxjbx 2·1

1| wb‡Pi iwk¸‡jv w`‡q mgvbcvZ †jL :

(K) 3 †KwR, 5 UvKv, 6 †KwR, 10 UvKv

(L) 9 eQi, 10 w`b, 18 eQi I 20 w`b

(M) 7 †m.wg., 15 †m‡KÛ, 28 †m.wg. I 1 wgwbU

(N) 12wU LvZv, 15wU †cwÝj, 20 UvKv I 25 UvKv

(O) 125 Rb QvÎ I 25 Rb wk¶K, 2500 UvKv I 500 UvKv

2| wb‡Pi µwgK mgvbcv‡Zi cvšxq iwk `BwU †`Iqv Av‡Q| mgvbcvZ ˆZwi Ki :

(K) 6, 24 (L) 25, 81 (M) 16, 49 (N) $\frac{5}{7}, 1\frac{2}{5}$ (O) 1·5, 13·5|

3| kb¨¯vb ciY Ki :

(K) 11 : 25 :: $\boxed{\quad}$: 50 (L) 7 : $\boxed{\quad}$ :: 8 : 64 (M) 2·5 : 5·0 :: 7 : $\boxed{\quad}$

(N) $\frac{1}{3} : \frac{1}{5} :: \boxed{\quad} : \frac{7}{10}$ (O) $\boxed{\quad}$: 12·5 :: 5 : 25

4| wb‡Pi iwk¸‡jvi 4_ mgvbcvZx wbYq Ki :

(K) 5, 7, 10 (L) 15, 25, 33 (M) 16, 24, 32

(N) 8, $8\frac{1}{2}$, 4 (O) 5, 4·5, 7

5| 15 †KwR Pv‡ji `vg 600 UvKv n‡j, Gi/c 25 †KwR Pv‡ji `vg KZ ?

6| GKwU Mv‡g›Um d¨v±wi‡Z ˆ`wbK 550 wU kvU ˆZwi nq | H d¨v±wi‡Z GKB nv‡i 1 mßv‡n KZwU kvU ˆZwi nq ?

7| Kwei mv‡n‡ei wZb c‡Îi eqm h_vµ‡g 5 eQi, 7 eQi I 9 eQi | wZwb 4200 UvKv wZb cÎ‡K Zv‡`i eqm Abcv‡Z fvM K‡i w`‡jb, †K KZ UvKv cv‡e ?

8| 2160 UvKv i/wg, †Rmwgb I KvKwj i g‡a¨ 1 : 2 : 3 Abcv‡Z fvM K‡i w`‡j †K KZ UvKv cv‡e?

9| wKQ UvKv jwee, mwg I wmqvg Gi g‡a¨ 5 : 4 : 2 Abcv‡Z fvM K‡i †`Iqv n‡jv| wmqvg 180 UvKv †c‡j jwee I mwg KZ UvKv cv‡e wbYq Ki |

10| meR, Wwjg I wjsKb wZb fvB| Zv‡`i wcZv 6300 UvKv Zv‡`i g‡a¨ fvM K‡i w`‡jb| G‡Z meR Wwj‡gi $\frac{3}{5}$ Ask Ges Wwjg wjsK‡bi wØ¸Y UvKv cvq| c‡Z¨‡Ki UvKvi cwigvY †ei Ki|

11| Zvgv, `¯‘v I i|cv wgwk‡q GK iK‡gi Mnbv ˆZwi Kiv n‡jv| H Mnbvq Zvgv I `¯‘vi AbycvZ 1 : 2 Ges `¯‘v I i|cvi AbycvZ 3 : 5 | 19 Mvg IR‡bi Mnbvq KZ Mvg i|cv Av‡Q?

12| `BwU mgvb gv‡ci Mvm kie‡Z cY Av‡Q| H kie‡Z cwb I wmiv‡ci AbycvZ h_vµ‡g c_g Mv‡m 3 : 2 I wØZxq Mv‡m 5 : 4 | H `BwU Mv‡mi kieZ GK‡Î wgkY Ki‡j cwb I wmiv‡ci AbycvZ wbYq Ki |

13| K : L = 4 : 7, L : M = 10 : 7 n‡j, K : L : M wbYq Ki|

14| 9600 UvKv mviv, gvBgbv I ivBmvi g‡a¨ 4 : 3 : 1 Abycv‡Z fvM K‡i w`‡j †K KZ UvKv cv‡e ?

15| wZbRb Qv‡Îi g‡a¨ 4200 UvKv Zv‡`i †kwY AbycvZ fvM K‡i †`Iqv n‡jv| Zviv hw` h_vµ‡g 6ô, 7g I 8g †kwYi wk¶v_x nq, Z‡e †K KZ UvKv cv‡e ?

16| †mvjvqgvb I mvjgv‡bi Av‡qi AbycvZ 5 : 7| mvjgvb I BDm‡di Av‡qi AbycvZ 4 : 5| †mvjvqgv‡bi Avq 120 UvKv n‡j BDm‡di Avq KZ?

2·3 jvf-¶wZ

GKRb †`vKvb`vi 1 WRb ej‡cb 60 UvKvq µq K‡i 72 UvKvq weµq Ki‡jb| GLv‡b †`vKvb`vi 12wU ej‡cb 60 UvKvq µq Ki‡jb| d‡j 1wU ej‡c‡bi µqgj¨ $\frac{60}{12}$ UvKv ev 5 UvKv| Avevi wZwb 12wU ej‡cb 72 UvKvq weµq Ki‡jb| d‡j 1wU ej‡c‡bi weµqgj¨ $\frac{72}{12}$ UvKv ev 6 UvKv|

1wU ej‡c‡bi µqgj¨ 5 UvKv I weµqgj¨ 6 UvKv|

†Kv‡bv wRwbm †h g‡j¨ µq Kiv nq, Zv‡K µqgj¨ Ges †h g‡j¨ weµq Kiv nq, Zv‡K weµqgj¨ e‡j|

µqg‡j¨i †P‡q weµqgj¨ †ewk n‡j, jvf nq|

jvf = weµqgj¨ – µqgj¨ = 6 UvKv – 5 UvKv ev 1 UvKv|

GLv‡b †`vKvb`vi cwZwU ej‡c‡b 1 UvKv K‡i jvf Ki‡jb|

Avevi g‡b Kwi, GKRb Kjwe‡µZv 1 nwj Kjv 20 UvKvq µq K‡i 18 UvKvq weµq Ki‡jb| µqg‡j¨i †P‡q weµqgj¨ Kg n‡j, ¶wZ ev †jvKmvb nq|

¶wZ = µqgj¨ – weµqgj¨ = (20–18) UvKv

= 2 UvKv

GLv‡b Kjwe‡µZv cwZ nwj‡Z 2 UvKv K‡i ¶wZ Ki‡jb|

g‡b Kwi, GKRb Kvco e¨emvqx gv‡K‡Ui GKwU †`vKvb fvov wb‡q 5 Rb KgPvix wb‡qvM w`‡jb| wZwb †`vKv‡bi fvov, KgPvix‡`i †eZb, †`vKv‡bi we`¨r wej I Ab¨vb¨ AvbIw½K LiP enb K‡ib| G mKj LiP Zvi Kvc‡oi µqg‡j¨i mv‡_ †hvM Kiv nq| GB †hvMdj‡KB wewb‡qvM e‡j| hw` H Kvco e¨emvqx gv‡m 2,00,000 UvKv wewb‡qvM K‡i gv‡m 2,50,000 UvKvi Kvco weµq K‡ib, Z‡e Zvi (2,50,000 – 2,00,000) UvKv ev 50,000 UvKv jvf n‡e| Avevi hw` gvm‡k‡l 1,80,000 UvKvi Kvco weµq K‡i _v‡Kb Zvn‡j Zvi (2,00,000 – 1,80,000) UvKv ev 20,000 UvKv ¶wZ ev †jvKmvb n‡e|

j¶ Kwi :

- jvf = weµqgj¨ – µqgj¨
  ev, weµqgj¨ = µqgj¨ + jvf
  ev, µqgj¨ = weµqgj¨ – jvf
- ¶wZ = µqgj¨ – weµqgj¨
  ev, µqgj¨ = weµqgj¨ + ¶wZ
  ev, weµqgj¨ = µqgj¨ – ¶wZ

jvf ev ¶wZ‡K Avgiv kZKivq cKvk Ki‡Z cwi| †hgb, Dc‡ii Av‡jvPbvq 5 UvKvq ej‡cb wK‡b 6 UvKvq weµq Kivq 1 UvKv jvf nq|

A_vr, 5 UvKvq jvf nq 1 UvKv

$\therefore$ 1 Ó Ó Ó $\frac{1}{5}$ Ó

$\therefore$ 100 Ó Ó Ó $\frac{1\times\cancel{100}^{20}}{\cancel{5}_{1}}$ Ó = 20 UvKv

$\therefore$ wb‡Yq jvf 20%|

Abiƒcfv‡e, Kjwe‡µZv 20 UvKvi Kjv wK‡b 18 UvKvq weµq Kivq 2 UvKv ¶wZ n‡q‡Q|

A_vr, 20 UvKvq ¶wZ nq 2 UvKv

$\therefore$ 1 Ó Ó Ó $\frac{2}{20}$ Ó

$\therefore$ 100 Ó Ó Ó $\frac{2\times\cancel{100}^{5}}{\cancel{20}_{1}}$ Ó ev 10 UvKv

$\therefore$ wb‡Yq ¶wZ 10%

D`vniY 9| GKRb Kgjwe‡µZv cwZkZ Kgjv 1000 UvKvq wK‡b 1200 UvKvq weµq Ki‡jb| Zvi KZ jvf n‡jv?

mgvavb : 100wU Kgjvi µqgj¨ 1000 UvKv

100wU Ó weµqgj¨ 1200 Ó

GLv‡b µqg‡j¨i †P‡q weµqgj¨ †ewk nIqvq jvf n‡q‡Q|

A_vr, jvf = weµqgj¨ – µqgj¨

= 1200 UvKv – 1000 UvKv

= 200 UvKv

wb‡Yq jvf 200 UvKv|

D`vniY 10| GKRb †`vKvb`vi 50 †KwRi 1 e¯—v Pvj 1600 UvKvq wKb‡jb| Pv‡ji `vg K‡g hvIqvq 1500 UvKvq weµq K‡ib, Zvi KZ ¶wZ n‡jv?

mgvavb : GLv‡b, 1 e¯—v Pv‡ji µqgj¨ 1600 UvKv

Ges 1 Ó Ó weµqgj¨ 1500 Ó

∴ µqg‡j¨i †P‡q weµqgj¨ Kg nIqvq ¶wZ n‡q‡Q|

∴ ¶wZ = µqgj¨ – weµqgj¨

= 1600 UvKv – 1500 UvKv = 100 UvKv

wb‡Yq ¶wZ 100 UvKv|

D`vniY 11| 75 UvKvq 15wU ej‡cb wK‡b 90 UvKvq weµq Ki‡j kZKiv KZ jvf n‡e?

mgvavb : GLv‡b, 15wU ej‡c‡bi µqgj¨ 75 UvKv

Ges 15wU Ó weµqgj¨ 90 UvKv

µqg‡j¨i †P‡q weµqgj¨ †ewk nIqvq jvf n‡q‡Q|

∴ jvf = weµqgj¨ – µqgj¨

= 90 UvKv – 75 UvKv = 15 UvKv

∴ 75 UvKvq jvf nq 15 UvKv

1 Ó Ó Ó $\frac{15}{75}$ Ó

∴ 100 Ó Ó Ó $\frac{{}^{1}\cancel{15}\times\cancel{100}^{20}}{\cancel{75}\,\cancel{5}_{1}}$ Ó ev 20 UvKv

AZGe jvf 20%|

D`vniY 12| GKRb gvQwe‡µZv cwZ nwj Bwjk gvQ 1600 UvKvq wK‡b cwZwU gvQ 350 UvKv K‡i weµq Ki‡jb| Zvi kZKiv KZ jvf ev ¶wZ n‡jv ?

mgvavb : cwZ nwj ev 4wU Bwj‡ki `vg = 1600 UvKv

∴ 1wU Ó Ó = $\frac{\cancel{1600}^{400}}{\cancel{4}_{1}}$ UvKv = 400 UvKv

Aevi, 1wU Bwj‡ki weµqgj¨ 350 UvKv

GLv‡b, µqgj¨i †P‡q weµqgj¨ Kg nIqvq ¶wZ n‡q‡Q|

∴ ¶wZ = µqgj¨ – weµqgj¨

= 400 UvKv – 350 UvKv = 50 UvKv

∴ 400 UvKvq ¶wZ nq 50 UvKv

1 Ó Ó Ó $\frac{50}{400}$ Ó

∴ 100 Ó Ó Ó $\frac{\cancel{50}^{25} \times \cancel{100}^{1}}{\cancel{400}_{\cancel{4}_{2}}}$ Ó ev $\frac{25}{2}$ UvKv ev $12\frac{1}{2}$ UvKv

∴ ¶wZ $12\frac{1}{2}$%

D`vniY 13| GKev· Av½i 2750 UvKvq weµq Kivq 450 UvKv ¶wZ n‡jv| H Av½i 3600 UvKvq weµq Ki‡j KZ jvf ev ¶wZ n‡Zv?

mgvavb : Av½‡ii weµqgj¨ = 2750 UvKv

¶wZ = 450 UvKv

µqgj¨ = 3200 UvKv

Aevi, weµqgj¨ = 3600 UvKv

µqgj¨ = 3200 UvKv

jvf = 400 UvKv

∴ jvf 400 UvKv|

D`vniY 14| GKRb Pv e¨emvqx GKev· Pv cvZv †KwR cwZ 80 UvKv wnmv‡e µq K‡ib| me Pv cvZv †KwR cwZ 75 UvKv `‡i weµq Kivq 500 UvKv ¶wZ nq| wZwb KZ †KwR Pv cvZv µq K‡iwQ‡jb?

mgvavb : †KwR cwZ Pv cvZvi µqgj¨ 80 UvKv
Ó Ó Ó Ó weµqgj¨ 75 UvKv
∴ 1 †KwR Pv cvZv weµq Ki‡j ¶wZ nq 5 UvKv

∴ 5 UvKv ¶wZ nq 1 †KwR‡Z
1 Ó Ó Ó $\frac{1}{5}$ Ó
500 Ó Ó Ó $\frac{1\times 50\cancel{0}^{100}}{\cancel{5}_{1}}$ Ó
= 100 †KwR‡Z
∴ Pv cvZv µq K‡iwQ‡jb 100 †KwR|

D`vniY 15| GKRb wWgwe‡µZv cwZ WRb wWg 101 UvKv `‡i 5 WRb Ges 90 UvKv `‡i 6 WRb wWg wK‡b KZ `‡i weµq Ki‡j Zvi WRb cwZ 3 UvKv jvf n‡e ?

mgvavb : 1 WRb wW‡gi µqgj¨ 101 UvKv
∴ 5 Ó Ó Ó 101× 5 UvKv ev 505 UvKv
Avevi, 1 WRb wW‡gi µqgj¨ 90 UvKv
∴ 6 Ó Ó Ó 90× 6 UvKv ev 540 UvKv
∴ (5+6) WRb ev 11 WRb wW‡gi µqgj¨ (505 + 540) UvKv ev 1045 UvKv
∴ 1 Ó Ó Ó $\frac{1045}{11}$ UvKv ev 95 UvKv
M‡o 1 WRb wW‡gi µqgj¨ 95 UKv
WRb cwZ 3 UvKv jv‡f 1 WRb wW‡gi weµqgj¨ (95 + 3) UvKv ev 98 UvKv
∴ cwZ WRb wW‡gi weµqgj¨ 98 UvKv n‡j WRb cwZ 3 UvKv jvf n‡e|

D`vniY 16| GKwU QvMj 10% ¶wZ‡Z weµq Kiv n‡jv| weµqgj¨ 450 UvKv †ewk n‡j 5% jvf n‡Zv| QvMjwUi µqgj¨ KZ?
mgvavb : g‡b Kwi, QvMjwUi µqgj¨ 100 UvKv
10% ¶wZ‡Z weµqgj¨ (100 – 10) UvKv ev, 90 UvKv
5% jv‡f weµqgj¨ (100 + 5) UvKv = 105 UvKv

5% jv‡f wеµqgj¨ – 10% ¶wZ‡Z wеµqgj¨

= (105 – 90) UvKv ev, 15 UvKv

∴ wеµqgj¨ 15 UvKv †ewk n‡j µqgj¨ 100 UvKv

1 Ó Ó Ó Ó Ó Ó $\frac{100}{15}$ Ó

∴ 450 Ó Ó Ó Ó Ó Ó $\frac{100\times \cancel{450}^{30}}{\cancel{15}_{1}}$ Ó

= 3000 UvKv

QvMjwUi µqgj¨ 3000 UvKv

D`vniY 17| bwej wgwói †`vKvb †_‡K 250 UvKv `‡i 2 †KwR m‡›`k µq Ki‡jv| f¨v‡Ui nvi 4 UvKv n‡j, m‡›`k µq eve` †m †`vKwb‡K KZ UvKv †`‡e?

mgvavb : 1 †KwR m‡›`‡ki `vg 250 UvKv

∴ 2 Ó Ó Ó (250 × 2) UvKv

= 500 UvKv

100 UvKvq f¨vU 4 UvKv

∴ 1 Ó Ó $\frac{4}{100}$ Ó

∴ 500 Ó Ó $\frac{4\times \cancel{500}^{5}}{\cancel{100}_{1}}$ Ó = 20 UvKv

∴ bwej m‡›`k µq eve` †`vKwb‡K †`‡e (500 + 20) UvKv ev 520 UvKv|

j¶Yxq : †Kv‡bv `‡e¨i µqg‡j¨i mv‡_ wbw`©ó nv‡i c`vbKZ Ki‡K f¨vU (VAT) e‡j|

KvR : 1| KYv kwoi †`vKv‡b wM‡q 1,200 UvKvq GKwU wm‡éi kwo I 1,800 UvKvq GKwU w_wcm µq Ki‡jv| f¨v‡Ui nvi 4 UvKv n‡j, †m †`vKwb‡K KZ UvKv †`‡e?

2| BkivK gwbnwi †`vKv‡b wM‡q GK WRb †cbwmj µq K‡i †`vKwb‡K 250 UvKv w`j| f¨v‡Ui nvi 4 UvKv n‡j, cwZwU †cbwm‡ji `vg KZ?

D`vniY 18| bwmi mv‡n‡ei gj †eZb 27,650 UvKv| ewlK †gvU Av‡qi c_g GK j¶ Awk nvRv‡i AvqKi 0 (kb¨) UvKv| cieZx UvKvi Dci AvqK‡ii nvi 10 UvKv n‡j, bwmi mv‡ne KZ UvKv AvqKi †`‡e?

mgvavb : 1 gv‡mi gj †eZb 27,650 UvKv

∴ 12 Ó Ó Ó (27,650 × 12) UvKv

= 3,31,800 UvKv

∴ Ki‡hvM¨ UvKvi cwigvY (3,31,800 – 1,80,000) UvKv ev 1,51,800 UvKv

100 UvKvq AvqKi 10 UvKv

∴ 1 Ó Ó $\frac{10}{100}$ Ó

∴ 1,51,800 Ó Ó $\frac{10\times \overset{1,51,8}{\cancel{1,51,800}}}{\underset{1}{\cancel{100}}}$ Ó ev 15,180 UvKv

∴ bwmi mv‡ne 15,180 UvKv AvqKi †`b|

D`vniY 19| c`xc †MMwi GKRb e¨emvqx| e¨emwqK c‡qvR‡b Zv‡K cw_exi wewfbœ †`‡k ågY Ki‡Z nq| d‡j Zv‡K mv‡_ K‡i BDGm Wjvi wb‡q †h‡Z nq| hw` 1 BDGm Wjvi = 81·50 UvKv nq Ges Zvi hw` 7000 Wjvi c‡qvRb nq, Z‡e evsjv‡`wk KZ UvKv jvM‡e?

mgvavb : 1 BDGm Wjvi 81·50 UvKv

7000 Ó Ó 81·50 × 7000 UvKv

= 5,70,500·00 UvKv

wb‡Yq UvKvi cwigvY = 5,70,500 UvKv|

Abkxjbx 2·2

1| GKRb †`vKvb`vi cwZ wgUvi 200 UvKv `‡i 5 wgUvi Kvco wK‡b cwZ wgUvi 225 UvKv `‡i weµq Ki‡j KZ jvf n‡q‡Q?

2| GKRb Kgjwe‡µZv cwZ nwj 60 UvKv `‡i 5 WRb Kgjv wK‡b cwZ nwj 50 UvKv `‡i weµq Ki‡j KZ ¶wZ n‡q‡Q?

3| iwe cwZ †KwR 40 UvKv `‡i 50 †KwR PvDj wK‡b 44 UvKv †KwR `‡i weµq Ki‡j KZ jvf ev ¶wZ n‡e?

4| cwZ wjUvi wgéwfUv `a 52 UvKvq wK‡b 55 UvKv `‡i weµq Ki‡j kZKiv KZ jvf nq?

5| cwZwU PK‡jU 8 UvKv wn‡m‡e µq K‡i 8·50 UvKv wn‡m‡e weµq K‡i 25 UvKv jvf n‡jv, †gvU KqwU PK‡jU µq Kiv n‡qwQj?

6| cwZ wgUvi 125 UvKv `‡i Kvco µq K‡i 150 UvKv `‡i weµq Ki‡j †`vKvb`v‡ii 2000 UvKv jvf nq| †`vKvb`vi †gvU KZ wgUvi Kvco µq K‡iwQ‡jb?

7| GKwU `e¨ 190 UvKvq µq K‡i 175 UvKvq weµq Ki‡j kZKiv KZ jvf ev ¶wZ n‡e ?

8| 25 wgUvi Kvco †h g‡j¨ µq K‡i, †mB g‡j¨ 20 wgUvi Kvco weµq Ki‡j kZKiv KZ jvf ev ¶wZ n‡e ?

9| 5 UvKvq 8wU Avgj wK µq K‡i 5 UvKvq 6wU `‡i weµq Ki‡j kZKiv KZ jvf ev ¶wZ n‡e ?

10| GKwU Mwoi weµqgj¨ MwowUi µqg‡j¨i $\frac{4}{5}$ As‡ki mgvb| kZKiv jvf ev ¶wZ wbYq Ki|

11| GKwU `e¨ 400 UvKvq weµq Ki‡j hZ ¶wZ nq 480 UvKvq weµq Ki‡j, Zvi wZb¸Y jvf nq| `e¨wUi µqgj¨ wbYq Ki|

12| GKwU Nwo 625 UvKvq weµq Ki‡j 10% ¶wZ nq| KZ UvKvq weµq Ki‡j 10% jvf n‡e ?

13| gvBkv 20 UvKv `‡i 15 wgUvi jvj wdZv µq Ki‡jv| f¨v‡Ui nvi 4 UvKv| †m †`vKwb‡K 500 UvKvi GKwU †bvU w`j| †`vKwb Zv‡K KZ UvKv †diZ †`‡eb|

14| wg. ivq GKRb miKvix KgKZv| wZwb Zx_¯vb cwi`k‡bi Rb¨ fvi‡Z hv‡eb| hw` evsjv‡`‡ki 1 UvKv mgvb fviZxq 0.63 iƒc nq, Z‡e fviZxq 3000 iƒci Rb¨ evsjv‡`‡ki KZ UvKv c‡qvRb n‡e ?

15| bxwjg GKRb PvKwiRxwe| Zvi gwmK gj‡eZb 22,250 UvKv| ewlK †gvU Av‡qi c_g GK j¶ Avwk nvRv‡i AvqKi 0 (kb¨) UvKv| cieZx UvKvi Dci AvqK‡ii nvi 10 UvKv n‡j bxwjg Ki eve` KZ UvKv cwi‡kva K‡ib?

2·4 MwZ welqK mgm¨v

w¯i cwb‡Z †bŠKvi MwZ‡eM n‡jv Gi cKZ MwZ‡eM| †¯vZw¯bx b`x‡Z †bŠKv †h MwZ‡e‡M P‡j Zv †bŠKvi KvhKix MwZ‡eM| †¯v‡Zi AbK‡j Pj‡j †bŠKvi cKZ MwZ‡e‡Mi mv‡_ †¯v‡Zi †eM †hvM K‡i KvhKix MwZ‡eM †ei Kiv nq| Avevi †¯v‡Zi cwZK‡j Pj‡j †bŠKvi cKZ †eM †_‡K †¯v‡Zi †eM we‡qvM K‡i †bŠKvi KvhKix †eM wbYq Kiv nq|

AZGe, †¯v‡Zi AbK‡j †bŠKvi KvhKix MwZ‡eM = †bŠKvi cKZ MwZ‡eM + †¯v‡Zi MwZ‡eM|

†¯v‡Zi cwZK‡j †bŠKvi KvhKix MwZ‡eM = †bŠKvi cKZ MwZ‡eM – †¯v‡Zi MwZ‡eM|

D`vniY 20| GKwU †bŠKv w¯i cwb‡Z NÈvq 6 wK.wg. †h‡Z cv‡i| †¯v‡Zi cwZK‡j 6 wK.wg. †h‡Z †bŠKvwUi 3 ¸Y mgq jv‡M| †¯v‡Zi AbK‡j 50 wK.wg. †h‡Z †bŠKvwUi KZ mgq jvM‡e?

mgvavb : †bŠKvwU w¯i cwb‡Z 6 wK.wg. hvq 1 NÈvq

Ó Ó Ó 1 Ó Ó $\frac{1}{6}$ Ó

†¯v‡Zi cwZK‡j 6 wK.wg. hvq 1×3 NÈvq ev 3 NÈvq

ckg‡Z, 3 NÈvq hvq 6 wK.wg.

$\therefore$ 1 Ó Ó $\frac{6}{3}$ Ó ev 2 wK.wg.

†¯v‡Zi cwZK‡j †bŠKvi KvhKix †eM = †bŠKvi cKZ †eM – †¯v‡Zi †eM

$\therefore$ †¯v‡Zi †eM = †bŠKvi cKZ †eM – †bŠKvi KvhKix †eM

= (6 – 2) wK.wg. ev 4 wK.wg. cwZ NÈvq

†mv‡Zi AbK‡j †bŠKvi KvhKix †eM = †bŠKvi cKZ MwZ‡eM + †¯v‡Zi †eM

= (6 + 4) wK.wg. ev 10 wK.wg. cwZ NÈvq

$\therefore$ †mv‡Zi AbK‡j 10 wK.wg. hvq 1 NÈvq

Ó Ó 1 Ó Ó $\frac{1}{10}$ Ó

$\therefore$ Ó Ó 50 Ó Ó $\frac{1\times\cancel{50}^{5}}{\cancel{10}_{1}}$ NÈvq ev 5 NÈvq

†¯v‡Zi AbK‡j †h‡Z 5 NÈv jvM‡e|

D`vniY 21| GKwU cvwbi U¨v‡¼ 2wU bj Av‡Q| GKwU bj Øviv cvwb wfZ‡i c‡ek K‡i Ges Ab¨ bj Øviv cvwb †ei nq| 1g bjwU Øviv Lvwj U¨v¼wU cY Ki‡Z mgq jv‡M 40 wgwbU Avi 2q bjwU Øviv cvwb cY U¨v¼wU Lwj n‡Z mgq jv‡M 50 wgwbU| GLb `BwU bj GK‡Î L‡j w`‡j KZ wgwb‡U U¨v¼wU cY n‡e?

mgvavb : 1g bj Øviv U¨v¼wU 40 wgwb‡U cvwb cY nq

$\therefore$ Ó Ó Ó Ó 1 Ó Ó Ó $\frac{1}{40}$ Ask

Avevi, 2q bj Øviv U¨v¼wU 50 wgwb‡U Lwj nq

$\therefore$ Ó Ó Ó Ó 1 Ó Ó $\frac{1}{50}$ Ask

bj `BwU GK‡Î L‡j w`‡j 1 wgwb‡U cvwb cY n‡e U¨v¼wUi $\left(\frac{1}{40}-\frac{1}{50}\right)$ Ask

$= \frac{5-4}{200}$ Ask $= \frac{1}{200}$ Ask

U¨v¼wUi $\frac{1}{200}$ Ask cwb cY nq 1 wgwb‡U

$\therefore$ Ó 1 Ó Ó Ó $\frac{1\times 200}{1}$ wgwb‡U

= 200 wgwbU = 3 NÈv 20 wgwb‡U

wb‡Yq mgq 3 NÈv 20 wgwbU|

D`vniY 22| 60 wgUvi `xN GKwU †U‡bi MwZ‡eM NÈvq 48 wK.wg.| †ijjvB‡bi cv‡ki GKwU LwU‡K AwZµg Ki‡Z †UbwUi KZ mgq jvM‡e ?

mgvavb : LwUwU AwZµg Ki‡Z †UbwU‡K wb‡Ri ˆ`‡N¨i mgvb `iZ AwZµg Ki‡Z n‡e|

48 wK.wg. = 48 $\times$ 1000 wgUvi ev 48000 wgUvi

†UbwU 48000 wg. AwZµg K‡i 1 NÈvq

Ó 1 Ó Ó Ó $\frac{1}{48000}$ NÈvq ev $\frac{1\times 60\times 60}{48000}$ †m‡K‡Û

Ó 60 Ó Ó Ó $\frac{1\times \cancel{60}\times \cancel{60}^{3}\times \cancel{60}^{3}}{\cancel{48000}\,\cancel{8}\,\cancel{4}\,2}$ †m‡K‡Û

$= \frac{9}{2}$ †m‡KÛ

$= 4\frac{1}{2}$ †m‡KÛ

†UbwU $4\frac{1}{2}$ †m‡K‡Û LwUwU AwZµg Ki‡e|

Abkxjbx 2·3

1| 5 : 4 Ges 6 : 7 Gi avivewnK AbcvZ †KvbwU ?

(K) 24 : 30 : 28 (L) 30 : 24 : 28

(M) 28 : 24 : 30 (N) 24 : 28 : 30

2| GKwU µwgK mgvbcv‡Zi 1g I 3q iwk h_vµ‡g 4 I 25 n‡j, ga¨ mgvbcvZx †KvbwU ?

(K) 8 (L) 50

(M) 10 (N) 20

3| 3, 5, 15-Gi PZ_ mgvbcvZx †KvbwU ?

(K) 20 (L) 25

(M) 10 (N) 35

4| GKRb †`vKvb`vi GKwU w`qvkjvB e· 1.50 UvKvq µq K‡i 2.00 UvKvq weµq Ki‡j Zvi kZKiv KZ jvf n‡e ?

(K) 20% (L) 15%

(M) 25% (N) $33\frac{1}{3}\%$

5| GKRb Kjwe‡µZv cwZ nvwj Kjv 25 UvKv `‡i µq K‡i cwZ nvwj 27 UvKv `‡i weµq Ki‡j, Zvi 50 UvKv jvf nq| †m KZ nvwj Kjv µq K‡iwQj ?

(K) 25 nvwj (L) 20 nvwj

(M) 50 nvwj (N) 27 nvwj

6| wb‡Pi iwk¸‡jv `vM †U‡b wgj Ki :

(K) µqgj¨ weµqgj¨i †P‡q †ewk n‡j	(K) Kg jv‡M
(L) µqgj¨ weµqgj¨i †P‡q Kg n‡j	(L) jvf nq
(M) †¯v‡Zi AbK‡j mgq	(M) †ewk jv‡M
(N) †¯v‡Zi cwZK‡j mgq	(N) ¶wZ nq

7| 5 Rb kwgK 6 w`‡b 8 weNv Rwgi dmj DVv‡Z cv‡i| 20 weNv Rwgi dmj DVv‡Z 25 Rb kwg‡Ki KZ w`b jvM‡e?

8| ¯cb GKwU KvR 24 w`‡b Ki‡Z cv‡i| iZb D³ KvR 16 w`‡b Ki‡Z cv‡i| ¯cb I iZb GK‡Î KvRwU KZ w`‡b †kl Ki‡Z cvi‡e?

9| nweev I nwjgv GKwU KvR GK‡Î 20 w`‡b Ki‡Z cv‡i| nweev I nwjgv GK‡Î 8 w`b KvR Kivi ci nweev P‡j †Mj| nwjgv ewK KvR 21 w`‡b †kl Kij| m¤úY KvRwU nwjgv KZ w`‡b Ki‡Z cviZ?

10| 30 Rb kwgK 20 w`‡b GKwU ewo ˆZwi Ki‡Z cv‡i| KvR ïi“i 10 w`b c‡i Lvivc Avenvlqvi Rb¨ 6 w`b KvR eÜ ivL‡Z n‡q‡Q| wbawiZ mg‡q KvRwU †kl Ki‡Z AwZwi³ KZRb kwgK jvM‡e?

11| GKwU KvR K I L GK‡Î 16 w`‡b, L I M GK‡Î 12 w`‡b Ges K I M GK‡Î 20 w`‡b Ki‡Z cv‡i| K, L I M GK‡Î KvRwU KZ w`‡b Ki‡Z cvi‡e?

12| GKwU †PŠev”Pvq `BwU bj Av‡Q| c_g I wØZxq bj Øviv h_vµ‡g 12 NÈv I 18 NÈvq Lvwj †PŠev”PwU cY nq| `BwU bj GK mv‡_ L‡j w`‡j Lvwj †PŠev”PwU KZ NÈvq cY n‡e?

13| †¯v‡Zi AbK‡j GKwU †bŠKv 4 NÈvq 36 wK.wg. c_ AwZµg K‡i| †¯v‡Zi †eM cwZNÈvq 3 wK.wg. n‡j, w¯i cwb‡Z †bŠKvi †eM KZ?

14| †¯v‡Zi cwZK‡j GKwU RvnvR 11 NÈvq 77 wK.wg. c_ AwZµg K‡i| w¯i cwb‡Z Rvnv‡Ri MwZ‡eM cwZNÈvq 9 wK.wg. n‡j, †¯v‡Zi MwZ‡eM cwZNÈvq KZ?

15| `vo †e‡q GKwU †bŠKv †¯v‡Zi AbK‡j 15 wgwb‡U 3 wK.wg. Ges †¯v‡Zi cwZK‡j 15 wgwb‡U 1 wK.wg. c_ AwZµg K‡i| w¯i cwb‡Z †bŠKv I †¯v‡Zi MwZ‡eM wbYq Ki|

16| GKRb KlK 5 †Rvov Mi“ Øviv 8 w`‡b 40 †n±i Rwg Pvl Ki‡Z cv‡ib| wZwb 7 †Rvov Mi“ Øviv 12 w`‡b KZ †n±i Rwg Pvl Ki‡Z cvi‡eb?

17| wjwj GKv GKwU KvR 10 NÈvq Ki‡Z cv‡ib| wgwj GKv H KvRwU 8 NÈvq Ki‡Z cv‡ib| wjwj I wgwj GK‡Î H KvRwU KZ NÈvq Ki‡Z cvi‡eb?

18| `BwU bj Øviv GKwU Lwj †PŠev”Pv h\_vµ‡g 20 wgwb‡U I 30 wgwb‡U cwb-cY Kiv hvq| †PŠev”PwU Lwj \_vKv Ae¯vq `BwU bj GK mv‡_ L‡j †`Iqv n‡jv| c_g bjwU KLb eÜ Ki‡j †PŠev”PwU 18 wgwb‡U cwb-cY n‡e?

19| 100 wgUvi `xN GKwU †U‡bi MwZ‡eM NÈvq 48 wK‡jwgUvi| H †UbwU 30 †m‡K‡Û GKwU †mZ AwZµg K‡i| †mZwUi ˆ`N¨ KZ?

20| 120 wgUvi `xN GKwU †Ub 330 wgUvi `xN GKwU †mZ AwZµg Ki‡e| †UbwUi MwZ‡eM NÈvq 30 wK.wg. n‡j, †mZwU AwZµg Ki‡Z †UbwUi KZ mgq jvM‡e?

21| Rwmg mv‡ne GKRb K›Uv±i| wZwb 2 wK.wg. iv¯v 30 w`‡b 2 j¶ UvKvq †giwg‡Zi Rb¨ KvR †c‡jb| wZwb GB KvRwU Kivi Rb¨ 20 Rb kwgK wb‡qvM w`‡jb| wKš 12 w`b ci Lvivc AvenvIqvi Kvi‡Y Zv‡K 4 w`b KvR eÜ †i‡L ewK KvR †kl Ki‡Z n‡jv| KvR †k‡l †`Lv †Mj 2,25,000 UvKv LiP n‡jv| GgZve¯vq wb‡Pi ck¸‡jvi DËi `vI :

(K) 12 w`‡b iv¯vi kZKiv KZ Ask m¤úb n‡qwQj?

(L) wbw`ó mg‡q ewK KvR Kivq AwZwi³ KZ Rb kwgK †j‡MwQj?

(M) AwZwi³ kwgKmsL¨v c`Ë kwgK msL¨vi kZKiv KZ?

(N) KvRwU m¤úb Kivq Zvi kZKiv KZ ¶wZ n‡jv?

ZZxq Aa¨vq

cwigvc

ˆ`bw›`b Rxe‡b Avgiv wewfbœ cKv‡ii †fvM¨cY¨ e¨enviKwi hvi g‡a¨ Av‡Q Pvj, Wvj, wPwb, jeY, djgj, `a, ˆZj, cwb BZ¨w`| e¨emvwqK I e¨enwiK †¶‡Î G¸‡jvi cwigvc c‡qvRb nq| c‡ei †kwY‡Z Avgiv ˆ`N©¨, IRb, †¶Îdj I mgq cwigv‡ci aviYv †c‡qwQ| ˆ`N©¨ ev `iZ cwigvc Kivi Rb¨ Avgiv GKUv wbw`ó gv‡ci ˆ`‡N©¨i mv‡_ Gi Zjbv Kwi| Zij e¨ZxZ Ab¨vb¨ `e¨ IRb w`‡q cwigvc Ki‡Z nq| wKš Zij c`v‡_i †Kv‡bv AvKvi †bB| GwU gvcvi Rb¨ wbw`ó AvKv‡ii gvcwb e¨envi Kiv nq| G Aa¨v‡q ˆ`N©¨, †¶Îdj, IRb I Zij c`v‡_i AvqZb cwigv‡ci wek` Av‡jvPbv Kiv n‡q‡Q|

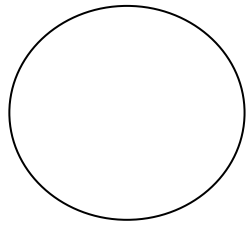
Aa¨vq †k‡l wk¶v_xiv-

- ➢ ˆ`N©¨ cwigv‡ci Avšÿm¤úK e¨vL¨v Ges G msµvšÍ mgm¨v mgvavb Ki‡Z cvi‡e|
- ➢ IRb I Zij c`v‡_i AvqZb cwigvc Kxfv‡e Kiv nq Zv e¨vL¨v Ki‡Z cvi‡e Ges G m¤úwKZ mgm¨v mgvavb Ki‡Z cvi‡e|
- ➢ †¯‹j e¨envi K‡i AvqZvKvi I eMvKvi †¶‡Îi ˆ`N©¨ I c¯' cwigvc K‡i †¶Îdj wbYq Ki‡Z cvi‡e|
- ➢ IRb cwigv‡ci wewfbœ cwigvcK e¨envi K‡i `e¨w`i IRb cwigvc Ki‡Z cvi‡e|
- ➢ Zij c`v‡_i AvqZb cwigv‡ci wewfbœ cwigvcK e¨envi K‡i †h‡Kv‡bv Zij c`v‡_i cwigvc Ki‡Z cvi‡e|
- ➢ ˆ`bw›`b Rxe‡b AvbgwbK cwigvc Ki‡Z cvi‡e|

3·1 ˆ`N©¨ cwigvc

Avgiv evRv‡i wM‡q Kvco, ˆe`¨wZK Zvi, iwk BZ¨w` wK‡b _wK| GKUv wbw`ó gv‡ci ˆ`‡N©¨i mv‡_ Zjbv K‡i G¸‡jv µq-weµq nq| Avevi ewo n‡Z ¯‹j, evRvi ev †÷kb KZ `i Zv-I Avgv‡`i Rvbvi c‡qvRb nq| GB `iZI Avgiv H wbw`ó gv‡ci ˆ`‡N©¨i mv‡_ Zjbv K‡i †ei Kwi| GB ˆ`N©¨‡K cwigv‡ci GKK ejv nq|

wewUk c×wZ‡Z ˆ`N©¨ cwigv‡ci GKK wn‡m‡e MR, dU, BwÂ Pvj Av‡Q| eZgv‡b cw_ex‡Z AwaKvsk †`‡k ˆ`N©¨ cwigvc wn‡m‡e e¨eüZ n‡”Q †gwUK c×wZ| cw_exi DËi †gi“ †_‡K dv‡Ýi ivRavbx c¨wi‡mi `wNgv eivei wele‡iLv chšÍ ˆ`‡N©¨i †KwUfv‡Mi GKfvM‡K 1 wgUvi wn‡m‡e MY¨ Kiv nq| †gwUK c×wZ‡Z ˆ`N©¨ cwigv‡ci GKK n‡”Q wgUvi|



cøwUbvg I BwiwWqvg avZi msvgk‡Y ˆZwi wgUv‡ii Avmj bgbv cw_exi me †`‡ki Rb¨ Av`k ev ÷¨vÛvW i/‡c MY¨ Kiv nq| GwU dv‡Ýi hv`N‡i msiw¶Z i‡q‡Q| wewfb †`‡ki c‡qvR‡b Av`k bgbv †_‡K ¯vbxq bgbv ˆZwi K‡i †bIqv nq|

1 wgUvi = DËi †gi“ †_‡K wele‡iLv chš— †gvU `i‡Zi 1 †KwU fv‡Mi 1 fvM

j¶ Kwi, 1982 mvj †_‡K evsjv‡`‡ki meÎ ˆ`N¨ gvcvi Rb¨, IRb wbY‡qi Rb¨ Ges Zij c`v‡_i AvqZb cwigv‡ci Rb¨ ÔAvšRwZK Av`kgvbÕ ev Ôwm‡÷g Ae B›Uvib¨vkbvj BDwbUÕ MnY Kiv n‡q‡Q|

ˆ`N¨ cwigv‡ci GKKvewj

†gwUK c×wZ			weªwUk c×wZ		
10 wgwjwgUvi (wg.wg.)	=	1 †mw›UwgUvi (†m. wg.)	12 BwÂ	=	1 dzU
10 †mw›UwgUvi	=	1 †WwmwgUvi (†Wwm. wg.)	3 dzU	=	1 MR
10 †WwmwgUvi	=	1 wgUvi (wg.)	1760 MR	=	1 gvBj
10 wgUvi	=	1 †WKwgUvi (†WKv. wg.)			
10 †WKwgUvi	=	1 †n‡±wgUvi (†n. wg.)			
10 †n‡±wgUvi	=	1 wK‡jwgUvi (wK. wg.)			

†gwUK I weªwUk cwigv‡ci m¤úK

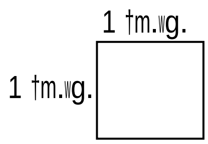
1 BwÂ	=	2·54 †m. wg. (cÖvq)
1 gvBj	=	1·61 wK. wg. (cÖvq)
1 wgUvi	=	39·37 BwÂ (cÖvq)
1 wK. wg.	=	0·62 gvBj (cÖvq)

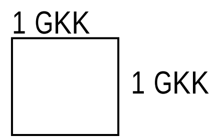
KvR : 1|	ˆ`bw›`b Rxe‡b e¨eüZ nq ev Kv‡R jv‡M Ggb wKQz e¯‘i bvg Ki, hv‡`i ˆ`N¨ cwigvc Ki‡Z nq|
2|	†¯‹j w`‡q †Zvgvi GKwU eB‡qi I †Uwe‡ji ˆ`N¨ I c¯' BwÂ‡Z Ges †mw›UwgUv‡i gvc| G n‡Z 1 BwÂ mgvb KZ †mw›UwgUvi Zv wbY©q Ki|
3|	gvcvi wdZv w`‡q †kÖwYK‡¶i ˆ`N¨ I c¯' cwigvc Ki|

3·2 †¶Îdj cwigvc

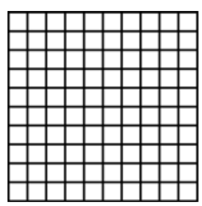
†¶Îdj cwigv‡ci avivYv Avgv‡`i Rxe‡b LeB ¸i“ZcY| emev‡mi Rb¨ Ni-ewo n‡Z ïi“ K‡i wk¶v cwZôvb, nvmcvZvj, miKwi wewfb feb BZ¨vw` Avgv‡`i LeB c‡qvRbxq ¯vcbv| G¸‡jv †h Rwgi Dci ˆZwi Ki‡Z nq Zvi †¶Îdj Rvbv Avgv‡`i GKvšÍ c‡qvRb| †Kv‡bv wbw`ó mxgv‡iLv Øviv Ave× ¯vb n‡jv †¶Î Ges GB †¶‡Îi cwigvc‡K Zvi †¶Îdj ev Kwj e‡j|

†h‡Kv‡bv †¶‡Îi mvaviYZ ˆ`N¨ I c¯ _v‡K| G Rb¨ †¶Îd‡ji GKK wn‡m‡e GK GKK ˆ`‡N¨i evûwewkó GKwU eM‡¶‡Îi †¶Îdj‡K aiv nq| †¶Îd‡ji GKK‡K eM GKK †jLv nq| †h eM‡¶‡Îi evûi ˆ`N¨ 1 wgUvi, Zvi †¶Îdj 1 eMwgUvi| Abiƒc 1 eMdU, 1 eM‡mw›UwgUvi, BZ¨vw` I †¶Îd‡ji GKK wn‡m‡e e¨eüZ nq|





†Kv‡bv †¶‡Îi †¶Îdj wbYq Ki‡Z n‡j, Gi g‡a¨ KZ¸‡jv eMGKK Av‡Q Zv †ei Ki‡Z nq|
g‡b Kwi, wb‡Pi eM‡¶‡Îi cwZevûi ˆ`N¨ 1 wgUvi| AZGe, Gi †¶Îdj 1 eMwgUvi| eM‡¶ÎwUi c‡Z¨K evû‡K mgvb 10 As‡k wef³ K‡i wecixZ we›`¸‡jv ci¯úi msh³ Kiv n‡jv|

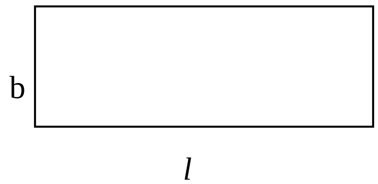


G‡¶‡Î c‡Z¨K As‡ki ˆ`N¨ 1 †WwmwgUvi| AZGe, 1wU ¶` e‡Mi †¶Îdj n‡jv,
1 †WwmwgUvi × 1 †WwmwgUvi = 1 eM‡WwmwgUvi | †`Lv hv‡”Q †h, wPÎwU‡Z 100wU ¶` eM‡¶Î i‡q‡Q|
AZGe, 1 eMwgUvi = 100 eM‡WwmwgUvi|

Z`ƒc, 1 †WwmwgUvi ˆ`‡N¨i evûwewkó eM‡¶Î wb‡q Gi c‡Z¨K evû‡K 10wU mgvb As‡k fvM K‡i Av‡Mi g‡Zv msh³ K‡i †`Lv‡bv hvq †h, 1 eM‡WwmwgUvi = (10×10) eM‡m.wg. ev 100 eM‡mw›UwgUvi| AZGe, 1 eMwgUvi = 10,000 eM‡mw›UwgUvi|
j¶ Kwi, 4 wgUvi eM Ges 4 eMwgUvi GK K_v bq| 4 wgUvi eM Øviv Ggb GKwU eM‡¶Î‡K †evSvq hvi c‡Z¨K evûi ˆ`N¨ 4 wgUvi Ges hvi †¶Îdj (4 × 4) eMwgUvi ev 16 eMwgUvi| wKš 4 eMwgUvi Øviv Ggb GKwU eM‡¶‡Îi †¶Îdj †evSvq hvi ˆ`N¨ I c¯ wgUv‡ii GK‡K †g‡c ¸Y Ki‡j 4 nq|

wb‡P K‡qKwU †¶‡Îi †¶Îd‡ji mÎ †`Iqv n‡jv :

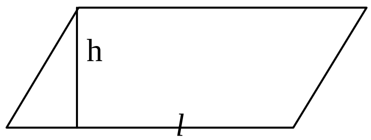
AvqZ



†¶Îdj = ˆ`N¨ × c¯

$= l \times b$

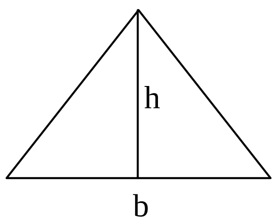
mvgvšÍwiK



†¶Îdj = fwg × D”PZv

$= l \times h$

wÎfR



†¶Îdj = $\frac{1}{2}$ × fwg × D”PZv

$= \frac{1}{2} \times (b \times h)$

†¶Îdj cwigv‡c †gwUK I weªwUk c×wZi m¤úK

weªwUk c×wZ‡Z

1 eMBwÂ	= 6.45 eM‡mw›UwgUvi (cÖvq)
1 eMdzU	= 929 eM‡mw›UwgUvi (cÖvq)
1 eMMR	= 0.84 eMwgUvi (cÖvq)

¯^vbxq c×wZ‡Z

1 eM‡mw›UwgUvi	=	0.155 eMBwÂ (cÖvq)
1 eMwgUvi	=	10.76 eMdzU (cÖvq)
1 †n±i	=	2.47 GKi (cÖvq)

KvR :

1| †¯‹j w`‡q †Zvgvi GKwU eB‡qi I covi †Uwe‡ji ˆ`N¨ †mw›UwgUv‡i †g‡c Gi †¶Îdj wbYq Ki|

2| `jMZfv‡e †Zvgvi †eÂ, †Uwej, `iRv, Rvbvjv BZ¨w`i ˆ`N¨ I c¯ †¯‹‡ji mvnv‡h¨ †g‡c †¶Îdj †ei Ki|

3·3 IRb cwigvc

c‡Z¨K e¯'i IRb Av‡Q| wewfbœ †`‡k wewfbœ GK‡Ki mvnv‡h¨ e¯' IRb Kiv nq|

IRb cwigv‡ci †gwUK GKKvewj

10 wgwjMvg (wg. Mv.)	=	1 †mw›UMvg (†m. Mv.)
10 †mw›UMvg	=	1 †WwmMvg (†WwmMv.)
10 †WwmMvg	=	1 Mvg (Mv.)
10 Mvg	=	1 †WKvMvg (†WKvMv.)

10 †WKvMvg	=	1 †n‡±vMvg (†n. Mv.)
10 †n‡±vMvg	=	1 wK‡jvMvg (†K. wR.)
100 wK‡jvMvg (†K. wR.)	=	1 KB›Uvj
1000 wK‡jvMvg ev 10 KB›Uvj	=	1 †gwUK Ub

IRb cwigv‡ci GKK : Mvg

1 wK‡jvMvg ev 1 †K.wR. = 1000 Mvg

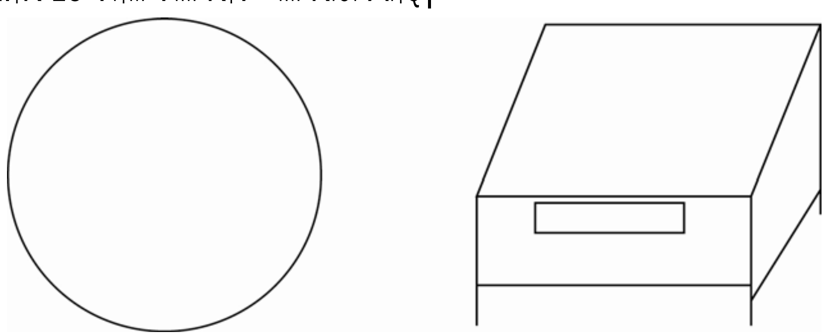
4^{0} †mjwmqvm ZvcgvÎvq 1 Nb †m. wg. wei× cvwbi IRb 1 Mvg|

†gwUK c×wZ‡Z IRb cwigv‡ci Rb¨ e¨eüZ Avi I `BwU GKK Av‡Q| AwaK cwigvY e¯‘i IR‡bi Rb¨ G `BwU GKK e¨envi Kiv nq| GKK `BwU n‡"Q KB›Uvj I †gwUK Ub|

kn‡i I Mv‡g IRb cwigv‡ci Rb¨ `wocvjøv I evULviv e¨envi Kiv nq| G evULviv 5 Mvg, 10 Mvg, 50 Mvg, 100 Mvg, 200 Mvg, 500 Mvg, 1 †K. wR., 2 †K. wR., 5 †K. wR., 10 †K. wR. BZ¨w` IR‡bi nq|

A‡bK †¶‡Î kn‡i `vMKvUv e¨v‡jÝ Øviv IRb cwigvc Kiv nq| GwU †`L‡Z A‡bKUvB GKwU KwZZ wciwg‡Wi wb‡Pi As‡ki g‡Zv hvi Dc‡i `e¨ ivLv hvq Ges hvi Mv‡q GKcv‡k †`qvjNwoi Wvqv‡ji `v‡Mi g‡Zv †MvjvKvi †iLvq `vM KvUv _v‡K| IR‡bi mgnv‡i wK‡jvMv‡gi gv‡c `v‡Mi cv‡k msL¨v emv‡bv _v‡K Ges Nwoi wgwb‡Ui KvUvi g‡Zv GKUv wb‡`kK KvUv _v‡K| gvcvi Rb¨ e¨v‡j‡Ýi Dci †Kv‡bv `e¨ emv‡jB KvUwU †h msL¨v‡K wb‡`k K‡i †m msL¨vB H e¯‘i IRb|

G‡Z cwZ †K. wR.‡K 10 fv‡M fvM K‡i `vM KvUv Av‡Q|



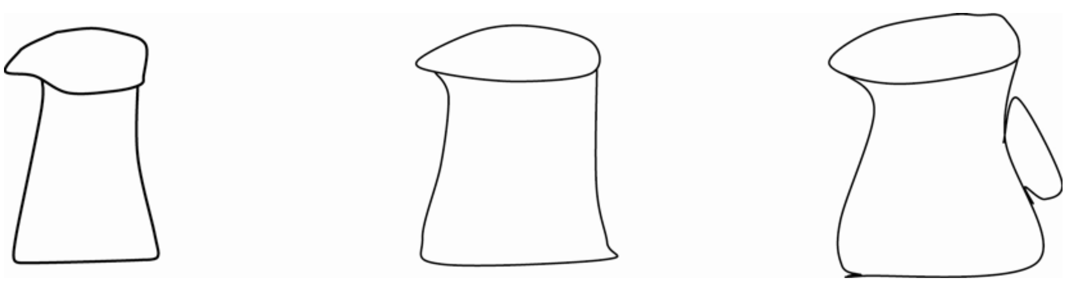
eZgv‡b `vMKvUv e¨v‡jÝ Gi ¯‘‡j wWwRUvj e¨v‡jÝ e¨eüZ n‡"Q| GwU GKwU †QvU ev‡· i g‡Zv hvi Mv‡q GK cv‡k msL¨vq Mv‡g IRb c`wkZ nq| Gi mvnv‡h¨ `‡e¨i gj¨I wbY‡qi e¨e¯'v Av‡Q| KviY GB e¨v‡j‡Ý K¨vjK‡jU‡ii mweavI _v‡K| cwZ wK‡jvMvg `‡e¨i gj¨gvb w`‡q c`wkZ msL¨v‡K K¨vjK‡jU‡ii wbq‡g ¸Y Ki‡jB `‡e¨i †gvU gj¨ cvIqv hvq| G Rb¨ GB e¨v‡jÝ e¨envi Kiv mweavRbK| Z‡e †ewk cwigvY `e¨ IRb Ki‡Z GLbI `wocvjøv e¨envi Kiv nq|

KvR :	`jxqfv‡e `woc¨vjøv A_ev wWwRUvj e¨v‡jÝ e¨envi K‡i †¯‹j, c¨vK, wUwdbe‡·i IRb cwigvc K‡i †gwUK c×wZ‡Z †jL|

3·4 Zij c`v‡_i AvqZb cwigvc

†Kv‡bv Zij c`v_ KZUv RvqMv R‡o _v‡K Zv Gi AvqZb\|

GKwU Nbe¯‘i ^`N¨, c¯', D"PZv Av‡Q\| wKš' †Kv‡bv Zij c`v‡_i Zv †bB\| †h cv‡Î ivLv nq †mB cv‡Îi AvKvi aviY K‡i\| G Rb¨ wbw`ó AvqZ‡bi †Kv‡bv Nbe¯'i AvKwZi gvcwb Øviv Zij c`v_ gvcv nq\| G †¶‡Î Avgiv mvaviYZ wjUvi gvcwb e¨envi Kwi\| G gvcwb¸‡jv $\frac{1}{4}, \frac{1}{2}, 1, 2, 3, 4, ...$ BZ¨w` wjUvi wewkó Gjywgwbqvg ev wUb wkU Øviv ^Zwi GK cKv‡ii †KvbK AvKwZi cvÎ ev wmwjÛvi AvKwZi gM\| Avevi ¯"Q Kv‡Pi ^Zwi 25, 50, 100, 200, 300, 500, 1000 wgwjwjUvi `vMKvUv Lvov cvÎI e¨envi Kiv nq\| mvaviYZ `a I ^Zj gvcvi †¶‡Î DwjøwLZ cvÎ¸‡jv e¨envi Kiv nq\|



1 wjUvi gvcwb 1 wjUvi `vM KvUv gM wPÎ 1 M¨vjb

†µZv-we‡µZvi mweav‡_ eZgv‡b †fvR¨‡Zj †evZjRvZ K‡i weµ n‡"Q\| G †¶‡Î 1, 2, 5 I 8 wjUv‡ii †evZj †ewk e¨eüZ nq\| wewfb cKv‡ii cvbxq 250, 500, 1000, 2000 wgwjwjUvi ev Ab¨vb¨ AvqZ‡b †evZjRvZ K‡i weµ Kiv nq\|

1 wjUvi †evZj 5 wjUvi †evZj

wPÎ

1 Nb †mw›UwgUvi‡K ms‡¶‡c Bs‡iwR‡Z wm. wm. (Cubic Centimetre) †jLv nq\|

1 Nb †m.wg. (wm.wm.) = 1 wgwjwjUvi	1 Nb BwÂ = 16·39 wgwjwjUvi (cvq)

AvqZb cwigv‡c †gwUK GKKvewj

1000 Nb †mw›UwgUvi (Nb †m. wg.)	=	1 Nb †WwmwgUvi (N. †Wwmwg.)
1000 Nb †WwmwgUvi	=	1 Nb wgUvi (N. wg.)
1000 Nb †mw›UwgUvi	=	1 wjUvi
1 wjUvi cwbi IRb	=	1 wK‡jvMvg

KvR :

1| GKwU cvbxqR‡ji cv‡Îi aviY¶gZv KZ wm. wm. Zv cwigvc Ki|

2| wk¶K KZK wbawiZ ARvbv AvqZ‡bi GKwU cv‡Îi AvqZb Abgvb Ki| Zvici Gi mwVK AvqZb †ei K‡i f‡ji cwigvY wbYq Ki|

D`vniY 1| 16 GKi Rwg‡Z 420 †gwUK Ub Avj Drcb n‡j, 1 GKi Rwg‡Z Kx cwigvY Avj Drcb nq ?

mgvavb : 16 GKi Rwg‡Z Drcb nq 420 †gwUK Ub Avj

∴ 1 Ó Ó Ó Ó $\frac{420}{16}$ Ó Ó Ó

= $26\frac{1}{4}$ †g. Ub ev 26 †gwUK Ub 250 †KwR Avj|

1 †g. Ub = 1000 †KwR

∴ 1 GK‡i Av‡ji Drcv`b 26 †gwUK Ub 250 †KwR|

D`vniY 2| ivqnvb GK GKi Rwg‡Z avb Pvl K‡i 400 †KwR avb †c‡q‡Q| cwZ †KwR av‡b 700 Mvg Pvj n‡j, †m Kx cwigvY Pvj †c‡j?

mgvavb : 1 †K. wR. av‡b Pvj nq 700 Mvg

∴ 400 Ó Ó Ó Ó 700 × 400 Ó

= 280000 Mvg

= 280 †KwR

∴ cvß Pv‡ji cwigvY 280 †KwR|

D`vniY 3| GKwU †gvUiMvwo 10 wjUvi wW‡R‡j 80 wK‡jwgUvi hvq| 1 wK‡jwgUvi †h‡Z Kx cwigvY wW‡R‡ji c‡qvRb ?

mgvavb : 80 wK‡jwgUvi hvq 10 wjUvi wW‡R‡j

∴ 1 Ó Ó $\frac{10}{80}$ Ó Ó $= \frac{1000}{8}$ wgwjwjUvi ev 125 wgwjwjUvi wW‡R‡j

∴ c‡qvRbxq wW‡R‡ji cwigvY 125 wgwjwjUvi|

D`vniY 4| GKwU wÎfRvKvi fwgi ^`N¨ 6 wgUvi I D”PZv 4 wgUvi| wÎfRvKvi †¶ÎwUi †¶Îdj KZ ?

mgvavb : wÎfRvKvi †¶ÎwUi †¶Îdj $= \frac{1}{2} \times$ (fwg × D”PZv)

$= \frac{1}{2} \times (6 \times 4)$ eMwgUvi = 12 eMwgUvi

∴ wÎfRvKvi †¶ÎwUi †¶Îdj 12 eMwgUvi|

D`vniY 5| GKwU wÎfRvKwZ Rwgi †¶Îdj 216 eMwgUvi| Gi fwg 18 wgUvi n‡j, D”PZv wbYq Ki|

mgvavb : Avgiv Rwb,

$\frac{1}{2} \times$ fwg × D”PZv = wÎf‡Ri †¶Îdj

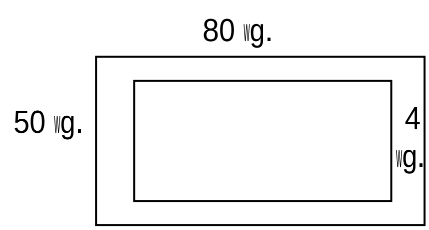
ev, $\frac{1}{2} \times$ 18 wgUvi × D”PZv = 216 eMwgUvi

ev, 9 wgUvi × D”PZv = 216 eMwgUvi

ev, D”PZv = $\frac{216}{9}$ wgUvi ev 24 wgUvi

∴ D”PZv 24 wgUvi|

D`vniY 6| cvomn GKwU cK‡ii ^`N¨ 80 wgUvi I c¯’ 50 wgUvi| hw` cK‡ii c‡Z¨K cv‡oi we¯—vi 4 wgUvi nq, Z‡e cKicv‡oi †¶Îdj KZ?



mgvavb :

cvo ev‡` cK‡ii ^`N¨ = {80 – (4 × 2)} wgUvi
= 72 wgUvi

cvo ev‡` cK‡ii c¯’ = {50 – (4 × 2)} wgUvi
= 42 wgUvi

GLb cvomn cK‡ii †¶Îdj = (80 × 50) eMwgUvi
= 4000 eMwgUvi

Ges cvo ev‡` cK‡ii †¶Îdj = (72 × 42) eMwgUvi
= 3024 eMwgUvi

∴ cKicv‡oi †¶Îdj = (4000 – 3024) eMwgUvi
= 976 eMwgUvi|

∴ cKicv‡oi †¶Îdj 976 eMwgUvi|

Abkxjbx 3

1| wK‡jvwgUv‡i cKvk Ki :

(K) 40390 †m. wg. (L) 75 wgUvi 250 wg. wg.

2| 5.37 †WKwgUvi‡K wgUvi I †WwmwgUv‡i cKvk Ki :

3| wb‡P K‡qKwU wÎfRvKvi †¶‡Îi fwg I D”PZv †`Iqv n‡jv| wÎfRvKvi †¶‡Îi †¶Îdj wbYq Ki :

(K) fwg 10wg. I D”PZv 6 wg.|

(L) fwg 25 †m .wg. I D”PZv 14 †m. wg.|

4| GKwU AvqZvKvi †¶‡Îi ˆ`N¨ c‡¯'i 3 ¸Y| Gi Pvwiw`‡K GKevi c`w¶Y Ki‡j 1 wK‡jvwgUvi nvUv nq| AvqZvKvi †¶ÎwUi ˆ`N¨ I c¯' wbYq Ki|

5| cwZ wgUvi 100 UvKv `‡i 100 wgUvi j¤v I 50 wgUvi PIov GKwU AvqZvKvi cv‡Ki Pvwiw`‡K †eov w`‡Z KZ LiP jvM‡e ?

6| GKwU mvgvšÍwiK †¶‡Îi fwg 40 wgUvi I D”PZv 50 wgUvi| Gi †¶Îdj wbYq Ki|

7| GKwU Nb‡Ki GKav‡ii ˆ`N¨ 4 wgUvi| NbKwUi Zj¸‡jvi †¶Îdj wbYq Ki|

8| †hv‡md Zvi GK LÊ Rwg‡Z 500 †K. wR. 700 Mvg Avj Drcv`b K‡ib| wZwb GKB †¶Îdj wewkó 11 LÊ Rwg‡Z Kx cwigvY Avj Drcv`b Ki‡eb ?

9| c‡i‡ki 16 GKi Rwg‡Z 28 †gwUK Ub avb Drcb n‡q‡Q| Zvi cwZ GKi Rwg‡Z Kx cwigvY avb n‡q‡Q ?

10| GKwU w÷j wg‡j GK gv‡m 20000 †gwUK Ub iW ˆZwi nq| H wg‡j ˆ`wbK Kx cwigvY iW ˆZwi nq ?

11| GK e¨emvqx †Kv‡bv GKw`b 20 †K. wR. 400 Mvg Wvj weµq K‡ib| G wnmv‡e Kx cwigvY Wvj wZwb GK gv‡m weµq Ki‡eb ?

12| GKLÐ Rwg‡Z 20 †K. wR. 850 Mvg mwilv Drcb n‡j, Abic 7 LÐ Rwg‡Z †gvU Kx cwigvY mwilv Drcb n‡e ?

13| GKwU g‡Mi wfZ‡ii AvqZb 1·5 wjUvi n‡j, 270 wjUv‡i KZ gM cwb n‡e ?

14| GK e¨emvqx †Kv‡bv GKw`b 18 †K. wR. 300 Mvg Pvj Ges 5 †K. wR. 750 Mvg jeY weµq K‡ib| G wnmv‡e gv‡m wZwb Kx cwigvY Pvj I jeY weµq K‡ib ?

15| †Kv‡bv cwiev‡i ˆ`wbK 1·25 wjUvi `a jv‡M| cwZ wjUvi `‡ai `vg 52 UvKv n‡j, H cwiev‡i 30 w`‡b KZ UvKvi `a jvM‡e ?

16| GKwU AvqZvKvi evMv‡bi ˆ`N¨ I c¯' h_vµ‡g 60 wgUvi, 40 wgUvi| Gi wfZ‡i PZw`‡K 2 wgUvi PIov iv¯Ív Av‡Q| iv¯ÍwUi †¶Îdj wbYq Ki|

17| GKwU N‡ii ˆ`N¨, c‡¯'i 3 ¸Y| cwZ eMwgUv‡i 7.50 UvKv `‡i N‡ii †g‡S Kv‡cU w`‡q go‡Z †gvU 1102.50 UvKv e¨q nq| NiwUi ˆ`N¨ I c¯' wbYq Ki|

PZ_ Aa¨vq

exRMwYZxq iwki ¸Y I fvM

MwY‡Zi Pvi‡U †gšwjK cÖµqv n‡jv †hvM, we‡qvM, ¸Y I fvM| we‡qvM n‡"Q †hv‡Mi wecixZ cÖµqv Avi fvM n‡"Q ¸‡Yi wecixZ cÖµqv| cvUMwY‡Z †Kej abvZ¥K wPýh³ msL¨v e¨envi Kiv nq| wKš exRMwY‡Z abvZ¥K I FYvZ¥K Dfq wPýh³ msL¨v Ges msL¨vmPK cZxKI e¨envi Kiv nq| Avgiv lô †kªwY‡Z wPýh³ iwki †hvM-we‡qvM Ges exRMwYZxq iwki †hvM I we‡qvM m¤‡Ü avi Yv †c‡qwQ| G Aa¨v‡q wPýh³ iwki ¸Y I fvM Ges exRMwYZxq iwki ¸Y I fvM cÖµqv m¤‡Ü Av‡jvPbv Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv –

- exRMwYZxq iwki ¸Y I fvM Ki‡Z cvi‡e|
- eÜbx e¨envi‡ii gva¨‡g exRMwYZxq iwki †hvM, we‡qvM, ¸Y I fvM msµvš— ^``bw›`b Rxe‡bi mgm¨vi mgvavb Ki‡Z cvi‡e|

4·1 exRMwYZxq iwki ¸Y

¸‡Yi wewbgq wewa :

Avgiv Rwb, $2\times 3=6$, Avevi $3\times 2=6$

$\therefore\ 2\times 3=3\times 2$, hv ¸‡Yi wewbgq wewa|

GKBfv‡e, a, b †h‡Kv‡bv `BwU exRMwYZxq iwk n‡j, $a\times b\ =\ b\times a$ A_vr, MY¨ I MY‡Ki ¯'vb wewbgq Ki‡j, ¸Yd‡ji †Kv‡bv cwieZb nq bv|

¸‡Yi ms‡hvM wewa :

$(2\times 3)\times 4=6\times 4=24$; Avevi, $2\times (3\times 4)=2\times 12=24$

$\therefore\ (2\times 3)\times 4=2\times (3\times 4)$, hv ¸‡Yi ms‡hvM wewa|

GKBfv‡e, a, b, c †h‡Kv‡bv wZbwU exRMwYZxq iwki Rb¨

$(a\times b)\times c=a\times (b\times c)$, hv ¸‡Yi ms‡hvM wewa |

4·2 wPýh³ iwki ¸Y

Avgiv Rwb, 2 †K 4 evi wb‡j $2+2+2+2=8=2\times 4$ nq| GLv‡b ejv hvq †h, 2 †K 4 Øviv ¸Y Kiv n‡q‡Q|

A_vr, $2\times 4=2+2+2+2=8$

†h‡Kv‡bv exRMwYZxq iwk a I b Gi Rb¨

$\boxed{a \times b = ab}$ ....(i)

Avevi, $(-2)\times 4 = (-2)+(-2)+(-2)+(-2) = -8 = -(2\times 4)$

A_vr, $(-2)\times 4 = -(2\times 4) = -8$

mvaviYfv‡e, $\boxed{(-a)\times b = -(a\times b) = -ab}$ ....(ii)

Avevi, $a\times(-b) = (-b)\times a$, ¸‡Yi wewbgq wewa

$$= -(b\times a)$$
$$= -(a\times b)$$
$$= -ab$$

A_vr, $\boxed{a\times(-b) = -(a\times b) = -ab}$ ....(iii)

mZivs, $(-a)\times(-b) = -\{(-a)\times b\}$ [(iii) Abhvqx]

$= -\{-(a\times b)\}$ [(ii) Abhvqx]

$= -(-ab)$

$= a\times b$ [$\because -x$ Gi †hvMvZ¥K wecixZ x]

$= ab$

A_vr, $\boxed{(-a)\times(-b) = ab}$ ....(iv)

j¶ Kwi :

* GKB wPýh³ `BwU iwki ¸Ydj (+) wPýh³ n‡e|
* wecixZ wPýh³ `BwU iwki ¸Ydj (–) wPýh³ n‡e|

$(+1)\times(+1)$	$=$	$+1$
$(-1)\times(-1)$	$=$	$+1$
$(+1)\times(-1)$	$=$	-1
$(-1)\times(+1)$	$=$	-1

¸‡Yi mPK wewa :

Avgiv Rvwb, $a\times a = a^2$, $a\times a\times a = a^3$, $a\times a\times a\times a = a^4$

$\therefore a^2\times a^4 = (a\times a)\times(a\times a\times a\times a) = a\times a\times a\times a\times a\times a = a^6 = a^{2+4}$

mvaviYfv‡e, $\boxed{a^m\times a^n = a^{m+n}}$ m, n †h‡Kv‡bv ¯vfvweK msL¨v|

GB cwµqv‡K ¸‡Yi mPK wewa ejv nq|

Avevi, $(a^3)^2 = a^3\times a^3 = a^6 = a^{3\times 2} = a^6$

mvaviYfv‡e, $\boxed{(a^m)^n = a^{mn}}$

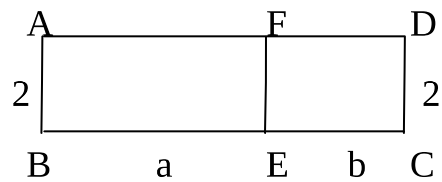
¸‡Yi eÈb wewa

Avgiv Rwb, $2(a+b)=(a+b)+(a+b)$ $[\because\ 2x=x+x]$

$$=(a+a)+(b+b)$$

$$=2a+2b$$

Avevi cv‡ki wPÎ n‡Z cvB,



$ABEF$ AvqZ‡¶ÎwUi †¶Îdj

= ˆ`N¨ × c¯ = $BE\times AB=a\times 2=2\times a=2a$

Avevi, $ECDF$ AvqZ‡¶ÎwUi †¶Îdj = ˆ`N¨ × c¯

= $EC\times CD=b\times 2=2\times b=2b$

$\therefore$ $ABCD$ AvqZ‡¶ÎwUi †¶Îdj

= $ABEF$ AvqZ‡¶‡Îi †¶Îdj $+ECDF$ AvqZ‡¶‡Îi †¶Îdj

$=2a+2b$

Avevi, $ABCD$ AvqZ‡¶‡Îi †¶Îdj

= ˆ`N¨ × c¯

$=BC\times AB$

$=AB\times(BE+EC)$

$=2\times(a+b)=2(a+b)$

$\therefore$ $2(a+b)=2a+2b.$

mvaviYfv‡e, $m(a+b+c+\ldots\ldots\ldots..)$ = $ma+mb+mc+\ldots\ldots\ldots..$

GB wbqg‡K ¸‡Yi eÈb wewa ejv nq|

4·3 GKc`x iwk‡K GKc`x iwk Øviv ¸Y

`BwU GKc`x iwki ¸‡Yi †¶‡Î Zv‡`i mvswL¨K mnMØq‡K wPýh³ msL¨vi ¸‡Yi wbq‡g ¸Y Ki‡Z nq| Dfqc‡` we`¨gvb exRMwYZxq cZxK¸‡jv‡K mPK wbq‡g ¸Y K‡i ¸Yd‡j wjL‡Z nq| Ab¨vb¨ cZxK¸‡jv AcwiewZZ Ae¯vq ¸Yd‡j †bIqv nq|

D`vniY 1| $5x^2y^4$ †K $3x^2y^3$ Øviv ¸Y Ki|

mgvavb : $5x^2y^4\times 3x^2y^3$

$= (5\times 3)\times(x^2\times x^2)\times(y^4\times y^3)$

$= 15x^4y^7$ [mPK wbqg Abhvqx]

wb‡Yq ¸Ydj $15x^4y^7$.

D`vniY 2| $12a^2xy^2$ †K $-6ax^3b$ Øviv ¸Y Ki|

mgvavb : $12a^2xy^2\times(-6ax^3b)$

$=12\times(-6)\times(a^2\times a)\times b\times(x\times x^3)\times y^2$

$= -72a^3bx^4y^2$

wb‡Yq ¸Ydj $-72a^3bx^4y^2$.

D`vniY 3| $-7a^2b^4c$ †K $4a^2c^3$d Øviv ¸Y Ki|

mgvavb : $(-7a^2b^4c)\times 4a^2c^3$d

$= (-7\times 4)\times(a^2\times a^2)\times b^4\times(c\times c^3)\times d$

$= -28a^4b^4c^4d$

wb‡Yq ¸Ydj $-28a^4b^4c^4d$.

D`vniY 4| $-5a^3bc^5$ †K $-4ab^5c^2$ Øviv ¸Y Ki|

mgvavb : $(-5a^3bc^5)\times(-4ab^5c^2)$

$= (-5)\times(-4)\times(a^3\times a)\times(b\times b^5)\times(c^5\times c^2)$

$= 20a^4b^6c^7$

wb‡Yq ¸Ydj $20a^4b^6c^7$.

KvR : 1| ¸Y Ki :

(K) $7a^2b^5$ †K $8a^5b^2$ Øviv
(L) $-10x^3y^4z$ †K $3x^2y^5$ Øviv
(M) $9ab^2x^3y$ †K $-5xy^2$ Øviv
(N) $-8a^3x^4by^2$ †K $-4abxy$ Øviv

4·4 eûc`x iwk‡K GKc`x iwk Øviv ¸Y

eûc`x iwk‡K GKc`x iwk Øviv ¸Y Ki‡Z n‡j ¸‡Y¨i (c_g iwk) c‡Z¨K c`‡K ¸YK (wØZxq iwk) Øviv ¸Y Ki‡Z nq|

D`vniY 5| $\left(5x^2y+7xy^2\right)$ †K $5x^3y^3$ Øviv ¸Y Ki|

mgvavb : $(5x^2y+7xy^2)\times 5x^3y^3$

$=(5x^2y\times 5x^3y^3)+(7xy^2\times 5x^3y^3)$ [eÈb wewa Abmv‡i]

$=(5\times 5)\times(x^2\times x^3)\times(y\times y^3)+(7\times 5)\times(x\times x^3)\times(y^2\times y^3)$

$=25x^5y^4+35x^4y^5$

wb‡Yq ¸Ydj $25x^5y^4+35x^4y^5$

weKí c×wZ :

$$\begin{array}{r} 5x^2y+7xy^2 \\ \times\ 5x^3y^3 \\ \hline 25x^5y^4+35x^4y^5 \end{array}$$

wb‡Yq ¸Ydj $25x^5y^4+35x^4y^5$

D`vniY 6| $2a^3 - b^3 + 3abc$ †K $a^4\text{b}^2$ Øviv ¸Y Ki|

mgvavb : $(2a^3 - b^3 + 3abc) \times a^4b^2$

$$= (2a^3 \times a^4b^2) - (b^3 \times a^4b^2) + (3abc \times a^4b^2)$$

$$= 2a^7b^2 - a^4b^5 + 3a^5b^3c$$

weKí c×wZ : $2a^3 - b^3 + 3abc$

$$\begin{array}{r} 2a^3 - b^3 + 3abc \\ \times\ a^4b^2 \\ \hline 2a^7b^2 - a^4b^5 + 3a^5b^3c \end{array}$$

wb‡Yq ¸Ydj $2a^7b^2 - a^4b^5 + 3a^5b^3c$.

D`vniY 7| $-3x^2zy^3 + 4z^3xy^2 - 5y^4x^3z^2$ †K $-6x^2y^2z$ Øviv ¸Y Ki|

mgvavb : $(-3x^2zy^3 + 4z^3xy^2 - 5y^4x^3z^2) \times (-6x^2y^2z)$

$$= (-3x^2zy^3) \times (-6x^2y^2z) + (4z^3xy^2) \times (-6x^2y^2z) - (5y^4x^3z^2) \times (-6x^2y^2z)$$

$$= \{(-3) \times (-6) \times x^2 \times x^2 \times y^3 \times y^2 \times z \times z\} + \{4 \times (-6) \times x \times x^2 \times y^2 \times y^2 \times z^3 \times z\}$$

$$- \{5 \times (-6) \times x^3 \times x^2 \times y^4 \times y^2 \times z^2 \times z\}$$

$$= 18x^4y^5z^2 + (-24x^3y^4z^4) - (-30x^5y^6z^3)$$

$$= 18x^4y^5z^2 - 24x^3y^4z^4 + 30x^5y^6z^3$$

wb‡Yq ¸Ydj $18x^4y^5z^2 - 24x^3y^4z^4 + 30x^5y^6z^3$.

KvR : 1| c_g iwk‡K wØZxq iwk Øviv ¸Y Ki :

(K) $5a^2 + 8b^2, 4ab$

(L) $3p^2q + 6pq^3 + 10p^3q^5,\ 8p^3q^2$

(M) $-2c^2d + 3d^3c - 5cd^2, -7c^3d^5$.

4·5 eûc`x iwk‡K eûc`x iwk Øviv ¸Y

eûc`x iwk‡K eûc`x iwk Øviv ¸Y Ki‡Z n‡j ¸‡Y¨i c‡Z¨K c`‡K ¸Y‡Ki c‡Z¨K c` Øviv Avjv`v Avjv`vfv‡e ¸Y K‡i m`k c`¸‡jv‡K wb‡P wb‡P mvwR‡q wjL‡Z nq| AZtci wPýh³ iwki †hv‡Mi wbq‡g †hvM Ki‡Z nq| wem`k c` _vK‡j †m¸‡jv‡K c_Kfv‡e wjL‡Z nq Ges ¸Yd‡j emv‡Z nq|

D`vniY 8| $3x+2y$ †K $x+y$ Øviv ¸Y Ki|

mgvavb :

$$\begin{array}{rl}
3x+2y & \leftarrow \text{¸Y¨} \\
x+y & \leftarrow \text{¸YK} \\
\hline
3x^2+2xy & \leftarrow x \text{ Øviv ¸Y} \\
3xy+2y^2 & \leftarrow y \text{ Øviv ¸Y} \\
\hline
3x^2+5xy+2y^2 & \leftarrow \text{¸Ydj}
\end{array}$$

†hvM K‡i, $3x^2+5xy+2y^2$ ¸Ydj

wb‡Yq ¸Ydj $3x^2+5xy+2y^2$.

	$3x$	$2y$
x	$3x^2$	$2xy$
y	$3xy$	$2y^2$

$(3x+2y)\times(x+y)$
$=3x^2+5xy+2y^2$.

D`vniY 9| $a^2-2ab+b^2$ †K $a-b$ Øviv ¸Y Ki|

mgvavb :

$$\begin{array}{rl}
a^2-2ab+b^2 & \leftarrow \text{¸Y¨} \\
a-b & \leftarrow \text{¸YK} \\
\hline
a^3-2a^2b+ab^2 & \leftarrow a \text{ Øviv ¸Y} \\
-a^2b+2ab^2-b^3 & \leftarrow -b \text{ Øviv ¸Y} \\
\hline
a^3-3a^2b+3ab^2-b^3 & \leftarrow \text{¸Ydj}
\end{array}$$

†hvM K‡i, $a^3-3a^2b+3ab^2-b^3$ ¸Ydj

wb‡Yq ¸Ydj $a^3-3a^2b+3ab^2-b^3$.

¸‡Yi wbqg :

(i) c_‡g ¸‡Y¨i c‡Z¨K c`‡K ¸Y‡Ki c_g c` Øviv ¸Y K‡i ¸Ydj wjL‡Z n‡e|

(ii) Gici ¸‡Y¨i c‡Z¨K c`‡K ¸Y‡Ki wØZxq c` Øviv ¸Y K‡i ¸Ydj †ei Ki‡Z n‡e| G ¸Ydj‡K Ggbfv‡e mwR‡q wjL‡Z n‡e †hb Dfq ¸Yd‡ji m`k c`¸‡jv wb‡P wb‡P c‡o|

(iii) cvß `BwU ¸Yd‡ji exRMwYZxq mgwóB n‡jv wb‡Yq ¸Ydj|

D`vniY 10| $2x^2+3x-4$ †K $3x^2-4x-5$ Øviv ¸Y Ki|

mgvavb :

$$\begin{array}{rl}
2x^2+3x-4 & \leftarrow \text{¸Y¨} \\
3x^2-4x-5 & \leftarrow \text{¸YK} \\
\hline
6x^4+9x^3-12x^2 & \leftarrow 3x^2 \text{ Øviv ¸Y} \\
-8x^3-12x^2+16x & \leftarrow -4x \text{ Øviv ¸Y} \\
-10x^2-15x+20 & \leftarrow -5 \text{ Øviv ¸Y} \\
\hline
6x^4+x^3-34x^2+x+20 & \leftarrow \text{¸Ydj}
\end{array}$$

†hvM K‡i, $6x^4+x^3-34x^2+x+20$ ¸Ydj

wb‡Yq ¸Ydj $6x^4+x^3-34x^2+x+20$.

KvR : 1g ivwk‡K 2q ivwk Øviv ¸Y Ki :

(K) $x+7,\ x+9$

(L) $a^2-ab+b^2,\ 3a+4b$

(M) $x^2-x+1,\ 1+x+x^2.$

Abkxjbx 4·1

1g ivwk‡K 2q ivwk Øviv ¸Y Ki (1 †_‡K 24) :

1| $3ab,\ 4a^3$

2| $5xy,\ 6az$

3| $5a^2x^2,\ 3ax^5y$

4| $8a^2b,\ -2b^2$

5| $-2abx^2,\ 10b^3xyz$

6| $-3p^2q^3,\ -6p^5q^4$

7| $-12m^2a^2x^3,\ -2ma^2x^2$

8| $7a^3bx^5y^2,\ -3x^5y^3a^2b^2$

9| $2x+3y,\ 5xy$

10| $5x^2-4xy,\ 9x^2y^2$

11| $2a^2-3b^2+c^2,\ a^3b^2$

12| $x^3-y^3+3xyz,\ x^4y$

13| $2a-3b,\ 3a+2b$

14| $a+b,\ a-b$

15| $x^2+1,\ x^2-1$

16| $a^2+b^2,\ a+b$

17| $a^2-ab+b^2,\ a+b$

18| $x^2+2xy+y^2,\ x+y$

19| $x^2-2xy+y^2,\ x-y$

20| $x^2+2x-3,\ x+3$

21| $a^2+ab+b^2,\ b^2-ab+a^2$

22| $a+b+c,\ a+b+c$

23| $x^2+xy+y^2,\ x^2-xy+y^2$

24| $y^2-y+1,\ 1+y+y^2$

25| $A=x^2+xy+y^2$ Ges $B=x-y$ n‡j, cgvY Ki †h, $AB=x^3-y^3$.

26| $A=a^2-ab+b^2$ Ges $B=a+b$ n‡j, $AB=$ KZ ?

27| †`LvI †h, $(a+1)(a-1)(a^2+1)=a^4-1.$

28| †`LvI †h, $(x+y)(x-y)(x^2+y^2)=x^4-y^4.$

4·6 exRMwYZxq iwki fvM

wPýh³ iwki fvM

Avgiv Rwb, $a\times(-b)=(-a)\times b=-ab$

mZivs, $-ab\div a=a\times(-b)\div a=-b$

GKBfv‡e, $-ab\div b=-a$

$-ab\div(-a)=b$

$-ab\div(-b)=a$

$$-\frac{ab}{a}=\frac{a\times(-b)}{a}=-b$$

$$\frac{-ab}{b}=\frac{(-a)\times b}{b}=-a$$

$$\frac{-ab}{-a}=\frac{(-a)\times b}{-a}=b$$

$$\frac{-ab}{-b}=\frac{a\times(-b)}{-b}=a$$

j¶ Kwi :

$$\boxed{\begin{array}{l}\dfrac{+\ 1}{+\ 1}=+\ 1\\ \dfrac{-\ 1}{-\ 1}=+\ 1\\ \dfrac{-\ 1}{+\ 1}=-\ 1\\ \dfrac{+\ 1}{-\ 1}=-\ 1\end{array}}$$

* GKB wPýh³ `BwU iwki fvMdj (+) wPýh³ n‡e|
* wecixZ wPýh³ `BwU iwki fvMdj (–) wPýh³ n‡e|

fv‡Mi mPK wewa

$$a^5\div a^2=\frac{a^5}{a^2}=\frac{a\times a\times a\times a\times a}{a\times a}=a\times a\times a$$ [je I ni †_‡K mvaviY Drcv`K eRb K‡i]|

$$=a^3=a^{5-2},\ a\neq 0$$

mvaviYfv‡e, $\boxed{a^m\div a^n=a^{m-n}}$, †hLv‡b m I n ¯^vfvweK msL¨v Ges $m>n, a\neq 0.$
GB cwµqv‡K fv‡Mi mPK wewa ejv nq|

j¶ Kwi : $a\neq 0$ n‡j,

$$a^m\div a^m=\frac{a^m}{a^m}=a^{m-m}=a^0$$

Avevi, $a^m\div a^m=\dfrac{a^m}{a^m}=1$

$\therefore\ a^0=1,\ (a\neq 0).$

Abym×vš— : $\boxed{\boldsymbol{a}^0=1, \boldsymbol{a}\neq 0.}$

4·7 GKc`x iwk‡K GKc`x iwk Øviv fvM

GKc`x iwk‡K GKc`x iwk Øviv fvM Ki‡Z n‡j, mvswL¨K mnM‡K cwUMwYZxq wbq‡g fvM Ges exRMwYZxq cZxK‡K mPK wbq‡g fvM Ki‡Z nq|

D`vniY 11| $10a^5b^7$ †K $5a^2b^3$ Øviv fvM Ki|

mgvavb : $\dfrac{10a^5b^7}{5a^2b^3} = \dfrac{10}{5} \times \dfrac{a^5}{a^2} \times \dfrac{b^7}{b^3}$

$= 2 \times a^{5-2} \times b^{7-3} = 2a^3b^4$

wb‡Yq fvMdj $2a^3b^4$

D`vniY 12| $40x^8y^{10}z^5$ †K $-8x^4y^2z^4$ Øviv fvM Ki|

mgvavb : $\dfrac{40x^8y^{10}z^5}{-8x^4y^2z^4} = \dfrac{40}{-8} \times \dfrac{x^8}{x^4} \times \dfrac{y^{10}}{y^2} \times \dfrac{z^5}{z^4}$

$= -5 \times x^{8-4} \times y^{10-2} \times z^{5-4} = -5x^4y^8z$

wb‡Yq fvMdj $-5x^4y^8z$.

D`vniY 13| $-45x^{13}y^9z^4$ †K $-5x^6y^3z^2$ Øviv fvM Ki|

mgvavb : $\dfrac{-45x^{13}y^9z^4}{-5x^6y^3z^2} = \dfrac{-45}{-5} \times \dfrac{x^{13}}{x^6} \times \dfrac{y^9}{y^3} \times \dfrac{z^4}{z^2}$

$= 9 \times x^{13-6} \times y^{9-3} \times z^{4-2} = 9x^7y^6z^2$

wb‡Yq fvMdj $9x^7y^6z^2$

KvR : c_g iwk‡K wØZxq iwk Øviv fvM Ki :

(K) $12a^3b^5c$, $3ab^2$ (L) $-28p^3q^2r^5$, $7p^2qr^3$

(M) $35x^5y^7$, $-5x^5y^2$ (N) $-40x^{10}y^5z^9$, $-8x^6y^2z^5$

4·8 eûc`x iwk‡K GKc`x iwk Øviv fvM

Avgiv Rwb, $a+b+c$ GKwU eûc`x iwk|

GLb $(a+b+c)\div d$

$$= (a+b+c)\times\frac{1}{d}$$

$$= a\times\frac{1}{d}+b\times\frac{1}{d}+c\times\frac{1}{d} \qquad [¸‡Yi eÈb wewa]$$

$$= \frac{a}{d}+\frac{b}{d}+\frac{c}{d}$$

Avevi, $(a+b+c)\div d$

$$= \frac{a+b+c}{d}=\frac{a}{d}+\frac{b}{d}+\frac{c}{d}$$

D`vniY 14| $10x^5y^3-12x^3y^8+6x^4y^7$ †K $2x^2y^2$ Øviv fvM Ki|

mgvavb : $\dfrac{10x^5y^3-12x^3y^8+6x^4y^7}{2x^2y^2}$

$$= \frac{10x^5y^3}{2x^2y^2}-\frac{12x^3y^8}{2x^2y^2}+\frac{6x^4y^7}{2x^2y^2}$$

$$= 5x^{5-2}y^{3-2}-6x^{3-2}y^{8-2}+3x^{4-2}y^{7-2}$$

$$= 5x^3y-6xy^6+3x^2y^5$$

wb‡Yq fvMdj $5x^3y-6xy^6+3x^2y^5$.

D`vniY 15| $35a^5b^4c+20a^6b^8c^3-40a^5b^6c^4$ †K $5a^2b^3c$ Øviv fvM Ki|

mgvavb : $\dfrac{35a^5b^4c+20a^6b^8c^3-40a^5b^6c^4}{5a^2b^3c}$

$$= \frac{35a^5b^4c}{5a^2b^3c}+\frac{20a^6b^8c^3}{5a^2b^3c}-\frac{40a^5b^6c^4}{5a^2b^3c}$$

$$= 7a^{5-2}b^{4-3}c^{1-1}+4a^{6-2}b^{8-3}c^{3-1}-8a^{5-2}b^{6-3}c^{4-1}$$

$$= 7a^3b+4a^4b^5c^2-8a^3b^3c^3 \qquad [\because c^{1-1}=c^0=1]$$

wb‡Yq fvMdj $7a^3b+4a^4b^5c^2-8a^3b^3c^3$.

KvR : 1| $9x^4y^5+12x^8y^5+21x^9y^6$ †K $3x^3y^2$ Øviv fvM Ki|

2| $28a^5b^6-16a^6b^8-20a^7b^5$ †K $4x^4y^3$ Øviv fvM Ki|

4·9 eûc`x iwk‡K eûc`x iwk Øviv fvM

eûc`x iwk‡K eûc`x iwk Øviv fvM Kivi †¶‡Î c_‡g fvR¨ I fvRK Df‡qi g‡a¨ Av‡Q Ggb GKwU exRMwYZxq cZx‡Ki NvZi Aatµg Abmv‡i iwkØq‡K mvRv‡Z n‡e| Gici cwUMwY‡Zi fvM cÖwµqvi g‡Zv wb‡Pi wbq‡g avc avc fvM Ki‡Z n‡e|

* fv‡R¨i c_g c`wU‡K fvR‡Ki c\_g c` Øviv fvM Ki‡j †h fvMdj nq Zv wb‡Yq fvMd‡ji c_g c`|
* fvMd‡ji H c_g c` Øviv fvR‡Ki c‡Z¨K c`‡K ¸Y K‡i ¸Ydj m`k c` Abhvqx fv‡R¨i wb‡P ewm‡q fvR¨ †_‡K we‡qvM Ki‡Z n‡e|
* we‡qvMdj bZb fvR¨ n‡e| we‡qvMdj Ggbfv‡e wjL‡Z n‡e †hb Zv Av‡Mi g‡Zv we‡eP¨ cZx‡Ki Aatµg Abmv‡i _v‡K|
* bZb fv‡R¨i c_g c`wU‡K fvR‡Ki c\_g c` Øviv fvM Ki‡j †h fvMdj nq Zv wb‡Yq fvMd‡ji wØZxq c`|
* Gfv‡e µgvš‡q fvM Ki‡Z n‡e|

D`vniY 16| $6x^2+x-2$ †K $2x-1$ Øviv fvM Ki|

mgvavb : GLv‡b fvR¨ I fvRK Df‡qB x Gi Nv‡Zi Aatµg Abmv‡i mvRv‡bv Av‡Q|

$$
\begin{array}{l}
2x-1\,)\,6x^2+x-2\,(\,3x+2\\
\qquad\quad 6x^2-3x\\
\qquad (-)\quad (+)\\
\hline
\qquad\qquad\quad 4x-2\\
\qquad\qquad\quad 4x-2\\
\qquad\qquad (-)\;(+)\\
\hline
\qquad\qquad\qquad 0
\end{array}
$$

1g avc : $6x^2 \div 2x = 3x$

2q avc : $4x \div 2x = 2$

wb‡Yq fvMdj $3x+2$.

D`vniY 17| $2x^2-7xy+6y^2$ †K $x-2y$ Øviv fvM Ki|

mgvavb : GLv‡b iwk `BwU x Gi Nv‡Zi Aatµg Abmv‡i mvRv‡bv Av‡Q|

$$
\begin{array}{l}
x-2y\,)\,2x^2-7xy+6y^2\,(\,2x-3y\\
\qquad\quad 2x^2-4xy\\
\qquad (-)\quad (+)\\
\hline
\qquad\qquad\quad -3xy+6y^2\\
\qquad\qquad\quad -3xy+6y^2\\
\qquad\qquad (+)\quad (-)\\
\hline
\qquad\qquad\qquad 0
\end{array}
$$

1g avc : $2x^2 \div x = 2x$

2q avc : $-3xy \div x = -3y$

wb‡Yq fvMdj $2x-3y$.

D`vniY 18| $16x^4+36x^2+81$ †K $4x^2-6x+9$ Øviv fvM Ki|
mgvavb : GLv‡b iwk `BwU x Gi Nv‡Zi Aatµg Abmv‡i mvRv‡bv Av‡Q|

$$4x^2-6x+9\,)\,16x^4+36x^2+81\,(\,4x^2+6x+9$$
$$16x^4+36x^2-24x^3$$
$$(-)\quad(-)\quad(+)$$
$$24x^3+81$$
$$24x^3-36x^2+54x$$
$$(-)\quad(+)\quad(-)$$
$$36x^2-54x+81$$
$$36x^2-54x+81$$
$$(-)\quad(+)\quad(-)$$
$$0$$

1g avc : $16x^4 \div 4x^2 = 4x^2$

2q avc : $24x^3 \div 4x^2 = 6x$

3q avc : $36x^2 \div 4x^2 = 9$

wb‡Yq fvMdj $4x^2+6x+9$.

gšÍe¨ : 2q av‡c bZb fvR¨‡K I x Gi Nv‡Zi Aatµg Abmv‡i mwR‡q †jLv n‡q‡Q|

D`vniY 19| $2x^4+110-48x$ †K $4x+11+x^2$ Øviv fvM Ki|

mgvavb : fvR¨ I fvRK Dfq‡K x Gi Nv‡Zi Aatµg Abmv‡i mwR‡q cvB,

fvR¨ = $2x^4+110-48x = 2x^4-48x+110$

fvRK = $4x+11+x^2 = x^2+4x+11$

GLb, $x^2+4x+11\,)\,2x^4-48x+110\,(\,2x^2-8x+10$
$$2x^4+8x^3+22x^2$$
$$-8x^3-22x^2-48x+110$$
$$-8x^3-32x^2-88x$$
$$10x^2+40x+110$$
$$10x^2+40x+110$$
$$0$$

wb‡Yq fvMdj $2x^2-8x+10$.

D`vniY 20| x^4-1 †K x^2+1 Øviv fvM Ki|

mgvavb : GLv‡b iwk `BwU x Gi Nv‡Zi Aatµg Abmv‡i mvRv‡bv Av‡Q|

$$\begin{array}{r|l} x^2+1\,) & x^4-1\,(\,x^2-1 \\ & \underline{x^4+x^2} \\ & \quad -x^2-1 \\ & \quad \underline{-x^2-1} \\ & \qquad 0 \end{array}$$

wb‡Yq fvMdj x^2-1.

KvR : 1| $2m^2-5mn+2n^2$ †K $2m-n$ Øviv fvM Ki|

2| $a^4+a^2b^2+b^4$ †K a^2-ab+b^2 Øviv fvM Ki|

3| $81p^4+q^4-22p^2q^2$ †K $9p^2+2pq-q^2$ Øviv fvM Ki|

Abkxjbx 4·2

c_g iwk‡K wØZxq iwk Øviv fvM Ki :

1| $45a^4,\ 9a^2$

2| $-24a^5,\ 3a^2$

3| $30a^4x^3,\ -6a^2x$

4| $-28x^4y^3z^2,\ 4xy^2z$

5| $-36a^3z^3y^2,\ -4ayz$

6| $-22x^3y^2z,\ -2xyz$

7| $3a^3b^2-2a^2b^3,\ a^2b^2$

8| $36x^4y^3+9x^5y^2,\ 9xy$

9| $a^3b^4-3a^7b^7,\ -a^3b^3$

10| $6a^5b^3-9a^3b^4,\ 3a^2b^2$

11| $15x^3y^3+12x^3y^2-12x^5y^3,\ 3x^2y^2$

12| $6x^8y^6z-4x^4yz+2x^2y^2z^2,\ 2x^2y^2z$

13| $24a^2b^2c-15a^4b^4c^4-9a^2b^6c^2,\ -3ab^2$

14| $a^3b^2+2a^2b^3,\ a+2b$

15| $6x^2+x-2,\ 2x-1$

16| $6y^2+3x^2-11xy,\ 3x-2y$

17| $x^3+y^3,\ x+y$

18| $a^2+4axyz+4x^2y^2z^2,\ a+2xyz$

19| $16p^4-81q^4,\ 2p+3q$

20| $64-a^3,\ a-4$

21| $x^2-8xy+16y^2,\ x-4y$

22| $x^4+8x^2+15,\ x^2+5$

23| $x^4+x^2+1,\ x^2-x+1$

24| $4a^4+b^4-5a^2b^2,\ 4a^2-b^2$

25| $2a^2b^2+5abd+3d^2,\ ab+d$

26| $x^4y^4-1,\ x^2y^2+1$

27| $1-x^6,\ 1-x+x^2$

28| $x^2-8abx+15a^2b^2,\ x-3ab$

29| $x^3y-2x^2y^2+axy,\ x^2-2xy+a$

30| $a^2bc+b^2ca+c^2ab,\ a+b+c$

31| $a^2x-4ax+3ax^2,\ a+3x-4$

32| $81x^4+y^4-22x^2y^2,\ 9x^2+2xy-y^2$

33| $12a^4+11a^2+2,\ 3a^2+2$

34| $x^4+x^2y^2+y^4,\ x^2-xy+y^2$

35| $a^5+11a-12,\ a^2-2a+3$

4·10 eÜbxi e¨envi

GKwU ¯‡ji g¨v‡bwRs KwgwU Zv‡`i ¯‡ji 10 Rb Mixe wk¶v_©xi Rb¨ `t¯' Kj¨vY Znwej †_‡K a UvKv eivÏ Kij| †mB UvKv †_‡K c‡Z¨K wk¶v_©x‡K cwZwU b UvKv g‡j¨i 2wU K‡i LvZv I cwZwU c UvKv g‡j¨i 1wU K‡i Kjg weZiY Kiv n‡jv| G‡Z wKQ UvKv DØË n‡jv| GB UvKvi mv‡_ Avil d UvKv †hvM K‡i Zv 2 Rb cwZeÜx wk¶v_©xi g‡a¨ mgvbfv‡e fvM K‡i †`Iqv n‡jv|

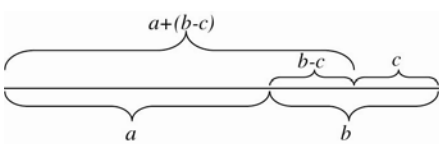
Dc‡ii ewYZ Z_¨¸‡jv‡K exRMwYZxq iwki gva¨‡g cKvk Ki‡Z cwi :

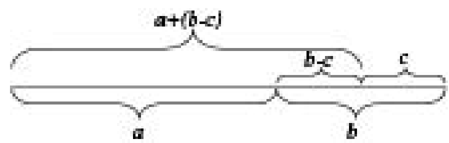
$$[\{a-(2b+c)\times 10\}+d]\div 2$$

GLv‡b, 1g eÜbx ( ), 2q eÜbx { }, 3q eÜbx [ ] e¨envi Kiv n‡q‡Q| eÜbx ¯’vc‡bi wbqg n‡”Q [{()}]| G Qvov I iwkwU‡Z cwµqv wPý +, −, × I ÷ e¨envi Kiv n‡q‡Q| Gi¸c iwki mijxKi‡Y '*BODMAS*' AbmiY Kiv nq| Avevi, eÜbxi †¶‡Î chvqµ‡g 1g, 2q I 3q eÜbxi KvR Ki‡Z nq|

eÜbx AcmviY :

j¶ Kwi : $b>c$

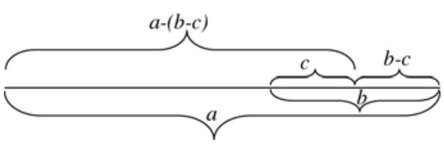


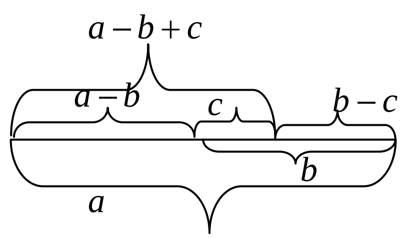


wP‡Î †`Lv hvq, $a+(b-c)=a+b-c$

eÜbxi Av‡M Ô+Õ wPý _vK‡j, eÜbx Acmvi‡Y eÜbxi wfZ‡ii c`¸‡jvi wP‡ýi cwieZb nq bv|

Avevi, j¶ Kwi : $b>c$, $a>b-c$





wP‡Î †`Lv hvq, $a-(b-c)=a-b+c$

eÜbxi Av‡M Ô–Õ wPý _vK‡j, eÜbx Acmvi‡Y eÜbxi wfZ‡ii c`¸‡jvi wP‡ýi cwieZb n‡q wecixZ wPýh³ nq|

KvR : wb‡Pi iwk¸‡jvi gvb AcwiewZZ †i‡L eÜbx ¯vcb Ki :			
iwk	eÜbxi Av‡Mi wPý	eÜbxi Ae¯vb	eÜbxh³ iwk
$7+5-2$	+	2q I 3q c` 1g eÜbxf³	
$7-5+2$	$-$	Ó Ó	
$a-b+c-d$	+	3q I 4_ c` 1g eÜbxf³	
$a-b-c-d$	$-$	Ó Ó	

KvR : wb‡Pi iwk¸‡jvi eÜbx AcmviY Ki :	
eÜbxh³ iwk	eÜbxg³ iwk
$8+(6-2)$	
$8-(6-2)$	
$p+q+(r-s)$	
$p+q-(r-s)$	

D`vniY 21| mij Ki : $6-2\{5-(8-3)+(5+2)\}$.

mgvavb : $6-2\{5-(8-3)+(5+2)\}$.

$$= 6-2\{5-5+7\}$$
$$= 6-2\{+7\}$$
$$= 6-14$$
$$= -8.$$

D`vniY 22| mij Ki : $a+\{b-(c-d)\}$.

mgvavb : $a+\{b-(c-d)\}$

$$= a+\{b-c+d\}$$
$$= a+b-c+d.$$

D`vniY 23| mij Ki : $a-[b-\{c-(d-e)\}-f]$

mgvavb : $a-[b-\{c-(d-e)\}-f]$

$$= a-[b-\{c-d+e\}-f]$$
$$= a-[b-c+d-e-f]$$
$$= a-b+c-d+e+f.$$

D`vniY 24| mij Ki : $3x-[5y-\{10z-(5x-10y+3z)\}]$.

mgvavb : $3x-[5y-\{10z-(5x-10y+3z)\}]$

$=3x-[5y-\{10z-5x+10y-3z\}]$

$=3x-[5y-\{7z-5x+10y\}]$

$=3x-[5y-7z+5x-10y]$

$=3x-[5x-5y-7z]$

$=3x-5x+5y+7z$

$=-2x+5y+7z$

$=5y-2x+7z.$

D`vniY 25| $3x-4y-8z+5$ Gi ZZxq I PZ_ c` eÜbxi Av‡M (–) wPý w`‡q c_g eÜbxf³ Ki| cieZx‡Z wØZxq c` I c_g eÜbxf³ iwk‡K wØZxq eÜbxf³ Ki †hb eÜbxi Av‡M (–) wPý _v‡K|

mgvavb : $3x-4y-8z+5$ iwkwUi ZZxq I PZ_ c` h_vµ‡g $8z$ I 5.
ckvbmv‡i, $3x-4y-(8z-5)$
Avevi, $3x-\{4y+(8z-5)\}$.

KvR : mij Ki :
1| $x-\{2x-(3y-4x+2y)\}$
2| $8x+y-[7x-\{5x-(4x-3x-y)+2y\}]$

Abkxjbx 4·3

1| $3a^2b$ Ges $-4ab^2$ Gi ¸Ydj wb‡Pi †KvbwU ?
(K) $-12a^2b^2$ (L) $-12a^3b^2$ (M) $-12a^2b^3$ (N) $-12a^3b^3$

2| $20a^6b^3$ †K $4a^3b$ Øviv fvM Ki‡j fvMdj wb‡Pi †KvbwU ?
(K) $5a^3b$ (L) $5a^6b^2$ (M) $5a^3b^2$ (N) $5a^3b^3$

3| $\dfrac{-25x^3y}{5xy^3}$ = KZ ?
(K) $-5x^2y^2$ (L) $5x^2y^2$ (M) $\dfrac{5x^2}{y^2}$ (N) $\dfrac{-5x^2}{y^2}$

4| $a=3, b=2$ n‡j, $(8a-2b)+(-7a+4b)$ Gi gvb KZ ?
(K) 3 (L) 4 (M) 7 (N) 15

5| $x=-1$ n‡j, x^3+2x^2-1 Gi gvb wb‡Pi †KvbwU ?

(K) 0 (L) -1 (M) 1 (N) -2

6| $10x^6y^5z^4$ †K $-5x^2y^2z^2$ Øviv fvM Ki‡j fvMdj KZ n‡e ?

(K) $-2x^4y^2z^3$ (L) $-2x^4y^3z^2$ (M) $-2x^3y^3z^3$ (N) $-2x^4y^3z^3$

7| $4a^4-6a^3+3a+14$ GKwU evRMwYZxq ivwk| GKRb wk¶v_x© ivwkwU †_‡K wb‡Pi Z_¨¸‡jv wjL‡jv|

(i) eûc`x ivwkwUi PjK a

(ii) eûc`xwUi gvÎv 4

(iii) a^3 Gi mnM 6

Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK ?

(K) i I ii (L) ii I iii (M) i I iii (N) i, ii I iii

8| 2 eQi c‡e evev‡ji eqm x eQi Ges Zvi gv‡qi eqm $5x$ eQi wQj| Zvn‡j

(1) gv‡qi eZ©gvb eqm KZ ?

(K) x eQi (L) $5x$ eQi (M) $(x+2)$ eQi (N) $(5x+2)$ eQi

(2) `BR‡bi eZ©gvb eq‡mi mgwó KZ ?

(K) $6x$ eQi (L) $(5x+4)$ eQi (M) $(6x+4)$ eQi (N) $(6x+2)$ eQi

(3) `BR‡bi eZ©gvb eq‡mi cv_©K¨ KZ ?

(K) $(6x-4)$ eQi (L) $(4x-2)$ eQi (M) $(x-2)$ eQi (N) $4x$ eQi

mij Ki (9 †_‡K 23) :

9| $7+2[-8-\{-3-(-2-3)\}-4]$

10| $-5-[-8-\{-4-(-2-3)\}+13]$

11| $7-2[-6+3\{-5+2(4-3)\}]$

12| $x-\{a+(y-b)\}$

13| $3x+(4y-z)-\{a-b-(2c-4a)-5a\}$

14| $-a+[-5b-\{-9c+(-3a-7b+11c)\}]$

15| $-a-[-3b-\{-2a-(-a-4b)\}]$

16| $\{2a-(3b-5c)\}-[a-\{2b-(c-4a)\}-7c]$

17| $-a+[-6b-\{-15c+(-3a-9b-13c)\}]$

18| $-2x-[-4y-\{-6z-(8x-10y+12z)\}]$

19| $3x-5y+[2+(3y-x)+\{2x-(x-2y)\}]$

20| $4x+[-5y-\{9z+(3x-7y+x)\}]$

21| $20-[\{(6a+3b)-(5a-2b)\}+6]$

22| $15a+2[3b+3\{2a-2(2a+b)\}]$

23| $[8b-3\{2a-3(2b+5)-5(b-3)\}]-3b$

24| eÜbxi c‡e (–) wPý w`‡q $a-b+c-d$ Gi 2q, 3q I 4\_ c` c_g eÜbxi wfZi ¯’vcb Ki|

25| $a-b-c+d-m+n-x+y$ iwk‡Z eÜbxi Av‡M (–) wPý w`‡q 2q, 3q I 4\_ c` I (+) wPý w`‡q 6ô I 7g c` c_g eÜbxf³ Ki|

26| $7x-5y+8z-9$ Gi ZZxq I PZ_ c` eÜbxi Av‡M (–) wPý w`‡q c_g eÜbxf³ Ki| c‡i wØZxq c` I c_g eÜbxf³ iwk‡K wØZxq eÜbxf³ Ki †hb eÜbxi Av‡M (+) wPý _v‡K|

27| $15x^2+7x-2$ Ges $5x-1$ `BwU exRMwYZxq iwk|

(K) c_g iwk †_‡K wØZxq iwk we‡qvM Ki|

(L) iwkØ‡qi ¸Ydj wbYq Ki|

(M) c_g iwk‡K wØZxq iwk Øviv fvM Ki|

28| $2x+y$, $3x-z$ Ges $x-4y-3z+2$ wZbwU exRMwYZxq iwk|

(K) c_g I wØZxq iwki †hvMdj †ei Ki|

(L) ZZxq iwki †hvMvZ¥K wecixZ iwk †jL Ges c_g I wØZxq iwki †hvMdj †_‡K cvß ZZxq iwk we‡qvM Ki|

(M) mij Ki : $7+[(2x+y)-\{(3x-z)-(x-4y-3z+2)+10\}$

(N) ZZxq iwk‡K c_g iwk Øviv ¸Y Ki|

cÂg Aa¨vq

exRMwYZxq mÎvewj I c‡qvM

exRMwYZxq cZxK Øviv cKwkZ †h‡Kv‡bv mvaviY wbqg ev wm×vš‡K exRMwYZxq mÎ ev ms‡¶‡c mÎ ejv nq| Avgiv wewfb †¶‡Î mÎ e¨envi K‡i _wK| G Aa¨v‡q c_g PviwU mÎ Ges G PviwU m‡Îi mvnv‡h¨ Abwm×vš wbY‡qi c×wZ †`Lv‡bv n‡q‡Q| G Qvov exRMwYZxq mÎ I Abwm×vš c‡qvM K‡i exRMwYZxq iwki gvb wbYq I Drcv`‡K we‡kwlY Dc¯vcb Kiv n‡q‡Q| Avevi exRMwYZxq iwki mvnv‡h¨ fvR¨, fvRK, ¸YbxqK, ¸wYZK m¤ú‡K avivYv †`Iqv n‡q‡Q Ges Kxfv‡e Aba¶ wZbwU exRMwYZxq iwki M.mv.¸. I j.mv.¸. wbYq Kiv hvq Zv Av‡jvPbv Kiv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv –

- eM wbY‡q exRMwYZxq m‡Îi eYbv I c‡qvM Ki‡Z cvi‡e|
- exRMwYZxq mÎ I Abwm×vš c‡qvM K‡i iwki gvb wbYq Ki‡Z cvi‡e|
- exRMwYZxq mÎ c‡qvM K‡i Drcv`‡K we‡kwlY Ki‡Z cvi‡e|
- ¸YbxqK I ¸wYZK Kx Zv e¨vL¨v Ki‡Z cvi‡e|
- Aba¶ wZbwU exRMwYZxq iwki mvswL¨K mnMmn M.mv.¸. I j.mv.¸. wbYq Ki‡Z cvi‡e|

5.1 exRMwYZxq mÎvewj

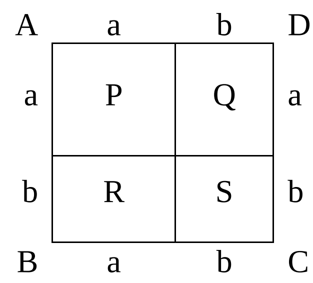
mÎ 1| $\quad (\boldsymbol{a}+\boldsymbol{b})^2 = \boldsymbol{a}^2 + 2\boldsymbol{ab} + \boldsymbol{b}^2$

cgvY : $\quad (a+b)^2$ Gi A_ $(a+b)$ †K $(a+b)$ Øviv ¸Y|

$$
\begin{aligned}
\therefore (a+b)^2 &= (a+b)(a+b) \\
&= a(a+b) + b(a+b) \\
&= a^2 + ab + ba + b^2 \\
&= a^2 + ab + ab + b^2
\end{aligned}
$$

$$\therefore (a+b)^2 = a^2 + 2ab + b^2$$

`BwU iwki †hvMd‡ji eM = 1g iwki eM + 2 × 1g iwk × 2q iwk + 2q iwki eM mÎwUi R¨wgwZK e¨vL¨v :



$ABCD$ GKwU eM‡¶Î hvi

AB evû $= a+b$

BC evû $= a+b$

eM‡¶ÎwU‡K a I b Øviv Ggbfv‡e fvM Kiv n‡q‡Q, †hLv‡b PviwU †¶Î P, Q, R, S cvIqv †M‡Q|

GLv‡b P I S eM‡¶Î Ges Q I R AvqZ‡¶Î|

Avgiv Rwb, eM‡¶‡Îi †¶Îdj = (ˆ`N¨)$^2$ Ges AvqZ‡¶‡Îi †¶Îdj = ˆ`N¨ × c¯

AZGe, P Gi †¶Îdj $= a\times a = a^2$

Q Gi †¶Îdj $= a\times b = ab$

R Gi †¶Îdj $= a\times b = ab$

S Gi †¶Îdj $= b\times b = b^2$

GLb, $ABCD$ eM‡¶‡Îi †¶Îdj = $(P+Q+R+S)$ Gi †¶Îdj

$$\therefore\ (a+b)^2 = a^2+ab+ab+b^2$$
$$= a^2+2ab+b^2$$
$$\therefore\ (a+b)^2 = a^2+2ab+b^2$$

Abwm×vš 1| $\boldsymbol{a^2+b^2=(a+b)^2-2ab}$

Avgiv Rwb, $(a+b)^2 = a^2+2ab+b^2$

ev, $(a+b)^2-2ab = a^2+2ab+b^2-2ab$ [Dfqc¶ †_‡K $2ab$ we‡qvM K‡i]

ev, $(a+b)^2-2ab = a^2+b^2$

$$\therefore\ a^2+b^2=(a+b)^2-2ab.$$

D`vniY 1| $(m+n)$ Gi eM wbYq Ki|

mgvavb : $(m+n)^2$
$$= (m)^2+2\times m\times n+(n)^2$$
$$= m^2+2mn+n^2$$

D`vniY 2| $(3x+4)$ Gi eM wbYq Ki|

mgvavb : $(3x+4)^2$
$$= (3x)^2+2\times 3x\times 4+(4)^2$$
$$= 9x^2+24x+16$$

D`vniY 3| $(2x+3y)$ Gi eM© wbYq Ki|

mgvavb : $(2x+3y)^2$

$= (2x)^2 + 2\times 2x\times 3y + (3y)^2$
$= 4x^2 + 12xy + 9y^2$

D`vniY 4| e‡Mi mÎ c‡qvM K‡i 105 Gi eM© wbYq Ki|

mgvavb : $(105)^2 = (100+5)^2$

$= (100)^2 + 2\times 100\times 5 + (5)^2$
$= 10000 + 1000 + 25$
$= 11025$

KvR : m‡Îi mvnv‡h¨ iwk¸‡jvi eM© wbYq Ki :
1| $x+2y$ 2| $3a+5b$ 3| $5+2a$ 4| 15   5| 103

mÎ 2| $\boldsymbol{(a-b)^2 = a^2 - 2ab + b^2}$

cgvY : $(a-b)^2$ Gi A_© $(a-b)$ †K $(a-b)$ Øviv ¸Y|

$\therefore (a-b)^2 = (a-b)(a-b)$
$= a(a-b) - b(a-b)$
$= a^2 - ab - ba + b^2$
$= a^2 - ab - ab + b^2$
$\therefore (a-b)^2 = a^2 - 2ab + b^2$

`BwU iwki we‡qvMd‡ji eM© = 1g iwki eM© – 2 × 1g iwk × 2q iwk + 2q iwki eM©

j¶ Kwi : wØZxq mÎwU c_g m‡Îi mvnv‡h¨I wbYq Kiv hvq|

Avgiv Rwb, $(a+b)^2 = a^2 + 2ab + b^2$

$\therefore \{(a+(-b)\}^2 = a^2 + 2\times a\times(-b) + (-b)^2$ [b Gi cwie‡Z© $-b$ ewm‡q]
$= a^2 - 2ab + b^2$

Abwm×vš— 2| $\boldsymbol{a^2 + b^2 = (a-b)^2 + 2ab}$

Avgiv Rwb, $(a-b)^2 = a^2 - 2ab + b^2$

ev, $(a-b)^2 + 2ab = a^2 - 2ab + b^2 + 2ab$ [Dfqc‡¶ $2ab$ †hvM K‡i]

ev, $(a-b)^2 + 2ab = a^2 + b^2$

$\therefore\ a^2 + b^2 = (a-b)^2 + 2ab$

D`vniY 5| $p-q$ Gi eM© wbY©q Ki|

mgvavb : $(p-q)^2$

$= (p)^2 - 2\times p\times q + (q)^2$

$= p^2 - 2pq + q^2$

D`vniY 6| $(5x-3y)$ Gi eM© wbY©q Ki|

mgvavb : $(5x-3y)^2$

$= (5x)^2 - 2\times 5x\times 3y + (3y)^2$

$= 25x^2 - 30xy + 9y^2$

D`vniY 7| e‡Mi mÎ c‡qvM K‡i 98 Gi eM© wbY©q Ki|

mgvavb : $(98)^2 = (100-2)^2$

$= (100)^2 - 2\times 100\times 2 + (2)^2$

$= 10000 - 400 + 4$

$= 9604$

KvR : m‡Îi mvnv‡h¨ iwk¸‡jvi eM© wbY©q Ki :

1| $5x-3$ 2| $ax-by$ 3| 95 4| $5x-6$

c_g I wØZxq m‡Îi Avi I K‡qKwU Abywm×vš— :

Abywm×vš— 3| $(a+b)^2 = a^2 + 2ab + b^2$

$= a^2 + b^2 - 2ab + 4ab \quad [\because +2ab = -2ab + 4ab]$

$= (a-b)^2 + 4ab$

$\therefore \quad \boldsymbol{(a+b)^2 = (a-b)^2 + 4ab}$

Abywm×vš— 4| $(a-b)^2 = a^2 - 2ab + b^2$

$= a^2 + b^2 + 2ab - 4ab \quad [\because -2ab = +2ab - 4ab]$

$= (a+b)^2 - 4ab$

$\therefore\ \boldsymbol{(a-b)^2 = (a+b)^2 - 4ab}$

Abywm×vš— 5| $(a+b)^2 + (a-b)^2 = (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2)$

$$= a^2 + 2ab + b^2 + a^2 - 2ab + b^2$$
$$= 2a^2 + 2b^2$$
$$= 2(a^2 + b^2)$$

$\therefore\ \boldsymbol{(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)}$

Abwm×vš 6| $(a+b)^2 - (a-b)^2 = (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$

$$= a^2 + 2ab + b^2 - a^2 + 2ab - b^2$$
$$= 4ab$$

$\therefore\ \boldsymbol{(a+b)^2 - (a-b)^2 = 4ab}$

D`vniY 8| $a+b=7$ Ges $ab=9$ n‡j, a^2+b^2 Gi gvb wbYq Ki|

mgvavb : $a^2 + b^2 = (a+b)^2 - 2ab$

$$= (7)^2 - 2\times 9$$
$$= 49 - 18$$
$$= 31$$

D`vniY 9| $a+b=5$ Ges $ab=6$ n‡j, $(a-b)^2$ Gi gvb wbYq Ki|

mgvavb : $(a-b)^2 = (a+b)^2 - 4ab$

$$= (5)^2 - 4\times 6$$
$$= 25 - 24$$
$$= 1$$

D`vniY 10| $p - \frac{1}{p} = 8$ n‡j, cgvY Ki †h, $p^2 + \frac{1}{p^2} = 66$.

mgvavb : $p^2 + \frac{1}{p^2} = \left(p - \frac{1}{p}\right)^2 + 2\times p \times \frac{1}{p}$ $\quad [\because a^2 + b^2 = (a-b)^2 + 2ab]$

$$= (8)^2 + 2$$
$$= 64 + 2$$
$$= 66$$ (cgwYZ)

weKí c×wZ :

†`Iqv Av‡Q , $p - \frac{1}{p} = 8$

$\therefore \left(p-\frac{1}{p}\right)^2 = (8)^2$ [Dfqc¶‡K eM K‡i]

ev, $p^2 - 2\times p\times\frac{1}{p} + \left(\frac{1}{p}\right)^2 = 64$

ev, $p^2+\frac{1}{p^2}-2=64$

ev, $p^2+\frac{1}{p^2}=64+2$

$\therefore p^2+\frac{1}{p^2}=66$ (cgwYZ)

KvR : 1| $a+b=4$ Ges $ab=2$ n‡j, $(a-b)^2$ Gi gvb wbYq Ki|

2| $a-\frac{1}{a}=5$ n‡j, †`LvI †h, $a^2+\frac{1}{a^2}=27.$

D`vniY 11| $a+b+c$ Gi eM wbYq Ki|

mgvavb : awi, $a+b=p$

$\therefore (a+b+c)^2$

$=\{(a+b)+c\}^2=(p+c)^2$

$=p^2+2pc+c^2$

$=(a+b)^2+2\times(a+b)\times c+c^2$ [p-Gi gvb emm‡q]

$=a^2+2ab+b^2+2ac+2bc+c^2$

$=a^2+b^2+c^2+2ab+2bc+2ca$

weKí mgvavb :

$(a+b+c)^2$

$=\{(a+b)+c\}^2$

$=(a+b)^2+2\times(a+b)\times c+c^2$

$=a^2+2ab+b^2+2ac+2bc+c^2$

$=a^2+b^2+c^2+2ab+2ac+2bc$

$=a^2+b^2+c^2+2ab+2bc+2ca$

KvR : 1| $a+b+c$ Gi eM wbYq Ki, †hLv‡b $(b+c)=m$

2| $a+b+c$ Gi eM wbYq Ki, †hLv‡b $(a+c)=n$

D`vniY 12| $(x+y-z)$ Gi eM wbYq Ki|

mgvavb : awi, $x+y=m$

$$\therefore (x+y-z)^2 = \{x+y)-z\}^2$$
$$= (m-z)^2$$
$$= m^2 - 2mz + z^2$$
$$= (x+y)^2 - 2\times(x+y)\times z + z^2$$ [m-Gi gvb emv‡q]
$$= x^2 + 2xy + y^2 - 2xz - 2yz + z^2$$
$$= x^2 + y^2 + z^2 + 2xy - 2xz - 2yz$$

D`vniY 13| $3x - 2y + 5z$ Gi eM© wbY©q Ki|

mgvavb : $(3x-2y+5z)^2$

$$= \{(3x-2y)+5z\}^2$$
$$= (3x-2y)^2 + 2\times(3x-2y)\times 5z + (5z)^2$$ [∵1g iwk $3x-2y$, 2q iwk= $5z$]
$$= (3x)^2 - 2\times 3x\times 2y + (2y)^2 + 2\times 5z(3x-2y) + 25z^2$$
$$= 9x^2 - 12xy + 4y^2 + 30xz - 20yz + 25z^2$$
$$= 9x^2 + 4y^2 + 25z^2 - 12xy + 30xz - 20yz.$$

D`vniY 14| mij Ki : $(2x+3y)^2 - 2(2x+3y)(2x-5y) + (2x-5y)^2$

mgvavb : awi, $2x+3y = a$ Ges $2x-5y = b$

c`Ë iwk = $a^2 - 2ab + b^2$

$$= (a-b)^2$$
$$= \{(2x+3y)-(2x-5y)\}^2$$ [a I b Gi gvb emv‡q]
$$= \{2x+3y-2x+5y\}^2$$
$$= (8y)^2$$
$$= 64y^2$$

D`vniY 15| $x = 7$ Ges $y = 6$ n‡j, $16x^2 - 40xy + 25y^2$ Gi gvb wbY©q Ki|

mgvavb : c`Ë iwk = $16x^2 - 40xy + 25y^2$

$$= (4x)^2 - 2\times 4x \times 5y + (5y)^2$$
$$= (4x-5y)^2$$
$$= (4\times 7 - 5\times 6)^2 \quad [x \text{ I } y \text{ Gi gvb ewm‡q}]$$
$$= (28-30)^2$$
$$= (-2)^2 = (-2)\times(-2)$$
$$= 4$$

KvR :
1| $3x-2y-z$ Gi eM wbYq Ki|
2| mij Ki : $(5a-7b)^2 + 2(5a-7b)(9b-4a) + (9b-4a)^2$
3| $x=3$ n‡j, $9x^2-24x+16$ Gi gvb KZ ?

Abkxjbx 5·1

m‡Îi mvnv‡h¨ eM wbYq Ki (1–16) :

1| $a+5$	2| $5x-7$	3| $3a-11xy$	4| $5a^2+9m^2$
5| 55	6| 990	7| $xy-6y$	8| $ax-by$
9| 97	10| $2x+y-z$	11| $2a-b+3c$	12| $x^2+y^2-z^2$
13| $a-2b-c$	14| $3x-2y+z$	15| $bc+ca+ab$	16| $2a^2+2b-c^2$

mij Ki (17–24) :

17| $(2a+1)^2 - 4a(2a+1) + 4a^2$

18| $(5a+3b)^2 + 2(5a+3b)(4a-3b) + (4a-3b)^2$

19| $(7a+b)^2 - 2(7a+b)(7a-b) + (7a-b)^2$

20| $(2x+3y)^2 + 2(2x+3y)(2x-3y) + (2x-3y)^2$

21| $(5x-2)^2 + (5x+7)^2 - 2(5x-2)(5x+7)$

22| $(3ab-cd)^2 + 9(cd-ab)^2 + 6(3ab-cd)(cd-ab)$

23| $(2x+5y+3z)^2 + (5y+3z-x)^2 - 2(5y+3z-x)(2x+5y+3z)$

24| $(2a-3b+4c)^2 + (2a+3b-4c)^2 + 2(2a-3b+4c)(2a+3b-4c)$

gvb wbYq Ki (25–28) :

25| $25x^2+36y^2-60xy$, hLb $x=-4, y=-5$

26| $16a^2-24ab+9b^2$, hLb $a=7, b=6.$

27| $9x^2+30x+25,$ hLb $x=-2.$

28| $81a^2+18ac+c^2,$ hLb $a=7, c=-67.$

29| $a-b=7$ Ges $ab=3$ n‡j, †`LvI †h, $(a+b)^2=61.$

30| $a+b=5$ Ges $ab=12$ n‡j, †`LvI †h, $a^2+b^2=1$

31| $x+\dfrac{1}{x}=5$ n‡j, cgvY Ki †h, $\left(x^2-\dfrac{1}{x^2}\right)^2=525$

32| $a+b=8$ Ges $a-b=4$ n‡j, $ab=$ KZ ?

33| $x+y=7$ Ges $xy=10$ n‡j, x^2+y^2+5xy Gi gvb KZ ?

34| $m+\dfrac{1}{m}=2$ n‡j, †`LvI †h, $m^4+\dfrac{1}{m^4}=2$

mÎ 3| $\boldsymbol{(a+b)(a-b)=a^2-b^2}$

cgvY : $(a+b)(a-b)=a(a-b)+b(a-b)$

$$= a^2-ab+ab-b^2$$

$$\therefore\ (a+b)(a-b)=a^2-b^2$$

`BwU iwki †hvMdj × G‡`i we‡qvMdj = iwk `BwUi e‡Mi we‡qvMdj

mÎ 4| $\boldsymbol{(x+a)(x+b)=x^2+(a+b)x+ab}$

cgvY : $(x+a)(x+b)=(x+a)x+(x+a)b$

$$=x^2+ax+bx+ab$$

$$=x^2+(a+b)x+ab$$

A_vr, $(x+a)(x+b)=x^2+($$a$ Ges b Gi exRMwYZxq †hvMdj$)$ $x+($$a$ Ges b Gi ¸Ydj$)$

D`vniY 16| m‡Îi mvnv‡h¨ $3x+2y$ †K $3x-2y$ Øviv ¸Y Ki|

mgvavb : $(3x+2y)(3x-2y)$

$$=(3x)^2-(2y)^2$$

$$=9x^2-4y^2$$

D`vniY 17| m‡Îi mvnv‡h¨ ax^2+b †K ax^2-b Øviv ¸Y Ki|

mgvavb : $(ax^2+b)(ax^2-b)$

$$=(ax^2)^2-(b)^2$$

$$=a^2x^4-b^2$$

D`vniY 18| m‡Îi mvnv‡h¨ $3x+2y+1$ †K $3x-2y+1$ Øviv ¸Y Ki|

mgvavb : $(3x+2y+1)(3x-2y+1)$

$$= \{(3x+1)+2y\}\{(3x+1)-2y\}$$
$$= (3x+1)^2 - (2y)^2$$
$$= 9x^2 + 6x + 1 - 4y^2$$
$$= 9x^2 - 4y^2 + 6x + 1$$

D`vniY 19| $a+3$ †K $a+2$ Øviv ¸Y Ki|

mgvavb : $(a+3)(a+2)$

$$= a^2 + (3+2)a + 3\times 2$$
$$= a^2 + 5a + 6$$

D`vniY 20| $px+3$ †K $px-5$ Øviv ¸Y Ki|

mgvavb : $(px+3)(px-5)$

$$= (px)^2 + \{3+(-5)\}px + 3\times(-5)$$
$$= p^2x^2 + (3-5)px - 15$$
$$= p^2x^2 + (-2)px - 15$$
$$= p^2x^2 - 2px - 15$$

D`vniY 21| p^2-2r †K p^2-3r Øviv ¸Y Ki|

mgvavb : $(p^2-2r)(p^2-3r)$

$$= (p^2)^2 + (-2r-3r)p^2 + (-2r)\times(-3r)$$
$$= p^4 - 5rp^2 + 6r^2$$
$$= p^4 - 5p^2r + 6r^2$$

KvR : 1| $(2a+3)$ †K $(2a-3)$ Øviv ¸Y Ki|
2| $(4x+5)$ †K $(4x+3)$ Øviv ¸Y Ki|
3| $(6a-7)$ †K $(6a+5)$ Øviv ¸Y Ki|

Abkxjbx 5·2

m‡Î i mvnv‡h¨ ¸Ydj wbYq Ki :

1| $(4x+3),\ (4x-3)$

2| $(13-12p),\ (13+12p)$

3| $(ab+3),\ (ab-3)$

4| $(10-xy),\ (10+xy)$

5| $(4x^2+3y^2),\ (4x^2-3y^2)$

6| $(a-b-c),\ (a+b+c)$

7| $(x^2-x+1),\ (x^2+x+1)$

8| $\left(x-\frac{1}{2}a\right),\ \left(x-\frac{5}{2}a\right)$

9| $\left(\frac{1}{4}x-\frac{1}{3}y\right),\ \left(\frac{1}{4}x+\frac{1}{3}y\right)$

10| $(a^4+3a^2x^2+9x^4),\ (9x^4-3a^2x^2+a^4)$

11| $(x+1),\ (x-1),\ (x^2+1)$ 12| $(9a^2+b^2),\ (3a+b),\ (3a-b)$

5·2 exRMwYZxq ivwki Drcv`K

Avgiv Rvwb, $6 = 2\times 3$.

GLv‡b, 2 I 3 n‡jv 6 Gi `BwU Drcv`K ev ¸YbxqK|

3 bs mÎ †_‡K Avgiv Rvwb, $a^2-b^2=(a+b)(a-b)$

Zvn‡j, $(a+b)$ I $(a-b)$ exRMwYZxq ivwk a^2-b^2 Gi `BwU Drcv`K ev ¸YbxqK|

†Kv‡bv exRMwYZxq ivwk `B ev Z‡ZwaK ivwki ¸Ydj n‡j, †k‡lv³ ivwk¸‡jvi c‡Z¨KwU‡K c_g ivwki Drcv`K ev ¸YbxqK ejv nq|

exRMwYZxq wewfb mÎ Ges ¸‡Yi wewbgqwewa, ms‡hvMwewa I eÈbwewa e¨envi K‡i exRMwYZxq ivwk‡K Drcv`‡K we‡klY Kiv nq|

D`vniY 22| $20x+4y$ †K Drcv`‡K we‡klY Ki|

mgvavb : $20x+4y = 4\times 5x+4\times y$

$= 4(5x+y)$ [¸‡Yi eÈbwewa Abhvqx]

D`vniY 23| $ax-by+ax-by$ †K Drcv`‡K we‡klY Ki|

mgvavb : $ax-by+ax-by = ax+ax-by-by$

$= 2ax-2by = 2(ax-by)$

D`vniY 24| Drcv`‡K we‡klY Ki : $2x-6x^2$

mgvavb : $2x-6x^2 = 2x(1-3x)$

D`vniY 25| Drcv`‡K we‡klY Ki : $x^2+4x+xy+4y$

mgvavb : $x^2+4x+xy+4y$

$= x(x+4)+y(x+4)$

$= (x+4)(x+y)$

j¶ Kwi : `BwU ivwk Ggbfv‡e wbevPb Ki‡Z n‡e †hb eÈbwewa c‡qvM K‡i cvß ivwk `BwUi g‡a¨ GKwU mvaviY Drcv`K cvIqv hvq|

KvR : Drcv`‡K we‡klY Ki :

1| $28a+7b$ 2| $15y-9y^2$ 3| $5a^2b^4-9a^4b^2$

4| $2a^2+3a+2ab+3b$ 5| $x^4+6x^2+4x^3+24x$

exRMwYZxq m‡Îi mvnv‡h¨ Drcv`‡K we‡klY :

D`vniY 26| Drcv`‡K we‡klY Ki : $25-9x^2$

mgvavb : $25-9x^2=(5)^2-(3x)^2=(5+3x)(5-3x)$

D`vniY 27| $8x^4-2x^2a^2$ Drcv`‡K we‡klY Ki|

mgvavb : $8x^4-2x^2a^2=2x^2(4x^2-a^2)$ [eÈbwewa Abhvqx]

$=2x^2\{(2x)^2-(a)^2\}=2x^2(2x+a)(2x-a)$

D`vniY 28| Drcv`‡K we‡klY Ki : $25(a+2b)^2-36(2a-5b)^2$

mgvavb : awi, $a+2b=x$ Ges $2a-5b=y$

$\therefore$ c`Ë ivwk $=25x^2-36y^2$

$=(5x)^2-(6y)^2$

$=(5x+6y)(5x-6y)$

$=\{5(a+2b)+6(2a-5b)\}\{5(a+2b)-6(2a-5b)\}$ [x I y Gi gvb ewm‡q]

$=(5a+10b+12a-30b)(5a+10b-12a+30b)$

$=(17a-20b)(40b-7a)$

D`vniY 29| Drcv`‡K we‡klY Ki : x^2+5x+6

mgvavb : x^2+5x+6

$=x^2+(2+3)x+2\times 3$

$=(x+2)(x+3)$

$\because (x+a)(x+b)$

$=x^2+(a+b)x+ab$

GLv‡b, $a=2$ Ges $b=3$

D`vniY 30| Drcv`‡K we‡klY Ki : $4x^2-4xy+y^2-z^2$

mgvavb : $4x^2-4xy+y^2-z^2$

$=(2x)^2-2\times 2x\times y+(y)^2-z^2$

$=(2x-y)^2-(z)^2$

$=(2x-y+z)(2x-y-z)$

D`vniY 31| Drcv`‡K we‡kø lY Ki : $2bd - a^2 - c^2 + b^2 + d^2 + 2ac$

mgvavb : $2bd - a^2 - c^2 + b^2 + d^2 + 2ac$

$$= b^2 + 2bd + d^2 - a^2 + 2ac - c^2 \quad \text{[mwR‡q]}$$
$$= (b^2 + 2bd + d^2) - (a^2 - 2ac + c^2)$$
$$= (b + d)^2 - (a - c)^2$$
$$= (b + d + a - c)(b + d - a + c)$$
$$= (a + b - c + d)(b - a + c + d)$$

KvR : Drcv`‡K we‡kølY Ki :

1| $a^2 - 81b^2$ 2| $25x^4 - 36y^4$ 3| $9x^2 - (2x + y)^2$

4| $x^2 + 7x + 10$ 5| $m^2 + m - 30$

Abkxjbx 5·3

Drcv`‡K we‡kølY Ki :

1| $x^2 + xy + zx + yz$

2| $a^2 + bc + ca + ab$

3| $ab(px + qy) + a^2qx + b^2py$

4| $4x^2 - y^2$

5| $9a^2 - 4b^2$

6| $a^2b^2 - 49y^2$

7| $16x^4 - 81y^4$

8| $a^2 - (x + y)^2$

9| $(2x - 3y + 5z)^2 - (x - 2y + 3z)^2$

10| $4 + 8a^2 + 9a^4$

11| $2a^2 + 6a - 80$

12| $y^2 - 6y - 91$

13| $p^2 - 15p + 56$

14| $45a^8 - 5a^4x^4$

15| $a^2 + 3a - 40$

16| $(x^2 + 1)^2 - (y^2 + 1)^2$

17| $x^2 + 11x + 30$

18| $a^2 - b^2 + 2bc - c^2$

19| $144x^7 - 25x^3a^4$

20| $4x^2 + 12xy + 9y^2 - 16a^2$

5·3 fvR¨, fvRK, ¸YbxqK I ¸wYZK

x, y I z wZbwU iwk| awi,

$x \div y = z$

fvR¨ fvRK fvMdj

GLv‡b GKwU fvM cwµqv †`Lv‡bv n‡q‡Q| x †K fvM Kiv n‡q‡Q, ZvB x fvR¨| Avevi, y Øviv fvM Kiv n‡q‡Q, d‡j y fvRK Ges z n‡jv fvMdj|

†hgb, $10 \div 2 = 5$

GLv‡b, $10 \longrightarrow$ fvR¨

$2 \longrightarrow$ fvRK

$5 \longrightarrow$ fvMdj

G‡¶‡Î $10, 2$ Gi GKwU ¸wYZK| Avevi $10, 5$ Gi I GKwU ¸wYZK|

GKwU iwk (fvR¨) Aci GKwU iwk (fvRK) Øviv wbt‡k‡l wefvR¨ n‡j, fvR¨‡K fvR‡Ki GKwU ¸wYZK (($Multiple$) ejv nq| Avi fvRK‡K ¸YbxqK ev Drcv`K ($Factor$) e‡j|

5·4 Mwiô mvaviY ¸YbxqK (M.mv.¸.)

cwUMwYZ †_‡K Avgiv †R‡bwQ,

12 Gi ¸YbxqK¸‡jv $1, 2, 3, 4, 6, 12$

18 Ó Ó $1, 2, 3, 6, 9, 18$

24 Ó Ó $1, 2, 3, 4, 6, 8, 12, 24$

$12, 18$ I 24 Gi mvaviY ¸YbxqK¸‡jv $2, 3$ I 6| G‡`i g‡a¨ eo ¸YbxqKwU 6|

$\therefore$ $12, 18$ I 24 Gi M.mv.¸. 6 |

exRMwY‡Z,

xyz Gi ¸YbxqK¸‡jv h_vµ‡g x, y, z

$5x$ Gi ¸YbxqK¸‡jv h_vµ‡g $5, x$

$3xp$ Gi ¸YbxqK¸‡jv h_vµ‡g $3, x, p$

$\therefore$ xyz, $5x$, $3xp$ iwk¸‡jvi mvaviY ¸YbxqK x

$\therefore$ iwk¸‡jvi M.mv.¸. x

†h iwk `B ev Z‡ZwaK iwki c‡Z¨KwUi ¸YbxqK, H iwk‡K c`Ë iwk¸‡jvi mvaviY ¸YbxqK ejv nq|

`B ev Z‡ZwaK iwki Mwiô mvaviY ¸YbxqK (M.mv.¸.) n‡jv Ggb GKwU iwk hv mvaviY ¸YbxqK¸‡jvi g‡a¨ me‡P‡q eo gv‡bi GKwU iwk Ges hv Øviv c`Ë iwk¸‡jv wbt‡k‡l wefvR¨ nq|

M.mv.¸. wbY‡qi wbqg

(K) cwUMwY‡Zi wbq‡g c`Ë iwk¸‡jvi mvswL¨K mn‡Mi M.mv.¸. wbYq Ki‡Z n‡e|

(L) exRMwYZxq iwk¸‡jvi †gŠwjK Drcv`K †ei Ki‡Z n‡e|

(M) mvswL¨K mn‡Mi M.mv.¸. Ges c`Ë iwk¸‡jvi m‡ev”P exRMwYZxq mvaviY †gŠwjK Drcv`K¸‡jvi avivevwnK ¸Ydj n‡”Q wb‡Yq M.mv.¸.|

D`vniY 32| $8x^2yz^2$ Ges $10x^3y^2z^3$ Gi M.mv.¸. wbYq Ki|

mgvavb : $8x^2yz^2 = 2\times 2\times 2\times x\times x\times y\times z\times z$

$$10x^3y^2z^3 = 2\times 5\times x\times x\times x\times y\times y\times z\times z\times z$$

mZivs, †`Lv hv‡"Q mvaviY ¸YbxqK¸‡jv $2, x, x, y, z, z.$

wb‡Yq M.mv.¸. $2\times x\times x\times y\times z\times z = 2x^2yz^2$

D`vniY 33| $2(a^2-b^2)$ Ges $(a^2-2ab+b^2)$ Gi M.mv.¸. wbYq Ki|

mgvavb : 1g ivwk = $2(a^2-b^2)=2(a+b)(a-b)$

2q ivwk = $a^2-2ab+b^2=(a-b)(a-b)$

GLv‡b mvswL¨K mnM 2 I 1Gi M.mv.¸. $=1.$

Ges mvaviY †gŠwjK Drcv`K ev ¸YbxqK $(a-b)$

wb‡Yq M.mv.¸. $(a-b)$

D`vniY 34| $x^2-4, 2x+4$ Ges x^2+5x+6 Gi M.mv.¸. wbYq Ki|

mgvavb : 1g ivwk $= x^2-4=(x+2)(x-2)$

2q ivwk $= 2x+4=2(x+2)$

3q ivwk $= x^2+5x+6=x^2+2x+3x+6$

$$= x(x+2)+3(x+2)=(x+2)(x+3)$$

GLv‡b c`Ë ivwk¸‡jvi mvswL¨K mnM 1, 2 Ges 1 Gi M.mv.¸. $=1$

mvaviY †gŠwjK Drcv`K $=(x+2)$

wb‡Yq M.mv.¸. $1\times(x+2)=(x+2)$

KvR : M.mv.¸. wbYq Ki :	
1| $3x^3y^2,\ 2x^2y^3$	2| $3xy,\ 6x^2y,\ 9xy^2$
3| $(x^2-25), (x-5)^2$	4| $x^2-9,\ x^2+7x+12,\ 3x+9$

5·5 jwNô mvaviY ¸wYZK (j.mv.¸.)

cwUMwY‡Z Avgiv Rwb,

4 Gi ¸wYZK¸‡jv n‡"Q 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

6 Ó Ó Ó 6, 12, 18, 24, 30, 36, ...

4 Ges 6 Gi mvaviY ¸wYZK n‡"Q 12, 24, 36 ,.............

4 Ges 6 Gi jwNô mvaviY ¸wYZK n‡"Q 12.

`B ev ZZ‡ZwaK msL¨vi j.mv.¸. n‡”Q Ggb GKwU msL¨v hv c`Ë msL¨v¸‡jvi mvaviY ¸wYZK¸‡jvi g‡a¨ me‡P‡q †QvU|

exRMwYZxq iwki †¶‡Î,

$x^2y^2 \div x^2y = y$

Ges $x^2y^2 \div xy^2 = x$

A_vr, x^2y I xy^2 Gi c‡Z¨KwU Øviv x^2y^2 wbt‡k‡l wefvR¨|

mZivs, x^2y^2 n‡jv x^2y I xy^2 Gi GKwU mvaviY ¸wYZK|

Avevi, $x^2y = x \times x \times y$

$xy^2 = x \times y \times y$

GLv‡b iwk `BwU‡Z $x$ Av‡Q m‡ev©”P `Bevi Ges y Av‡Q m‡ev©”P `Bevi|

$\therefore$ j.mv.¸. = $x \times x \times y \times y = x^2y^2$

gšÍe¨ : j.mv.¸. = mvaviY Drcv`K × mvaviY bq Gi¨c Drcv`K|

`B ev ZZ‡ZwaK iwki m¤¢ve¨ mKj Drcv`‡Ki m‡ev©”P NvZi ¸Ydj‡K iwk¸‡jvi jwNô mvaviY ¸wYZK (j.mv.¸.) ejv nq|

j.mv.¸. wbY‡qi wbqg

j.mv.¸. wbYq Kivi Rb¨ c_‡g mvswL¨K mnM¸‡jvi j.mv.¸. †ei Ki‡Z n‡e| Gici Drcv`‡Ki m‡ev©”P NvZ †ei Ki‡Z n‡e| AZtci Df‡qi ¸Ydj B n‡e c`Ë iwk¸‡jvi j.mv.¸.|

D`vniY 35| $4x^2y^3z$, $6xy^3z^2$ Ges $8x^3yz^3$ Gi j.mv.¸. wbYq Ki|

mgvavb : iwk¸‡jvi mvswL¨K mnM 4, 6 I 8 Gi j.mv.¸. 24

c`Ë iwk¸‡jvi Aš‘f³ $x, y, z$ Drcv`K¸‡jvi m‡ev©”P NvZ h_vµ‡g x^3, y^3 I z^3

wb‡Yq j.mv.¸. $24x^3y^3z^3$

D`vniY 36| $a^2 - b^2$ I $a^2 + 2ab + b^2$ Gi j.mv.¸. wbYq Ki|

mgvavb : 1g iwk $= a^2 - b^2 = (a+b)(a-b)$

2q iwk $= a^2 + 2ab + b^2 = (a+b)^2$

c`Ë iwk¸‡jvi m¤¢ve¨ Drcv`K¸‡jvi m‡ev©”P NvZ $(a-b)$ I $(a+b)^2$

wb‡Yq j.mv.¸. $(a-b)(a+b)^2$

D`vniY 37| $2x^2y + 4xy^2$, $4x^3y - 16xy^3$ Ges $5x^2y^2(x^2 + 4xy + 4y^2)$ Gi j.mv.¸. wbYq Ki|

mgvavb : 1g iwk $= 2x^2y + 4xy^2 = 2xy(x+2y)$

2q iwk $= 4x^3y - 16xy^3 = 4xy(x^2 - 4y^2) = 4xy(x+2y)(x-2y)$

3q iwk $=5x^2y^2(x^2+4xy+4y^2)=5x^2y^2(x+2y)^2$

mvswL¨K mnM 2, 4 I 5 Gi j.mv.¸. 20

c`Ë iwk¸‡jv‡Z m¤ve¨ Drcv`K¸‡jvi m‡ev©P NvZ h_vµ‡g $x^2, y^2, (x+2y)^2, (x-2y)$

wb‡Yq j.mv.¸. $20x^2y^2(x-2y)(x+2y)^2$

KvR : j.mv.¸. wbYq Ki :	
1| $3x^2y^3$, $9x^3y^2$ I $12x^2y^2$	2| $3a^2+9$, a^4-9 I a^4+6a^2+9
3| $x^2+10x+21$, x^4-49x^2	4| $a-2, a^2-4$, a^2-a-2

Abkxjbx 5·4

1| 11 Gi eM KZ ?

(K) 22 (L) 101 (M) 111 (N) 121

2| $a-5$ Gi eM †KvbwU ?

(K) $a^2+10a+25$ (L) $a^2-10a+25$ (M) $a^2+5a+25$ (N) $a^2-5a+25$

3| $(2x+3)$ I $(2x-3)$ Gi ¸Ydj KZ ?

(K) $4x^2-9$ (L) $4x^2+12x-9$ (M) $4x^2-12x-9$ (N) $4x^2+9$

4| $(x+y)^2+2(x+y)(x-y)+(x-y)^2$ Gi gvb †KvbwU ?

(K) $8x^2$ (L) $8y^2$ (M) $4x^2$ (N) $4y^2$

5| $a+b=4$ Ges $a-b=2$ n‡j, ab Gi gvb KZ ?

(K) 3 (L) 8 (M) 12 (N) 16

6| GKwU iwk Aci GKwU iwk Øviv wbt‡k‡l wefvR¨ n‡j, fvR¨‡K fvR‡Ki Kx ejv nq ?

(K) fvMdj (L) fvM‡kl (M) ¸wYZK (N) ¸YbxqK

7| $a, a^2, a(a+b)$ Gi jwNô mvaviY ¸wYZK †KvbwU ?

(K) a (L) a^2 (M) $a(a+b)$ (N) $a^2(a+b)$

8| $2a$ I $3b$ Gi M.mv.¸. KZ ?

(K) 1 (L) 6 (M) a (N) b

9| (i) $(a+b)^2=a^2+2ab+b^2$

(ii) $4ab=(a+b)^2+(a-b)^2$

(iii) $a^2-b^2=(a+b)(a-b)$

Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK ?

(K) i I ii (L) i I iii

(M) ii I iii (N) i, ii I iii

10| (i) j.mv.¸. Gi cY i/c n‡jv jwNô mvaviY ¸wYZK

(ii) j.mv.¸. wbY‡qi Rb¨ iwk¸‡jvi mvaviY ¸wYZK wbYq Ki‡Z nq|

(iii) M.mv.¸. Gi cY i/c n‡jv Mwiô mvaviY ¸wYZK

Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK ?

(K) i I ii (L) i I iii

(M) ii I iii (N) i, ii I iii

11| (i) x^2-16 $(ii)x^2+3x-4$ `BwU exRMwYwZK iwk–

(1) $x=1$ n‡j, (i) I (ii) Gi Aši wb‡Pi †KvbwU ?

(K) 0 (L) -15

(M) 15 (N) 16

(2) (ii) Gi Drcv`‡K we‡kwlZ i/c wb‡Pi †KvbwU ?

(K) $(x-1)(x+4)$ (L) $(x+1)(x-4)$

(M) $(-x+1)(x+4)$ (N) $(-x+1)(4-x)$

(3) (i) I (ii) Gi mvaviY Drcv`K wb‡Pi †KvbwU ?

(K) $(x-4)$ (L) $(x-1)$

(M) $(x+1)$ (N) $(x+4)$

12| (x^3y-xy^3) I $(x-y)(x+2y)$ `BwU exRMwYZxq iwk| Zvn‡j,

(1) c_g iwki Drcv`‡K we‡kwlZ i/c wb‡Pi †KvbwU?

(K) $(x+y)(x-y)$ (L) $x(x+y)(x-y)$

(M) $y(x+y)(x-y)$ (N) $xy(x+y)(x-y)$

(2) exRMwYwZK iwk `BwUi M.mv.¸. wb‡Pi †KvbwU ?

(K) $(x+y)$ (L) $(x-y)$

(M) $y(x+y)$ (N) $x(x-y)$

(3) exRMwYwZK iwk `BwUi j.mv.¸. wb‡Pi †KvbwU ?

(K) $x(x+y)(x-y)$ (L) $y(x+y)(x-y)$

(M) $xy(x^2-y^2)(x+2y)$ (N) $xy(x+y)(x+2y)$

M.mv.¸. wbYq Ki (13 – 22) :

13| $3a^3b^2c, 6ab^2c^2$

14| $5ab^2x^2, 10a^2by^2$

15| $3a^2x^2, 6axy^2, 9ay^2$

16| $16a^3x^4y, 40a^2y^3x, 28ax^3$

17| a^2+ab, a^2-b^2

18| $x^3y-xy^3, (x-y)^2$

19| $x^2+7x+12, x^2+9x+20$

20| $a^3-ab^2, a^4+2a^3b+a^2b^2$

21| $a^2-16, 3a+12, a^2+5a+4$

22| $xy-y, x^3y-xy, x^2-2x+1$

j.mv.¸. wbYq Ki (23 – 32) :

23| $6a^3b^2c, 9a^4bd^2$

24| $5x^2y^2, 10xz^3, 15y^3z^4$

25| $2p^2xy^2, 3pq^2, 6pqx^2$

26| $(b^2-c^2), (b+c)^2$

27| x^2+2x, x^2+3x+2

28| $9x^2-25y^2, 15ax-25ay$

29| $x^2-3x-10, x^2-10x+25$

30| $a^2-7a+12, a^2+a-20, a^2+2a-15$

31| $x^2-8x+15, x^2-25, x^2+2x-15$

32| $x+5, x^2+5x, x^2+7x+10$

33| $a=2x-3$ Ges $b=2x+5$ n‡j-

(K) $a+b$ Gi gvb wbYq Ki|

(L) m‡Îi mvnv‡h¨ a^2 Gi gvb wbYq Ki|

(M) m‡Îi mvnv‡h¨ a I b Gi ¸Ydj wbYq Ki| $x=2$ n‡j, $ab=$ KZ ?

34| x^4-625 Ges $x^2+3x-10$ `BwU exRMwYZxq ivwk| Zvn‡j-

(K) c_g ivwk‡K Drcv`‡K we‡kÐY Ki‡Z n‡j, †Kvb mÎwU e¨envi Ki‡Z n‡e ?

(L) wØZxq ivwk‡K Drcv`‡K we‡kÐY Ki|

(M) ivwk `BwUi M.mv.¸ wbYq Ki|

(N) ivwk `BwUi j.mv.¸. wbYq Ki|

lô Aa¨vq

exRMwYZxq fMvsk

fMvsk A_ fvOv Ask| Avgiv ˆ`bw›`b Rxe‡b GKwU m¤úY wRwb‡mi mv‡_ Gi AskI e¨envi Kwi| ZvB fMvsk, MwY‡Zi GKwU Acwinvh welq| cwUMwYZxq fMvs‡ki g‡Zv exRMwYZxq fMvs‡kI jNKiY I mvaviY niwewkóKiY ¸iZcY fwgKv iv‡L| cwUMwYZxq fMvs‡ki A‡bK RwUj mgm¨v exRMwYZxq fMvs‡ki gva¨‡g mn‡R mgvavb Kiv hvq| Kv‡RB wk¶v_x‡`i exRMwYZxq fMvsk m¤ú‡K m¯úó aviYv _vKv c‡qvRb| G Aa¨v‡q exRMwYZxq fMvs‡ki jNKiY, mvaviY niwewkóKiY Ges †hvM I we‡qvM Dc¯vcb Kiv n‡q‡Q|

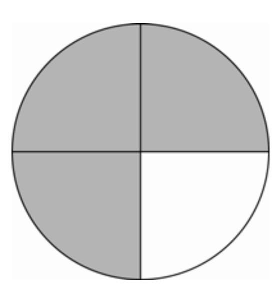
Aa¨vq †k‡l wk¶v_xiv –

- ➢ exRMwYZxq fMvsk Kx Zv e¨vL¨v Ki‡Z cvi‡e|
- ➢ exRMwYZxq fMvs‡ki jNKiY I mvaviY niwewkóKiY Ki‡Z cvi‡e|
- ➢ exRMwYZxq fMvs‡ki †hvM, we‡qvM I mijxKiY Ki‡Z cvi‡e|

6·1 fMvsk

Awei GKwU Av‡cj mgvb `Bfv‡M fvM K‡i GK fvM Zvi fvB Kwei‡K w`j| Zvn‡j `B fvB‡qi c‡Z¨‡K †cj Av‡cjwUi A‡aK, A_vr $\frac{1}{2}$ Ask| GB $\frac{1}{2}$ GKwU fMvsk|

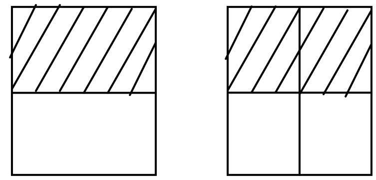
Avevi aiv hvK, wUbv GKwU e‡Ëi 4 fv‡Mi 3 fvM Kv‡jv is Ki‡jv| Zvn‡j, Zvi is Kiv n‡jv m¤úY eËwUi $\frac{3}{4}$ Ask| GLv‡b $\frac{1}{2}$, $\frac{3}{4}$ G¸‡jv cwUMwYZxq fMvsk hv‡`i je 1, 3 Ges ni 2, 4| hw` †Kv‡bv fMvs‡ki ïa je ev ïa ni ev je I ni Dfq‡K exRMwYZxq cZxK ev iwk Øviv cKvk Kiv nq, Z‡e Zv n‡e exRMwYZxq fMvsk| †hgb,



$\frac{a}{4}, \frac{5}{b}, \frac{a}{b}, \frac{2a}{a+b}, \frac{a}{5x}, \frac{x}{x+1}, \frac{2x+1}{x-3}$, BZ¨vw` exRMwYZxq fMvsk|

6·2 mgZj fMvsk :

j¶ Kwi, `BwU mgvb eMvKvi †¶‡Îi 1bs wP‡Î `B fv‡Mi GK fvM, A_vr $\frac{1}{2}$ Ask Kv‡jv is Kiv n‡q‡Q Ges 2bs wP‡Î Pvi fv‡Mi `B fvM, A_vr $\frac{2}{4}$ Ask Kv‡jv is Kiv n‡q‡Q| wKš‘ †`Lv hvq, `B wP‡Îi †gvU Kv‡jv is Kiv Ask mgvb|



1bs wPÎ 2bs wPÎ

AZGe, Avgiv wjL‡Z cvwi, $\frac{1}{2}=\frac{1\times 2}{2\times 2}=\frac{2}{4}$; Avevi, $\frac{1}{2}=\frac{1\times 3}{2\times 3}=\frac{3}{6}$

Gfv‡e, $\frac{1}{2}$ = $\frac{2}{4}$ = $\frac{3}{6}$ = $\frac{5}{10}$ = ..., G¸‡jv ci¯úi mgZj fMvsk|

GKBfv‡e exRMwYZxq fMvs‡ki †¶‡Î, $\frac{a}{b}=\frac{a\times c}{b\times c}=\frac{ac}{bc}$ [je I ni‡K c Øviv ¸Y K‡i, $c\neq o$]

Avevi, $\frac{ac}{bc}$ = $\frac{ac\div c}{bc\div c}$ = $\frac{a}{b}$ [je I ni‡K c Øviv fvM K‡i, $c\neq o$]

$\therefore$ $\frac{a}{b}$ Ges $\frac{ac}{bc}$ ci¯úi mgZj fMvsk|

j¶Yxq †h, †Kv‡bv fMvs‡ki je I ni‡K kb¨ Qvov GKB iwk Øviv ¸Y ev fvM Ki‡j, fMvs‡ki gv‡bi †Kv‡bv cwieZb nq bv|

KvR : $\frac{2}{5}$ Ges $\frac{a}{x}$ Gi wZbwU K‡i mgZj fMvsk †jL|

6·3 fMvs‡ki jNKiY

wb‡Pi Lwj Ni¸‡jv ciY Ki (`BwU K‡i †`Lv‡bv n‡jv) :

$\frac{9}{12}$ = $\frac{3\times 3}{2\times 2\times 3}$ = $\frac{3}{4}$	$\frac{2^3}{2^4}$ =
$\frac{a^2b}{ab^2}$ =	$\frac{x^3}{x^2}=\frac{x\times x\times x}{x\times x}=x$
$\frac{3x}{6xy}$=	$\frac{2mn}{4m^2}$ =

†Kv‡bv fMvs‡ki jNKi‡Yi A_ n‡jv fMvskwU‡K jwNô AvKv‡i cwiYZ Kiv| G Rb¨ je I ni‡K G‡`i mvaviY ¸YbxqK ev Drcv`K Øviv fvM Kiv nq| †Kv‡bv fMvs‡ki je I n‡ii g‡a¨ †Kv‡bv mvaviY ¸YbxqK ev Drcv`K bv _vK‡j Gi¦c fMvsk‡K jwNô AvKv‡ii fMvsk ejv nq|

D`vniY 1| $\dfrac{4a^2bc}{6ab^2c}$ †K jNKiY Ki|

mgvavb : $\dfrac{4a^2bc}{6ab^2c} = \dfrac{2\times 2\times a\times a\times b\times c}{2\times 3\times a\times b\times b\times c} = \dfrac{2a}{3b}$.

weKí c×wZ : $\dfrac{4a^2bc}{6ab^2c} = \dfrac{2abc\times 2a}{2abc\times 3b} = \dfrac{2a}{3b}$. [je I n‡ii M.mv.¸. $2abc$]

D`vniY 2| $\dfrac{2a^2+3ab}{4a^2-9b^2}$ †K jwNô AvKv‡i cwiYZ Ki|

mgvavb : $\dfrac{2a^2+3ab}{4a^2-9b^2} = \dfrac{2a^2+3ab}{(2a)^2-(3b)^2}$

$= \dfrac{a(2a+3b)}{(2a+3b)(2a-3b)} = \dfrac{a}{2a-3b}$. $\left[\because x^2-y^2=(x+y)(x-y)\right]$

D`vniY 3| jNKiY Ki : $\dfrac{x^2+5x+6}{x^2+3x+2}$

mgvavb : $\dfrac{x^2+5x+6}{x^2+3x+2} = \dfrac{x^2+2x+3x+6}{x^2+x+2x+2}$

$= \dfrac{x(x+2)+3(x+2)}{x(x+1)+2(x+1)} = \dfrac{(x+2)(x+3)}{(x+1)(x+2)} = \dfrac{x+3}{x+1}$.

6·4 mvaviY niwewkó fMvsk

mvaviY niwewkó fMvsk‡K mgniwewkó fMvskI e‡j| G‡¶‡Î c`Ë fMvsk¸‡jvi ni mgvb Ki‡Z nq|

$\dfrac{a}{2b}$ I $\dfrac{m}{3n}$ fMvsk `BwU we‡ePbv Kwi| fMvsk `BwUi ni $2b$ Ges $3n$ Gi j.mv.¸. $6bn$.

AZGe, `BwU fMvs‡kiB ni $6bn$ Ki‡Z n‡e|

GLv‡b, $\dfrac{a}{2b} = \dfrac{a\times 3n}{2b\times 3n}$ $\left[\because 6bn\div 2b=3n\right]$

$= \dfrac{3an}{6bn}$

Ges $\frac{m}{3n} = \frac{m\times 2b}{3n\times 2b}$ $[\because 6bn \div 3n = 2b]$

$= \frac{2bm}{6bn}$.

$\therefore$ mvaviY niwewkó fMvsk `BwU $\frac{3an}{6bn}, \frac{2bm}{6bn}$.

mvaviY niwewkó fMvs‡k cKvk Kivi wbqg :

1| fMvsk¸‡jvi n‡ii j.mv.¸. †ei Ki‡Z n‡e|

2| j.mv.¸. †K c‡Z¨K fMvs‡ki ni Øviv fvM K‡i fvMdj †ei Ki‡Z n‡e|

3| cvß fvMdj Øviv mswkó fMvs‡ki je I ni‡K ¸Y Ki‡Z n‡e|

D`vniY 4| mvaviY niwewkó fMvs‡k cKvk Ki : $\frac{a}{4x}, \frac{b}{2x^2}$.

mgvavb : ni $4x$ Ges $2x^2$ Gi j.mv.¸. = $4x^2$

$\therefore \frac{a}{4x} = \frac{a\times x}{4x\times x}$ $[\because 4x^2 \div 4x = x]$

$= \frac{ax}{4x^2}$.

Ges $\frac{b}{2x^2} = \frac{b\times 2}{2x^2\times 2}$ $[\because 4x^2 \div 2x^2 = 2]$

$= \frac{2b}{4x^2}$.

$\therefore$ mvaviY niwewkó fMvsk `BwU $\frac{ax}{4x^2}, \frac{2b}{4x^2}$.

D`vniY 5| mvaviY niwewkó fMvs‡k iƒcvši Ki : $\frac{2}{a^2-4}, \frac{5}{a^2+3a-10}$

mgvavb : 1g fMvs‡ki ni $= a^2-4 = (a+2)(a-2)$

2q fMvs‡ki ni $= a^2+3a-10 = a^2-2a+5a-10$

$= a(a-2)+5(a-2) = (a-2)(a+5)$

ni `BwUi j.mv.¸. $(a+2)(a-2)(a+5)$

$\therefore \frac{2}{a^2-4} = \frac{2}{(a+2)(a-2)} = \frac{2\times(a+5)}{(a+2)(a-2)\times(a+5)}$ [je I ni‡K $(a+5)$ Øviv ¸Y K‡i]

$= \frac{2(a+5)}{(a^2-4)(a+5)}$

Ges $\frac{5}{a^2+3a-10} \quad = \frac{5}{(a-2)(a+5)} = \frac{5\times(a+2)}{(a-2)(a+5)\times(a+2)}$ [je I ni‡K $(a+2)$ Øvi v ¸Y K‡i]

$$= \frac{5(a+2)}{(a^2-4)(a+5)}$$

∴ wb‡Yq fMvsk `BwU $\frac{2(a+5)}{(a^2-4)(a+5)}, \frac{5(a+2)}{(a^2-4)(a+5)}$

D`vniY 6| mvaviY niwewkó fMvs‡k cwiYZ Ki :

$$\frac{1}{x^2+3x}, \frac{2}{x^2+5x+6}, \frac{3}{x^2-x-12}.$$

mgvavb : 1g fMvs‡ki ni = $x^2+3x = x(x+3)$

2q fMvs‡ki ni = $x^2+5x+6 = x^2+2x+3x+6$

$$= x(x+2)+3(x+2) = (x+2)(x+3)$$

3q fMvs‡ki ni = $x^2-x-12 = x^2+3x-4x-12$

$$= x(x+3)-4(x+3) = (x+3)(x-4)$$

ni¸wji j.mv.¸. $x(x+2)(x+3)(x-4)$

∴ 1g fMvsk $= \frac{1}{x^2+3x} \quad = \frac{1\times(x+2)(x-4)}{x(x+3)\times(x+2)(x-4)} = \frac{(x+2)(x-4)}{x(x+2)(x+3)(x-4)}$

2q fMvsk = $\frac{2}{x^2+5x+6} \quad = \frac{2}{(x+2)(x+3)} = \frac{2\times x(x-4)}{(x+2)(x+3)\times x(x-4)}$

$$= \frac{2x(x-4)}{x(x+2)(x+3)(x-4)}$$

3q fMvsk = $\frac{3}{x^2-x-12} \quad = \frac{3}{(x+3)(x-4)} = \frac{3\times\ x(x+2)}{(x+3)(x-4)\times\ x(x+2)}$

$$= \frac{3x(x+2)}{x(x+2)(x+3)(x-4).}$$

∴ wb‡Yq fMvsk¸wjU h_vµ‡g

$$\frac{(x+2)(x-4)}{x(x+2)(x+3)(x-4)}, \frac{2x(x-4)}{x(x+2)(x+3)(x-4)}, \frac{3x(x+2)}{x(x+2)(x+3)(x-4).}$$

KvR :

1| Drcv`‡K we‡kIY Ki : $a^2-9b^2,\ x^2+x-6.$

2| iwk wZbwUi j.mv.¸. wbYq Ki : $a^2+3a,\ a^2+5a+6,\ a^2-a-12.$

3| mvaviY niwewkó fMvs‡k cKvk Ki : $\frac{a}{2x},\ \frac{b}{4y}$

Abkxjbx 6·1

jwNô AvKv‡i cKvk Ki (1-10) :

1| $\frac{a^2b}{a^3c}$ 2| $\frac{a^2bc}{ab^2c}$ 3| $\frac{x^3y^3z^3}{x^2y^2z^2}$ 4| $\frac{x^2+x}{xy+y}$ 5| $\frac{4a^2b}{6a^3b}$ 6| $\frac{2a-4ab}{1-4b^2}$

7| $\frac{2a+3b}{4a^2-9b^2}$ 8| $\frac{a^2+4a+4}{a^2-4}$ 9| $\frac{x^2-y^2}{(x+y)^2}$ 10| $\frac{x^2+2x-15}{x^2+9x+20}$

mvaviY niwewkó fMvs‡k cKvk Ki (11-20) :

11| $\frac{a}{bc},\frac{a}{ac}$ 12| $\frac{x}{pq},\frac{y}{pr}$ 13| $\frac{2x}{3m},\frac{3y}{2n}$ 14| $\frac{a}{a-b},\frac{b}{a+b}$

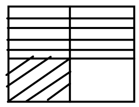
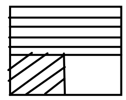
15| $\frac{x^2}{a^2-2ab},\ \frac{y^2}{a+2b}$ 16| $\frac{3}{a^2-4},\frac{2}{a(a+2)}$ 17| $\frac{a}{a^2-9},\frac{b}{a+3}$

18| $\frac{a}{a+b},\ \frac{b}{a-b},\ \frac{c}{a-c}$ 19| $\frac{a}{a-b},\ \frac{b}{a+b},\ \frac{c}{a(a+b)}$

20| $\frac{2}{x^2-x-2},\frac{3}{x^2+x-6}$

6·5 exRMwYZxq fMœvs‡ki †hvM, we‡qvM I mijxKiY

j¶ Kwi :

পাটিগণিত	exRMwYZ
m¤úY eMvKvi †¶ÎwU‡K $\mathbf{1}$ aiv n‡j, Gi	m¤úY eMvKvi †¶ÎwU‡K x aiv n‡j, Gi
Kv‡jv Ask = $\mathbf{1}$ Gi $\frac{2}{4} = \frac{2}{4}$ 	 Kv‡jv Ask = x Gi $\frac{2}{4} = \frac{2x}{4}$
`vMUvbv Ask = $\mathbf{1}$ Gi $\frac{1}{4} = \frac{1}{4}$	`vMUvbv Ask = x Gi $\frac{1}{4} = \frac{x}{4}$
$\therefore$ †gvU isKiv Ask = $\boxed{\frac{2}{4}+\frac{1}{4}}$	$\therefore$ †gvU isKiv Ask = $\boxed{\frac{2x}{4}+\frac{x}{4}}$
$= \frac{2+1}{4} = \frac{3}{4}$	$= \frac{2x+x}{4} = \frac{3x}{4}$
$\therefore$ mv`v Ask = $\left(1-\frac{3}{4}\right) = \boxed{\frac{4}{4}-\frac{3}{4}}$	$\therefore$ mv`v Ask = $x-\frac{3x}{4} = \boxed{\frac{4x}{4}-\frac{3x}{4}}$
$= \frac{4-3}{4} = \frac{1}{4}$	$= \frac{4x-3x}{4} = \frac{x}{4}$

j¶ Kwi, cwZwU N‡ii fMœvsk¸‡jv mvaviY niwewkó|

exRMwYZxq fMœvs‡ki †hvM I we‡qv‡Mi wbqg :

(1) fMœvsk¸‡jv‡K jwNô mvaviY niwewkó Ki‡Z n‡e|

(2) †hvMd‡ji ni n‡e jwNô mvaviY ni Ges je n‡e iƒcvšwiZ fMœvsk¸‡jvi j‡ei †hvMdj|

(3) we‡qvMd‡ji ni n‡e jwNô mvaviY ni Ges je n‡e iƒcvšwiZ fMœvsk¸‡jvi j‡ei we‡qvMdj|

exRMwYZxq fMœvs‡ki †hvM

D`vniY 7| †hvM Ki : $\frac{x}{a}$ Ges $\frac{y}{a}$

mgvavb : $\frac{x}{a}+\frac{y}{a}=\frac{x+y}{a}$

D`vniY 8| $\frac{a}{m}$ Ges $\frac{b}{n}$ †hvM Ki|

mgvavb : $\frac{a}{m} + \frac{b}{n} = \frac{a\times n}{m\times n}+\frac{b\times m}{n\times m}$

$= \frac{an+bm}{mn}$

D`vniY 9| †hvMdj wbYq Ki : $\frac{3a}{2x}+\frac{b}{2y}$.

mgvavb : $\frac{3a}{2x}+\frac{b}{2y}=\frac{3a\times y}{2x\times y}+\frac{b\times x}{2y\times x}=\frac{3ay+bx}{2xy}$ [mgn‡ii Rb¨ $2x, 2y$ Gi j.mv.¸. $2xy$ wb‡q]

exRMwYZxq fMvs‡ki we‡qvM

D`vniY 10| we‡qvM Ki : $\frac{a}{x}$ †_‡K $\frac{b}{x}$

mgvavb : $\frac{a}{x}-\frac{b}{x}=\frac{a-b}{x}$

D`vniY 11| $\frac{2a}{3x}$ †_‡K $\frac{b}{3y}$ we‡qvM Ki|

mgvavb : $\frac{2a}{3x}-\frac{b}{3y}=\frac{2a\times y}{3xy}-\frac{b\times x}{3xy}=\frac{2ay-bx}{3xy}$

D`vniY 12| we‡qvMdj wbYq Ki : $\frac{1}{a+2}-\frac{1}{a^2-4}$.

mgvavb : $\frac{1}{a+2}-\frac{1}{a^2-4}=\frac{1}{a+2}-\frac{1}{(a+2)(a-2)}=\frac{1\times(a-2)}{(a+2)\times(a-2)}-\frac{1}{(a+2)(a-2)}$

$=\frac{(a-2)-1}{(a+2)(a-2)}=\frac{a-2-1}{(a+2)(a-2)}=\frac{a-3}{a^2-4}$.

KvR : wb‡Pi QKwU ciY Ki :	
$\frac{1}{5}+\frac{3}{5}=$	$\frac{4}{5}-\frac{2}{5}=$
$\frac{3}{m}+\frac{2}{n}=$	$\frac{5}{ab}-\frac{1}{a}=$
$\frac{2}{x}+\frac{5}{2x}=$	$\frac{7}{xyz}-\frac{2z}{xy}=$
$\frac{3}{m}+\frac{2}{m^2}=$	$\frac{5}{p^2}-\frac{2}{3p}=$

exRMwYZxq fMvs‡ki mijxKiY :

cwµqv wPý Øviv msh³ `B ev Z‡ZwaK exRMwYZxq fMvsk‡K GKwU fMvs‡k ev iwk‡Z cwiYZ KivB n‡jv fMvs‡ki mijxKiY| G‡Z cvß fMvskwU‡K jwNô AvKv‡i cKvk Kiv nq|

D`vniY 13| mij Ki : $\frac{a}{a+b}+\frac{b}{a-b}$.

mgvavb : $\frac{a}{a+b}+\frac{b}{a-b} = \frac{a\times(a-b)+b\times(a+b)}{(a+b)(a-b)} = \frac{a^2-ab+ab+b^2}{(a+b)(a-b)}$

$= \frac{a^2+b^2}{a^2-b^2}$.

D`vniY 14| mij Ki : $\frac{x+y}{xy}-\frac{y+z}{yz}$.

mgvavb : $\frac{x+y}{xy}-\frac{y+z}{yz} = \frac{z\times(x+y)-x\times(y+z)}{xyz} = \frac{zx+zy-xy-xz}{xyz}$

$= \frac{yz-xy}{xyz} = \frac{y(z-x)}{xyz} = \frac{z-x}{xz}$.

D`vniY 15| mij Ki : $\frac{x-y}{xy}+\frac{y-z}{yz}-\frac{z-x}{zx}$

mgvavb : $\frac{x-y}{xy}+\frac{y-z}{yz}-\frac{z-x}{zx} = \frac{(x-y)\times z+(y-z)\times x-(z-x)\times y}{xyz}$

$= \frac{zx-yz+xy-zx-yz+xy}{xyz} = \frac{2xy-2yz}{xyz} = \frac{2y(x-z)}{xyz} = \frac{2(x-z)}{xz}$

Abkxjbx 6·2

1| $\frac{ab}{xy}$ Gi mgZzj fMvsk wb‡Pi †KvbwU ?

(K). $\frac{abc}{xyz}$ (L). $\frac{a^2b}{x^2y}$ (M). $\frac{abz}{xyz}$ (N). $\frac{a}{x}$

2| $\frac{2x+x^2}{6x}$ Gi jwNô AvKvi wb‡Pi †KvbwU ?

(K). $\frac{1}{3}$ (L). $\frac{2+x}{6}$ (M). $\frac{x}{6}$ (N). $\frac{1+x}{3}$

3| $\frac{2}{3a}$ I $\frac{3}{5ab}$ Gi mgniwewkó fMvsk wb‡Pi †KvbwU ?

(K). $\frac{10b}{15ab}, \frac{9}{15ab}$ (L). $\frac{6}{15ab}, \frac{b}{15ab}$ (M). $\frac{2}{15ab}, \frac{3}{15ab}$ (N). $\frac{10a}{15a^2b}, \frac{9a}{15a^2b}$

4| $\frac{x}{yz}$ I $\frac{y}{zx}$ Gi mvaviY niwewkó fMvsk wb‡Pi †KvbwU ?

(K). $\frac{zx^2}{xyz^2}, \frac{y^2z}{xyz^2}$ (L). $\frac{x^2}{xyz^2}, \frac{y^2}{xyz^2}$ (M). $\frac{x}{xyz}, \frac{y}{xyz}$ (N). $\frac{x^2}{xyz}, \frac{y^2}{xyz}$

5| wb‡Pi Z_¨¸‡jv j¶ Ki :

i. $\frac{ac}{bd}+1 = \frac{ac+1}{bd+1}$; ii. $\frac{a}{2b}+\frac{a}{4b}=\frac{3a}{4b}$; iii. $\frac{3x}{y}-\frac{2x}{5y}=\frac{13x}{5y}$

Dc‡ii Z‡_¨i Av‡jv‡K wb‡Pi †KvbwU mZ¨ ?

(K). i I ii (L). ii I iii (M). i I iii (N). i, ii I iii

6| $\frac{a}{x+1}, \frac{a}{2x+2}, \frac{3a}{x^2-1}$ wZbwU exRMwYZxq fMvsk|

wb‡Pi ck¸‡jvi DËi `vI :

(1) 1g fMvsk †_‡K 2q fMvsk we‡qvM Ki‡j we‡qvMdj wb‡Pi †KvbwU ?

(K). $\frac{1}{2x+2}$ (L). $\frac{2a}{x+2}$ (M). $\frac{a}{x+1}$ (N). $\frac{a}{2(x+1)}$

(2) ni wZbwUi j.mv.¸. wb‡Pi †KvbwU ?

(K). $2(x^2-1)$ (L). $(x+1)^3(x-1)$ M. $2(x^2+1)$ (N). $2(x+1)$

(3) fMvsk wZbwU‡K mgniwewkó fMvs‡k i|cvšÍi Ki‡j 2q fMvskwU Kx n‡e?

K. $\dfrac{a}{2(x^2-1)}$ L. $\dfrac{a(x-1)}{2(x^2-1)}$ M. $\dfrac{a(x-1)}{2(x+1)}$ N. $\dfrac{2a(x-1)}{x^2-1}$

†hvMdj wbYq Ki (7-12) :

7| $\dfrac{3a}{5}+\dfrac{2b}{5}$ 8| $\dfrac{1}{5x}+\dfrac{2}{5x}$ 9| $\dfrac{x}{2a}+\dfrac{y}{3b}$ 10| $\dfrac{2a}{x+1}+\dfrac{2a}{x-2}$ 11| $\dfrac{a}{a+2}+\dfrac{2}{a-2}$

12| $\dfrac{3}{x^2-4x-5}+\dfrac{4}{x+1}$

we‡qvMdj wbYq Ki (13-18) :

13| $\dfrac{2a}{7}-\dfrac{4b}{7}$ 14| $\dfrac{2x}{5a}-\dfrac{4y}{5a}$ 15| $\dfrac{a}{8x}-\dfrac{b}{4y}$

16| $\dfrac{3}{x+3}-\dfrac{2}{x+2}$ 17| $\dfrac{p+q}{pq}-\dfrac{q+r}{qr}$ 18| $\dfrac{2x}{x^2-4y^2}-\dfrac{x}{xy+2y^2}$

mij Ki : (19-24) :

19| $\dfrac{5}{a^2-6a+5}+\dfrac{1}{a-1}$ 20| $\dfrac{1}{x+2}-\dfrac{1}{x^2-4}$ 21| $\dfrac{a}{3}+\dfrac{a}{6}-\dfrac{3a}{8}$

22| $\dfrac{a}{b}-\dfrac{3a}{2b}+\dfrac{2a}{3b}$ 23| $\dfrac{x}{yz}-\dfrac{y}{zx}+\dfrac{z}{xy}$ 24| $\dfrac{x-y}{xy}+\dfrac{y-z}{yz}+\dfrac{z-x}{zx}$

25| wZbwU exRMwYZxq fMvsk : $\dfrac{x}{x+y},\ \dfrac{x}{x-4y},\ \dfrac{y}{x^2-3xy-4y^2}$

K. 3q fMvs‡ki ni‡K Drcv`‡K we‡kIY Ki|

L. 1g I 2q fMvsk‡K mgniwewkó fMvs‡k cKvk Ki|

M. fMvsk wZbwUi †hvMdj wbYq Ki|

26| wZbwU exRMwYZxq fMvsk : $\dfrac{1}{a(a+2)},\ \dfrac{1}{a^2+5a+6},\ \dfrac{1}{a^2-a-6}$

K. 3q fMvs‡ki ni‡K Drcv`‡K we‡kIY Ki|

L. 2q I 3q fMvsk‡K mvaviY niwewkó fMvs‡k iƒcvši Ki|

M. 2q I 3q fMvs‡ki †hvMdj †_‡K 1g fMvsk we‡qvM Ki|

mßg Aa¨vq

mij mgxKiY

Avgiv lô †kÖwY‡Z mgxKiY I mij mgxKiY Kx Zv †R‡bwQ Ges ev¯ÍewfwËK mgm¨v †_‡K mgxKiY MVb K‡i Zv mgvavb Ki‡Z wk‡LwQ| mßg †kÖwYi G Aa¨v‡q Avgiv mgxKiY mgvav‡bi wKQz wewa I G‡`i c‡qvM m¤ú‡K Rvbe Ges ev¯Íe mgm¨vi wfwË‡Z mgxKiY MVb K‡i Zv mgvavb Kiv wkLe| G QvovI G Aa¨v‡q †jLwPÎ m¤ú‡K cÖv_wgK aviYv †`Iqv n‡q‡Q Ges mgxKi‡Yi mgvavb †jLwP‡Î †`Lv‡bv n‡q‡Q|

Aa¨vq †k‡l wk¶v_xiv –

- mgxKi‡Yi c¶vš—i wewa, eRb wewa, Avo¸Yb wewa, cÖZmvg¨ wewa e¨vL¨v Ki‡Z cvi‡e|
- mgxKi‡Yi wewamgn c‡qvM K‡i mgxKiY mgvavb Ki‡Z cvi‡e|
- mij mgxKiY MVb I mgvavb Ki‡Z cvi‡e|
- †jLwPÎ Kx Zv e¨vL¨v Ki‡Z cvi‡e|
- †jLwP‡Îi A¶ I mweavRbK GKK wb‡q we›`cvZb Ki‡Z cvi‡e|
- †jLwP‡Îi mvnv‡h¨ mgxKi‡Yi mgvavb Ki‡Z cvi‡e|

7·1 ce cv‡Vi cbiv‡jvPbv

(1) †hv‡Mi I ¸‡Yi wewbgq wewa :

a, b Gi †h‡Kv‡bv gv‡bi Rb¨, $a+b=b+a$ Ges $ab=ba$

(2) ¸‡Yi eÈb wewa :

a, b, c Gi †h‡Kv‡bv gv‡bi Rb¨, $a(b+c)=ab+ac, (b+c)a=ba+ca$

Avgiv mgxKiYwU j¶ Kwi : $x+3=7.$

(K) mgxKiYwUi AÁvZ iwk ev PjK †KvbwU?

(_) mgxKiYwUi cwµqv wPý †KvbwU?

(M) mgxKiYwU mij mgxKiY wK bv?

(N) mgxKiYwUi gj KZ?

Avgiv Rwb PjK, cwµqv wPý I mgvb wPý mseywjZ MwYwZK evK¨‡K mgxKiY e‡j| Avi Pj‡Ki GK NvZ wewkó mgxKiY‡K mij mgxKiY e‡j| mij mgxKiY GK ev GKwaK PjKwewkó n‡Z cv‡i|

†hgb, $x+3=7, \quad 2y-1=y+3, \quad 3z-5=0, \quad 4x+3=x-1,$

$x+4y-1=0, \; 2x-y+1=x+y$ BZ¨w`, G¸‡jv mij mgxKiY|

Avgiv G Aa¨v‡q ïa GK PjKwewkó mij mgxKiY wb‡q Av‡jvPbv Kie|
mgxKiY mgvavb K‡i Pj‡Ki †h gvb cvIqv hvq, G‡K mgxKiYwUi gj e‡j| gjwU Øviv mgxKiYwU wm× nq| A_©vr, PjKwUi H gvb mgxKi‡Y emv‡j mgxKiYwUi `By c¶ mgvb nq|

mgxKiY mgvav‡bi Rb¨ PviwU ¯^Ztwm× Av‡Q, Zv Avgiv Rwb| G¸‡jv n‡jv :

(1) ci¯úi mgvb iwki c‡Z¨KwUi mv‡_ GKB iwk †hvM Ki‡j †hvMdj¸‡jv ci¯úi mgvb nq|
(2) ci¯úi mgvb iwki c‡Z¨KwU †_‡K GKB iwk we‡qvM Ki‡j we‡qvMdj¸‡jv ci¯úi mgvb nq|
(3) ci¯úi mgvb iwki c‡Z¨KwU‡K GKB iwk Øviv ¸Y Ki‡j ¸Ydj¸‡jv ci¯úi mgvb nq|
(4) ci¯úi mgvb iwki c‡Z¨KwU‡K Akb¨ GKB iwk Øviv fvM Ki‡j fvMdj¸‡jv ci¯úi mgvb nq|

KvR :
$2x-1=0$ mgxKiYwUi NvZ KZ ? Gi cwµqv wPý †KvbwU wjL| mgxKiYwUi gj KZ?

7·2 mgxKi‡Yi wewamgn

(1) c¶vši wewa :

mgxKiY-1 cieZx avc

(K) $x-5+5=3+5$ [¯Ztwm× (1)]

$x-5=3$

(L) $x=3+5$

mgxKiY-2 cieZx avc

(K) $4x-3x=3x+7-3x$ [¯Ztwm× (2)]

$4x=3x+7$

(L) $4x-3x=7$

mgxKiY-1 G (L) Gi †¶‡Î 5 Gi wPý cwiewZZ n‡q evgc¶ †_‡K Wvbc‡¶ †M‡Q| mgxKiY-2 G (L) Gi †¶‡Î $3x$ Gi wPý cwiewZZ n‡q Wvbc¶ †_‡K evgc‡¶ †M‡Q|

†Kv‡bv mgxKi‡Yi †h‡Kv‡bv c`‡K GK c¶ †_‡K wPý cwieZb K‡i Acic‡¶ miv‡miwi ¯’vbvšÍi Kiv hvq| GB ¯’vbvšÍi‡K e‡j c¶vši wewa|

(2) eRb wewa :

(a) †hv‡Mi eRb wewa :

mgxKiY-1 $2x+3=a+3$ cieZx avc

(K) $2x+3-3=a+3-3$ [¯Ztwm× (2)]

(L) $2x=a$

mgxKiY-2 $7x-5=2a-5$ cieZx avc

(K) $7x-5+5=2a-5+5$ [¯Ztwm× (1)]

(L) $7x=2a$

mgxKiY-1 G (L) Gi †¶‡Î Dfqc¶ †_‡K 3 eRb Kiv n‡q‡Q|

mgxKiY-2 G (L) Gi †¶‡Î Dfqc¶ †_‡K -5 eRb Kiv n‡q‡Q|

†Kv‡bv mgxKi‡Yi Dfqc¶ †_‡K GKB wPýh³ m`k c` mivmwi eRb Kiv hvq| G‡K ejv nq †hv‡Mi (ev we‡qv‡Mi) eRb wewa|

(b) ¸‡Yi eRb wewa :

mgxKiY $4(2x+1)=4(x-2)$ cieZx avc

(K) $\dfrac{4(2x+1)}{4}=\dfrac{4(x-2)}{4}$ [¯Z:wm× (4)]

(L) $2x+1=x-2$

mgxKiYwUi (L) Gi †¶‡Î Dfqc¶ †_‡K mvaviY Drcv`K mivmwi eRb Kiv hvq| G‡K ejv nq ¸‡Yi eRb wewa|

(3) Avo¸Yb wewa :

mgxKiY $\dfrac{x}{2}=\dfrac{5}{3}$ cieZx avc

(K) $\dfrac{x}{2}\times 6=\dfrac{5}{3}\times 6$ [Dfqc¶‡K ni 2 I 3 Gi j.mv.¸. 6 Øviv ¸Y Kiv n‡q‡Q]

(L) $3\times x=2\times 5$

mgxKiYwUi (L) Gi †¶‡Î wjL‡Z cwii,

evgc‡¶i je × Wvbc‡¶i ni = evgc‡¶i ni × Wvbc‡¶i je
G‡K ejv nq Avo¸Yb wewa|

(4) cwZmvg¨ wewa :

mgxKiY : $2x+1=5x-8$

ev, $5x-8=2x+1$

GKB mv‡_ evgc‡¶i me¸‡jv c` Wvbc‡¶ I Wvbc‡¶i me¸‡jv c` evgc‡¶ †Kv‡bv wPý cwieZb bv K‡i ¯’vbvšÍi Kiv hvq| G‡K ejv nq cwZmvg¨ wewa|

DwjøwLZ ¯‹twm×mgn I wewamgn c‡qvM K‡i GKwU mgxKiY‡K Aci GKwU mnR mgxKi‡Y iƒcvšÍi K‡i me‡k‡l Zv $x=a$ AvKv‡i cvIqv hvq| A_vr, PjK x Gi gvb a wbYq Kiv nq|

D`vniY 1| mgvavb Ki : $x+3=9$.

mgvavb : $x+3=9$

ev, $x=9-3$ [c¶všÍi K‡i]

ev, $x=6$

∴ mgvavb : $x=6$

weKí wbqg : $x+3=9$

ev, $x+3-3=9-3$ [Dfqc¶ †_‡K 3 we‡qvM K‡i]

ev, $x=6$

∴ mgvavb : $x=6$

D`vniY 2| mgvavb Ki I ïw× cix¶v Ki : $4y-5=2y-1$.

mgvavb : $4y-5=2y-1$.

ev, $4y-2y=-1+5$ [c¶všÍi K‡i]

ev, $2y=4$

ev, $2y=2\times 2$

ev, $y=2$ [Dfqc¶ †_‡K mvaviY Drcv`K 2 eRb K‡i]

∴ mgvavb : $y=2$

ïw× cix¶v : c`Ë mgxKi‡Y y Gi gvb 2 emv‡q cvB,

evgc¶ = $4y-5=4\times 2-5=8-5=3$

Wvbc¶ = $2y-1=2\times 2-1=4-1=3$.

∴ evgc¶ = Wvbc¶

∴ mgxKiYwUi mgvavb ï× n‡q‡Q|

D`vniY 3| mgvavb Ki : $\frac{2z}{3}-\frac{z}{6}=-\frac{3}{4}$

mgvavb : $\frac{2z}{3}-\frac{z}{6}=-\frac{3}{4}$

ev, $\frac{4z-z}{6}=-\frac{3}{4}$ [evgc‡¶ ni 3, 6 Gi j.mv.¸. 6]

ev, $\frac{3z}{6}=-\frac{3}{4}$

ev, $\frac{z}{2}=-\frac{3}{4}$

ev, $4\times z=2\times(-3)$ [Avo¸Yb K‡i]

ev, $2\times 2z=2\times(-3)$

ev, $2z=-3$ [Dfqc¶ †_‡K mvaviY Drcv`K 2 eRb K‡i]

ev, $\frac{2z}{2}=-\frac{3}{2}$ [Dfqc¶‡K 2 Øviv fvM K‡i]

ev, $z=-\frac{3}{2}$

∴ mgvavb : $z=-\frac{3}{2}$.

D`vniY 4| mgvavb Ki : $2(5+x)=16$.

mgvavb : $2(5+x)=16$

ev, $2\times 5+2\times x=16$ [eÈb wewa Abmv‡i]

ev, $10+2x=16$

ev, $2x+10-10=16-10$ [Dfqc¶ †_‡K 10 we‡qvM K‡i]

ev, $2x=6$

ev, $\frac{2x}{2}=\frac{6}{2}$ [Dfqc¶‡K 2 Øviv fvM K‡i]

ev, $x=3$.

∴ mgvavb $x=3$

D`vniY 5| mgvavb Ki : $\frac{3x+7}{4}+\frac{5x-4}{7}=x+3\frac{1}{2}$

mgvavb : $\frac{3x+7}{4}+\frac{5x-4}{7}=x+3\frac{1}{2}$

ev, $\frac{3x+7}{4}+\frac{5x-4}{7}-x=\frac{7}{2}$ [c¶vši K‡i]

ev, $\frac{7(3x+7)+4(5x-4)-28x}{28}=\frac{7}{2}$ [evgc‡¶ ni 4, 7 Gi j.mv.¸. 28]

ev, $\frac{21x+49+20x-16-28x}{28}=\frac{7}{2}$ [eÈb wewa Abmv‡i]

ev, $\frac{13x+33}{28}=\frac{7}{2}$

ev, $28\times\frac{13x+33}{28}=28\times\frac{7}{2}$ [Dfqc¶‡K 28 Øviv ¸Y K‡i]

ev, $13x+33=98$

ev, $13x=98-33$

ev, $13x=65$

ev, $\frac{13x}{13}=\frac{65}{13}$ [Dfqc¶‡K 13 Øviv fvM K‡i]

ev, $x=5$

$\therefore$ mgvavb : $x=5$

KvR : mgvavb Ki :
1| $2x-1=0$ 2| $\frac{x}{2}+1=3$ 3| $4(y-3)=8$

Abkxjbx 7·1

mgvavb Ki :

1| $4x+1=2x+7$ 2| $5x-3=2x+3$

3| $3y+1=7y-1$ 4| $7y-5=y-1$

5| $17-2z=3z+2$ 6| $13z-5=3-2z$

7| $\frac{x}{4}=\frac{1}{3}$ 8| $\frac{x}{2}+1=3$

9| $\frac{x}{3}+5=\frac{x}{2}+7$

10| $\frac{y}{2}-\frac{y}{3}=\frac{y}{5}-\frac{1}{6}$

11| $\frac{y}{5}-\frac{2}{7}=\frac{5y}{7}-\frac{4}{5}$

12| $\frac{2z-1}{3}=5$

13| $\frac{5x}{7}+\frac{4}{5}=\frac{x}{5}+\frac{2}{7}$

14| $\frac{y-2}{4}+\frac{2y-1}{3}=y-\frac{1}{3}$

15| $\frac{3y+1}{5}=\frac{3y-7}{3}$

16| $\frac{x+1}{2}-\frac{x-2}{3}-\frac{x-3}{5}=2$

17| $2(x+3)=10$

18| $5(x-2)=3(x-4)$

19| $7(3-2y)+5(y-1)=34$

20| $(z-1)(z+2)=(z+4)(z-2)$

7·3 mij mgxKiY MVb I mgvavb

GKRb †µZv 3 †KwR cvUwj ¸o wKb‡Z Pvb| †`vKvb`vi x †KwR IR‡bi GKwU eo cvUwji A‡aK gvc‡jb| wKš G‡Z 3 †KwRi Kg n‡jv| Avi‡iv 1 †KwR †`Iqvq 3 †KwR n‡jv| Avgiv GLb †ei Ki‡Z PvB, m¤úY cvUwjwUi IRb KZ wQj, A_vr x Gi gvb KZ ? G Rb¨ mgm¨wU †_‡K GKwU mgxKiY MVb Ki‡Z n‡e| G‡¶‡Î mgxKiYwU n‡e $\frac{x}{2}+1=3$| mgxKiYwU mgvavb Ki‡j x Gi gvb cvIqv hv‡e| A_vr, ¸‡oi m¤úY cvUwji IRb Rvbv hv‡e|

KvR : c`Ë Z_¨ †_‡K mgxKiY MVb Ki (GKwU K‡i †`Iqv n‡jv) :	
c`Ë Z_¨	mgxKiY
1| GKwU msL¨v x Gi cvP¸Y †_‡K 25 we‡qvM Ki‡j we‡qvMdj n‡e 190	
2| c‡Îi eZgvb eqm y eQi, wcZvi eqm c‡Îi eq‡mi Pvi¸Y Ges Zv‡`i eZgvb eq‡mi mgwó 45 eQi|	$y+4y=45$
3| GKwU AvqZvKvi cK‡ii ˆ`N¨ $x$ wgUvi, ˆ`N¨ A‡c¶v c¯' 3 wgUvi Kg Ges cKwiUi cwimxgv 26 wgUvi|	

D`vniY 7| Anbv GKwU cix¶vq Bs‡iwR‡Z I MwY‡Z †gvU 176 b¤i †c‡q‡Q Ges Bs‡iwR A‡c¶v MwY‡Z 10 b¤i †ewk †c‡q‡Q| †m †Kvb wel‡q KZ b¤i †c‡q‡Q?

mgvavb : awi, Anbv Bs‡iwR‡Z x b¤i †c‡q‡Q|

mZivs, †m MwY‡Z †c‡q‡Q $x+10$ b¤i|

ckg‡Z,

$$x + x + 10 = 176$$

ev, $2x + 10 = 176$

ev, $2x = 176 - 10$ [c¶vši K‡i]

ev, $2x = 166$

ev, $\frac{2x}{2} = \frac{166}{2}$ [Dfqc¶‡K 2 Øviv fvM K‡i]

ev, $x = 83$

$\therefore$ $x + 10 = 83 + 10 = 93$

$\therefore$ Anbv Bs‡iwR‡Z †c‡q‡Q 83 b¤i Ges MwY‡Z †c‡q‡Q 93 b¤i|

D`vniY 8| k¨vgj †`vKvb †_‡K wKQ Kjg wKbj| †m¸‡jvi $\frac{1}{2}$ Ask Zvi †evb‡K I $\frac{1}{3}$ Ask Zvi fvB‡K w`j| Zvi Kv‡Q Avi 5wU Kjg iBj| k¨vgj KqwU Kjg wK‡bwQj?

mgvavb : awi, k¨vgj xwU Kjg wK‡bwQj|

$\therefore$ k¨vgj Zvi †evb‡K †`q $x$ Gi $\frac{1}{2}$wU ev $\frac{x}{2}$wU Kjg Ges Zvi fvB‡K †`q x Gi $\frac{1}{3}$ wU ev $\frac{x}{3}$wU Kjg|

kZvbmv‡i, $x - \left(\frac{x}{2} + \frac{x}{3}\right) = 5$

ev, $x - \frac{x}{2} - \frac{x}{3} = 5$

ev, $\frac{6x - 3x - 2x}{6} = 5$ [evgc‡¶ ni 2, 3 Gi j.mv.¸. 6]

ev, $\frac{x}{6} = 5$

ev, $x = 5 \times 6$ [Avo¸Yb K‡i]

ev, $x = 30$

$\therefore$ k¨vgj 30wU Kjg wK‡bwQj|

D`vniY 9| GKwU evm NÈvq 25 wK.wg. MwZ‡e‡M XvKvi MveZjx †_‡K AwiPv †cŠQvj| Avevi evmwU NÈvq 30 wK.wg. MwZ‡e‡M AwiPv †_‡K MveZjx wd‡i Gj| hvZvqv‡Z evmwUi †gvU $5\frac{1}{2}$ NÈv mgq jvMj| MveZjx †_‡K AwiPvi `iZ KZ?

mgvavb : g‡b Kwi, MveZjx †_‡K AwiPvi `iZ d wK.wg. |

$\therefore$ MveZjx †_‡K AwiPv †h‡Z mgq jv‡M $\frac{d}{25}$ NÈv|

Avevi AwiPv †_‡K MveZjx wd‡i Avm‡Z mgq jv‡M $\frac{d}{30}$ NÈv|

$\therefore$ hvZvqv‡Z evmwUi †gvU mgq jvMj $\left(\frac{d}{25}+\frac{d}{30}\right)$ NÈv|

ckg‡Z, $\frac{d}{25}+\frac{d}{30}=5\frac{1}{2}$

ev, $\frac{6d+5d}{150}=\frac{11}{2}$

ev, $11d=\cancelto{75}{150}\times\frac{11}{\cancelto{1}{2}}$

ev, $d=75$

$\therefore$ MveZjx †_‡K AwiPvi `iZ 75wK.wg.|

Abkxjbx 7·2

wb‡Pi mgm¨v¸‡jv †_‡K mgxKiY MVb K‡i mgvavb Ki :

1| †Kvb msL¨vi wØ¸‡Yi mv‡_ 5 †hvM Ki‡j †hvMdj 25 n‡e?

2| †Kvb msL¨v †_‡K 27 we‡qvM Ki‡j we‡qvMdj -21 n‡e?

3| †Kvb msL¨vi GK-ZZxqvsk 4 Gi mgvb n‡e?

4| †Kvb msL¨v †_‡K 5 we‡qvM Ki‡j we‡qvMd‡ji 5 ¸Y mgvb 20 n‡e?

5| †Kvb msL¨vi A‡aK †_‡K Zvi GK-ZZxqvsk we‡qvM Ki‡j we‡qvMdj 6 n‡e?

6| wZbwU µwgK ¯vfweK msL¨vi mgwó 63 n‡j, msL¨v wZbwU †ei Ki|

7| `BwU msL¨vi †hvMdj 55 Ges eo msL¨wUi 5 ¸Y †QvU msL¨wUi 6 ¸‡Yi mgvb| msL¨v `BwU wbYq Ki|

8| MxZv, wiZv I wgZvi GK‡Î 180 UvKv Av‡Q| wiZvi †P‡q MxZvi 6 UvKv Kg I wgZvi 12 UvKv †ewk Av‡Q| Kvi KZ UvKv Av‡Q?

9| GKwU LvZv I GKwU Kj‡gi †gvU `vg 75 UvKv| LvZvi `vg 5 UvKv Kg I Kj‡gi `vg 2 UvKv †ewk n‡j, LvZvi `vg Kj‡gi `v‡gi wØ¸Y n‡Zv| LvZv I Kj‡gi †KvbwUi `vg KZ?

10| GKRb dj we‡µZvi †gvU d‡ji $\frac{1}{2}$ Ask Av‡cj, $\frac{1}{3}$ Ask Kgjv‡je I 40wU Avg Av‡Q| Zvi wbKU †gvU KZ¸‡jv dj Av‡Q?

11| wcZvi eZgvb eqm c‡Îi eZgvb eq‡mi 6 ¸Y| 5 eQi ci Zv‡`i eq‡mi mgwó n‡e 45 eQi| wcZv I c‡Îi eZgvb eqm KZ?

12| wjRv I wkLvi eq‡mi AbcvZ 2:3| Zv‡`i `BR‡bi eq‡mi mgwó 30 eQi n‡j, Kvi eqm KZ ?

13| GKwU wµ‡KU †Ljvq Bgb I mg‡bi †gvU ivbmsL¨v 58| Bg‡bi ivbmsL¨v mg‡bi ivbmsL¨vi wØ¸‡Yi †P‡q 5 ivb Kg| H †Ljvq Bg‡bi ivbmsL¨v KZ?

14| GKwU †Ub NÈvq 30 wK.wg. †e‡M P‡j Kgjvci †÷kb †_‡K bvivqYMÄ †÷k‡b †cŠQvj| †UbwUi †eM NÈvq 25 wK.wg. n‡j 10 wgwbU mgq †ewk jvMZ| `B †÷k‡bi g‡a¨ `iZ KZ?

15| GKwU AvqZ‡Kvi Rwgi ˆ`N¨ c‡¯'i wZb¸Y Ges Rwgi cwimxgv 40wgUvi| RwgwUi ˆ`N¨ I c¯' wbYq Ki|

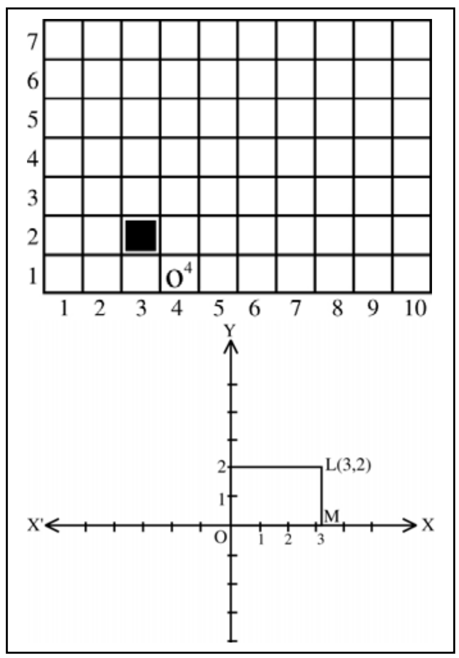
†jLwPÎ

7·4 ¯'vbv‡¼i aviYv

dv‡Ýi weL¨vZ MwYZwe` †i‡b †`Kv‡Z (Rene Descartes : 1596–1650) : mec_g ¯'vbv‡¼i aviYv †`b| wZwb `BwU ci¯úi‡Q`x j¤^‡iLvi mv‡c‡¶ we›`i Ae¯'vb e¨vL¨v K‡ib|

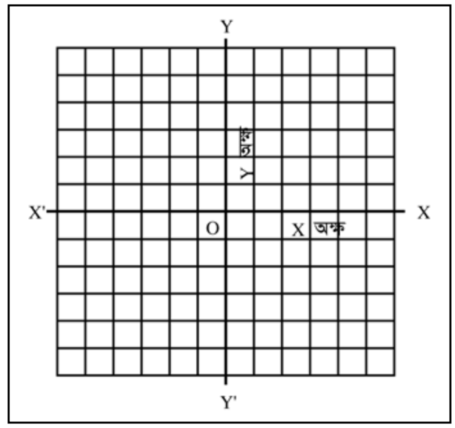
GKwU †kwYK‡¶ GKK Avmbwebv¨‡m GKRb wk¶v_xi Ae¯'vb †Kv_vq Rvb‡Z n‡j AbfwgK †iLv ev kqvb †iLv eivei †Kv_vq Av‡Q Ges Djø¤^ †iLv ev Lvov †iLv eivei †Kv_vq Av‡Q Zv Rvbv `iKvi|

awi, †kwYK‡¶ GKRb wk¶v_x wjRv (L)-Gi Ae¯'vb Rvb‡Z PvB| wjRvi Ae¯'vb‡K GKwU we›` (·) wn‡m‡e we‡ePbv Kiv hvq| wP‡Î j¶ Kwi, wjRv GKwU wbw`ó we›` $O$ †_‡K AbfwgK †iLv $OX$ eivei 3 GKK `‡i M we›`‡Z Ges †mLvb †_‡K Djø¤^ †iLv $OY$ Gi mgvšivj †iLv eivei Dciw`‡K 2 GKK `‡i $L$ we›`‡Z Ae¯'vb Ki‡Q| Zvi G Ae¯'vb‡K $(3, 2)$ Øviv cKvk Kiv nq|



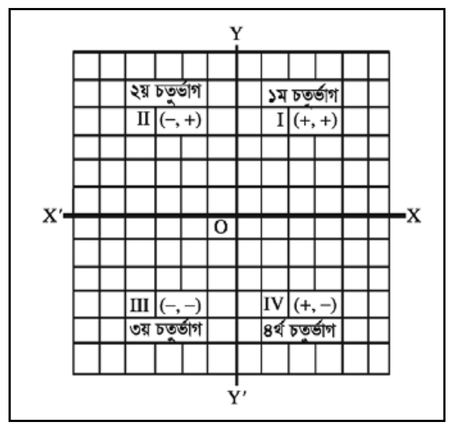
7·5 wet` cvZb

QK KvM‡R mgvb `‡i ci¯úi‡Q`x mgvšivj mij‡iLv Øviv †QvU †QvU e‡M wef³ Kiv _v‡K| QK KvM‡R †Kv‡bv we›`i Ae¯vb †`Lv‡bv‡K ev †Kv‡bv we›` ¯vcb Kiv‡K we›` cvZb e‡j| we›` cvZ‡bi Rb¨ mweavg‡Zv `BwU ci¯úi j¤ mij‡iLv †bIqv nq| wP‡Î XOX' I YOY' †iLvØq ci¯úi j¤fv‡e O we›`‡Z †Q` K‡i‡Q| O we›`‡K ejv nq gjwe›`| Abfw̄gK †iLv XOX' †K x-A¶ Ges Djø¤ †iLv YOY' †K y-A¶ ejv nq|



cavbZ QK KvM‡Ri ¶`Zg eM‡¶‡Îi evûi ˆ`N¨‡K GKK wn‡m‡e aiv nq| mvaviYfv‡e †h‡Kv‡bv we›`i ¯vbv¼‡K $(x, y)$ †jLv nq| $x$-†K ejv nq we›`wUi x-¯vbv¼ ev fR Ges y-†K ejv nq we›`wUi $y$-¯vbv¼ ev †KwU| ¯úóZB gjwe›` O Gi ¯vbv¼ n‡e $(0, 0)$|

gjwe›` †_‡K $x$-A‡¶i Wvbw`K abvZ¥K w`K I evgw`K FYvZ¥K w`K| Avevi, gjwe›` †_‡K y-A‡¶i Dc‡ii w`K abvZ¥K w`K I wb‡Pi w`K FYvZ¥K w`K| d‡j QKwU A¶Øq Øviv PviwU fv‡M wef³ n‡q‡Q| GBfvM PviwU Nwoi KvUvi NY‡bi wecixZ w`K Abhvqx 1g, 2q, 3q I 4_ PZfvM wn‡m‡e cwiwPZ| c_g PZfv‡M †h‡Kv‡bv we›`i x ¯vbv¼ I y ¯vbv¼ DfqB abvZ¥K, wØZxq PZfv‡M †h‡Kv‡bv we›`i $x$ ¯vbv¼ FYvZ¥K I $y$ ¯vbv¼ abvZ¥K, ZZxq PZfv‡M †h‡Kv‡bv we›`i x ¯vbv¼ FYvZ¥K I y ¯vbv¼ FYvZ¥K Ges PZ_ PZfv‡M †h‡Kv‡bv we›`i x ¯vbv¼ abvZ¥K I y ¯vbv¼ FYvZ¥K|



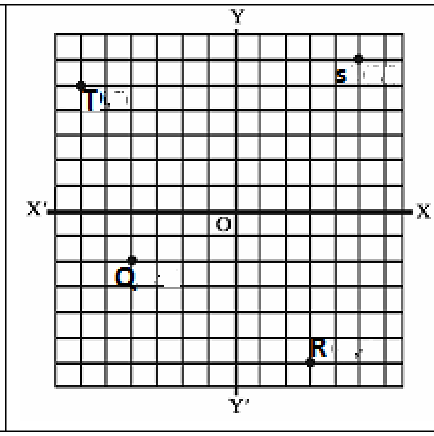
c‡ei Ab‡”Q‡` Av‡jwPZ wjRvi Ae¯vb $(3, 2)$ wbYq Kivi Rb¨ c_‡g $x$-A¶ eivei Wvbw`‡K 3 GKK `i‡Z th‡Z n‡e| Zvici †mLvb †_‡K Lvov Dci w`‡K 2 GKK `i‡Z th‡Z n‡e| Zv n‡j wjRvi Ae¯vb $L$ we›`i ¯vbv¼ n‡e $(3, 2)$| Abicfv‡e wP‡Î P we›`i ¯vbv¼ $(-2, 4)$|

D`vniY 1| QK KvM‡R wb‡Pi c_g PviwU we›` ¯vcb K‡i Zxi wPý Abhvqx †hvM Ki : $(3,2) \to (6,2) \to (6,4) \to (3,4)$| wP‡ÎwUi R¨wgwZK AvKwZ Kx n‡e?

mgvavb : awi, we›` PviwU h_vµ‡g A, B, C, D| A_vr. $A(3, 2), B(6, 2), C(6, 4)$ Ges $D(3, 4)$| QK KvM‡R Dfq A‡¶

¶`Zg eM‡¶‡Î i cwZ evûi ^`N¨‡K GKK awi| A we›`yU ¯’vcb Ki‡Z gjwe›` O †_‡K x-A‡¶i Wvbw`K eivei 3wU †QvU e‡Mi evûi mgvb `‡i wM‡q Dc‡ii w`‡K 2wU †QvU e‡Mi evûi mgvb D‡V †M‡j †h we›`yU cvIqv hv‡e, Zv A we›`| Abiƒcfv‡e c`Ë Awkó we›`mgn ¯’vcb Kwi| Zvici $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$ Gfv‡e we›`¸‡jv †hvM Kwi| G‡Z $ABCD$ wPÎwU cvIqv †Mj| †`Lv hvq †h, $ABCD$ wPÎwU GKwU AvqZ|

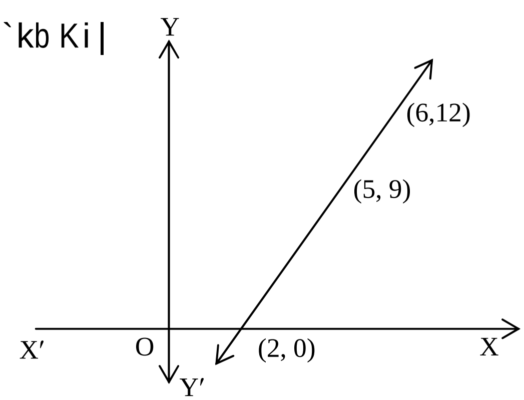
KvR :

wPÎ †_‡K †Zvgiv Q, R, S, T we›`i ¯’vbv¼ wbYq Ki|	

7.6 †jLwP‡Î mgxKi‡Yi mgvavb

†jLwP‡Îi mvnv‡h¨ mn‡RB mgxKi‡Yi mgvavb †ei Kiv hvq| g‡b Kwi, $2x-5=0$ mgxKiYwU mgvavb Ki‡Z n‡e| mgxKi‡Yi evgc¶ $2x-5$ iwk‡Z x-Gi wewfbœ gvb emv‡j iwkwUi wewfbœ gvb cvIqv hvq| †jLwP‡Î cwZwU x †K fR Ges iwkwUi gvb‡K †KwU a‡i GKwU K‡i we›` cvIhv hv‡e| we›`¸‡jv †hvM K‡i GKwU mij‡iLv Aw¼Z n‡e| mij‡iLwU †h we›`‡Z A¶‡K †Q` K‡i, †mB we›`i fRB wb‡Yq mgvavb| †Kbbv, x-Gi GB gv‡bi Rb¨ iwkwUi gvb 0 nq, hv mgxKi‡Yi Wvbc‡¶i gv‡bi mgvb nq| G †¶‡Î mgxKiYwUi mgvavb $x=\frac{5}{2}$|

D`vniY 2| $3x-6=0$ mgvavb Ki Ges †jLwP‡Î mgvavb c`kb Ki|



mgvavb : $3x-6=0$

ev, $3x=6$ [c¶všÍi K‡i]

ev, $\frac{3x}{3}=\frac{6}{3}$ [Dfqc¶‡K 3 Øviv fvM K‡i]

ev, $x=2$

$\therefore$ mgvavb : $x=2$

†jLwPÎ A¼b : c`Ë mgxKiY $3x-6=0$

x Gi K‡qKwU gvb wb‡q $3x-6$ Gi Abi¦c gvb †ei Kwi Ges wb‡Pi QKwU ˆZwi Kwi :

x	$3x-6$	$(x,3x-6)$
2	0	$(2,0)$
5	9	$(5,9)$
6	12	$(6,12)$

†jLwPÎ A¼‡bi Rb¨ wZbwU we›` $(2,0),(5,9)$ I $(6,12)$ †bIqv n‡jv|

g‡b Kwi, ci¯úi j¤ XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges 0 gjwe›`|

QK KvM‡R Dfq A‡¶ ¶`Zg eM©‡¶‡Îi GK evûi ˆ`N¨‡K GKK a‡i $(2,0),(5,9)$, $(6,12)$ we›`¸‡jv ¯vcb Kwi| Zvici we›`¸‡jv cici ms‡hvM Kwi| †jLwP‡Î GKwU mij‡iLv cvB| mij‡iLwU x-A¶‡K $(2,0)$ we›`‡Z †Q` K‡i| we›`wUi fR n‡jv 2 | mZivs c`Ë mgxKi‡Yi mgvavb $x=2$|

D`vniY 3| †jLwP‡Îi mvnv‡h¨ mgvavb Ki : $3x-4=-x+4$

mgvavb : c`Ë mgxKiY $3x-4=-x+4$

x Gi K‡qKwU gvb wb‡q $3x-4$ Gi Abi¦c gvb †ei Kwi Ges cv‡ki QK-1 ˆZwi Kwi :

QK-1

x	$3x-4$	$(x, 3x-4)$
0	-4	$(0,-4)$
2	2	$(2,2)$
4	8	$(4,8)$

∴ $3x-4$ Gi †j‡Li Dci wZbwU we›` $(0,-4),(2,2)$, $(4,8)$ wbB|

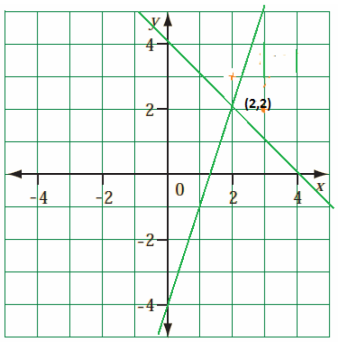
Avevi, x Gi K‡qKwU gvb wb‡q $-x+4$ Gi Abi¦c gvb †ei Kwi Ges cv‡ki QK-2 ˆZwi Kwi :

∴ $-x+4$ Gi †j‡Li Dci wZbwU we›` $(0,4),(2,2)$, $(4,0)$ wbB|

QK-2

x	$-x+4$	$(x,-x+4)$
0	4	$(0,4)$
2	2	$(2,2)$
4	0	$(4,0)$

g‡b Kwi, ci¯úi j¤ XOX' I YOY' h_vµ‡g x-A¶ I y-A¶ Ges 0 gjwe›`| GLb, QK-1 G cvß $(0,-4)$, $(2,2)$, $(4,8)$ we›` wZbwU ¯vcb Kwi Ges G‡`i cici ms‡hvM Kwi| †jLwP‡Î GKwU mij‡iLv cvB| Avevi, QK-2G cvß $(0,4),(2,2)$, $(4,0)$ we›` wZbwU ¯vcb Kwi I G‡`i cici ms‡hvM Kwi| G‡¶‡ÎI †jLwP‡Î GKwU mij‡iLv cvB|



j¶ Kwi, mij‡iLv `BwU ci¯úi $(2,2)$ we›`y‡Z †Q` K‡i‡Q| †Q`we›`y‡Z $3x-4$ I $-x+4$ Gi gvb ci¯úi mgvb| mZivs, c`Ë mgxKi‡Yi mgvavb n‡jv $(2,2)$ we›`y‡Z f‡Ri gvb, A_vr $x=2$ |

KvR : wb‡Pi mgxKiY¸‡jvi mgvav‡bi †jLwPÎ AvK :
1| $2x-1=0$ 2| $3x+5=2$

Abkxjbx 7·3

1| $\frac{x}{2}=\frac{1}{3}$ mgxKi‡Yi gj wb‡Pi †KvbwU?

K. $\frac{1}{2}$ L. $\frac{2}{3}$ M. $\frac{3}{2}$ N. 6

2| $\frac{x}{3}-3=0$ mgxKi‡Yi gj wb‡Pi †KvbwU?

K. $\frac{1}{3}$ L. 3 M. 9 N. -9

3| GKwU wÎf‡Ri evû wZbwUi ˆ`N©¨ $(x+1)$ †m.wg., $(x+2)$ †m.wg. I $(x+3)$ †m.wg. $(x>0)$ | wÎfRwUi cwimxgv 15 †m.wg. n‡j, x Gi gvb KZ?
K. 1 †m.wg. L. 2 †m.wg. M. 3 †m.wg. N. 6 †m.wg.

4| †Kvb msL¨vi GK-PZ_vsk 4 Gi mgvb n‡e?

K. 16 L. 12 M. 4 N. $\frac{1}{4}$

5| wb‡Pi Z_¨¸‡jv j¶ Ki :
i. mgxKi‡Yi Dfqc¶ †_‡K mvaviY Drcv`K eRb Kiv hvq|
ii. $2x+1=x-3$ GKwU ØNvZ mgxKiY|
iii. $x+2=2$ mgxKi‡Yi gj 0.
Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK?
K. *i* I *ii* L. *i* I *iii* M. *ii* I *iii* N. *i, ii* I *iii*

6| Kb‡Ki wbKU 8wU I †Kqvi wbKU 12wU PK‡jU Av‡Q| Zvn‡j wb‡Pi cÖ‡kœi DËi `vI :

(1) †Kqv KbK‡K xwU PK‡jU w`‡j Zv‡`i PK‡jU msL¨v mgvb n‡e| †m †¶‡Î wb‡Pi †Kvb mgxKiYwU mwVK?

K. $8+x=12$ L. $8=12-x$ M. $8+x=12-x$ N. $8-x=x-12$

(2) x Gi gvb KZ n‡j Zv‡`i PK‡jU msL¨v mgvb n‡e?

K. 2 L. 4 M. 6 N. 10

(3) KbK †Kqv‡K KqwU PK‡jU w`‡j †Kqvi PK‡jU Kb‡Ki PK‡j‡Ui Pvi¸Y n‡e?

K. 2 L. 4 M. 6 N. 10

7| wPÎ †_‡K wb‡Pi QKwU cziY Ki :

(Dfq A‡¶ ¶`Zg eM©‡¶‡Îi evûi ˆ`N©¨‡K GKK a‡i)

we›`	¯'vbv¼
A	$(4, 3)$
B	$(-2,\ \ )$
C	$(\ \ , -5)$
D	$(\ \ ,\ \ )$
O	$(\ \ ,\ \ )$
P	$(\ \ , 0)$
Q	$(0,\ \ )$

8| wb‡Pi we›`¸‡jv QK KvM‡R ¯'vcb K‡i ZxiwPý Abhvqx †hvM Ki I wPÎwUi R¨vwgwZK bvgKiY Ki :

(K) $(2, 2) \rightarrow (6, 2), \rightarrow (6, 6) \rightarrow (2, 6) \rightarrow (2, 2),$

(L) $(0, 0) \rightarrow (-6, -6), \rightarrow (8, 6) \rightarrow (0, 0)$

9| mgvavb Ki Ges mgvavb †jLwP‡Î †`LvI :

(K) $x-4=0$ (L) $2x+4=0$ (M) $x+3=8$

(N) $2x+1=x-3$ (O) $3x+4=5x$

10| GKwU wÎf‡Ri wZb evûi ˆ`N©¨ $(x+2)$ †m.wg. $(x+4)$ †m.wg. I $(x+6)$ †m.wg. $(x>0)$ Ges wÎfRwUi cwimxgv 18 †m.wg.|

K. c`Ë kZvbhvqx AvbycwZK wPÎ AvK|

L. mgxKiY MVb K‡i mgvavb Ki|

M. mgvav‡bi †jLwPÎ AvK|

11| XvKv I Awi‡Pi ga¨eZx `iZ 77 wK.wg.| GKwU evm NÈvq 30 wK.wg. †e‡M XvKv †_‡K Awi‡Pi c‡_ iIbv w`j| Aci GKwU evm NÈvq 40 wK.wg. †e‡M Awi‡Pv †_‡K XvKvi c‡_ GKB mg‡q iIbv w`j I evm `BwU XvKv †_‡K x wK.wg. `‡i wgwjZ n‡jv|

K. evm `BwU Awi‡Pv †_‡K KZ `‡i wgwjZ n‡e Zv x Gi gva¨‡g cKvk Ki|

L. x Gi gvb wbYq Ki|

M. Mše¨¯'‡b †cŠQv‡Z †Kvb ev‡mi KZ mgq jvM‡e?

Aóg Aa¨vq

mgvšivj mij‡iLv

ˆ`bw›`b Rxe‡b Avgv‡`i Pvicv‡k hv wKQ †`wL I e¨envi Kwi Gi wKQ Pvi‡Kvbv, wKQ †MvjvKvi| Avgv‡`i Nievo, `vjvb‡KvVv, `iRv-Rvbvjv, LvU-Avjgvwi, †Uwej-†Pqvi, eB-LvZv BZ¨vw` meB Pvi‡Kvbv| G‡`i avi¸‡jv mij‡iLv wn‡m‡e we‡ePbv Ki‡j †`Lv hvq †h, Giv mg`ieZx ev mgvšivj|

Aa¨vq †k‡l wk¶v_xiv –

- ➢ mgvšivj mij‡iLv I †Q`K Øviv Drcb †Kv‡Yi ˆewkó¨ e¨vL¨v Ki‡Z cvi‡e|
- ➢ `BwU mij‡iLv mgvšivj nIqvi kZ eYbv Ki‡Z cvi‡e|
- ➢ `BwU mij‡iLv mgvšivj nIqvi kZ cgvY Ki‡Z cvi‡e|

8·1 R¨vwgwZK hw³ c×wZ

cwZÁv : R¨vwgwZ‡Z †h mKj wel‡qi Av‡jvPbv Kiv nq, mvaviYfv‡e Zv‡`i cwZÁv ejv nq|

m¤úv`¨ : †h cwZÁvq †Kv‡bv R¨vwgwZK welq A¼b K‡i †`Lv‡bv nq Ges hw³ Øviv A¼‡bi wbfjZv cgvY Kiv hvq, G‡K m¤úv`¨ ejv nq|

m¤úv‡`¨i wewfb Ask:

(K) DcvË : m¤úv‡`¨ hv †`Iqv _v‡K, ZvB DcvË|

(L) A¼b : m¤úv‡`¨ hv KiYxq, ZvB A¼b|

(M) cgvY : hw³ Øviv A¼‡bi wbfjZv hvPvB n‡jv cgvY|

Dccv`¨ : †h cwZÁvq †Kv‡bv R¨vwgwZK welq‡K hw³ Øviv cwZwôZ Kiv nq, G‡K Dccv`¨ e‡j| Dccv‡`¨i wewfb Ask:

(K) mvaviY wbePb: G As‡k cwZÁvi welqwU mij fv‡e eYbv Kiv nq|

(L) we‡kl wbePb: G As‡k cwZÁvi welqwU wPÎ Øviv we‡klfv‡e †`Lv‡bv nq|

(M) A¼b: G As‡k cwZÁv mgvav‡bi ev cgv‡Yi Rb¨ AwZwi³ A¼b Ki‡Z nq|

(N) cgvY: G As‡k ¯Ztwm×¸‡jv Ges c‡e MwVZ R¨vwgwZK mZ¨ e¨envi K‡i Dch³ hw³ Øviv c¯ weZ welqwU‡K cwZwôZ Kiv nq|

Abwm×vš : †Kv‡bv R¨vwgwZK cwZÁv cwZwôZ K‡i Gi wm×vš †_‡K GK ev GKwaK †h bZb wm×vš MnY Kiv hvq, G‡`i‡K Abwm×vš ejv nq|

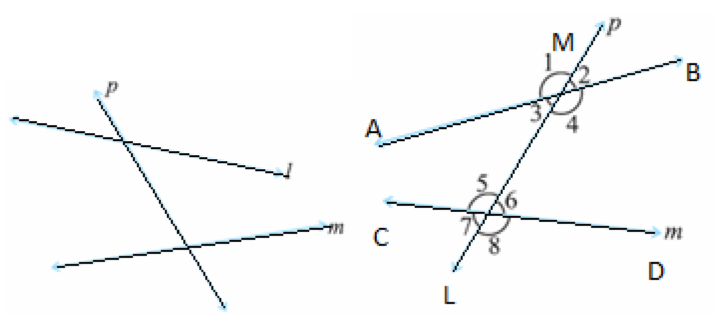
AvawbK hw³gjK R¨vwgwZi Av‡jvPbvi Rb¨ wKQ †gšwjK ¯xKvh, msÁv I wP‡ýi c‡qvRb nq|

R¨wgwZ‡Z e¨eüZ wPýmgn

wPý	A_	wPý	A_
+	†hvM	$\angle$	†KvY
=	mgvb	$\perp$	j¤
$>$	enËi	Δ	wÎfR
$<$	¶`Zi	$\odot$	eË
$\cong$	memg	$\because$	†h‡nZ
$\parallel$	mgvšivj	$\therefore$	mZivs , AZGe

8.2 †Q`K

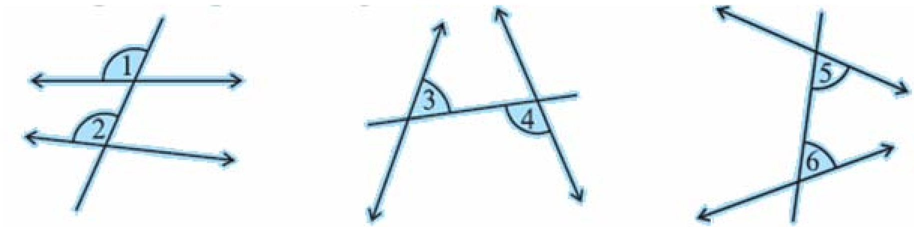
†Kv‡bv mij‡iLv `B ev Z‡ZwaK mij‡iLv‡K wewfbœ we›`‡Z †Q` Ki‡j G‡K †Q`K e‡j| wP‡Î, AB I CD `BwU mij‡iLv Ges $LM$ mij‡iLv †m¸‡jv‡K `BwU wfbœ we›` $P,Q$ †Z †Q` K‡i‡Q| LM mij‡iLv AB I CD mij‡iLvØ‡qi †Q`K| †Q`KwU AB I CD mij‡iLv `BwUi mv‡_ †gvU AvUwU †KvY ˆZwi K‡i‡Q| †KvY¸‡jv‡K $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$ Øviv wb‡`k Kwi| †KvY¸‡jv‡K Ašt¯' I ewnt¯', Abyi I GKvšÍi GB Pvi †kÖwY‡Z fvM Kiv hvq|



Ašt¯' †KvY	$\angle 3, \angle 4, \angle 5, \angle 6$
ewnt¯' †KvY	$\angle 1, \angle 2, \angle 7, \angle 8$
Abyi †KvY †Rvov	$\angle 1$ Ges $\angle 5$, $\angle 2$ Ges $\angle 6$ $\angle 3$ Ges $\angle 7$, $\angle 4$ Ges $\angle 8$
Ašt¯' GKvšÍi †KvY †Rvov	$\angle 3$ Ges $\angle 6$, $\angle 4$ Ges $\angle 5$
ewnt¯' GKvšÍi †KvY †Rvov	$\angle 1$ Ges $\angle 8$, $\angle 2$ Ges $\angle 7$
†Q`‡Ki GKB cv‡k Ašt¯' †KvY †Rvov	$\angle 3$ Ges $\angle 5$, $\angle 4$ Ges $\angle 6$

Abic †KvY¸‡jvi ˆewkó¨: (K) kxl©we›` Avjv`v (L) †Q`‡Ki GKB cv‡k Aew¯Z|
GKvši †KvY¸‡jvi ˆewkó¨: (K) kxl©we›` Avjv`v (L) †Q`‡Ki wecixZ cv‡k Aew¯Z
(M) mij‡iLv `BwUi g‡a¨ Aew¯Z|

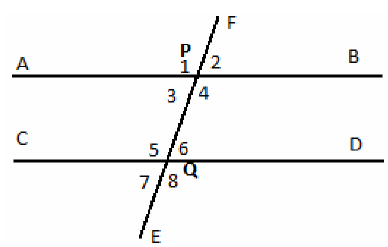
KvR
1|(K) wP‡Î i †KvY¸‡jv †Rvovq †Rvovq kbv³ Ki|
(L) $\angle 3$ I $\angle 6$ Gi Abic †KvY †`LvI|
(M) $\angle 4$ Gi wecZxc †KvY Ges $\angle 1$ Gi m¤úiK †KvY wb‡`k Ki|



8·3 †Rvov mgvšivj mij‡iLv

Avgiv †R‡bwQ †h, GKB mgZ‡j Aew¯Z `BwU mij‡iLv G‡K Aci‡K †Q` bv Ki‡j †m¸‡jv mgvšivj mij‡iLv | `BwU mgvšivj mij‡iLv †_‡K †h‡Kv‡bv `BwU †iLvsk wb‡j, †iLvsk `BwUI ci¯úi mgvšivj nq| `BwU mgvšivj mij‡iLvi GKwUi †h‡Kv‡bv we›` †_‡K AciwUi j¤^`iZ me©`v mgvb| Avevi `BwU mij‡iLvi GKwUi †h‡Kv‡bv `BwU we›` †_‡K AciwUi j¤^`iZ ci¯úi mgvb n‡j I †iLØq mgvšivj| GB j¤^`iZ‡K `BwU mgvšivj †iLØ‡qi `iZ ejv nq|

j¶ Kwi, †Kv‡bv wbw`ó mij‡iLvi Dci Aew¯Z bq Gic we›`i ga¨ w`‡q H mij‡iLvi mgvšivj K‡i GKwU gvÎ mij‡iLv AvKv hvq|



Dc‡ii wP‡Î, AB I CD `BwU mgvšivj mij‡iLv Ges $EF$ mij‡iLv †m¸‡jv‡K `BwU we›` $P$ I $Q$ †Z †Q` K‡i‡Q| EF mij‡iLv AB I CD mij‡iLvØ‡qi †Q`K| †Q`KwU AB I CD mij‡iLv `BwUi mv‡_ $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5, \angle 6, \angle 7, \angle 8$ †gvU AvUwU †KvY ˆZwi K‡i‡Q| G †KvY¸‡jvi g‡a¨

(K) $\angle 1$ Ges $\angle 5$, $\angle 2$ Ges $\angle 6$, $\angle 3$ Ges $\angle 7$, $\angle 4$ Ges $\angle 8$ ci¯úi Abic †KvY|
(L) $\angle 3$ Ges $\angle 6$, $\angle 4$ Ges $\angle 5$ n‡jv ci¯úi GKvši †KvY|
(M) $\angle 3, \angle 4, \angle 5, \angle 6$ Ašt¯’ †KvY|

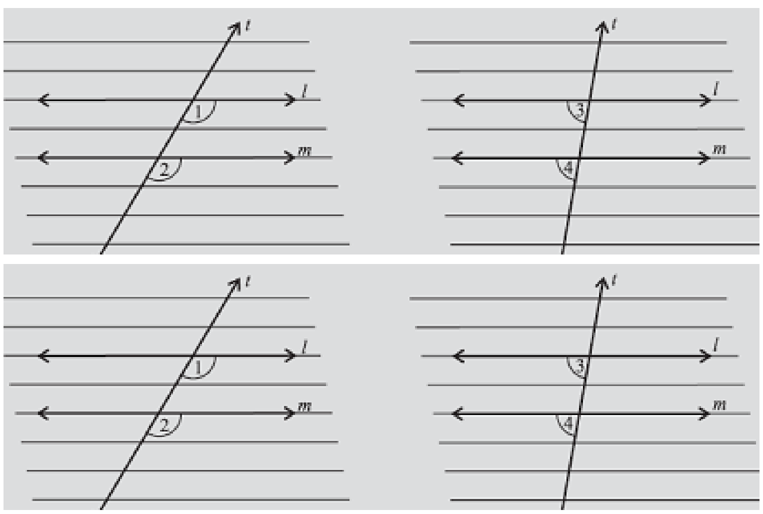
GB GKvši I Abic †KvY¸‡jvi g‡a¨ m¤úK i‡q‡Q| GB m¤úK †ei Kivi Rb¨ `jMZfv‡e wb‡Pi KvRwU Ki:

KvR :

1| i“jUvbv GKcôv KvM‡R wP‡Îi b¨vq `BwU mgvšivj mij‡iLv I G‡`i GKwU †Q`K AvK| `B †Rvov Abic †KvY wPwýZ Ki| cwZ‡Rvov Abic †KvY mgvb wKbv hvPvB Ki| mgvb n‡q‡Q wK?

2| `B †Rvov GKvši †KvY wPwýZ Ki| cwZ †Rvov GKvši †KvY mgvb wKbv hvPvB Ki| mgvb n‡q‡Q wK?

3| mgvšivj mij‡iLvØ‡qi †Q`‡Ki GKB cv‡ki Ašt¯ †KvY `BwU cwigvc Ki| †KvY `BwUi cwigv‡ci †hvMdj †ei Ki| †hvMdj †Zvgvi mncvVx‡`i †ei Kiv †hvMd‡ji mv‡_ Zjbv Ki| †Zvgv‡`i †hvMdj mvgvb¨ Kg-†ewk 180° n‡q‡Q wK?



Kv‡Ri djvdj chv‡jvPbv K‡i Avgiv wb‡Pi wm×v‡š— DcbxZ nB:

- `BwU mgvšivj mij‡iLvi GKwU †Q`K Øviv Drcb c‡Z¨K Abic †KvY †Rvov mgvb n‡e|
- `BwU mgvšivj mij‡iLvi GKwU †Q`K Øviv Drcb c‡Z¨K GKvši †KvY †Rvov mgvb n‡e|
- `BwU mgvšivj mij‡iLvi GKwU †Q`K Øviv Drcb †Q`‡Ki GKB cv‡ki Ašt¯ †KvY `BwU ci¯úi m¤úiK|

welqwU mn‡R g‡b ivLvi Rb¨ j¶ Ki :

Abic †KvY †Rvov *F* e‡Y Avi GKvši †KvY †Rvov *Z* e‡Y wPwýZ|

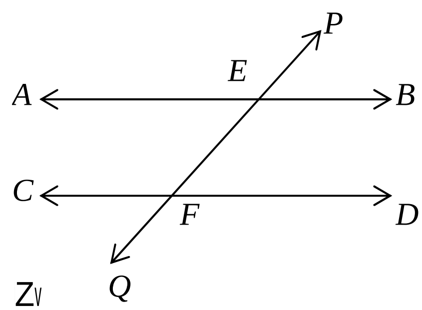
mgvšivj mij‡iLvi GB wZbwU ag Avjv`vfv‡e cgvY Kiv hvq bv| G‡`i †h‡Kv‡bv GKwU‡K mij‡iLvi msÁv wn‡m‡e we‡ePbv K‡i ewK `BwU ag cgvY Kiv hvq|

msÁv : `BwU mij‡iLvi GKwU †Q`K Øviv Drcb I Abi¡c †KvY †Rvov mgvb n‡j †iLvØq mgvšivj|

Dccv`¨ 1

`BwU mgvšÍiv‡j mij‡iLv‡K G‡`i GKwU mij‡iLv †Q` Ki‡j GKvšÍi †KvY †Rvov mgvb|

we‡kl wbePb : g‡b Kwi, $AB \parallel CD$ Ges PQ †Q`K Zv‡`i h_vµ‡g E I F we›`y‡Z †Q` K‡i‡Q| cgvY Ki‡Z n‡e †h, $\angle AEF$ = GKvšÍi $\angle EFD$ |



cgvY :

avc : h_v_Zv

(1) $\angle PEB$ = Abi~íc $\angle EFD$ [mgvšÍivj †iLvi msÁvbmv‡i Abi~íc †KvY mgvb]

(2) $\angle PEB$ = wecZxc $\angle AEF$ [wecZxc †KvYØq ci¯úi mgvb]

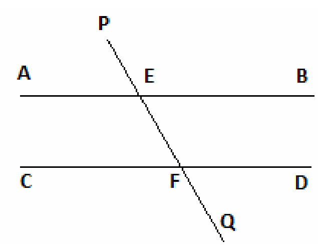
$\therefore\ \angle AEF = \angle EFD$ [(1) I (2) †_‡K]

[cgwYZ]

KvR :

1| cgvY Ki †h, `BwU mgvšÍivj mij‡iLvi GKwU †Q`K Øviv Drcb †Q`‡Ki GKB cv‡ki Aš¯— †KvYØq ci¯úi mgvb|

wP‡Î, $AB \parallel CD$ Ges PQ †Q`K Zv‡`i h_vµ‡g E I F we›`y‡Z †Q` K‡i‡Q|



mZivs, (K) $\angle AEF$ = GKvšÍi $\angle EFD$

(L) $\angle PEB$ = Abic $\angle EFD$

(M) $\angle BEF + \angle EFD$ = `B mg‡KvY|

KvR :

1| GKwU mij‡iLvi Dci `BwU we›` bvI| †iLwUi we›` `BwU‡Z GKB w`‡K 60° Gi mgvb `BwU †KvY AvK| †KvYØ‡qi Aw¼Z evû `BwU mgvšÍivj wKbv hvPvB Ki|

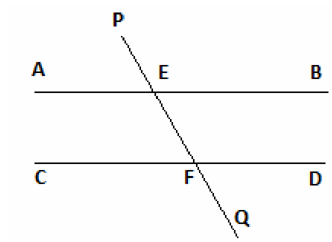
Kv‡Ri djvdj chv‡jvPbv K‡i Avgiv wb‡Pi wm×v‡šÍ DcbxZ nB:

`BwU mij‡iLv Aci GKwU mij‡iLv‡K †Q` Ki‡j hw` Abic †KvY¸‡jv ci¯úi mgvb nq, Z‡e H mij‡iLv `BwU ci¯úi mgvšÍivj|

`BwU mij‡iLv Aci GKwU mij‡iLv‡K †Q` Ki‡j hw` GKvšÍi †KvY¸‡jv ci¯úi mgvb nq, Z‡e H mij‡iLv `BwU ci¯úi mgvšÍivj|

`BwU mij‡iLv Aci GKwU mij‡iLv‡K †Q` Ki‡j hw` †Q`‡Ki GKB cv‡ki Aš—t¯’ †KvY `BwUi mgwó `B mg‡Kv‡Yi mgvb nq, Z‡e H mij‡iLv `BwU ci¯úi mgvšÍivj|

wP‡Î, AB I CD †iLvØq‡K PQ †iLv h_vµ‡g E I F we›`‡Z †Q` K‡i‡Q Ges



(K) $\angle AEF$ = GKvši $\angle EFD$

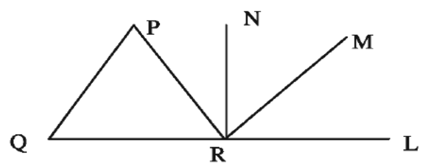
A_ev, (L) $\angle PEB$ = Abic $\angle EFD$

A_ev, (M) $\angle BEF + \angle EFD$ = `B mg‡KvY|

mZivs, AB I CD †iLv `BwU ci¯úi mgvšivj|

Abkxjbx 8

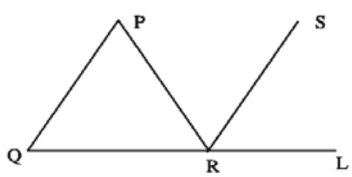
1|



wP‡Î, $\angle PQR = 55^\circ, \angle LRN = 90^\circ$ Ges $PQ \parallel MR$ n‡j, $\angle MRN$ Gi gvb wb‡Pi †KvbwU ?

K. 35° L. 45° M. 55° N. 90°

2|



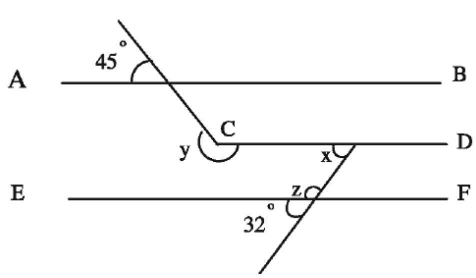
wPÎ, $PQ \parallel SR$, $PQ = PR$ Ges $\angle PRQ = 50^\circ$ n‡j, $\angle LRS$ Gi gvb wb‡Pi †KvbwU ?

K. 80^0 L. 50° M. 55° N. 75°

3| ABC mgwØevû wÎf‡R fyg BC Gi mgvšivj EF †iLv AB Ges AC †K E, F we›`‡Z †Q` K‡i‡Q| $\angle B = 52^\circ$ n‡j, $\angle A + \angle F$ Gi gvb wb‡Pi †KvbwU ?

K. 76° L. 104° M. 128° N. 156°

4|



$AB \parallel CD \parallel EF$

(1) $\angle X$ Gi gvb wb‡Pi †KvbwU ?

K. 28° L. 32° M. 45° N. 58°

(2) $\angle Z$ Gi gvb wb‡Pi †KvbwU ?

K. 58° L. 103° M. 122° N. 148°

(3) wb‡Pi †KvbwU $y-z$ Gi gvb ?

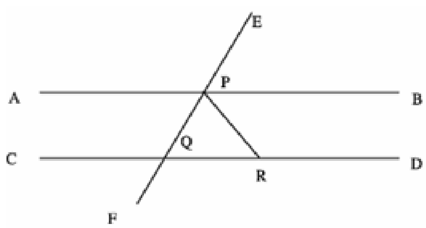
K. 58° L. 77° M. 103° N. 122°

5| i. GKB †iLvi Dci Aew¯Z `BwU mwbwnZ †KvY ci¯úi mgvb n‡Z cv‡i|
ii. wecZxc †KvYØ‡qi mgwØLÐK GKB mij‡iLvq Aew¯Z|
iii. GKwU †iLvi ewnt¯ GKwU we›` w`‡q H †iLvi mgvšivj GKwaK †iLv AvKv hvq|

Dc‡ii Z‡_¨i wfwË‡Z wb‡Pi †KvbwU mwVK ?

K. i I ii L. i I iii M. ii I iii N. i, ii I iii

6|



wP‡Î, $AB \parallel CD$, $\angle BPE = 60^\circ$ Ges $PQ = PR$.

K. †`LvI †h, $\frac{1}{2}\angle APE = 60^\circ$

L. $\angle CQF$ Gi gvb †ei Ki|

M. cgvY Ki †h, PQR GKwU mgevû wÎfR|

beg Aa¨vq

wÎfR

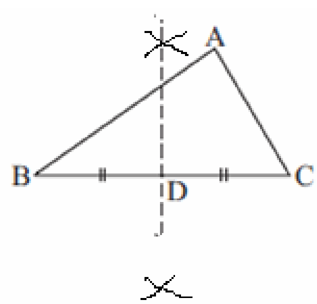
Avgiv †R‡bwQ, wZbwU †iLvsk Øviv Ave× †¶‡Îi mxgv‡iLv‡K wÎfR ejv nq Ges †iLvsk¸‡jv‡K wÎf‡Ri evû e‡j| †h‡Kv‡bv `BwU evûi mvaviY we›`y‡K kxlwe›` ejv nq| `BwU evû kxlwe›`‡Z †h †KvY Drcb K‡i Zv wÎf‡Ri GKwU †KvY| wÎf‡Ri wZbwU evû I wZbwU †KvY Av‡Q| evû‡f‡` wÎfR wZb cÖKvi: mgevû, mgwØevû I welgevû| Avevi †KvY‡f‡` I wÎfR wZb cÖKvi: m²‡KvYx, ¯j‡KvYx I mg‡KvYx| wÎf‡Ri evû wZbwUi ˆ`‡N©¨i mgwó‡K wÎf‡Ri cwimxgv ejv nq| Gi Av‡jv‡K wÎf‡Ri Ab¨vb¨ ˆewkó¨ Ges wÎfR msµvšÍ †gŠwjK Dccv`¨ I A¼b wel‡q Av‡jvPbv Kiv n‡q‡Q|

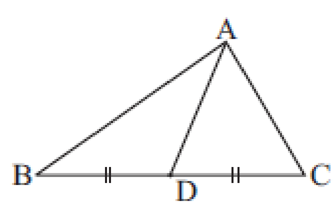
Aa¨vq †k‡l wk¶v_©xiv –

- wÎf‡Ri Aš—¯’ I ewnt¯’ †KvY eYbv Ki‡Z cvi‡e|
- wÎf‡Ri †gŠwjK Dccv`¨¸‡jv cgvY Ki‡Z cvi‡e|
- wewfbœ kZ©mv‡c‡¶ wÎfR AvK‡Z cvi‡e|
- wÎf‡Ri evû I †Kv‡Yi cvi¯úwiK m¤úK© e¨envi K‡i Rxebwfwˇ mgm¨vi mgvavb Ki‡Z cvi‡e|
- wÎfR †¶‡Îi fvg I D”PZv †g‡c †¶Îdj cwigvc Ki‡Z cvi‡e|

9·1 wÎf‡Ri ga¨gv

cv‡ki wP‡Î, ABC GKwU wÎfR| A, B, C wÎfRwUi wZbwU kxlwe›`| $AB, BC, CA$ wÎfRwUi wZbwU evû Ges $\angle A, \angle B, \angle C$ wZbwU †KvY| wÎfRwUi †h‡Kv‡bv GKwU evû $BC$ Gi ga¨we›` D wbYq Kwi Ges D n‡Z wecixZ kxlwe›` A chš— †iLvsk AwuK| AD, ABC wÎf‡Ri GKwU ga¨gv|

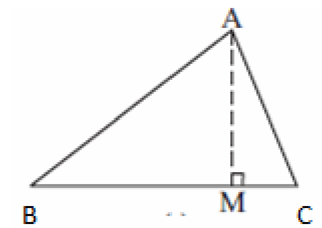




wÎf‡Ri kxlwe›` †_‡K wecixZ evûi ga¨we›` chš— Aw¼Z †iLvsk ga¨gv|

9·2 wÎf‡Ri D”PZv

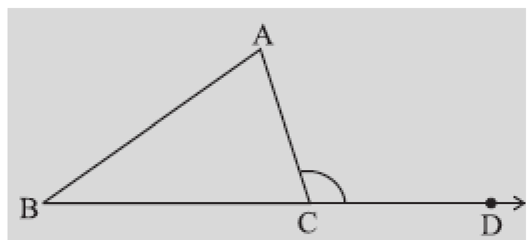
cv‡ki wP‡Î, ABC GKwU wÎfR| A kxl©we›`y n‡Z wecixZ evû $BC$ Gi j¤ `iZB wÎf‡Ri D”PZv| A n‡Z BC Gi Dci j¤ AM A¼b Kwi | AM, ABC wÎf‡Ri D”PZv| c‡Z¨K kxl©we›`y n‡Z wÎf‡Ri D”PZv wbYq Kiv hvq|



9·3 wÎf‡Riewnt¯ I Ašt¯ †KvY

†Kv‡bv wÎf‡Ri GKwU evû ewaZ Ki‡j †h †KvY Drcb nq Zv wÎfRwUi GKwU ewnt¯ †KvY| GB †Kv‡Yi mwbwnZ †KvYwU Qvov wÎf‡Ri Aci `BwU †KvY‡K GB ewnt¯ †Kv‡Yi wecixZ Ašt¯ †KvY ejv nq|

cv‡ki wP‡Î, ΔABC Gi BC evû‡K D chš ewaZ Kiv n‡q‡Q| $\angle ACD$ wÎfRwUi GKwU ewnt¯ †KvY| $\angle ABC, \angle BAC$ I $\angle ACB$ wÎfRwUi wZbwU Ašt¯ †KvY| $\angle ACB$ †K $\angle ACD$ Gi †cwi¶‡Z mwbwnZ Ašt¯ †KvY ejv nq| $\angle ABC$ I $\angle BAC$ Gi c‡Z¨K‡K $\angle ACD$ Gi wecixZ Ašt¯ †KvY ejv nq|



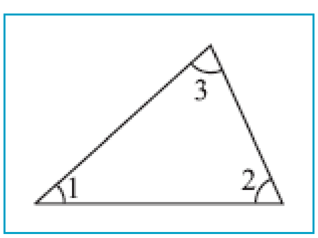
KvR :

1| wÎf‡Ri KqwU ga¨gv ? KqwU D”PZv?
2| ga¨gv I D”PZv wK me`vB wÎf‡Ri Af¨š‡i _vK‡e?
3| GKwU wÎfR AvK, hvi D”PZv I ga¨gv GKB †iLvsk|

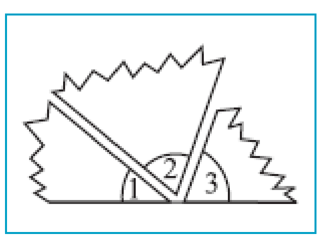
†KvY¸‡jv‡K wb‡q wÎf‡Ri GKwU Amvavi Y ag i‡q‡Q| wb‡Pi wZbwU KvR Kwi Ges djvdj ch‡e¶Y Kwi|

KvR :

1| GKwU wÎfR AvK| Gi †KvY wZbwU †K‡U wPÎ (ii) Gi b¨vq mvRvI| wZbwU †KvY wg‡j GLb GKwU †KvY n‡jv| †KvYwU mij †KvY Ges Gi cwigvc 180° | wÎf‡Ri wZbwU †Kv‡Yi mgwó 180°|

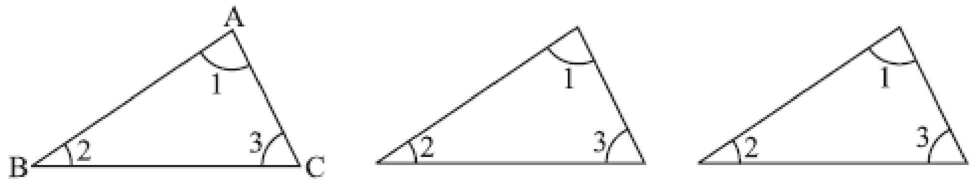


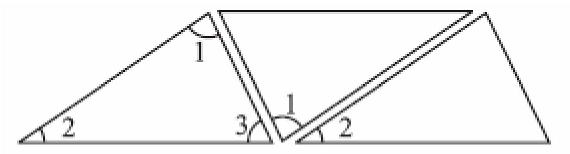
(i)



(ii)

2| GKwU wÎfR AvK Ges Gi Abi|c Avil `BwU wÎfR AvK| wÎfR wZbwU wP‡Îi b¨vq mvRvI| †KvY wZbwU GK‡Î mij †KvY ˆZwi K‡i wK?





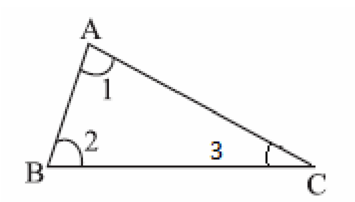
3| †h †Kv‡bv wZbwU wÎfR A¼b Ki| Pv`vi mvnv‡h¨ cwZwU wÎf‡Ri †KvY¸‡jv cwigvc Ki Ges wb‡Pi mviwYwU ciY Ki|

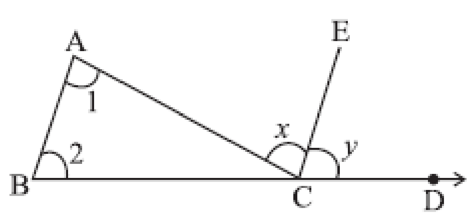
wÎfR	†Kv‡Yi cwigvc	†KvY¸‡jvi †hvMdj
ΔABC	$\angle A =$ $\angle B =$ $\angle C =$	$\angle A + \angle B + \angle C$

cwZwU †¶‡Î †KvY wZbwUi †hvMdj Avbygwb K 180° n‡q‡Q wK?

9·4 wÎf‡Ri wZb †Kv‡Yi †hvMdj

Dccv`¨ 1| wÎf‡Ri wZb †Kv‡Yi mgwó `B mg‡Kv‡Yi mgvb|





we‡kl wbePb : g‡b Kwi, ABC GKwU wÎfR|

cgvY Ki‡Z n‡e †h, $\angle BAC + \angle ABC + \angle ACB =$ `B mg‡KvY

A¼b : BC evû‡K D chš— ewaZ Kwi Ges BA †iLvi mgvš—ivj K‡i CE †iLv AwK|

cgvY :

avc	h_v_Zv
(1) $\angle BAC = \angle ACE$	[BA || CE Ges AC †iLv Zv‡`i †Q`K|] [∵ GKvši †KvY `BwU mgvb|]
(2) $\angle ABC = \angle ECD$	[BA || CE Ges BD †iLv Zv‡`i †Q`K|] [∵ Abic †KvY `BwU mgvb|]
(3) $\angle BAC + \angle ABC = \angle ACE + \angle ECD = \angle ACD$	
(4) $\angle BAC + \angle ABC + \angle ACB = \angle ACD + \angle ACB$	[Dfqc‡¶ $\angle ACB$ †hvM K‡i]
(5) $\angle ACD + \angle ACB =$ `B mg‡KvY	[mij †KvY Dccv`¨]

$\therefore \angle BAC + \angle ABC + \angle ACB =$ `B mg‡KvY\| [cgwYZ]

Abywm×vš— 1\| wÎf‡Ri GKwU evû‡K ewaZ Ki‡j †h ewnt¯' †KvY Drcb nq, Zv Gi wecixZ Aš—¯' †KvYØ‡qi mgwói mgvb\|

Abywm×vš— 2\| wÎf‡Ri GKwU evû‡K ewaZ Ki‡j †h ewnt¯' †KvY Drcb nq, Zv Gi Aš—¯' wecixZ †KvY `BwUi c‡Z¨KwU A‡c¶v enËi\|

Abywm×vš— 3\| mg‡KvYx wÎf‡Ri m²‡KvYØq ci¯úi ciK\|

Abywm×vš— 4\| mgevû wÎf‡Ri c‡Z¨KwU †Kv‡Yi cwigvY 60^0.

Abkxjbx 9·1

1\| wP‡Î, ΔABC Gi $\angle ABC = 90^\circ$, $\angle BAC = 48^\circ$ Ges BD, AC Gi Dci j¤\| Aewkó †KvY¸‡jvi gvb wbYq Ki\|

2\| GKwU mgwØevû wÎf‡Ri kxlwe›`‡Z Aew¯'Z †KvYwUi gvb $50^0$\| Aewkó †KvY `BwUi gvb wbYq Ki\|

3\| cgvY Ki †h, PZf‡Ri PviwU †Kv‡Yi mgwó Pvi mg‡Kv‡Yi mgvb\|

4\| `BwU †iLv $PQ$ Ges $RS$ ci¯úi $O$ we›`‡Z †Q` K‡i\| $PQ$ Ges $RS$ Gi Dci h_vµ‡g $L$ I $M$ Ges $E$ I $F$ PviwU we›`, †hb, $LM \perp RS$, $EF \perp PQ$. cgvY Ki †h, $\angle MLO = \angle FEO$.

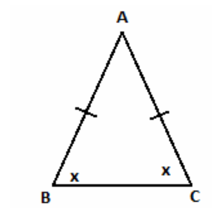
5\| ΔABC-Gi $AC \perp BC$; E, AC Gi ewaZvs‡ki Dci †h‡Kv‡bv we›` Ges $ED \perp AB$. $ED$ Ges $BC$ ci¯úi‡K $O$ we›`‡Z †Q` K‡i\| cgvY Ki †h, $\angle CEO = \angle DBO$.

9·5 wÎf‡Ri evû I †Kv‡Yi m¤úK

wÎf‡Ri evû I †Kv‡Yi g‡a¨ m¤úK i‡q‡Q| weIqwU †evSvi Rb¨ wb‡Pi KvRwU Ki|

KvR :   1| †h‡Kv‡bv GKwU †KvY AvK| †KvYwUi kxI©we›`y †_‡K Dfq evû‡Z mgvb `i‡Z `BwU we›`y wPwýZ Ki| we›`y `BwU hy³ Ki| GKwU mgwØevû wÎfR Aw¼Z n‡jv| Pv`vi mvnv‡h¨ fvg msjM †KvY `BwU cwigvc Ki| †KvY `BwU wK mgvb ?

hw` †Kv‡bv wÎf‡Ri `BwU evû ci¯úi mgvb nq, Z‡e G‡`i wecixZ †KvY `BwUI ci¯úi mgvb| cieZx Aa¨v‡q GB cwZÁwUi hyw³gjK cgvY Kiv n‡e| A_vr, ABC wÎf‡R $AB = AC$ n‡j, $\angle ABC = \angle ACB$ n‡e | mgwØevû wÎf‡Ri G ˆewkó¨ wewfbœ hyw³gjK cgv‡Y c‡qvM Kiv nq|



KvR :   1| †h‡Kv‡bv wZbwU wÎfR AvK| i“jv‡ii mvnv‡h¨ cwZwU wÎf‡Ri wZbwU evûi ˆ`N¨ I Pv`vi mvnv‡h¨ wZbwU †KvY cwigvc Ki Ges wb‡Pi mviwYwU cY Ki|

wÎfR	evûi cwigvc	†Kv‡Yi cwigvc	evûi Zjbv	†Kv‡Yi Zjbv
ΔABC	$AB =$ $BC =$ $CA =$	$\angle A =$ $\angle B =$ $\angle C =$		

cwZwU †¶‡Î †Kv‡bv `BwU evû I G‡`i wecixZ †KvY¸‡jv Zjbv Ki| G †_‡K Kx wm×v‡šÍ DcbxZ nIqv hvq?

Dccv`¨ 2

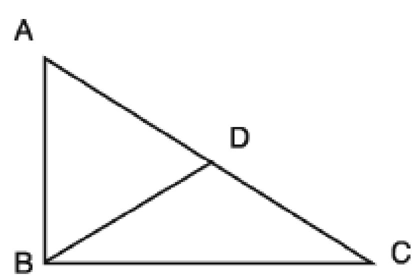
†Kv‡bv wÎf‡Ri GKwU evû Aci GKwU evû A‡c¶v enËi n‡j, enËi evûi wecixZ †KvY ¶`Zi evûi wecixZ †KvY A‡c¶v enËi n‡e|

we‡kl wbePb: g‡b Kwi, ΔABC -G $AC > AB$.

cgvY Ki‡Z n‡e †h, $\angle ABC > \angle ACB$.

A¼b : AC †_‡K AB Gi mgvb K‡i

AD Ask KwU Ges B, D †hvM Kwi|

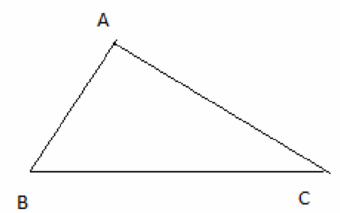


cgvY:

avc	h_v_Zv
(1) ΔABD -G $AB = AD$.	
$\therefore \angle ADB = \angle ABD$.	[mgwØevû wÎf‡Ri fwg msjM †KvYØq mgvb|]
(2) ΔBDC -G ewnt¯ $\angle ADB > \angle BCD$	[ewnt¯ †KvY wecixZ Ašt¯ †KvY `BwUi c‡Z¨KwU A‡c¶v enËi]
$\therefore \angle ABD > \angle BCD$ বা $\angle ABD > \angle ACB$	
(3) $\angle ABC > \angle ABD$	[$\angle ABD$ †KvYwU $\angle ABC$ Gi GKwU Ask]
mZivs, $\angle ABC > \angle ACB$ (cgwYZ)|	

Dccv`¨ 3

†Kv‡bv wÎf‡Ri GKwU †KvY Aci GKwU †KvY A‡c¶v enËi n‡j, enËi †Kv‡Yi wecixZ evû ¶`Zi †Kv‡Yi wecixZ evû A‡c¶v enËi|

we‡kl wbePb: g‡b Kwi, ΔABC Gi $\angle ABC > \angle ACB$ cgvY Ki‡Z n‡e †h, $AC > AB$ cgvY:	
avc	h_v_Zv
(1) hw` AC evû AB evû A‡c¶v enËi bv nq, Z‡e (i) $AC = AB$ A_ev (ii) $AC < AB$ n‡e|	
(i) hw` $AC = AB$ nq, $\angle ABC = \angle ACB$ wKš kZvbhvqx $\angle ABC > \angle ACB$ Zv c`Ë kZwe‡ivax|	[mgwØevû wÎf‡Ri fwg msjM †KvYØq mgvb]
(ii) Avevi, hw` $AC < AB$ nq, Z‡e $\angle ABC < \angle ACB$ n‡e| wKš Zv-I c`Ë kZwe‡ivax|	[¶`Zi evûi wecixZ †KvY ¶`Zi]
(2) mZivs, AC evû AB Gi mgvb ev AB †_‡K ¶`Zi n‡Z cv‡i bv| $\therefore$ $AC > AB$ (cgwYZ)|	

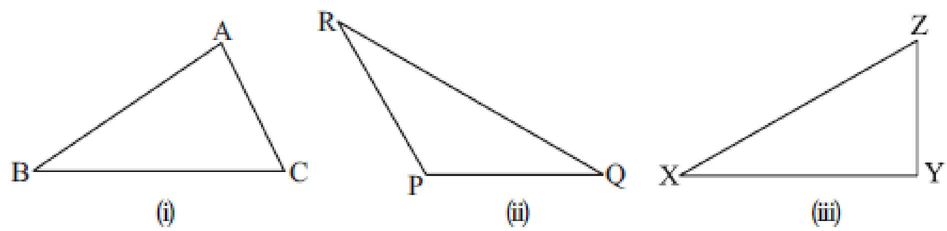
9·6 wÎf‡Ri `B evûi ˆ`‡N¨i †hvMdj

wÎf‡Ri †h‡Kv‡bv `B evûi ˆ`‡N¨i mgwói mv‡_ ZZxq evûi ˆ`‡N¨i m¤úK i‡q‡Q| m¤úKwU Abave‡bi Rb¨ `jMZfv‡e wb‡Pi KvRwU Ki|

KvR

1| 15wU wewfb gv‡ci KwV †RvMvo Ki| G‡`i †h‡Kv‡bv wZbwU w`‡q GKwU wÎfR ˆZwi Kivi †Póv Ki| †Zvgiv wK cwZeviB wÎfR ˆZwi Ki‡Z cvi‡Qv? KLb cvi‡Qv bv Zvi e¨vL¨v `vI|

2| †h‡Kv‡bv wZbwU wÎfR ΔABC, ΔPQR I ΔXYZ AvK|



i“jv‡ii mvnv‡h¨ wÎf‡Ri evû¸‡jvi ˆ`N¨ gvc Ges wb‡Pi mviwYwU ciY Ki:

wÎfR	wZb evûi ˆ`N¨	mZ¨ wKbv	mZ¨/wg_¨v
ΔABC	AB ___ BC ___ CA ___	$AB - BC < CA$ ___ + ___ > ___ $BC - CA < AB$ ___ + ___ > ___ $CA - AB < BC$ ___ + ___ > ___	
ΔPQR	PQ ___ QR ___ RP ___	$PQ - QR < RP$ ___ + ___ > ___ $QR - RP < PQ$ ___ + ___ > ___ $RP - PQ < QR$ ___ + ___ > ___	
ΔXYZ	XY ___ YZ ___ ZX ___	$XY - YZ < ZX$ ___ + ___ > ___ $YZ - ZX < XY$ ___ + ___ > ___ $ZX - XY < YZ$ ___ + ___ > ___	

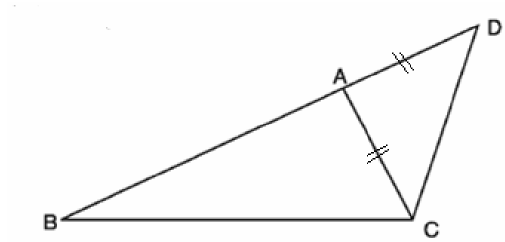
j¶ Kwi, †h‡Kv‡bv wÎf‡Ri †h‡Kv‡bv `B evûi ˆ`‡N¨i †hvMdj Gi ZZxq evûi ˆ`N¨ A‡c¶v †ewk| Avgiv Avi I j¶ Kwi, †h‡Kv‡bv wÎf‡Ri †h‡Kv‡bv `B evûi ˆ`‡N¨i we‡qvMdj Gi ZZxq evûi ˆ`N¨ A‡c¶v Kg|

Dccv`¨ 4

wÎf‡Ri †h‡Kv‡bv `B evûi ˆ`‡N¨i mgwó Gi ZZxq evûi ˆ`N¨ A‡c¶v enËi|

we‡kl wbePb: g‡b Kwi, ABC GKwU wÎfR| cgvY Ki‡Z n‡e †h, ΔABC Gi †h‡Kv‡bv `B evûi ˆ`‡N¨i mgwó Gi ZZxq evûi ˆ`N¨ A‡c¶v enËi|

awi, BC wÎfRwUi enËg evû| Zvn‡j $AB+AC>BC$ cgvY Ki‡Z n‡e|



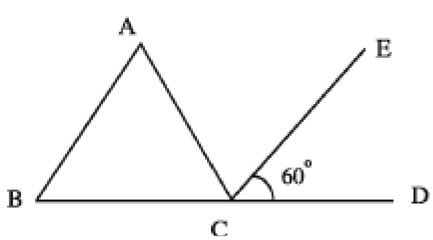
A¼b : BA †K D chš— ewaZ Kwi, †hb $AD=AC$ nq| C, D †hvM Kwi|

cgvY :

avc	h_v_Zv
(1) ΔADC - G $AD=AC$.	[mgwØevû wÎf‡Ri fwg msjMœ †KvYØq mgvb]
$\therefore \angle ACD=\angle ADC. \therefore \angle ACD=\angle BDC.$	
(2) $\angle BCD>\angle ACD.$	[KviY $\angle ACD, \angle BCD$ Gi GKwU Ask]
$\therefore \quad \angle BCD>\angle BDC.$	
(৩) ΔBCD G $\angle BCD>\angle BDC.$	[enËi †Kv‡Yi wecixZ evû enËi]
$\therefore \quad BD>BC.$	
(4) wKš‘ $BD=AB+AD=AB+AC$	[†h‡nZ $AC=AD$]
$\therefore \quad AB+AC>BC.$ (cgwYZ)	

Abkxjbx 9·2

wb‡Pi Z‡_¨i wfwË‡Z 1-3 b¤i c‡ki DËi `vI :



wP‡Î, ABC Gi BC evû‡K D chš— ewaZ Kiv n‡q‡Q| $CE, \angle ACD$ Gi mgwØLÐK| $AB \parallel CE$ Ges $\angle ECD=60^\circ$

1| $\angle BAC$ Gi gvb wb‡Pi †KvbwU?

K. 30° L. 45° M. 60° N. 120°

2| $\angle ACD$ Gi gvb wb‡Pi †KvbwU?

K. 60° L. 90° M. 120° N. 180°

3| ΔABC †Kvb ai‡bi wÎfR?

K. ¯j‡KvYx L. mgwØevû M. mgevû N. mg‡KvYx

4. ΔABC G $\angle A = 70^\circ$, $\angle B = 40^\circ$ n‡j ΔABC Kx ai‡bi wÎfR?

K. ¯j‡KvYx L. mg‡KvYx M. mgevû N. mgwØevû

5| GKwU wÎf‡Ri `BwU evû h_vµ‡g 5 †m.wg. Ges 4 †m.wg. wÎf‡Ri Aci evûwU wb‡Pi †KvbwU n‡Z cv‡i?

K. 1 †m.wg. L. 4 †m.wg. M. 9 †m.wg. N. 10 †m.wg.

6| mgwØevû wÎf‡Ri mgvb evûØq‡K ewaZ Ki‡j Drcb ewnt¯ †KvYØ‡qi GKwU 120° n‡j, AciwU KZ?

K. 120° L. 90° M. 60° N. 30°

7| mg‡KvYx wÎf‡Ri m²‡KvYØ‡qi GKwU 40° n‡j, Aci m²‡Kv‡Yi gvb wb‡Pi †KvbwU ?

K. 40° L. 45° M. 50° N. 60°

8| †Kv‡bv wÎf‡Ri GKwU †KvY Aci `BwU †Kv‡Yi mgwói mgvb n‡j, wÎfRwU Kx ai‡bi n‡e?

K. mgevû L. m²‡KvYx M. mg‡KvYx N. ¯j‡KvYx

9| ΔABC-G $AB > AC$ Ges $\angle B$ I $\angle C$ Gi mgwØLÐKØq ci¯úi P we›`‡Z †Q` K‡i‡Q| cgvY Ki †h, $PB > PC$.

10| ABC GKwU mgwØevû wÎfR Ges Gi $AB = AC; BC$ †K †h‡Kv‡bv `i‡Z D chš evov‡bv n‡jv| cgvY Ki †h, $AD > AB$.

11| $ABCD$ PZf‡R $AB = AD, BC = CD$ Ges $CD > AD$.

cgvY Ki †h, $\angle DAB > \angle BCD$.

12| ΔABC - G $AB = AC$ Ges D, BC -Gi Dci GKwU we›`y| cgvY Ki †h, $AB > AD$.

13| ΔABC - G $AB \perp AC$ Ges D, AC -Gi Dci GKwU we›`y| cgvY Ki †h, $BC > BD$.

14| cgvY Ki †h, mg‡KvYx wÎfy‡Ri AwZfyRB enËg evû|

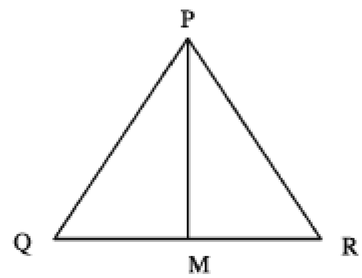
15| cgvY Ki †h, wÎfy‡Ri enËg evûi wecixZ †KvY enËg|

16| wP‡Î, $PM \perp QR$, $\angle QPM = \angle RPM$ Ges $\angle QPR = 90^\circ$

K. $\angle QPM$ Gi gvb wbYq Ki|

L. $\angle PQM$ I $\angle PRM$ Gi gvb KZ?

M. $PQ = 6$ †m.wg. n‡j, PR Gi gvb wbYq Ki|



9·7 wÎfyR A¼b

c‡Z¨K wÎfy‡Ri QqwU Ask Av‡Q; wZbwU evû Ges wZbwU †KvY| wÎfy‡Ri GB QqwU As‡ki K‡qKwU Aci GKwU wÎfy‡Ri Abic As‡ki mgvb n‡j `BwU wÎfyR memg n‡Z cv‡i| mZivs †Kej H Ask¸‡jv †`Iqv _vK‡j wÎfyRwUi AvKvi wbw`ó nq Ges wÎfyRwU AvKv hvq| wb‡Pi DcvË¸‡jv Rvbv _vK‡j GKwU wbw`ó wÎfyR mn‡RB AvKv hvq:

(1) wZbwU evû,

(2) `BwU evû I G‡`i Ašfy³ †KvY,

(3) GKwU evû I G‡`i msjMœ `BwU †KvY,

(4) `BwU †KvY I G‡`i GKwUi wecixZ evû,

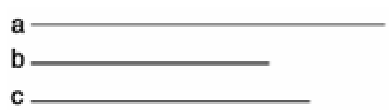
(5) `BwU evû I G‡`i GKwU wecixZ †KvY,

(6) mg‡KvYx wÎfy‡Ri AwZfyR I Aci GKwU evû A_ev †KvY|

m¤úv`¨ 1

†Kv‡bv wÎfy‡Ri wZbwU evû †`Iqv Av‡Q, wÎfyRwU AvK‡Z n‡e|

g‡b Kwi, GKwU wÎfy‡Ri wZbwU evû a, b, c †`Iqv Av‡Q| wÎfyRwU AvK‡Z n‡e|



A¼b :

(1) †h‡Kv‡bv iwk¥ BD †_‡K a Gi mgvb K‡i BC †K‡U wbB|

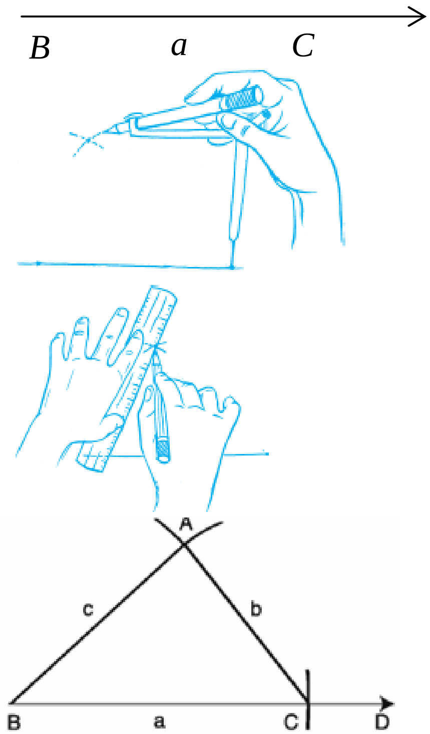
(2) B I C we›`y‡K †K›`ª K‡i h_vµ‡g b Ges c Gi mgvb e¨vmva© wb‡q BC Gi GKB cv‡k `yBwU eËPvc AuwK| eËPvc `yBwU ci¯úi A we›`y‡Z †Q` K‡i|

(3) A, B Ges A, C †hvM Kwi|

Zvn‡j ΔABC-B DwÏó wÎfR|

cÖgvY : A¼bvbmv‡i, ΔABC-G $BC = a,\ AC = b$ Ges $AB = c.$

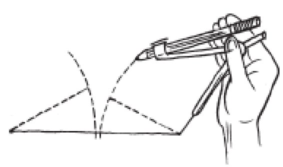
$\therefore\ \Delta ABC$ c`Ë evûÎq wÎfR|



KvR :

1| 8 †m.wg., 5 †m.wg. I 6 †m.wg ˆ`‡N¨i wZbwU evûwewkó GKwU wÎfR AuvK|

2| 8 †m.wg., 5 †m.wg. I 3 †m.wg ˆ`‡N¨i wZbwU evûwewkó GKwU wÎfR A¼‡bi †Póv Ki|

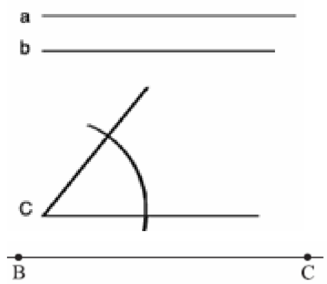


†Zvgvi †Póv mdj n‡q‡Q wK?

gše¨ : wÎf‡Ri `yB evûi mgwó Gi ZZxq evû A‡cÿv enËi| ZvB c`Ë evû¸‡jv Ggb n‡Z n‡e †h, †h‡Kv‡bv `yBwUi ˆ`‡N¨i mgwó ZZxqwUi ˆ`N¨ A‡cÿv enËi nq| Zvn‡jB wÎfRwU AuvKv m¤¢e n‡e|

m¤úv`¨ 2

†Kv‡bv wÎf‡Ri `ywU evû I G‡`i Aš—f©³ †KvY †`Iqv Av‡Q, wÎfRwU AvK‡Z n‡e|

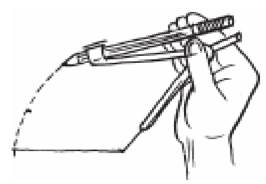


g‡b Kwi, GKwU wÎf‡Ri `ywU evû $a$ I $b$ Ges Zv‡`i Aš—f©³ †KvY $\angle C$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

A¼b :

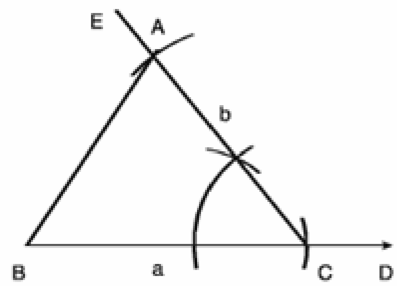
(1) †h‡Kv‡bv iwk¥ BD †_‡K a Gi mgvb K‡i BC wbB|

(2) BC †iLvski C we›`y‡Z c`Ë $\angle C$ Gi mgvb $\angle BCE$ AvwK|



(3) CE †iLvsk †_‡K b Gi mgvb K‡i CA wbB|

(8) A, B †hvM Kwi|



Zvn‡j ΔABC-B DwÏó wÎfR|

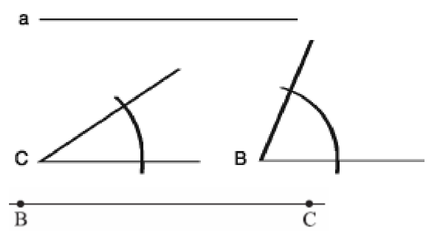
cÖgvY : A¼b Abmv‡i,

ΔABC - G $BC = a, CA = b$ Ges $\angle ACB = \angle C.$

$\therefore$ ΔABC-B wbw`ó wÎfR|

m¤úv`¨ 3

†Kv‡bv wÎf‡Ri GKwU evû I Gi msjMœ `ywU †KvY †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

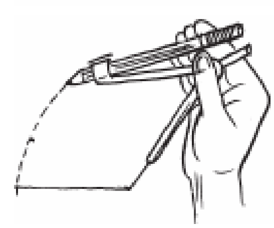


g‡b Kwi, GKwU wÎf‡Ri GKwU evû a Ges Gi msjMœ `ywU †KvY $\angle B$ I $\angle C$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|

A¼b :

(1) †h‡Kv‡bv iwk¥ BD †_‡K a Gi mgvb K‡i BC wbB|

(2) BC †iLvs‡ki B I C we›`y‡Z h_vµ‡g $\angle CBE = \angle B$ Ges $\angle BCF = \angle C$ AvwK| $BE$ I $CF$ ci¯úi $A$ we›`y‡Z †Q` K‡i|



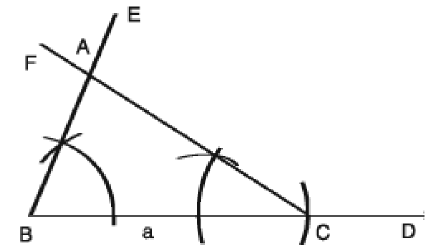
(3) A, B I A, C †hvM Kwi|

Zvn‡j ΔABC -B DwÏó wÎfR|

cgvY : A¼b Abmv‡i,

ΔABC - G $BC = a, \angle ABC = \angle B$ Ges $\angle ACB = \angle C.$

$\therefore$ ΔABC -B wbw`ó wÎfR|



gšÍe¨ : wÎf‡Ri wZb †Kv‡Yi mgwó `B mg‡Kv‡Yi mgvb, ZvB c`Ë †KvY `BwU Ggb n‡Z n‡e †hb G‡`i mgwó `B mg‡KvY A‡c¶v †QvU nq| GB kZ cvjb Kiv bv n‡j †Kv‡bv wÎfR AvKv m¤¢e n‡e bv|

KvR :

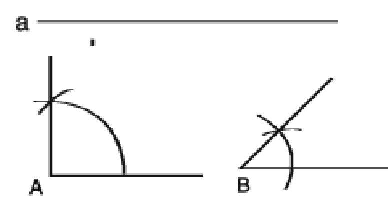
1| 7 †m.wg. ^`‡N¨i evû I $50°$ I $60°$ †KvYwewkó GKwU wÎfR AvK|

2| 6 †m.wg. ^`‡N¨i evû I $140°$ I $70°$ †KvYwewkó GKwU wÎfR A¼‡bi †Póv Ki| †Zvgvi †Póv mdj n‡q‡Q wK? †Kb e¨vL¨v Ki|

m¤úv`¨ 4

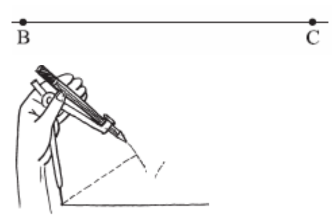
†Kv‡bv wÎf‡Ri `BwU †KvY Ges G‡`i GKwUi wecixZ evû †`Iqv Av‡Q, wÎfRwU AvK‡Z n‡e|

g‡b Kwi, GKwU wÎf‡Ri `BwU †KvY $\angle A$ I $\angle B$ Ges $\angle A$ Gi wecixZ evû $a$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|



A¼b :

(1) †h‡Kv‡bv iwk¥ BD †_‡K a Gi mgvb K‡i BC wbB|



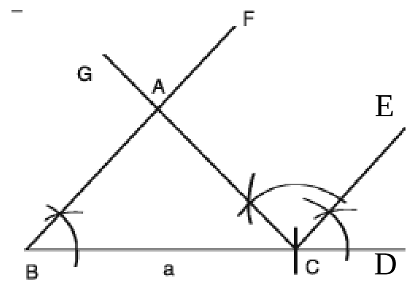
(2) BC †iLvs‡ki B I C we›`‡Z $\angle B$ Gi mgvb K‡i $\angle CBF$ I $\angle DCE$ AwwK|

(3) Avevi CE †iLvi C we›`‡Z Gi †h cv‡k $\angle B$ Aew¯Z Zvi wecixZ cv‡k $\angle A$ Gi mgvb K‡i $\angle ECG$ AwwK|

CG I BF †iLv A we›`‡Z †Q` K‡i|

$\therefore$ wÎfR ABC B DwÏó wÎfR|

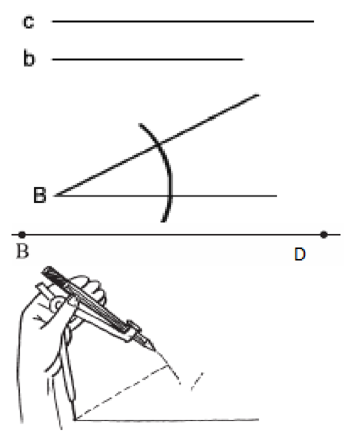
cgvY : A¼bvbmv‡i, $\angle ABC = \angle ECD$. GB †KvY `BwU Abic e‡j $BF \parallel CE$ ev $BA \parallel CE$ |
GLb $BA \parallel CE$ Ges AC G‡`i †Q`K|
$\therefore \angle BAC =$ GKvši $\angle ACE = \angle A$.
GLb ΔABC G $\angle BAC = \angle A, \angle ABC = \angle B$ Ges $BC = a$. mZivs, ABC wÎfRwU kZg‡Z Aw¼Z n‡jv|



m¤úv`¨ 5

†Kv‡bv wÎf‡Ri `BwU evû Ges G‡`i GKwUi wecixZ †KvY †`Iqv Av‡Q, wÎfRwU AvK‡Z n‡e|

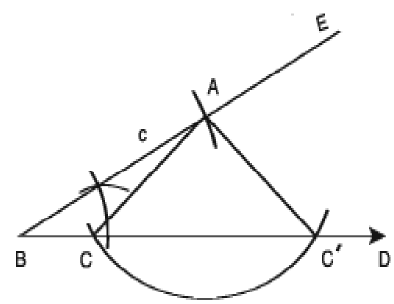
g‡b Kwi, GKwU wÎf‡Ri `BwU evû $b$ I $c$ Ges $b$ evûi wecixZ †KvY $\angle B$ †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|



A¼b :

(1) †h‡Kv‡bv iwk¥ BD AwK|
(2) B we›`‡Z c`Ë $\angle B$ Gi mgvb K‡i $\angle DBE$ AwK|

(3) BE †iLv †_‡K c Gi mgvb K‡i BA wbB|
(4) GLb A we›`‡K †K›`ª K‡i b Gi ^`‡N¨i mgvb e¨vmva wb‡q GKwU eËPvc AwK| eËPvcwU $BD$ †iLv‡K $C$ I $C'$ we›`‡Z †Q` K‡i|
(5) A, C Ges A, C' †hvM Kwi|
Zvn‡j ΔABC Ges $\Delta ABC'$-Dfq wÎfR c`Ë kZ ciY K‡i Aw¼Z|

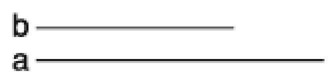


cgvY : A¼bvbmv‡i, ΔABC - G $BA = c, AC = b$ Ges $\angle ABC = \angle B$ |
Avevi, $\Delta ABC'$ - G $BA = c, AC' = b$ Ges $\angle ABC' = \angle B$ |
†`Lv hvq, $\Delta ABC$ Ges $\Delta ABC'$ DfqB c`Ë kZmgn ciY K‡i|
Zvn‡j ΔABC ev $\Delta ABC'$-B DwÏó wÎfR|

m¤úv`¨ 6

†Kv‡bv mg‡KvYx wÎf‡Ri AwZfR I Aci GKwU evû †`Iqv Av‡Q, wÎfRwU AvK‡Z n‡e|

g‡b Kwi, GKwU wÎf‡Ri AwZfR a I Aci GK evû b †`Iqv Av‡Q| wÎfRwU AvK‡Z n‡e|



A¼b :

(1) †h‡Kv‡bv iwk¥ BD †_‡K b Gi mgvb K‡i BC wbB|

(2) B we›`‡Z BE j¤ AwK|

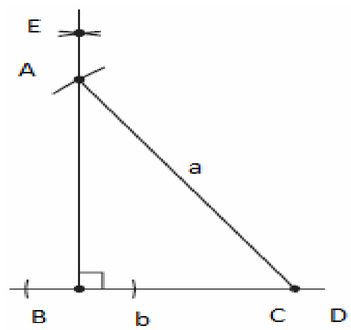
(3) C †K †K›`` K‡i a Gi mgvb e¨vmva wb‡q GKwU eËPvc AwK, †hb GwU BE-†K A we›`‡Z †Q` K‡i|

(4) A I C †hvM Kwi|

Zvn‡j ΔABC-B DwÏó wÎfR|

cgvY : A¼bvbmv‡i, $AC = a, BC = b$ Ges $\angle ABC =$ GK mg‡KvY|

$\therefore$ ΔABC-B wb‡Yq wÎfR|



Abkxjbx 9·3

1| †Kv‡bv wÎf‡Ri `BwU evû Ges G‡`i GKwU wecixZ †KvY †`Iqv _vK‡j, mewaK KqwU wÎfR AvKv hv‡e?

K. 1 L. 2 M. 3 N. 4

2| †Kvb †¶‡Î wÎfR AvKv m¤¢e hLb wZbwU evûi ˆ`N¨ h_vµ‡g-

K. 1 †m.wg., 2 †m.wg. 3 †m.wg. L. 3 †m.wg., 4 †m.wg. 5 †m.wg.
M. 2 †m.wg., 4 †m.wg. 6 †m.wg. N. 3 †m.wg., 4 †m.wg. 7 †m.wg.

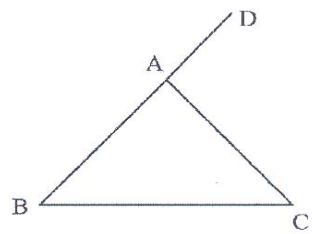
3| *i*. GKwU wÎf‡Ri `BwU evû Ges Zv‡`i Ašf ³ †KvY †`Iqv _vK‡j, wÎfRwU AvKv hvq|
ii. `BwU evûi mgwó ZZxq evû A‡c¶v enËi n‡j, wÎfRwU AvKv hvq|
iii. †Kv‡bv wÎf‡Ri GKwaK ¯j‡KvY _vK‡Z cv‡i|

Dc‡ii Z_¨ Abmv‡i wb‡Pi †KvbwU mwVK

K. *i*I　　L. *ii* I *iii*

M. *i* I *iii*　　N. *i*, *ii* I *iii*

wb‡Pi wPÎ Abmv‡i 4-5 b¤i c‡ki DËi `vI :



4| C we›`‡Z BA †iLvi mgvšivj †iLv AvK‡Z n‡j, †Kvb †Kv‡Yi mgvb †KvY AvK‡Z n‡e?

K. $\angle ABC$　　L. $\angle ACB$　　M. $\angle BAC$　　N. $\angle CAD$

5| $\angle CAD$ Gi mgvb wb‡Pi †KvbwU?

K. $\angle BAC + \angle ACB$　　L. $\angle ABC + \angle ACB$

M. $\angle ABC + \angle ACB + \angle BAC$　　N. $\angle ABC + \angle BAC$

6| GKwU wÎf‡Ri wZbwU evûi ˆ`N© †`Iqv Av‡Q| wÎfRwU AvK|

(K) 3 †m.wg., 4 †m.wg., 6 †m.wg　　(L) 3.5 †m.wg., 4.7 †m.wg., 5.6 †m.wg

7| GKwU wÎf‡Ri `BwU evû I G‡`i Ašf©³ †KvY †`Iqv Av‡Q| wÎfRwU AvK|

(K) 3 †m.wg., 4 †m.wg., 60°　　(L) 3.8 †m.wg., 4.7 †m.wg., 45°

8| GKwU wÎf‡Ri GKwU evû I Gi msjMœ `BwU †KvY †`Iqv Av‡Q| wÎfRwU AvK|

(K) 5 †m.wg., 30°, 45°　　(L) 4.5 †m.wg., 45°, 60°

9| GKwU wÎf‡Ri `BwU †KvY I c_g †Kv‡Yi wecixZ evû †`Iqv Av‡Q| wÎfRwU AvK|

(K) 120°, 30°, 5 †m.wg.　　(L) 60°, 30°, 4 †m.wg.

10| GKwU wÎf‡Ri `BwU evû I c_g evûi wecixZ †KvY †`Iqv Av‡Q | wÎfRwU AvK|

(K) 5.3 †m.wg., 6 †m.wg., 60°　　(L) 4 †m.wg., 5 †m.wg., 30°

11| GKwU mg‡KvYx wÎf‡Ri AwZfR I Gi msjMœ evûi ˆ`N© †`Iqv Av‡Q| wÎfRwU AvK|

(K) 7.2 †m.wg., 4.5 †m.wg.　　(L) 4.7 †m.wg., 3 †m.wg.

12| GKwU mg‡KvYx wÎf‡Ri GKwU wbw`©ó evû 5.3 †m.wg. Ges GKwU m²‡KvY 45° †`Iqv Av‡Q| wÎfRwU AvK|

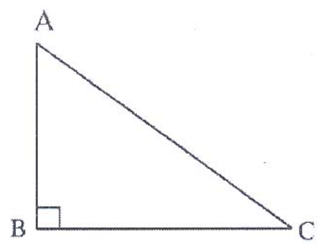
13| GKB mij‡iLvq Aew¯’Z bq Ggb wZbwU we›` A, B I C.

K. we›` wZbwU w`‡q GKwU wÎfR AvK|

L. Aw¼Z wÎf‡Ri kxIwe›` †_‡K fwgi Ici j¤ AvK|

M. Aw¼Z wÎf‡Ri fwg, mg‡KvYx mgwØevû wÎf‡Ri AwZfR n‡j, wÎfRwU AvK|

14|



K. wP‡Îi wÎfRwUi AwZfR †KvbwU?

L. AwZf‡Ri cwigvY †mw›UwgUv‡i wbYq Ki Ges $\angle ACB$ Gi mgvb K‡i GKwU †KvY AvK|

L. GKwU mg‡KvYx wÎfR AvK, hvi AwZfR wP‡Î Aw¼Z wÎf‡Ri AwZfR A‡c¶v 2 †m.wg. eo Ges GKwU †KvY, $\angle ACB$ Gi mgvb nq|

15| GKwU wÎf‡Ri `BwU evû $a = 3 \cdot 2$ †m.wg., $b = 4 \cdot 5$ †m.wg. Ges GKwU †KvY $\angle B = 30^\circ$

K. $\angle B$ Gi mgvb GKwU †KvY AvK|

L. GKwU wÎfR AvK, hvi `B evû a I b Gi mgvb Ges Ašf³ $\angle B$ Gi mgvb nq|

M. Ggb GKwU wÎfR AvK, hvi GKwU evû b Ges $\angle B$ Gi wecixZ evû .. nq|

16| wÎf‡Ri GKwU evûi ˆ`N¨ 4 †m.wg. Ges evû msjM †KvY `BwU 37° I 46°.

K. wÎf‡Ri Aci †Kv‡Yi cwigvY KZ?

L. wÎfRwU Kx ai‡bi Ges †Kb?

M. wÎfRwU AvK|

`kg Aa¨vq

memgZv I m`kZv

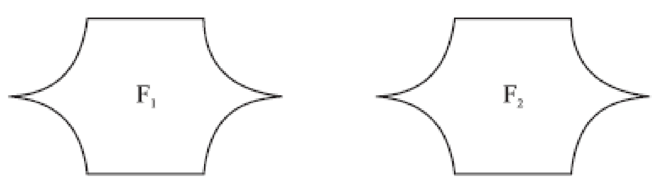
Avgiv `ˆbw›`b Rxe‡b A‡bK wRwbm ‡`L‡Z cvB| ‡hgb G‡`i wKQy Dch~³ Mvjvb, Avevi wKQy ‡`L‡Z GKB iKg, wKš' mgvb bq| ‡Zgwb‡`i ‡kwYi wk¶v_x©‡`i MwYZ cvV¨c~¯—Kwi AvKwZ, AvKvi I IR‡b GKB, ‡m¸‡jv mew`K w`‡q mgvb ev memg| Avevi GKwU Mv‡Qi cvZv¸‡jvi AvKwZ GKB n‡jI AvKv‡ii wfbœ, cvZv¸‡jv ‡`L‡Z GK iKg ev m`k| d‡UvMwdi ‡`vKv‡b hLb Avgiv gjKwci AwZwi³ Kwc PvB Zv gjKwci ûeû mgvb, eo ev ‡QvU K‡i PvB‡Z cwi| KwcwU hw` gjKwci mgvb nq ‡m‡¶‡Î Kwc `BwU memg| Avi ‡¯^j wVK ‡i‡L KwcwU hw` gjKwci ‡P‡q eo ev ‡QvU nq ‡m‡¶‡Î Kwc `BwU m`k| GB Aa¨v‡q Avgiv AZ¨š— ¸i"ZcY© GB `B R¨vwgwZK aviYv wb‡q Av‡jvPbv Kie| Avgiv AvcvZZ mgZjxq ‡¶‡Îi memgZv I m`kZv we‡ePbv Kie|

Aa¨vq ‡k‡l wk¶v_x©iv Ñ

- ➢ wewfbœ R¨vwgwZK AvKvi I AvKwZ n‡Z memg Ges m`k AvKvi I AvKwZ wPwýZ Ki‡Z cvi‡e|
- ➢ memgZv I m`kZvi g‡a¨ cv_©K¨ Ki‡Z cvi‡e|
- ➢ wÎf‡Ri memgZv cgvY Ki‡Z cvi‡e|
- ➢ wÎfR I PZf‡Ri m`kZv e¨vL¨v Ki‡Z cvi‡e|
- ➢ memgZv I m`kZvi ˆewkó¨i wfwË‡Z mnR mgm¨vi mgvavb Ki‡Z cvi‡e|

10·1 memgZv

wb‡Pi mgZjxq wPÎ `BwU ‡`L‡Z GKB AvKwZ I AvKv‡ii| wPÎ `BwU memg wKbv wbwðZ nIqvi Rb¨ Dcwicvrb c×wZ MnY Kiv hvq| G c×wZ‡Z c_g wP‡Îi GKwU Abywjwc Kwc K‡i wØZxqwUi Dci iwL| hw` wPÎ¸‡jv ci¯úi‡K m¤úYi‡c AveZ K‡i, Z‡e Giv memg| wPÎ F_1, wPÎ F_2 Gi memg n‡j Avgiv $F_1 \cong F_2$ Øviv cKvk Kwi|



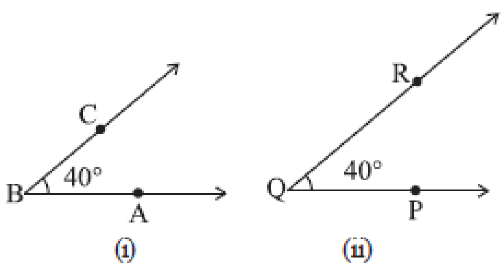
(i) (ii)

`BwU ‡iLvsk KLb memg n‡e? wP‡Î `B ‡Rvov ‡iLvsk AvKv n‡q‡Q| Dcwicvrb c×wZ‡Z AB Gi Abywjwc Kwc CD Gi Dci ‡i‡L ‡`wL ‡h, $AB$ ‡iLvsk $CD$ ‡iLvsk‡K ‡X‡K w`‡q‡Q Ges A I B we›`` h_vµ‡g

C I D we›`i Dci cwZZ n‡q‡Q| mZivs †iLvsk `BwU memg| GKB KvR wØZxq †Rvov mij‡iLvi Rb¨ K‡i †`wL †h, †iLvsk `BwU memg bq| j¶ Kwi, †Kej c_g †Rvov †iLvs‡ki ^`N¨ mgvb|

`BwU †iLvs‡ki ^`N¨ mgvb n‡j †iLvsk `BwU memg| Avevi wecixZfv‡e, `BwU †iLvsk memg n‡j G‡`i ^`N¨ mgvb|

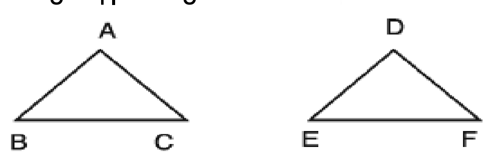
`BwU †KvY KLb memg n‡e? wP‡Î 40° `BwU †KvY AvKv n‡q‡Q| DcwicvZb c×wZ MnY K‡i c_g wP‡Îi GKwU Abiƒc Kwc K‡i wØZxqwUi Dci iwL| B we›` $Q$ we›`i Dci Ges BA iwk¥ QP iwk¥i Ici cwZZ n‡q‡Q| j¶ Kwi, †KvY `BwUi cwigvc mgvb e‡j BC iwk¥ QR iwk¥i Dci cwZZ n‡q‡Q| A_vr $\angle ABC \cong \angle PQR$



`BwU †Kv‡Yi cwigvc mgvb n‡j †KvY `BwU memg| Avevi wecixZfv‡e, `BwU †KvY memg n‡j G‡`i cwigvcI mgvb|

10·2 wÎf‡Ri memgZv

GKwU wÎfR‡K Aci GKwU wÎf‡Ri Dci ¯vcb Ki‡j hw` wÎfR `BwU me‡Zvfv‡e wg‡j hvq, Z‡e wÎfR `BwU memg nq| memg wÎf‡Ri Abic evû I Abic †KvY¸‡jv mgvb| wb‡Pi ΔABC I ΔDEF memg|



ΔABC I ΔDEF memg n‡j Ges A, B, C kxl h_vµ‡g D, E, F kx‡li Dci cwZZ n‡j

$AB = DE, AC = DF, BC = EF.$

$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ n‡e|

ΔABC I ΔDEF memg †evSv‡Z $\Delta ABC \cong \Delta DEF$ †jLv nq|

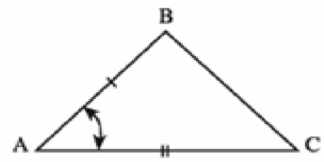
wÎf‡Ri memgZv cgv‡Yi Rb¨ Kx Z_¨ c‡qvRb ? G Rb¨ `jMZfv‡e wb‡Pi KvRwU Ki:

KvR :
1| ΔABC GKwU wÎfR AvK †hb $AB = 5$ †m.wg., $BC = 6$ †m.wg.Ges $\angle B = 60^\circ$ nq|
(K) wÎf‡Ri ZZxq evûi ^`N¨ Ges Ab¨ †KvY `BwU cwigvc Ki|
(L) †Zvgv‡`i cwigvc¸‡jv Zjbv Ki| Kx †`L‡Z cv"Q?

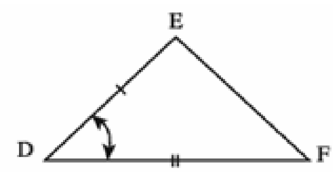
Dccv`¨ 1 (evû-†KvY-evû Dccv`¨)

hw` `BwU wÎf‡Ri GKwUi `B evû h_vµ‡g AciwUi `B evûi mgvb nq Ges evû `BwUi Ašf ³ †KvY `BwU ci¯úi mgvb nq, Z‡e wÎfR `BwU memg nq|

we‡kl wbePb: g‡b Kwi,



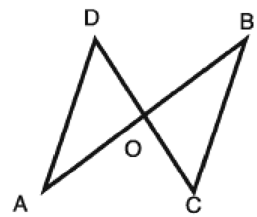
ΔABC I ΔDEF G $AB = DE, AC = DF$
Ges Ašf ³ $\angle BAC$ = Ašf ³ $\angle EDF$
cgvY Ki‡Z n‡e †h, $\Delta ABC \cong \Delta DEF$



cgvY :

avc	h_v_Zv
(1) ΔABC †K ΔDEF Gi Dci Ggbfv‡e ¯vcb Kwi †hb A we›` $D$ we›`i Dci I AB evû DE evû eivei Ges DE evûi †h cv‡k F Av‡Q C we›` Hcv‡k c‡o| GLb $AB = DE$ e‡j $B$ we›` Aek¨B E we›`i Dci co‡e|	[evûi memgZv]
(2) †h‡nZ $\angle BAC = \angle EDF$ Ges AB evû DE evûi Dci c‡o, mZivs AC evû DF evû eivei co‡e|	[†Kv‡Yi memgZv]
(3) $AC = DF$ e‡j C we›` Aek¨B $F$ we›`i Dci co‡e|	[evûi memgZv]
(4) GLb B we›` $E$ we›`i Dci Ges C we›` $F$ we›`i Dci c‡o e‡j BC evû Aek¨B EF evûi mv‡_ c‡ivcwi wg‡j hv‡e|	[`BwU we›`i ga¨ w`‡q GKwU gvÎ mij‡iLv A¼b Kiv hvq]

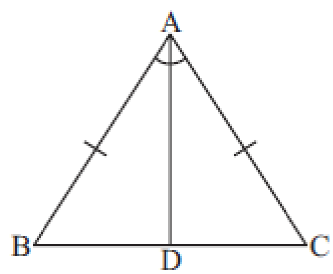
AZGe, $\Delta ABC, \Delta DEF$ Gi Dci mgvcwZZ n‡e|
$\Delta ABC \cong \Delta DEF$ (cgwYZ)



D`vniY 1| wP‡Î, $AO = OB, CO = OD$.
cgvY Ki †h, $\Delta AOD \cong \Delta BOC$.
cgvY : ΔAOD Ges ΔBOC G
$AO = OB, CO = OD$ †`Iqv Av‡Q
Ges Zv‡`i Ašf©³ $\angle AOD =$ Ašf©³ $\angle BOC$
[wecZxc †KvY ci¯úi mgvb]|
$\therefore \Delta AOD \cong \Delta BOC$ [evû-†KvY-evû Dccv`¨] (cgwYZ)

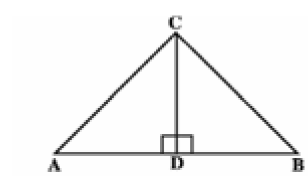
Dccv`¨ 2

hw` †Kv‡bv wÎf‡Ri `BwU evû ci¯úi mgvb nq, Z‡e G‡`i wecixZ †KvY `BwUI ci¯úi mgvb n‡e|



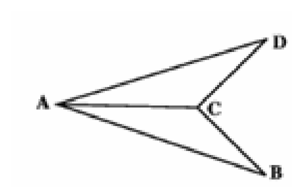
we‡kl wbePb : g‡b Kwi, ABC wÎf‡R $AB = AC$|
cgvY Ki‡Z n‡e †h, $\angle ABC = \angle ACB$|
A¼b : $\angle BAC$ Gi mgwØLÐK AD AwK †hb Zv BC †K D we›`‡Z †Q` K‡i|
cgvY : ΔABD Ges ΔACD G
(1) $AB = AC$ (c`Ë)
(2) AD mvaviY evû Ges
(3) Ašf©³ $\angle BAD =$ Ašf©³ $\angle CAD$ (A¼bvbmv‡i)
mZivs, $\Delta ABD \cong \Delta ACD$ [evû-†KvY-evû Dccv`¨]
$\therefore \angle ABD = \angle ACD$ A_vr, $\angle ABC = \angle ACB$ (cgwYZ)

Abkxjbx 10·1

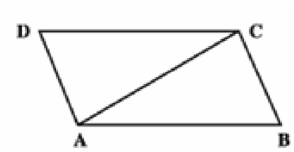


1| wP‡Î, CD, AB Gi j¤ mgwØLÐK,

cgvY Ki †h $\Delta ADC \cong \Delta BDC$.



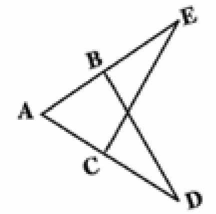
2| wP‡Î, $CD = CB$ Ges $\angle DCA = \angle BCA$
cgvY Ki †h, $AB = AD$



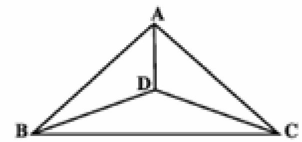
3| wP‡Î, $\angle BAC = \angle ACD$ Ges $AB = DC$
cgvY Ki †h, $AD = BC$, $\angle CAD = \angle ACB$
Ges $\angle ADC = \angle ABC$.

4| cgvY Ki †h, mgwØevû wÎf‡Ri fwg‡K Dfqw`‡K ewaZ Ki‡j Drcb ewnt¯' †KvY `BwU ci¯úi mgvb|

5| wP‡Î, $AD = AE, BD = CE$
Ges $\angle AEC = \angle ADB$
cgvY Ki †h, $AB = AC$



6| wP‡Î, ΔABC Ges ΔDBC `BwU mgwØevû wÎfR|
cgvY Ki †h, $\Delta ABD = \Delta ACD$

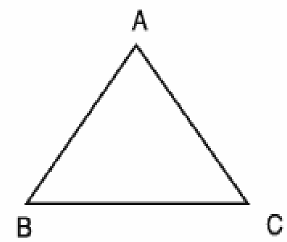


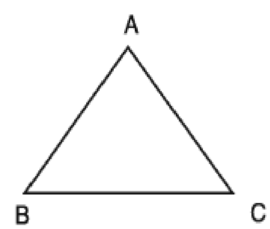
7| cgvY Ki †h, mgwØevû wÎf‡Ri fwgi cvšÍwe›` †_‡K wecixZ evûØ‡qi Dci Aw¼Z ga¨gØq mgvb|
8| cgvY Ki †h, mgevû wÎf‡Ri †KvY¸‡jv ci¯úi mgvb|

Dccv`¨ 3 (evû-evû-evû Dccv`¨)

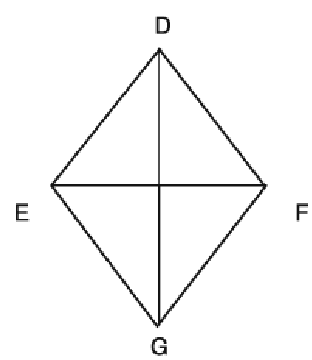
hw` GKwU wÎf‡Ri wZb evû Aci GKwU wÎf‡Ri wZb evûi mgvb nq, Z‡e wÎfR `BwU memg n‡e|

we‡kl wbePb : g‡b Kwi, ΔABC Ges ΔDEF G
$AB = DE, AC = DF$ Ges $BC = EF$,
cgvY Ki‡Z n‡e †h, $\Delta ABC \cong \Delta DEF$.





cgvY : g‡b Kwi, BC Ges EF evû h_vµ‡g ΔABC Ges ΔDEF Gi enËg evûØq|
GLb ΔABC †K ΔDEF Gi Dci Ggbfv‡e ¯vcb Kwi, †hb B we›` $E$ we›`i Dci Ges BC evû Gi mgvb EF evû eivei Ges EF †iLvi †h cv‡k D we›` Av‡Q, $A$ we›`‡K Gi wecixZ cv‡k ¯vcb Kwi| g‡b Kwi, G we›` $A$ we›`i bZb Ae¯vb|
†h‡nZ $BC = EF$, C we›` $F$ we›`i Dci co‡e| mZivs ΔGEF n‡e ΔABC Gi bZb Ae¯vb|
A_vr, $EG = BA, FG = CA$ I $\angle EGF = \angle BAC$.
D, G †hvM Kwi|



avc h_v_Zv

(1) ΔEGD G $EG = ED$ [KviY $EG = BA = ED$] [Dccv`¨-2]

AZGe, $\angle EDG = \angle EGD$

(2) ΔFGD G $FG = FD$ [Dccv`¨-2]

AZGe, $\angle FDG = \angle FGD$.

(3) mZivs, $\angle EDG + \angle FDG = \angle EGD + \angle FGD$ [evû-†KvY-evû Dccv`¨]

ev, $\angle EDF = \angle EGF$

A_vr, $\angle BAC = \angle EDF$

AZGe, ΔABC I ΔDEF -G $AB = DE, AC = DF$ Ges

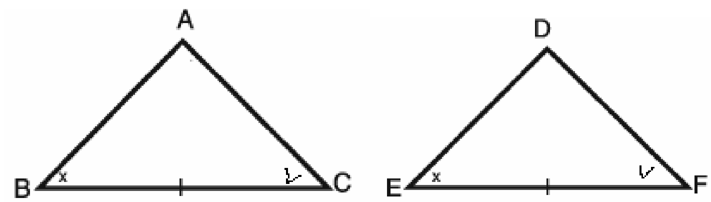
Ašf³ $\angle BAC =$ Ašf³ $\angle EDF$

$\therefore$ $\Delta ABC \cong \Delta DEF$ (cgwYZ)|

Dccv`¨ 4 (†KvY-evû-†KvY Dccv`¨)

hw` GKwU wÎf‡Ri `BwU †KvY I †KvY msjM evû h_vµ‡g Aci GKwU wÎf‡Ri `BwU †KvY I †KvY msjM evûi mgvb nq, Z‡e wÎfR `BwU memg n‡e|

we‡kl wbePb: g‡b Kwi, ΔABC I ΔDEF -G $\angle B = \angle E$, $\angle C = \angle F$ Ges †KvY msjM BC evû = Abic EF evû|



cgvY Ki‡Z n‡e †h, $\Delta ABC \cong \Delta DEF$.

cgvY :

avc h_v_Zv

(1) ΔABC †K ΔDEF Gi Dci Ggbfv‡e ¯'vcb Kwi †hb, B we›` $E$ we›`i Dci BC evû EF evû eivei Ges EF †iLvi †h cv‡k D Av‡Q A we›` †hb Hcv‡k c‡o| †h‡nZ $BC = EF$, AZGe $C$ we›` F we›`i Dci Aek¨B co‡e| [evûi memgZv]

(2) Avevi, $\angle B = \angle E$ e‡j, BA evû DE evû eivei co‡e Ges $\angle C = \angle F$ e‡j, CA evû FD evû eivei co‡e| [†Kv‡Yi memgZv]

(3) $\therefore BA$ Ges CA evûi mvaviY we›` $A$, $BD$ I $FD$ evûi mvaviY we›` D Gi Dci co‡e|

A_vr, $\Delta ABC, \Delta DEF$ Gi Dci mgvcwZZ n‡e|

$\therefore \Delta ABC \cong \Delta DEF$ (cgwYZ)

D`vniY 1| cgvY Ki †h, †Kv‡bv wÎf‡Ri wkit†Kv‡Yi mgwØLÐK hw` fwgi Dci j¤ nq, Z‡e wÎfRwU mgwØevû|

we‡kl wbePb : wP‡Î, ΔABC Gi wkit‡KvY A-Gi mgwØLÐK AD fwg BC Gi D we›`‡Z j¤|

cgvY Ki‡Z n‡e †h, $AB = AC.$

cgvY : ΔABD Ges ΔACD G

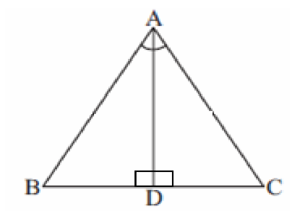
$\angle BAD = \angle CAD$ [$\because AD$, $\angle BAC$ Gi mgwØLÐK]

$\angle ADB = \angle ADC$ [$\because AD$, BC Gi Dci j¤]

Ges AD mvaviY evû|

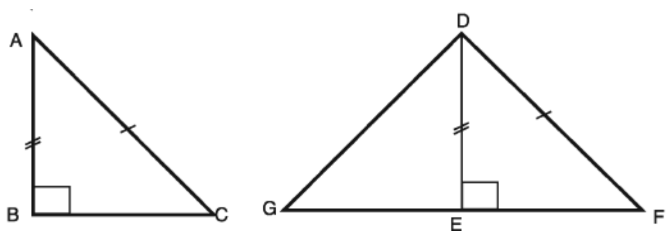
mZivs $\Delta ABD = \Delta ACD$ [Dccv`¨ 4]

GZGe, $AB = AC$ [cgwYZ]



Dccv`¨ 5 (mg‡KvYx AwZfR-evû Dccv`¨)

`BwU mg‡KvYx wÎf‡Ri AwZfRØq mgvb n‡j Ges GKwUi GK evû AciwUi Aci GK evûi mgvb n‡j, wÎfRØq memg n‡e|



we‡kl wbePb : g‡b Kwi, ABC I DEF mg‡KvYx wÎfRØ‡q

AwZfR AC =AwZfR DF Ges $AB = DE.$

cgvY Ki‡Z n‡e †h, $\Delta ABC \cong \Delta DEF$

cgvY :

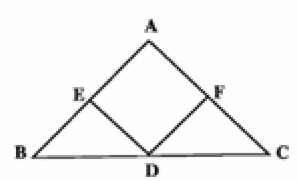
avc	h_v_Zv
(1) ΔABC †K ΔDEF Gi Dci Ggbfv‡e ¯'vcb Kwi †hb, B we›` $E$ we›`i Dci, BA evû ED evû eivei Ges C we›` $ED$ Gi †h cv‡k $F$ we›` Av‡Q Gi wecixZ cv‡k c‡o|	[evûi memgZv]
awi, G we›` $C$ we›`i bZb Ae¯'vb| †h‡nZ $AB = DE, A$ we›` $D$ we›`i Dci co‡e| d‡j ΔDEG n‡e ΔABC Gi bZb Ae¯'vb|	
mZivs, $DG = AC = DF, \angle DEG = \angle DEF = \angle ABC$ = GK mg‡KvY Ges $\angle DGE = \angle ACB$ |	
(2) †h‡nZ $\angle DEF + \angle DEG =$ 1 mg‡KvY + 1 mg‡KvY = 2 mg‡KvY, $\therefore GEF$ GKwU mij‡iLv|	
GLb, †h‡nZ ΔDGF -G $DG = DF$	
$\therefore$ $\angle DFG = \angle DGF$ ev $\angle DFE = \angle DGF$	
mZivs $\angle DFE = \angle ACB$	[Dccv`¨ 2]
(3) GLb, ΔABC I ΔDEF -G	
$\angle ABC = \angle DEF$ [$\because$ c‡Z¨‡K GK mg‡KvY]	
$\angle ACB = \angle DFE$ Ges AB evû = Abic DE evû|	
mZivs, $\Delta ABC \cong \Delta DEF$ (cgwYZ)	[†KvY-evû-†KvY Dccv`¨]

Abkxjbx 10·2

1\| ΔABC G $AB = AC$ Ges O, ABC Gi Af¨š‡i Ggb GKwU we›` †hb $OB = OC$ ev cgvY Ki †h, $\angle AOB = \angle AOC.$

2\| ΔABC Gi AB I AC evû‡Z h_vµ‡g D I E Ggb `BwU we›` †hb $BD = CE$ Ges $BE = CD$. cgvY Ki †h, $\angle ABC = \angle ACB.$

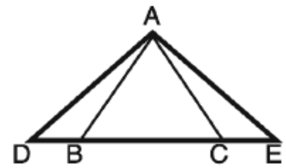
3\| wP‡Î, ΔABC -G $AB = AC, BD = DC$ Ges $BE = CF$ \| cgvY Ki †h, $\angle EDB = \angle FDC$



4| wP‡Î, ΔABC-G $AB = AC$ Ges

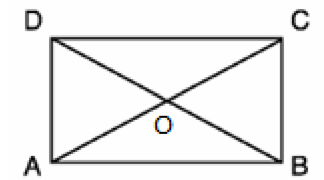
$\angle BAD = \angle CAE$ | cgvY Ki †h,

$AD = AE$



5| $ABCD$ PZf©‡R $AC, \angle BAD$ Ges $\angle BCD$ Gi mgwØLÐK| cgvY Ki †h, $\angle B = \angle D$.

6| wP‡Î, $ABCD$ PZf©‡Ri AB Ges CD ci¯úi mgvb I mgvšÍivj Ges AC I BD KY `BwU $O$ we›`‡Z †Q` K‡i‡Q| cgvY Ki †h, $AD = BC$.



7| cgvY Ki †h, mgwØevû wÎf©‡Ri fwgi cvšÍwe›`Øq †_‡K wecixZ evûi Dci Aw¼Z j¤Øq ci¯úi mgvb|

8| cgvY Ki †h, †Kv‡bv wÎf©‡Ri fwgi cvš we›`Øq †_‡K wecixZ evûi Dci Aw¼Z j¤Øq hw` mgvb nq, Z‡e wÎfRwU mgwØevû|

9| $ABCD$ PZf©‡Ri $AB = AD$ Ges $\angle B = \angle D =$ GK mg‡KvY|

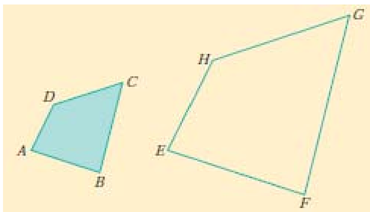
cgvY Ki †h, $\Delta ABC \cong \Delta ADC$.

10·3 m`kZv

wb‡Pi wPÎ¸‡jv GKB wP‡Îi †QvU-eo AvKvi| G‡`i wewfbœ As‡ki AvKvi GKB, wKš Abic `B we›`i `iZ mgvb bq| wPÎ¸‡jv‡K m`k wPÎ ejv nq|

KvR :

1| (K) wP‡Îi PZfR `BwU wK m`k e‡j g‡b nq?

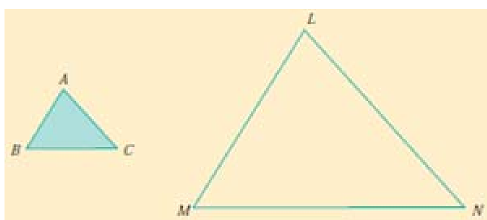


†KvY		evû	
A	E	AB =	EF =
B	F	BC =	FG =
C	G	CA =	GH =
D	H	AD =	EH =

(L) wPÎ `BwUi †KvY¸‡jv †g‡c mvivwYwU ciY Ki| †KvY¸‡jvi g‡a¨ †Kv‡bv m¤úK Av‡Q wK ?

(M) wPÎ `BwUi Abic evû¸‡jv †g‡c mvivwYwU ciY Ki| evû¸‡jvi g‡a¨ †Kv‡bv m¤úK Av‡Q wK ?

2| *ABC* wÎfR‡K *LMN* ewaZ K‡i wÎfRwU AvKv n‡q‡Q|



(K)Abic †KvY¸‡jv wb‡`k Ki Ges cwigvc Ki|

(L)Abic evû¸‡jv wb‡`k Ki Ges cwigvc K‡i AbcvZ †ei Ki| AbcvZ¸‡jv wK mgvb ?

m`k wPÎ GKB AvKwZi wKš AvKv‡i mgvb bvI n‡Z cv‡i| m`k wP‡Îi AvKvi mgvb n‡j Zv memg wP‡Î ciYZ nq| mZivs memgZv m`kZvi we‡kl ic|

`BwU wÎfR ev eûfR m`k n‡j

- Abic †KvY¸‡jv mgvb|
- Abic evû¸‡jv mgvbcwZK|

m`k wP‡Îi evû¸‡jvi AbcvZ Øvivi gj wP‡Îi Zjbvq Ab¨ wP‡Îi eab A_ev m‡¼vPb †evSvq|

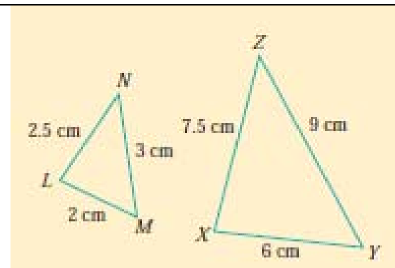
10·4 m`k wÎfR

`BwU m`k wÎf‡Ri Abic †KvY¸‡jv mgvb Ges Abic evû¸‡jv mgvbycvwZK| `BwU wÎfR m`k n‡q‡i Rb¨ b¨bZg kZ †ei Kwi|

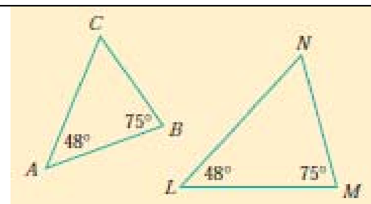
KvR :

1| wZb-Pvi R‡bi `j MVb K‡i wb‡Pi KvR¸‡jv Ki :

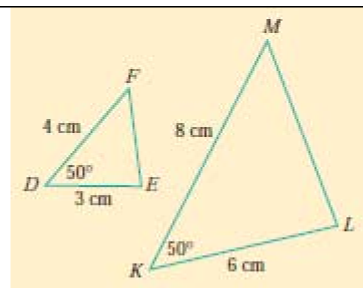
1| (K) ΔLMN wÎfRwU AvK, hvi $LM =$ 2 †m.wg., $MN = 3$ †m.wg., $LN =$ 2.5 †m.wg.| G wÎfRwU wK Abb¨?
(L) ΔXYZ wÎfRwU AvK, hvi $XY =$ 6 †m.wg., $YZ = 9$ †m.wg., $XZ =$ 7.5 †m.wg.|
(M) ΔLMN I ΔXYZ wÎf‡Ri Abic evû¸‡jvi AbycvZ mgvb wK ?
(N) ΔLMN I ΔXYZ m`k wK?



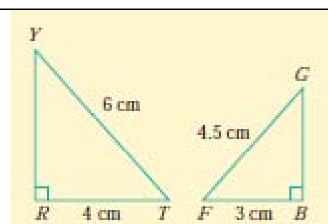
2| (K) ΔABC wÎfRwU AvK, hvi $\angle A =$ 48°, $\angle B =$ 75°.
(L) Gevi ΔLMN wÎfRwU AvK, hvi $\angle L =$ 48°, $\angle M =$ 75°.
(M) ΔABC I ΔLMN m`k wK? †Kb?
(N) †Zvgvi AvKv wÎfR¸‡jv Ab¨ wk¶v_©x‡`i AvKv wÎfR¸‡jvi mv‡_ Zjbv Ki| †m¸‡jv wK m`k?



3| (K) ΔDEF wÎfRwU AvK, hvi $DE =$ 3 †m.wg., $DF = 4$ †m.wg. I Ašf³ †KvY $\angle D = 50°$.
(L) ΔKLM wÎfRwU AvK, hvi $KL =$ 6 †m.wg., $KM = 8$ †m.wg. I Ašf³ †KvY $\angle K = 50°$.
(M) ΔDEF I ΔKLM wÎf‡Ri Abic evû¸‡jv wK mgvbycvwZK ?
(N) ΔDEF I ΔKLM m`k wK? e¨vL¨v Ki|



4| (K) ΔRTY wÎfRwU AvK, hvi $RT =$ 4 †m.wg., $\angle R = 90°$ I AwZfR $TY = 6$ †m.wg.|
(L) (K) ΔBFG wÎfRwU AvK, hvi $BF =$ 3 †m.wg., $\angle B = 90°$ I AwZfR $FG = 4.5$ †m.wg.|
(M) ΔRTY I ΔBFG wÎf‡Ri Abic evû¸‡jvi AbycvZ †ei Ki| Zviv mgvb wK ?
(N) ΔLMN I ΔXYZ m`k wK?

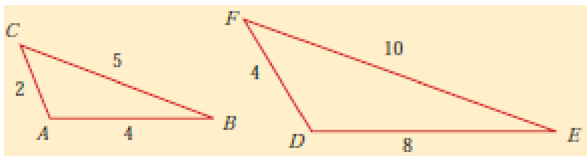


10.5 wÎf‡Ri m`kZvi kZ

Dc‡ii Av‡jvPbv †_‡K Avgiv wÎf‡Ri m`kZvi KwZcq kZ wbaviY Ki‡Z cwi| kZ¸‡jv wb¤œic:

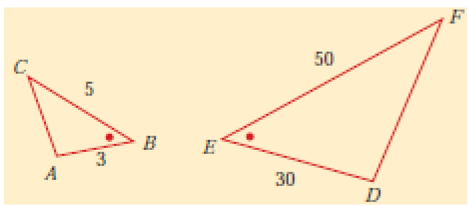
kZ 1| (evû-evû-evû)

hw` GKwU wÎf‡Ri wZb evû Aci GKwU wÎf‡Ri wZb evûi mgvbcwZK nq, Z‡e wÎfR `BwU m`k|



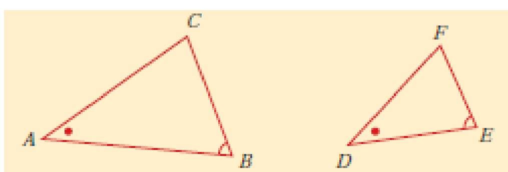
kZ 2| (evû-†KvY-evû)

hw` `BwU wÎf‡Ri GKwUi `B evû h_vµ‡g AciwUi `B evûi mgvbcwZK nq Ges evû `BwUi Ašf³ †KvY `BwU ci¯úi mgvb nq, Z‡e wÎfR `BwU m`k|



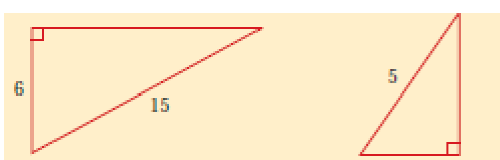
kZ 3| (†KvY-†KvY)

hw` `BwU wÎf‡Ri GKwUi `BwU †KvY h_vµ‡g AciwUi `BwU †Kv‡Yi mgvb nq, Z‡e wÎfR `BwU m`k|



kZ 4| (AwZfR-evû)

hw` `BwU mg‡KvYx wÎf‡Ri GKwUi AwZfR I GKwU evû h_vµ‡g AciwUi AwZfR I Abic evûi mgvbcwZK nq, Z‡e wÎfR `BwU m`k|



10·6 m`k PZfR

`BwU m`k Pff‡Ri Abic †KvY¸‡jv mgvb Ges Abic evû¸‡jv mgvbcwZK| `BwU PZfR m`k nIqvi kZ wbYq Kwi|

KvR :

wZb-Pvi R‡bi `j MVb K‡i wb‡Pi KvR¸‡jv Ki:

1| (K) $KLMN$ PZfRwU AvK, hvi $\angle K = 45°$, $KL =$ 3 †m.wg., $LM =$ 2 †m.wg., $MN = 3$ †m.wg., $NK =$ 2.5 †m.wg.|

[Bw½Z ; c_‡g $\angle K$ †KvYwU AvK Ges †Kv‡Yi evû `BwU †_‡K $KL$ I $LM$ mgvb `i‡Z `BwU we›` wPwýZ Ki|AZtci Aci `B evû AvK|]

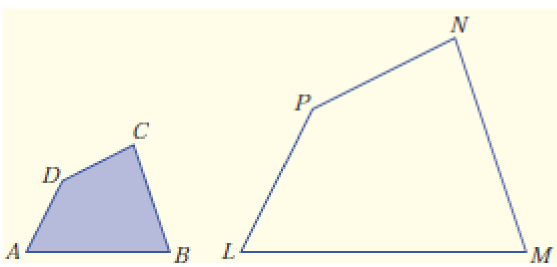
(L) $WXYZ$ PZfRwU AvK, hvi $WX =$ 8 †m.wg., $XY =$ 4 †m.wg., $YZ = 6$ †m.wg., $ZX =$ 5 †m.wg., $\angle L = 45°$. G PZfRwU wK Abb¨?

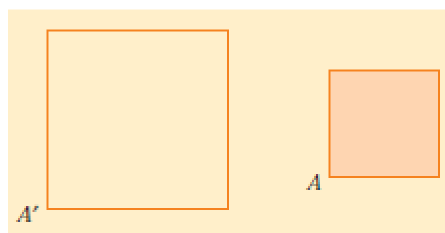
(M) $KLMN$ I $WXYZ$ PZf‡Ri Abic evû¸‡jvi AbcvZ mgvb wK?

(N) $KLMN$ I $WXYZ$ PZf‡Ri Abic †KvY¸‡jv cwigvc Ki| †m¸‡jv wK ci¯úi mgvb ?

(N) $KLMN$ I $WXYZ$ m`k wK?

2| †Zvgvi cQ›`gZ †KvY I evû wb‡q wb‡Pi KvRwU cbivq Ki| PZfR¸‡jv m`k wK?





`BwU PZf‡Ri Abic evû¸‡jv mgvbcwZK n‡j PZfR `BwU m`k|

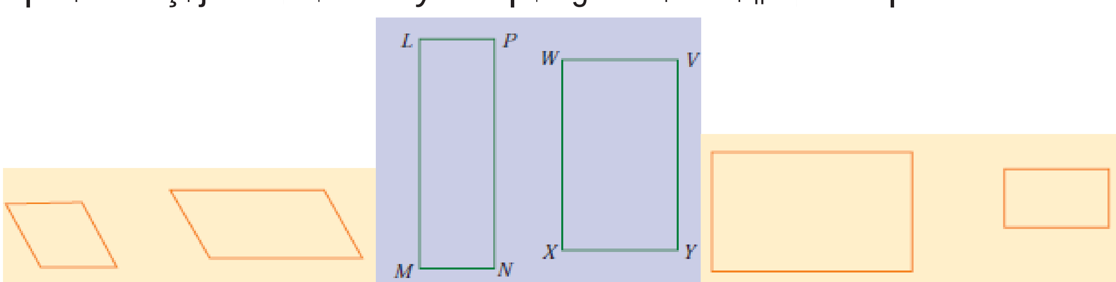
j¶Yxq †h, `BwU m`k PZf‡Ri

(K) Abic †KvY¸‡jv mgvb Ges

(L) Abic evû¸‡jv mgvbcwZK|

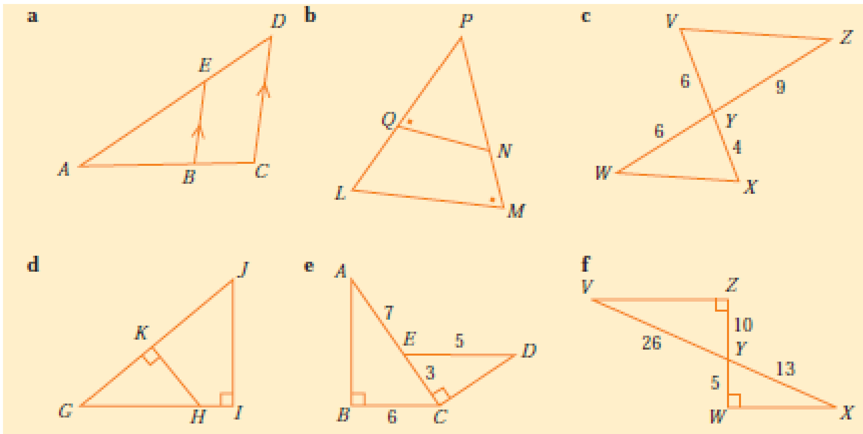
KvR :

1| wb‡Pi wPÎ¸‡jvi m`k †Rvo wPwýZ Ki| †Zvgvi DË‡ii c‡¶ hw³ `vI|

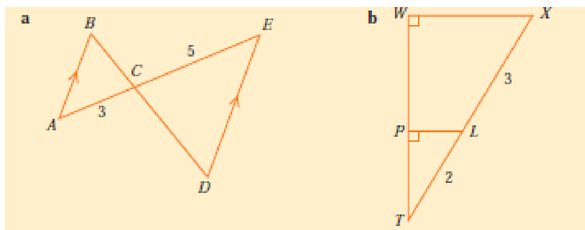


Abkxjbx 10·3

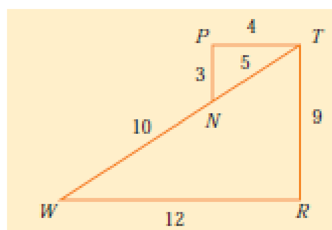
1| wb‡Pi cwZwU wP‡Î wÎfR `BwUi m`kZvi KviY eYbv Ki|



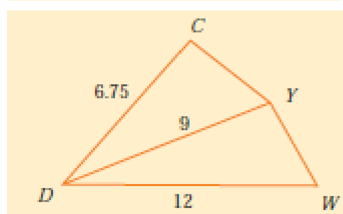
2| cgvY Ki †h, wb‡Pi cwZwU wP‡Îi wÎfR `BwU m`k|



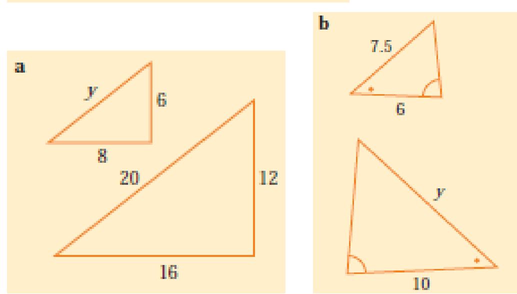
3| †`LvI †h, $\Delta PTN$ Ges $\Delta RWT$ m`k|



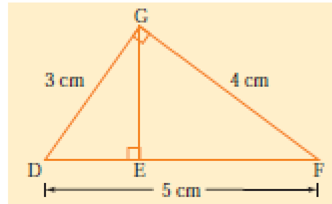
4| DY †iLvsk $\angle CDW$ †KvYwUi wØLÐK| †`LvI †h, $\Delta CDY$ I $\Delta YDW$ m`k|



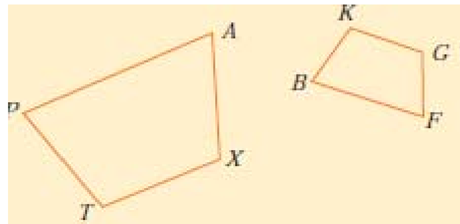
5| wb‡Pi cwZwU m`k wÎfR †Rvov †_‡K y Gi gvb †ei Ki|



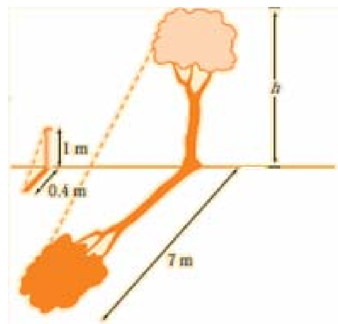
6| cgvY Ki †h, wP‡Îi wÎfR wZbwU m`k|



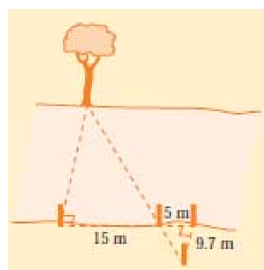
7| PZfR `BwUi Abic †KvY I Abic evû¸‡jv wPwýZ Ki| PZfR `BwU m`k wK-bv hvPvB Ki|



8|1 wgUvi ^`‡N¨i GKwU jwV gwU‡Z `Ðvqgvb Ae¯vq 0.4 wgUvi Qvqv †d‡j| GKwU Lvov Mv‡Qi Qvqvi ^`N¨ 7 wgUvi n‡j MvQwUi D”PZv KZ ?



9| wknve b`x cvi bv n‡q b`xi c¯ gvc‡Z Pvq| G Rb¨ †m wVK Aci cv‡o GKwU MvQ †e‡Q wb‡q b`xi cv‡o wP‡Îi b¨vq wKQ gvc‡RvK Kij| b`xi c¯ wbYq Ki|



GKv`k Aa¨vq

Z_¨ I DcvË

cvPxbKvj †_‡KB †Kv‡bv wbw`ó D‡Ï‡k¨ ev ¯e Rxe‡bi A‡bK NUbv ev Z_¨vewj MwYwZK msL¨vi gva¨‡g cKvk Kiv n‡Zv| eZgv‡b ^`bw›`b Rxe‡bi wewfb NUbv ev Z_¨mgn msL¨vi gva¨‡g cKv‡ki e¨vcKZv ew× †c‡q‡Q| Avi msL¨vevPK Z_¨mgn n‡”Q cwimsL¨vb| ^`bw›`b Rxe‡b e¨eüZ wewfb cwimsL¨vb mnR‡eva¨ I AvKlYxq Kivi Rb¨ Zv wewfb ai‡bi †jLwP‡Îi mvnv‡h¨ Dc¯vcb Kiv nq| Avi Gme †jLwPÎ †`‡L Dc¯wcZ NUbv m¤‡Ü Avgiv m¯úó aviYv cvB I eS‡Z cwi| G Aa¨v‡q Avgiv Z_¨ I Dcv‡Ëi AvqZ‡jL m¤‡Ü Rvbe| ZvQvov Aweb¨¯— DcvË web¨¯— Kivi Rb¨ †kwY e¨eav‡bi gva¨‡g Kxfv‡e MYmsL¨v mviwY MVb Kiv nq Zv Rvbe| cwimsL¨v‡bi GB wel‡q¸‡jv wk¶v_x‡`i ^`bw›`b Rxe‡b e¨vcK e¨eüZ nq weavq G m¤‡Ü Zv‡`i cwi®vi Ávb _vKv Acwinvh|

Aa¨vq †k‡l wk¶v_xiv –

- MYmsL¨v mviwY Kx Zv e¨vL¨v Ki‡Z cvi‡e|
- †kwY e¨eav‡bi gva¨‡g Aweb¨¯— DcvË web¨¯— AvKv‡i cKvk Ki‡Z cvi‡e|
- AvqZ‡jL A¼b Ki‡Z cvi‡e|
- Aw¼Z AvqZ‡jL n‡Z cPiK †ei Ki‡Z cvi‡e|
- Aw¼Z AvqZ‡jL n‡Z DcvË m¤ú‡K e¨vL¨v Ki‡Z cvi‡e|

11·1 Z_¨ I DcvË

Iô †kwY‡Z Avgiv Z_¨ I DcvË m¤‡Ü †R‡bwQ| msL¨wfwËK †Kv‡bv Z_¨ ev NUbv n‡”Q GKwU cwimsL¨vb| Avi Z_¨ ev NUbv wb‡`kK msL¨v¸‡jv n‡”Q cwimsL¨v‡bi DcvË| aiv hvK, †Kv‡bv GK cix¶vq mßg †kwY‡Z Aa¨qbiZ 35 Rb wk¶v_xi MwY‡Z cvß b¤i n‡jv -

80, 60, 65, 75, 80, 60, 60, 90, 95, 70, 100, 95, 85, 60, 85, 85, 90, 98, 85, 55, 50, 95, 90, 90, 98, 65, 70, 70, 75, 85, 95, 75, 65, 75, 65|

GLv‡b, msL¨v Øviv wb‡`©wkZ b¤imgn H cix¶vi GKwU cwimsL¨vb| msL¨v Øviv wb‡`©wkZ b¤i¸‡jv n‡jv cwimsL¨v‡bi DcvË| Zvn‡j Avgiv ej‡Z cwi, cwimsL¨v‡bi DcvËmgn msL¨vi gva¨‡g Dc¯vcb Ki‡Z nq| Z‡e †Kv‡bv wew”Qb msL¨v‡K cwimsL¨vb ejv nq bv| †hgb, GKRb Qv‡Îi cvß b¤i 85 ejv n‡j Zv cwimsL¨vb n‡e bv|

11·2 cwimsL¨vb DcvË

cwimsL¨vb DcvË `B ai‡bi| h_v,
(1) cv_wgK DcvË ev cZ¨¶ DcvË I (2) gva¨wgK DcvË ev c‡iv¶ DcvË|

(1) cv_wgK DcvË : c‡e ewYZ †Kv‡bv GK cix¶vq MwY‡Z cvß b¤i¸‡jv cv_wgK DcvË| Gic DcvË c‡qvRb Abhvqx AbmÜvbKvix mivmwi Drm †_‡K msMn Ki‡Z cv‡i| mZivs Drm †_‡K mivmwi †h DcvË msMnxZ nq ZvB n‡jv cv_wgK DcvË| mivmwi msMnxZ weavq cv_wgK Dcv‡Ëi wbf©i‡hvM¨Zv A‡bK †ewk|

(2) gva¨wgK DcvË : cw_exi K‡qKwU kn‡ii †Kv‡bv GK gv‡mi ZvcgvÎv Avgv‡`i c‡qvRb| †hfv‡e MwY‡Zi cvß b¤i¸‡jv Avgiv msMn K‡iwQ †mfv‡e ZvcgvÎvi Z_¨ Avgv‡`i c‡¶ msMn Kiv m¤e bq| G‡¶‡Î †Kv‡bv cwZôv‡bi msMnxZ DcvË Avgiv Avgv‡`i c‡qvR‡b e¨envi Ki‡Z cwi| mZivs GLv‡b Drm n‡"Q c‡iv¶| c‡iv¶ Drm †_‡K msMnxZ DcvË n‡"Q gva¨wgK DcvË| AbmÜvbKvix †h‡nZ wb‡Ri c‡qvRb Abhvqx mivmwi DcvË msMn Ki‡Z cv‡i bv †m‡nZ Zvi wbKU Gfv‡e msMnxZ Dcv‡Ëi wbf©i‡hvM¨Zv A‡bK Kg|

11·3 Awebv¨¯Í I webv¨¯Í DcvË

Awebv¨¯Í DcvË : c‡e ewYZ wk¶v_x©‡`i MwY‡Z cvß b¤i¸‡jv n‡jv Awebv¨¯Í DcvË| GLv‡b b¤i¸‡jv G‡jv‡g‡jvfv‡e Av‡Q| b¤i¸‡jv gv‡bi †Kv‡bv µ‡g mvRv‡bv †bB|

webv¨¯Í DcvË : Dc‡ii ewYZ b¤i¸‡jv gv‡bi Ea¶µg Abmv‡i mvRv‡j Avgiv cvB, 50, 55, 60, 60, 60, 60, 65, 65, 65, 65, 70, 70, 70, 75, 75, 75, 75, 80, 80, 85, 85, 85, 85, 85, 90, 90, 90, 90, 95, 95, 95, 95, 98, 98, 100|

Gfv‡e mvRv‡bv DcvËmgn‡K webv¨¯Í DcvË e‡j| DcvËmgn Av‡iv mnRfv‡e mvwiwYf³ K‡i webv¨¯Í Kiv hvq hv wb‡P †`Lv‡bv n‡jv|

Awebv¨¯Í DcvË‡K webv¨¯Í Kivi mnR wbqg :
Dc‡i ewYZ cvß mewbg b¤i 50 Ges m‡ev©"P b¤i 100| GLb †kwYweb¨vm Kivi Rb¨ 50 Gi Kg mweavRbK †h‡Kv‡bv GKwU msL¨v aiv hvq| mZivs Avgiv hw` 46 †_‡K ïi K‡i cwZ 5 b¤‡ii e¨eav‡bi Rb¨ GKwU †kwY MVb Kwi Zvn‡j KqwU †kwY n‡e Zv wbavib Ki‡Z cwi| D‡jøL¨, Dcv‡Ëi msL¨vi Dci wfwË K‡i mweavRbK e¨eavb wb‡q KZK¸‡jv †kwY‡Z fvM Kiv nq| †kwY‡Z fvM Kivi wbawiZ †Kv‡bv wbqg †bB| Z‡e mPviPi c‡Z¨K †kwYi e¨eavb ev e¨wßi mewb¤ 5 I m‡ev©"P 15 Gi g‡a¨ mxgve× ivLv nq| msL¨v †kwY wbavi‡Yi Rb¨ Dcv‡Ëi cwimi wbYq Ki‡Z nq|

cwimi = (m‡ev”P msL¨v – mewb¤ msL¨v) + 1

GLv‡b †kwYe¨wß 5 Gi Rb¨ Av‡jvP¨ DcvË‡i †kwYmsL¨v = $\dfrac{(\text{m‡ev”P msL¨v} - \text{mewb¤ msL¨v}) + 1}{5}$

$= \dfrac{(100-50)+1}{5}$ ev $\dfrac{51}{5}$ = 10·2 = 11|

mZivs 46 †_‡K Avi¤ K‡i cwZ 5 b¤^‡ii Rb¨ e¨eavb †kwY ˆZwi Ki‡j †kwYmsL¨v n‡e 11wU| c_‡g evgc‡k GKwU Kjv‡g b¤^img‡ni †kwY¸‡jv †jLv n‡e| Gici cvß b¤^i¸‡jv G‡K G‡K we‡ePbv Kwi Ges c_g b¤^i †h †kwY‡Z co‡e Zvi Rb¨ H †kwYi Wv‡b Avi GKwU Kjv‡g U¨wj (Tally) wPý Ô|Õ w`B| †Kv‡bv †kwY‡Z hw` Pv‡ii †ewk U¨wj wPý c‡o Z‡e cÂg U¨wjwPýwU PviwU wPý R‡o AvovAwo fv‡e w`‡Z n‡e| Gfv‡e †kwYweb¨vm †kl n‡j U¨wjwPý MYbv K‡i †kwY Abhvqx b¤^icvß wk¶v_xi msL¨v wbaviY Kiv nq| †Kv‡bv †kwY‡Z hZRb QvÎ Ašf³ n‡e ZvB n‡e H †kwYi NUbmsL¨v ev MYmsL¨v| MYmsL¨v msewjZ mviwY n‡e MYmsL¨v mviwY| Dc‡ii Av‡jvPbvq ewYZ DcvË‡i web¨¯— mviwY wb‡P †`Iqv n‡jv :

b¤^‡ii †kwY (†kwY e¨eavb/e¨wß = 5)	U¨wj wPý	MYmsL¨v ev NUbmsL¨v (wk¶v_xi msL¨v)
46 – 50	I	1
51 – 55	I	1
56 – 60	IIII	4
61 – 65	IIII	4
66 – 70	III	3
71 – 75	IIII	4
76 – 80	II	2
81 – 85	IIII	5
86 – 90	IIII	4
91 – 95	IIII	4
96 – 100	III	3
	†gvU	35

j¶ Kwi : GLv‡b †kwY e¨eavb ev e¨wß aiv n‡q‡Q 5| c‡qvR‡b Ges DcvË web¨v‡mi mweavi Rb¨ †kwY e¨eavb †h‡Kv‡bv msL¨v aiv †h‡Z cv‡i| Z‡e wnmv‡ei mweav‡_ †kwY e¨eavb 5 †_‡K 15 Gi g‡a¨ mxgve× ivLv nq|

D`vniY 1| †Kv‡bv kn‡ii Rvbqvwi gv‡mi 31 w`‡bi ZvcgvÎv (wWwM †mjwmqvm) wb‡P †`Iqv n‡jv| MYmsL¨v mviwY ˆZwi Ki (ZvcgvÎv¸‡jv cYmsL¨vq)|
20, 18, 14, 21, 11, 14, 12, 10, 15, 18, 12, 14, 16, 15, 12, 14, 18, 20, 22, 9, 11, 10, 14, 12, 18, 20, 22, 14, 25, 20, 10|

mgvavb : GLv‡b ZvcgvÎvi mewbg msL¨vgvb 9 Ges m‡ev”P msL¨vgvb 25| mZivs c`Ë Dcv‡Ëi cwimi = (25 – 9) + 1 = 17| mZivs 5 wWwM †mjwmqvm Gi Rb¨ †kwYmsL¨v $\frac{17}{5} = 3\cdot 4$

$\therefore$ †kwYmsL¨v n‡e 4|

c`Ë Dcv‡Ëi MYmsL¨v mviwY n‡jv :

ZvcgvÎvi †kwY	U¨vwjwPý	MYmsL¨v
9 – 13	~~IIII~~ ~~IIII~~	10
14 – 18	~~IIII~~ ~~IIII~~ III	13
19 – 23	~~IIII~~ II	7
24 – 28	I	1
	†gvU	31

KvR : 1| †Zvgv‡`i †kwYi 30 Rb K‡i wk¶v_©x wb‡q GK GKwU `j MVb Ki| c‡Z¨K `‡ji m`m¨MY wb‡R wbR `‡ji m`m¨‡`i D”PZv (†mw›UwgUv‡i) cwigvc Ki| cvß Dcv‡Ëi MYmsL¨v mviwY ˆZwi Ki|

11·4 MYmsL¨v AvqZ‡jL

†Kv‡bv cwimsL¨vb hLb †jLwP‡Îi gva¨‡g Dc¯’vcb Kiv nq ZLb Zv †evSv I wm×vš— †bIqvi Rb¨ †hgb mnR nq †Zgwb wPËvKlK nq| GB †c¶vc‡U cwimsL¨v‡b †jLwP‡Îi gva¨‡g MYmsL¨v mviwY Dc¯’vcb eûj cPwjZ c×wZ| Avi AvqZ‡jL ev MYmsL¨v AvqZ‡jL n‡”Q MYmsL¨v mviwYi GKwU †jLwPÎ| MYmsL¨v AvqZ‡jL AvKvi Rb¨ wb‡Pi avc¸‡jv AbmiY Kiv nq :

1| GKwU MYmsL¨v mviwYi †kwY e¨wß x-A¶ eivei †jLv nq Ges †kwY e¨wß fwg a‡i AvqZ AvKv nq| mweavRbK †¯‹‡j †kwY e¨wß †bIqv nq|

2| mweavRbK †¯‹‡j y-A¶ eivei MYmsL¨vi gvb †bIqv nq Ges MYmsL¨v nq AvqZ‡Zi D”PZv| Dfq A‡¶i Rb¨ GKB ev c_K mweavRbK †¯‹j †bIqv hvq|

D`vniY 2| †Zvgv‡`i ¯‹‡ji 10g †kÖYxi 60 Rb wk¶v_©xi IR‡bi (Avmb wK‡jvMÖvg) MYmsL¨v mviwY wb‡P †`Iqv n‡jv| MYmsL¨v mviwY †_‡K Dcv‡Ëi AvqZ‡jL AvK Ges AvqZ‡jL †`‡L cPiK (Avmb gvb) wbYq Ki|

†kÖY e¨wß	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
MYmsL¨v	8	15	25	10	2

mgvavb : x-A¶ I y-A¶ eivei QK KvM‡Ri (Graph Paper) ¶`Zg e‡Mi cwZ Ni‡K †kÖYe¨wßi GK GKK Ges y-A¶ eivei QK KvM‡Ri cwZ 2 Ni‡K MYmsL¨vi 5 GKK a‡i MYmsL¨v AvqZ‡jL AvKv n‡q‡Q| x-A¶ eivei †kÖYe¨wß Ges y-A¶ eivei MYmsL¨v aiv n‡q‡Q| †h‡nZ †kÖYe¨wß x-A¶ eivei 41 †_‡K Avi¤¢ Kiv n‡q‡Q, †m‡nZ x-A‡¶i gj we›` †_‡K 41 chš— fvOv wPý w`‡q †evSv‡bv n‡q‡Q †h, ewK Ni¸‡jv we`¨gvb Av‡Q|

wPÎ

Dc‡ii AvqZ‡jL †_‡K cZxqgvb nq †h, MYmsL¨vi cvPh 50–55 †kÖY‡Z| mZivs cPiK GB †kÖY‡Z we`¨gvb| cPiK wbaviY Kivi Rb¨ Avq‡Zi DcwifvM †KŠwYK we›` †_‡K `BwU AvovAwo †iLvsk Av‡Mi I c‡ii Avq‡Zi DcwifvM †KŠwYK we›` mv‡_ ms‡hvM Kiv nq| G‡`i †Q`we›` †_‡K mswkó fwgi Dci j¤^ Uvbv nq| j¤^ x-A‡¶i †hLv‡b wgwjZ nq Gi e¨wß wbaviY Kiv nq| wbawiZ e¨wß n‡jv cPiK| wPÎ †_‡K †`Lv hvq 52 Dcv‡Ëi cPiK|

wb‡Yq cPiK 52 †KwR|

D`vniY 3| †Kv‡bv we`¨vj‡qi 10g †kÖY‡Z Aa¨qbiZ 125 Rb wk¶v_©xi MwYZ welq cvß b¤^‡ii MYmsL¨v we‡kIY (Frequency Distribution) mviwY wb‡P †`Iqv n‡jv| GKwU AvqZ‡jL AvK Ges AvqZ‡jL †_‡K cPiK (Avmb) wbYq Ki|

†kÖYe¨wß	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
wk¶v_©xi msL¨v (MYmsL¨v)	5	12	30	40	20	13	3	2

mgvavb : c_‡g QK KvM‡R x-A¶ I y-A¶ AvKv n‡q‡Q, y-A¶ eivei wk¶v_xi msL¨v (MYmsL¨v) Ges x-A¶ eivei †kwYe¨wß a‡i AvqZ‡jLwU AvKv n‡q‡Q| GLv‡b x I y Dfq A‡¶ QK KvM‡Ri GK Ni mgvb 2 GKK aiv n‡q‡Q| x-A‡¶ 0 †_‡K 20 chš— Av‡Q †evSv‡Z fvOv wPý †`Iqv n‡q‡Q|

wPÎ

GLv‡b wPÎwqZ AvqZ‡jL †_‡K †`Lv hvq, †ewk msL¨K wk¶v_xi cvß b¤i 50 †_‡K 60 Gi g‡a¨ Ges †Q`we›` †_‡K x A‡¶i Dci †h j¤ Uvbv n‡q‡Q Gi e¨wß 50 I 60 Gi ga¨we›`| ZvB wk¶v_x‡`i cvß b¤‡ii cPiK n‡jv 55|

KvR : 1| †Zvgv‡`i †kwY‡Z Aa¨qbiZ wk¶v_x‡`i wb‡q `BwU `j MVb Ki| `‡ji bvg `vI| †hgb, kvcjv I iRbxMÜv| †Kv‡bv ˆÎgvwmK/AaewlK cix¶vq (K) kvcjv `‡ji evsjvq cvß b¤‡ii MYmsL¨v mviwY ˆZwi K‡i AvqZ‡jL AvK| (L) iRbxMÜv `‡ji Bs‡iwR‡Z cvß b¤‡ii MYmsL¨v mviwY ˆZwi K‡i AvqZ‡jL AvK|

Abkxjbx 11

1| DcvË ej‡Z Kx †evSvq Zv D`vni‡Yi gva¨‡g wjL|

2| DcvË KZ cKv‡ii? c‡Z¨K cKv‡ii DcvË Kxfv‡e msMn Kiv nq Ges c‡Z¨K cKvi DcvË msM‡ni mweav I Amweav wjL|

3| Aweb¨¯— DcvË Kx? D`vniY `vI|

4| GKwU Aweb¨¯— DcvË wjL| gv‡bi µgvbmv‡i mwR‡q web¨¯— Dcv‡Ë i/cvši Ki|

5| †Kv‡bv †kwYi 60 Rb wk¶v_xi MwYZ welq cvß b¤i wb‡P †`Iqv n‡jv| MYmsL¨v mviwY ˆZwi Ki|

50, 84, 73, 56, 97, 90, 82, 83, 41, 92, 42, 55, 62, 63, 96, 41, 71, 77, 78, 22, 48, 46, 33, 44, 61, 66, 62, 63, 64, 53, 60, 50, 72, 67, 99, 83, 85, 68, 69, 45, 22, 22, 27, 31, 67, 65, 64, 64, 88, 63, 47, 58, 59, 60, 72, 71, 73, 49, 75, 64|

6| wb‡P 50wU †`vKv‡bi gwmK weµ‡qi cwigvY (nvRvi UvKvq) †`Iqv n‡jv| 5 †kwYe¨wß a‡i MYmsL¨v mviwY ˆZwi Ki|

132, 140, 130, 140, 150, 133, 149, 141, 138, 162, 158, 162, 140, 150, 144, 136, 147, 146, 150, 143, 148, 150, 160, 140, 146, 159, 143, 145, 152, 157, 159, 132, 161, 148, 146, 142, 157, 150, 178, 141, 149, 151, 146, 147, 144, 153, 137, 154, 152, 148|

7| †Zvgv‡`i we`¨vj‡qi 8g †kwYi 30 Rb Qv‡Îi IRb (†KwR‡Z) wb‡P †`Iqv n‡jv :
40, 55, 42, 42, 45, 50, 50, 56, 50, 45, 42, 40, 43, 47, 43, 50, 46, 45, 42, 43, 44, 52, 44, 45, 40, 45, 40, 44, 50, 40|
(K) gv‡bi µgvbmv‡i mvRvI|
(L) Dcv‡Ëi MYmsL¨v mviwY ˆZwi Ki|

8| †Kv‡bv GjvKvi 35wU cwiev‡ii †jvKmsL¨v wb‡P †`Iqv n‡jv :
6, 3, 4, 7, 10, 8, 5, 6, 4, 3, 2, 6, 8, 9, 5, 4, 3, 7, 6, 5, 3, 4, 8, 5, 9, 3, 5, 7, 6, 9, 5, 8, 4, 6, 10|
†kwYe¨wß 2 wb‡q MYmsL¨v MVb Ki|

9| 30 Rb kwg‡Ki NÈv cwZ gRwi (UvKvq) wb‡P †`Iqv n‡jv :
20, 22, 30, 25, 28, 30, 35, 40, 25, 20, 28, 40, 45, 50, 40, 35, 40, 35, 25, 35, 35, 40, 25, 20, 30, 35, 50, 40, 45, 50|
†kwY e¨eavb 5 wb‡q MYmsL¨v mviwY MVb Ki|

10| wb‡Pi MYmsL¨v mviwY n‡Z AvqZ‡jL AvK Ges cPiK wbYq Ki :

†kwYe¨wß	11-20	21-30	31-40	41-50	51-60	61-70	71-80	81-90	91-100
MYmsL¨v	10	20	35	20	15	10	8	5	3

11| AvšRwZK gv‡bi T-20 wµ‡KU †Ljvq †Kv‡bv `‡ji msMnxZ ivb Ges DB‡KU cZ‡bi cwimsL¨vb wb‡Pi mviwY‡Z †`Iqv n‡jv| AvqZ‡jL AvK|

Ifvi	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
ivb	6	8	10	8	12	8	6	12	7	15	10	12	14	10	8	12	8	14	8	6
DB‡KU cZb	0	0	0	0	0	1	0	0	0	0	1	0	0	1	1	1	2	0	0	0

[Bw½Z : x-A¶ eivei Ifvi Ges y-A¶ eivei ivb a‡i AvqZ‡jL AvK| †h Ifv‡i DB‡KU cZb nq †mB Ifv‡i msMnxZ iv‡bi Dc‡i ô•ô wPý w`‡q DB‡KU cZb †evSvb hvq|

12| †Zvgv‡`i †kwYi 30 Rb wk¶v_xi D”PZv (†m.wg.) wb‡P †`Iqv n‡jv| D”PZvi AvqZ‡jL AvK Ges Gi †_‡K cPiK wbYq Ki|
145, 160, 150, 155, 148, 152, 160, 165, 170, 160, 175, 165, 180, 175, 160, 165, 145, 155, 175, 170, 165, 175, 145, 170, 165, 160, 180, 170, 165, 150|

DËigvjv

Abkxjbx: 1·1

1| (K) 13, (L) 23, (M) 39, (N) 105 ; 2| (K) 15, (L) 31, (M) 63 (N) 102 ; 3| (K) 3, (L) 6, (M) 30, (N) 5 ; 4| (K) 3, (L) 6, (M) 7 ; 5| 15 ; 6| 20|

Abkxjbx: 1·2

1| (L) ; 2| (M) ; 3| 1)(N), 2) (K) 3) (K) ; 4| (N) ; 5| (K) 7140 (L) 19wU (M) 16 ; 6| (K) ·6, (L) 1·5, (M) 0·07, (N) 25·32, (O) 0·024, (P) 12·035 ; 7| (K) 2·65, (L) 4·82, (M) 0·19 ; 8| (K) $\frac{1}{8}$, (L) $\frac{7}{11}$, (M) $3\frac{5}{12}$, (N) $5\frac{13}{18}$; 9| (K) 0·926, (L) 1·683, (M) 2·774 ; 10| 84 Rb, 393 Rb ; 11| 52 Rb ; 12| 32 Rb ; 13| 42wU ; 14| 225 ; 15| 25 Rb ; 16| 18, 19 ; 17| 4, 5 ; 18| (K) 1, 2, 3, 6 (L) 10 (M) 10 Rb|

Abkxjbx 2·1

1| (K) 3 : 6 :: 5 : 10, (L) 9 : 18 :: 10 : 20, (M) 7 : 28 :: 15 : 60
(N) 12 : 15 :: 20 : 25, (O) 125 : 25 :: 2500 : 500

2| (K) 6 : 12 :: 12 : 24, (L) 25 : 45 :: 45 : 81, (M) 16 : 28 :: 28 : 49
(N) $\frac{5}{7}$: 1 :: 1 : $\frac{7}{5}$, (O) 1·5 : 4.5 ::4.5 : 13·5

3| (K) 22, (L) 56, (M) 14, (N) $\frac{7}{6}$, (O) 2.5

4| (K) 14, (L) 55, (M) 48, (N) $\frac{17}{4}$ (O) 6·30

5| 1000 UvKv 6| 3850 wU 7| 1000 UvKv, 1400 UvKv, 1800 UvKv

8| i"wg cv‡e 360 UvKv, †Rmwgb cv‡e 720 UvKv Ges KvKwj cv‡e 1080 UvKv

9| jwee cv‡e 450 UvKv, mwg cv‡e 360 UvKv

10| meR cv‡e 1800 UvKv, Wwjg cv‡e 3000 UvKv I Avbvi cv‡e 1500 UvKv 11| 10 Mvg

12| 26 : 19 13| 40 : 70 : 49 14| mviv cv‡e 4800 UvKv, gvBgbv cv‡e 3600 UvKv Ges ivBmv cv‡e 1200 UvKv 15| 6ô †kwYi QvÎ cv‡e 1200 UvKv, 7g †kwYi QvÎ cv‡e 1400 UvKv Ges 8g †kwYi QvÎ cv‡e 1600 UvKv 16| BDm‡di Avq 210 UvKv

Abkxjbx 2·2

1| jvf 125 UvKv 2| ¶wZ 150 UvKv 3| jvf 200 UvKv 4| jvf $5\frac{10}{13}$%

5| 50 wU P‡Kv‡jU 6| 80 wgUvi 7| ¶wZ $7\frac{17}{19}$% 8| jvf 20% 9| jvf $33\frac{1}{3}$%

10| ¶wZ 20% 11| 420 UvKv 12| $763\frac{8}{9}$ UvKv 13| 188 UvKv 14| 4,761.90 UvKv

15| 8,700 UvKv|

Abkxjbx 2·3

7| 3 w`‡b, 8| $9\frac{3}{5}$ w`‡b, 9| 35 w`‡b, 10| 45 Rb, 11| $10\frac{10}{47}$ w`‡b, 12| $7\frac{1}{5}$ NÈvq, 13| 6 wK.wg./NÈv, 14| 2 wK.wg./NÈv 15| w¯i cwb‡Z †bŠKvi †eM 8 wK.wg./NÈv, †¯v‡Zi cwb‡Z †bŠKvi †eM 4 wK.wg./NÈv 16| 84 †n±i, 17| $4\frac{4}{9}$ NÈvq, 18| 8 wgwbU ci,
19| 300 wgUvi, 20| 54 †m‡KÐ|

Abkxjbx 3

1| (K) 0.4039 wK.wg. (L) 0.07525 wK.wg.
2| 53.7 wgUvi, 537 †WwmwgUvi
3| (K) 30 eMwgUvi, (L) 175 eM‡mw›UwgUvi
4| ˆ`N¨ 475 eMwgUvi, c¯ 125 wgUvi 5| 30000 UvKv 6| 2000 e.wg. 7| 96 eMwgUvi
8| 5 †gwUK Ub 507 †K.wR. 700 Mvg 9| 1 †gwUK Ub 750 †K.wR.
10| 666 †gwUK Ub 666 †K.wR. $666\frac{2}{3}$ Mvg 11| 612 †K.wR.
12| 145 †K.wR. 950 Mvg 13| 180 gM 14| 549 †K.wR. Pvj Ges 172 †K.wR. 500 Mvg jeY
15| 1950 UvKv 16| 384 eMwgUvi 17| ˆ`N¨ 21 wgUvi I c¯ 7 wgUvi

Abkxjbx 4·1

1| $12a^4b$ 2| $30axyz$ 3| $15a^3x^7y$ 4| $-16a^2b^3$ 5| $-20ab^4x^3yz$ 6| $18p^7q^7$
7| $24m^3a^4x^5$ 8| $-21a^5b^3x^{10}y^5$ 9| $10x^2y+15xy^2$ 10| $45x^4y^2-36x^3y^3$
11| $2a^5b^2-3a^3b^4+a^3b^2c^2$ 12| $x^7y-x^4y^4+3x^5y^2z$ 13| $6a^2-5ab-6b^2$
14| a^2-b^2 15| x^4-1 16| $a^3+a^2b+ab^2+b^3$ 17| a^3+b^3
18| $x^3+3x^2y+3xy^2+y^3$ 19| $x^3-3x^2y+3xy^2-y^3$ 20| x^3+5x^2+3x-9
21| $a^4+a^2b^2+b^4$ 22| $a^2+b^2+c^2+2ab+2bc+2ca$ 23| $x^4+x^2y^2+y^4$
24| y^4+y^2+1 26| a^3+b^3

Abkxjbx 4·2

1| $5a^2$ 2| $-8a^3$ 3| $-5a^2x^2$ 4| $-7x^3yz$ 5| $9a^2yz^2$ 6| $11x^2y$
7| $3a-2b$ 8| $4x^3y^2+x^4y$ 9| $-b+3a^4b^4$ 10| $2a^3b-3ab^2$ 11| $5xy+4x-4x^3$
12| $3x^6y^4-2x^2yz+z$ 13| $-8ac+5a^3b^2c^4+3ab^4c^2$ 14| a^2b^2 15| $3x+2$
16| $x-3y$ 17| x^2-xy+y^2 18| $a+2xyz$ 19| $8p^3-12p^2q+18pq^2-27q^3$
20| $-a^2-4a-16$ 21| $x-4y$ 22| x^2+3 23| x^2+x+1 24| a^2-b^2
25| $2ab+3d$ 26| x^2y^2-1 27| $1+x-x^3-x^4$ 28| $x-5ab$ 29| xy
30| abc 31| ax 32| $9x^2-2xy-y^2$ 33| $4a^2+1$ 34| x^2+xy+y^2
35| $a^3+2a^2+a-4.$

Abkxjbx 4·3

1| (N) 2| (M) 3| (N) 4| (M) 5| (K) 6| (L) 7| (K) 8| (1)(N) (2)(M) (3)(N)
9| -21 10| -9 11| 37 12| $x-y-a+b$ 13| $3x+4y-z+b+2c$
14| $2a+2b-2c$ 15| $7b-2a$ 16| $5a-b+11c$ 17| $2a+3b+28c$
18| $-10x+14y-18z$ 19| $3x+2$ 20| $2y-9z$ 21| $14-a-5b$ 22| $3a-6b$
23| $38b-6a$ 24| $a-(b-c+d)$ 25| $a-(b+c-d)-m+(n-x)+y$
26| $7x+\{-5y-(-8z+9)\}$ 27| (K) $15x^2+2x-1$ (L) $75x^3+20x^2-17x+2$ (M) $3x+2$
28| (L) $5x+y-z$ (L) $-x+4y+3z-2, 6x-3y-4z+2$ (M) $-3y-2z-1$
(N) $2x^2-7xy-6xz-3yz+4x+2y-4y^2$

Abkxjbx 5·1

1| $a^2+10a+25$ 2| $25x^2-70x+49$ 3| $9a^2-66axy+121x^2y^2$
4| $25a^4+90a^2m^2+81m^4$ 5| 3025 6| 980100 7| $x^2y^2-12xy^2+36y^2$
8| $a^2x^2-2abxy+b^2y^2$ 9| 9409 10| $4x^2+y^2+z^2+4xy-4xz-2yz$
11| $4a^2+b^2+9c^2-4ab+12ac-6bc$ 12| $x^4+y^4+z^4+2x^2y^2-2x^2z^2-2y^2z^2$
13| $a^2+4b^2+c^2-4ab-2ac+4bc$ 14| $9x^2+4y^2+z^2-12xy+6xz-4yz$
15| $b^2c^2+c^2a^2+a^2b^2+2abc^2+2ab^2c+2a^2bc$ 16| $4a^4+4b^2+c^4+8a^2b-4a^2c^2-4bc^2$
17| 1 18| $81a^2$ 19| $4b^2$ 20| $16x^2$ 21| 81 22| $4c^2d^2$ 23| $9x^2$ 24| $16a^2$
25| 100 26| 100 27| 1 28| 16 32| 12 33| 79

Abkxjbx 5·2

1| $16x^2-9$ 2| $169-144p^2$ 3| a^2b^2-9 4| $100-x^2y^2$ 5| $16x^4-9y^4$
6| $a^2-b^2-c^2-2bc$ 7| x^4+x^2+1 8| $x^2-3ax+\frac{5}{4}a^2$ 9| $\frac{x^2}{16}-\frac{y^2}{9}$
10| $a^8+81x^8+9a^4x^4$ 11| x^4-1 12| $81a^4-b^4$

Abkxjbx 5·3

1| $x(x+y+z+yz)$ 2| $(a+b)(a+c)$ 3| $(ax+by)(bp+aq)$ 4| $(2x+y)(2x-y)$
5| $(3a+2b)(3a-2b)$ 6| $(ab+7y)(ab-7y)$ 7| $(2x+3y)(2x-3y)(4x^2+9y^2)$
8| $(a+x+y)(a-x-y)$ 9| $(3x-5y+8z)(x-y+2z)$ 10| $(3a^2+2a+2)(3a^2-2a+2)$
11| $2(a+8)(a-5)$ 12| $(y+7)(y-13)$ 13| $(p-8)(p-7)$
14| $5a^4(3a^2+x^2)(3a^2-x^2)$ 15| $(a+8)(a-5)$ 16| $(x+y)(x-y)(x^2+y^2+2)$
17| $(x+5)(x+6)$ 18| $(a+b-c)(a-b+c)$ 19| $x^3(12x^2+5a^2)(12x^2-5a^2)$
20| $(2x+3y+4a)(2x+3y-4a)$

Abkxjbx 5·4

1| (N) 2| (L) 3| (K) 4| (M) 5| (K) 6| (M) 7| (N) 8| (K) 9| (L) 10| (K)

13| $3ab^2c$ 14| $5ab$

15| $3a$ 16| $4ax$ 17| $(a+b)$ 18| $(x-y)$ 19| $(x+4)$ 20| $a(a+b)$ 21| $(a+4)$

22| $(x-1)$ 23| $18a^4b^2cd^2$ 24| $30x^2y^3z^4$ 25| $6p^2q^2x^2y^2$ 26| $(b-c)(b+c)^2$

27| $x(x^2+3x+2)$ 28| $5a(9x^2-25y^2)$ 29| $(x+2)(x-5)^2$ 30| $(a+5)(a^2-7a+12)$

31| $(x-3)(x^2-25)$ 32| $x(x+2)(x+5)$

33| (K) $2(2x+1)$ (L) $4x^2-12x+9$ (M) $4x^2+4x-15,\ 9$

34| (K) $a^2-b^2=(a+b)(a-b)$ (L) $(x+5)(x-2)$ (M) $(x+5)$ (N) $(x^4-625)(x-2)$

Abkxjbx 6·1

1| $\frac{b}{ac}$ 2| $\frac{a}{b}$ 3| xyz 4| $\frac{x}{y}$ 5| $\frac{2}{3a}$ 6| $\frac{2a}{1+2b}$ 7| $\frac{1}{2a-3b}$ 8| $\frac{a+2}{a-2}$ 9| $\frac{x-y}{x+y}$

10| $\frac{x-3}{x+4}$ 11| $\frac{a^2}{abc},\frac{ab}{abc}$ 12| $\frac{rx}{pqr},\frac{qy}{pqr}$ 13| $\frac{4nx}{6mn},\frac{9my}{6mn}$ 14| $\frac{a(a+b)}{a^2-b^2},\frac{b(a-b)}{a^2-b^2}$

15| $\frac{(a+2b)x}{a(a^2-4b^2)},\frac{a(a-2b)y^2}{a(a^2-4b^2)}$ 16| $\frac{3a}{a(a^2-4)},\frac{2(a-2)}{a(a^2-4)}$ 17| $\frac{a}{a^2-9},\frac{b(a-3)}{a^2-9}$

18| $\frac{a(a-b)(a-c)}{(a^2-b^2)(a-c)},\frac{b(a+b)(a-c)}{(a^2-b^2)(a-c)},\frac{c(a+b)(a-b)}{(a^2-b^2)(a-c)}$

19| $\frac{a^2(a+b)}{a(a^2-b^2)},\frac{ab(a-b)}{a(a^2-b^2)},\frac{c(a-b)}{a(a^2-b^2)}$ 20| $\frac{2(x+3)}{(x+1)(x-2)(x+3)},\frac{3(x+1)}{(x+1)(x-2)(x+3)}$

Abkxjbx 6·2

1| M 2| L 3| K 4| N 5| L 6| (1) N 6| (2) K 6| (3) L

7| $\frac{3a+2b}{5}$ 8| $\frac{3}{5x}$ 9| $\frac{3bx+2ay}{6ab}$ 10| $\frac{2a(2x-1)}{(x+1)(x-2)}$ 11| $\frac{a^2+4}{a^2-4}$ 12| $\frac{4x-17}{(x+1)(x-5)}$

13| $\frac{2a-4b}{7}$ 14| $\frac{2x-4y}{5a}$ 15| $\frac{ay-2bx}{8xy}$ 16| $\frac{x}{(x+2)(x+3)}$ 17| $\frac{q(r-p)}{pqr}$,

18| $\frac{x(4y-x)}{y(x^2-4y^2)}$ 19| $\frac{a}{a^2-6a+5}$ 20| $\frac{x-3}{x^2-4}$ 21| $\frac{a}{8}$ 22| $\frac{a}{6b}$ 23| $\frac{x^2-y^2+z^2}{xyz}$

24| 0 25| K. $(x+y)(x-4y)$ L. $\frac{x(x-4y)}{(x+y)(x-4y)},\frac{x(x+y)}{(x+y)(x-4y)}$

M. $\frac{2x^2-3xy+y}{(x+y)(x-4y)}$ 26| K. $(a+2)(a-3)$

L. $\frac{a-3}{(a+2)(a+3)(a-3)},\frac{a+3}{(a+2)(a+3)(a-3)}$ M. $\frac{a^2+9}{a(a+2)(a^2-9)}$

Abkxjbx 7·1

1| 3 2| 2 3| $\frac{1}{2}$ 4| $\frac{2}{3}$ 5| 3 6| $\frac{8}{15}$ 7| $\frac{4}{3}$ 8| 4 9| -12 10| 5 11| 1

12| 8 13| -1 14| -6 15| $\frac{19}{3}$ 16| -7 17| 2 18| -1 19| -2 20| 6

Abkxjbx 7·2

1| 10 2| 6 3| 12 4| 9 5| 36 6| 20,21,22 7| 25,30 8| MxZv 52 UvKv, wiZv 58 UvKv, wgZv 70 UvKv 9| LvZv 53 UvKv, Kjg 22 UvKv 10| 240wU 11| wcZvi eqm 30 eQi, c‡Îi eqm 5 eQi 12| wjRvi eqm 12 eQi, wkLvi eqm 18 eQi 13| 37 ivb 14| 25 wK.wg. 15| ˆ`N¨ 15 wgUvi, c¯— 5 wgUvi|

Abkxjbx 7·3

1| L 2| M 3| M 4| K 5| L 6| (1) M 6| (2) (K) 6| (3) (L)

9| (K) 4 (L) -2 (M) 5 (N) -4 (O) 2 10| L. 2 11| K. $(77 - x)$ wK.wg. L. 33

+M. XvKv †_‡K AwiPv : 2 NÈv 34 wgwbU, AwiPv †_‡K XvKv : 1 NÈv 55 wgwbU 30 †m‡KÛ|

Abkxjbx 8

1| K 2| K 3| M 4| (1) L, (2) N, (3) L 5| K

Abkxjbx 9·2

1| M 2| M 3| M 4| N 5| L 6| K 7| M 8| M

Abkxjbx 9·3

1| L 2| L 3| K 4| K 5| L

মুদ্রণে :

২০১৩
শিক্ষাবর্ষ
৭–গণিত

সমৃদ্ধ বাংলাদেশ গড়ে তোলার জন্য যোগ্যতা অর্জন কর
- মাননীয় প্রধানমন্ত্রী শেখ হাসিনা

আলস্য দোষের আকর

২০১০ শিক্ষাবর্ষ থেকে সরকার কর্তৃক বিনামূল্যে বিতরণের জন্য

মুদ্রণে :