

# 1 Surroundings

## 1.1 bashrc

```
1 |oj() {
2 |    g++ -o "/tmp/out" "$1" && "/tmp/out"
3 |}
```

# 2 Data\_Structure

## 2.1 Dynamic\_KD\_tree

```
1 template<typename T,size_t kd>//有kd個維度
2 struct kd_tree{
3     struct point{
4         T d[kd];
5         T dist(const point &x)const{
6             T ret=0;
7             for(size_t i=0;i<kd;++i)ret+=abs(d[i]-x.d[i]);
8             return ret;
9         }
10        bool operator==(const point &p){
11            for(size_t i=0;i<kd;++i)
12                if(d[i]!=p.d[i])return 0;
13            return 1;
14        }
15        bool operator<(const point &b)const{
16            return d[0]<b.d[0];
17        }
18    };
19 private:
20     struct node{
21         node *l,*r;
22         point pid;
23         int s;
24         node(const point &p):l(0),r(0),pid(p),s(1){}
25         ~node(){delete l;delete r;}
26         void up(){s=(l?l->s:0)+1+(r?r->s:0);}
27     }*root;
28     const double alpha,loga;
29     const T INF;//記得要給INF,表示極大值
30     int maxn;
31     struct __cmp{
32         int sort_id;
33         bool operator()(const node*x,const node*y)const{
34             return operator()(x->pid,y->pid);
35         }
36         bool operator()(const point &x,const point &y)const{
37             if(x.d[sort_id]!=y.d[sort_id])
38                 return x.d[sort_id]<y.d[sort_id];
39             for(size_t i=0;i<kd;++i)
40                 if(x.d[i]!=y.d[i])return x.d[i]<y.d[i];
41             return 0;
42         }
43     }cmp;
44     int size(node *o){return o?o->s:0;}
45     vector<node*> A;
46     node* build(int k,int l,int r){
```

```
47     if(l>r) return 0;
48     if(k==kd) k=0;
49     int mid=(l+r)/2;
50     cmp.sort_id = k;
51     nth_element(A.begin()+l,A.begin()+mid,A.begin()+r+1,cmp);
52     node *ret=A[mid];
53     ret->l = build(k+1,l,mid-1);
54     ret->r = build(k+1,mid+1,r);
55     ret->up();
56     return ret;
57 }
58 bool isbad(node*o){
59     return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
60 }
61 void flatten(node *u,typename vector<node*>::iterator &it){
62     if(!u)return;
63     flatten(u->l,it);
64     *it=u;
65     flatten(u->r,++it);
66 }
67 void rebuild(node*&u,int k){
68     if((int)A.size()<u->s)A.resize(u->s);
69     auto it=A.begin();
70     flatten(u,it);
71     u=build(k,0,u->s-1);
72 }
73 bool insert(node*&u,int k,const point &x,int dep){
74     if(!u) return u=new node(x), dep<=0;
75     ++u->s;
76     cmp.sort_id=k;
77     if(insert(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x,dep-1)){
78         if(!isbad(u))return 1;
79         rebuild(u,k);
80     }
81     return 0;
82 }
83 node *findmin(node*o,int k){
84     if(!o)return 0;
85     if(cmp.sort_id==k)return o->l?findmin(o->l,(k+1)%kd):o;
86     node *l=findmin(o->l,(k+1)%kd);
87     node *r=findmin(o->r,(k+1)%kd);
88     if(l&&!r)return cmp(l,o)?l:o;
89     if(!l&&r)return cmp(r,o)?r:o;
90     if(!l&&!r)return o;
91     if(cmp(l,r))return cmp(l,o)?l:o;
92     return cmp(r,o)?r:o;
93 }
94 bool erase(node *&u,int k,const point &x){
95     if(!u)return 0;
96     if(u->pid==x){
97         if(u->r);
98         else if(u->l) u->r=u->l, u->l=0;
99         else return delete(u),u=0, 1;
100        --u->s;
101        cmp.sort_id=k;
102        u->pid=findmin(u->r,(k+1)%kd)->pid;
103        return erase(u->r,(k+1)%kd,u->pid);
104    }
105    cmp.sort_id=k;
106    if(erase(cmp(x,u->pid)?u->l:u->r,(k+1)%kd,x))
107        return --u->s, 1;
108    return 0;
109 }
110 T heuristic(const T h[]){
111     T ret=0;
112     for(size_t i=0;i<kd;++i)ret+=h[i];
```

```
113     return ret;
114 }
115 int qM;
116 priority_queue<pair<T,point>> pQ;
117 void nearest(node *u,int k,const point &x,T *h,T &mndist){
118     if(u==0||heuristic(h)==mndist)return;
119     T dist=u->pid.dist(x),old=h[k];
120     /*mndist=std::min(mndist,dist);*/
121     if(dist<mndist){
122         pQ.push(std::make_pair(dist,u->pid));
123         if((int)pQ.size()==qM+1)
124             mndist=pQ.top().first,pQ.pop();
125     }
126     if(x.d[k]<u->pid.d[k]){
127         nearest(u->l,(k+1)%kd,x,h,mndist);
128         h[k] = abs(x.d[k]-u->pid.d[k]);
129         nearest(u->r,(k+1)%kd,x,h,mndist);
130     }else{
131         nearest(u->r,(k+1)%kd,x,h,mndist);
132         h[k] = abs(x.d[k]-u->pid.d[k]);
133         nearest(u->l,(k+1)%kd,x,h,mndist);
134     }
135     h[k]=old;
136 }
137 vector<point>in_range;
138 void range(node *u,int k,const point&mi,const point&ma){
139     if(!u)return;
140     bool is=1;
141     for(int i=0;i<kd;++i)
142         if(u->pid.d[i]<mi.d[i]||ma.d[i]<u->pid.d[i])
143             { is=0;break; }
144     if(is) in_range.push_back(u->pid);
145     if(mi.d[k]<u->pid.d[k])range(u->l,(k+1)%kd,mi,ma);
146     if(ma.d[k]>u->pid.d[k])range(u->r,(k+1)%kd,mi,ma);
147 }
148 public:
149     kd_tree(const T &INF,double a=0.75):
150         root(0),alpha(a),loga(log2(1.0/a)),INF(INF),maxn(1){}
151     ~kd_tree(){delete root;}
152     void clear(){delete root;root=0,maxn=1;}
153     void build(int n,const point *p){
154         delete root,A.resize(maxn=n);
155         for(int i=0;i<n;++i)A[i]=new node(p[i]);
156         root=build(0,0,n-1);
157     }
158     void insert(const point &x){
159         insert(root,0,x,__lg(size(root))/loga);
160         if(root->s>maxn)maxn=root->s;
161     }
162     bool erase(const point &p){
163         bool d=erase(root,0,p);
164         if(root&&root->s<alpha*maxn)rebuild();
165         return d;
166     }
167     void rebuild(){
168         if(root)rebuild(root,0);
169         maxn=root->s;
170     }
171     T nearest(const point &x,int k){
172         qM=k;
173         T mndist=INF,h[kd]={};
174         nearest(root,0,x,h,mndist);
175         mndist=pQ.top().first;
176         pQ = priority_queue<pair<T,point>>();
177         return mndist;//回傳離x第k近的點的距離
```

```

178 }
179 const vector<point> &range(const point&mi,const point&ma){
180     in_range.clear();
181     range(root,0,mi,ma);
182     return in_range;//回傳介於mi到ma之間的點vector
183 }
184 int size(){return root?root->s:0;}
185 };

```

## 2.2 FenwickTree

```

1 // 區間加值 BIT 只支援 1-based O(Q*log(N)) 閉區間
2 class RangeUpdateBIT {
3     private:
4         ll d[maxn], dd[maxn];
5         ll sum(int i) {
6             ll s = 0, ss = 0;
7             int c = i + 1;
8             while (i > 0) s += d[i], ss += dd[i], i -= i & -i;
9             return c * s - ss;
10        }
11        void add(int i, ll v) {
12            int c = i;
13            while (i < maxn)
14                d[i] += v, dd[i] += c * v, i += i & -i;
15        }
16    public:
17        RangeUpdateBIT() {
18            memset(d, 0, sizeof(d));
19            memset(dd, 0, sizeof(dd));
20        }
21        ll sum(int l, int r) { return sum(r) - sum(l - 1); }
22        void add(int l, int r, ll v) {
23            add(l, v), add(r + 1, -v);
24        }
25    };

```

## 2.3 FenwickTree2D

```

1 /** 支援單點增值和區間查詢，O((A+Q)*log(A))，A
2  * 是矩陣面積。只能用於 1-based **/
3 const int R = 256, C = 256;
4 class BIT2D {
5     private:
6         ll a[R + 1][C + 1];
7         ll sum(int x, int y) {
8             ll ret = 0;
9             for (int i = x; i; i -= (i & -i))
10                 for (int j = y; j; j -= (j & -j))
11                     ret += a[i][j];
12             return ret;
13        }
14    public:
15        // 建立元素都是零的 R*C 大小的矩陣。
16        BIT2D() { memset(a, 0, sizeof(a)); }
17        // 單點增值，注意 1-based。
18        void add(int x, int y, ll v) {
19            for (int i = x; i <= R; i += (i & -i))
20                for (int j = y; j <= C; j += (j & -j))

```

```

21         a[i][j] += v;
22     }
23     // 區間和，注意 1-based。二維都是閉區間。
24     ll sum(int x0, int y0, int x1, int y1) {
25         return sum(x1, y1) - sum(x0 - 1, y1) -
26                sum(x1, y0 - 1) + sum(x0 - 1, y0 - 1);
27     }
28 };

```

## 2.4 HeavyLight

```

1 #include<vector>
2 #define MAXN 100005
3 int siz[MAXN], max_son[MAXN], pa[MAXN], dep[MAXN];
4 int link_top[MAXN], link[MAXN], cnt;
5 vector<int> G[MAXN];
6 void find_max_son(int u){
7     siz[u]=1;
8     max_son[u]=-1;
9     for(auto v:G[u]){
10         if(v==pa[u])continue;
11         pa[v]=u;
12         dep[v]=dep[u]+1;
13         find_max_son(v);
14         if(max_son[u]==-1||siz[v]>siz[max_son[u]])max_son[u]=v;
15         siz[u]+=siz[v];
16     }
17 }
18 void build_link(int u,int top){
19     link[u]=++cnt;
20     link_top[u]=top;
21     if(max_son[u]==-1)return;
22     build_link(max_son[u],top);
23     for(auto v:G[u]){
24         if(v==max_son[u]||v==pa[u])continue;
25         build_link(v,v);
26     }
27 }
28 int find_lca(int a,int b){
29     //求LCA，可以在過程中對區間進行處理
30     int ta=link_top[a],tb=link_top[b];
31     while(ta!=tb){
32         if(dep[ta]<dep[tb]){
33             swap(ta,tb);
34             swap(a,b);
35         }
36         //這裡可以對a所在的鏈做區間處理
37         //區間為(link[ta],link[a])
38         ta=link_top[a=pa[ta]];
39     }
40     //最後a,b會在同一條鏈，若a!=b還要在進行一次區間處理
41     return dep[a]<dep[b]?a:b;
42 }

```

## 2.5 Link\_Cut\_Tree

```

1 struct splay_tree{
2     int ch[2],pa; //子節點跟父母
3     bool rev; //反轉的懶惰標記

```

```

4     splay_tree():pa(0),rev(0){ch[0]=ch[1]=0;}
5 };
6 vector<splay_tree> nd;
7 //有的時候用vector會TLE，要注意
8 //這邊以node[0]作為null節點
9 bool isroot(int x){ //判斷是否為這棵splay tree的根
10     return nd[nd[x].pa].ch[0]!=x&&nd[nd[x].pa].ch[1]!=x;
11 }
12 void down(int x){ //懶惰標記下推
13     if(nd[x].rev){
14         if(nd[x].ch[0])nd[nd[x].ch[0]].rev^=1;
15         if(nd[x].ch[1])nd[nd[x].ch[1]].rev^=1;
16         swap(nd[x].ch[0],nd[x].ch[1]);
17         nd[x].rev=0;
18     }
19 }
20 void push_down(int x){ //所有祖先懶惰標記下推
21     if(!isroot(x))push_down(nd[x].pa);
22     down(x);
23 }
24 void up(int x){ //將子節點的資訊向上更新
25 void rotate(int x){ //旋轉，會自行判斷轉的方向
26     int y=nd[x].pa,z=nd[y].pa,d=(nd[y].ch[1]==x);
27     nd[x].pa=z;
28     if(!isroot(y))nd[z].ch[nd[z].ch[1]==y]=x;
29     nd[y].ch[d]=nd[x].ch[d^1];
30     nd[nd[y].ch[d]].pa=y;
31     nd[y].pa=x,nd[x].ch[d^1]=y;
32     up(y),up(x);
33 }
34 void splay(int x){ //將x伸展到splay tree的根
35     push_down(x);
36     while(!isroot(x)){
37         int y=nd[x].pa;
38         if(!isroot(y)){
39             int z=nd[y].pa;
40             if((nd[z].ch[0]==y)^(nd[y].ch[0]==x))rotate(y);
41             else rotate(x);
42         }
43         rotate(x);
44     }
45 }
46 int access(int x){
47     int last=0;
48     while(x){
49         splay(x);
50         nd[x].ch[1]=last;
51         up(x);
52         last=x;
53         x=nd[x].pa;
54     }
55     return last; //access後splay tree的根
56 }
57 void access(int x,bool is=0){ //is=0就是一般的access
58     int last=0;
59     while(x){
60         splay(x);
61         if(is&&!nd[x].pa){
62             //printf("%d\n",max(nd[last].ma,nd[nd[x].ch[1]].ma));
63         }
64         nd[x].ch[1]=last;
65         up(x);
66         last=x;
67         x=nd[x].pa;

```

```

68 }
69 }
70 void query_edge(int u,int v){
71     access(u);
72     access(v,1);
73 }
74 void make_root(int x){
75     access(x),splay(x);
76     nd[x].rev^=1;
77 }
78 void make_root(int x){
79     nd[access(x)].rev=1;
80     splay(x);
81 }
82 void cut(int x,int y){
83     make_root(x);
84     access(y);
85     splay(y);
86     nd[y].ch[0]=0;
87     nd[x].pa=0;
88 }
89 void cut_parents(int x){
90     access(x);
91     splay(x);
92     nd[nd[x].ch[0]].pa=0;
93     nd[x].ch[0]=0;
94 }
95 void link(int x,int y){
96     make_root(x);
97     nd[x].pa=y;
98 }
99 int find_root(int x){
100     x=access(x);
101     while(nd[x].ch[0])x=nd[x].ch[0];
102     splay(x);
103     return x;
104 }
105 int query(int u,int v){
106     //傳回uv路徑splay tree的根結點
107     //這種寫法無法求LCA
108     make_root(u);
109     return access(v);
110 }
111 int query_lca(int u,int v){
112     //假設求鏈上點權的總和，sum是子樹的權重和，data是節點的權重
113     access(u);
114     int lca=access(v);
115     splay(u);
116     if(u==lca){
117         //return nd[lca].data+nd[nd[lca].ch[1]].sum
118     }else{
119         //return nd[lca].data+nd[nd[lca].ch[1]].sum+nd[u].sum
120     }
121 }
122 struct EDGE{
123     int a,b,w;
124 }e[10005];
125 int n;
126 vector<pair<int,int>> G[10005];
127 //first表示子節點，second表示邊的編號
128 int pa[10005],edge_node[10005];
129 //pa是父母節點，暫存用的，edge_node是每個編被存在哪個點裡面的
    陣列
130 void bfs(int root){

```

```

131 //在建構的時候把每個點都設成一個splay tree
132 queue<int > q;
133 for(int i=1;i<=n;++i)pa[i]=0;
134 q.push(root);
135 while(q.size()){
136     int u=q.front();
137     q.pop();
138     for(auto P:G[u]){
139         int v=P.first;
140         if(v!=pa[u]){
141             pa[v]=u;
142             nd[v].pa=u;
143             nd[v].data=e[P.second].w;
144             edge_node[P.second]=v;
145             up(v);
146             q.push(v);
147         }
148     }
149 }
150 }
151 void change(int x,int b){
152     splay(x);
153     //nd[x].data=b;
154     up(x);
155 }

```

## 2.6 MaxSumSegmentTree

```

1 /** 計算最大子區間連續和的線段樹，限定 1-based 。
2  * 複雜度 O(Q*log(N)) */
3 #define ls i << 1
4 #define rs i << 1 | 1
5 class MaxSumSegmentTree {
6     private:
7     struct node {
8         ll lss, rss, ss, ans;
9         void set(ll v) { lss = rss = ss = ans = v; }
10    };
11    int n;
12    vector<node> a; // 萬萬不可用普通陣列，要用 vector
13    vector<ll> z;
14    void pull(int i) {
15        a[i].ss = a[ls].ss + a[rs].ss;
16        a[i].lss = max(a[ls].lss, a[ls].ss + a[rs].lss);
17        a[i].rss = max(a[rs].rss, a[rs].ss + a[ls].rss);
18        a[i].ans = max(max(a[ls].ans, a[rs].ans),
19                        a[ls].rss + a[rs].lss);
20    }
21    void build(int i, int l, int r) {
22        if (l == r) return a[i].set(z[l]), void();
23        int m = (l + r) >> 1;
24        build(ls, l, m), build(rs, m + 1, r), pull(i);
25    }
26    void set(int i, int l, int r, int q, ll v) {
27        if (l == r) return a[i].set(v), void();
28        int m = (l + r) >> 1;
29        if (q <= m) set(ls, l, m, q, v);
30        else set(rs, m + 1, r, q, v);
31        pull(i);
32    }
33    node query(int i, int l, int r, int ql, int qr) {
34        if (ql <= l && r <= qr) return a[i];

```

```

35     int m = (l + r) >> 1;
36     if (qr <= m) return query(ls, l, m, ql, qr);
37     if (m < ql) return query(rs, m + 1, r, ql, qr);
38     node lo = query(ls, l, m, ql, qr),
39           ro = query(rs, m + 1, r, ql, qr), ans;
40     ans.ss = lo.ss + ro.ss;
41     ans.lss = max(lo.lss, lo.ss + ro.lss);
42     ans.rss = max(ro.rss, ro.ss + lo.rss);
43     ans.ans = max(max(lo.ans, ro.ans), lo.rss + ro.lss);
44     return ans;
45 }
46
47 public:
48     MaxSumSegmentTree(int n) : n(n) {
49         a.resize(n << 2), z.resize(n << 2);
50         build(1, 1, n);
51     }
52     // 單點設值。限定 1-based 。
53     inline void set(int i, ll v) { set(1, 1, n, i, v); }
54     // 問必區間 [l, r] 的最大子區間連續和。限定 1-based 。
55     inline ll query(int l, int r) {
56         return query(1, 1, n, l, r).ans;
57     }
58 };

```

## 2.7 PersistentSegmentTree

```

1 int a[maxn], b[maxn], root[maxn], cnt;
2 struct node {
3     int sum, L_son, R_son;
4 } tree[maxn << 5];
5 int create(int _sum, int _L_son, int _R_son) {
6     int idx = ++cnt;
7     tree[idx].sum = _sum, tree[idx].L_son = _L_son, tree[idx]
8     ].R_son = _R_son;
9     return idx;
10 }
11 void Insert(int &root, int pre_rt, int pos, int L, int R) {
12     root = create(tree[pre_rt].sum+1, tree[pre_rt].L_son,
13                 tree[pre_rt].R_son);
14     if(L==R) return;
15     int M = (L+R)>>1;
16     if(pos<=M) Insert(tree[root].L_son, tree[pre_rt].L_son,
17                     pos, L, M);
18     else Insert(tree[root].R_son, tree[pre_rt].R_son, pos, M
19                 +1, R);
20 }
21 int query(int L_id, int R_id, int L, int R, int K) {
22     if(L==R) return L;
23     int M = (L+R)>>1;
24     int s = tree[tree[R_id].L_son].sum - tree[tree[L_id].
25         L_son].sum;
26     if(K<=s) return query(tree[L_id].L_son, tree[R_id].L_son,
27                         L, M, K);
28     return query(tree[L_id].R_son, tree[R_id].R_son, M+1, R,
29                 K-s);
30 }
31 int main() {
32     int n,m; cin >> n >> m
33     for(int i=1; i<=n; i++) {
34         cin >> a[i]; b[i] = a[i];
35     } sort(b+1,b+1+n); //離散化

```

```

29 int b_sz = unique(b+1, b+1+n) - (b+1);
30 cnt = root[0] = 0;
31 for(int i=1; i<=n; i++) {
32     int pos = lower_bound(b+1, b+1+b_sz, a[i]) - b;
33     Insert(root[i], root[i-1], pos, 1, b_sz);
34 }
35 while(m--) {
36     int l, r, k; cin >> l >> r >> k;
37     int pos = query(root[l-1], root[r], 1, b_sz, k);
38     cout << b[pos] << endl;
39 } return 0;
40 }

```

## 2.8 RangeUpdateSegmentTree

```

1 //閉區間，1-based
2 #define ls i << 1
3 #define rs i << 1 | 1
4 const ll rr = 0x6891139; // 亂數，若跟題目碰撞會吃 WA 或 RE
5 class RangeUpdateSegmentTree {
6 private:
7     struct node { //s : sum, x : max
8         int l, r; ll adt = 0, stt = rr, s = 0, x = 0;
9     };
10    vector<node> a; // 萬萬不可以用普通陣列，要用 vector
11    void push(int i) {
12        if (a[i].stt != rr) {
13            a[ls].stt = a[rs].stt = a[i].stt;
14            a[ls].adt = a[rs].adt = 0;
15            a[ls].x = a[rs].x = a[i].stt;
16            a[ls].s = (a[ls].r - a[ls].l + 1) * a[i].stt;
17            a[rs].s = (a[rs].r - a[rs].l + 1) * a[i].stt;
18            a[i].stt = rr;
19        }
20        if (a[i].adt) {
21            a[ls].adt += a[i].adt, a[rs].adt += a[i].adt;
22            a[ls].x += a[i].adt, a[rs].x += a[i].adt;
23            a[ls].s += a[i].adt * (a[ls].r - a[ls].l + 1);
24            a[rs].s += a[i].adt * (a[rs].r - a[rs].l + 1);
25            a[i].adt = 0;
26        }
27    }
28    void pull(int i) {
29        a[i].s = a[ls].s + a[rs].s;
30        a[i].x = max(a[ls].x, a[rs].x);
31    }
32    void build(int l, int r, int i) {
33        a[i].l = l, a[i].r = r;
34        if (l == r) return;
35        int mid = (l + r) >> 1;
36        build(l, mid, ls), build(mid + 1, r, rs);
37    }
38 public:
39    RangeUpdateSegmentTree(int n) : a(n << 2) {
40        build(1, n, 1);
41    }
42    void set(int l, int r, ll val, int i = 1) {
43        if (a[i].l >= l && a[i].r <= r) {
44            a[i].s = val * (a[i].r - a[i].l + 1);
45            a[i].x = a[i].stt = val;
46            a[i].adt = 0;
47            return;

```

```

48        }
49        push(i);
50        int mid = (a[i].l + a[i].r) >> 1;
51        if (l <= mid) set(l, r, val, ls);
52        if (r > mid) set(l, r, val, rs);
53        pull(i);
54    }
55    void add(int l, int r, ll val, int i = 1) {
56        if (a[i].l >= l && a[i].r <= r) {
57            a[i].s += val * (a[i].r - a[i].l + 1);
58            a[i].x += val;
59            a[i].adt += val;
60            return;
61        }
62        push(i);
63        int mid = (a[i].l + a[i].r) >> 1;
64        if (l <= mid) add(l, r, val, ls);
65        if (r > mid) add(l, r, val, rs);
66        pull(i);
67    }
68    ll maxx(int l, int r, int i = 1) {
69        if (l <= a[i].l && a[i].r <= r) return a[i].x;
70        push(i);
71        ll ret = -9e18;
72        int mid = (a[i].l + a[i].r) >> 1;
73        if (l <= mid) ret = max(ret, maxx(l, r, ls));
74        if (r > mid) ret = max(ret, maxx(l, r, rs));
75        pull(i);
76        return ret;
77    }
78    ll sum(int l, int r, int i = 1) {
79        if (l <= a[i].l && a[i].r <= r) return a[i].s;
80        push(i);
81        ll ret = 0;
82        int mid = (a[i].l + a[i].r) >> 1;
83        if (l <= mid) ret += sum(l, r, ls);
84        if (r > mid) ret += sum(l, r, rs);
85        pull(i);
86        return ret;
87    }
88 };

```

## 2.9 SparseTable

```

1 #define flg(a) floor(log2(a))
2 struct SparseTable {
3     vector<vector<ll>> a;
4     SparseTable(vector<ll>& data) {
5         int n = data.size();
6         a.assign(flg(n) + 1, vector<ll>(n));
7         a[0] = data;
8         for (int i = 1; (1 << i) <= n; i++)
9             for (int j = 0, k = n - (1 << i); j <= k; j++)
10                 a[i][j] = max(a[i-1][j],
11                               a[i-1][j + (1 << (i-1))]);
12     }
13     ll maxx(int l, int r) { // [l, r], 0/1-based
14         int k = flg(r - l + 1);
15         return max(a[k][l], a[k][r - (1 << k) + 1]);
16     }
17 };

```

## 2.10 Treap

```

1 // 區間加值、反轉、rotate、刪除、插入元素、求區間
2 // srand(time(0))
3 class Treap {
4 private:
5     struct Node {
6         int pri = rand(), size = 1;
7         ll val, mn, inc = 0; bool rev = 0;
8         Node *lc = 0, *rc = 0;
9         Node(ll v) { val = mn = v; }
10    };
11    Node* root = 0;
12    void rev(Node* t) {
13        if (!t) return;
14        swap(t->lc, t->rc), t->rev ^= 1;
15    }
16    void update(Node* t, ll v) {
17        if (!t) return;
18        t->val += v, t->inc += v, t->mn += v;
19    }
20    void push(Node* t) {
21        if (t->rev) rev(t->lc), rev(t->rc), t->rev = 0;
22        update(t->lc, t->inc), update(t->rc, t->inc);
23        t->inc = 0;
24    }
25    void pull(Node* t) {
26        t->size = 1 + size(t->lc) + size(t->rc);
27        t->mn = t->val;
28        if (t->lc) t->mn = min(t->mn, t->lc->mn);
29        if (t->rc) t->mn = min(t->mn, t->rc->mn);
30    }
31    void discard(Node* t) { // 看要不要釋放記憶體
32        if (!t) return;
33        discard(t->lc), discard(t->rc);
34        delete t;
35    }
36    void split(Node* t, Node*& a, Node*& b, int k) {
37        if (!t) return a = b = 0, void();
38        push(t);
39        if (size(t->lc) < k) {
40            a = t;
41            split(t->rc, a->rc, b, k - size(t->lc) - 1);
42            pull(a);
43        } else {
44            b = t;
45            split(t->lc, a, b->lc, k);
46            pull(b);
47        }
48    }
49    Node* merge(Node* a, Node* b) {
50        if (!a || !b) return a ? a : b;
51        if (a->pri > b->pri) {
52            push(a);
53            a->rc = merge(a->rc, b);
54            pull(a);
55            return a;
56        } else {
57            push(b);
58            b->lc = merge(a, b->lc);
59            pull(b);
60            return b;
61        }
62    }
63    inline int size(Node* t) { return t ? t->size : 0; }

```

```

64 public:
65 int size() { return size(root); }
66 void add(int l, int r, ll val) {
67     Node *a, *b, *c, *d;
68     split(root, a, b, r);
69     split(a, c, d, l - 1);
70     update(d, val);
71     root = merge(merge(c, d), b);
72 }
73 // 反轉區間 [l, r]
74 void reverse(int l, int r) {
75     Node *a, *b, *c, *d;
76     split(root, a, b, r);
77     split(a, c, d, l - 1);
78     swap(d->lc, d->rc);
79     d->rev ^= 1;
80     root = merge(merge(c, d), b);
81 }
82 // 區間 [l, r] 向右 rotate k 次, k < 0 表向左 rotate
83 void rotate(int l, int r, int k) {
84     int len = r - l + 1;
85     Node *a, *b, *c, *d, *e, *f;
86     split(root, a, b, r);
87     split(a, c, d, l - 1);
88     k = (k + len) % len;
89     split(d, e, f, len - k);
90     root = merge(merge(c, merge(f, e)), b);
91 }
92 // 插入一個元素 val 使其 index = i <= size
93 void insert(int i, ll val) {
94     if (i == size() + 1) {
95         push_back(val); return;
96     }
97     assert(i <= size());
98     Node *a, *b;
99     split(root, a, b, i - 1);
100     root = merge(merge(a, new Node(val)), b);
101 }
102 void push_back(ll val) {
103     root = merge(root, new Node(val));
104 }
105 void remove(int l, int r) {
106     int len = r - l + 1;
107     Node *a, *b, *c, *d;
108     split(root, a, b, l - 1);
109     split(b, c, d, len);
110     discard(c); // 看你要不要釋放記憶體
111     root = merge(a, d);
112 }
113 ll minn(int l, int r) {
114     Node *a, *b, *c, *d;
115     split(root, a, b, r);
116     split(a, c, d, l - 1);
117     int ans = d->mn;
118     root = merge(merge(c, d), b);
119     return ans;
120 }
121 };

```

## 3 Flow\_Matching

### 3.1 Dinic

```

1 class Dinic {
2 private:
3     struct edge { int d, r; ll c; };
4     vector<vector<edge>> adj; vector<int> lv, ve; int n;
5     bool mklv(int s, int d) {
6         lv.assign(n, -1); lv[s] = 0; queue<int> q({s});
7         while (!q.empty()) {
8             int v = q.front(); q.pop();
9             for (auto& e : adj[v]) {
10                 if (e.c == 0 || lv[e.d] != -1) continue;
11                 lv[e.d] = lv[v] + 1, q.push(e.d);
12             }
13         }
14         return lv[d] > 0;
15     }
16     ll aug(int v, ll f, int d) {
17         if (v == d) return f;
18         for (; ve[v] < adj[v].size(); ve[v]++) {
19             auto& e = adj[v][ve[v]];
20             if (lv[e.d] != lv[v] + 1 || !e.c) continue;
21             ll sent = aug(e.d, min(f, e.c), d);
22             if (sent > 0) {
23                 e.c -= sent, adj[e.d][e.r].c += sent;
24                 return sent;
25             }
26         }
27         return 0;
28     }
29 public:
30     // 建空圖。n 為節點數量 (含 source 和 sink)。
31     Dinic(int n) : n(n + 1) { clear(); }
32     void clear() { adj.assign(n, {}); }
33     void add_edge(int src, int dst, ll cap) {
34         edge ss{dst, (int)adj[dst].size(), cap};
35         edge dd{src, (int)adj[src].size(), 0};
36         adj[src].push_back(ss), adj[dst].push_back(dd);
37     }
38     ll max_flow(int s, int d) {
39         ll ret = 0;
40         while (mklv(s, d)) {
41             ve.assign(n, 0);
42             while (ll f = aug(s, inf, d)) ret += f;
43         }
44         return ret;
45     }
46 };

```

### 3.2 Ford\_Fulkerson

```

1 const int maxn = 1e5 + 10, INF = 1e9;
2 const long long INF64 = 1e18;
3 struct edge { int to, cap, rev; };
4 vector<edge> G[maxn];
5 int n, m, s, t, a, b, c;
6 bool vis[maxn];
7 int dfs(int v, int t, int f) {

```

```

8     cout << v << ' ' << t << ' ' << f << '\n';
9     if (v == t) return f;
10    vis[v] = true;
11    for (edge &e : G[v]) {
12        if (!vis[e.to] && e.cap > 0) {
13            int d = dfs(e.to, t, min(f, e.cap));
14            if (d > 0) {
15                e.cap -= d, G[e.to][e.rev].cap += d;
16                return d;
17            }
18        }
19    }
20    return 0;
21 }
22 int ford_fulkerson(int s, int t) {
23     int flow = 0, f;
24     for (int i = 0; i < n; i++) {
25         cout << i << " : ";
26         for (edge e : G[i])
27             cout << '(' << e.to << ', ' << e.cap << ')' << ' '
28             ;
29         cout << '\n';
30     }
31     do {
32         memset(vis, false, sizeof(vis));
33         f = dfs(s, t, INF);
34         for (int i = 0; i < n; i++) {
35             cout << i << " : ";
36             for (edge e : G[i])
37                 cout << '(' << e.to << ', ' << e.cap << ')' << ' '
38                 ;
39             cout << '\n';
40         }
41         flow += f;
42     } while (f > 0);
43     return flow;
44 }
45 void init(int n) {
46     for (int i = 0; i < n; i++) G[i].clear();
47 }
48 int main() {
49     cin >> n >> m >> s >> t;
50     init(n);
51     while (m--) {
52         cin >> a >> b >> c;
53         G[a].push_back((edge){b, c, (int)G[b].size()});
54         G[b].push_back((edge){a, 0, (int)G[a].size() - 1});
55     }
56     cout << ford_fulkerson(s, t) << '\n';
57     return 0;
58 }

```

### 3.3 Hopcroft\_Karp

```

1 // 匈牙利算法的優化，二分圖最大匹配 O(EV)
2 int n, m, vis[maxn], level[maxn], pr[maxn], pr2[maxn];
3 vector<int> edge[maxn]; // for Left
4 bool dfs(int u) {
5     vis[u] = true;
6     for (vector<int>::iterator it = edge[u].begin();
7         it != edge[u].end(); ++it) {
8         int v = pr2[*it];

```

```

9     if (v == -1 ||
10         (!vis[v] && level[u] < level[v] && dfs(v))) {
11         pr[u] = *it, pr2[*it] = u;
12         return true;
13     }
14     } return false;
15 }
16 int hopcroftKarp() {
17     memset(pr, -1, sizeof(pr));
18     memset(pr2, -1, sizeof(pr2));
19     for (int match = 0;;) {
20         queue<int> Q;
21         for (int i = 1; i <= n; ++i) {
22             if (pr[i] == -1) level[i] = 0, Q.push(i);
23             else level[i] = -1;
24         }
25         while (!Q.empty()) {
26             int u = Q.front(); Q.pop();
27             for (vector<int>::iterator it = edge[u].begin();
28                 it != edge[u].end(); ++it) {
29                 int v = pr2[*it];
30                 if (v != -1 && level[v] < 0)
31                     level[v] = level[u] + 1, Q.push(v);
32             }
33         }
34         for (int i = 1; i <= n; ++i) vis[i] = false;
35         int d = 0;
36         for (int i = 1; i <= n; ++i)
37             if (pr[i] == -1 && dfs(i)) ++d;
38         if (d == 0) return match;
39         match += d;
40     }
41 }

```

### 3.4 Hungarian

```

1 // Time: O(VE)
2 const int INF = 2e9;
3 const int N = ?; // 男女總人數; 女 id: 0 ~ p, 男 id: p
4 // +1 ~ N-1
5 int vis[N], rnd, m[N]; // 跑完匈牙利後配對結果儲存於此, -1
6 // 表示人醜
7 vector<int> g[N]; // 關係表
8 int dfs(int s) {
9     for (int x : g[s]) {
10         if (vis[x]) continue;
11         vis[x] = 1;
12         if (m[x] == -1 || dfs(m[x])) {
13             m[x] = s, m[s] = x;
14             return 1;
15         }
16     } return 0;
17 }
18 int hungarian(int p) { // p : 女性人數
19     memset(m, -1, sizeof(m));
20     int c = 0;
21     for (int i = 0; i < p; ++i) {
22         if (m[i] == -1) {
23             memset(vis, 0, sizeof(vis));
24             c += dfs(i);
25         }
26     } return c; // 成功結婚對數

```

25 }

### 3.5 KM

```

1 /* 時間複雜度 O(N^3)
2 求完美匹配中的最大權匹配
3 如果不存在完美匹配, 求最大匹配
4 如果存在數個最大匹配, 求數個最大匹配當中最大權匹配 */
5 const ll INF = 5e18;
6 const int N = ?; // maxn
7 int n; // count of vertex (one side)
8 ll g[N][N]; // weights
9 class KM {
10 private:
11     ll lx[N], ly[N], s[N];
12     int px[N], py[N], m[N], p[N];
13     void adj(int y) { // 把增廣路上所有邊反轉
14         m[y] = py[y];
15         if (px[m[y]] != -2)
16             adj(px[m[y]]);
17     }
18     bool dfs(int x) { // DFS找增廣路
19         for (int y = 0; y < n; ++y) {
20             if (py[y] != -1) continue;
21             ll t = lx[x] + ly[y] - g[x][y];
22             if (t == 0) {
23                 py[y] = x;
24                 if (m[y] == -1) {
25                     adj(y);
26                     return 1;
27                 }
28                 if (px[m[y]] != -1) continue;
29                 px[m[y]] = y;
30                 if (dfs(m[y])) return 1;
31             } else if (s[y] > t) {
32                 s[y] = t, p[y] = x;
33             }
34         } return 0;
35     }
36 public:
37     ll max_weight() {
38         memset(ly, 0, sizeof(ly));
39         memset(m, -1, sizeof(m));
40         for (int x = 0; x < n; ++x) {
41             lx[x] = -INF;
42             for (int y = 0; y < n; ++y)
43                 lx[x] = max(lx[x], g[x][y]);
44         }
45         for (int x = 0; x < n; ++x) {
46             for (int y = 0; y < n; ++y) s[y] = INF;
47             memset(px, -1, sizeof(px));
48             memset(py, -1, sizeof(py));
49             px[x] = -2;
50             if (dfs(x)) continue;
51             bool flag = 1;
52             while (flag) {
53                 ll cut = INF;
54                 for (int y = 0; y < n; ++y)
55                     if (py[y] == -1 && cut > s[y]) cut = s[y];
56             }
57         }

```

```

58     for (int j = 0; j < n; ++j) {
59         if (px[j] != -1) lx[j] -= cut;
60         if (py[j] != -1) ly[j] += cut;
61         else s[j] -= cut;
62     }
63     for (int y = 0; y < n; ++y) {
64         if (py[y] == -1 && s[y] == 0) {
65             py[y] = p[y];
66             if (m[y] == -1) {
67                 adj(y);
68                 flag = 0;
69                 break;
70             }
71             px[m[y]] = y;
72             if (dfs(m[y])) {
73                 flag = 0;
74                 break;
75             }
76         }
77     }
78     }
79     ll ans = 0;
80     for (int y = 0; y < n; ++y)
81         if (g[m[y]][y] != -INF) ans += g[m[y]][y];
82     return ans;
83 }
84 };
85

```

### 3.6 Min\_Cost\_Max\_Flow

```

1 class MCMF { // 0/1-based
2 private:
3     struct edge { int to, r; ll rest, c; };
4     int n; ll f = 0, c = 0;
5     vector<vector<edge>> g;
6     vector<int> pre, prel;
7     bool run(int s, int t) {
8         vector<ll> dis(n, inf); vector<bool> vis(n);
9         dis[s] = 0; queue<int> q; q.push(s);
10        while (q.size()) {
11            int u = q.front(); q.pop(); vis[u] = 0;
12            for (int i = 0; i < g[u].size(); ++i) {
13                int v = g[u][i].to; ll w = g[u][i].c;
14                if (g[u][i].rest <= 0 ||
15                    dis[v] <= dis[u] + w) continue;
16                pre[v] = u, prel[v] = i;
17                dis[v] = dis[u] + w;
18                if (!vis[v]) vis[v] = 1, q.push(v);
19            }
20        }
21        if (dis[t] == inf) return 0;
22        ll tf = inf;
23        for (int v = t, u, l; v != s; v = u) {
24            u = pre[v], l = prel[v];
25            tf = min(tf, g[u][l].rest);
26        }
27        for (int v = t, u, l; v != s; v = u) {
28            u = pre[v], l = prel[v], g[u][l].rest -= tf;
29            g[v][g[u][l].r].rest += tf;
30        }
31        c += tf * dis[t], f += tf;
32        return 1;

```



```

33 }
34 public:
35 MCMF(int n) // 建空圖, n 節點數 (含 src 和 sink)
36 : n(n + 1), g(n + 1), pre(n + 1), prel(n + 1) {}
37 // 加有向邊 u->v, cap 容量 cost 成本
38 void add_edge(int u, int v, ll cap, ll cost) {
39     g[u].push_back({v, (int)g[v].size(), cap, cost});
40     g[v].push_back({u, (int)g[u].size() - 1, 0, -cost});
41 }
42 pair<ll, ll> query(int src, int sink) {
43     while (run(src, sink));
44     return {f, c}; // {min cost, max flow}
45 }
46 };

```

### 3.7 SW\_MinCut

```

1 // all pair min cut, global min cut
2 struct SW { // O(V^3)
3     static const int MXN = 514;
4     int n, vst[MXN], del[MXN];
5     int edge[MXN][MXN], wei[MXN];
6     void init(int _n){
7         n = _n; FZ(edge); FZ(del);
8     }
9     void addEdge(int u, int v, int w) {
10         edge[u][v] += w; edge[v][u] += w;
11     }
12     void search(int &s, int &t) {
13         FZ(vst); FZ(wei);
14         s = t = -1;
15         while (true){
16             int mx=-1, cur=0;
17             for (int i=0; i<n; i++)
18                 if (!del[i] && !vst[i] && mx<wei[i])
19                     cur = i, mx = wei[i];
20             if (mx == -1) break;
21             vst[cur] = 1;
22             s = t; t = cur;
23             for (int i=0; i<n; i++)
24                 if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
25         }
26     }
27     int solve() {
28         int res = 2147483647;
29         for (int i=0, x, y; i<n-1; i++) {
30             search(x, y);
31             res = min(res, wei[y]);
32             del[y] = 1;
33             for (int j=0; j<n; j++)
34                 edge[x][j] = (edge[j][x] += edge[y][j]);
35         }
36         return res;
37     }
38 } graph;

```

## 4 Geometry

### 4.1 ClosestPair

```

1 typedef pair<ll, ll> pii;
2 #define x first
3 #define y second
4 ll dd(const pii& a, const pii& b) {
5     ll dx = a.x - b.x, dy = a.y - b.y;
6     return dx * dx + dy * dy;
7 }
8 const ll inf = 1e18;
9 ll dac(vector<pii>& p, int l, int r) {
10     if (l >= r) return inf;
11     int m = (l + r) / 2;
12     ll d = min(dac(p, l, m), dac(p, m + 1, r));
13     vector<pii> t;
14     for (int i = m; i >= l && p[m].x - p[i].x < d; i--)
15         t.push_back(p[i]);
16     for (int i = m + 1; i <= r && p[i].x - p[m].x < d; i++)
17         t.push_back(p[i]);
18     sort(t.begin(), t.end());
19     [(pii& a, pii& b) { return a.y < b.y; }]);
20     int n = t.size();
21     for (int i = 0; i < n - 1; i++)
22         for (int j = 1; j < 4 && i + j < n; j++)
23             // 這裡可以知道是哪兩點是最小點對
24             d = min(d, dd(t[i], t[i + j]));
25     return d;
26 }
27 // 給一堆點, 求最近點對的距離「的平方」。
28 ll closest_pair(vector<pii>& pp) {
29     sort(pp.begin(), pp.end());
30     return dac(pp, 0, pp.size() - 1);
31 }

```

### 4.2 Geometry

```

1 //Copy from linkela
2 const double PI=atan2(0.0,-1.0);
3 template<typename T>
4 struct point{
5     T x,y;
6     point(){}
7     point(const T&x,const T&y):x(x),y(y){}
8     point operator+(const point &b)const{
9         return point(x+b.x,y+b.y); }
10    point operator-(const point &b)const{
11        return point(x-b.x,y-b.y); }
12    point operator*(const T &b)const{
13        return point(x*b,y*b); }
14    point operator/(const T &b)const{
15        return point(x/b,y/b); }
16    bool operator==(const point &b)const{
17        return x==b.x&&y==b.y; }
18    T dot(const point &b)const{
19        return x*b.x+y*b.y; }
20    T cross(const point &b)const{
21        return x*b.y-y*b.x; }
22    point normal()const{//求法向量

```

```

23     return point(-y,x); }
24    T abs2()const{//向量長度的平方
25     return dot(*this); }
26    T rad(const point &b)const{//兩向量的弧度
27    return fabs(atan2(fabs(cross(b)),dot(b))); }
28    T getA()const{//對x軸的弧度
29    T A=atan2(y,x); //超過180度會變負的
30    if(A<=-PI/2)A+=PI*2;
31    return A;
32 }
33 };
34 template<typename T>
35 struct line{
36     line(){}
37     point<T> p1,p2;
38     T a,b,c;//ax+by+c=0
39     line(const point<T>&x,const point<T>&y):p1(x),p2(y){}
40     void pton()const{//轉成一般式
41         a=p1.y-p2.y;
42         b=p2.x-p1.x;
43         c=-a*p1.x-b*p1.y;
44     }
45     T ori(const point<T> &p)const{//點和有向直線的關係, >0左
46         //邊, =0在線上<0右邊
47         return (p2-p1).cross(p-p1);
48     }
49     T btw(const point<T> &p)const{//點投影落在線段上<=0
50         return (p1-p).dot(p2-p);
51     }
52     bool point_on_segment(const point<T>&p)const{//點是否在線段
53         //上
54         return ori(p)==0&&btw(p)<=0;
55     }
56     T dis2(const point<T> &p,bool is_segment=0)const{//點跟直線
57         //線段的距離平方
58         point<T> v=p2-p1,v1=p-p1;
59         if(is_segment){
60             point<T> v2=p-p2;
61             if(v.dot(v1)<=0)return v1.abs2();
62             if(v.dot(v2)>=0)return v2.abs2();
63         }
64         T tmp=v.cross(v1);
65         return tmp*tmp/v.abs2();
66     }
67     T seg_dis2(const line<T> &l)const{//兩線段距離平方
68         return min({dis2(l.p1,1),dis2(l.p2,1),l.dis2(p1,1),l.dis2(p2,1)});
69     }
70     point<T> projection(const point<T> &p)const{//點對直線的投
71         //影
72         point<T> n=(p2-p1).normal();
73         return p-n*(p-p1).dot(n)/n.abs2();
74     }
75     point<T> mirror(const point<T> &p)const{
76         //點對直線的鏡射, 要先呼叫pton轉成一般式
77         point<T> R;
78         T d=a*a+b*b;
79         R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
80         R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
81         return R;
82     }
83     bool equal(const line &l)const{//直線相等
84         return ori(l.p1)==0&&ori(l.p2)==0;
85     }

```

```

81 }
82 bool parallel(const line &l) const {
83     return (p1-p2).cross(l.p1-l.p2)==0;
84 }
85 bool cross_seg(const line &l) const {
86     return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;
87     //直線是否交線段
88 }
89 int line_intersect(const line &l) const { //直線相交情況, -1無
90     //限多點、1交於一點、0不相交
91     return parallel(l)?(ori(l.p1)==0?-1:0):1;
92 }
93 int seg_intersect(const line &l) const {
94     T c1=ori(l.p1), c2=ori(l.p2);
95     T c3=l.ori(p1), c4=l.ori(p2);
96     if(c1==0&&c2==0) { //共線
97         bool b1=btw(l.p1)>=0, b2=btw(l.p2)>=0;
98         T a3=1.btw(p1), a4=1.btw(p2);
99         if(b1&&b2&&a3==0&&a4==0) return 2;
100         if(b1&&b2&&a3>0&&a4==0) return 3;
101         if(b1&&b2&&a3>0&&a4>0) return 0;
102         return -1; //無限交點
103     } else if(c1*c2<=0&&c3*c4<=0) return 1;
104     return 0; //不相交
105 }
106 point<T> line_intersection(const line &l) const { /*直線交點*/
107     point<T> a=p2-p1, b=l.p2-l.p1, s=l.p1-p1;
108     //if(a.cross(b)==0) return INF;
109     return p1+a*(s.cross(b)/a.cross(b));
110 }
111 point<T> seg_intersection(const line &l) const { //線段交點
112     int res=seg_intersect(l);
113     if(res<=0) assert(0);
114     if(res==2) return p1;
115     if(res==3) return p2;
116     return line_intersection(l);
117 }
118 };
119 template<typename T>
120 struct polygon {
121     polygon() {}
122     vector<point<T>> > p; //逆時針順序
123     T area() const { //面積
124         T ans=0;
125         for(int i=p.size()-1, j=0; j<(int)p.size(); i=j++)
126             ans+=p[i].cross(p[j]);
127         return ans/2;
128     }
129     point<T> center_of_mass() const { //重心
130         T cx=0, cy=0, w=0;
131         for(int i=p.size()-1, j=0; j<(int)p.size(); i=j++) {
132             T a=p[i].cross(p[j]);
133             cx+=(p[i].x+p[j].x)*a;
134             cy+=(p[i].y+p[j].y)*a;
135             w+=a;
136         }
137         return point<T>(cx/3/w, cy/3/w);
138     }
139     char ahas(const point<T>& t) const { //點是否在簡單多邊形內,
140         //是的話回傳1、在邊上回傳-1、否則回傳0
141         bool c=0;
142         for(int i=0, j=p.size()-1; i<p.size(); j=i++)
143             if(line<T>(p[i], p[j]).point_on_segment(t)) return -1;
144         else if((p[i].y>t.y) != (p[j].y>t.y) &&

```



```

255 while(L<R&&s[i].ori(px[R-1])<=0)--R;
256 while(L<R&&s[i].ori(px[L])<=0)++L;
257 q[++R]=s[i];
258 if(q[R].parallel(q[R-1])){
259     --R;
260     if(q[R].ori(s[i].p1)>0)q[R]=s[i];
261 }
262 if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
263 }
264 while(L<R&&q[L].ori(px[R-1])<=0)--R;
265 p.clear();
266 if(R-L<=1)return 0;
267 px[R]=q[R].line_intersection(q[L]);
268 for(int i=L;i<=R;++i)p.push_back(px[i]);
269 return R-L+1;
270 }
271 };
272 template<typename T>
273 struct triangle{
274     point<T> a,b,c;
275     triangle(){
276         triangle(const point<T> &a,const point<T> &b,const point<T> &c):a(a),b(b),c(c){}
277     }
278     T area()const{
279         T t=(b-a).cross(c-a)/2;
280         return t>0?-t;
281     }
282     point<T> barycenter()const{//重心
283         return (a+b+c)/3;
284     }
285     point<T> circumcenter()const{//外心
286         static line<T> u,v;
287         u.p1=(a+b)/2;
288         u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
289         v.p1=(a+c)/2;
290         v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
291         return u.line_intersection(v);
292     }
293     point<T> incenter()const{//內心
294         T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).abs2());
295         return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B+C);
296     }
297     point<T> perpencenter()const{//垂心
298         return barycenter()*3-circumcenter()*2;
299     }
300 };
301 template<typename T>
302 struct point3D{
303     T x,y,z;
304     point3D(){
305         point3D(const T&x,const T&y,const T&z):x(x),y(y),z(z){}
306     }
307     point3D operator+(const point3D &b)const{
308         return point3D(x+b.x,y+b.y,z+b.z);
309     }
310     point3D operator-(const point3D &b)const{
311         return point3D(x-b.x,y-b.y,z-b.z);
312     }
313     point3D operator*(const T &b)const{
314         return point3D(x*b,y*b,z*b);
315     }
316     point3D operator/(const T &b)const{
317         return point3D(x/b,y/b,z/b);
318     }
319     bool operator==(const point3D &b)const{
320         return x==b.x&&y==b.y&&z==b.z;
321     }
322     T dot(const point3D &b)const{
323         return x*b.x+y*b.y+z*b.z;
324     }
325     point3D cross(const point3D &b)const{
326         return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
327     }
328     T abs2()const{//向量長度的平方
329         return dot(*this);
330     }
331     T area2(const point3D &b)const{//和b、原點圍成面積的平方
332         return cross(b).abs2()/4;
333     }
334 };
335 template<typename T>
336 struct line3D{
337     point3D<T> p1,p2;
338     line3D(){
339         line3D(const point3D<T> &p1,const point3D<T> &p2):p1(p1),p2(p2){}
340     }
341     T dis2(const point3D<T> &p,bool is_segment=0)const{//點跟直線/線段的距離平方
342         point3D<T> v=p2-p1,v1=p-p1;
343         if(is_segment){
344             point3D<T> v2=p-p2;
345             if(v.dot(v1)<=0)return v1.abs2();
346             if(v.dot(v2)>=0)return v2.abs2();
347         }
348         point3D<T> tmp=v.cross(v1);
349         return tmp.abs2()/v.abs2();
350     }
351     pair<point3D<T>,point3D<T>> closest_pair(const line3D<T> &l)const{
352         point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
353         point3D<T> N=v1.cross(v2),ab(p1-l.p1);
354         //if(N.abs2()==0)return NULL;平行或重合
355         T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//最近點對距離
356         point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2),G=l.p1-p1;
357         ;
358         T t1=(G.cross(d2)).dot(D)/D.abs2();
359         T t2=(G.cross(d1)).dot(D)/D.abs2();
360         return make_pair(p1+d1*t1,l.p1+d2*t2);
361     }
362     bool same_side(const point3D<T> &a,const point3D<T> &b)const{
363         return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
364     }
365 };
366 template<typename T>
367 struct plane{
368     point3D<T> p0,n;//平面上的點和法向量
369     plane(){
370         plane(const point3D<T> &p0,const point3D<T> &n):p0(p0),n(n){}
371     }
372     T dis2(const point3D<T> &p)const{//點到平面距離的平方
373         T tmp=(p-p0).dot(n);
374         return tmp*tmp/n.abs2();
375     }
376     point3D<T> projection(const point3D<T> &p)const{
377         return p-n*(p-p0).dot(n)/n.abs2();
378     }
379     point3D<T> line_intersection(const line3D<T> &l)const{
380         T tmp=n.dot(l.p2-l.p1);//等於0表示平行或重合該平面
381         return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
382     }
383     line3D<T> plane_intersection(const plane &p1)const{
384         point3D<T> e=n.cross(p1.n),v=n.cross(e);
385         T tmp=p1.n.dot(v);//等於0表示平行或重合該平面
386         point3D<T> q=p0+(v*(p1.p0-p0))/tmp;
387         return line3D<T>(q,q+e);
388     }
389 };
390 template<typename T>
391 struct triangle3D{
392     point3D<T> a,b,c;
393     triangle3D(){
394         triangle3D(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c):a(a),b(b),c(c){}
395     }
396     bool point_in(const point3D<T> &p)const{//點在該平面上的投影在三角形中
397         return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
398     }
399 };
400 template<typename T>
401 struct tetrahedron{//四面體
402     point3D<T> a,b,c,d;
403     tetrahedron(){
404         tetrahedron(const point3D<T> &a,const point3D<T> &b,const point3D<T> &c,const point3D<T> &d):a(a),b(b),c(c),d(d){}
405     }
406     T volume6()const{//體積的六倍
407         return (d-a).dot((b-a).cross(c-a));
408     }
409     point3D<T> centroid()const{
410         return (a+b+c+d)/4;
411     }
412     bool point_in(const point3D<T> &p)const{
413         return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,d,a).point_in(p);
414     }
415 };
416 template<typename T>
417 struct convexhull3D{
418     static const int MAXN=1005;
419     struct face{
420         int a,b,c;
421         face(int a,int b,int c):a(a),b(b),c(c){}
422     };
423     vector<point3D<T>> pt;
424     vector<face> ans;
425     int fid[MAXN][MAXN];
426     void build(){
427         int n=pt.size();
428         ans.clear();
429         memset(fid,0,sizeof(fid));
430         ans.emplace_back(0,1,2);//注意不能共線
431         ans.emplace_back(2,1,0);
432         int ftop = 0;
433         for(int i=3, ftop=1; i<n; ++i, ++ftop){
434             vector<face> next;
435             for(auto &f:ans){
436                 T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[f.a]).cross(pt[f.c]-pt[f.a]));
437                 if(d<=0) next.push_back(f);
438                 int ff=0;
439                 if(d>0) ff=ftop;
440                 else if(d<0) ff=-ftop;
441                 fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c][f.a]=ff;
442             }
443             for(auto &f:ans){
444                 if(fid[f.a][f.b]>0 && fid[f.a][f.b]!=fid[f.b][f.a])
445                     next.emplace_back(f.a,f.b,i);
446                 if(fid[f.b][f.c]>0 && fid[f.b][f.c]!=fid[f.c][f.b])
447                     next.emplace_back(f.b,f.c,i);
448                 if(fid[f.c][f.a]>0 && fid[f.c][f.a]!=fid[f.a][f.c])
449                     next.emplace_back(f.c,f.a,i);
450             }
451             ans=next;
452         }
453     }
454 };

```

```

433     next.emplace_back(f.c,f.a,i);
434 }
435 ans=next;
436 }
437 }
438 point3D<T> centroid()const{
439     point3D<T> res(0,0,0);
440     T vol=0;
441     for(auto &f:ans){
442         T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]));
443         res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
444         vol+=tmp;
445     }
446     return res/(vol*4);
447 }
448 };

```

### 4.3 HyperbolaGeometry

```

1 #define x first
2 #define y second
3 // 看要做整數運算還是浮點數運算(甚至分數運算,請自行實作加減
  乘除)
4 // 若是整數運算,某些運算可能失真(如求兩直線交點)
5 #define T double
6
7 // 兩個 eq 選一個
8 inline bool eq(double a, double b) { return abs(a - b) < 1e
  -7; }
9 inline bool eq(long long a, long long b) { return a == b; }
10
11 #define point vec
12 struct vec {
13     T x, y; // 向量或坐標的x,y值
14     vec operator+(vec o) { return {x + o.x, y + o.y}; }
15     vec operator-(vec o) { return {x - o.x, y - o.y}; }
16     vec operator*(T o) { return {x * o, y * o}; }
17     vec operator/(T o) { return {x / o, y / o}; }
18     T operator%(vec o) { return x * o.x + y * o.y; } //
    內積
19     T operator*(vec o) { return x * o.y - y * o.x; } //
    外積
20     T abs() { return x * x + y * y; } //
    絕對值平方
21     bool samedir(vec o) { return eq(x * o.y, y * o.x); } //
    兩向量方向是否相同或相反
22 };
23 vec makevec(point src, point dst) { return {dst.x - src.x,
  dst.y - src.y}; }
24
25 #define seg line
26 struct line {
27     point s, t; // 此直線經過s,t;或此線段始於s且止於t
28     vec d; // 此直線的向量
29     T a, b, c; // ax+by=c
30
31     line(point p, point q) { // 此直線經過p,q;或此線段為始
    於p且止於q
32         s = p, t = q, d = makevec(p, q);
33         a = p.y - q.y, b = q.x - p.x, c = a * p.x + b * p.y;
34     }

```

```

35 // 點是否在直線上
36 bool passLine(point p) { return d.samedir(p - s); }
37 bool passSeg(point p) { // 點是否在線段上
38     vec ap = makevec(s, p), bp = makevec(t, p);
39     return passLine(p) && ap % bp < 0;
40 }
41 // 兩直線是否重合
42 bool sameLine(line o) { return d.samedir(o.d) && passLine
  (o.s); }
43 // 兩直線是否平行且不重合
44 bool para(line o) { return d.samedir(o.d) && !passLine(o.
  s); }
45 point proj(point p) { // 求某點在此直線上的投影座標
46     vec e = {p - s};
47     T t = e % d / d.abs();
48     vec dst = {d.x * t, d.y * t};
49     return s + dst;
50 }
51 // 點與直線距離平方
52 T dist2(point p) { return (proj(p) - p).abs(); }
53 // 兩平行直線距離平方
54 T dist2(line o) { return (o.proj(s) - s).abs(); }
55 // 此直線是否將兩點隔開
56 bool split(point p, point q) { return (a * p.x + b * p.y
  < 0) != (a * q.x + b * q.y < 0); }
57 // 兩非平行線段是否相交
58 bool meet(seg o) { return split(o.s, o.t) && o.split(s, t
  ); }
59 point intersect(line o) { // 兩非平行直線相交座標
60     return {(c * o.b - b * o.c) / (a * o.b - b * o.a),
61             (a * o.c - c * o.a) / (a * o.b - b * o.a)};
62 }
63 double cosangle(line o) { // 兩直線夾角之 cos 值
64     return (d % o.d) / (sqrt(d.abs() * o.d.abs()));
65 }
66 };
67
68 #define rr (r * r) // 半徑平方
69 #define usevars
70 //打字加速
71 double x1 = c.x, x2 = o.c.x, y1 = c.y, y2 = o.c.y;
72 double r1 = r, r2 = o.r, r12 = r1 * r1, r22 = r2 * r2; \
73 double dx = x2 - x1, dy = y2 - y1, dd = dx * dx + dy * dy
  , d = sqrt(dd);
74 const double PI = acos(-1);
75 struct circle {
76     point c; // 圓心
77     double r; // 半徑
78 // 求直線與圓的交點並回傳交點數量。若有兩點,存於ans1與
  ans2,若有一點,存於ans1。
79 int meetLine(line l, point& ans1, point& ans2) {
80     double d2 = l.dist2(c);
81     if (eq(d2, rr)) return ans1 = l.proj(c), 1; // 交於
    一點
82     if (d2 > rr) return 0; // 無交
    點
83     l = {l.s - c, l.t - c};
84     double s = l.a * l.a + l.b * l.b, w = rr - l.c * l.c
  / s, m = sqrt(w / s);
85     double x = -l.a * l.c / s, y = -l.b * l.c / s;
86     ans1 = {x + l.b * m, y - l.a * m}, ans2 = {x - l.b *
  m, y + l.a * m};
87     ans1 = ans1 + c, ans2 = ans2 + c;

```

```

88     return 2;
89 }
90 // 求線段與圓的交點並回傳交點數量。
91 int meetSeg(seg l, point& ans1, point& ans2) {
92     int res = meetLine(l, ans1, ans2);
93     if (res == 0) return 0;
94     if (res == 1) return l.passSeg(ans1);
95     return (int)l.passSeg(ans1) + l.passSeg(ans2);
96 }
97 // 求圓與圓的交點並回傳交點數量。
98 int meetCircle(circle o, point& ans1, point& ans2) {
99     usevars;
100     if (d > r1 + r2) return 0; // 互斥
101     if (d < abs(r1 - r2)) return 0; // 完全包含
102     point A = {(x1 + x2) / 2, (y1 + y2) / 2};
103     double f = (r12 - r22) / (2 * dd);
104     point B = {dx * f, dy * f};
105     double h = (r12 - r22);
106     f = sqrt(2 * (r12 + r22) / dd - h * h / (dd * dd) -
  1) / 2;
107     point C = {dy * f, -dx * f};
108     ans1 = A + B + C, ans2 = A + B - C;
109     return eq(d, r1 + r2) ? 1 : 2;
110 }
111 double coverArea(circle o) { // 求兩圓重疊部分面積
112     if (r < o.r) return o.coverArea(*this);
113     usevars;
114     if (d > r1 + r2) return 0; // 互斥
115     if (d < abs(r1 - r2)) return PI * r2 * r2; // 完全包
    含
116     double d1 = (r12 - r22 + dd) / (2 * d), d2 = d - d1;
117     return r12 * acos(d1 / r1) - d1 * sqrt(r12 - d1 * d1)
  + r22 * acos(d2 / r2) - d2 * sqrt(r22 - d2 * d2
  );
118 }
119 };
120 double len(point a, point b) { return sqrt((a - b).abs()); }
121 // 打字加速
122 struct tri {
123     point a, b, c;
124     T area2() { return abs((b - a) * (c - a)); } // 求面積之
    兩倍
125     point barycenter() { return (a + b + c) / 3; } // 重心
126     point perpendcenter() { return barycenter() * 3 -
    circumcenter() * 2; } // 垂心
127     point circumcenter() { // 外心
128         point p1 = (a + b) / 2, p2 = {p1.x - a.y + b.y, p1.y
  + a.x - b.x};
129         line u = {p1, p2};
130         p1 = (a + c) / 2, p2 = {p1.x - a.y + c.y, p1.y + a.x
  - c.x};
131         line v = {p1, p2};
132         return u.intersect(v);
133     }
134     point incentre() { // 內心
135         T A = len(b, c), B = len(a, c), C = len(a, b);
136         point p = {A * a.x + B * b.x + C * c.x, A * a.y + B *
  b.y + C * c.y};
137         return p / (A + B + C);
138     }
139 // 費馬點

```

```

139 // 若有一角 >= 120 (cos(x) <= -0.5) , 費馬點為該角對應的
      點
140 // 否則三角型三條邊對外做正三角形, 得到三個頂點 A', B', C
      '
141 // 費馬點為 AA' BB' CC' 三線之交點
142 };

```

## 4.4 MinRect

```

1 // 全部浮點數運算, 先製作凸包, 然後呼叫 minrect
2 typedef long double dd;
3 typedef pair<dd, dd> pii;
4 #define x first
5 #define y second
6 #define in inline
7 #define cp const pii&
8 #define op operator
9 #define ab (cp a, cp b)
10 const dd eps = 1e-8;
11 in pii op+ab { return {a.x + b.x, a.y + b.y}; }
12 in pii op-ab { return {a.x - b.x, a.y - b.y}; }
13 in pii op*(cp p, dd v) { return {v * p.x, v * p.y}; }
14 in dd op^ab { return a.x * b.x + a.y * b.y; }
15 in dd op*ab { return a.x * b.y - a.y * b.x; }
16 in dd op%ab {
17     dd dx = a.x - b.x, dy = a.y - b.y;
18     return dx * dx + dy * dy;
19 }
20 in dd crzf(cp o, cp a, cp b) { return (a - o) * (b - o); }
21 in dd dotf(cp o, cp a, cp b) { return (a - o) ^ (b - o); }
22
23 #define judge \
24     crzf(ret.back(), pp[i]) <= eps
25 vector<pii> makepoly(vector<pii>& pp) {
26     sort(pp.begin(), pp.end());
27     pp.erase(unique(pp.begin(), pp.end()), pp.end());
28     int n = pp.size(); vector<pii> ret;
29     for (int i = 0; i < n; i++) {
30         while (ret.size() >= 2 && judge) ret.pop_back();
31         ret.push_back(pp[i]);
32     }
33     for (int i = n - 2, s = ret.size() + 1; i >= 0; i--) {
34         while (ret.size() >= s && judge) ret.pop_back();
35         ret.push_back(pp[i]);
36     }
37     if (n >= 2) ret.pop_back(); return ret;
38 }
39
40 // 給凸包, 問最小覆蓋矩形面積以及該矩形頂點座標 (存於 rec)
41 // 。頂點座標按照凸包製作方式排序。如果不需要矩形座標, 把跟
42 // rec 有關的程式碼移除。
43 #define xx(i) ((i + 1) % n)
44 in pii foot(cp s1, cp s2, cp q) {
45     return s1 + (s2 - s1) * dotf(s1, s2, q) * (1 / (s1 % s2));
46 }
47 dd minrect(const vector<pii>& poly, vector<pii>& rec) {
48     int n = poly.size(); if (n < 3) return 0;
49     dd minn = 1e50; rec.resize(4);
50     int j = 1, k = 1, r;
51     for (int i = 0; i < n; i++) {
52         while (crzf(poly[i], poly[xx(i)], poly[xx(j)]) -
53             crzf(poly[i], poly[xx(i)], poly[j]) > -eps)

```

```

54         j = xx(j);
55         while (dotf(poly[i], poly[xx(i)], poly[xx(k)]) -
56             dotf(poly[i], poly[xx(i)], poly[k]) > -eps)
57             k = xx(k);
58         if (i == 0) r = k;
59         while (dotf(poly[i], poly[xx(i)], poly[xx(r)]) -
60             dotf(poly[i], poly[xx(i)], poly[r]) < eps)
61             r = xx(r);
62         dd a = crzf(poly[i], poly[xx(i)], poly[j]) *
63             (dotf(poly[i], poly[xx(i)], poly[k]) -
64             dotf(poly[i], poly[xx(i)], poly[r])) /
65             (poly[i] % poly[xx(i)]);
66         a = abs(a); if (a < minn) { minn = a;
67             rec[0] = foot(poly[i], poly[xx(i)], poly[r]);
68             rec[1] = foot(poly[i], poly[xx(i)], poly[k]);
69             pii toss = foot(poly[i], poly[xx(i)], poly[j]);
70             rec[2] = poly[j] + rec[0] - toss;
71             rec[3] = poly[j] + rec[1] - toss;
72         }
73     }
74     rec = makepoly(rec); return minn;
75 }

```

## 4.5 Rectangle\_Union\_Area

```

1 const int maxn = 1e5 + 10;
2 struct rec{
3     int t, b, l, r;
4 } r[maxn];
5 int n, cnt[maxn << 2];
6 long long st[maxn << 2], ans = 0;
7 vector<int> x, y;
8 vector<pair<pair<int, int>, pair<int, int>>> v;
9 void modify(int t, int l, int r, int ql, int qr, int v) {
10     if (ql <= l && r <= qr) cnt[t] += v;
11     else {
12         int m = (l + r) >> 1;
13         if (qr <= m) modify(t << 1, l, m, ql, qr, v);
14         else if (ql >= m) modify(t << 1 | 1, m, r, ql, qr, v);
15         else modify(t << 1, l, m, ql, m, v), modify(t << 1 | 1, m, r, m, qr, v);
16     }
17     if (cnt[t]) st[t] = y[r] - y[l];
18     else if (r - l == 1) st[t] = 0;
19     else st[t] = st[t << 1] + st[t << 1 | 1];
20 }
21 int main() {
22     cin >> n;
23     for (int i = 0; i < n; i++) {
24         cin >> r[i].l >> r[i].r >> r[i].b >> r[i].t;
25         if (r[i].l > r[i].r) swap(r[i].l, r[i].r);
26         if (r[i].b > r[i].t) swap(r[i].b, r[i].t);
27         x.push_back(r[i].l);
28         x.push_back(r[i].r);
29         y.push_back(r[i].b);
30         y.push_back(r[i].t);
31     }
32     sort(x.begin(), x.end());
33     sort(y.begin(), y.end());
34     x.erase(unique(x.begin(), x.end()), x.end());
35     y.erase(unique(y.begin(), y.end()), y.end());
36     for (int i = 0; i < n; i++) {

```

```

37         r[i].l = lower_bound(x.begin(), x.end(), r[i].l) - x.
            begin();
38         r[i].r = lower_bound(x.begin(), x.end(), r[i].r) - x.
            begin();
39         r[i].b = lower_bound(y.begin(), y.end(), r[i].b) - y.
            begin();
40         r[i].t = lower_bound(y.begin(), y.end(), r[i].t) - y.
            begin();
41         v.emplace_back(make_pair(r[i].l, 1), make_pair(r[i].b
            , r[i].t));
42         v.emplace_back(make_pair(r[i].r, -1), make_pair(r[i].
            b, r[i].t));
43     }
44     sort(v.begin(), v.end(), [](pair<pair<int, int>, pair<int
            , int>>> a, pair<pair<int, int>, pair<int, int>>> b){
45         if (a.first.first != b.first.first) return a.first.
            first < b.first.first;
46         return a.first.second > b.first.second;
47     });
48     for (int i = 0; i < v.size(); i++) {
49         if (i) ans += (x[v[i].first.first] - x[v[i - 1].first
            .first]) * st[1];
50         modify(1, 0, y.size(), v[i].second.first, v[i].second
            .second, v[i].first.second);
51     }
52     cout << ans << '\n';
53     return 0;
54 }

```

## 4.6 SmallestCircle

```

1 using PT = point<T>;
2 using CPT = const PT;
3 PT circumcenter(CPT &a, CPT &b, CPT &c) {
4     PT u = b-a, v = c-a;
5     T c1 = u.abs2()/2, c2 = v.abs2()/2;
6     T d = u.cross(v);
7     return PT(a.x+(v.y*c1-u.y*c2)/d, a.y+(u.x*c2-v.x*c1)/d);
8 }
9 void solve(PT p[], int n, PT &c, T &r2){
10     random_shuffle(p,p+n);
11     c = p[0]; r2 = 0; // c,r2 = 圓心,半徑平方
12     for(int i=1; i<n; i++){
13         if( (p[i]-c).abs2() > r2) {
14             c=p[i]; r2=0;
15             for(int j=0; j<i; j++){
16                 if( (p[j]-c).abs2() > r2) {
17                     c.x = (p[i].x+p[j].x)/2;
18                     c.y = (p[i].y+p[j].y)/2;
19                     r2 = (p[j]-c).abs2();
20                     for(int k=0; k<j; k++){
21                         if( (p[k]-c).abs2() > r2) {
22                             c = circumcenter(p[i], p[j], p[k]);
23                             r2 = (p[i]-c).abs2();
24                         }
25                     }
26                 }
27             }

```

## 4.7 旋轉卡尺

```

1 typedef pair<ll, ll> pii;
2 #define x first
3 #define y second
4 #define ii (i + 1) % n // 打字加速!
5 inline pii operator-(const pii& a, const pii& b) {
6     return {a.x - b.x, a.y - b.y};
7 } // const 不可省略
8 inline ll operator*(const pii& a, const pii& b) {
9     return a.x * b.y - a.y * b.x;
10 }
11 inline ll crzf(const pii& o, const pii& a, const pii& b) {
12     return (a - o) * (b - o);
13 }
14 inline ll dd(const pii& a, const pii& b) {
15     ll dx = a.x - b.x, dy = a.y - b.y;
16     return dx * dx + dy * dy;
17 }
18 // 給平面上任意個點，求其凸包。返回順序為逆時針。此方法會移除
    重複點。
19 #define jud \
20     crzf(ret[ret.size() - 2], ret.back(), pp[i]) <= 0
21 vector<pii> makepoly(vector<pii>& pp) {
22     int n = pp.size();
23     sort(pp.begin(), pp.end());
24     pp.erase(unique(pp.begin(), pp.end()), pp.end());
25     vector<pii> ret;
26     for (int i = 0; i < n; i++) {
27         while (ret.size() >= 2 && jud) ret.pop_back();
28         ret.push_back(pp[i]);
29     }
30     for (int i = n - 2, t = ret.size() + 1; i >= 0; i--) {
31         while (ret.size() >= t && jud) ret.pop_back();
32         ret.push_back(pp[i]);
33     }
34     if (n >= 2) ret.pop_back();
35     return ret;
36 }
37 // (shoelace formula)
38 // 給凸包，問其面積「的兩倍」。若凸包少於三個點，回傳零。
39 ll area(vector<pii>& poly) {
40     int n = poly.size();
41     ll ret = 0;
42     for (int i = 0; i < n; i++)
43         ret += (poly[i].x * poly[i+1].y);
44     for (int i = 0; i < n; i++)
45         ret -= (poly[i].y * poly[i+1].x);
46     return ret;
47 }
48 // 給凸包，問其兩點最遠距離「的平方」。若要問平面上任意個點的
    兩點最遠
49 // 距離，請先轉成凸包。若凸包少於兩個點，回傳零。
50 #define kk (k + 1) % n
51 ll maxdist(vector<pii>& poly) {
52     int k = 1, n = poly.size();
53     if (n < 2) return 0;
54     if (n == 2) return dd(poly[0], poly[1]);
55     ll ret = 0;
56     for (int i = 0; i < n; i++) {
57         while (abs(crzf(poly[kk], poly[i], poly[i+1])) >=
58             abs(crzf(poly[k], poly[i], poly[i+1])))
59             k = kk;
60         ret = max(ret, max(dd(poly[i], poly[k]),
61             dd(poly[i+1], poly[k])));
62     }

```

```

63     return ret;
64 }

```

## 5 Graph

### 5.1 BCC\_edge

```

1 邊雙連通
2 任意兩點間至少有兩條不重疊的路徑連接，找法：
3 1. 標記出所有的橋
4 2. 對全圖進行 DFS，不走橋，每一次 DFS 就是一個新的邊雙連通
5 // from BCW
6 struct BccEdge {
7     static const int MXN = 100005;
8     struct Edge { int v, eid; };
9     int n, m, step, par[MXN], dfn[MXN], low[MXN];
10    vector<Edge> E[MXN];
11    DisjointSet djs;
12    void init(int _n) {
13        n = _n; m = 0;
14        for (int i = 0; i < n; i++) E[i].clear();
15        djs.init(n);
16    }
17    void add_edge(int u, int v) {
18        E[u].PB({v, m});
19        E[v].PB({u, m});
20        m++;
21    }
22    void DFS(int u, int f, int f_eid) {
23        par[u] = f;
24        dfn[u] = low[u] = step++;
25        for (auto it: E[u]) {
26            if (it.eid == f_eid) continue;
27            int v = it.v;
28            if (dfn[v] == -1) {
29                DFS(v, u, it.eid);
30                low[u] = min(low[u], low[v]);
31            } else {
32                low[u] = min(low[u], dfn[v]);
33            }
34        }
35    }
36    void solve() {
37        step = 0;
38        memset(dfn, -1, sizeof(int)*n);
39        for (int i = 0; i < n; i++) {
40            if (dfn[i] == -1) DFS(i, i, -1);
41        }
42        djs.init(n);
43        for (int i = 0; i < n; i++) {
44            if (low[i] < dfn[i]) djs.uni(i, par[i]);
45        }
46    }
47 } graph;

```

### 5.2 LCA

```

1 /* 三種 0/1-based。只支援無向樹 */
2 /* Time: O(N+Q) Space: O(N^2) online */
3 class SsadbTarjan {
4 private:
5     int n;
6     vector<int> par, dep; vector<vector<int>>> ca;
7     int dfs(int u, vector<vector<int>>& edge, int d) {
8         dep[u] = d;
9         for (int a = 0; a < n; a++)
10             if (dep[a] != -1)
11                 ca[a][u] = ca[u][a] = parent(a);
12         for (int a : edge[u]) {
13             if (dep[a] != -1) continue;
14             dfs(a, edge, d + 1);
15             par[a] = u;
16         }
17     }
18     int parent(int x) {
19         if (par[x] == x) return x;
20         return par[x] = parent(par[x]);
21     }
22 public:
23     SsadbTarjan(vector<vector<int>>& edge, int root)
24         : n(edge.size()) {
25         dep.assign(n, -1); par.resize(n);
26         ca.assign(n, vector<int>(n));
27         for (int i = 0; i < n; i++) par[i] = i;
28         dfs(root, edge, 0);
29     }
30     int lca(int a, int b) { return ca[a][b]; }
31     int dist(int a, int b) {
32         return dep[a] + dep[b] - 2 * dep[ca[a][b]];
33     }
34 };
35 /* Time: O(N+Q) Space: O(N+Q) only offline */
36 #define x first
37 #define y second
38 class OfflineTarjan {
39 private:
40     vector<int> par, anc, dep, ans, rank;
41     vector<vector<pii>>> qry;
42     vector<vector<int>>& edge; // 安全考量可把 & 去掉
43     int root, n;
44     void merge(int a, int b) {
45         a = parent(a), b = parent(b);
46         if (rank[a] < rank[b]) swap(a, b);
47         else if (rank[a] == rank[b]) rank[a]++;
48         par[b] = a;
49     }
50     void dfs(int u, int d) {
51         anc[parent(u)] = u, dep[u] = d;
52         for (int a : edge[u]) {
53             if (dep[a] != -1) continue;
54             dfs(a, d + 1);
55             merge(a, u);
56             anc[parent(u)] = u;
57         }
58         for (auto q : qry[u])
59             if (dep[q.first] != -1)
60                 ans[q.second] = anc[parent(q.first)];
61     }
62     int parent(int x) {
63         if (par[x] == x) return x;
64         return par[x] = parent(par[x]);
65     }

```

```

66 void solve(vector<pii>& query) {
67     dep.assign(n, -1), rank.assign(n, 0);
68     par.resize(n), anc.resize(n), qry.resize(n);
69     for (int i = 0; i < n; i++) anc[i] = par[i] = i;
70     ans.resize(query.size());
71     for (int i = 0; i < query.size(); i++) {
72         auto& q = query[i];
73         qry[q.first].emplace_back(q.second, i);
74         qry[q.second].emplace_back(q.first, i);
75     }
76     dfs(root, 0);
77 }
78 public:
79 // edge 是傳 reference，完成所有查詢不可改。
80 OfflineTarjan(vector<vector<int>>& edge, int root)
81 : edge(edge), root(root), n(edge.size()) {}
82 // 離線查詢，query 陣列包含所有詢問 {src, dst}。呼叫一
83 // 次無
84 // 論 query 量多少，複雜度都是 O(N)。所以應盡量只呼叫一
85 // 次。
86 vector<int> lca(vector<pii>& query) {
87     solve(query); return ans;
88 }
89 vector<int> dist(vector<pii>& query) {
90     solve(query);
91     for (int i = 0; i < query.size(); i++) {
92         auto & q = query[i];
93         ans[i] = dep[q.first] + dep[q.second]
94             - 2 * dep[ans[i]];
95     } return ans;
96 }
97 /* Udchen Time: O(QlgN) Space: O(NlgN)。支援非離線。*/
98 class SparseTableTarjan {
99 private:
100     int maxlg;
101     vector<vector<int>> anc;
102     vector<int> dep;
103     void dfs(int u, vector<vector<int>>& edge, int d) {
104         dep[u] = d;
105         for (int i = 1; i < maxlg; i++)
106             if (anc[u][i - 1] == -1) break;
107             else anc[u][i] = anc[anc[u][i - 1]][i - 1];
108         for (int a : edge[u]) {
109             if (dep[a] != -1) continue;
110             anc[a][0] = u;
111             dfs(a, edge, d + 1);
112         }
113     }
114 public:
115     SparseTableTarjan(vector<vector<int>>& edge, int root) {
116         int n = edge.size();
117         maxlg = ceil(log2(n));
118         anc.assign(n, vector<int>(maxlg, -1));
119         dep.assign(n, -1);
120         dfs(root, edge, 0);
121     }
122     int lca(int a, int b) {
123         if (dep[a] > dep[b]) swap(a, b);
124         for (int k = 0; dep[b] - dep[a]; k++)
125             if (((dep[b] - dep[a]) >> k) & 1) b = anc[b][k];
126         if (a == b) return a;
127         for (int k = maxlg - 1; k >= 0; k--)
128             if (anc[a][k] != anc[b][k])
129                 a = anc[a][k], b = anc[b][k];

```

```

129         return anc[a][0];
130     }
131     int dist(int a, int b) {
132         return dep[a] + dep[b] - 2 * dep[lca(a, b)];
133     }
134 };

```

## 5.3 MahattanMST

```

1 #define REP(i,n) for(int i=0;i<n;i++)
2 typedef long long LL;
3 const int N=200100;
4 int n,m;
5 struct PT {int x,y,z,w,id;}; p[N];
6 inline int dis(const PT &a,const PT &b){return abs(a.x-b.x)+
7     abs(a.y-b.y);}
8 inline bool cpx(const PT &a,const PT &b)
9 {return a.x!=b.x? a.x>b.x:a.y>b.y;}
10 inline bool cpz(const PT &a,const PT &b){return a.z<b.z;}
11 struct E{int a,b,c;}; e[8*N];
12 bool operator<(const E&a,const E&b){return a.c<b.c;}
13 struct Node{ int L,R,key; } node[4*N];
14 int s[N];
15 int F(int x) {return s[x]==x? x : s[x]=F(s[x]); }
16 void U(int a,int b) {s[F(b)]=F(a);}
17 void init(int id,int L,int R) {
18     node[id] = (Node){L,R,-1};
19     if(L==R)return;
20     init(id*2,L,(L+R)/2);
21     init(id*2+1,(L+R)/2+1,R);
22 }
23 void ins(int id,int x) {
24     if(node[id].key==-1 || p[node[id].key].w>p[x].w)
25         node[id].key=x;
26     if(node[id].L==node[id].R) return;
27     if(p[x].z<=(node[id].L+node[id].R)/2) ins(id*2,x);
28     else ins(id*2+1,x);
29 }
30 int Q(int id,int L,int R){
31     if(R<node[id].L || L>node[id].R)return -1;
32     if(L<=node[id].L && node[id].R<=R)return node[id].key;
33     int a=Q(id*2,L,R),b=Q(id*2+1,L,R);
34     if(b!=-1 || (a!=-1 && p[a].w<p[b].w)) return a;
35     else return b;
36 }
37 void calc() {
38     REP(i,n) {
39         p[i].z = p[i].y-p[i].x;
40         p[i].w = p[i].x+p[i].y;
41     }
42     sort(p,p+n,cpz);
43     int cnt = 0, j, k;
44     for(int i=0; i<n; i=j){
45         for(j=i+1; p[j].z==p[i].z && j<n; j++);
46         for(k=i, cnt++; k<j; k++) p[k].z = cnt;
47     }
48     init(1,1,cnt);
49     sort(p,p+n,cpx);
50     REP(i,n) {
51         j=Q(1,p[i].z,cnt);
52         if(j!=-1) e[m++] = (E){p[i].id, p[j].id, dis(p[i],p[j])};
53         ins(1,i);

```

```

53     }
54 }
55 LL MST() {
56     LL r=0;
57     sort(e, e+m);
58     REP(i, m) {
59         if(F(e[i].a)==F(e[i].b)) continue;
60         U(e[i].a, e[i].b);
61         r += e[i].c;
62     }
63     return r;
64 }
65 int main() {
66     int ts;
67     scanf("%d", &ts);
68     while (ts--) {
69         m = 0;
70         scanf("%d",&n);
71         REP(i,n) {scanf("%d",&p[i].x,&p[i].y);p[i].id=s[i]=i;}
72         calc();
73         REP(i,n)p[i].y= -p[i].y;
74         calc();
75         REP(i,n)swap(p[i].x,p[i].y);
76         calc();
77         REP(i,n)p[i].x=-p[i].x;
78         calc();
79         printf("%lld\n",MST()*2);
80     }
81     return 0;
82 }

```

## 5.4 MinMeanCycle

```

1 #include<cstdio> //for DBL_MAX
2 int dp[MXN][MXN]; // 1-base,0(NM)
3 vector<tuple<int,int,int>> edge;
4 double mmc(int n){ //allow negative weight
5     const int INF = 0x3f3f3f3f;
6     for(int t=0; t<n; ++t){
7         memset(dp[t+1],0x3f,sizeof(dp[t+1]));
8         for(const auto &e:edge) {
9             int u, v, w; tie(u,v,w) = e;
10             dp[t+1][v] = min(dp[t+1][v],dp[t][u]+w);
11         }
12     }
13     double res = DBL_MAX;
14     for(int u=1; u<=n; ++u) {
15         if(dp[n][u]==INF) continue;
16         double val = -DBL_MAX;
17         for(int t=0;t<n;++t)
18             val = max(val,(dp[n][u]-dp[t][u])*1.0/(n-t));
19         res = min(res,val);
20     } return res;
21 }

```

## 5.5 Tarjan



```

2 點 u 為割點 if and only if 滿足 1. or 2.
3 1. u 為樹根，且 u 有多於一個子樹。
4 2. u 不為樹根，且滿足存在 (u,v) 為樹枝邊（或稱父子邊，即 u 為
   v 在搜索樹中的父親），使得 DFN(u) <= Low(v)。
5 -----
6 橋
7 一條無向邊 (u,v) 是橋 if and only if (u,v) 為樹枝邊，且滿足
   DFN(u) < Low(v)。
8 // 0 base
9 struct TarjanSCC{
10     static const int MAXN = 1000006;
11     int n, dfn[MAXN], low[MAXN], scc[MAXN], scn, count;
12     vector<int> G[MAXN];
13     stack<int> stk;
14     bool ins[MAXN];
15     void tarjan(int u) {
16         dfn[u] = low[u] = ++count;
17         stk.push(u);
18         ins[u] = true;
19         for(auto v:G[u]) {
20             if(!dfn[v]) {
21                 tarjan(v);
22                 low[u] = min(low[u], low[v]);
23             } else if(ins[v]) {
24                 low[u] = min(low[u], dfn[v]);
25             }
26         }
27         if(dfn[u] == low[u]) {
28             int v;
29             do {
30                 v = stk.top(); stk.pop();
31                 scc[v] = scn;
32                 ins[v] = false;
33             } while(v != u);
34             scn++;
35         }
36     }
37     void getSCC(){
38         memset(dfn,0,sizeof(dfn));
39         memset(low,0,sizeof(low));
40         memset(ins,0,sizeof(ins));
41         memset(scc,0,sizeof(scc));
42         count = scn = 0;
43         for(int i = 0; i < n; i++)
44             if(!dfn[i]) tarjan(i);
45     }
46 } SCC;

```

## 5.6 Two\_SAT

```

1 const int N = 5010 * 2; // 變數最大數量的兩倍
2 namespace Two_Sat {
3     vector<int> a[N], b[N], stk;
4     int vis[N], res[N];
5     void dfs(int u, vector<int>* g, int sc) {
6         vis[u] = 1, res[u] = sc;
7         for(int v : g[u]) if(!vis[v]) dfs(v, g, sc);
8         if(g == a) stk.push_back(u);
9     }
10 // 先呼叫 imply 來設定約束，然後呼叫 scc 跑分析。
11 // var[x] 的真值對應 i = x * 2 ; var[x] 的假值對應 i = x * 2
    + 1

```

```

12 // e.g. 若 var[3] 為真則 var[6] 必為假，則呼叫 imply(6, 13)
13 void imply(int u, int v) { // if u then v
14     a[u].push_back(v), b[v].push_back(u);
15 }
16 // 跑 two_sat，回傳 true 表示有解。解答存於 Two_Sat::res
17 // e.g. 若 res[13] == 1 表 var[6] 必為假
18 // e.g. 若 res[0] == 1 且 res[1] == 1，表 var[0] 必為真且必
    為假，矛盾，無解。
19 int scc(int n /*變數實際數量的兩倍*/) {
20     memset(vis, 0, sizeof(vis));
21     for(int i = 0; i < n; i++) if(!vis[i]) dfs(i, a, -1);
22     memset(vis, 0, sizeof(vis));
23     int sc = 0;
24     while(!stk.empty()) {
25         if(!vis[stk.back()]) dfs(stk.back(), b, sc++);
26         stk.pop_back();
27     }
28     for(int i = 0; i < n; i += 2) {
29         if(res[i] == res[i + 1]) return 0;
30         if(res[i] > res[i + 1]) res[i] = 1, res[i + 1] = 0;
31         else res[i] = 0, res[i + 1] = 1;
32     }
33     return 1;
34 }
35 // namespace Two_Sat

```

## 6 Math

### 6.1 ax+by=gcd(a,b)

```

1 // 給 a,b，解 ax+by=gcd(a,b)
2 typedef pair<ll, ll> pii;
3 pii extgcd(ll a, ll b) {
4     if(b == 0) return {1, 0};
5     ll k = a / b;
6     pii p = extgcd(b, a - k * b);
7     return {p.second, p.first - k * p.second};
8 }

```

### 6.2 Discrete\_sqrt

```

1 int order(ll b, ll p) {
2     if(__gcd(b, p) != 1) return -1;
3     int ret = 2;
4     while(++ret)
5         if(fastpow(b, ret, p) == 1) break;
6     return ret;
7 }
8 // 把 fastpow 也抄過來，會用到。
9 // 問 (x^2 = y) mod p 的解。回傳 -1 表示 x 無解。
10 ll dsqrt(ll y, ll p) {
11     if(__gcd(y, p) != 1) return -1;
12     if(fastpow(y, (p - 1) / 2, p) == p - 1) return -1;
13     int e = 0;
14     ll s = p - 1;
15     while(!(s & 1)) s >>= 1, e++;
16     int q = 2;

```

```

17 while(1)
18     if(fastpow(q, (p - 1) / 2, p) == p - 1)
19         break;
20     else q++;
21 ll x = fastpow(y, (s + 1) / 2, p);
22 ll b = fastpow(y, s, p);
23 ll g = fastpow(q, s, p);
24 while(1) {
25     int m;
26     for(m = 0; m < e; m++) {
27         int o = order(p, b);
28         if(o == -1) return -1;
29         if(o == fastpow(2, m, p)) break;
30     }
31     if(m == 0) return x;
32     x = x * fastpow(g, fastpow(2, e - m - 1, p) % p);
33     g = fastpow(g, fastpow(2, e - m, p), p);
34     b = b * g % p;
35     if(b == 1) return x;
36     e = m;
37 }
38 }

```

## 6.3 EulerFunction

```

1 // 查詢 phi(x) 亦即比 x 小且與 x 互質的數的數量。
2 int phi(int x) {
3     int r = x;
4     for(int p = 2; p * p <= x; p++) {
5         if(x % p == 0) {
6             while(x % p == 0) x /= p;
7             r -= r / p;
8         }
9     }
10    if(x > 1) r -= r / x;
11    return r;
12 }
13 // 查詢所有 phi(x), x in [0, n] 回傳陣列。
14 vector<int> phi_in(int n) {
15     vector<bool> p(n, 1); vector<int> r(n);
16     for(int i = 0; i < n; i++) r[i] = i;
17     r[1] = p[0] = p[1] = 0;
18     for(int i = 2; i < n; i++) {
19         if(!p[i]) continue;
20         r[i]--;
21         for(int j = i * 2; j < n; j += i)
22             p[j] = 0, r[j] = r[j] / i * (i - 1);
23     } return r;
24 }

```

## 6.4 Expression

```

1 /*支援處理四則運算的工具。給四則運算的字串，檢查格式並計算
2 其值。如果格式不合法，會丟出錯誤。複雜度 O(字串長度)。
3 支援的符號有四則運算和求餘數，先乘除後加減。可以使用括號
4 、或前置正負號。數字開頭可以為零或禁止為零。可以兼容或禁
5 止多重前置號（例如 --1 視為 1、+++1 視為 -1）。
6 空字串視為不合法。運算範圍限於 long long。如果試圖除
7 以零或對零求餘也會丟出錯誤。*/

```

```

8 void req(bool b) { if (!b) throw ""; }
9 const int B = 2; // 可以調整成 B 進位
10 class Expr {
11 private:
12 deque<char> src;
13 Expr(const string& s) : src(s.begin(), s.end()) {}
14 inline char top() {
15     return src.empty() ? '\0' : src.front();
16 }
17 inline char pop() {
18     char c = src.front(); src.pop_front(); return c;
19 }
20 ll n() {
21     ll ret = pop() - '0';
22     // 若要禁止數字以 0 開頭，加上這行
23     // req(ret || !isdigit(top()));
24     while (isdigit(top())) ret = B * ret + pop() - '0';
25     return ret;
26 }
27 ll fac() {
28     if (isdigit(top())) return n();
29     if (top() == '-') { pop(); return -fac(); }
30     if (top() == '(') {
31         pop();
32         ll ret = expr(1);
33         req(pop() == ')');
34         return ret;
35     }
36     // 若要允許前置正號，加上這行
37     // if(top() == '+') { pop(); return fac(); }
38     throw "";
39 }
40 ll term() {
41     ll ret = fac(); char c = top();
42     while (c == '*' || c == '/' || c == '%') {
43         pop();
44         if (c == '*') ret *= fac();
45         else {
46             ll t = fac(); req(t);
47             if (c == '/') ret /= t; else ret %= t;
48         }
49         c = top();
50     } return ret;
51 }
52 ll expr(bool k) {
53     ll ret = term();
54     while (top() == '+' || top() == '-')
55         if (pop() == '+') ret += term();
56         else ret -= term();
57     req(top() == (k ? ')' : '\0'));
58     return ret;
59 }
60 public:
61 // 給定數學運算的字串，求其值。若格式不合法，丟出錯誤。
62 static ll eval(const string& s) {
63     // 若要禁止多重前置號，加上這四行
64     // req(s.find("--") == -1); // 禁止多重負號
65     // req(s.find("-+") == -1);
66     // req(s.find("+-") == -1);
67     // req(s.find("++") == -1);
68     return Expr(s).expr(0);
69 }
70 };

```

## 6.5 FFT

```

1 // int(complex.real() + 0.05) // .imag()
2 template <typename T, typename VT = vector<complex<T>>>
3 struct FFT {
4     const T pi;
5     FFT(const T pi = acos((T)-1.0)) : pi(pi) {}
6     unsigned bit_reverse(unsigned a, int len) {
7         a = ((a&0x55555555U)<<1) | ((a&0xAAAAAAAAU)>>1);
8         a = ((a&0x33333333U)<<2) | ((a&0xCCCCCCCCU)>>2);
9         a = ((a&0x0F0F0F0FU)<<4) | ((a&0xF0F0F0FU)>>4);
10        a = ((a&0x00FF00FFU)<<8) | ((a&0xFF00FF00U)>>8);
11        a = ((a&0x0000FFFFU)<<16) | ((a&0xFFFF0000U)>>16);
12        return a >> (32-len);
13    }
14    void fft(bool is_inv, VT &in, VT &out, int N) {
15        int bitlen = __lg(N), num = is_inv ? -1 : 1;
16        for(int i = 0; i < N; ++i)
17            out[bit_reverse(i, bitlen)] = in[i];
18        for(int step = 2, mh = 1; step <= N; step <= 1, mh
19            <= 1){
20            for(int i = 0; i < mh; ++i){
21                complex<T> wi = exp(complex<T>(0, i * num *
22                    pi / mh));
23                for(int j = i, k = i + mh; j < N; j += step,
24                    k += step) {
25                    complex<T> u = out[j], t = wi * out[k];
26                    out[j] = u + t, out[k] = u - t;
27                }
28            }
29        }
30        for (int i = 0; is_inv && i < N; ++i)
31            out[i] /= N;
32    };
33    int main () { // polynomial multiplication
34        FFT<double> F; int n = 4;
35        vector<complex<double>> a = {1, 2, 0, 0};
36        vector<complex<double>> b = {2, 3, 0, 0};
37        vector<complex<double>> a_fft(n), b_fft(n), ab_fft(n), ab(n)
38            );
39        F.fft(0, a, a_fft, 4), F.fft(0, b, b_fft, 4);
40        for (int i = 0; i < n; i++)
41            ab_fft[i] = a_fft[i] * b_fft[i];
42        F.fft(1, ab_fft, ab, n);
43        for (auto p : ab)
44            cout << int(p.real() + 1e-6) << " ";
45        return 0;
46    }
47 };

```

## 6.6 FindRealRoot

```

1 // an*x^n + ... + a1x + a0 = 0;
2 int sign(double x){
3     return x < -eps ? -1 : x > eps;
4 }
5 double get(const vector<double>&coef, double x){
6     double e = 1, s = 0;
7     for(auto i : coef) s += i*e, e *= x;
8     return s;
9 }

```

```

10 double find(const vector<double>&coef, int n, double lo,
11     double hi){
12     double sign_lo, sign_hi;
13     if( !(sign_lo = sign(get(coef,lo))) ) return lo;
14     if( !(sign_hi = sign(get(coef,hi))) ) return hi;
15     if(sign_lo * sign_hi > 0) return INF;
16     for(int stp = 0; stp < 100 && hi - lo > eps; ++stp){
17         double m = (lo+hi)/2.0;
18         int sign_mid = sign(get(coef,m));
19         if(!sign_mid) return m;
20         if(sign_lo*sign_mid < 0) hi = m;
21         else lo = m;
22     }
23     return (lo+hi)/2.0;
24 }
25 vector<double> cal(vector<double>coef, int n){
26     vector<double>res;
27     if(n == 1){
28         if(sign(coef[1])) res.pb(-coef[0]/coef[1]);
29         return res;
30     }
31     vector<double>dcoef(n);
32     for(int i = 0; i < n; ++i) dcoef[i] = coef[i+1]*(i+1);
33     vector<double>droot = cal(dcoef, n-1);
34     droot.insert(droot.begin(), -INF);
35     droot.pb(INF);
36     for(int i = 0; i+1 < droot.size(); ++i){
37         double tmp = find(coef, n, droot[i], droot[i+1]);
38         if(tmp < INF) res.pb(tmp);
39     }
40     return res;
41 }
42 int main () {
43     vector<double>ve;
44     vector<double>ans = cal(ve, n);
45     // 視情況把答案 +eps，避免 -0

```

## 6.7 Fraction

```

1 #define cfl(str) (const frac& f) const { return str; }
2 #define cll(str) (ll l) const { return str; }
3 #define lfl(str) (ll l, const frac& f) { return str; }
4 #define ff inline frac operator
5 #define bb inline bool operator
6 #define fff inline friend frac operator
7 #define fbb inline friend bool operator
8
9 class frac {
10 private: ll x, y;
11 public:
12     frac() : x(0), y(1) {}
13     frac(ll v) : x(v), y(1) {}
14     frac(ll xx, ll yy, bool f = 0) : x(xx), y(yy) {
15         assert(y != 0);
16         if (!f) {
17             ll g = __gcd(x, y);
18             x /= g, y /= g;
19             if (y < 0) x *= -1, y *= -1;
20         }
21     }
22     // 以下斟酌使用，不必全抄

```

```

23 ff = (ll l) { return frac(l); }
24 ff - () const { return frac(-x, y, 1); }
25 ff ! () const { // 倒數
26     return x > 0 ? frac(y, x, 1) : frac(-y, -x, 1);
27 }
28
29 bb > cfl(x * f.y > y * f.x)
30 bb < cfl(x * f.y < y * f.x)
31 bb <= cfl(x * f.y <= y * f.x)
32 bb >= cfl(x * f.y >= y * f.x)
33 bb == cfl(x == f.x && y == f.y)
34 bb != cfl(x != f.x || y != f.y)
35 ff + cfl(frac(x * f.y + y * f.x, y * f.y))
36 ff - cfl(frac(x * f.y - y * f.x, y * f.y))
37 ff * cfl(frac(x * f.x, y * f.y))
38 ff / cfl(frac(x * f.y, y * f.x))
39
40 bb > cll(x > 1 * y)
41 bb < cll(x < 1 * y)
42 bb >= cll(x >= 1 * y)
43 bb <= cll(x <= 1 * y)
44 bb == cll(x == 1 * y)
45 bb != cll(x != 1 * y)
46 ff + cll(frac(x + 1 * y, y))
47 ff - cll(frac(x - 1 * y, y))
48 ff * cll(frac(1 * x, y))
49 ff / cll(frac(x, 1 * y))
50
51 fbb < lfl(f > 1)
52 fbb > lfl(f < 1)
53 fbb <= lfl(f >= 1)
54 fbb >= lfl(f <= 1)
55 fbb == lfl(f == 1)
56 fbb != lfl(f != 1)
57 fff + lfl(f + 1)
58 fff - lfl(-f + 1)
59 fff * lfl(f * 1)
60 fff / lfl(!f * 1)
61
62 inline operator double() { return (double)x / y; }
63 inline friend frac abs(const frac& f) {
64     return frac(abs(f.x), f.y, 1);
65 }
66 inline friend ostream& operator <<
67     (ostream & out, const frac& f) {
68     out << f.x;
69     if (f.y != 1) out << '/' << f.y;
70     return out;
71 }
72 };

```

## 6.8 Karatsuba

```

1 // N is power of 2
2 template<typename Iter>
3 void DC(int N, Iter tmp, Iter A, Iter B, Iter res){
4     fill(res, res+2*N, 0);
5     if (N<=32){
6         for (int i=0; i<N; i++)
7             for (int j=0; j<N; j++)
8                 res[i+j] += A[i]*B[j];
9         return;
10    }

```

```

11    int n = N/2;
12    auto a = A+n, b = A;
13    auto c = B+n, d = B;
14    DC(n, tmp+n, a, c, res+2*N);
15    for (int i=0; i<N; i++){
16        res[i+N] += res[2*N+i];
17        res[i+n] -= res[2*N+i];
18    }
19    DC(n, tmp+n, b, d, res+2*N);
20    for (int i=0; i<N; i++){
21        res[i] += res[2*N+i];
22        res[i+n] -= res[2*N+i];
23    }
24    auto x = tmp;
25    auto y = tmp+n;
26    for (int i=0; i<n; i++) x[i] = a[i]+b[i];
27    for (int i=0; i<n; i++) y[i] = c[i]+d[i];
28    DC(n, tmp+n, x, y, res+2*N);
29    for (int i=0; i<N; i++)
30        res[i+n] += res[2*N+i];
31 }
32 // DC(1<=16, tmp.begin(), A.begin(), B.begin(), res.begin());

```

## 6.9 Matrix

```

1 struct Matrix {
2     int r, c;
3     vector<vector<ll>> m;
4     Matrix(int r, int c): r(r), c(c), m(r, vector<ll>(c)) {}
5     vector<ll> &operator[](int i) { return m[i]; }
6     Matrix operator+(const Matrix &a) {
7         Matrix rev(r, c);
8         for (int i = 0; i < r; ++i)
9             for (int j = 0; j < c; ++j)
10                 rev[i][j] = m[i][j] + a.m[i][j];
11         return rev;
12    }
13    Matrix operator-(const Matrix &a) {
14        Matrix rev(r, c);
15        for (int i = 0; i < r; ++i)
16            for (int j = 0; j < c; ++j)
17                rev[i][j] = m[i][j] - a.m[i][j];
18        return rev;
19    }
20    Matrix operator*(const Matrix &a) {
21        Matrix rev(r, a.c);
22        Matrix tmp(a.c, a.r);
23        for (int i = 0; i < a.r; ++i)
24            for (int j = 0; j < a.c; ++j)
25                tmp[j][i] = a.m[i][j];
26        for (int i = 0; i < r; ++i)
27            for (int j = 0; j < a.c; ++j)
28                for (int k = 0; k < c; ++k)
29                    rev.m[i][j] += m[i][k] * tmp[j][k];
30        return rev;
31    }
32    // 回傳反矩陣。注意這是 const 方法所以原矩陣不受影響
33    Matrix inverse() const {
34        Matrix t(r, r + c);
35        for (int y = 0; y < r; y++) {
36            t.m[y][c + y] = 1;
37            for (int x = 0; x < c; x++) t.m[y][x] = m[y][x];
38        }

```

```

39        if (!t.gauss()) return Matrix(0, 0);
40        Matrix ret(c, r);
41        for (int y = 0; y < r; y++)
42            for (int x = 0; x < c; x++)
43                ret[y][x] = t.m[y][c + x] / t.m[y][y];
44        return ret;
45    }
46    // 做高斯消去 (最高次係數應置於最左, 常數應置於最右)
47    // 回傳 det * 0(n^3)。如果不是方陣, 回傳值無意義。
48    ll gauss() {
49        vector<ll> lazy(r, 1);
50        bool sign = false;
51        for (int i = 0; i < r; ++i) {
52            if (m[i][i] == 0) {
53                int j = i + 1;
54                while (j < r && !m[j][i]) j++;
55                if (j == r) continue;
56                m[i].swap(m[j]); sign = !sign;
57            }
58            for (int j = 0; j < r; ++j) {
59                if (i == j) continue;
60                lazy[j] = lazy[j] * m[i][i];
61                ll mx = m[j][i];
62                for (int k = 0; k < c; ++k)
63                    m[j][k] =
64                        m[j][k] * m[i][i] - m[i][k] * mx;
65            }
66        }
67        ll det = sign ? -1 : 1;
68        for (int i = 0; i < r; ++i) {
69            det = det * m[i][i] / lazy[i];
70            for (auto &j : m[i]) j /= lazy[i];
71        }
72        return det;
73    }
74 };

```

## 6.10 MillerRabin

```

1 //From jacky860226
2 typedef long long LL;
3 inline LL mul(LL a, LL b, LL m){/*a*b%m
4     return (a%m)*(b%m)%m;
5 }
6 /*LL mul(LL a, LL b, LL m){/*a*b%m
7     a %= m, b %= m;
8     LL y = (LL)((double)a*b/m+0.5); //fast for m < 2^58
9     LL r = (a*b-y*m)%m;
10    return r<0 ? r+m : r;
11 }*/
12 template<typename T> T pow(T a, T b, T mod) { //a^b%mod
13     T ans = 1;
14     while(b) {
15         if(b&1) ans = mul(ans, a, mod);
16         a = mul(a, a, mod);
17         b >>= 1;
18     } return ans;
19 }
20 template<typename T> bool isprime(T n, int num) { //num = 3,7
21     int sprp[3] = {2,7,61}; //int範圍可解
22     //int ll sprp[7] =
23         {2,325,9375,28178,450775,9780504,1795265022}; //至少

```

```

        unsigned long long範圍
23 if(n==2) return true;
24 if(n<2 || n%2==0) return false;
25 //n-1 = u * 2^t
26 int t = 0; T u = n-1;
27 while(u%2==0) u >>= 1, t++;
28 for(int i=0; i<num; i++) {
29     T a = sprp[i]%n;
30     if(a==0 || a==1 || a==n-1) continue;
31     T x = pow(a,u,n);
32     if(x==1 || x==n-1) continue;
33     for(int j=1; j<t; j++) {
34         x = mul(x,x,n);
35         if(x==1) return false;
36         if(x==n-1) break;
37     }
38     if(x!=n-1) return false;
39 } return true;
40 }

```

## 6.11 ModInv

```

1 int phi(int x) {
2     int r = x;
3     for (int p = 2; p * p <= x; p++) {
4         if (x % p == 0) {
5             while (x % p == 0) x /= p;
6             r -= r / p;
7         }
8     }
9     if (x > 1) r -= r / x;
10    return r;
11 }
12 // 解 (ax == 1) mod b。a、b 互質整數，否則不存在modinv。
13 ll modinv(ll a, ll b){
14     if(__gcd(a, b) != 1) return -1;
15     // Euler 定理: a^phi(b) == 1 (mod b)
16     // -> a^(phi(b) - 1) is the mod inverse to b of a
17     int mod_inv_pow = phi(b) - 1;
18     int ans = 1, base = a % b;
19     while(mod_inv_pow > 0){
20         if(mod_inv_pow & 1)
21             ans = ans * base % b;
22         base = base * base % b;
23         mod_inv_pow >>= 1;
24     } return ans;
25 }
26 ll modinv(ll a, ll p) { //(ax == 1)mod p, p質數, a正整數
27     if (p == 1) return 0;
28     ll pp = p, y = 0, x = 1;
29     while (a > 1) {
30         ll q = a / p, t = p;
31         p = a % p, a = t, t = y, y = x - q * y, x = t;
32     }
33     if (x < 0) x += pp;
34     return x;
35 }
36 // 解 (ax == b) mod p。p 必須是質數, a 和 b 是正整數。
37 ll modinv(ll a, ll b, ll p) {
38     ll ret = modinv(a, p);
39     return ret * b % p;
40 }

```

## 6.12 NTT

```

1 template<typename T, typename VT=std::vector<T> >
2 struct NTT{
3     const T P,G;
4     NTT(T p=(1<<23)*7*17+1,T g=3):P(p),G(g){
5         inline unsigned int bit_reverse(unsigned int a,int len){
6             a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
7             a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
8             a=((a&0x0F0F0F0FU)<<4)|((a&0xFF0F0F0FU)>>4);
9             a=((a&0x00FF00FFU)<<8)|((a&0xFFFF00FFU)>>8);
10            a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
11            return a>>(32-len);
12        }
13        inline T pow_mod(T n,T k,T m){
14            T ans=1;
15            for(n=(n>=m?n%m:n);k;k>>=1){
16                if(k&1)ans=ans*n%m;
17                n=n*n%m;
18            } return ans;
19        }
20        inline void ntt(bool is_inv,VT &in,VT &out,int N){
21            int bitlen=std::__lg(N);
22            for(int i=0;i<N;++i)out[bit_reverse(i,bitlen)]=in[i];
23            for(int step=2,id=1;step<=N;step<=1,++id){
24                T wn=pow_mod(G,(P-1)>>id,P),wi=1,u,t;
25                const int mh=step>>1;
26                for(int i=0;i<mh;++i){
27                    for(int j=i;j<N;j+=step){
28                        u = out[j], t = wi*out[j+mh]%P;
29                        out[j] = u+t;
30                        out[j+mh] = u-t;
31                        if(out[j]>=P)out[j]-=P;
32                        if(out[j+mh]<0)out[j+mh]+=P;
33                    }
34                    wi = wi*wn%P;
35                }
36            }
37            if(is_inv){
38                for(int i=1;i<N/2;++i)std::swap(out[i],out[N-i]);
39                T invn=pow_mod(N,P-2,P);
40                for(int i=0;i<N;++i)out[i]=out[i]*invn%P;
41            }
42        }
43    };
44    #endif

```

## 6.13 PrimeList

```

1 12721      13331      14341      75577
2 123457     222557     556679     880301
3 999983     1e6+99      1e9+9      2e9+99
4 1e12+39    1e15+37      1e9+7      1e7+19
5 1097774749 1076767633    100102021
6 999997771 1001010013    1000512343
7 987654361 999991231     999888733
8 98789101 987777733   999991921
9 1010101333 1010102101
10 2305843009213693951 4611686018427387847
11 9223372036854775783 18446744073709551557

```

## 6.14 SG

```

1 Anti Nim (取走最後一個石子者敗) :
2 先手必勝 if and only if
3 1. 「所有」堆的石子數都為 1 且遊戲的 SG 值為 0。
4 2. 「有些」堆的石子數大於 1 且遊戲的 SG 值不為 0。
5 -----
6 Anti-SG (決策集合為空的遊戲者贏) :
7 定義 SG 值為 0 時，遊戲結束，
8 則先手必勝 if and only if
9 1. 遊戲中沒有單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數為 0。
10 2. 遊戲中某個單一遊戲的 SG 函數大於 1 且遊戲的 SG 函數不為 0。
11 -----
12 Sprague-Grundy :
13 1. 雙人、回合制
14 2. 資訊完全公開
15 3. 無隨機因素
16 4. 可在有限步內結束
17 5. 沒有和局
18 6. 雙方可採取的行動相同
19
20 SG(S) 的值為 0 : 後手(P)必勝
21 不為 0 : 先手(N)必勝
22 int mex(set S) {
23     // find the min number >= 0 that not in the S
24     // e.g. S = {0, 1, 3, 4} mex(S) = 2
25 }
26 state = []
27 int SG(A) {
28     if (A not in state) {
29         S = sub_states(A)
30         if( len(S) > 1 ) state[A] = reduce(operator.xor, [SG(B)
31             for B in S])
32         else state[A] = mex(set(SG(B) for B in next_states(A)))
33     } return state[A]
34 }

```

## 6.15 Simplex

```

1 /*target:
2 max \sum_{j=1}^n A_{0,j}*x_j
3 condition:
4 \sum_{j=1}^n A_{i,j}*x_j <= A_{i,0} | i=1~m
5 x_j >= 0 | j=1~n
6 VDB = vector<double>*/
7 template<class VDB>
8 VDB simplex(int m,int n,vector<VDB> a){
9     vector<int> left(m+1), up(n+1);
10    iota(left.begin(), left.end(), n);
11    iota(up.begin(), up.end(), 0);
12    auto pivot = [&](int x, int y){
13        swap(left[x], up[y]);
14        auto k = a[x][y]; a[x][y] = 1;
15        vector<int> pos;
16        for(int j = 0; j <= n; ++j){
17            a[x][j] /= k;
18            if(a[x][j] != 0) pos.push_back(j);
19        }
20    };

```

```

19     }
20     for(int i = 0; i <= m; ++i){
21         if(a[i][y]==0 || i == x) continue;
22         k = a[i][y], a[i][y] = 0;
23         for(int j : pos) a[i][j] -= k*a[x][j];
24     }
25 };
26 for(int x,y;;){
27     for(int i=x=1; i <= m; ++i)
28         if(a[i][0]<a[x][0]) x = i;
29     if(a[x][0]>=0) break;
30     for(int j=y=1; j <= n; ++j)
31         if(a[x][j]<a[x][y]) y = j;
32     if(a[x][y]>=0) return VDB();//infeasible
33     pivot(x, y);
34 }
35 for(int x,y;;){
36     for(int j=y=1; j <= n; ++j)
37         if(a[0][j] > a[0][y]) y = j;
38     if(a[0][y]<=0) break;
39     x = -1;
40     for(int i=1; i<=m; ++i) if(a[i][y] > 0)
41         if(x == -1 || a[i][0]/a[i][y]
42            < a[x][0]/a[x][y]) x = i;
43     if(x == -1) return VDB();//unbounded
44     pivot(x, y);
45 }
46 VDB ans(n + 1);
47 for(int i = 1; i <= m; ++i)
48     if(left[i] <= n) ans[left[i]] = a[i][0];
49 ans[0] = -a[0][0];
50 return ans;
51 }

```

## 6.16 外星模運算

```

1 //a[0]^(a[1]^a[2]^...)
2 #define maxn 100000
3 int euler[maxn+5];
4 bool is_prime[maxn+5];
5 void init_euler(){
6     is_prime[1] = 1; //一不是質數
7     for(int i=1; i<=maxn; i++) euler[i]=i;
8     for(int i=2; i<=maxn; i++) {
9         if(!is_prime[i]) { //是質數
10             euler[i]--;
11             for(int j=i<1; j<=maxn; j+=i) {
12                 is_prime[j]=1;
13                 euler[j] = euler[j]/i*(i-1);
14             }
15         }
16     }
17 }
18 LL pow(LL a, LL b, LL mod) { //a^b%mod
19     LL ans=1;
20     for(; b; a=a*a%mod, b>>=1)
21         if(b&1) ans = ans*a%mod;
22     return ans;
23 }
24 bool isless(LL *a, int n, int k) {
25     if(*a==1)return k>1;
26     if(--n==0)return *a<k;

```

```

27     int next=0;
28     for(LL b=1;b<k;++next)
29         b *= *a;
30     return isless(a+1, n, next);
31 }
32 LL high_pow(LL *a, int n, LL mod){
33     if(*a==1||--n==0)return *a%mod;
34     int k = 0, r = euler[mod];
35     for(LL tma=1;tma!=pow(*a,k+r,mod);++k)
36         tma = tma*(a)%mod;
37     if(isless(a+1,n,k))return pow(*a,high_pow(a+1,n,k),mod);
38     int tmd = high_pow(a+1,n,r), t = (tmd-k+r)%r;
39     return pow(*a,k+t,mod);
40 }
41 LL a[1000005]; int t,mod;
42 int main(){
43     init_euler();
44     scanf("%d", &t);
45     #define n 4
46     while(t--){
47         for(int i=0;i<n;++i)scanf("%lld", &a[i]);
48         scanf("%d", &mod);
49         printf("%lld\n", high_pow(a,n,mod));
50     }
51     return 0;
52 }

```

## 6.17 質因數分解

```

1 LL func(const LL n,const LL mod,const int c) {
2     return (LLmul(n,n,mod)+c+mod)%mod;
3 }
4 LL pollorrho(const LL n, const int c) { //循環節長度
5     LL a=1, b=1;
6     a=func(a,n,c)%n;
7     b=func(b,n,c)%n; b=func(b,n,c)%n;
8     while(gcd(abs(a-b),n)==1) {
9         a=func(a,n,c)%n;
10        b=func(b,n,c)%n; b=func(b,n,c)%n;
11    }
12    return gcd(abs(a-b),n);
13 }
14 void prefactor(LL &n, vector<LL> &v) {
15     for(int i=0;i<12;++i) {
16         while(n%prime[i]==0) {
17             v.push_back(prime[i]);
18             n/=prime[i];
19         }
20     }
21 }
22 void smallfactor(LL n, vector<LL> &v) {
23     if(n<MAXPRIME) {
24         while(isp[(int)n]) {
25             v.push_back(isp[(int)n]);
26             n/=isp[(int)n];
27         }
28         v.push_back(n);
29     } else {
30         for(int i=0;i<primecnt&&prime[i]*prime[i]<=n;++i) {
31             while(n%prime[i]==0) {
32                 v.push_back(prime[i]);
33                 n/=prime[i];
34             }

```

```

35     }
36     if(n!=1) v.push_back(n);
37 }
38 }
39 void comfactor(const LL &n, vector<LL> &v) {
40     if(n<1e9) {
41         smallfactor(n,v);
42         return;
43     }
44     if(Isprime(n)) {
45         v.push_back(n);
46         return;
47     }
48     LL d;
49     for(int c=3;++c) {
50         d = pollorrho(n,c);
51         if(d!=n) break;
52     }
53     comfactor(d,v);
54     comfactor(n/d,v);
55 }
56 void Factor(const LL &x, vector<LL> &v) {
57     LL n = x;
58     if(n==1) { puts("Factor 1"); return; }
59     prefactor(n,v);
60     if(n==1) return;
61     comfactor(n,v);
62     sort(v.begin(),v.end());
63 }
64 void AllFactor(const LL &n,vector<LL> &v) {
65     vector<LL> tmp;
66     Factor(n,tmp);
67     v.clear();
68     v.push_back(1);
69     int len;
70     LL now=1;
71     for(int i=0;i<tmp.size();++i) {
72         if(i==0 || tmp[i]!=tmp[i-1]) {
73             len = v.size();
74             now = 1;
75         }
76         now*=tmp[i];
77         for(int j=0;j<len;++j)
78             v.push_back(v[j]*now);
79     }
80 }

```

## 7 String

### 7.1 AC 自動機

```

1 template<char L='a',char R='z'>
2 class ac_automaton{
3     struct joe{
4         int next[R-L+1], fail, efl, ed, cnt_dp, vis;
5         joe():ed(0),cnt_dp(0),vis(0){
6             for(int i=0; i<=R-L; i++) next[i]=0;
7         }
8     };
9     public:
10     std::vector<joe> S;

```



```

11 std::vector<int> q;
12 int qs, qe, vt;
13 ac_automaton(): S(1), qs(0), qe(0), vt(0){
14 void clear(){
15     q.clear();
16     S.resize(1);
17     for(int i=0; i<=R-L; i++) S[0].next[i] = 0;
18     S[0].cnt_dp = S[0].vis = qs = qe = vt = 0;
19 }
20 void insert(const char *s){
21     int o = 0;
22     for(int i=0, id; s[i]; i++){
23         id = s[i]-L;
24         if(!S[o].next[id]){
25             S.push_back(joe());
26             S[o].next[id] = S.size()-1;
27         }
28         o = S[o].next[id];
29     }
30     ++S[o].ed;
31 }
32 void build_fail(){
33     S[0].fail = S[0].efl = -1;
34     q.clear();
35     q.push_back(0);
36     ++qe;
37     while(qs!=qe){
38         int pa = q[qs++], id, t;
39         for(int i=0; i<=R-L; i++){
40             t = S[pa].next[i];
41             if(!t) continue;
42             id = S[pa].fail;
43             while(~id && !S[id].next[i]) id = S[id].fail;
44             S[t].fail = ~id ? S[id].next[i] : 0;
45             S[t].efl = S[S[t].fail].ed ? S[t].fail : S[S[t].fail].efl;
46             q.push_back(t);
47             ++qe;
48         }
49     }
50 }
51 /*DP出每個前綴在字串s出現的次數並傳回所有字串被s匹配成功的
   次數O(N*M)*/
52 int match_0(const char *s){
53     int ans = 0, id, p = 0, i;
54     for(i=0; s[i]; i++){
55         id = s[i]-L;
56         while(!S[p].next[id] && p) p = S[p].fail;
57         if(!S[p].next[id]) continue;
58         p = S[p].next[id];
59         ++S[p].cnt_dp; /*匹配成功則它所有後綴都可以被匹配(DP計算)*/
60     }
61     for(i=qe-1; i>=0; --i){
62         ans += S[q[i]].cnt_dp * S[q[i]].ed;
63         if(~S[q[i]].fail) S[S[q[i]].fail].cnt_dp += S[q[i]].cnt_dp;
64     }
65     return ans;
66 }
67 /*多串匹配走efl邊並傳回所有字串被s匹配成功的次數O(N*M^1.5)*/
68 int match_1(const char *s) const{
69     int ans = 0, id, p = 0, t;
70     for(int i=0; s[i]; i++){

```

```

71     id = s[i]-L;
72     while(!S[p].next[id] && p) p = S[p].fail;
73     if(!S[p].next[id]) continue;
74     p = S[p].next[id];
75     if(S[p].ed) ans += S[p].ed;
76     for(t=S[p].efl; ~t; t=S[t].efl){
77         ans += S[t].ed; /*因為都走efl邊所以保證匹配成功*/
78     }
79 }
80 return ans;
81 }
82 /*枚舉(s的字子串⊆A)的所有相異字串各恰一次並傳回次數O(N*M
   ^{(1/3)})*/
83 int match_2(const char *s){
84     int ans=0, id, p=0, t;
85     ++vt;
86     /*把戳記vt+=1, 只要vt沒溢位, 所有S[p].vis==vt就會變成false
   這種利用vt的方法可以O(1)歸零vis陣列*/
87     for(int i=0; s[i]; i++){
88         id = s[i]-L;
89         while(!S[p].next[id] && p) p = S[p].fail;
90         if(!S[p].next[id]) continue;
91         p = S[p].next[id];
92         if(S[p].ed && S[p].vis!=vt){
93             S[p].vis = vt;
94             ans += S[p].ed;
95         }
96     }
97     for(t=S[p].efl; ~t && S[t].vis!=vt; t=S[t].efl){
98         S[t].vis = vt;
99         ans += S[t].ed; /*因為都走efl邊所以保證匹配成功*/
100     }
101 }
102 return ans;
103 }
104 /*把AC自動機變成真的自動機*/
105 void evolution(){
106     for(qs=1; qs!=qe;){
107         int p = q[qs++];
108         for(int i=0; i<=R-L; i++){
109             if(S[p].next[i]==0) S[p].next[i] = S[S[p].fail].next[i];
110         }
111     }
112 }

```

## 7.2 BWT

```

1 const int N = 8; // 字串長度
2 int s[N+M+1] = "suffixes"; // 字串, 後面預留一倍空間。
3 int sa[N]; // 後綴陣列
4 int pivot;
5 int cmp(const void* i, const void* j) {
6     return strcmp(s+*(int*)i, s+*(int*)j, N);
7 }
8 // 此處便宜行事, 採用 O(N^2 log N) 的後綴陣列演算法。
9 void BWT() {
10     strncpy(s + N, s, N);
11     for (int i=0; i<N; ++i) sa[i] = i;
12     qsort(sa, N, sizeof(int), cmp);
13     // 當輸入字串的所有字元都相同, 必須當作特例處理。

```

```

14 // 或者改用stable sort。
15 for (int i=0; i<N; ++i)
16     cout << s[(sa[i] + N-1) % N];
17 for (int i=0; i<N; ++i)
18     if (sa[i] == 0) {
19         pivot = i;
20         break;
21     }
22 }
23 // Inverse BWT
24 const int N = 8; // 字串長度
25 char t[N+1] = "xuffessi"; // 字串
26 int pivot;
27 int next[N];
28 void IBWT() {
29     vector<int> index[256];
30     for (int i=0; i<N; ++i)
31         index[t[i]].push_back(i);
32     for (int i=0, n=0; i<256; ++i)
33         for (int j=0; j<index[i].size(); ++j)
34             next[n++] = index[i][j];
35     int p = pivot;
36     for (int i=0; i<N; ++i)
37         cout << t[p = next[p]];
38 }

```

## 7.3 Kmp

```

1 // KMP fail function.
2 int* kmp_fail(string& s) {
3     int* f = new int[s.size()]; int p = f[0] = -1;
4     for (int i = 1; s[i]; i++) {
5         while (p != -1 && s[p+1] != s[i]) p = f[p];
6         if (s[p+1] == s[i]) p++;
7         f[i] = p;
8     }
9     return f;
10 }
11 // 問 sub 在 str 中出現幾次。
12 int kmp_count(string& str, string& sub) {
13     int* fail = kmp_fail(sub); int p = -1, ret = 0;
14     for (int i = 0; i < str.size(); i++) {
15         while (p != -1 && sub[p+1] != str[i]) p = fail[p];
16         if (sub[p+1] == str[i]) p++;
17         if (p == sub.size() - 1) p = fail[p], ret++;
18     }
19     delete[] fail; return ret;
20 }
21 // 問 sub 在 str 第一次出現的開頭 index。-1 表示找不到。
22 int kmp(string& str, string& sub) {
23     int* fail = kmp_fail(sub);
24     int i, j = 0;
25     while (i < str.size() && j < sub.size()) {
26         if (sub[j] == str[i]) i++, j++;
27         else if (j == 0) i++;
28         else j = fail[j-1] + 1;
29     }
30     delete[] fail;
31     return j == sub.size() ? (i - j) : -1;
32 }

```

## 7.4 LPS

```

1 char t[1001];           // 原字串
2 char s[1001 * 2];       // 穿插特殊字元之後的t
3 int z[1001 * 2], L, R;   // 源自Gusfield's Algorithm
4 // 由a往左、由b往右，對稱地作字元比對。
5 int extend(int a, int b) {
6     int i = 0;
7     while (a-i>=0 && b+i<N && s[a-i] == s[b+i]) i++;
8     return i;
9 }
10 void longest_palindromic_substring() {
11     int N = strlen(t);
12     // t穿插特殊字元，存放到s。
13     // （實際上不會這麼做，都是細算索引值。）
14     memset(s, '.', N*2+1);
15     for (int i=0; i<N; ++i) s[i*2+1] = t[i];
16     N = N*2+1;
17     // s[N] = '\0'; // 可做可不做
18     // Manacher's Algorithm
19     z[0] = 1; L = R = 0;
20     for (int i=1; i<N; ++i) {
21         int ii = L - (i - L); // i的映射位置
22         int n = R + 1 - i;
23         if (i > R) {
24             z[i] = extend(i, i);
25             L = i;
26             R = i + z[i] - 1;
27         } else if (z[ii] == n) {
28             z[i] = n + extend(i-n, i+n);
29             L = i;
30             R = i + z[i] - 1;
31         } else z[i] = min(z[ii], n);
32     }
33     // 尋找最長迴文子串的長度。
34     int n = 0, p = 0;
35     for (int i=0; i<N; ++i)
36         if (z[i] > n) n = z[p = i];
37     // 記得去掉特殊字元。
38     cout << "最長迴文子串的長度是" << (n-1) / 2;
39     // 印出最長迴文子串，記得別印特殊字元。
40     for (int i=p-z[p]+1; i<=p+z[p]-1; ++i)
41         if (i & 1) cout << s[i];
42 }

```

## 7.5 Manacher

```

1 // Longest Palindromic Substring
2 int manacher (string str) { // O(n)
3     int len = (s.length() << 1) | 1;
4     vector<int> z(len);
5     string s(len, '$');
6     for (int i = 1; i < len; i += 2)
7         s[i] = str[i >> 1];
8     int r = 0, p = 0, ans = 0;
9     for (int i = 0, j = p << 1; i < len; i++, j--) {
10         z[i] = (i >= r) ? 1 : min(z[j], r - i + 1);
11         while(0 <= i - z[i] && i + z[i] < len && s[i - z[i]] == s[i + z[i]])
12             z[i]++;

```

```

13         if (r < i + z[i] - 1)
14             r = i + z[i] - 1, p = i;
15         ans = max(ans, z[i]);
16     }
17     return ans - 1;
18 }

```

## 7.6 RollHash

```

1 // 問 pat 在 str 第一次出現的開頭 index 。 -1 表示找不到。
2 int rollhash(string& str, string& pat) {
3     const ll x = 1e6 + 99; // 隨意大質數，建議 1e6
4     const ll m = 1e9 + 9; // 隨意大質數，建議 1e9
5     assert(pat.size()); // pat 不能是空字串
6     ll xx = 1, sh = 0;
7     for (char c : pat)
8         sh = (sh * x + c) % m, xx = xx * x % m;
9     deque<ll> hash = {0};
10    int ret = 0;
11    for (char c : str) {
12        hash.push_back((hash.back() * x + c) % m);
13        if (hash.size() <= pat.size()) continue;
14        ll h = hash.back() - hash.front() * xx;
15        h = (h % m + m) % m;
16        if (h == sh) return ret;
17        hash.pop_front();
18        ret++;
19    }
20    return -1;

```

## 7.7 suffix\_array

```

1 // qsort suffix array, 0-based only, O(T * log^2 T)
2 const int N = ?; // 字串最大長度
3 namespace SA {
4     int sa[N], t0[N], t1[N];
5     struct CMP {
6         int *r, n, X;
7         bool operator()(int i, int j) {
8             if (r[i] != r[j]) return r[i] < r[j];
9             int a = (i + n < X) ? r[i + n] : -1;
10            int b = (j + n < X) ? r[j + n] : -1;
11            return a < b;
12        }
13    };
14    // str = 字串，可為 vector 或 string 或 char[] 等
15    // n = 字串長(含$)
16    // 結果存在 SA::sa
17    template <typename T>
18    void build(const T &str) {
19        int n = str.size();
20        int *a = t0, *aa = t1;
21        for (int i = 0; i < n; ++i) sa[i] = i, a[i] = str[i];
22        for (int m = 2; m <= n; m *= 2) {
23            CMP cmp = {a, m / 2, n};
24            sort(sa, sa + n, cmp);
25            int r = 0;
26            aa[sa[0]] = r;

```

```

27         for (int i = 1; i < n; ++i) {
28             if (cmp(sa[i - 1], sa[i])) r++;
29             aa[sa[i]] = r;
30         }
31         swap(a, aa);
32         if (r == n - 1) break;
33     }
34 }
35 // namespace SA
36
37 // 掛長的 IS suffix array，0-based only
38 // N = 字串最大長度，A = 最大字元 ascii
39 // 複雜度 O(N+A)
40 const int N = ?, A = ?;
41 namespace SA {
42     #define pushS(x) sa[--b[s[x]]] = x
43     #define pushL(x) sa[b[s[x]]++] = x
44     #define induce_sort(v)
45     {
46         fill_n(sa, n, 0);
47         copy_n(bb, A, b);
48         for (i = n1 - 1; ~i; --i) pushS(v[i]);
49         copy_n(bb, A - 1, b + 1);
50         for (i = 0; i < n; ++i)
51             if (sa[i] && !t[sa[i] - 1]) pushL(sa[i] - 1);
52         copy_n(bb, A, b);
53         for (i = n - 1; ~i; --i)
54             if (sa[i] && t[sa[i] - 1]) pushS(sa[i] - 1);
55     }
56 template <typename T>
57 void sais(const T s, int n, int *sa, int *bb, int *p, bool *t
58         , int A) {
59     int *r = p + n, *s1 = p + n / 2, *b = bb + A;
60     int n1 = 0, i, j, x = t[n - 1] = 1, y = r[0] = -1, cnt =
61         -1;
62     for (i = n - 2; ~i; --i) t[i] = (s[i] == s[i + 1] ? t[i +
63         1] : s[i] < s[i + 1]);
64     for (i = 1; i < n; ++i) r[i] = t[i] && !t[i - 1] ? (p[n1]
65         = i, n1++) : -1;
66     fill_n(bb, A, 0);
67     for (i = 0; i < n; ++i) ++bb[s[i]];
68     for (i = 1; i < A; ++i) bb[i] += bb[i - 1];
69     induce_sort(p);
70     for (i = 0; i < n; ++i)
71         if (~(x = r[sa[i]]))
72             j = y < 0 || memcmp(s + p[x], s + p[y], (p[x + 1]
73                 - p[x]) * sizeof(s[0])) ? s1[y = x] = cnt +=
74                 j;
75     if (cnt + 1 < n1)
76         sais(s1, n1, sa, b, r, t + n, cnt + 1);
77     else
78         for (i = 0; i < n1; ++i) sa[s1[i]] = i;
79     for (i = 0; i < n1; ++i) s1[i] = p[sa[i]];
80     induce_sort(s1);
81 }
82 int sa[N];
83 int b[N + A], p[N * 2];
84 bool t[N * 2];
85 // 計算 suffix array，字串須為 char[] 或 int[]，不可為
86 // string 或 vector
87 // s = 字串
88 // n = 字串長度(含$)
89 // 結果存在 SA::sa
90 template <typename T>

```

```

84 void build(const T s, int n) { sais(s, n, sa, b, p, t, A); }
85 } // namespace SA

```

## 7.8 Trie

```

1 class Trie {
2 private:
3     struct Node {
4         int cnt = 0, sum = 0;
5         Node *tr[128] = {};
6         ~Node() {
7             for (int i = 0; i < 128; i++)
8                 if (tr[i]) delete tr[i];
9         }
10    };
11    Node *root;
12 public:
13    void insert(char *s) {
14        Node *ptr = root;
15        for (; *s; s++) {
16            if (!ptr->tr[*s]) ptr->tr[*s] = new Node();
17            ptr = ptr->tr[*s];
18            ptr->sum++;
19        }
20        ptr->cnt++;
21    }
22    inline int count(char *s) {
23        Node *ptr = find(s);
24        return ptr ? ptr->cnt : 0;
25    }
26    Node *find(char *s) {
27        Node *ptr = root;
28        for (; *s; s++) {
29            if (!ptr->tr[*s]) return 0;
30            ptr = ptr->tr[*s];
31        } return ptr;
32    }
33    bool erase(char *s) {
34        Node *ptr = find(s);
35        if (!ptr) return false;
36        int num = ptr->cnt;
37        if (!num) return false;
38        ptr = root;
39        for (; *s; s++) {
40            Node *tmp = ptr;
41            ptr = ptr->tr[*s];
42            ptr->sum -= num;
43            if (!ptr->sum) {
44                delete ptr;
45                tmp->tr[*s] = 0;
46                return true;
47            }
48        }
49    }
50    Trie() { root = new Node(); }
51    ~Trie() { delete root; }
52 };

```

## 7.9 Z

```

1 void z_build(string &s, vector<int> &z) {
2     int bst = z[0] = 0;
3     for (int i = 1; s[i]; i++) {
4         if (z[bst] + bst < i) z[i] = 0;
5         else z[i] = min(z[bst] + bst - i, z[i - bst]);
6         while (s[z[i]] == s[i + z[i]]) z[i]++;
7         if (z[i] + i > z[bst] + bst) bst = i;
8     }
9 }
10 // Queries how many times s appears in t
11 int z_match(string &s, string &t) {
12     int ans = 0;
13     int lens = s.length(), lent = t.length();
14     vector<int> z(lens + lent + 1);
15     string st = s + "$" + t;
16     z_build(st, z);
17     for (int i = lens + 1; i <= lens + lent; i++)
18         if (z[i] == lens) ans++;
19     return ans;
20 }

```

## 8 DP

### 8.1 Bounded\_Knapsack

```

1 namespace {
2     static const int MAXW = 1000005;
3     static const int MAXN = 1005;
4     struct BB {
5         int w, v, c;
6         BB(int w = 0, int v = 0, int c = 0): w(w), v(v), c(c) {}
7         bool operator<(const BB &x) const {
8             return w * c < x.w * x.c;
9         }
10    };
11    static int run(BB A[], int dp[], int W, int N) {
12        static int MQ[MAXW][2];
13        for (int i = 0, sum = 0; i < N; i++) {
14            int w = A[i].w, v = A[i].v, c = A[i].c;
15            sum = min(sum + w*c, W);
16            for (int j = 0; j < w; j++) {
17                int l = 0, r = 0;
18                MQ[l][0] = 0, MQ[l][1] = dp[j];
19                for (int k = 1, tw = w+j, tv = v; tw <= sum
20                    && k <= c; k++, tw += w, tv += v) {
21                    int dpv = dp[tw] - tv;
22                    while (1 <= r && MQ[r][1] <= dpv) r--;
23                    r++;
24                    MQ[r][0] = k, MQ[r][1] = dpv;
25                    dp[tw] = max(dp[tw], MQ[l][1] + tv);
26                }
27                for (int k = c+1, tw = (c+1)*w+j, tv = (c+1)*
28                    v; tw <= sum; k++, tw += w, tv += v) {
29                    if (k - MQ[l][0] > c) l++;
30                    int dpv = dp[tw] - tv;
31                    while (1 <= r && MQ[r][1] <= dpv) r--;
32                    r++;
33                    MQ[r][0] = k, MQ[r][1] = dpv;
34                    dp[tw] = max(dp[tw], MQ[l][1] + tv);
35                }
36            }
37        }
38    }
39 }

```

```

34     }
35 }
36 }
37 static int knapsack(int C[][3], int N, int W) { // O(WN)
38     vector<BB> A;
39     for (int i = 0; i < N; i++) {
40         int w = C[i][0], v = C[i][1], c = C[i][2];
41         A.push_back(BB(w, v, c));
42     }
43     assert(N < MAXN);
44     static int dp1[MAXW+1], dp2[MAXW+1];
45     BB Ar[2][MAXN];
46     int ArN[2] = {};
47     memset(dp1, 0, sizeof(dp1[0])*(W+1));
48     memset(dp2, 0, sizeof(dp2[0])*(W+1));
49     sort(A.begin(), A.end());
50     int sum[2] = {};
51     for (int i = 0; i < N; i++) {
52         int ch = sum[1] < sum[0];
53         Ar[ch][ArN[ch]] = A[i];
54         ArN[ch]++;
55         sum[ch] = min(sum[ch] + A[i].w*A[i].c, W);
56     }
57     run(Ar[0], dp1, W, ArN[0]);
58     run(Ar[1], dp2, W, ArN[1]);
59     int ret = 0;
60     for (int i = 0, j = W, mx = 0; i <= W; i++, j--) {
61         mx = max(mx, dp2[i]);
62         ret = max(ret, dp1[j] + mx);
63     }
64     return ret;
65 }
66 }
67 int main() {
68     int W, N;
69     assert(scanf("%d %d", &W, &N) == 2);
70     int C[MAXN][3];
71     for (int i = 0; i < N; i++)
72         assert(scanf("%d %d %d", &C[i][1], &C[i][0], &C[i][2]) == 3);
73     printf("%d\n", knapsack(C, N, W));
74     return 0;
75 }

```

### 8.2 DP\_1D1D

```

1 int t, n, L, p;
2 char s[MAXN][35];
3 ll sum[MAXN] = {0};
4 long double dp[MAXN] = {0};
5 int prevd[MAXN] = {0};
6 long double pw(long double a, int n) {
7     if (n == 1) return a;
8     long double b = pw(a, n/2);
9     if (n & 1) return b*b*a;
10    else return b*b;
11 }
12 long double f(int i, int j) {
13     // cout << (sum[i] - sum[j]+i-j-1-L) << endl;
14     return pw(abs(sum[i] - sum[j]+i-j-1-L), p) + dp[j];
15 }
16 struct INV {
17     int L, R, pos;

```

```

18 };
19 INV stk[MAXN*10];
20 int top = 1, bot = 1;
21 void update(int i) {
22     while ( top > bot && i < stk[top].L && f(stk[top].L, i) <
23           f(stk[top].L, stk[top].pos) ) {
24         stk[top - 1].R = stk[top].R;
25         top--;
26     }
27     int lo = stk[top].L, hi = stk[top].R, mid, pos = stk[top]
28         ].pos;
29     // if ( i >= lo ) lo = i + 1;
30     while ( lo != hi ) {
31         mid = lo + (hi - lo) / 2;
32         if ( f(mid, i) < f(mid, pos) ) hi = mid;
33         else lo = mid + 1;
34     }
35     if ( hi < stk[top].R ) {
36         stk[top + 1] = (INV) { hi, stk[top].R, i };
37         stk[top++].R = hi;
38     }
39 }
40 int main() {
41     cin >> t;
42     while ( t-- ) {
43         cin >> n >> L >> p;
44         dp[0] = sum[0] = 0;
45         for ( int i = 1 ; i <= n ; i++ ) {
46             cin >> s[i];
47             sum[i] = sum[i-1] + strlen(s[i]);
48             dp[i] = numeric_limits<long double>::max();
49         }
50         stk[top] = (INV) {1, n + 1, 0};
51         for ( int i = 1 ; i <= n ; i++ ) {
52             if ( i >= stk[bot].R ) bot++;
53             dp[i] = f(i, stk[bot].pos);
54             update(i);
55             // cout << (11) f(i, stk[bot].pos) << endl;
56         }
57         if ( dp[n] > 1e18 ) {
58             cout << "Too hard to arrange" << endl;
59         } else {
60             vector<PI> as;
61             cout << (11)dp[n] << endl;
62         }
63     }
64     return 0;
65 }

```

### 8.3 LCIS

```

1 vector<int> LCIS(vector<int> a, vector<int> b) {
2     int n = a.size(), m = b.size();
3     int dp[LEN][LEN] = {}, pre[LEN][LEN] = {};
4     for(int i=1; i<=n; i++) {
5         int p = 0;
6         for(int j=1; j<=m; j++)
7             if(a[i-1]!=b[j-1]) {
8                 dp[i][j] = dp[i-1][j], pre[i][j] = j;
9                 if( a[i-1]>b[j-1] && dp[i-1][j]>dp[i-1][p] )
10                     p = j;
11             } else {
12                 dp[i][j] = dp[i-1][p]+1, pre[i][j] = p;
13             }
14     }
15 }

```

```

14 }
15 int len = 0, p = 0;
16 for(int j=1; j<=m; j++)
17     if(dp[n][j]>len) len = dp[n][j], p = j;
18 vector<int> ans;
19 for(int i=n; i>=1; i--) {
20     if(a[i-1]==b[p-1] && p!=pre[i][p])
21         ans.push_back(b[p-1]);
22     p = pre[i][p];
23 }
24 reverse(ans.begin(), ans.end());
25 return ans;
26 }

```

## 9 Other

### 9.1 BuiltIn

```

1 //gcc專用
2 //unsigned int ffs
3 //unsigned long ffs1
4 //unsigned long long ffs11
5 unsigned int x; scanf("%u",&x)
6 printf("右起第一個1的位置");
7 printf("%d\n",__builtin_ffs(x));
8 printf("左起第一個1之前0的個數:");
9 printf("%d\n",__builtin_clz(x));
10 printf("右起第一個1之後0的個數:");
11 printf("%d\n",__builtin_ctz(x));
12 printf("1的個數:");
13 printf("%d\n",__builtin_popcount(x));
14 printf("1的個數的奇偶性:");
15 printf("%d\n",__builtin_parity(x));

```

### 9.2 CNF

```

1 #define MAXN 55
2 struct CNF{
3     int s,x,y;//s->xy | s->x, if y==1
4     int cost;
5     CNF(){}
6     CNF(int s,int x,int y,int c):s(s),x(x),y(y),cost(c){}
7 };
8 int state;//規則數量
9 map<char,int> rule;//每個字元對應到的規則，小寫字母為終端字符
10 vector<CNF> cnf;
11 void init(){
12     state=0;
13     rule.clear();
14     cnf.clear();
15 }
16 void add_to_cnf(char s,const string &p,int cost){
17     //加入一個s -> <p>的文法，代價為cost
18     if(rule.find(s)==rule.end())rule[s]=state++;
19     for(auto c:p)if(rule.find(c)==rule.end())rule[c]=state++;
20     if(p.size()==1){

```

```

21     cnf.push_back(CNF(rule[s],rule[p[0]],-1,cost));
22 }else{
23     int left=rule[s];
24     int sz=p.size();
25     for(int i=0;i<sz-2;++i){
26         cnf.push_back(CNF(left,rule[p[i]],state,0));
27         left=state++;
28     }
29     cnf.push_back(CNF(left,rule[p[sz-2]],rule[p[sz-1]],cost));
30     ;
31 }
32 vector<long long> dp[MAXN][MAXN];
33 vector<bool> neg_INF[MAXN][MAXN];//如果花費是負的可能會有無限
34 //小的情形
35 void relax(int l,int r,const CNF &c,long long cost,bool neg_c
36     =0){
37     if(!neg_INF[l][r][c.s]&&(neg_INF[l][r][c.x]||cost<dp[l][r][
38         c.s])){
39         if(neg_c||neg_INF[l][r][c.x]){
40             dp[l][r][c.s]=0;
41             neg_INF[l][r][c.s]=true;
42         }else dp[l][r][c.s]=cost;
43     }
44 }
45 void bellman(int l,int r,int n){
46     for(int k=1;k<=state;++k)
47         for(auto c:cnf)
48             if(c.y==1)relax(l,r,c,dp[l][r][c.x]+c.cost,k==n);
49 }
50 void cyk(const vector<int> &tok){
51     for(int i=0;i<(int)tok.size();++i){
52         for(int j=0;j<(int)tok.size();++j){
53             dp[i][j]=vector<long long>(state+1,INT_MAX);
54             neg_INF[i][j]=vector<bool>(state+1,false);
55         }
56         dp[i][i][tok[i]]=0;
57         bellman(i,i,tok.size());
58     }
59 }
60 for(int r=1;r<(int)tok.size();++r){
61     for(int l=r-1;l>=0;--l){
62         for(int k=1;k<r;++k)
63             for(auto c:cnf)
64                 if(~c.y)relax(l,r,c,dp[l][k][c.x]+dp[k+1][r][c.y]+c
65                     .cost);
66         bellman(l,r,tok.size());
67     }
68 }
69 }

```

### 9.3 HeapsAlgo

```

1 void heaps(int k, vector<int> &s, int n) {
2     if (k == 1) {
3         // for (int i = 0; i < n; i++)
4         // cout << s[i] << " \n"[i == n - 1];
5         return;
6     }
7     for (int i = 0; i < k - 1; ++i) {
8         heaps(k - 1, s, n);
9         if (k & 1) swap(s[i], s[k - 1]);
10        else swap(s[0], s[k - 1]);

```

```

11 }
12 heaps(k - 1, s, n);
13 }
14 void permutation(int n) {
15     vector<int> v(n);
16     for (int i = 0; i < n; i++) v[i] = i;
17     heaps(n, v, n);
18 }

```

## 9.4 Reminder

### 9.4.1 Complexity

#### 1. LCA

Method.....	Time.....	Space.....	離線
SsadbTarjan	$O(N + Q)$	$O(N^2)$	不須離線
OfflineTarjan	$O(N + Q)$	$O(N + Q)$	須離線
SparseTable	$O(N + Q \log N)$	$O(N \log N)$	不須離線

#### 2. Dinic

Graph.....	Space.....	Time
Gernal	$O(V + E)$	$O(EV^2)$
Bipartite	$O(V + E)$	$O(E\sqrt{V})$
UnitNetwork	$O(V + E)$	$O(E \min(V^{1.5}, \sqrt{E}))$

### 9.4.2 二分圖匹配

- 最大匹配數：給定二分圖  $G$ ，在  $G$  的子圖  $M$  中， $M$  的任兩條邊都沒有公共節點，則  $M$  成為此二分圖的匹配， $|EM|$  最大的匹配則成為最大匹配。
- 最小點覆蓋：在  $VG$  中選取最少的點，形成子集合  $V$ ，使  $E$  為所有與  $V$  中的點 incident 的邊形成的集合。
- 最大獨立集：在  $VG$  中選取最多的點，形成子集合  $V$ ，且任兩個  $V$  中的 vertices 都不相鄰。
- Konig 定理：對於任意二分圖，滿足以下兩個條件

- 最大匹配數 = 最小點覆蓋的頂點數
- 最大獨立集之頂點數 = 總頂點數 - 最大匹配數

### 9.4.3 Pick 公式

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2-1

### 9.4.4 圖論

- For planner graph,  $F = E - V + C + 1$ ,  $C$  是連通分量數
- For planner graph,  $E < 3V - 6$
- 對於連通圖  $G$ ，最大獨立點集的大小設為  $I(G)$ ，最大匹配大小設為  $M(G)$ ，最小點覆蓋設為  $C_v(G)$ ，最小邊覆蓋設為  $C_e(G)$ 。對於任意連通圖：
  - $I(G) + C_v(G) = |V|$
  - $M(G) + C_e(G) = |V|$
- 對於連通二分圖：
  - $I(G) = C_v(G)$
  - $M(G) = C_e(G)$

#### 5. 最大權閉合圖：

- $C(u, v) = \infty, (u, v) \in E$
- $C(S, v) = W_v, W_v > 0$
- $C(v, T) = -W_v, W_v < 0$
- $\text{ans} = \sum_{W_v > 0} W_v - \text{flow}(S, T)$

#### 6. 最大密度子圖：

- 求  $\max \left( \frac{W_e + W_v}{|V|} \right), e \in E', v \in V'$
- $U = \sum_{v \in V} 2W_v + \sum_{e \in E} W_e$
- $C(u, v) = W_{(u, v)}, (u, v) \in E$ ，雙向邊
- $C(S, v) = U, v \in V$
- $D_u = \sum_{(u, v) \in E} W_{(u, v)}$
- $C(v, T) = U + 2g - D_v - 2W_v, v \in V$
- 二分搜  $g$ ：  
 $l = 0, r = U, \text{eps} = 1/n^2$   
 if  $((U \times |V| - \text{flow}(S, T))/2 > 0) \quad l = \text{mid}$   
 else  $r = \text{mid}$
- $\text{ans} = \min\_cut(S, T)$
- $|E| = 0$  要特殊判斷

#### 7. 弦圖：

- 點數大於 3 的環都要有一條弦
- 完美消除序列從後往前依次給每個點染色，給每個點染上可以染的最小顏色
- 最大團大小 = 色數
- 最大獨立集：完美消除序列從前往後能選就選
- 最小團覆蓋：最大獨立集的點和他延伸的邊構成
- 區間圖是弦圖
- 區間圖的完美消除序列：將區間按造又端點由小到大排序
- 區間圖染色：用線段樹做

### 9.4.5 0-1 分數規劃

$x_i \in \{0, 1\}$ ,  $x_i$  可能會有其他限制，求  $\max \left( \frac{\sum B_i x_i}{\sum C_i x_i} \right)$

- $D(i, g) = B_i - g \times C_i$
- $f(g) = \sum D(i, g) x_i$
- $f(g) = 0$  時  $g$  為最佳解， $f(g) < 0$  沒有意義
- 因為  $f(g)$  單調可以二分搜  $g$
- 或用 Dinkelbach 通常比較快

```

1 binary_search(){
2     while(r-l>eps){
3         g=(l+r)/2;
4         for(i:所有元素)D[i]=B[i]-g*C[i]; //D(i,g)
5         找出一組合法x[i]使f(g)最大;
6         if(f(g)>0) l=g;
7         else r=g;
8     }
9     Ans = r;
10 }
11 Dinkelbach(){
12     g=任意狀態(通常設為0);
13     do{
14         Ans=g;
15         for(i:所有元素)D[i]=B[i]-g*C[i]; //D(i,g)
16         找出一組合法x[i]使f(g)最大;
17         p=0,q=0;
18         for(i:所有元素)

```

```

19         if(x[i])p+=B[i],q+=C[i];
20         g=p/q; //更新解，注意q=0的情況
21     }while(abs(Ans-g)>EPS);
22     return Ans;
23 }

```

### 9.4.6 Math

- $\sum_{d|n} \phi(n) = n$
- Harmonic series  $H_n = \ln(n) + \gamma + 1/(2n) - 1/(12n^2) + 1/(120n^4)$
- Gray Code =  $n \oplus (n >> 1)$
- $SG(A + B) = SG(A) \oplus SG(B)$
- Rotate Matrix  $M(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$
- $\sum_{d|n} \mu(n) = [n == 1]$
- $g(m) = \sum_{d|m} f(d) \Leftrightarrow f(m) = \sum_{d|m} \mu(d) \times g(m/d)$
- $\sum_{i=1}^n \sum_{j=1}^m \text{互質數量} = \sum \mu(d) \lfloor \frac{n}{d} \rfloor \lfloor \frac{m}{d} \rfloor$
- $\sum_{i=1}^n \sum_{j=1}^n lcm(i, j) = n \sum_{d|n} d \times \phi(d)$
- Josephus Problem  
 $f(1, k) = 0, f(n, k) = (f(n-1, k) + k) \% n$
- Mobius  

$$u(n) = \begin{cases} 1 & , n = 1 \\ 0 & , n \text{有平方數因數} \\ (-1)^k & , n = p_1 p_2 p_3 \dots p_k \end{cases}$$
 $u(ab) = u(a)u(b), \sum_{d|n} u(d) = [n == 1]$
- Mobius Inversion  
 $f(m) = \sum_{d|n} g(d) \Leftrightarrow g(n) = \sum_{d|n} u(d) \times f(n/d) = \sum_{d|n} u(n/d) \times f(d)$
- 排組公式

- n-Catalan  $C_0 = 1, C_{n+1} = \frac{2(2n+1)C_n}{n+2}$
- kn-Catalan  $\frac{C_n^{kn}}{n(k-1)+1}, C_m^n = \frac{n!}{m!(n-m)!}$
- Stirling number of  $2^nd, n$  人分  $k$  組方法數目
  - $S(0, 0) = S(n, n) = 1$
  - $S(n, 0) = 0$
  - $S(n, k) = kS(n-1, k) + S(n-1, k-1)$
- Bell number,  $n$  人分任意多組方法數目
  - $B_0 = 1$
  - $B_n = \sum_{i=0}^{n-1} C_{n-1}^i S(n, i)$
  - $B_{n+1} = \sum_{k=0}^n C_n^k B_k$
  - $B_{p+n} \equiv B_n + B_{n+1} \text{mod } p$ ,  $p$  is prime
  - $B_{p+m} \equiv mB_n + B_{n+1} \text{mod } p$ ,  $p$  is prime
  - From  $B_0 : 1, 1, 2, 5, 15, 52,$   
 $203, 877, 4140, 21147, 115975$
- Derangement, 錯排, 沒有人在自己位置上
  - $D_n = n!(1 - \frac{1}{1} + \frac{1}{2!} - \frac{1}{3!} \dots + (-1)^n \frac{1}{n!})$
  - $D_n = (n-1)(D_{n-1} + D_{n-2}), D_0 = 1, D_1 = 0$
  - From  $D_0 : 1, 0, 1, 2, 9, 44,$   
 $265, 1854, 14833, 133496$
- Binomial Equality
  - $\sum_k \binom{r}{m+k} \binom{s}{n-k} = \binom{r+s}{m+n}$
  - $\sum_k \binom{m+k}{l} \binom{n}{s-k} = \binom{m+n}{l+s}$
  - $\sum_k \binom{m+k}{l} \binom{n}{s-k} (-1)^k = (-1)^{l+m} \binom{s-m}{l-m-n}$
  - $\sum_{k \leq l} \binom{m}{l-k} \binom{n}{k} (-1)^k = (-1)^{l+m} \binom{n-l-m-1}{l+n-m}$
  - $\sum_{0 \leq k \leq l} \binom{l-k}{m} \binom{n}{q+k} = \binom{l+q+1}{m+n+1}$
  - $\binom{r}{k} = (-1)^k \binom{r-k}{m-k}$
  - $\binom{r}{m} \binom{m}{r+k} = \binom{r}{m-k} \binom{r-k}{m-n+1}$
  - $\sum_{k \leq n} \binom{r}{k} = \binom{n+1}{m+1}$
  - $\sum_{0 \leq k \leq n} \binom{m}{k} = \binom{n+1}{m+1}$
  - $\sum_{k \leq m} \binom{m}{k} x^k y^{m-k} = \sum_{k \leq m} \binom{-r}{k} (-x)^k (x+y)^{m-k}$



## 14. LinearAlgebra

- (a)  $tr(A) = \sum_i A_{i,i}$   
 (b) eigen vector:  $(A - cI)x = 0$

## 15. 冪次, 冪次和

- (a)  $a^b \% P = a^{b \% \varphi(P) + \varphi(P)}, b \geq \varphi(P)$   
 (b)  $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$   
 (c)  $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n^5}{5} + \frac{n^4}{2} + \frac{n^3}{3} - \frac{n}{30}$   
 (d)  $1^5 + 2^5 + 3^5 + \dots + n^5 = \frac{n^6}{6} + \frac{n^5}{2} + \frac{5n^4}{12} - \frac{n^2}{12}$   
 (e)  $0^k + 1^k + 2^k + \dots + n^k = P(k), P(k) = \frac{(n+1)^{k+1} - \sum_{i=0}^{k-1} C_i^{k+1} P(i)}{k+1}, P(0) = n+1$   
 (f)  $\sum_{k=0}^{m-1} k^n = \frac{1}{n+1} \sum_{k=0}^n C_k^{n+1} B_k m^{n+1-k}$   
 (g)  $\sum_{j=0}^m C_j^{m+1} B_j = 0, B_0 = 1$   
 (h) 除了  $B_1 = -1/2$ , 剩下的奇數項都是 0  
 (i)  $B_2 = 1/6, B_4 = -1/30, B_6 = 1/42, B_8 = -1/30, B_{10} = 5/66, B_{12} = -691/2730, B_{14} = 7/6, B_{16} = -3617/510, B_{18} = 43867/798, B_{20} = -174611/330,$

## 16. Chinese Remainder Theorem

- (a)  $\gcd(m_i, m_j) = 1$   
 (b)  $x \% m_1 = a_1$   
 $x \% m_2 = a_2$   
 $\vdots$   
 $x \% m_n = a_n$   
 (c)  $M = m_1 m_2 \dots m_n, M_i = M / m_i$   
 (d)  $t_i m_i = 1 \pmod{m_i}$   
 (e)  $x = a_1 t_1 * M_1 + \dots + a_n t_n * M_n + kM, k \in N$

## 9.4.7 Burnside's lemma

- $|X/G| = \frac{1}{|G|} \sum_{g \in G} |X^g|$
- $X^g = t^{c(g)}$
- $G$  表示有幾種轉法,  $X^g$  表示在那種轉法下, 有幾種是會保持對稱的,  $t$  是顏色數,  $c(g)$  是循環節不動的面數。
- 正立方體塗三顏色, 轉 0 有  $3^6$  個元素不變, 轉 90 有 6 種, 每種有  $3^3$  不變, 180 有  $3 \times 3^4$ , 120(角) 有  $8 \times 3^2$ , 180(邊) 有  $6 \times 3^3$ , 全部  $\frac{1}{24} (3^6 + 6 \times 3^3 + 3 \times 3^4 + 8 \times 3^2 + 6 \times 3^3) = 57$

## 9.4.8 Tree Counting

- Rooted tree:  $s_{n+1} = \frac{1}{n} \sum_{i=1}^n (i \times a_i \times \sum_{j=1}^{\lfloor n/i \rfloor} a_{n+1-i \times j})$
- Unrooted tree:
  - Odd:  $a_n - \sum_{i=1}^{n/2} a_i a_{n-i}$
  - Even:  $Odd + \frac{1}{2} a_{n/2} (a_{n/2} + 1)$
- Spanning Tree
  - Cayley:  $n^{n-2}$  (Complete Graph)
  - Kirchhoff:  $M[i][i] = \deg(V_i), M[i][j] = E(i, j) ? -1 : 0$ . delete any one row and col in  $A$ ,  $ans = \det(A)$

## 9.5 莫隊算法 — 區間眾數

```

1 using namespace std;
2 const int maxn = 1e6 + 10;
3 struct query { int id, bk, l, r; };
4 int arr[maxn], cnt[maxn], d[maxn], n, m, bk, mx;
5 pair<int,int> ans[maxn];
6 vector<query> q;
7 bool cmp(query x, query y) {
8     return (x.bk < y.bk || (x.bk == y.bk) && x.r < y.r);
9 }
10 void add(int pos) {
11     d[cnt[arr[pos]]]--;
12     cnt[arr[pos]]++;
13     d[cnt[arr[pos]]]++;
14     if(d[mx + 1] > 0) mx++;
15 }
16 void del(int pos) {
17     d[cnt[arr[pos]]]--;
18     cnt[arr[pos]]--;
19     d[cnt[arr[pos]]]++;
20     if(d[mx] == 0) mx--;
21 }
22 void mo(int n, int m) {
23     sort(q.begin(), q.end(), cmp);
24     for(int i = 0, cl = 1, cr = 0; i < m; i++) {
25         while(cr < q[i].r) add(++cr);
26         while(cl > q[i].l) add(--cl);
27         while(cr > q[i].r) del(cr--);
28         while(cl < q[i].l) del(cl++);
29         ans[q[i].id] = make_pair(mx, d[mx]);
30     }
31 }
32 int main(){
33     cin >> n >> m;
34     bk = (int)sqrt(n + 0.5);
35     for(int i = 1; i <= n; i++) cin >> arr[i];
36     q.resize(m);
37     for(int i = 0; i < m; i++) {
38         cin >> q[i].l >> q[i].r;
39         q[i].id = i, q[i].bk = (q[i].l - 1) / bk;
40     }
41     mo(n, m);
42     for(int i = 0; i < m; i++)
43         cout << ans[i].first << ' ' << ans[i].second << '\n';
44     return 0;
45 }
```

# NCTU-PUSHEEN

## CODEBOOK

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2.6	MaxSumSegmentTree	3	<b>5</b>	<b>Graph</b>	<b>12</b>	7.6	RollHash	20
2.7	PersistentSegmentTree	3	5.1	BCC_edge	12	7.7	suffix_array	20
2.8	RangeUpdateSegmentTree	4	5.2	LCA	12	7.8	Trie	21
2.9	SparseTable	4	5.3	MahattanMST	13	7.9	Z	21
2.10	Treap	4	5.4	MinMeanCycle	13	<b>8</b>	<b>DP</b>	<b>21</b>
<b>3</b>	<b>Flow_Matching</b>	<b>5</b>	5.5	Tarjan	13	8.1	Bounded_Knapsack	21
3.1	Dinic	5	5.6	Two_SAT	14	8.2	DP_1D1D	21
3.2	Ford_Fulkerson	5	<b>6</b>	<b>Math</b>	<b>14</b>	8.3	LCIS	22
3.3	Hopcroft_Karp	5	6.1	$ax+by=\gcd(a,b)$	14	<b>9</b>	<b>Other</b>	<b>22</b>
3.4	Hungarian	6	6.2	Discrete_sqrt	14	9.1	BuiltIn	22
3.5	KM	6	6.3	EulerFunction	14	9.2	CNF	22
3.6	Min_Cost_Max_Flow	6	6.4	Expression	14	9.3	HeapsAlgo	22
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