Problem 1. Grade Distribution

(Time Limit: 2 seconds)

Problem Description

An English teacher wants to know the English proficiency of this class. He decides to hold an exam at the beginning of the semester, and students are divided into four groups A, B, C, D, by scores. Each group is separated by a certain score. We hope that the difference of the numbers of students among these four groups is as small as possible. If the total number of students is N, the best situation would be $N/4$ students in each group. Please design an algorithm to divide the students into four groups with minimizing the sum of the differences between $N/4$ and the number of students in each group and print out the boundary scores in ascending order.

Example: If there are 9 students in the class, their scores are 95, 0, 40, 41, 80, 85, 90, 91, and 95. If we choose 41, 45, and 85 as the boundary scores, there would be 3, 0, 2, 3 students in group A, B, C, and, D, accordingly. The sum of differences is $d = |3 - 2.25| + |2.25 - 0| + |2.25 - 2| + |3 - 2.25| = 4$.

Input Format

The first line is the number of students $N (0 < N \leq 10000)$. The next N lines are the score of students X $(0 \leq X < 100)$.

Output Format

Output three integers, a, b and c $(0 \leq a < b < c < 100)$ denoting the boundary scores in ascending order. Use space to separate each integer. If there are multiple possible boundary scores with the same value of d, pick the solution with the smallest value of a. If there still exists a tie break, apply the same rule to b and then to c.

Example

Sample Input:	Sample Output:
9 95 0 40 41 80 85 90 91 95	40 80 90

Problem 2. Cutting a Cake

(Time Limit: 3 seconds)

Problem Description

Shik got a large rectangular cake on his birthday. This cake is of size m by n. There are several strawberries on the cake. After frozen the cake in the fridge for so many days, Shik decided to split the cake into two halves for his friend HH.

HH wants to eat the cake. However, HH does not want to eat an uneven cake with an uneven shape. There are several restrictions on cutting the cake. First, the cut can only be made from the lower left corner all the way up to the upper right corner. All edges along the cut should be either parallel or perpendicular to the rectangle. Moreover, the length of the cut should be the shortest. That is, you can either move your knife up or right. These edges along the cut must have integer length. Finally, the size of the two cake pieces should be the same and there must be an equal number of strawberries on both sides of the cake.

Given the layout of Shik's birthday cake, in how many ways he can fairly cut the cake?

Technical Specification

- The number of test cases $T \leq 20$
- $1 \leq m, n \leq 50$, and $m \times n$ must be an even number.
- There are at most 50 strawberries on the cake.

Input Format

The first line contains an integer T indicating the number of the test cases. For each test case, there are two integers m, n in the first line. Then m lines follow. Each line contains a string of length n. The character '.' indicates that there is no strawberry on that position, whereas the character '#' indicates that there is a strawberry on that position.

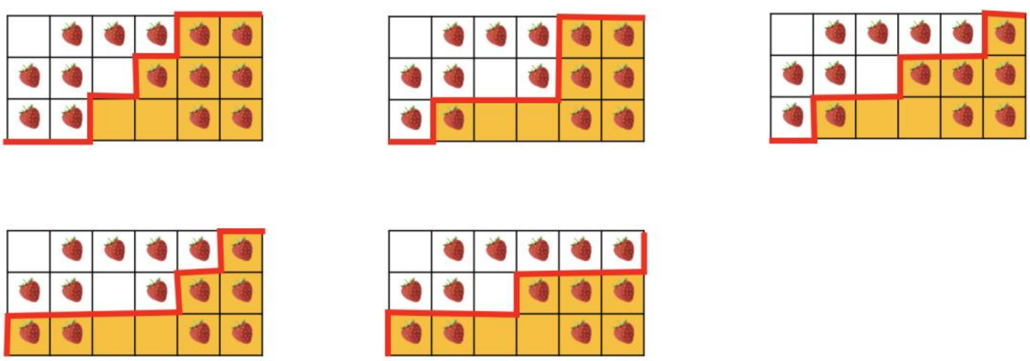
Output Format

For each test case, please output the number of ways to fairly split the cake modulo $10^9 + 7$ (from the lower left corner to the upper right corner.)

Example

Sample Input:	Sample Output:
4	8
3 6	0
......	7
......	5
......	
3 6	
.##...	
......	
......	
3 6	
...#..	
......	
....#.	
3 6	
.#####	
##.###	
##..##	

Explanation to Example 4: There are 5 ways to split the cake evenly.



Problem 3. Airport and railway

(Time Limit: 1 second)

Problem Description

In this task, we are considering how to build a transportation network to connect N cities such that people can travel from any city to another, directly or indirectly. To this aim, we can build railways between some pairs of cities. Besides, we can build airports at some cities. It is assumed that there will be an airline between any two airports. In other words, cities with airports are considered connected all together. However, the building costs of airports may differ for different cities.

On the other hand, all the railway stations can only be built at the cities and a railway can only connect two cities. That is, people can only transfer between two railways at the joint city of the two railways. All the railways are bidirectional. There are M candidate pairs of cities such that one can build a railway between the city pair, and the costs of the railways may be different.

Given the costs of building airports, as well as the building costs of candidate pairs of railways, the goal is to compute the minimal cost of a transportation network.

Technical Specification

- The number of test cases is at most 10.
- For each test case, $1 \leq N \leq 10000$ and $1 \leq M \leq 100000$.
- The cities are labelled from 0 to $N-1$.
- The cost of each airport or railway is a positive integer at most 10000.

Input Format

The test file contains several test cases. The first line contains an integer indicating the number of test cases. The first line of each test case contains two integers N followed by M. The next line contains N positive integers $c[0], c[1], \ldots, c[N-1]$, where $c[i]$ is the cost of building an airport at city i. Starting from the third line, there are M lines for the candidate pairs of building railways. Each line contains three integers u, v, and $w(u, v)$, which means one can build a railway between cities u and v with cost $w(u, v)$. There may be more than one candidate between the same pair of cities.

Output Format

For each test case, output the minimum cost in one line.

Example

Sample Input:	Sample Output:
2 3 2 1 1 5 0 1 3 1 2 4 4 3 5 6 7 8 0 1 2 3 2 4 0 3 3	6 9

For the first case in the sample, the best solution is to build airports at cities 0 and 1, so build a railway between cities 1 and 2. The cost is $1 + 1 + 4 = 6$.

For the second case, the best solution is to build all the three railways with cost $2 + 4 + 3 = 9$.

Problem 4. Make Money from Money

(Time Limit: 1 second)

Problem Description

It is well-known that making money from money is easy. The following game of money will show you how easy it is. At the start of the game (round 0), the player can invest a fixed amount of money (say, u units of digital coins) into his account. Then, at round 1, the system will add another amount of money (say, v) to the player's account as the bonus. After that, the player does not have to do anything and just waits for the money in his account to increase. Since then, at each round, the amount of money that has existed more than one round in the account will double. That is, at round 2, the total amount of money in the player's account will become $2u + v$, at round 3, it will become $2(u + v) + u$, and so on. Now, the player knows that at round n and m, the amounts of money in his account are x and y respectively. He wondered what will be the amount of money at round k. Can you help him?

Input Format

The first line of the input file contains an integer $T(T \leq 1000)$ which denotes the total number of test cases. The description of each test case is given below: Five integers n, x, m, y and k where ($0 < n, x, m, y, k < 10^9$ and $n \neq m$).

Output Format

For each test case produce one line of output giving either the number which is desired amount of money (modulo 1000000007) or the string 'Impossible'. You output 'Impossible' if the given input is not possible.

Example

Sample Input:	Sample Output:
2 3 18 5 47 6 3 32 7 56 6	76 Impossible

Problem 5. Anonymization

(Time Limit: 1 second)

Problem Description

Privacy is an important issue when publishing data. Even if some sensitive labels have been removed, data with unique features is easier to be re-identified. Therefore, anonymization is necessary before publishing data.

For simplicity, a data instance is just a sequence of N integers $\{A[i]: 0 \le i \le N-1\}$. Let K be a positive integer. A sequence of integers is K-anonymous if each appeared integer appears at least K times. For example, the sequence $(1,5,1,3,3,3,5)$ is 2-anonymous but not 3-anonymous. The K-anonymization is a process to modify a sequence $\{A[i]: 0 \le i \le N-1\}$ to a K-anonymous sequence $\{B[i]: 0 \le i \le N-1\}$, and the K-anonymization cost is the total difference between the original sequence and the K-anonymous sequence, i.e., $\sum_{i=0}^{N-1} |A[i] - B[i]|$. Given a sequence A, the goal is to find the minimum cost over any K-anonymous sequence B.

For example, $A = (1,4,2,3,3,3,5)$ and $K = 2$, we can find $B = (1,5,1,3,3,3,5)$, and the cost is 2; and if $B = (2,5,2,3,3,3,5)$, then the cost is still 2. But we cannot find any modification with cost less than 2. Thus, the minimum cost is 2. Meanwhile, for the same sequence A but $\mathrm{K} = 3$, if $B = (3,4,3,3,3,4,4)$, the cost is 5; and the cost is still 5 for $B = (2,4,2,2,4,4,4)$ or $(2,3,2,2,3,3,3)$. In fact, the minimum cost is 5.

Technical Specification

- The number of test cases is at most 9.
- For each test case, $K \le N \le 50000$.
- All numbers are nonnegative integers at most 10000000.
- The answer of each test case is less than 10^{15}.

Input Format

The test file contains several test cases. The first line contains an integer indicating the number of test cases. The first line of each test case contains N and K. The next line contains N nonnegative integers which are the elements of the sequence to be K-anonymized.

Output Format

For each test case, output the minimum K-anonymizing cost in one line.

Example

Sample Input:	Sample Output:
2 7 2 1 4 2 3 3 3 5 7 3 1 4 2 3 3 3 5	2 5