

NCTU_Yggdarsill

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1 Building Environment

1.1 Default

```
1 #define F(n) Fi(i,n)
2 #define Fi(i,n) Fl(i,0,n)
3 #define Fl(i,l,n) for(int i=(l);i<(int)(n);++i)
4 #include <bits/stdc++.h>
5 #include <bits/extc++.h>
6 // #include <ext/pb_ds/assoc_container.hpp>
7 // #include <ext/pb_ds/priority_queue.hpp>
8 using namespace std;
9 using namespace __gnu_pbds;
10 const double PI = acos(-1);
11 main() {
12     ios_base::sync_with_stdio(false);
13     cin.tie(NULL);
14     cout << fixed << setprecision(7) << PI << endl;
15 }
```

1.2 Vimrc

```
1 set tabstop=4
2 set autoindent
3
```

```

4 map <F9> :w<LF>:!g++ -O2 -std=c++11 -o %.out % && echo "----Start----" &&
. /%.out<LF>
5 imap <F9> <ESC><F9>

```

2 Convolution

2.1 FFT

```

1 #include <stdio>
2 #include <cstring>
3 #include <cmath>
4 const double PI=acos(-1.0);
5 typedef struct {
6     double real;
7     double im;
8 } COMPLEX;
9 COMPLEX X[66000],Y[66000],A[66000];
10 COMPLEX EE(COMPLEX a,COMPLEX b)
11 {
12     COMPLEX c;
13     c.real=a.real*b.real-a.im*b.im;
14     c.im=a.real*b.im+a.im*b.real;
15     return c;
16 }
17 /* 1 FFT , -1 IFFT */
18 void fft(COMPLEX x[],int nfft,int isign)
19 {
20     int i,j=0,k;
21     COMPLEX t;
22     for(i=1, j = nfft / 2;i<nfft-1;i++)
23     {
24         if(i<j)
25         {
26             t=x[j];
27             x[j]=x[i];
28             x[i]=t;
29         }
30         k=nfft/2;
31         while(k<=j)
32         {
33             j-=k;
34             k/=2;
35         }
36         if (j < k)
37             j+=k;
38     }
39     int le,lei,ip;
40     COMPLEX u,w, v;
41     for(le=2;le<=nfft;le *= 2)
42     {
43         lei=le/2;
44         w.real=cos(2.0*PI*isign/le);

```

```

45         w.im=sin(2.0*PI*isign/le);
46         for(i=0;i<nfft;i+=le)
47         {
48             u.real=1.0;
49             u.im=0.0;
50             for(j = i ; j < i + lei ; ++j)
51             {
52                 ip=j+lei;
53                 v = x[j];
54                 t=EE(u, x[ip]);
55                 x[j].real=v.real+t.real;
56                 x[j].im=v.im+t.im;
57                 x[ip].real=v.real-t.real;
58                 x[ip].im=v.im-t.im;
59                 u=EE(u,w);
60             }
61         }
62     }
63 }
64 void FFT(COMPLEX x[], int nfft)
65 {
66     fft(x,nfft,1);
67 }
68 void IFFT(COMPLEX x[],int nfft)
69 {
70     int i;
71     fft(x,nfft,-1);
72 }
73 for(i=0;i<nfft;i++)
74 {
75     x[i].real /= nfft;
76     x[i].im /= nfft;
77 }
78 }
79 int main(void) {
80     int t_num;
81     int i,ii,iii;
82     int p_num;
83     int Nx;
84     int NFFT;
85     int temp;
86     scanf("%d",&t_num);
87     for(i=0;i<t_num;i++){
88         scanf("%d",&p_num);
89         Nx=p_num*2-1;
90         NFFT = 2 << (int)log2(Nx);
91         for(ii=0;ii<p_num;++ii){
92             scanf("%d",&temp);
93             X[ii].real=(double)temp;
94             X[ii].im=0.0;
95         }
96         for(iii=0;iii<p_num;++iii){
97
98             scanf("%d",&temp);
99             Y[iii].real=(double)temp;
100             Y[iii].im=0.0;

```

```

101     }
102     for(ii=p_num;ii<NFFT;ii++)
103     {
104         X[ii].real=0.0;
105         X[ii].im=0.0;
106         Y[ii].real=0.0;
107         Y[ii].im=0.0;
108     }
109     FFT(X,NFFT);
110     FFT(Y,NFFT);
111     for(ii=0;ii<NFFT;ii++){
112         A[ii] = EE(X[ii], Y[ii]);
113     }
114     IFFT(A,NFFT);
115     for(ii=0;ii<Nx;ii++){
116         printf("%d ", (int)round(A[ii].real));
117     }
118     printf("\n");
119 }
120 return 0;
121 }

```

2.2 SunMoon FFT

```

1 #ifndef SUNMOON_FFT
2 #define SUNMOON_FFT
3 #include<vector>
4 #include<complex>
5 #include<algorithm>
6 template<typename T,typename VT=std::vector<std::complex<T> > >
7 struct FFT{
8     const T pi;
9     FFT(const T pi=acos((T)-1)):pi(pi){}
10    inline unsigned int bit_reverse(unsigned int a,int len){
11        a=((a&0x55555555U)<<1)|((a&0xAAAAAAAAU)>>1);
12        a=((a&0x33333333U)<<2)|((a&0xCCCCCCCCU)>>2);
13        a=((a&0x0F0F0F0FU)<<4)|((a&0xFF0F0F0FU)>>4);
14        a=((a&0x00FF00FFU)<<8)|((a&0xFFFF0000U)>>8);
15        a=((a&0x0000FFFFU)<<16)|((a&0xFFFF0000U)>>16);
16        return a>>(32-len);
17    }
18    inline void fft(bool is_inv,VT &in,VT &out,int N){
19        int bitlen=std::__lg(N),num=is_inv?-1:1;
20        for(int i=0;i<N;++i)out[bit_reverse(i,bitlen)]=in[i];
21        for(int step=2;step<=N;step<=1){
22            const int mh=step>>1;
23            for(int i=0;i<mh;++i){
24                std::complex<T> wi=exp(std::complex<T>(0,i*num*pi/mh));
25                for(int j=i;j<N;j+=step){
26                    int k=j+mh;
27                    std::complex<T> u=out[j],t=wi*out[k];
28                    out[j]=u+t;
29                    out[k]=u-t;
30                }

```

```

31     }
32 }
33 if(is_inv)for(int i=0;i<N;++i)out[i]/=N;
34 }
35 };
36 #endif

```

3 Geometry

3.1 Geometry

```

1 const double eps = 1e-10;
2 const double INF = 1.0/0.0;
3 const double SIDE = 10000;
4 const double PI = acos(-1.0);
5 const int MAXN = 500000 + 10;
6 struct PT{
7     double x,y;
8     PT(){}
9     PT(double x,double y):x(x),y(y){}
10    PT operator + (const PT& p)const{
11        return PT(x+p.x,y+p.y);
12    }
13    PT operator - (const PT& p)const{
14        return PT(x-p.x,y-p.y);
15    }
16    PT operator * (double c)const{
17        return PT(x*c,y*c);
18    }
19    PT operator / (double c)const{
20        return PT(x/c,y/c);
21    }
22    PT rot(double a)const{return PT(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a));}
23    double operator *(const PT& p)const{
24        return x*p.x+y*p.y;
25    }
26    double operator ^ (const PT& p)const{
27        return x*p.y-y*p.x;
28    }
29    bool operator ==(const PT& p)const{
30        return fabs(x-p.x)<eps&&fabs(y-p.y)<eps;
31    }
32    double len2()const{return x*x+y*y;}
33    double len()const{return sqrt(len2());}
34 }poi[MAXN],stk[MAXN];
35 struct LINE{
36     PT a,b;
37     double angle;
38     LINE(){}
39     LINE(PT _a,PT _b):a(_a),b(_b),angle(atan2(_b.y-_a.y,_b.x-_a.x)){}
40 }line[MAXN],deq[MAXN];
41 int top;

```

```

42 inline int ori(const PT& p1, const PT& p2, const PT& p3) {
43     double a = (p2 - p1) ^ (p3 - p1);
44     if (a > -eps && a < eps) return 0;
45     return a > 0 ? 1 : -1;
46 }
47 inline bool btw(const PT& p1, const PT& p2, const PT& p3) {
48     return (p2 - p1) * (p3 - p1) < eps;
49 }
50 //segment intersection
51 inline bool intersection(const PT& p1, const PT& p2, const PT& p3, const PT& p4)
52 {
53     int a123 = ori(p1, p2, p3);
54     int a124 = ori(p1, p2, p4);
55     int a341 = ori(p3, p4, p1);
56     int a342 = ori(p3, p4, p2);
57     if (a123 == 0 && a124 == 0) return btw(p1, p3, p4) || btw(p2, p3, p4) || btw(p3, p1, p2) ||
58         btw(p4, p1, p2);
59     return a123 * a124 <= 0 && a341 * a342 <= 0;
60 }
61 inline PT intersectionPoint(const PT& p1, const PT& p2, const PT& p3, const PT&
62     p4) {
63     double a123 = (p2 - p1) ^ (p3 - p1);
64     double a124 = (p2 - p1) ^ (p4 - p1);
65     return (p4 * a123 - p3 * a124) / (a123 - a124);
66 }
67 //line intersection
68 inline PT intersectionPoint(const LINE& l1, const LINE& l2) {
69     PT p1 = l1.a, p2 = l1.b, p3 = l2.a, p4 = l2.b;
70     double a123 = (p2 - p1) ^ (p3 - p1);
71     double a124 = (p2 - p1) ^ (p4 - p1);
72     return (p4 * a123 - p3 * a124) / (a123 - a124);
73 }
74 PT foot(const LINE& l, const PT& p) {
75     PT m(l.b.y - l.a.y, l.a.x - l.b.x);
76     return p + m * (l.a - p ^ l.b - p) / ((l.b - l.a).len2());
77 }
78 PT mirror(const LINE& l, const PT& p) {
79     PT m(l.b.y - l.a.y, l.a.x - l.b.x);
80     return p + m * (l.a - p ^ l.b - p) / ((l.b - l.a).len2()) * 2;
81 }
82 //segment-point distance
83 inline double sp_dis(PT a, PT l1, PT l2) {
84     if ((a - l1) * (l2 - l1) < 0) return (l1 - a).len();
85     else if ((a - l2) * (l1 - l2) < 0) return (l2 - a).len();
86     return fabs((l1 - a) ^ (l2 - a)) / ((l2 - l1).len());
87 }
88 struct cir {
89     point c;
90     double r;
91 } o[10];
92 double out_ang(cir a, cir b) { //a.c + (b.c - a.c).unit().rot(ang) * b.r
93     return acos((a.r - b.r) / (a.c - b.c).len());
94 }
95 double in_ang(cir a, cir b) {
96     return acos((a.r + b.r) / (a.c - b.c).len());
97 }

```

```

95 }
96 int main() {
97     double tmp, sum;
98     if (fabs(o[i].r - o[j].r) < (o[j].c - o[i].c).len()) {
99         tmp = out_ang(o[i], o[j]);
100         sum = ang_add(cl, tmp);
101         pi = o[i].c + point(o[i].r * cos(sum), o[i].r * sin(sum));
102         pj = o[j].c + point(o[j].r * cos(sum), o[j].r * sin(sum));
103         sum = ang_add(cl, -tmp);
104         pi = o[i].c + point(o[i].r * cos(sum), o[i].r * sin(sum));
105         pj = o[j].c + point(o[j].r * cos(sum), o[j].r * sin(sum));
106     }
107     if (o[i].r + o[j].r < (o[j].c - o[i].c).len()) {
108         tmp = in_ang(o[i], o[j]);
109         sum = ang_add(cl, tmp);
110         pi = o[i].c + point(o[i].r * cos(sum), o[i].r * sin(sum));
111         pj = o[j].c - point(o[j].r * cos(sum), o[j].r * sin(sum));
112         sum = ang_add(cl, -tmp);
113         pi = o[i].c + point(o[i].r * cos(sum), o[i].r * sin(sum));
114         pj = o[j].c - point(o[j].r * cos(sum), o[j].r * sin(sum));
115     }
116 }
117 inline double dist(const PT& p1, const PT& p2) {
118     return sqrt((p2 - p1) * (p2 - p1));
119 }
120 inline double tri(const PT& p1, const PT& p2, const PT& p3) {
121     return fabs((p2 - p1) ^ (p3 - p1));
122 }
123 inline double getPerimeter() {
124     double res = 0.0;
125     poi[top++] = poi[0];
126     for (int i = 0; i < top - 1; i++) res += dist(poi[i], poi[i + 1]);
127     return res;
128 }
129 inline double getarea() {
130     double res = 0.0;
131     for (int i = 1; i < top - 1; i++) res += tri(poi[0], poi[i], poi[i + 1]);
132     return 0.5 * res;
133 }
134 //convex hull
135 inline bool cmp_convex(const PT &a, const PT &b) {
136     if (a.x != b.x) return a.x < b.x;
137     return a.y < b.y;
138 }
139 inline void convex_hull(PT a[], int &n) {
140     top = 0;
141     sort(a, a + n, cmp_convex);
142     for (int i = 0; i < n; i++) {
143         while (top >= 2 && ori(stk[top - 2], stk[top - 1], a[i]) >= 0) top--;
144         stk[top++] = a[i];
145     }
146     for (int i = n - 2, t = top + 1; i >= 0; i--) {
147         while (top >= t && ori(stk[top - 2], stk[top - 1], a[i]) >= 0) top--;
148         stk[top++] = a[i];
149     }
150 }

```

```

151     }
152     top--;
153     for(int i=0;i<top;i++)poi[i]=stk[i];
154 }
155 //half plane intersection
156 inline bool cmp_half_plane(const LINE &a,const LINE &b){
157     if(fabs(a.angle-b.angle)<eps)return ori(a.a,a.b,b.a)<0;
158     return a.angle > b.angle;
159 }
160 inline void half_plane_intersection(LINE a[],int &n){
161     int m=1,front=0,rear=1;
162     sort(a,a+n,cmp_half_plane);
163     for(int i=1;i<n;i++){
164         if(fabs(a[i].angle-a[m-1].angle)>eps)a[m++]=a[i];
165     }
166     deq[0]=a[0],deq[1]=a[1];
167     for(int i=2;i<m;i++){
168         while(front<rear&&ori(a[i].a,a[i].b,intersectionPoint(deq[rear],deq[
169             rear-1]))<0)rear--;
170         while(front<rear&&ori(a[i].a,a[i].b,intersectionPoint(deq[front],deq[
171             front+1]))<0)front++;
172         deq[++rear]=a[i];
173     }
174     while(front<rear&&ori(deq[front].a,deq[front].b,intersectionPoint(deq[rear
175         ],deq[rear-1]))<0)rear--;
176     while(front<rear&&ori(deq[rear].a,deq[rear].b,intersectionPoint(deq[front
177         ],deq[front+1]))<0)front++;
178     if(front==rear)return;
179 }
180
181
182
183
184 //smallest cover rectangle
185 double ans1,ans2;
186 void rotating_calipers(){
187     ans1=ans2=INF;
188     int j=1,k=1,l=1;
189     poi[top]=poi[0];
190     for(int i=0;i<top;i++){
191         while(tri(poi[i],poi[i+1],poi[j])<tri(poi[i],poi[i+1],poi[j+1])) j=(j
192             +1)%top;
193         while(((poi[i+1]-poi[i])*(poi[k+1]-poi[k]))>eps)k=(k+1)%top;
194         if(i==0)l=(k+1)%top;
195         while(((poi[i+1]-poi[i])*(poi[l+1]-poi[l]))<-eps)l=(l+1)%top;
196         double tmp1 = tri(poi[i],poi[i+1],poi[j])/dist(poi[i],poi[i+1]);
197         double tmp2 = (((poi[k]-poi[i])*(poi[i+1]-poi[i]))-((poi[l]-poi[i])*(
198             poi[i+1]-poi[i])))/dist(poi[i],poi[i+1]));
199         if((tmp1+tmp2)*2.0<ans1)ans1=(tmp1+tmp2)*2.0;
200         if(tmp1*tmp2<ans2)ans2=tmp1*tmp2;
201     }
202 }

```

```

201 int main(){
202     int n,m;
203     while(~scanf("%d",&n)&&n){
204         for(int i=0;i<n;i++)scanf("%lf%lf",&poi[i].x,&poi[i].y);
205         convex_hull(poi,n);
206         rotating_calipers();
207         printf("%.2f %.2f\n",ans2,ans1);
208     }
209 }
210
211 inline bool online(const LINE &L,const PT &p){
212     return ori(p,L.a,L.b)==0&&btw(p,L.a,L.b);
213 }
214 inline bool on_convex(const PT& p){
215     for(int i=0;i<top;i++){
216         if(p==poi[i])return 1;
217     }
218     poi[top]=poi[0];
219     for(int i=0;i<top;i++){
220         line[i].a=poi[i];
221         line[i].b=poi[i+1];
222     }
223     for(int i=0;i<top;i++){
224         if(online(line[i],p))return 1;
225     }
226 //originally in long long, should be modified
227 bool in_simple_polygon(PT b[],int k){
228     bool flag=false;
229     for(int j=0;j<k;j++){
230         if(((p-b[j])^(p-b[(j+1)%k]))==0&&(p-b[j])*(p-b[(j+1)%k])<=0){
231             flag=true;
232             break;
233         }
234         if((b[j].y<p.y)^(b[(j+1)%k].y<p.y)){
235             long long xss=(b[j]-p)^(b[(j+1)%k]-p);
236             if((xss<0)^(b[j].y<b[(j+1)%k].y)){
237                 flag^=1;
238             }
239         }
240     }
241     return flag;
242 }

```

3.2 K-closet Pair

```

1 #define F(n) Fi(i,n)
2 #define Fi(i,n) Fl(i,0,n)
3 #define Fl(i,l,n) for(int i=(l);i<(int)(n);++i)
4 #include <bits/stdc++.h>
5 // #include <ext/pb_ds/assoc_container.hpp>
6 // #include <ext/pb_ds/priority_queue.hpp>
7 using namespace std;
8 // using namespace __gnu_pbds;
9 typedef long long ll;

```

```

10 struct point {
11     point(ll x_ = 0, ll y_ = 0): x(x_), y(y_) {} ll x, y;
12     inline bool operator<(const point &e_) const {
13         return (x != e_.x ? x < e_.x : y < e_.y);
14     }
15     inline friend istream& operator>>(istream &is_, point& e_) {
16         is_ >> e_.x >> e_.y;
17         return is_;
18     }
19 };
20 int k;
21 priority_queue<ll> PQ;
22 inline ll dist2(const point &e1, const point &e2) {
23     ll res = (e1.x-e2.x)*(e1.x-e2.x)+(e1.y-e2.y)*(e1.y-e2.y);
24     PQ.push(res);
25     if (PQ.size() > k) {
26         PQ.pop();
27     }
28     return res;
29 }
30 #define N 500005
31 point p[N];
32 queue<point> Q;
33 ll closet_point(int l, int m, int r, ll delta2) {
34     ll xmid = p[m-1].x;
35     while (!Q.empty()) {
36         Q.pop();
37     }
38     for (int i = l, j = m; i < m; ++i) {
39         if ((p[i].x-xmid)*(p[i].x-xmid) >= delta2) {
40             continue;
41         }
42         while (j < r && p[j].y < p[i].y && (p[j].y-p[i].y)*(p[j].y-p[i].y) < delta2) {
43             if ((p[j].x-xmid)*(p[j].x-xmid) < delta2) {
44                 Q.push(p[j]);
45             }
46             ++j;
47         }
48         while (!Q.empty() && Q.front().y < p[i].y && (Q.front().y-p[i].y)*(Q.front().y-p[i].y) > delta2) {
49             Q.pop();
50         }
51         while (!Q.empty()) {
52             delta2 = min(delta2, dist2(p[i], Q.front()));
53             Q.pop();
54         }
55     }
56     return delta2;
57 }
58 ll find_distance(int l, int r) {
59     if (r - l <= 3000) {
60         ll ans = 0x3f3f3f3f3f3f3f3f;
61         for (int i = l; i < r; ++i)
62             for (int j = i+1; j < r; ++j)
63                 ans = min(ans, dist2(p[i], p[j]));

```

```

64     return ans;
65 }
66 int m = (l+r)/2;
67 ll delta2 = min(find_distance(l, m), find_distance(m, r));
68 return min(delta2, closet_point(l, m, r, delta2));
69 }
70 int main() {
71     ios_base::sync_with_stdio(false);
72     cin.tie(NULL);
73     int n;
74     cin >> n >> k;
75     F(n) cin >> p[i];
76     sort(p, p+n);
77     find_distance(0, n);
78     cout << PQ.top() << '\n';
79 }

```

3.3 MinimumCoveringCircle

```

1 #define F(n) Fi(i,n)
2 #define Fi(i,n) Fl(i,0,n)
3 #define Fl(i,l,n) for(int i=(l);i<(int)(n);++i)
4 #include <bits/stdc++.h>
5 using namespace std;
6 const double eps = 1e-6;
7 #define x first
8 #define y second
9 typedef pair<double, double> point;
10 inline double dq(const point& p1, const point& p2) {
11     return sqrt((p1.x-p2.x)*(p1.x-p2.x)+(p1.y-p2.y)*(p1.y-p2.y));
12 }
13 inline point oc(const point& pa, const point& pb, const point& pc) {
14     double a, b, c, d, e, f, delta, dx, dy;
15     // ax + by = c
16     // dx + ey = f
17     a = pa.x - pb.x;
18     b = pa.y - pb.y;
19     c = a*(pa.x+pb.x)/2 + b*(pa.y+pb.y)/2;
20     d = pa.x - pc.x;
21     e = pa.y - pc.y;
22     f = d*(pa.x+pc.x)/2 + e*(pa.y+pc.y)/2;
23     delta = a*e-b*d;
24     dx = c*e-f*b;
25     dy = a*f-d*c;
26     return point(dx/delta, dy/delta);
27 }
28 inline point enc(const vector<point>& tmp) {
29     random_shuffle(tmp.begin(), tmp.end());
30     point O = tmp[0];
31     double r = 0;
32     Fl(i, 1, tmp.size()) if (dq(O, tmp[i]) - r > eps) {
33         O = tmp[i], r = 0;
34         Fi(j, i) if (dq(O, tmp[j]) - r > eps) {
35             O = point((tmp[i].x+tmp[j].x)/2, (tmp[i].y+tmp[j].y)/2);

```

```

36     r = dq(O, tmp[j]);
37     Fi(k, j) if (dq(O, tmp[k]) - r > eps)
38         O = oc(tmp[i], tmp[j], tmp[k]), r = dq(O, tmp[k]);
39 }
40 }
41 return O;
42 }
43 int n;
44 vector<point> v;
45 int main() {
46     ios_base::sync_with_stdio(false);
47     cin.tie(NULL);
48     while (cin >> n) {
49         if (!n) break;
50         v.clear();
51         F(n) {
52             point tp;
53             cin >> tp.x >> tp.y;
54             v.push_back(tp);
55         }
56         point ct = enc(v);
57         cout << setprecision(2) << fixed << ct.x << ' ' << ct.y << ' ' << dq(ct,
58             v[0]) << '\n';
59     }

```

4 GNU Black Magic

4.1 GNU Bitwise Operation

```

1 int __builtin_ffs (unsigned int x)
2 int __builtin_ffsl (unsigned long)
3 int __builtin_ffsll (unsigned long long)
4 // 返回右起第一個1的位置
5 // Returns one plus the index of the least significant 1-bit of x, or if x is
   zero, returns zero.
6
7 int __builtin_clz (unsigned int x)
8 int __builtin_clzl (unsigned long)
9 int __builtin_clzll (unsigned long long)
10 // 返回左起第一個1之前0的個數
11 // Returns the number of leading 0-bits in x, starting at the most
   significant bit position. If x is 0, the result is undefined.
12
13 int __builtin_ctz (unsigned int x)
14 int __builtin_ctzl (unsigned long)
15 int __builtin_ctzll (unsigned long long)
16 // 返回右起第一個1之後的0的個數
17 // Returns the number of trailing 0-bits in x, starting at the least
   significant bit position. If x is 0, the result is undefined.
18
19 int __builtin_popcount (unsigned int x)

```

```

20 int __builtin_popcountl (unsigned long)
21 int __builtin_popcountll (unsigned long long)
22 // 返回1的個數
23 // Returns the number of 1-bits in x.
24
25 int __builtin_parity (unsigned int x)
26 int __builtin_parityl (unsigned long)
27 int __builtin_parityll (unsigned long long)
28 // 返回1的個數的奇偶性(1的個數 mod 2的值)
29 // Returns the parity of x, i.e. the number of 1-bits in x modulo 2.

```

5 Graph

5.1 2-SAT

```

1 const int MAXN = 2020;
2
3 struct TwoSAT{
4     static const int MAXv = 2*MAXN;
5     vector<int> GO[MAXv],BK[MAXv],stk;
6     bool vis[MAXv];
7     int SC[MAXv];
8
9     void imply(int u,int v){ // u imply v
10         GO[u].push_back(v);
11         BK[v].push_back(u);
12     }
13     int dfs(int u,vector<int>*G,int sc){
14         vis[u]=1, SC[u]=sc;
15         for (int v:G[u])if (!vis[v])
16             dfs(v,G,sc);
17         if (G==GO)stk.push_back(u);
18     }
19     int scc(int n=MAXv){
20         memset(vis,0,sizeof(vis));
21         for (int i=0; i<n; i++)if (!vis[i])
22             dfs(i,GO,-1);
23         memset(vis,0,sizeof(vis));
24         int sc=0;
25         while (!stk.empty()){
26             if (!vis[stk.back()])
27                 dfs(stk.back(),BK,sc++);
28             stk.pop_back();
29         }
30     }
31 }SAT;
32
33 int main(){
34     SAT.scc(2*n);
35     bool ok=1;
36     for (int i=0; i<n; i++){
37         if (SAT.SC[2*i]==SAT.SC[2*i+1])ok=0;

```

```

38     }
39     if (ok){
40         for (int i=0; i<n; i++){
41             if (SAT.SC[2*i]>SAT.SC[2*i+1]){
42                 cout << i << endl;
43             }
44         }
45     }
46     else puts("NO");
47 }

```

5.2 Articulation Point

```

1 void tarjan(int u, int p)
2 { // u 為當前點, p 為當前點之母節點
3     // cnt 為 DFS 次序
4     low[u] = dfn[u] = ++cnt;
5     int i, v;
6     for (i = 0 ; i < G[u].size() ; ++i) {
7         v = G[u][i];
8         if (u == rt && !dfn[v]) ++c;
9         if (!dfn[v]){
10             // (u, v) 為 Tree Edge
11             tarjan(v, u);
12             low[u] = min(low[u], low[v]);
13             // To check if u is AP or not.
14             if (dfn[u] <= low[v] && u != rt) ge[u] = 1;
15         }
16         // 注意不可以同一條邊走兩次, 且根節點特判
17         if (v != p && p != -1) low[u] = min(low[u], dfn[v]);
18     }
19 }

```

5.3 BCC

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 const int MAXN = 10000;
4 vector<int> adja[MAXN];
5 int gcnt, top, timeStamp, dfn[MAXN], low[MAXN], depth[MAXN];
6 pair<int, int> stk[MAXN], ans[MAXN];
7 set<int> group[MAXN];
8 bool cut[MAXN];
9 void BCC(int now, int nextv){
10     int sf, st;
11     group[gcnt].clear();
12     do{
13         sf = stk[top-1].first, st = stk[top-1].second;
14         group[gcnt].insert(sf);
15         group[gcnt].insert(st);
16         --top;

```

```

17     }while(sf != now || st != nextv);
18     ++gcnt;
19 }
20 void tarjan(int now, int parent, int d){
21     int child = 0;
22     dfn[now] = low[now] = ++timeStamp, depth[now] = d;
23     for(int i = 0; i < adja[now].size(); i++){
24         int nextv = adja[now][i];
25         if(nextv == parent) continue;
26         if(dfn[nextv] == 0){
27             stk[top++] = make_pair(now, nextv);
28             tarjan(nextv, now, d+1);
29             low[now] = min(low[now], low[nextv]);
30             ++child;
31             if( (parent != -1 && low[nextv] >= dfn[now]) || (parent == -1 &&
child >= 2)){
32                 cut[now] = true;
33                 if(parent != -1) BCC(now, nextv);
34             }
35             if(parent == -1) BCC(now, nextv);
36         }
37         else if(depth[nextv] < depth[now]-1){
38             stk[top++] = make_pair(now, nextv);
39             low[now] = min(low[now], dfn[nextv]);
40         }
41     }
42 }
43 int main(){
44     int n,m,x,y,cnt=0;
45     while(~scanf("%d",&n)){
46         cnt=timeStamp=top=gcnt=0;
47         memset(cut, 0, sizeof(cut));
48         memset(dfn, 0, sizeof(dfn));
49         for(int i=0;i<n;i++)adja[i].clear();
50         for(int i=0;i<n;i++){
51             scanf("%d ",&x);
52             scanf("(%d",&m);
53             while(m--){
54                 scanf("%d",&y);
55                 adja[x].push_back(y);
56             }
57         }
58         for(int i=0;i<n;i++)
59             if(dfn[i]==0)tarjan(i, -1, 1);
60         for(int i=0;i<gcnt;i++){
61             if(group[i].size()==2){
62                 //critical links
63             }
64         }
65     }
66 }

```

5.4 Heavy Light Decomposition


```

1 //with set value && query sum, 1-based with n points
2 //remove vis in DFS, add it back if something weird happen(I don't think it
  , s required)
3 using namespace std;
4 int sz[N], top[N], up[N], dep[N];
5 int lightval[N]; //value on light edge
6 struct node{
7     node(){}
8     node(int _l, int _r): val(1), l(_l), r(_r), lc(NULL), rc(NULL){}
9     int l, r;
10    node *lc, *rc;
11    int sum;
12    int val;
13    int qsum(){return val>=0?val*(r-l):sum;}
14    void push(){
15        if(val>=0){
16            sum=val*(r-l);
17            lc->val=rc->val=val;
18            val=-1;
19        }
20    }
21    void pull(){
22        sum=lc->qsum()+rc->qsum();
23    }
24 };
25 node* tr[N];
26 node* build(int l, int r){
27     node *now=new node(l, r);
28     if(r-l>1){
29         now->lc=build(l, (l+r)/2);
30         now->rc=build((l+r)/2, r);
31     }
32     return now;
33 }
34 //partial
35 int qry(node* now, int l, int r){
36     if(l>=r) return 0;
37     if(l==now->l&&r==now->r){
38         return now->qsum();
39     }
40     int m=(now->l+now->r)/2;
41     now->push();
42     if(l>=m){
43         return qry(now->rc, l, r);
44     }
45     else if(r<=m){
46         return qry(now->lc, l, r);
47     }
48     else return qry(now->lc, l, m)+qry(now->rc, m, r);
49 }
50 void set0(node *now, int l, int r){
51     if(l>=r) return;
52     if(l==now->l&&r==now->r){
53         now->val=0;
54         return;
55 }

```

```

56 int m=(now->l+now->r)/2;
57 now->push();
58 if(l>=m){
59     set0(now->rc, l, r);
60 }
61 else if(r<=m){
62     set0(now->lc, l, r);
63 }
64 else{
65     set0(now->lc, l, m);
66     set0(now->rc, m, r);
67 }
68 now->pull();
69 }
70 vector<int> g[N];
71 void DFS(int u, int p, int d){
72     dep[u]=d;
73     sz[u]=1;
74     for(int i=0; i<g[u].size(); i++){
75         int v=g[u][i];
76         if(v==p) continue;
77         DFS(v, u, d+1);
78         sz[u]+=sz[v];
79     }
80 }
81 void decom(int u, int p, bool istop){
82     bool ed=true;
83     if(istop) top[u]=u, up[u]=p, lightval[u]=1;
84     else top[u]=top[p], up[u]=up[p];
85     for(int i=0; i<g[u].size(); i++){
86         int v=g[u][i];
87         if(v==p) continue;
88         if(sz[v]>=sz[u]-sz[v]){
89             decom(v, u, false);
90             ed=false;
91         }
92         else decom(v, u, true);
93     }
94     if(ed){
95         tr[top[u]]=build(dep[top[u]], dep[u]);
96     }
97 }
98 //global
99 int qry(int u, int v){
100     int res=0;
101     while(top[u]!=top[v]){
102         if(dep[top[u]]>dep[top[v]]) swap(u, v);
103         res+=qry(tr[top[v]], dep[top[v]], dep[v]);
104         res+=lightval[top[v]];
105         v=up[top[v]];
106     }
107     if(dep[u]>dep[v]) swap(u, v);
108     res+=qry(tr[top[v]], dep[u], dep[v]);
109     return res;
110 }
111 void set0(int u, int v){

```

```

112 while(top[u]!=top[v]){
113     if(dep[top[u]]>dep[top[v]]) swap(u,v);
114     set0(tr[top[v]],dep[top[v]],dep[v]);
115     lightval[top[v]]=0;
116     v=up[top[v]];
117 }
118 if(dep[u]>dep[v]) swap(u,v);
119 set0(tr[top[v]],dep[u],dep[v]);
120 }
121 int main(){
122     DFS(1,0,0);
123     decom(1,0,true);
124 }

```

5.5 K-D Tree (Insert)

```

1 #ifndef SUNMOON_DYNEMIC_KD_TREE
2 #define SUNMOON_DYNEMIC_KD_TREE
3 #include<algorithm>
4 #include<vector>
5 #include<queue>
6 #include<cmath>
7 template<typename T,size_t kd> //kd表示有幾個維度
8 class kd_tree{
9     public:
10         struct point{
11             T d[kd];
12             inline T dist(const point &x) const{
13                 T ret=0;
14                 for(size_t i=0;i<kd;++i) ret+=std::abs(d[i]-x.d[i]);
15                 return ret;
16             }
17             inline bool operator<(const point &b) const{
18                 return d[0]<b.d[0];
19             }
20         };
21     private:
22         struct node{
23             node *l,*r;
24             point pid;
25             int s;
26             node(const point &p):l(0),r(0),pid(p),s(1){}
27             inline void up(){
28                 s=(l?l->s:0)+1+(r?r->s:0);
29             }
30         };
31         const double alpha,loga;
32         const T INF; //記得要給INF，表示極大值
33         std::vector<node*> A;
34         int qM;
35         std::priority_queue<std::pair<T,point>> pQ;
36         struct __cmp{
37             int sort_id;
38             inline bool operator()(const node*x,const node*y) const{

```

```

39             return x->pid.d[sort_id]<y->pid.d[sort_id];
40         }
41     } cmp;
42     void clear(node *o){
43         if(!o) return;
44         clear(o->l);
45         clear(o->r);
46         delete o;
47     }
48     inline int size(node *o){
49         return o?o->s:0;
50     }
51     node* build(int k,int l,int r){
52         if(l>r) return 0;
53         if(k==kd) k=0;
54         int mid=(l+r)/2;
55         cmp.sort_id=k;
56         std::nth_element(A.begin()+l,A.begin()+mid,A.begin()+r+1,cmp);
57         node *ret=A[mid];
58         ret->l=build(k+1,l,mid-1);
59         ret->r=build(k+1,mid+1,r);
60         ret->up();
61         return ret;
62     }
63     inline bool isbad(node*o){
64         return size(o->l)>alpha*o->s||size(o->r)>alpha*o->s;
65     }
66     void flatten(node *u,typename std::vector<node*>::iterator &it){
67         if(!u) return;
68         flatten(u->l,it);
69         *it=u;
70         flatten(u->r,++it);
71     }
72     bool insert(node*&u,int k,const point &x,int dep){
73         if(!u){
74             u=new node(x);
75             return dep<=0;
76         }
77         ++u->s;
78         if(insert(x.d[k]<u->pid.d[k]?u->l:u->r,(k+1)%kd,x,dep-1)){
79             if(!isbad(u)) return 1;
80             if((int)A.size()<u->s) A.resize(u->s);
81             typename std::vector<node*>::iterator it=A.begin();
82             flatten(u,it);
83             u=build(k,0,u->s-1);
84         }
85         return 0;
86     }
87     inline T heuristic(const T h[]) const{
88         T ret=0;
89         for(size_t i=0;i<kd;++i) ret+=h[i];
90         return ret;
91     }
92     void nearest(node *u,int k,const point &x,T *h,T &mndist){
93         if(u==0||heuristic(h)>=mndist) return;
94         T dist=u->pid.dist(x),old=h[k];

```

```

95     /*mndist=std::min(mndist,dist);*/
96     if(dist<mndist){
97         pQ.push(std::make_pair(dist,u->pid));
98         if((int)pQ.size()==qM+1){
99             mndist=pQ.top().first,pQ.pop();
100         }
101     }
102     if(x.d[k]<u->pid.d[k]){
103         nearest(u->l,(k+1)%kd,x,h,mndist);
104         h[k]=std::abs(x.d[k]-u->pid.d[k]);
105         nearest(u->r,(k+1)%kd,x,h,mndist);
106     }else{
107         nearest(u->r,(k+1)%kd,x,h,mndist);
108         h[k]=std::abs(x.d[k]-u->pid.d[k]);
109         nearest(u->l,(k+1)%kd,x,h,mndist);
110     }
111     h[k]=old;
112 }
113 public:
114     kd_tree(const T &INF,double a=0.75):root(0),alpha(a),loga(log2(1.0/a)),
115     INF(INF){}
116     inline void clear(){
117         clear(root),root=0;
118     }
119     inline void build(int n,const point *p){
120         clear(root),A.resize(n);
121         for(int i=0;i<n;++i)A[i]=new node(p[i]);
122         root=build(0,0,n-1);
123     }
124     inline void insert(const point &x){
125         insert(root,0,x,std::__lg(size(root))/loga);
126     }
127     inline T nearest(const point &x,int k){
128         qM=k;
129         T mndist=INF,h[kd]={};
130         nearest(root,0,x,h,mndist);
131         mndist=pQ.top().first;
132         pQ=std::priority_queue<std::pair<T,point >>());
133         return mndist; /*回傳離x第k近的點的距離*/
134     }
135     inline int size(){return root?root->s:0;}
136 };
137 #endif

```

5.6 SCC

```

1 // Kosaraju - Find SCC by twice dfs, and the SCC DAG is in the Topology
2 // ordering.
3 // Owner: samsam2310
4 //
5 #include <bits/stdc++.h>
6 #define N 300002 // Maximum number of vertices
7 using namespace std;
8 vector<int> forward_graph[N]; // original graph

```

```

9 vector<int> backward_graph[N]; // reverse graph
10 vector<int> dag_graph[N]; // result dag graph(graph of scc)
11 int scc[N]; // SCC index of a vertex
12 bool visit[N];
13 void init() {
14     fill(forward_graph, forward_graph + N, vector<int>());
15     fill(backward_graph, backward_graph + N, vector<int>());
16     fill(dag_graph, dag_graph + N, vector<int>());
17 }
18 void dfs(vector<int> &graph, int now, int scc_id,
19         stack<int> *leave_order = NULL) {
20     visit[now] = true;
21     if (scc != -1) {
22         scc[now] = scc_id;
23     }
24     for (int v : graph[now]) {
25         if (!visit[v]) {
26             dfs(graph, v, scc_id, leave_order);
27         }
28     }
29     if (leave_order) {
30         leave_order->push(now);
31     }
32 }
33 int main(int argc, char *argv[]) {
34     ios_base::sync_with_stdio(false);
35     cin.tie(0);
36     init();
37     cin >> n;
38     for (int i = 0; i < n; ++i) {
39         int a, b; // edge of a -> b
40         cin >> a >> b;
41         forward_graph[a].push_back(b);
42         backward_graph[b].push_back(a);
43     }
44     // Find the SCC.
45     memset(visit, 0, sizeof(visit));
46     stack<int> leave_order;
47     for (int i = 0; i < n; ++i) {
48         if (!visit[i]) {
49             dfs(forward_graph, i, -1, &leave_order);
50         }
51     }
52     memset(visit, 0, sizeof(visit));
53     int scc_id = 0;
54     while (!leave_order.empty()) {
55         int v = leave_order.top();
56         leave_order.pop();
57         if (!visit[v]) {
58             dfs(backward_graph, v, scc_id, NULL);
59             ++scc_id;
60         }
61     }
62     // Build the SCC DAG.
63     for (int i = 0; i < n; ++i) {
64         for (int v : forward_graph[i]) {

```

```

65         if (scc[i] != scc[v]) {
66             dag_graph[scc[i]].push_back(scc[v]);
67         }
68     }
69 }
70 return 0;
71 }

```

```

45     split(t->l, k, a, b->l);
46     pull(b);
47 }
48 }
49 }

```

5.7 Treap

```

1 struct Treap{
2     Treap *l,*r;
3     int pri,sz,val,add;
4     Treap(int _val):pri(rand()),sz(1),val(_val),add(0),l(NULL),r(NULL){}
5 };
6
7 int size(Treap *t){
8     return t?t->sz:0;
9 }
10 void pull(Treap *t){
11     t->sz=size(t->l)+size(t->r)+1;
12 }
13 void push(Treap *t){
14     t->val+=t->add;
15     if(t->l) t->l->add+=t->add;
16     if(t->r) t->r->add+=t->add;
17     t->add=0;
18 }
19 Treap* merge(Treap *a,Treap *b){
20     if(!a||!b) return a?a:b;
21     if(a->pri > b->pri){
22         push(a);
23         a->r = merge(a->r,b);
24         pull(a);
25         return a;
26     }
27     else{
28         push(b);
29         b->l = merge(a,b->l);
30         pull(b);
31         return b;
32     }
33 }
34 void split(Treap *t,int k,Treap *&a,Treap *&b){
35     if(!t) a=b=NULL;
36     else{
37         push(t);
38         if(size(t->l) < k){
39             a=t;
40             split(t->r,k-size(t->l)-1,a->r,b);
41             pull(a);
42         }
43         else{
44             b=t;

```

6 Java

6.1 Big Integer

```

1 import java.math.*;
2 import java.io.*;
3 import java.util.*;
4 public class Main{
5     public static void main(String []argv){
6         c[0][0]=BigInteger.ONE;
7         for(int i=1;i<3001;i++){
8             c[i][0]=BigInteger.ONE;
9             c[i][i]=BigInteger.ONE;
10            for(int j=1;j<i;j++)c[i][j]=c[i-1][j].add(c[i-1][j-1]);
11        }
12        Scanner scanner = new Scanner(System.in);
13        int T = scanner.nextInt();
14        BigInteger x;
15        BigInteger ans;
16        while(T-- > 0){
17            ans = BigInteger.ZERO;
18            int n = scanner.nextInt();
19            for(int i=0;i<n;i++){
20                x = new BigInteger(scanner.next());
21                if(i%2 == 1)ans=ans.subtract(c[n-1][i].multiply(x));
22                else ans=ans.add(c[n-1][i].multiply(x));
23            }
24            if(n%2 == 0)ans=BigInteger.ZERO.subtract(ans);
25            System.out.println(ans);
26        }
27    }
28 }

```

6.2 Prime

```

1 import java.math.*;
2 import java.io.*;
3 import java.util.*;
4 public class Main{
5     public static void main(String []argv){
6         Scanner scanner = new Scanner(System.in);
7         int T = scanner.nextInt();
8         for (int cs = 0 ; cs < T ; cs++){

```

```

9         if (cs != 0) {
10             System.out.println("");
11         }
12         int a = scanner.nextInt();
13         int b = scanner.nextInt();
14         for (int i = a ; i <= b ; i++) {
15             BigInteger x = BigInteger.valueOf(i);
16             if (x.isProbablePrime(5) == true) {
17                 System.out.println(x);
18             }
19         }
20     }
21 }
22 }
23 }

```

7 Matching

7.1 Bipartite Matching

```

1 #include<bits/stdc++.h>
2 #define V 20100
3 #define inf 0x3f3f3f3f
4 int mx[V],my[V],dis[V],que[V];
5 bool vis[V];
6 vector<int> g[V];
7 bool DFS(int u){
8     vis[u]=true;
9     for(int i=0;i<g[u].size();i++){
10         int v=my[g[u][i]];
11         if(v==-1||!vis[v]&&dis[v]==dis[u]+1&&DFS(v)){
12             mx[u]=g[u][i];
13             my[g[u][i]]=u;
14             return true;
15         }
16     }
17     return false;
18 }
19 // n is the size of left hand side
20 int Hopcroft_Karp(int n){
21     int matching=0,qt,qf,sp,i,u,v;
22     bool flag=true;
23     memset(mx,-1,sizeof(mx));
24     memset(my,-1,sizeof(my));
25     while(flag){
26         flag=false;
27         qt=qf=0;
28         sp=inf;
29         for(i=0;i<n;i++){
30             if(mx[i]==-1){
31                 dis[i]=0;
32                 que[qt++]=i;

```

```

33     }
34     else dis[i]=inf;
35 }
36 while(qf<qt){
37     u=que[qf++];
38     if(dis[u]>=sp) continue;
39     for(i=0;i<g[u].size();i++){
40         v=my[g[u][i]];
41         if(v==-1){
42             if(dis[u]+1<sp){
43                 sp=dis[u]+1;
44                 flag=true;
45             }
46         }
47         else if(dis[u]+1<dis[v]){
48             dis[v]=dis[u]+1;
49             que[qt++]=v;
50         }
51     }
52 }
53 if(flag){
54     memset(vis,0,sizeof(vis));
55     for(i=0;i<n;i++){
56         if(dis[i]==0&&DFS(i)) matching++;
57     }
58 }
59 }
60 return matching;
61 }

```

7.2 Blossom

```

1 #define MAXN 505
2 vector<int>g[MAXN]; //用vector存圖
3 int pa[MAXN],match[MAXN],st[MAXN],S[MAXN],vis[MAXN];
4 int t,n;
5 inline int lca(int u,int v){ //找花的花托
6     for(++t;;swap(u,v)){
7         if(u==0) continue;
8         if(vis[u]==t) return u;
9         vis[u]=t; //這種方法可以不用清空vis陣列
10        u=st[pa[match[u]]];
11    }
12 }
13 #define qpush(u) q.push(u),S[u]=0
14 inline void flower(int u,int v,int l,queue<int> &q){
15     while(st[u]!=1){
16         pa[u]=v; //所有未匹配邊的pa都是雙向的
17         if(S[v==match[u]]==1) qpush(v); //所有奇點變偶點
18         st[u]=st[v]=1,u=pa[v];
19     }
20 }
21 inline bool bfs(int u){
22     for(int i=1;i<=n;++i) st[i]=i; //st[i]表示第i個點的集合

```

```

23  memset(S+1,-1,sizeof(int)*n); //-1:沒走過 0:偶點 1:奇點
24  queue<int>q; qpush(u);
25  while(q.size()){
26      u=q.front(),q.pop();
27      for(size_t i=0;i<g[u].size();++i){
28          int v=g[u][i];
29          if(S[v]==-1){
30              pa[v]=u,S[v]=1;
31              if(!match[v]){ //有增廣路直接擴充
32                  for(int lst;u;v=lst,u=pa[v]){
33                      lst=match[u],match[u]=v,match[v]=u;
34                      return 1;
35                  }
36                  qpush(match[v]);
37              }else if(!S[v]&&st[v]!=st[u]){
38                  int l=lca(st[v],st[u]); //遇到花，做花的處理
39                  flower(v,u,l,q),flower(u,v,l,q);
40              }
41          }
42      }
43      return 0;
44  }
45  inline int blossom(){
46      memset(pa+1,0,sizeof(int)*n);
47      memset(match+1,0,sizeof(int)*n);
48      int ans=0;
49      for(int i=1;i<=n;++i)
50          if(!match[i]&&bfs(i)) ++ans;
51      return ans;
52  }

```

7.3 Dinic

```

1  //Dinic
2  #define V 1000
3  struct edge{
4      edge(){}
5      edge(int a,int b,int c):to(a),cap(b),rev(c){}
6      int to,cap,rev;
7  };
8  vector<edge> g[V];
9  int level[V];
10 int iter[V];
11 void add_edge(int from,int to,int cap){
12     g[from].push_back(edge(to,cap,g[to].size()));
13     g[to].push_back(edge(from,0,g[from].size()-1));
14 }
15 void bfs(int s){
16     memset(level,-1,sizeof(level));
17     queue<int>que;
18     level[s]=0;
19     que.push(s);
20     while(!que.empty()){
21         int v=que.front();

```

```

22         que.pop();
23         for(int q=0;q<g[v].size();q++){
24             edge &e=g[v][q];
25             if(e.cap>0&&level[e.to]<0){
26                 level[e.to]=level[v]+1;
27                 que.push(e.to);
28             }
29         }
30     }
31 }
32 int dfs(int v,int t,int f){
33     if(v==t) return f;
34     for(int &q=iter[v];q<g[v].size();++q){
35         edge &e=g[v][q];
36         if(e.cap>0&&level[v]<level[e.to]){
37             int d=dfs(e.to,t,min(f,e.cap));
38             if(d>0){
39                 e.cap-=d;
40                 g[e.to][e.rev].cap+=d;
41                 return d;
42             }
43         }
44     }
45     return 0;
46 }
47 int max_flow(int s,int t){
48     int flow=0;
49     for(;;){
50         bfs(s);
51         if(level[t]<0) return flow;
52         memset(iter,0,sizeof(iter));
53         int f;
54         while((f=dfs(s,t,1e9))>0)
55             flow+=f;
56     }
57 }

```

7.4 General Weighted Matching

```

1  #include <iostream>
2  #include <cstdio>
3  #include <algorithm>
4  #include <vector>
5  using namespace std;
6
7  typedef long long s64;
8
9  const int INF = 2147483647;
10
11 const int MaxN = 400;
12 const int MaxM = 79800;
13
14 template <class T>
15 inline void tension(T &a, const T &b)

```

```

16 {
17     if (b < a)
18         a = b;
19 }
20 template <class T>
21 inline void relax(T &a, const T &b)
22 {
23     if (b > a)
24         a = b;
25 }
26 template <class T>
27 inline int size(const T &a)
28 {
29     return (int)a.size();
30 }
31
32 inline int getint()
33 {
34     char c;
35     while (c = getchar(), '0' > c || c > '9');
36
37     int res = c - '0';
38     while (c = getchar(), '0' <= c && c <= '9')
39         res = res * 10 + c - '0';
40     return res;
41 }
42
43 const int MaxNX = MaxN + MaxN;
44
45 struct edge
46 {
47     int v, u, w;
48
49     edge(){}
50     edge(const int &_v, const int &_u, const int &_w)
51         : v(_v), u(_u), w(_w){}
52 };
53
54 int n, m;
55 edge mat[MaxNX + 1][MaxNX + 1];
56
57 int n_matches;
58 s64 tot_weight;
59 int mate[MaxNX + 1];
60 int lab[MaxNX + 1];
61
62 int q_n, q[MaxN];
63 int fa[MaxNX + 1], col[MaxNX + 1];
64 int slackv[MaxNX + 1];
65
66 int n_x;
67 int bel[MaxNX + 1], blofrom[MaxNX + 1][MaxN + 1];
68 vector<int> bloch[MaxNX + 1];
69
70 inline int e_delta(const edge &e) // does not work inside blossoms
71 {

```

```

72     return lab[e.v] + lab[e.u] - mat[e.v][e.u].w * 2;
73 }
74 inline void update_slackv(int v, int x)
75 {
76     if (!slackv[x] || e_delta(mat[v][x]) < e_delta(mat[slackv[x]][x]))
77         slackv[x] = v;
78 }
79 inline void calc_slackv(int x)
80 {
81     slackv[x] = 0;
82     for (int v = 1; v <= n; v++)
83         if (mat[v][x].w > 0 && bel[v] != x && col[bel[v]] == 0)
84             update_slackv(v, x);
85 }
86
87 inline void q_push(int x)
88 {
89     if (x <= n)
90         q[q_n++] = x;
91     else
92     {
93         for (int i = 0; i < size(bloch[x]); i++)
94             q_push(bloch[x][i]);
95     }
96 }
97 inline void set_mate(int xv, int xu)
98 {
99     mate[xv] = mat[xv][xu].u;
100     if (xv > n)
101     {
102         edge e = mat[xv][xu];
103         int xr = blofrom[xv][e.v];
104         int pr = find(bloch[xv].begin(), bloch[xv].end(), xr) - bloch[xv].begin();
105
106         if (pr % 2 == 1)
107         {
108             reverse(bloch[xv].begin() + 1, bloch[xv].end());
109             pr = size(bloch[xv]) - pr;
110         }
111
112         for (int i = 0; i < pr; i++)
113             set_mate(bloch[xv][i], bloch[xv][i ^ 1]);
114         set_mate(xr, xu);
115
116         rotate(bloch[xv].begin(), bloch[xv].begin() + pr, bloch[xv].end());
117     }
118 }
119 inline void set_bel(int x, int b)
120 {
121     bel[x] = b;
122     if (x > n)
123     {
124         for (int i = 0; i < size(bloch[x]); i++)
125             set_bel(bloch[x][i], b);
126     }
127 }

```

```

127
128 inline void augment(int xv, int xu)
129 {
130     while (true)
131     {
132         int xnu = bel[mate[xv]];
133         set_mate(xv, xu);
134         if (!xnu)
135             return;
136         set_mate(xnu, bel[fa[xnu]]);
137         xv = bel[fa[xnu]], xu = xnu;
138     }
139 }
140 inline int get_lca(int xv, int xu)
141 {
142     static bool book[MaxNX + 1];
143     for (int x = 1; x <= n_x; x++)
144         book[x] = false;
145     while (xv || xu)
146     {
147         if (xv)
148         {
149             if (book[xv])
150                 return xv;
151             book[xv] = true;
152             xv = bel[mate[xv]];
153             if (xv)
154                 xv = bel[fa[xv]];
155         }
156         swap(xv, xu);
157     }
158     return 0;
159 }
160
161 inline void add_blossom(int xv, int xa, int xu)
162 {
163     int b = n + 1;
164     while (b <= n_x && bel[b])
165         b++;
166     if (b > n_x)
167         n_x++;
168
169     lab[b] = 0;
170     col[b] = 0;
171
172     mate[b] = mate[xa];
173
174     bloch[b].clear();
175     bloch[b].push_back(xa);
176     for (int x = xv; x != xa; x = bel[fa[bel[mate[x]]]])
177         bloch[b].push_back(x), bloch[b].push_back(bel[mate[x]]), q_push(bel[mate[x]]);
178     reverse(bloch[b].begin() + 1, bloch[b].end());
179     for (int x = xu; x != xa; x = bel[fa[bel[mate[x]]]])
180         bloch[b].push_back(x), bloch[b].push_back(bel[mate[x]]), q_push(bel[mate[x]]);
181
182     set_bel(b, b);
183
184     for (int x = 1; x <= n_x; x++)
185     {
186         mat[b][x].w = mat[x][b].w = 0;
187         blofrom[b][x] = 0;
188     }
189     for (int i = 0; i < size(bloch[b]); i++)
190     {
191         int xs = bloch[b][i];
192         for (int x = 1; x <= n_x; x++)
193             if (mat[b][x].w == 0 || e_delta(mat[xs][x]) < e_delta(mat[b][x]))
194                 mat[b][x] = mat[xs][x], mat[x][b] = mat[x][xs];
195         for (int x = 1; x <= n_x; x++)
196             if (blofrom[xs][x])
197                 blofrom[b][x] = xs;
198     }
199     calc_slackv(b);
200 }
201 inline void expand_blossom1(int b) // lab[b] == 1
202 {
203     for (int i = 0; i < size(bloch[b]); i++)
204         set_bel(bloch[b][i], bloch[b][i]);
205
206     int xr = blofrom[b][mat[b][fa[b]].v];
207     int pr = find(bloch[b].begin(), bloch[b].end(), xr) - bloch[b].begin();
208     if (pr % 2 == 1)
209     {
210         reverse(bloch[b].begin() + 1, bloch[b].end());
211         pr = size(bloch[b]) - pr;
212     }
213
214     for (int i = 0; i < pr; i += 2)
215     {
216         int xs = bloch[b][i], xns = bloch[b][i + 1];
217         fa[xs] = mat[xns][xs].v;
218         col[xs] = 1, col[xns] = 0;
219         slackv[xs] = 0, calc_slackv(xns);
220         q_push(xns);
221     }
222     col[xr] = 1;
223     fa[xr] = fa[b];
224     for (int i = pr + 1; i < size(bloch[b]); i++)
225     {
226         int xs = bloch[b][i];
227         col[xs] = -1;
228         calc_slackv(xs);
229     }
230
231     bel[b] = 0;
232 }
233 inline void expand_blossom_final(int b) // at the final stage
234 {
235     for (int i = 0; i < size(bloch[b]); i++)
236     {

```



```

237     if (bloch[b][i] > n && lab[bloch[b][i]] == 0)
238         expand_blossom_final(bloch[b][i]);
239     else
240         set_bel(bloch[b][i], bloch[b][i]);
241 }
242 bel[b] = 0;
243 }
244
245 inline bool on_found_edge(const edge &e)
246 {
247     int xv = bel[e.v], xu = bel[e.u];
248     if (col[xu] == -1)
249     {
250         int nv = bel[mate[xu]];
251         fa[xu] = e.v;
252         col[xu] = 1, col[nv] = 0;
253         slackv[xu] = slackv[nv] = 0;
254         q_push(nv);
255     }
256     else if (col[xu] == 0)
257     {
258         int xa = get_lca(xv, xu);
259         if (!xa)
260         {
261             augment(xv, xu), augment(xu, xv);
262             for (int b = n + 1; b <= n_x; b++)
263                 if (bel[b] == b && lab[b] == 0)
264                     expand_blossom_final(b);
265             return true;
266         }
267         else
268             add_blossom(xv, xa, xu);
269     }
270     return false;
271 }
272
273 bool match()
274 {
275     for (int x = 1; x <= n_x; x++)
276         col[x] = -1, slackv[x] = 0;
277
278     q_n = 0;
279     for (int x = 1; x <= n_x; x++)
280         if (bel[x] == x && !mate[x])
281             fa[x] = 0, col[x] = 0, slackv[x] = 0, q_push(x);
282     if (q_n == 0)
283         return false;
284
285     while (true)
286     {
287         for (int i = 0; i < q_n; i++)
288         {
289             int v = q[i];
290             for (int u = 1; u <= n; u++)
291                 if (mat[v][u].w > 0 && bel[v] != bel[u])
292                     {

```

```

293                     int d = e_delta(mat[v][u]);
294                     if (d == 0)
295                     {
296                         if (on_found_edge(mat[v][u]))
297                             return true;
298                     }
299                     else if (col[bel[u]] == -1 || col[bel[u]] == 0)
300                         update_slackv(v, bel[u]);
301                     }
302             }
303
304     int d = INF;
305     for (int v = 1; v <= n; v++)
306         if (col[bel[v]] == 0)
307             tension(d, lab[v]);
308     for (int b = n + 1; b <= n_x; b++)
309         if (bel[b] == b && col[b] == 1)
310             tension(d, lab[b] / 2);
311     for (int x = 1; x <= n_x; x++)
312         if (bel[x] == x && slackv[x])
313         {
314             if (col[x] == -1)
315                 tension(d, e_delta(mat[slackv[x]][x]));
316             else if (col[x] == 0)
317                 tension(d, e_delta(mat[slackv[x]][x]) / 2);
318         }
319
320     for (int v = 1; v <= n; v++)
321     {
322         if (col[bel[v]] == 0)
323             lab[v] -= d;
324         else if (col[bel[v]] == 1)
325             lab[v] += d;
326     }
327     for (int b = n + 1; b <= n_x; b++)
328         if (bel[b] == b)
329         {
330             if (col[bel[b]] == 0)
331                 lab[b] += d * 2;
332             else if (col[bel[b]] == 1)
333                 lab[b] -= d * 2;
334         }
335
336     q_n = 0;
337     for (int v = 1; v <= n; v++)
338         if (lab[v] == 0) // all unmatched vertices' labels are zero! cheers!
339             return false;
340     for (int x = 1; x <= n_x; x++)
341         if (bel[x] == x && slackv[x] && bel[slackv[x]] != x && e_delta(mat[
342             slackv[x]][x]) == 0)
343         {
344             if (on_found_edge(mat[slackv[x]][x]))
345                 return true;
346         }
347     for (int b = n + 1; b <= n_x; b++)
348         if (bel[b] == b && col[b] == 1 && lab[b] == 0)

```

```

348     expand_blossom1(b);
349 }
350 return false;
351 }
352
353 void calc_max_weight_match()
354 {
355     for (int v = 1; v <= n; v++)
356         mate[v] = 0;
357
358     n_x = n;
359     n_matches = 0;
360     tot_weight = 0;
361
362     bel[0] = 0;
363     for (int v = 1; v <= n; v++)
364         bel[v] = v, bloch[v].clear();
365     for (int v = 1; v <= n; v++)
366         for (int u = 1; u <= n; u++)
367             blofrom[v][u] = v == u ? v : 0;
368
369     int w_max = 0;
370     for (int v = 1; v <= n; v++)
371         for (int u = 1; u <= n; u++)
372             relax(w_max, mat[v][u].w);
373     for (int v = 1; v <= n; v++)
374         lab[v] = w_max;
375
376     while (match())
377         n_matches++;
378
379     for (int v = 1; v <= n; v++)
380         if (mate[v] && mate[v] < v)
381             tot_weight += mat[v][mate[v]].w;
382 }
383
384 int main()
385 {
386     n = getint(), m = getint();
387
388     for (int v = 1; v <= n; v++)
389         for (int u = 1; u <= n; u++)
390             mat[v][u] = edge(v, u, 0);
391
392     for (int i = 0; i < m; i++)
393     {
394         int v = getint(), u = getint(), w = getint();
395         mat[v][u].w = mat[u][v].w = w;
396     }
397
398     calc_max_weight_match();
399
400     printf("%lld\n", tot_weight);
401     for (int v = 1; v <= n; v++)
402         printf("%d ", mate[v]);
403     printf("\n");

```

```

404
405     return 0;
406 }

```

7.5 KM

```

1 #define MAXN 100
2 #define INF INT_MAX
3 int g[MAXN][MAXN], lx[MAXN], ly[MAXN], slack_y[MAXN];
4 int px[MAXN], py[MAXN], match_y[MAXN], par[MAXN];
5 int n;
6 void adjust(int y) { //把增廣路上所有邊反轉
7     match_y[y] = py[y];
8     if (px[match_y[y]] != -2)
9         adjust(px[match_y[y]]);
10 }
11 bool dfs(int x) { //DFS找增廣路
12     for (int y = 0; y < n; ++y) {
13         if (py[y] != -1) continue;
14         int t = lx[x] + ly[y] - g[x][y];
15         if (t == 0) {
16             py[y] = x;
17             if (match_y[y] == -1) {
18                 adjust(y);
19                 return 1;
20             }
21             if (px[match_y[y]] != -1) continue;
22             px[match_y[y]] = y;
23             if (dfs(match_y[y])) return 1;
24         } else if (slack_y[y] > t) {
25             slack_y[y] = t;
26             par[y] = x;
27         }
28     }
29     return 0;
30 }
31 inline int km() {
32     memset(ly, 0, sizeof(int) * n);
33     memset(match_y, -1, sizeof(int) * n);
34     for (int x = 0; x < n; ++x) {
35         lx[x] = -INF;
36         for (int y = 0; y < n; ++y) {
37             lx[x] = max(lx[x], g[x][y]);
38         }
39     }
40     for (int x = 0; x < n; ++x) {
41         for (int y = 0; y < n; ++y) slack_y[y] = INF;
42         memset(px, -1, sizeof(int) * n);
43         memset(py, -1, sizeof(int) * n);
44         px[x] = -2;
45         if (dfs(x)) continue;
46         bool flag = 1;
47         while (flag) {
48             int cut = INF;

```

```

49     for(int y=0;y<n;++y)
50         if(py[y]==-1&&cut>slack_y[y])cut=slack_y[y];
51     for(int j=0;j<n;++j){
52         if(px[j]!=-1)lx[j]-=cut;
53         if(py[j]!=-1)ly[j]+=cut;
54         else slack_y[j]-=cut;
55     }
56     for(int y=0;y<n;++y){
57         if(py[y]==-1&&slack_y[y]==0){
58             py[y]=par[y];
59             if(match_y[y]==-1){
60                 adjust(y);
61                 flag=0;
62                 break;
63             }
64             px[match_y[y]]=y;
65             if(dfs(match_y[y])){
66                 flag=0;
67                 break;
68             }
69         }
70     }
71 }
72 }
73 int ans=0;
74 for(int y=0;y<n;++y)if(g[match_y[y]][y]!=-INF)ans+=g[match_y[y]][y];
75 return ans;
76 }

```

7.6 Min Cost Flow

```

1 #define maxnode (1000+10)
2 #define maxedge (40000+10)
3 #define INF 1023456789
4 #include<bits/stdc++.h>
5 using namespace std;
6 int node, src, dest, nedge;
7 int head[maxnode], point[maxedge], nxt[maxedge], flow[maxedge], capa[maxedge], wt[maxedge];
8 int dist[maxnode], in[maxnode], from[maxnode], mf[maxnode];
9 //set number of node, source, and destination (one base)
10 void init(int _node, int _src, int _dest) {
11     node = _node;
12     src = _src;
13     dest = _dest;
14     nedge = 0;
15     memset(point, -1, sizeof(point));
16     for (int i = 1; i <= node; i++) head[i] = -1;
17     nedge = 0;
18 }
19 void add_edge(int u, int v, int c1, int w) {
20     point[nedge] = v, capa[nedge] = c1, flow[nedge] = 0, nxt[nedge] = head[u], wt[nedge]=w, head[u] = (nedge++);

```

```

21     point[nedge] = u, capa[nedge] = 0, flow[nedge] = 0, nxt[nedge] = head[v], wt[nedge]=-w, head[v] = (nedge++);
22 }
23 int sp(int &left){
24     for(int i=1;i<=node;i++) dist[i]=INF;
25     queue<int> que;
26     que.push(src);
27     in[src]=1;
28     mf[src]=left;
29     dist[src]=0;
30     while(!que.empty()){
31         int u=que.front();
32         que.pop();
33         in[u]=0;
34         if(dist[u]>=dist[dest]) continue;
35         for(int v=head[u];v!=-1;v=nxt[v]){
36             if(flow[v]==capa[v]) continue;
37             if(dist[u]+wt[v]<dist[point[v]]){
38                 dist[point[v]]=dist[u]+wt[v];
39                 from[point[v]]=u;
40                 mf[point[v]]=min(mf[u],capa[v]-flow[v]);
41                 if(!in[point[v]]){
42                     in[point[v]]=1;
43                     que.push(point[v]);
44                 }
45             }
46         }
47     }
48     left-=mf[dest];
49     if(dist[dest]<INF){
50         for(int u=dest;u!=src;u=point[from[u]^1]){
51             flow[from[u]]+=mf[dest];
52             flow[from[u]^1]-=mf[dest];
53         }
54     }
55     return dist[dest];
56 }
57 int min_cost_flow(){
58     int res=0,tmp,maxflow=2;
59     while(maxflow&&(tmp=sp(maxflow))<INF) res+=tmp;
60     return res;
61 }
62 int main(){
63     int n,m,x,y,z;
64     while(scanf("%d%d",&n,&m)==2){
65         init(n,1,n);
66         for(int i=0;i<m;i++){
67             scanf("%d%d%d",&x,&y,&z);
68             add_edge(x,y,1,z);
69             add_edge(y,x,1,z); //undirected
70         }
71         printf("%d\n",min_cost_flow());
72     }
73     return 0;
74 }

```

7.7 Stable Marriage

```

1 #define F(n) Fi(i, n)
2 #define Fi(i, n) Fl(i, 0, n)
3 #define Fl(i, l, n) for(int i = l ; i < n ; ++i)
4 #include <bits/stdc++.h>
5 using namespace std;
6 int D, quota[205], weight[205][5];
7 int S, scoretoDep[12005][205], score[5];
8 int P, prefer[12005][85], iter[12005];
9 int ans[12005];
10 typedef pair<int, int> PII;
11 map<int, int> samescore[205];
12 typedef priority_queue<PII, vector<PII>, greater<PII>> QQQ;
13 QQQ pri[205];
14 void check(int d) {
15     PII t = pri[d].top();
16     int v;
17     if (pri[d].size() - samescore[d][t.first] + 1 <= quota[d]) return;
18     while (pri[d].top().first == t.first) {
19         v = pri[d].top().second;
20         ans[v] = -1;
21         --samescore[d][t.first];
22         pri[d].pop();
23     }
24 }
25 void push(int s, int d) {
26     if (pri[d].size() < quota[d]) {
27         pri[d].push(PII(scoretoDep[s][d], s));
28         ans[s] = d;
29         ++samescore[s][scoretoDep[s][d]];
30     } else if (scoretoDep[s][d] >= pri[d].top().first) {
31         pri[d].push(PII(scoretoDep[s][d], s));
32         ans[s] = d;
33         ++samescore[s][scoretoDep[s][d]];
34         check(d);
35     }
36 }
37 void f() {
38     int over;
39     while (true) {
40         over = 1;
41         Fi (q, S) {
42             if (ans[q] != -1 || iter[q] >= P) continue;
43             push(q, prefer[q][iter[q]++]);
44             over = 0;
45         }
46         if (over) break;
47     }
48 }
49 int main() {
50     ios::sync_with_stdio(false);
51     cin.tie(NULL);
52     int sadmit, stof, d exceed, dfew;
53     while (cin >> D, D) { // Beware of the input format or judge may troll us.
54         sadmit = stof = d exceed = dfew = 0;

```

```

55     memset(iter, 0, sizeof(iter));
56     memset(ans, 0, sizeof(ans));
57     Fi (q, 205) {
58         pri[q] = QQQ();
59         samescore[q].clear();
60     }
61     cin >> S >> P;
62     Fi (q, D) {
63         cin >> quota[q];
64         Fi (w, 5) cin >> weight[q][w];
65     }
66     Fi (q, S) {
67         Fi (w, 5) cin >> score[w];
68         Fi (w, D) {
69             scoretoDep[q][w] = 0;
70             F (5) scoretoDep[q][w] += weight[w][i] * score[i];
71         }
72     }
73     Fi (q, S) Fi (w, P) {
74         cin >> prefer[q][w];
75         --prefer[q][w];
76     }
77     f();
78     Fi (q, D) sadmit += pri[q].size();
79     Fi (q, S) if (ans[q] == prefer[q][0]) ++stof;
80     Fi (q, D) if (pri[q].size() > quota[q]) ++d exceed;
81     Fi (q, D) if (pri[q].size() < quota[q]) ++dfew;
82     cout << sadmit << ' ' << stof << ' ' << d exceed << ' ' << dfew << '\n';
83 }
84 }

```

8 Mathematics

8.1 Extended GCD

```

1 long long extgcd(long long a, long long b, long long &x, long long &y) {
2     long long d = a;
3     if (b != 0) {
4         d = extgcd(b, a % b, y, x);
5         y -= (a / b) * x;
6     }
7     else x = 1, y = 0;
8     return d;
9 }
10 int main() {
11     int T;
12     long long a, b, m, GCD, x, y;
13     while (~scanf("%d", &T))
14         while (T--) {
15             scanf("%lld%lld%lld", &m, &a, &b);
16             GCD = extgcd(a, m, x, y);
17             if (GCD != 1) printf("No inverse, gcd(a,m)=%lld\n", GCD);

```

```

18         else{
19             b=((-b*x)%m+m)%m;
20             printf("%lld %lld\n", (x%m+m)%m,b);
21         }
22     }
23 }

```

8.2 Lucas's Theorem

```

1 bigM = int(1e9+7)
2 fac = [1]*10001
3 for i in range(1, 10001):
4     fac[i] = fac[i-1]*i
5 ifac = [pow(fac[i], bigM-2, bigM) for i in range(10001)]
6 def f(a, b, M):
7     if b == 0 or b == a:
8         return 1
9     elif a < b:
10        return 0
11    elif a < M:
12        return fac[a]*ifac[b]*ifac[a-b]%bigM
13    else:
14        return f(a//M, b//M, M) * f(a%M, b%M, M) % bigM
15 t = int(input())
16 for cases in range(t):
17     a, b, M = [int(x) for x in input().split()]
18     print(f(a, b, M))

```

8.3 Miller-Rabin

```

1 inline long long mod_mul(long long a,long long b,long long m){
2     a%=m,b%=m;
3     long long y=(long long)((double)a*b/m+0.5);/* fast for m < 2^58 */
4     long long r=(a*b-y*m)%m;
5     return r<0?r+m:r;
6 }
7 template<typename T>
8 inline T pow(T a,T b,T mod){//a^b%mod
9     T ans=1;
10    for(;b;a=mod_mul(a,a,mod),b>>=1)
11        if(b&1)ans=mod_mul(ans,a,mod);
12    return ans;
13 }
14 int sprp[3]={2,7,61};//int範圍可解
15 int llsprp[7]={2,325,9375,28178,450775,9780504,1795265022};//至少unsigned
16    long long範圍
17 template<typename T>
18 inline bool isprime(T n,int *sprp,int num){
19     if(n==2)return 1;
20     if(n<2||n%2==0)return 0;
21     int t=0;

```

```

21     T u=n-1;
22     for(;u%2==0;++t)u>>=1;
23     for(int i=0;i<num;++i){
24         T a=sprp[i]%n;
25         if(a==0||a==1||a==n-1)continue;
26         T x=pow(a,u,n);
27         if(x==1||x==n-1)continue;
28         for(int j=0;j<t;++j){
29             x=mod_mul(x,x,n);
30             if(x==1)return 0;
31             if(x==n-1)break;
32         }
33         if(x==n-1)continue;
34         return 0;
35     }
36     return 1;
37 }

```

8.4 Tonelli Shanks

```

1 #include<cstdio>
2 #include<cassert>
3 #include<cstdlib>
4 using namespace std;
5 int pow_mod(int a,int p,int q){ //a^p mod q
6     int r=1;
7     while(p){
8         if(p&1) r=1LL*r*a%q;
9         a=1LL*a*a%q;
10        p>>=1;
11    }
12    return r;
13 }
14 int Jacobi(int q,int p){ //q/p
15     if(p==1) return 1;
16     q%=p;
17     int c2=0,m2;
18     while(!(q&1)){
19         q>>=1;
20         c2^=1;
21     }
22     if((p&7)==7||(p&7)==1||!c2) m2=1;
23     else m2=-1;
24     if((p&2)&&(q&2)) m2*=-1;
25     return m2*Jacobi(p,q);
26 }
27 int Tonelli_Shanks(int a,int p){ //p is prime,gcd(a,p)=1
28     if(p==2) return 1;
29     if(Jacobi(a,p)==-1) return -1;
30     int s=0,q=p-1,z=2;
31     while(!(q&1)) q>>=1,s++;
32     while(Jacobi(z,p)==1) z++;
33     z = pow_mod(z, q, p);
34     int zp[30]={z};

```

```

35 for(int i=1;i<s;i++) zp[i]=1LL*zp[i-1]*zp[i-1]%p;
36 int r = pow_mod(a, (q+1)>>1, p), t = pow_mod(a, q, p);
37 while(t!=1){
38     int m=0;
39     for(int i=t;i!=1;i=1LL*i*i%p) m++;
40     r=1LL*r*zp[s-m-1]%p;
41     t=1LL*t*zp[s-m]%p;
42 }
43 return r;
44 }
45 int main(){
46     for(int i=0;i<37;i++){
47     }
48 }
49 return 0;
50 }

```

9 String

9.1 AC Automaton

```

1 #ifndef SUNMOON_AHO_CORASICK_AUTOMATON
2 #define SUNMOON_AHO_CORASICK_AUTOMATON
3 #include<queue>
4 #include<vector>
5 template<char L='a',char R='z'>
6 class ac_automaton{
7     private:
8         struct joe{
9             int next[R-L+1],fail,efl,ed,cnt_dp,vis;
10             joe():ed(0),cnt_dp(0),vis(0){
11                 for(int i=0;i<=R-L;++i)next[i]=0;
12             }
13         };
14     public:
15         std::vector<joe> S;
16         std::vector<int> q;
17         int qs,qe,vt;
18         ac_automaton():S(1),qs(0),qe(0),vt(0){
19             inline void clear(){
20                 q.clear();
21                 S.resize(1);
22                 for(int i=0;i<=R-L;++i)S[0].next[i]=0;
23                 S[0].cnt_dp=S[0].vis=qs=qe=vt=0;
24             }
25             inline void insert(const char *s){
26                 int o=0;
27                 for(int i=0,id;s[i];++i){
28                     id=s[i]-L;
29                     if(!S[o].next[id]){
30                         S.push_back(joe());
31                         S[o].next[id]=S.size()-1;

```

```

32         }
33         o=S[o].next[id];
34     }
35     ++S[o].ed;
36 }
37 inline void build_fail(){
38     S[0].fail=S[0].efl=-1;
39     q.clear();
40     q.push_back(0);
41     ++qe;
42     while(qs!=qe){
43         int pa=q[qs++],id,t;
44         for(int i=0;i<=R-L;++i){
45             t=S[pa].next[i];
46             if(!t)continue;
47             id=S[pa].fail;
48             while(~id&&!S[id].next[i])id=S[id].fail;
49             S[t].fail=~id?S[id].next[i]:0;
50             S[t].efl=S[S[t].fail].ed?S[t].fail:S[S[t].fail].efl;
51             q.push_back(t);
52             ++qe;
53         }
54     }
55 }
56 /*DP出每個前綴在字串s出現的次數並傳回所有字串被s匹配成功的次數O(N+M)*/
57 inline int match_0(const char *s){
58     int ans=0,id,p=0,i;
59     for(i=0;s[i];++i){
60         id=s[i]-L;
61         while(!S[p].next[id]&&p) p=S[p].fail;
62         if(!S[p].next[id])continue;
63         p=S[p].next[id];
64         ++S[p].cnt_dp; /*匹配成功則它所有後綴都可以被匹配(DP計算)*/
65     }
66     for(i=qe-1;i>=0;--i){
67         ans+=S[q[i]].cnt_dp*S[q[i]].ed;
68         if(~S[q[i]].fail)S[S[q[i]].fail].cnt_dp+=S[q[i]].cnt_dp;
69     }
70     return ans;
71 }
72 /*多串匹配走efl邊並傳回所有字串被s匹配成功的次數O(N*M^1.5)*/
73 inline int match_1(const char *s) const{
74     int ans=0,id,p=0,t;
75     for(int i=0;s[i];++i){
76         id=s[i]-L;
77         while(!S[p].next[id]&&p) p=S[p].fail;
78         if(!S[p].next[id])continue;
79         p=S[p].next[id];
80         if(S[p].ed)ans+=S[p].ed;
81         for(t=S[p].efl;~t;t=S[t].efl){
82             ans+=S[t].ed; /*因為都走efl邊所以保證匹配成功*/
83         }
84     }
85     return ans;
86 }
87 /*枚舉(s的子字串nA)的所有相異字串各恰一次並傳回次數O(N*M^(1/3))*/

```

```

88 inline int match_2(const char *s){
89     int ans=0,id,p=0,t;
90     ++vt;
91     /*把戳記vt+=1，只要vt沒溢位，所有S[p].vis==vt就會變成false
92     這種利用vt的方法可以O(1)歸零vis陣列*/
93     for(int i=0;s[i];++i){
94         id=s[i]-L;
95         while(!S[p].next[id]&&p=S[p].fail;
96         if(!S[p].next[id])continue;
97         p=S[p].next[id];
98         if(S[p].ed&&S[p].vis!=vt){
99             S[p].vis=vt;
100             ans+=S[p].ed;
101         }
102         for(t=S[p].efl;~t&&S[t].vis!=vt;t=S[t].efl){
103             S[t].vis=vt;
104             ans+=S[t].ed; /*因為都走efl邊所以保證匹配成功*/
105         }
106     }
107     return ans;
108 }
109 /*把AC自動機變成真的自動機*/
110 inline void evolution(){
111     for(qs=1;qs!=qe;){
112         int p=q[qs++];
113         for(int i=0;i<=R-L;++i)
114             if(S[p].next[i]==0)S[p].next[i]=S[S[p].fail].next[i];
115     }
116 }
117 };
118 #endif

```

```

20
21     for (int i=0; i<N; ++i)
22         cout << s[(sa[i] + N-1) % N];
23
24     for (int i=0; i<N; ++i)
25         if (sa[i] == 0)
26         {
27             pivot = i;
28             break;
29         }
30 }
31
32 // Inverse BWT
33 const int N = 8;           // 字串長度
34 char t[N+1] = "xuffessi"; // 字串
35 int pivot;
36 int next[N];
37
38 void IBWT()
39 {
40     vector<int> index[256];
41     for (int i=0; i<N; ++i)
42         index[t[i]].push_back(i);
43
44     for (int i=0, n=0; i<256; ++i)
45         for (int j=0; j<index[i].size(); ++j)
46             next[n++] = index[i][j];
47
48     int p = pivot;
49     for (int i=0; i<N; ++i)
50         cout << t[p = next[p]];
51 }

```

9.2 BWT

```

1 // BWT
2 const int N = 8;           // 字串長度
3 int s[N+N+1] = "suffixes"; // 字串，後面預留一倍空間。
4 int sa[N];                 // 後綴陣列
5 int pivot;
6
7 int cmp(const void* i, const void* j)
8 {
9     return strcmp(s+(int*)i, s+(int*)j, N);
10 }
11
12 // 此處便宜行事，採用 O(N²logN) 的後綴陣列演算法。
13 void BWT()
14 {
15     strncpy(s + N, s, N);
16     for (int i=0; i<N; ++i) sa[i] = i;
17     qsort(sa, N, sizeof(int), cmp);
18     // 當輸入字串的所有字元都相同，必須當作特例處理。
19     // 或者改用stable sort。

```

9.3 Suffix Array

```

1 //should initialize s and n first
2 #define N 301000
3 using namespace std;
4 char s[N]; //string=s,suffix array=sar,longest common prefix=lcp
5 int rk[2][N],id[2][N];
6 int n,p;
7 int cnt[N];
8 int len[N],od[N],sar[N];
9 inline int sr(int i,int t){ //rank of shifted position
10     return i+t<n?rk[p][i+t]:-1;
11 }
12 inline bool check_same(int i,int j,int t){
13     return rk[p][i]==rk[p][j]&&sr(i,t)==sr(j,t);
14 }
15 bool cmp(int i,int j){
16     return s[i]<s[j];
17 }
18 void sa(){ //length of array s
19     int i,t,now,pre;

```

```

20  memset(cnt,0,sizeof(cnt));
21  for(i=0;i<n;i++){
22      id[p][i]=i;
23      rk[p][i]=s[i];
24      cnt[s[i]]++;
25  }
26  for(i=1;i<128;i++) cnt[i]+=cnt[i-1];
27  sort(id[p],id[p]+n,cmp);
28  for(t=1;t<n;t<=1){
29      //least significant bit is already sorted
30      for(i=n-1;i>=0;i--){
31          now=id[p][i]-t;
32          if(now>=0) id[p^1][--cnt[rk[p][now]]]=now;
33      }
34      for(i=n-t;i<n;i++){
35          id[p^1][--cnt[rk[p][i]]]=i;
36      }
37      memset(cnt,0,sizeof(cnt));
38      now=id[p^1][0];
39      rk[p^1][now]=0;
40      cnt[0]++;
41      for(i=1;i<n;i++){
42          pre=now;
43          now=id[p^1][i];
44          if(check_same(pre,now,t)){
45              rk[p^1][now]=rk[p^1][pre];
46          }
47          else{
48              rk[p^1][now]=rk[p^1][pre]+1;
49          }
50          cnt[rk[p^1][now]]++;
51      }
52      p^=1;
53      if(rk[p][now]==n-1) break;
54      for(i=1;i<n;i++) cnt[i]+=cnt[i-1];
55  }
56  memcpy(sar,id[p],sizeof(sar));
57 }
58 void lcp(){
59     int i,l,pre;
60     for(i=0;i<n;i++) od[sar[i]]=i;
61     for(i=0;i<n;i++){
62         if(i) l=len[od[i-1]]?len[od[i-1]]-1:0;
63         else l=0;
64         if(od[i]){
65             pre=sar[od[i]-1];
66             while(pre+l<n&&i+l<n&&s[pre+l]==s[i+l]) l++;
67             len[od[i]]=l;
68         }
69         else len[0]=0;
70     }
71 }

```

9.4 Suffix Automaton

```

1  #include<bits/stdc++.h>
2  #define C 96
3  #define N 200100
4  using namespace std;
5  struct SAM{
6      struct node{
7          node *nxt[C],*pre;
8          int len;
9          vector<int> pos;
10     };
11     node mem[N*2],*root,*ed;
12     int top;
13     SAM(){
14         top = 0;
15         root = new_node(0);
16         ed = root;
17     }
18     node *new_node(int l){
19         for(int i=0;i<C;i++) mem[top].nxt[i]=NULL;
20         mem[top].pre=NULL;
21         mem[top].len=l;
22         mem[top].pos.clear();
23         return mem+(top++);
24     }
25     node *split_node(int l,node *p){
26         for(int i=0;i<C;i++) mem[top].nxt[i]=p->nxt[i];
27         mem[top].pre = p->pre;
28         mem[top].len = l;
29         mem[top].pos.assign()
30         p->pre = mem+top;
31         return mem+(top++);
32     }
33     void push(char c){
34         node *nw = new_node(ed->len+1),*ptr=ed->pre;
35         ed->nxt[c] = nw;
36         nw->pos.push_back(ed->len);
37         for(ptr;ptr=ptr->pre){
38             if(ptr->nxt[c]){
39                 if(ptr->nxt[c]->len==ptr->len+1){
40                     nw->pre = ptr->nxt[c];
41                 }
42                 else{
43                     node *tmp=ptr->nxt[c];
44                     nw->pre = split_node(ptr->len+1,tmp);
45                     while(ptr && ptr->nxt[c]==tmp){
46                         ptr->nxt[c] = nw->pre;
47                         ptr = ptr->pre;
48                     }
49                 }
50             }
51             break;
52         }
53         else{
54             ptr->nxt[c] = nw;
55         }

```



```

55     }
56     if(!nw->pre) nw->pre = root;
57     ed = ed->nxt[c];
58 }
59 void init(){
60     while(top){
61         mem[--top].pos.clear();
62     }
63     root = new_node(0);
64     ed = root;
65 }
66 void push(char *s){
67     for(int i=0;s[i];i++) push(s[i]-32);
68 }
69 long long count(){
70     long long ans=0;
71     for(int i=1;i<top;i++){
72         ans+=mem[i].len-mem[i].pre->len;
73     }
74     return ans;
75 }
76 }sam;
77 char S[N];
78 int main(){
79     int T;
80     scanf("%d",&T);
81     while(T--){
82         scanf("%s",S);
83         sam.build(S);
84         printf("%lld\n",sam.count());
85     }
86     return 0;
87 }

```

9.5 Z Algorithm

```

1 void Zalg(char *s, int *z, int n) {
2     z[0]=n;
3     for(int L=0, R=0, i=1; i<n; i++) {
4         if(i<=R && z[i-L]<=R-i) z[i]=z[i-L];
5         else {
6             L=i;
7             if(i>R) R=i;
8             while(R<n && s[R-L]==s[R]) R++;
9             z[i]=(R--)-L;
10        }
11    }
12 }

```

10 無權邊的生成樹個數 Kirchhoff's Theorem

1. 定義 $n \times m$ 矩陣 $E = (a_{i,j})$, n 為點數, m 為邊數, 若 i 點在 j 邊上, i 為小點 $a_{i,j} = 1$, i 為大點 $a_{i,j} = -1$, 否則 $a_{i,j} = 0$ 。

(證明省略)

4. 令 $E(E^T) = Q$, 他是一種有負號的 kirchhoff 的矩陣, 取 Q 的子矩陣即為 $F(F^T)$

結論: 做 Q 取子矩陣算 \det 即為所求。(除去第一行第一列 by mz)

11 monge

$$i \leq i' < j \leq j'$$

$$m(i, j) + m(i', j') \leq m(i', j) + m(i, j')$$

$$k(i, j-1) \leq k(i, j) \leq k(i+1, j)$$

12 四心

$$\frac{sa \cdot A + sb \cdot B + sc \cdot C}{sa + sb + sc}$$

外心 $\sin 2A : \sin 2B : \sin 2C$

內心 $\sin A : \sin B : \sin C$

垂心 $\tan A : \tan B : \tan C$

重心 $1 : 1 : 1$

13 Runge-Kutta

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(t_n, y_n)$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

14 Householder Matrix

$$I - 2 \frac{vv^T}{v^T v}$$