Machine Learning Worksheet 09

Shang-Hsin Yu – 03681048 – shanghsin.yu@tum.de

Problem 1

$$f_{jt}\begin{pmatrix} y_1 \\ y_2 \\ f(x_*) \end{pmatrix} = \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} K(x_1,x_1) & K(x_1,x_2) & K(x_1,x_*) \\ K(x_2,x_1) & K(x_2,x_2) & K(x_2,x_*) \\ K(x_*,x_1) & K(x_*,x_2) & K(x_*,x_*) \end{bmatrix}\right)$$

$$\approx \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0.5353 & 0.8825 \\ 0.5353 & 1 & 0.6065 \\ 0.8825 & 0.6565 & 1 \end{bmatrix}\right)$$

Problem 2

$$p(f_* \mid y, X, X_*) = \mathcal{N}(\mu_* + K_*^T K^{-1}(y-\mu), K_{**} - K_*^T K^{-1} K_*)$$

$$\mu_* + K_*^T K^{-1}(y-\mu) = 0 + \begin{bmatrix} 0.8825 & 0.6065 \end{bmatrix} \begin{bmatrix} 1.4016 & -0.7503 \\ -0.7503 & 1.4016 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} \approx 2.5034$$

$$\mu_* + K_*^T K^{-1}(y-\mu) = 1 - \begin{bmatrix} 0.8825 & 0.6065 \end{bmatrix} \begin{bmatrix} 1.4016 & -0.7503 \\ -0.7503 & 1.4016 \end{bmatrix} \begin{bmatrix} 0.8825 \\ 0.6065 \end{bmatrix} \approx 0.1960$$

Problem 3

$$p(y, f(x_1)) = \begin{pmatrix} y_1 \\ y_2 \\ f(x_1) \end{pmatrix} = \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} K(x_1,x_1)+\sigma_n^2 & K(x_1,x_2) & K(x_1,x_1) \\ K(x_2,x_1) & K(x_2,x_2)+\sigma_n^2 & K(x_2,x_1) \\ K(x_1,x_1) & K(x_1,x_2) & K(x_1,x_1) \end{bmatrix}\right)$$

$$\approx \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1+\sigma_n^2 & 0.5353 & 1 \\ 0.5353 & 1+\sigma_n^2 & 0.5353 \\ 1 & 0.5353 & 1 \end{bmatrix}\right)$$