Machine Learning Worksheet 06

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Problem 1

The derivatives of the log likelihood is:

$$\nabla_W E_{\mathcal{D}}(W) = \sum_{n=1}^{N} (Z_n - \sigma(W^T \Phi_n))\Phi_n$$

Where we can see that even after we have achieved linear separation, $\sigma(W^T\Phi_n)$ will continue to try to be as close to Z_n as possible, which is in this case 0 and 1. Since it is only reachable when the parameter of the sigmoid function reaches infinity, it will drive W towards it.

Problem 2

$$\phi(x1, x2) = x1 \cdot x2$$

This puts all the circles > 0 and all the crosses < 0.

Problem 3

$$W^T X_n + w_0,\ W = 5, 2,\ w_0 = -10$$

The decision boundary is given as above. We need two feature parameters only.

Problem 4

The rest is coding

Implementation exercise: Linear Classification

In [31]:

```
import numpy as np
from sklearn import datasets
import matplotlib.pyplot as plt
import math
%matplotlib inline
```

Some helper functions for visualisation

In [32]:

```
def plot_decision_boundary(X, Z, W=None, b=None):
    fig, ax = plt.subplots(1, 1, figsize=(5, 5))
    ax.scatter(X[:,0], X[:,1], c=Z, cmap=plt.cm.cool)
    ax.set_autoscale_on(False)

    a = - W[0, 0] / W[0, 1]
    xx = np.linspace(-30, 30)
    yy = a * xx - (b[0]) / W[0, 1]

    ax.plot(xx, yy, 'k-', c=plt.cm.cool(1.0/3.0))
```

Dataset Loader

In [33]:

```
def loadDataset(split, X=[] , XT=[], Z = [], ZT = []):
    dataset = datasets.load_iris()
    c = list(zip(dataset['data'], dataset['target']))
    np.random.seed(224)
    np.random.shuffle(c)
    x, t = zip(*c)
    sp = int(split*len(c))
    #sp = 3
    X = x[:sp]
    XT = x[sp:]
    Z = t[:sp]
    ZT = t[sp:]
    names = ['Sepal. length', 'Sepal. width', 'Petal. length', 'Petal. width']
    return np.array(X), np.array(XT), np.array(Z), np.array(ZT), names
```

In [34]:

```
# prepare data
split = 0.67
X, XT, Z, ZT, names = loadDataset(split)

# combine two of the 3 classes for a 2 class problem
Z[Z==2] = 1
ZT[ZT==2] = 1

# only look at 2 dimensions of the input data for easy visualisation
X = X[:,:2]
XT = XT[:,:2]
```

Exercise 1: Calculate probability of class 1

Compute the probability of class 1 given the data and the parameters.

arguments:

- *X*: data
- *W*: weight matrix, part of the parameters
- *b*: bias, part of the parameters

returns:

- *rate*: probabiliy of the predicted class 1

In [35]:

```
def pred(X, W, b):
    result = []
    for x in X:
        a = 0.0 + b
        a += W[0].dot(x)
        result.append(1.0 / (1.0 + np.exp(-a)))
    return result
```

Exercise 2: Calculate the log-likelihood given the target

Compute the logarithm of the likelihood for logistic regression. The negative log-likelihood is our loss function.

arguments:

- *X*: data
- *Z*: target
- *W*: weight matrix, part of the parameters
- *b*: bias, part of the parameters

returns:

- *log likelihood*: logarithm of the likelihood

In [36]:

```
def loglikelihood(X, Z, W, b):
    result = []
    p = pred(X, W, b)
    for i in xrange(len(p)):
        result.append(Z[i]*np.log(p[i])+(1-Z[i])*np.log(1-p[i]))
    return np.array(result)
```

Exercise 3: Implement the gradient of the loss/log-likelihood

Compute the gradient of the loss with respect to the parameters

arguments:

- *X*: data
- *Z*: target
- *W*: weight matrix, part of the parameters
- *b*: bias, part of the parameters

returns:

- *dLdW*: gradient of loss wrt to W
- *dLdb*: gradient of loss wrt to b

In [37]:

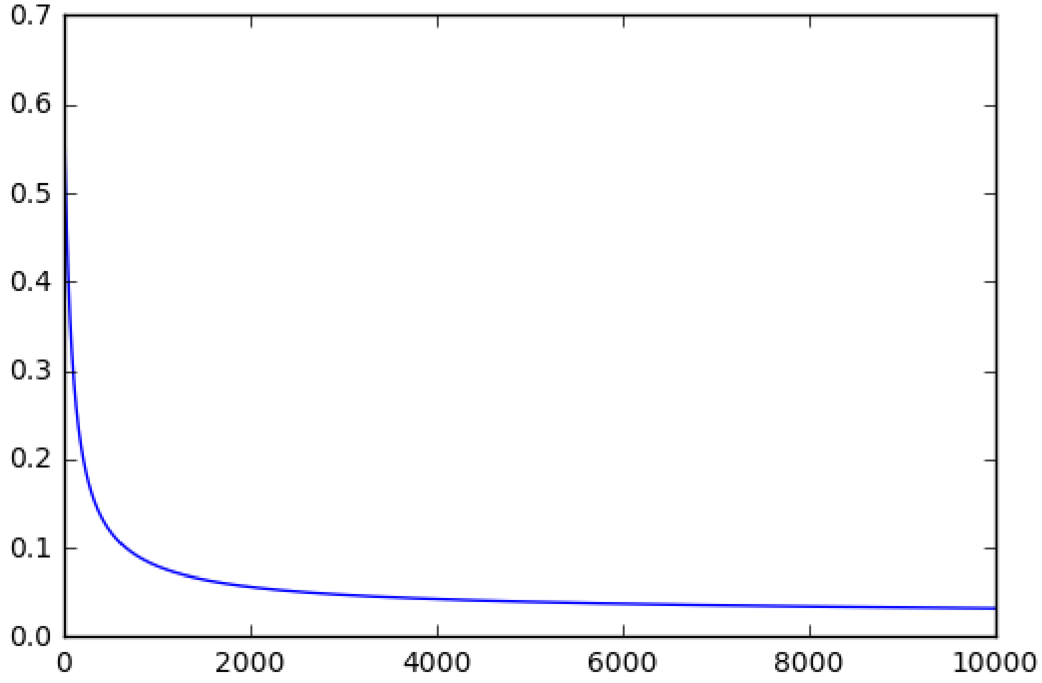
```
def grad(X, Z, W, b):
    resultW = [0.0, 0.0]
    resultb = 0.0
    p = pred(X, W, b)
    for i in xrange(len(X)):
        mul = p[i]-Z[i]
        #print "mul : ", mul
        resultW += mul*X[i]
        resultb += mul
    return np.array(resultW), np.array(resultb)
```

Exercise 4: Test everything

Run the provied simple gradient descent algorithm to optimize the model parameters and plot the resuling decision boundary.

In [38]:

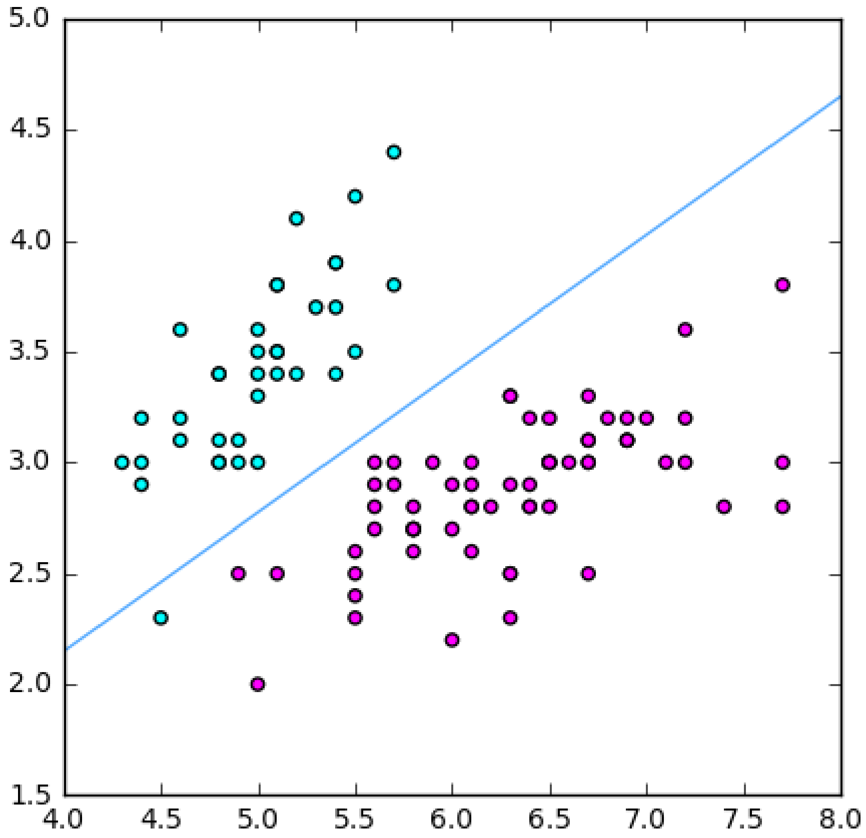
```
W = np.random.randn(1,2) * 0.01
b = np.random.randn(1) * 0.01

learning_rate = 0.001
train_loss = []
validation_loss = []
#print X
#print Z
for i in range(10000):
    dLdW, dLdb = grad(X, Z, W, b)
    W -= learning_rate * dLdW
    b -= learning_rate * dLdb
    #print W
    #print b
    #plot(X, W, b)
    train_loss.append( -loglikelihood(X, Z, W, b).mean())

#print train_loss[-1]
_ = plt.plot(train_loss)
```



Decision boundary on the training set

In [39]:

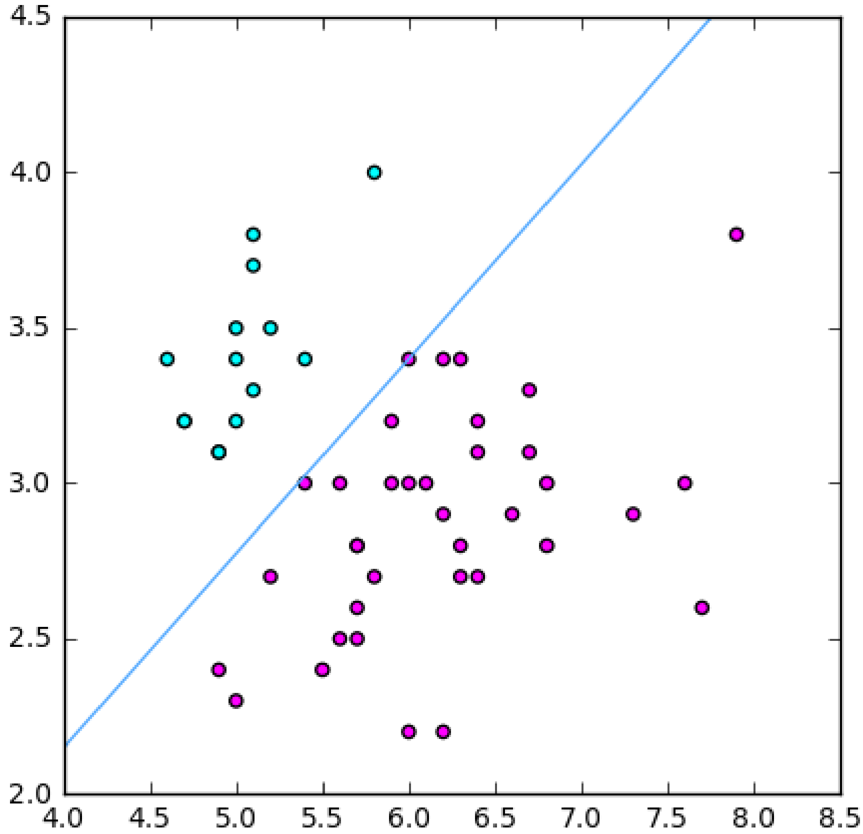
```
plot_decision_boundary(X, Z, W=W, b=b)
```



Decision boundary on the test set

In [40]:

```
plot_decision_boundary(XT, ZT, W=W, b=b)
```



In [ ]:

```
```