Machine Learning Worksheet 04

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Problem 1

Since Λ is a diagonal matrix, its transpose is simply a diagonal matrix where $\Lambda_{ii}^{-1} = \frac{1}{\Lambda_{ii}}$ on the diagonals and 0 everywhere else. Assume a 2x2 scenario, where

$$
\begin{aligned}
U\Lambda^{-1}U^T &= \begin{bmatrix} u_{00} & u_{10} \\ u_{01} & u_{11} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\lambda_0} & 0 \\ 0 & \frac{1}{\lambda_1} \end{bmatrix} \cdot \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{bmatrix} \\
&= \begin{bmatrix} \frac{u_{00}}{\lambda_0} & \frac{u_{10}}{\lambda_1} \\ \frac{u_{01}}{\lambda_0} & \frac{u_{11}}{\lambda_1} \end{bmatrix} \cdot \begin{bmatrix} u_{00} & u_{01} \\ u_{10} & u_{11} \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{\lambda_0}u_{00}u_{00} + \frac{1}{\lambda_1}u_{10}u_{10} & \frac{1}{\lambda_0}u_{00}u_{01} + \frac{1}{\lambda_1}u_{10}u_{11} \\ \frac{1}{\lambda_0}u_{00}u_{01} + \frac{1}{\lambda_1}u_{10}u_{11} & \frac{1}{\lambda_0}u_{01}u_{01} + \frac{1}{\lambda_1}u_{11}u_{11} \end{bmatrix} \\
&= \frac{1}{\lambda_0}u_0u_0^T + \frac{1}{\lambda_1}u_1u_1^T
\end{aligned}
$$

This can extend to any dimension.

Problem 2

We can sample from random variable Y through X by applying linear transformation of a sample of X: $y = Lx$. and by rearranging we get $x(y) = L^{-1}y$.
Then apply chang of variable theorem, assuming X has mean μ and variance Σ:

$$
\begin{aligned}
f(x) = f(x(y)) \mid \frac{dx}{dy} \mid &\propto e^{-\frac{1}{2}(L^{-1}y-\mu)^T\Sigma^{-1}(L^{-1}y-\mu)} \\
&= e^{-\frac{1}{2}(L^{-1}y-\mu)^TL^TL^{-T}\Sigma^{-1}L^{-1}L(L^{-1}y-\mu)} \\
&= e^{-\frac{1}{2}(y-L^{-1}\mu)^TL^{-T}\Sigma^{-1}L^{-1}(y-L\mu)} \\
&= e^{-\frac{1}{2}(y-L^{-1}\mu)^T(L\Sigma L^T)^{-1}(y-L\mu)}
\end{aligned}
$$

We get that Y is also a Gaussian distribution with mean $L\mu$ and variance $L\Sigma L^T$

Problem 3

I rewrote this and problem 4 three times and realized I have no idea what I'm doing instead of trying to piece equations together and force them to work. I gave up and decided to wait for the homework tutorial.

Problem 4