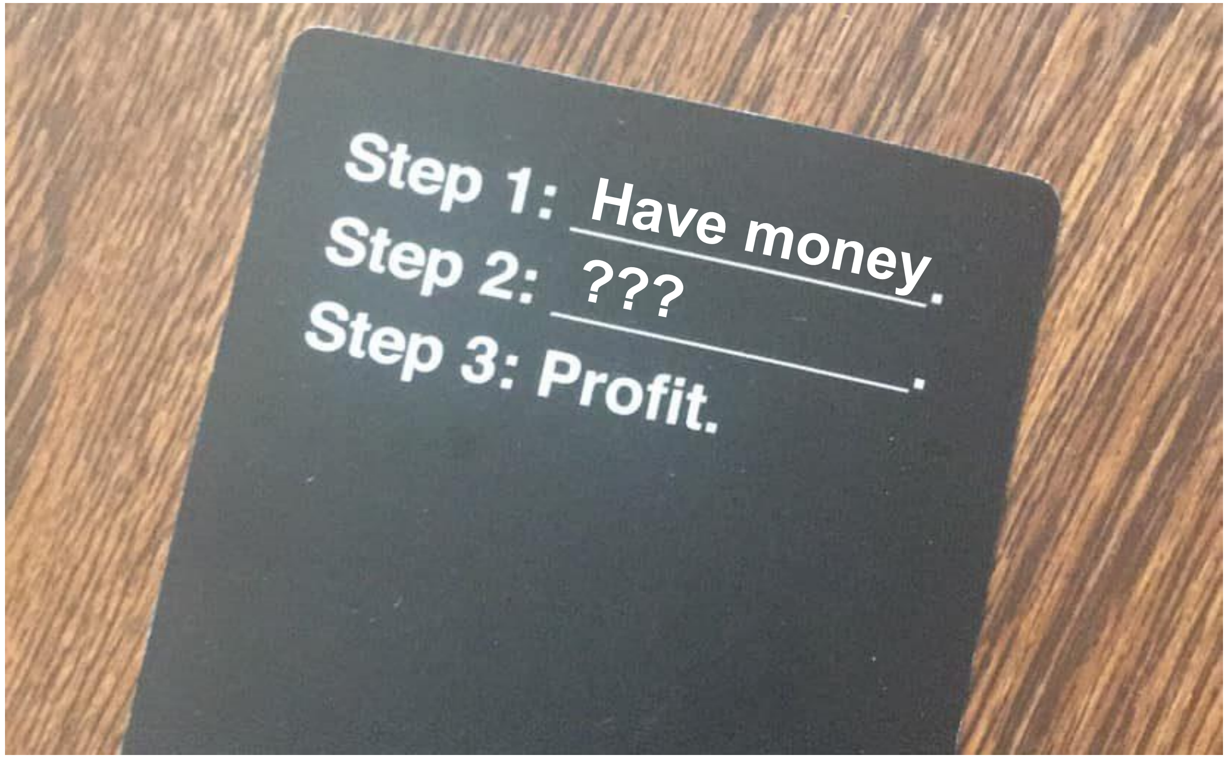
Pairs Trading for Financial Markets

A GENTLE INTRODUCTION INTO ALGORITHMIC TRADING

Making money in financial markets is hard



Today, I want to discuss a possible technique for step 2 ☺

What this session will cover

(I've tried to keep it accessible and intuitive, rather than too theoretical)

High level topics

What pairs trading is

Why it's profitable and how it minimises risk

General principles and how to do it

Asset universe selection

Plenty of further reading

More technical topics

Orders of integration and stationarity

Cointegration

Hypothesis tests for cointegration and stationarity

A notebook with a simple trading algorithm

Lots of reasons the simple algorithm sucks

Trading terminology

SUPER-QUICK PRIMER

Long and short positions

Pretty much any security type is suitable for pairs trading, as long as you can open both *long* and *short* positions on the underlying asset.

Long – buying an asset and selling it at a later date

- If the price of the underlying asset **increases**, your position is more valuable (and vice versa)

Short – borrowing an asset, selling it, and buying it back later

- If the price of the underlying asset **decreases**, your position is more valuable (and vice versa)

Aside: a problem with shorting

Losses on *long* positions are bounded by the price of the asset

- e.g. if you *long* 1 share of PEP at $500, you only lose $500 if PEP goes bust

Losses on *short* positions are theoretically unbounded

- If you *short* PEP at $500, and PEP's price hits $3000, you lose $2500

A poorly-chosen short position can be disastrous without mitigation through hedging

Hedging

Typically traders will limit short losses by [hedging]() – usually taking an offsetting position in a corresponding security or derivative

- A short position can be “hedged” by taking out a long position on the same asset

Hedging can be complex, especially if it involves trading derivatives

There are other tools to limit financial losses from risky positions (stop loss, etc)

- These aren’t hugely relevant for today so I’m going to pretend they don’t exist ☺

Market neutrality

Market neutral portfolios are hedging taken to a logical extreme

An MNP is not affected by any market movement… at all

Owning an MNP is a good way to insulate against shocks, market crashes, etc

- But they are also difficult to profit from
  - Market-neutral portfolios don't drop in value if the market drops (good)
  - … but they also don't appreciate in value if the market booms (less good)

The best way to make money from an MNP is *statistical arbitrage*

- Exploiting inefficiencies in the market created by stochastic processes
- Using statistics to make inferences about what is likely to happen to the MNP's value

Still awake?

Pairs trading

Pairs trading involves betting on whether an MNP will diverge or converge in price.

It relies on the principle of mean reversion to *predictably* make money.

Basic process:

Create a market-neutral (long *and* short) portfolio of suitable asset pairs

This portfolio will have a combined value (called a "spread") that *reverts to some mean* over a predictable period of time

When the neutrality temporarily weakens (i.e. the assets converge or diverge on a short-term basis), it opens an opportunity to make money

Setting up a pairs trade

1. Find two assets that are historically cointegrated using statistics
2. Compute the spread between the two series using a “hedge ratio” (basically just a weighting factor) that can be found by OLS regression
3. Test the spread to check for stationarity
4. Create some appropriately-timed trading signals to buy/sell
5. Profit… 🤞

Some of this might not make sense yet

DON’T PANIC

Orders of integration

The *order of integration* of a time series is simply how many *differences* are required to obtain a stationary series (i.e. some mean-reverting process).

It’s essentially a measure of the “statefulness” of a time-series

- i.e. dependency on previous values (related to the Markov property)

I(0) processes are essentially just noise around a mean

I(1) processes depend on their previous values

- e.g. random walks, Brownian motion, Wiener processes, stock prices, etc

Other orders exist – I(2), I(3), I(…), I(N), etc

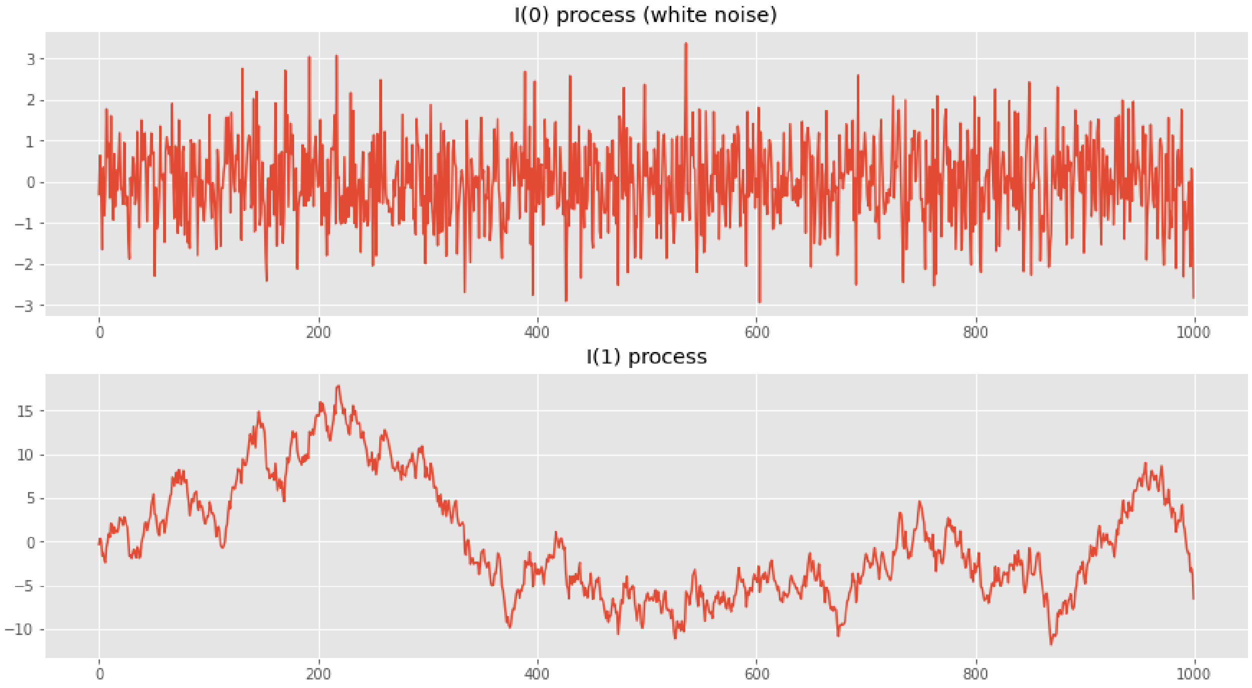
- But we’re only interested in I(1) and I(0) to trade pairs

I(0) vs. I(1)

I(0) is simply Gaussian noise

I(1) is the cumulative sum of I(0)

- i.e. *integrating* over I(0)



…I(1) looks like a stock price, right?

The trick

The key to pairs trading is to find some pair of tradeable assets such that, when combined, their prices form a time-series of I(0).

I(0) processes predictably revert to a mean

When traders can predict things, it allows them to make money

When we *know* a pair of assets will go up or down, we can trade profitably

So… how to create an I(0) portfolio from I(1) assets, without differencing?

Cointegration

A pair of time series of I(1) are *cointegrated* when there exists some *linear combination* such that the result is a stationary process of I(0)

Asset prices are normally I(1), so there usually exists some linear combination of some assets that is I(0) – i.e. that yields a stationary time-series

Obviously not all assets are cointegrated – it's actually quite unusual

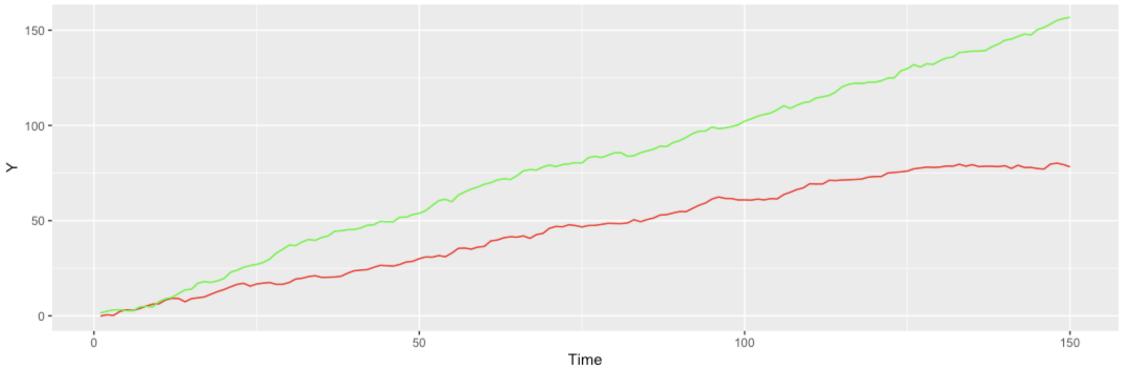
NB: Cointegration is not correlation (even if they are similar)

- Cointegrated timeseries can be uncorrelated
- Correlated time series can be non-cointegrated

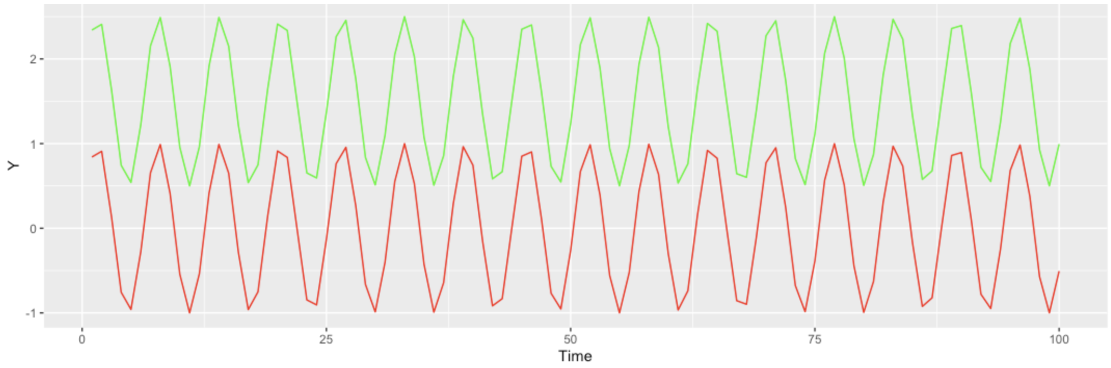
Correlation vs. cointegration

Images from: https://medium.com/ro-data-team-blog/measuring-correlation-ii-cointegration-for-time-series-analysis-f0f5e6f65f5

Correlated, but not cointegrated



Cointegrated, but uncorrelated

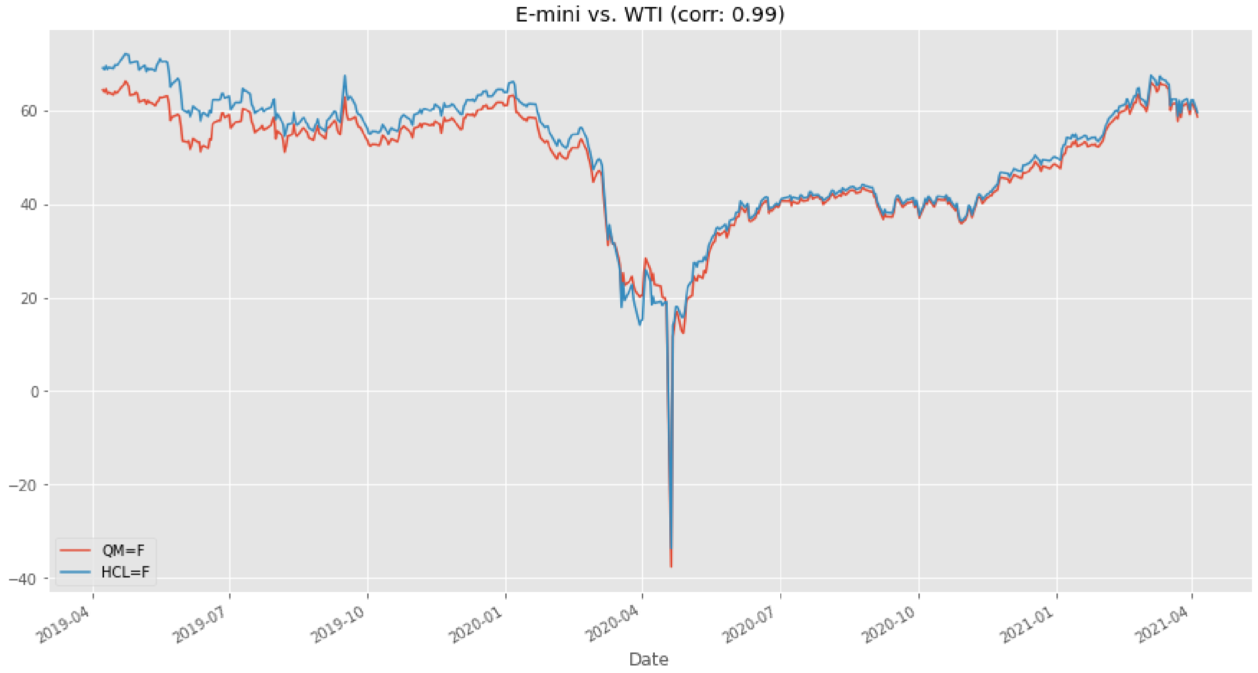


An example of cointegration

E-mini & WTI crude oil futures

Sanity checks for asset pairs:

- Same or similar industries?
- Both affected by same phenomena?
- Long historical cointegration?
- Response to market crashes?



The Augmented Dickey-Fuller (ADF) test

Used to check if a time series has a unit root (i.e. is stationary)

H_0: the time series is *not* stationary

H_A: the time series is stationary (or trend-stationary)

- The alternative hypothesis depends on what you want to test
- Normally in finance, the time series is assumed to be stationary (no trend)

Tests the distribution of residuals on an autoregressive process

The Engle-Granger cointegration test

H_0: the two time series are *not* cointegrated

H_A: the two time series are cointegrated

Basic process:

- Regression of one time series (x) as a linear combination of the other (y), s.t.
  - $y = \beta x$ where β is estimated with OLS
- Run a stationarity test on the residuals (e.g. Augmented Dickey-Fuller)
- If the p-value is below the chosen threshold, the residuals are stationary and thus x and y are cointegrated

Engle and Granger were awarded the Nobel prize for Economics in 2003

Ornstein-Uhlenbeck processes

The OU process is a model for stochastic time-series processes

- I'm not going to go into huge detail because I don't want to bore you to sleep
- The Wikipedia page is [actually very good] *(yes it's Wikipedia don't judge me)*

TLDR;

An OU process is a time-series process where:

- The long-term value reverts to some mean
- The speed of mean reversion is *proportional* to the distance from the mean

The OU process enables us to make inferences about *when prices will mean-revert*

- This allows us to make predictable trades

The goal of pairs trading is to make a pair of I(1) assets behave as an I(0) OU process

Finding pairs

STATISTICAL TESTING PROCEDURES

Testing a single pair for cointegration

Find a pair of assets that you think may be cointegrated, e.g.
- E.g. similar stocks like NVDA/AMD, GOOG/AMZN
- Oil futures

Take the first difference of each asset and test with ADF to ensure both are I(1)

Test for cointegration using the Engle-Granger two-step method (built into statsmodels)

Estimate the hedge ratio using OLS and compute the spread

Test the spread for stationarity using ADF

If all these tests pass at your desired significance level – SUCCESS ☺

Multiple comparison bias

The previous method only applies for testing a single pair

If you wanted to programmatically *discover* pairs, you'd need to do thousands of individual tests… which exposes you (the trader) to a risk of Type I errors (FP)

- E.g. testing 50 stocks for pairs requires 1225 comparisons
- At a significance threshold of $p = 0.05$, this would yield an expected 61.25 false positives

False positives are not very profitable ☺ you can correct p-values using these methods;

- Holm-Sidak (FWER control)
- Bonferroni (FWER control)
- Benyamini-Yekutieli (FDR control)

Multiple comparisons

IT’S JUPYTER TIME

Trading a pair

MORE JUPYTER NOTEBOOKS

How quickly do the prices mean-revert?

With a bit of rearranging, it's quite easy to compute the mean-reversion half life of an OU process…

$$h_l = \frac{\ln 2}{\lambda}$$

where λ is the coefficient from an AR(1) process (a regression on t-1)

- i.e. $p_t \cong \lambda p_{t-1}$

Fin