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# Topological and Geometric Deep Learning

Research project

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# Subject Area Overview

## Subject Area

Clique

Convolutional Graph Network

Graph Attention Network

Tasks solved by GNNs

Purpose and objectives

Relevance

## Experiments and Results

3-Clique merge with insertion

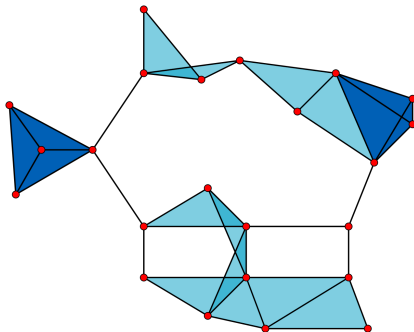
Infinite 3-clique merge with insertion

Prospects



## Clique

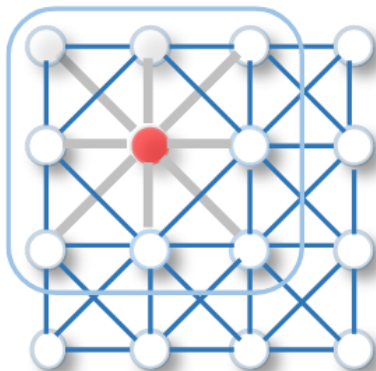
A graph clique is a subset of its nodes such that it is fully connected.  
We will work mainly with 3-cliques, and often will refer to them as triangles.





## Convolutional Graph Network I

A type of GNN which generalizes the convolution operation to graphs.



Convolution on image



Convolution on graph



## Convolutional Graph Network II

Assume we have a graph of  $N$  nodes, where each node has  $F$  features. We can construct an  $N \times F$  matrix called feature matrix. The first layer takes the feature matrix, and performs the following operation:  $Z = D^{-\frac{1}{2}} A D^{-\frac{1}{2}} X W$ , where:

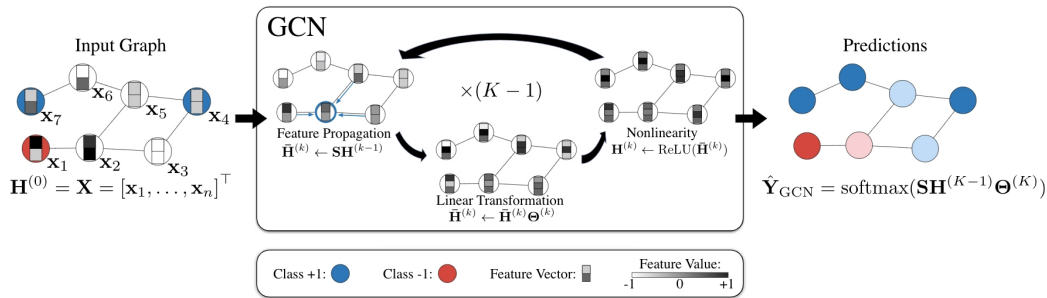
- $Z$  is resulting  $N \times C$  signal
- $D$  is  $N \times N$  degree matrix
- $A$  is  $N \times N$  adjacency matrix with self-loops
- $X$  is  $N \times F$  feature matrix (input signal)
- $W$  is  $F \times C$  learnable weight matrix

The last (output) layer usually applies softmax function to each row resulting in a new matrix  $S$ . Then, in order to classify a node  $v_i$  we simply take the index of maximum of  $S_i$ .



## Convolutional Graph Network III

The architecture of a graph convolutional network is presented on the figure below.





## Graph Attention Network I

A type of GNN which uses attention mechanism (also borrowed from ‘casual’ neural networks) which allows us to work with inputs of variable sizes and to focus on the most important features. The attention mechanism is a function  $\alpha : \mathbb{R}^C \times \mathbb{R}^C \rightarrow \mathbb{R}$  which takes two feature vectors  $X_i, X_j$  and returns a scalar representing how tight the connection between  $v_i$  and  $v_j$  is.



## Graph Attention Network II

We introduce an  $N \times N$  matrix  $\mathbf{e}$  storing the attention between the nodes:

$$\mathbf{e}_{i,j} = \alpha(W \cdot \mathbf{x}_i, W \cdot \mathbf{x}_j).$$

Don't calculate all pairwise attentions! One suggested solution is to use a neighborhood  $\mathcal{N}_i$  of a vertex  $\mathbf{v}_i$  and then compute the attentions between  $\mathbf{v}_i$  and its' neighbors. Existing models uses neighborhood of size 1, and perform great.

One might also want to normalize the coefficients. In order to do that, we can apply softmax function:  $\mathbf{c}_{i,j} = \text{softmax}_j(\mathbf{e}_{i,j})$ .





## Graph Attention Network III

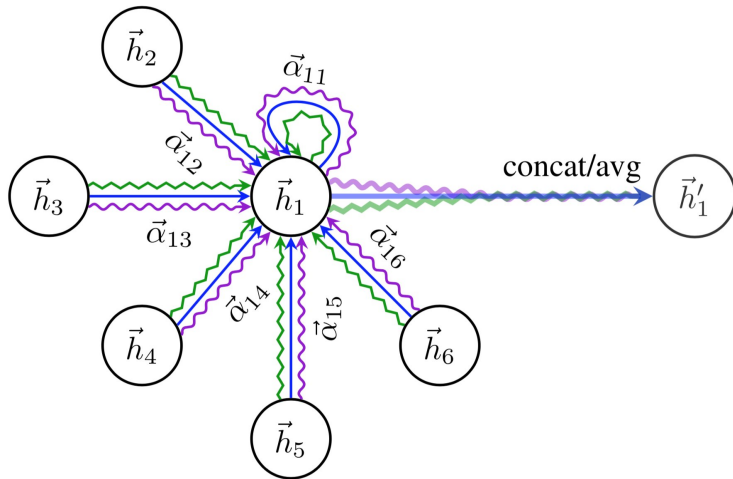


Figure: An example of multi-head attention in a neighborhood of size 1



## Tasks solved by GNNs

- Node classification
- Graph classification
- Link prediction



## Purpose and objectives

- Study the foundation of Deep Learning models on graph data
- Research a few topological structures and methods
- Implement a topology-motivated preprocessing
- Conduct the experiments



## Relevance

- The field is new, therefore, there is a lot of space for improvement
- GNNs allow us to analyze complex relations with great precision
- We can improve the performance by preprocessing the data
- We can improve models by tweaking them



# Experiments and Results Overview

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## Method

Planetoid/Cora dataset:

- Node classification problem
- 2708 nodes (7 classes)
- 10556 edges (undirected)
- 1433 features per node (number of occurrences of words)
- 5% training node label rate
- 100 runs with 300 epochs



## 3-Clique merge with insertion Description

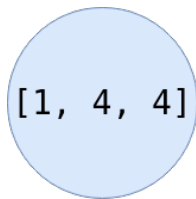
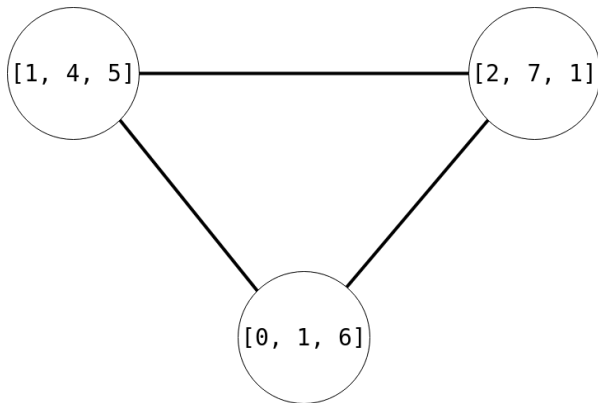
1. Find all 3-cliques from a graph and save them in a list
2. Sort them according to sum of pairwise distances
3. Repeat the following steps until there are any nodes in a list:
  - 3.1 Take the 'top' 3-clique
  - 3.2 Save all edges coming in/out of the clique
  - 3.3 Compute a 'generic feature vector' by taking average of their feature vectors
  - 3.4 Delete them from the graph
  - 3.5 Create a new 'merged' node with a generic feature vector and all saved edges
  - 3.6 Filter out the deleted nodes from list of 3-cliques



## 3-clique Merge Example I

The following figures show exactly how a 3-clique is merged: the resulting feature vector is the average of the feature vectors of initial nodes:

$$\frac{1}{3} \cdot ((1, 4, 5) + (2, 7, 1) + (0, 1, 6)) = \frac{1}{3} \cdot (3, 12, 12) = (1, 4, 4)$$







## 3-clique Merge Example II

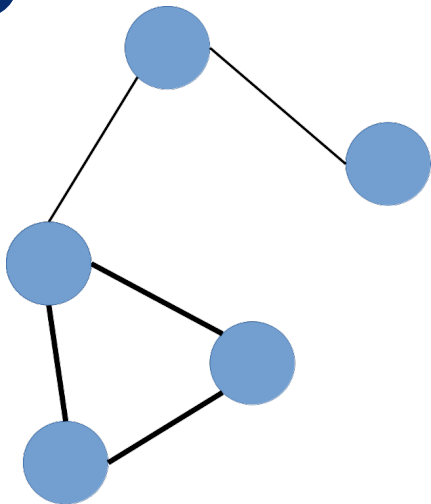


Figure: A part of some graph

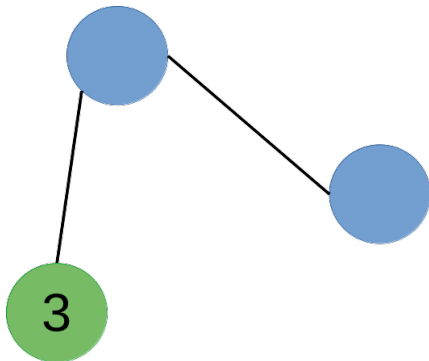


Figure: Three nodes from the left image united in 3-simplex having properties of the initial vertices



## Results I

Feature	Before	After	Delta
Node num	2708	2120	21.71%
Edge num	10556	4568	56.73%
Learning time	4.171s	2.939s	29.5%
Accuracy	0.803	0.746	7%



## Results II

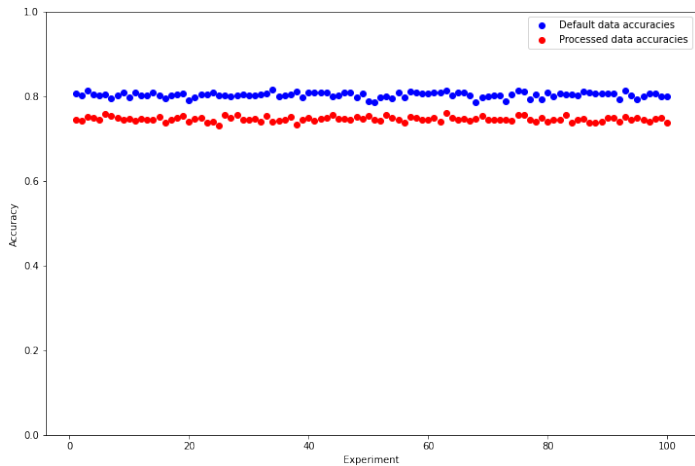


Figure: Average accuracies of default and processed data



## Infinite 3-clique merge with insertion Description

In this experiment, the methodology and the algorithm are similar to the ‘finite 3-clique merge’. The only difference is that we allow merged nodes to participate in merge process again.



## Results I

The proposed method allowed us to remove the majority of information from a graph.

Feature	Before	After	Delta
Node num	2708	1080	60.12%
Edge num	10556	656	93.79%
Learning time	2.537s	1.059s	58%
Accuracy	0.809	0.517	36%



## Results II

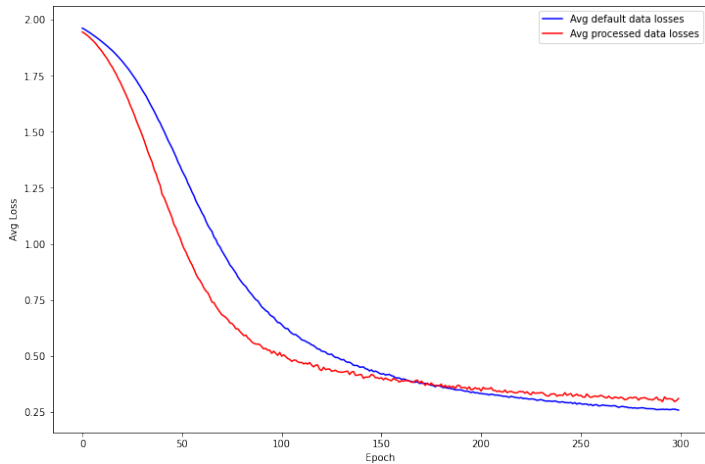


Figure: Average loss for 'default' and processed data



In future we want to focus on topological aspect of the transformation presented in the paper. In particular, we are interested in ‘infinite merge’, where ‘merged nodes’ can be merged again. We also want to see how the transformation presented can be used in other tasks, for example, in graph classification.



## References I

- [1] Vijay Prakash Dwivedi et al. “Benchmarking Graph Neural Networks”. In: CoRR abs/2003.00982 (2020). arXiv: 2003.00982. URL: <https://arxiv.org/abs/2003.00982>.
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## References II

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Any questions?