Q. об-но ли шаг = 1 ?

A. да (не бывает $y_{1.5}$)
нет

MA(2)

$$y_t = 5 + u_t + \cancel{0 \cdot u_{t-1}} + 2 \cdot u_{t-2}$$

$$u_t \sim N(0; 16)$$

ур-е – разностное

(u_t) - б. шум

A: $y_t = y_{t-1} - 6 y_{t-2} + u_t$

B: $y_t = y_{t-1} - 0{,}24 y_{t-2} + u_t$

а) Есть ли у этих ур-ий стац-ые решения?

б) если да, то имеют ли они вид MA(∞)

→ да (все $|\lambda| \neq 1$)

A: $\lambda^2 = \lambda - 6$

B: → не вида MA(∞)

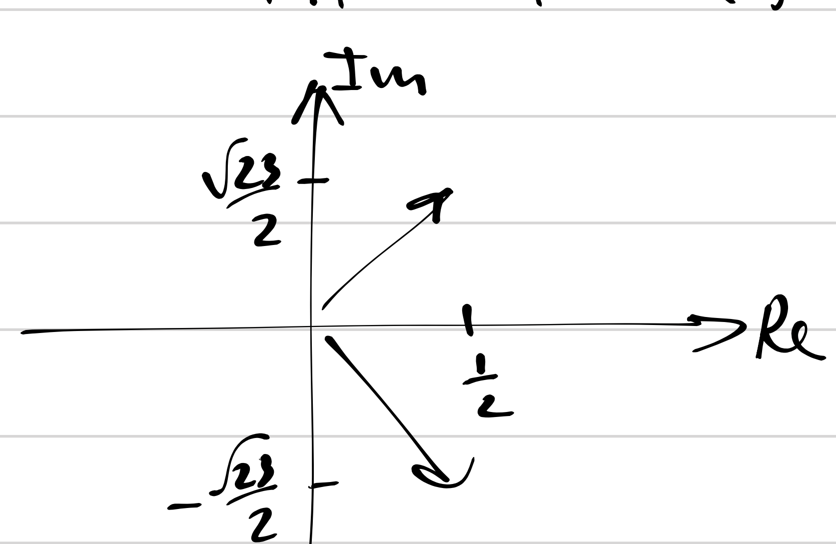
$y_t = \lambda^t$ (и сокр-ть по макс-му)

$$\lambda^2 - \lambda + 6 = 0$$

$$D = 1^2 - 24 = -23$$

$$\lambda_{1,2} = \frac{1 \pm i\sqrt{23}}{2} = \frac{1}{2} \pm i \frac{\sqrt{23}}{2}$$

$$|\lambda_1| = |\lambda_2| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{23}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{23}{4}} > 1$$



$\lambda^2 - \lambda + 0{,}24 = 0$ (B)

$\lambda_1 = 0{,}4$ все $|\lambda_i| \neq 1$

$\lambda_2 = 0{,}6$ все $|\lambda_i| < 1$

есть стац-ое решение вида MA(∞).

Упр. ARMA(1,1) процесс – стац-ое решение вида MA(∞) отн-но б. шума (u_t)

$Var(u_t)=\sigma^2$

$$(y_t-5)=0{,}3(y_{t-1}-5)+u_t+2u_{t-1}$$

а) $\gamma_0, \gamma_1, \gamma_2, \gamma_3 \ldots$ $\qquad \gamma_k = Cov(y_t, y_{t-k})$

б) $\rho_1, \rho_2, \rho_3, \rho_4 \ldots$ $\qquad \rho_k = Corr(y_t, y_{t-k})$

в) $\varphi_{11}, \varphi_{22}$ $\qquad \varphi_{kk} = pCorr(y_t, y_{t-k}; y_{t-1}, y_{t-2}, \ldots y_{t-k+1})$

LHS = RHS

(1) $$(y_t-5)=0{,}3(y_{t-1}-5)+u_t+2u_{t-1}$$

$$Cov(y_t-5, y_t) = Cov(0{,}3(y_{t-1}-5)+u_t+2u_{t-1}, y_t)$$

$Cov(LHS, y_t) = Cov(RHS, y_t)$

$Cov(LHS, y_{t-1}) = Cov(RHS, y_{t-1})$

$Cov(LHS, y_{t-2}) = Cov(RHS, y_{t-2})$

(2) $$\boxed{\gamma_0 = 0{,}3\cdot\gamma_1 + Cov(u_t, y_t) + 2Cov(u_{t-1}, y_t)}$$

$$Cov(LHS, u_{t-1}) = Cov(RHS, u_{t-1})$$

$$Cov(y_t, u_{t-1}) = 0{,}3\,\boxed{Cov(y_{t-1}, u_{t-1})} + \cancel{Cov(u_t, u_{t-1})} + 2Cov(u_{t-1}, u_{t-1})$$

MA(∞) отн-но (u_t)

$$y_t = \mu + u_t + \alpha_1 \cdot u_{t-1} + \alpha_2 u_{t-2} + \ldots$$

$$Cov(y_t, u_t) = \boxed{\sigma^2} \quad !!$$

(3) $$\boxed{\gamma_0 = 0{,}3\gamma_1 + \sigma^2 + 4.6\sigma^2}$$

$$Cov(y_t, u_{t-1}) = 0{,}3\sigma^2 + 2\cdot\sigma^2 = 2.3\cdot\sigma^2$$

(*) $(y_t-5) = 0{,}3\,(y_{t-1}-5) + u_t + 2u_{t-1}$ $\quad E(y_t)$?

$E(LHS) = E(RHS)$

$\mu - 5 = 0{,}3\cdot(\mu-5) + 0 + 0$

$\mu = 5$

$Cov(LHS, y_{t-1}) = Cov(RHS, y_{t-1})$

$\gamma_1 = 0{,}3\cdot\gamma_0 + \boxed{Cov(u_t, y_{t-1})} + 2\,Cov(u_{t-1}, y_{t-1})$

0

$y_{t-1} = \mu + u_{t-1} + \alpha_1 u_{t-2} + \dots$

$Cov(u_{t-1}, y_{t-1}) = \sigma^2$

$\gamma_1 = 0{,}3\,\gamma_0 + 2\cdot\sigma^2$

$Cov(LHS, y_{t-2}) = Cov(RHS, y_{t-2})$

$\gamma_2 = 0{,}3\gamma_1 + \underbrace{Cov(u_t, y_{t-2})}_{0} + 2\,\underbrace{Cov(u_{t-1}, y_{t-2})}_{0}$

$\gamma_2 = 0{,}3\,\gamma_1$

$Cov(LHS, y_{t-3}) = Cov(RHS, y_{t-3})$

$\gamma_3 = 0{,}3\,\gamma_2 + 0 + 2\cdot 0$

...

$$\begin{cases} (1) & \boxed{\gamma_0 = 0{,}3\gamma_1 + \sigma^2 + 4.6\sigma^2} \\ (2) & \gamma_1 = 0{,}3\,\gamma_0 + 2\sigma^2 \\ (3) & \gamma_2 = 0{,}3\gamma_1 \\ & \vdots \end{cases}$$

$\gamma_0 = 0{,}3\gamma_1 + 5.6\sigma^2$

$\gamma_1 = 0{,}09\gamma_1 + 3{,}68\sigma^2$

$\boxed{\gamma_1 = \frac{3.68\sigma^2}{0{,}91}}$

$\gamma_0 = 0{,}09\gamma_0 + 6.2\sigma^2$

$\boxed{\gamma_2 = 0{,}3\cdot\frac{3.68\sigma^2}{0{,}91}}$

$\boxed{\gamma_0 = \frac{6{,}2\cdot\sigma^2}{0{,}91}}$

$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{3.68}{6.2}$; $\rho_2 = 0{,}3\cdot\frac{3.68}{6.2}$ $\quad \rho_3 = 0{,}3^2\cdot\frac{3.68}{6.2}$ ...

$$\rho_k = Corr(y_t, y_{t-k}) = \frac{Cov(y_t, y_{t-k})}{\sqrt{Var(y_t)\,Var(y_{t-k})}} = \frac{\gamma_k}{\sqrt{\gamma_0\cdot\gamma_0}} = \frac{\gamma_k}{\gamma_0}$$

$\varphi_{11} = \text{pCorr}(y_t, y_{t-1}; \emptyset) = \rho_1 = \ldots$

$\varphi_{22} = \text{pCorr}(y_t, y_{t-2}; y_{t-1})$

φ_{22} находим её из предиктивного разложения (Yule-Walker system of eq-s)

$$\boxed{(y_t - 5) = \varphi_{21} \cdot (y_{t-1} - 5) + \boxed{\varphi_{22}} \cdot (y_{t-2} - 5) + w_t}$$

$$\begin{cases} Cov(w_t, y_{t-1}) = 0 \\ Cov(w_t, y_{t-2}) = 0 \end{cases} \to \varphi_{22}$$

$$\begin{cases} Cov(LHS, y_{t-1}) = Cov(RHS, y_{t-1}) \\ Cov(LHS, y_{t-2}) = Cov(RHS, y_{t-2}) \end{cases}$$

$$\begin{cases} \gamma_1 = \varphi_{21} \cdot \gamma_0 + \varphi_{22} \cdot \gamma_1 + 0 \\ \gamma_2 = \varphi_{21} \cdot \gamma_1 + \varphi_{22} \cdot \gamma_0 + 0 \end{cases} \quad / \gamma_0$$

$$\begin{cases} \rho_1 = \varphi_{21} \cdot 1 + \varphi_{22} \cdot \rho_1 \\ \rho_2 = \varphi_{21} \cdot \rho_1 + \varphi_{22} \cdot 1 \end{cases}$$

$$\begin{pmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \varphi_{21} \\ \varphi_{22} \end{pmatrix} = \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

Yule-Walker

Линал возвр-ся !! ААААА!

$$\varphi_{22} = \frac{\det\begin{pmatrix} 1 & \rho_1 \\ \rho_1 & \rho_2 \end{pmatrix}}{\det\begin{pmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{pmatrix}} =$$

правило Крамера

$$\varphi_{21} = \frac{\det\begin{pmatrix} \rho_1 & \rho_1 \\ \rho_2 & 1 \end{pmatrix}}{\det\begin{pmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{pmatrix}} = \frac{\rho_1 - \rho_1\rho_2}{1 - \rho_1^2} = 0{,}61$$

$$\varphi_{22} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} = -0{,}36$$

ρ_1, ρ_2 смотри раньше !!

$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{3.68}{6.2} = 0{,}59$ $\quad \rho_2 = 0{,}3 \cdot \frac{3.68}{6.2} = 0{,}12$

$$\varphi_{21} = \frac{\det\begin{pmatrix} \rho_1 & \rho_1 \\ \rho_2 & 1 \end{pmatrix}}{\det\begin{pmatrix} 1 & \rho_1 \\ \rho_1 & 1 \end{pmatrix}} = \frac{\rho_1 - \rho_1\rho_2}{1-\rho_1^2} = 0{,}81$$

$$\varphi_{22} = \frac{\rho_2 - \rho_1^2}{1-\rho_1^2} = -0{,}36$$

ρ_1, ρ_2 смотри раньше !!

$$\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{3.68}{6.2} = 0{,}59 \qquad \rho_2 = 0{,}3 \cdot \frac{3.68}{6.2} = 0{,}12$$

г) $y_{100} = 7$ постройте наилучший линейный прогноз для y_{101}, y_{102}

д) $y_{100} = 7$ $y_{99} = 11$ — // — y_{101}

г) $\hat{y}_{101} = ? + ?\, y_{100}$ $\qquad$ $\hat{y}_{102} = ? + ? \cdot y_{102}$

$$\boxed{(y_{101} - 5) = 0{,}59 \cdot (y_{100} - 5) + W_{101}} \quad (1)$$

(2)

$$\boxed{\hat{y}_{101} = 5 + 0{,}59 \cdot (7-5)}$$

$$(y_{102} - 5) = ? \cdot (y_{100} - 5) + \boxed{W_{102}} \leftarrow \text{ош. прогноза.}$$

$$\rho_2 = 0{,}12$$

$$\hat{y}_{102} = 5 + 0{,}12\,(7-5)$$

д) $(y_{101} - 5) = ? \cdot (y_{100} - 5) + ?\,(y_{99} - 5) + W^*_{101}$

$$\varphi_{21} = 0{,}81 \qquad \varphi_{22} = -0{,}36$$

$$\hat{y}_{101} = 5 + 0{,}81 \cdot (7-5) - 0{,}36 \cdot (11-5)$$

идея док-ва:

$$\hat{\beta} = \frac{\widehat{Cov}(y_t, y_{t-1})}{\widehat{Var}(y_{t-1})} = \frac{\hat{\gamma}_1}{\hat{\gamma}_0} = \hat{\rho}_1$$