Hi!! W02 - 6

ACF, PACF → теор. / выборочная

[работаем со стац-ми процесса-ми]

теоретич. выборочная

ACF – autocorrelation function.

теоретич.

$$\rho_k = \boxed{Corr(y_t, y_{t+k})} = Corr(y_t, y_{t-k})$$

у нестац → зависит и от t, и от k.

Упр

(u_t) – б. шум с $Var(u_t) = \sigma^2$

$$y_t = \sqrt{3} + u_t + 0,6u_{t-1} + 0,2u_{t-2}$$

а) правда ли, что (y_t) – стац-ый?

[слабо стац-ый]

б) если да, то найдите ACF ρ_k.

стац: $E(y_t) = \mu$ $\quad Var(y_t) = \gamma_0$

$Cov(y_t, y_{t+k}) = \gamma_k$ [не зав-т от t]

$$E(y_t) = E(\sqrt{3} + u_t + 0.6u_{t-1} + 0,2u_{t-2}) = \sqrt{3}$$

б.шум (u_t) $\quad E(u_t) = 0 \quad Var(u_t) = \sigma^2$

$Cov(u_t, u_{t+k}) = 0$ при $k \neq 0$

$$\gamma_0 = Cov(y_t, y_t) = Var(y_t) = Var(\sqrt{3} + u_t + 0,6u_{t-1} + 0,2u_{t-2}) = 0 + \sigma^2 + 0.6^2 \cdot \sigma^2 + 0,2^2 \cdot \sigma^2 = \sigma^2(1,40)$$

$$\gamma_1 = Cov(y_t, y_{t+1}) = Cov(u_t + 0{,}6u_{t-1} + 0{,}2u_{t-2},\ u_{t+1} + 0{,}6u_t + 0{,}2u_{t-1}) =$$

$$cov(y_t, y_{t-1}) =$$

[Смотрим у д. индекса совп-ие индексов]

$$= 0{,}6\cdot\sigma^2 + 0.6\cdot 0{,}2\cdot\sigma^2 = 0{,}72\cdot\sigma^2$$

$$\gamma_2 = Cov(y_t, y_{t+2}) = Cov(u_t + 0{,}6u_{t-1} + 0{,}2u_{t-2},\ u_{t+2} + 0{,}6u_{t+1} + 0{,}2u_t) =$$

$$= 0{,}2\,\sigma^2$$

$$\gamma_3 = Cov(y_t, y_{t+3}) = Cov(u_t + 0.6u_{t-1} + 0{,}2u_{t-2},\ u_{t+3} + 0{,}6u_{t+2} + 0{,}2u_{t+1}) = 0$$

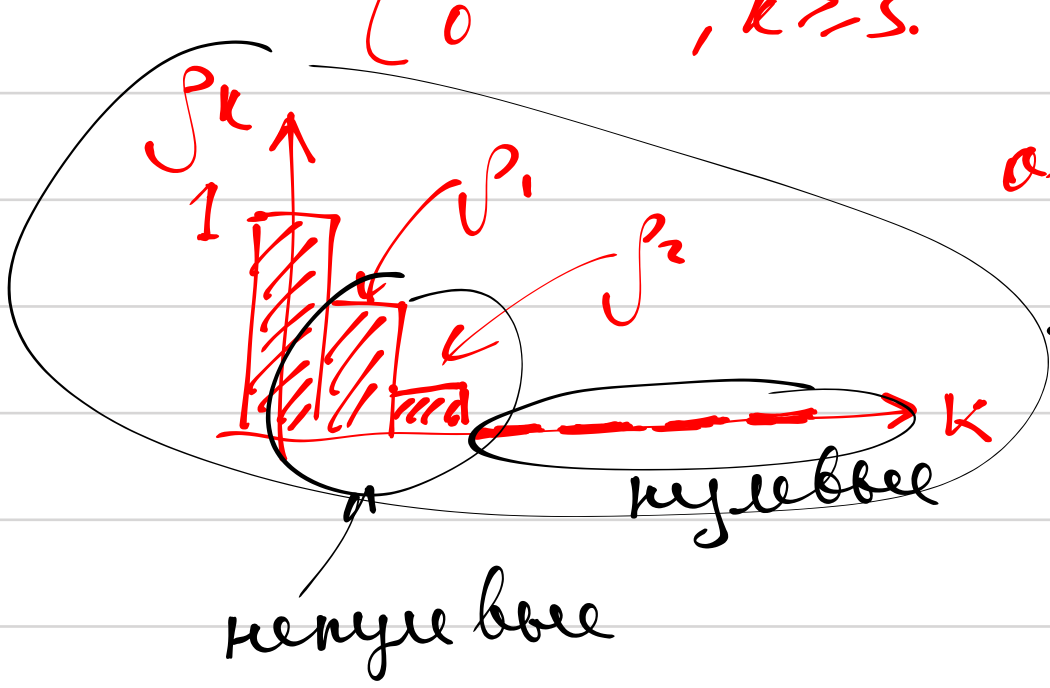
$$\gamma_4 = 0 \qquad \gamma_5 = 0 \ \ldots.$$

$$\gamma_k = \begin{cases} 1{,}4\,\sigma^2, & \text{если } k=0 \quad [Var] \\ 0{,}72\,\sigma^2, & k=1 \\ 0{,}2\,\sigma^2, & k=2 \\ 0, & k \geq 3 \end{cases}$$

$$\rho_k = corr(y_t, y_{t+k}) = \frac{Cov(y_t, y_{t+k})}{\sqrt{Var(y_t)\cdot Var(y_{t+k})}} = \frac{\gamma_k}{\sqrt{\gamma_0\cdot\gamma_0}} = \frac{\gamma_k}{\gamma_0}$$

$$\rho_k = \begin{cases} 1, & \text{если } k=0 \\ 0{,}72/1.4, & k=1 \\ 0{,}2/1.4, & k=2 \\ 0, & k \geq 3. \end{cases}$$

$[corr(y_t, y_t) = 1]$



автокорреляц. ф-ия.

исходя из теор-го ур-ия.

[на практике]

$$\begin{array}{c} y_1 \\ y_2 \\ \vdots \\ y_T \end{array}$$

sACF sample ACF

$$\hat\rho_k = \widehat{Corr}(y_t, y_{t+k})$$

$\hat\rho_2$

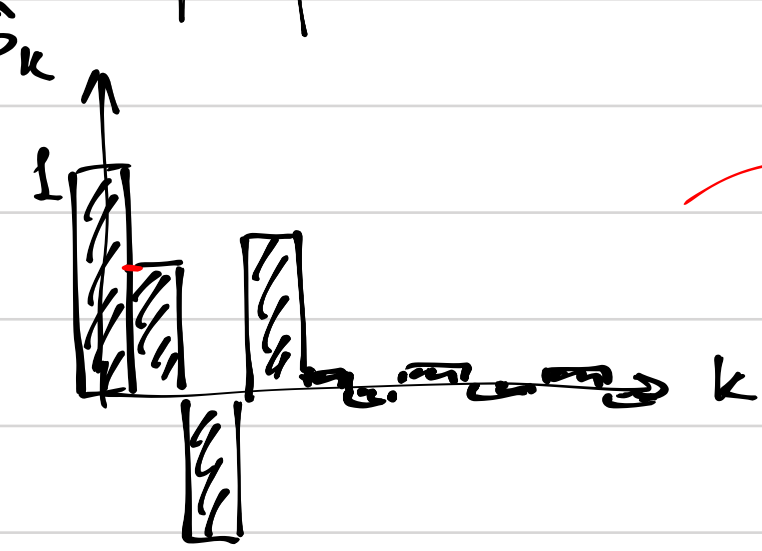
$$\begin{array}{cc} y_1 & - \\ y_2 & - \\ y_3 & y_1 \\ y_4 & y_3 \\ y_5 & y_4 \\ \vdots & \vdots \\ y_T & y_{T-2} \\ - & y_{T-1} \\ - & y_T \end{array} \rightsquigarrow \begin{array}{c} a \\ y_3 \\ \vdots \\ y_T \end{array} \quad \begin{array}{c} b \\ y_1 \\ y_2 \\ \vdots \\ y_{T-2} \end{array}$$

$$\hat\rho_2 = s\widehat{Corr}(a, b) = \frac{\sum(a_i-\bar a)(b_i-\bar b)}{\sqrt{\sum(a_i-\bar a)^2\cdot\sum(b_i-\bar b)^2}}$$

с точки зрения теории можно уточнить:

$$\hat\rho_2^* = \frac{\sum(a_i-\bar y)\cdot(b_i-\bar y)}{\sqrt{\sum(a_i-\bar y)^2\cdot\sum(b_i-\bar y)^2}}$$

Упр. видим графики sACF:
[качественный]



$\hat\rho_1 \approx 0.5$ $\hat\rho_2 \approx -0.5$
$\hat\rho_3 \approx 0.6$
$\hat\rho_4 \approx \hat\rho_5 \approx \hat\rho_6 \approx \ldots 0$

какую модель для ряда (y_t) нужно оценить?
[почему?]

а) MA(1): $y_t = \mu + u_t + \alpha_1 u_{t-1}$ где (u_t) — б. шум

б) MA(2): $y_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2}$

(!!) в) MA(3): $y_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3}$

moving average.

PACF P=partial $\rightarrow$ теоретич. $\rightarrow$ выборочная.

$\varphi_{11} = pCorr(y_t, y_{t+1})$

$\varphi_{22} = pCorr(y_t, y_{t+2}; y_{t+1})$

$\varphi_{33} = pCorr(y_t, y_{t+3}; y_{t+1}, y_{t+2})$

$\varphi_{44} = pCorr(y_t, y_{t+4}; y_{t+1}, y_{t+2}, y_{t+3})$

$y_t, y_{t+1}, y_{t+2}, y_{t+3}, y_{t+4}$?

def

$$pCorr(\underbrace{L, R}_{\text{между}}; \underbrace{M_1, M_2, M_3}_{\text{фиксируем}}) = Corr(\tilde{L}, \tilde{R}),\ \text{где}$$

очищаем от эффекта M_1, M_2, M_3

$\tilde{L}$ – это часть L, не корр-ая с M_1, M_2, M_3

$$\tilde{L} = L - (\alpha_1 M_1 + \alpha_2 M_2 + \alpha_3 M_3)$$

$\tilde{R}$ — // — R, не корр-ая с M_1, M_2, M_3

$$\tilde{R} = L - (\beta_1 M_1 + \beta_2 M_2 + \beta_3 M_3)$$

Упр. (u_t) – б.шум

$$y_t = \sqrt{3} + u_t + 0{,}6u_{t-1} + 0{,}2u_{t-2}$$

$$Cov(y_t, y_{t+k}) = \gamma_k = \begin{cases} 1{,}4\,\sigma^2 & k=0 \\ 0{,}72\,\sigma^2 & k=1 \\ 0{,}2\,\sigma^2 & k=2 \\ 0 & k\geq 3 \end{cases}$$

$$\varphi_{22} = pCorr(y_t, y_{t+2}; \boxed{y_{t+1}})\ ? =$$

$$= Corr(\tilde{y}_t, \tilde{y}_{t+2})$$

$\tilde{y}_t = y_t - \alpha \cdot y_{t+1}$ и

$\tilde{y}_t$ не корр с y_{t+1}

$Cov(\tilde{y}_t, y_{t+1}) = 0$

$Cov(y_t - \alpha y_{t+1}, y_{t+1}) = 0$

$\gamma_1 - \alpha \cdot \gamma_0 = 0$

$$\alpha = \frac{\gamma_1}{\gamma_0} = \frac{0{,}72}{1{,}4} = \frac{72}{140};\quad \tilde{y}_t = y_t - \frac{72}{140}y_{t+1}$$

Пояс. при незав: u_1, u_2, u_3

$L = u_1 + 2u_2 \quad R = 3u_2 + u_3$

$Corr(L, R) \overset{?}{\neq} 0$

$pCorr(L, R; u_2) \overset{?}{=} Corr(\tilde{L}, \tilde{R}) =$

$\tilde{L} = L - ?u_2 \quad corr(\tilde{L}, u_2) = 0$

$\tilde{R} = R - ?u_2 \quad Corr(\tilde{R}, u_2) = 0$

$= Corr(L - 2u_2, R - 3u_2) = Corr(u_1, u_3) = 0.$

$\tilde{y}_{t+2} = y_{t+2} - \beta \cdot y_{t+1}$

$\tilde{y}_{t+2}$ — это часть y_{t+2}, не корр-ная с y_{t+1}

мы очистили y_{t+2} от вклада y_{t+1}

$Cov(\tilde{y}_{t+2}, y_{t+1}) = 0$

$Cov(y_{t+2} - \beta y_{t+1}, y_{t+1}) = 0$

$\gamma_1 - \beta \cdot \gamma_0 = 0 \qquad \beta = \frac{\gamma_1}{\gamma_0} = \frac{72}{140}$

$\tilde{y}_{t+2} = y_{t+2} - \frac{72}{140} \cdot y_{t+1}$

$pCorr(y_t, y_{t+2}; y_{t+1}) = Corr(\tilde{y}_t, \tilde{y}_{t+2}) =$

$= Corr\left(y_t - \frac{72}{140} y_{t+1}, y_{t+2} - \frac{72}{140} y_{t+1}\right) =$

$= \frac{Cov\left(y_t - \frac{72}{140} y_{t+1}, y_{t+2} - \frac{72}{140} y_{t+1}\right)}{\sqrt{\boxed{Var\left(y_t - \frac{72}{140} y_{t+1}\right)} \cdot Var\left(y_{t+2} - \frac{72}{140} y_{t+1}\right)}}$

$$= \frac{Cov\left(y_t - \frac{72}{140} y_{t+1},\ y_{t+2} - \frac{72}{140} y_{t+1}\right)}{\sqrt{\boxed{Var\left(y_t - \frac{72}{140} y_{t+1}\right)} \cdot Var\left(y_{t+2} - \frac{72}{140} y_{t+1}\right)}} =$$

$$Var\left(y_t - \frac{72}{140} y_{t+1}\right) = \underbrace{\gamma_0}_{Var(y_t)} - 2\cdot\frac{72}{140}\cdot\gamma_1 + \left(\frac{72}{140}\right)^2\cdot\gamma_0$$

$$Var\left(y_{t+2} - \frac{72}{140}\cdot y_{t+1}\right) = \underbrace{\gamma_0}_{Var(y_{t+2})} - 2\cdot\frac{72}{140}\cdot\gamma_1 + \left(\frac{72}{140}\right)^2\cdot\gamma_0$$

$$= \frac{\gamma_2 - \frac{72}{140}\cdot\gamma_1 - \frac{72}{140}\cdot\gamma_1 + \left(\frac{72}{140}\right)^2\gamma_0}{\gamma_0 - \frac{2\cdot 72}{140}\cdot\gamma_1 + \left(\frac{72}{140}\right)^2\cdot\gamma_0} = \ldots \text{ арифм-ка.}$$

$$\varphi_{22} = \frac{\rho_2 - \frac{72\cdot 2}{140}\cdot\rho_1 + \left(\frac{72}{140}\right)^2\cdot 1}{1 - \frac{2\cdot 72}{140}\cdot\rho_1 + \left(\frac{72}{140}\right)^2}$$

теор. PACF

$$\left\{\begin{array}{l} \gamma_k = Cov(y_t, y_{t+k}) \\ \rho_k = Corr(y_t, y_{t+k}) \\ \rho_k = \frac{\gamma_k}{\gamma_0} \qquad \rho_0 = 1 \end{array}\right.$$

Упр.

(u_t) - б.шум

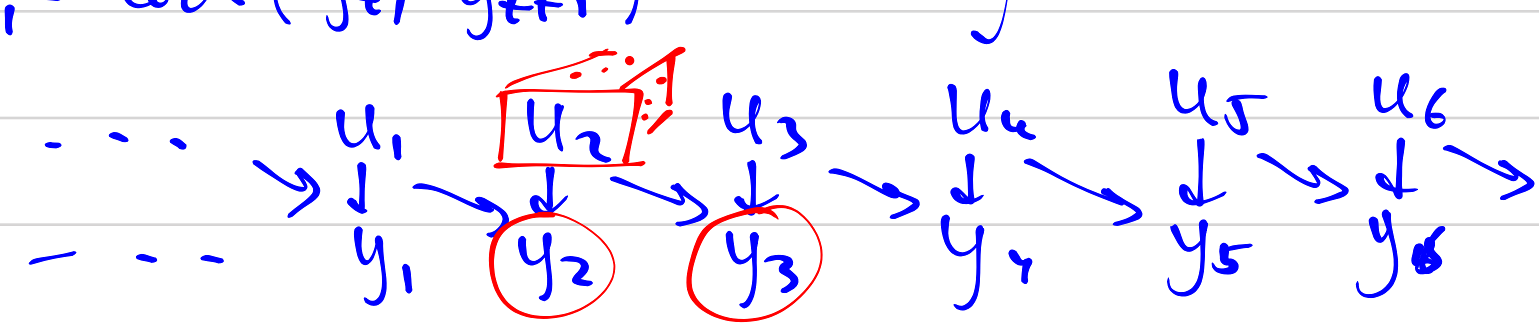
$y_t = u_t + u_{t+1}$

Инт.-но [без вычисл-ий] определите знаки

$\rho_1, \rho_2, \rho_3 \ldots$

$\varphi_{11}, \varphi_{22}, \varphi_{33}, \ldots$

$\rho_1 = Corr(y_t, y_{t+1}) > 0$ $\qquad$ $\rho_2 = Corr(y_t, y_{t+2}) = 0$



$$\begin{array}{ccccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \\ y_1 & y_2 & y_3 & y_4\ ? & y_5 & y_6 & y_7 \ldots \end{array}$$

$\varphi_{11} = \text{pCorr}(y_t, y_{t+1}) = \text{Corr}(y_t, y_{t+1}) = \rho_1 > 0$

$\varphi_{22} = \text{pCorr}(y_t, y_{t+2}; y_{t+1}) < 0$

φ_{33} [инт] > 0

$\varphi_{33} = \text{pCorr}(y_t, y_{t+3}; y_{t+1}, y_{t+2})$

$\varphi_{32} = \text{pCorr}(y_t, y_{t+2}; y_{t+1}, y_{t+3})$

макс. глубина фикс — между наличием

$y_t \quad y_{t+1} \quad y_{t+2} \quad y_{t+3}$