Home Assignment 1

1. Consider the Markov chain with the transition matrix

$$P = \begin{pmatrix} 0.2 & 0.9 \\ 0.7 & 0.3 \end{pmatrix}.$$

The hedgehog starts at the first state and moves randomly according to transition matrix P.

   (a) Draw the graph of this chain.

   (b) What is the probability that the hedgehog will be in state 2 after 3 moves?

   (c) What is the stationary distribution of this chain?

2. Consider iid sequence X_1, X_2, ... of uniform on $[0; 10]$ random variables. Find the following probability limits:

$$L_1 = \text{plim}\,\frac{X_1 + X_2 + \ldots + X_n}{2n},\ L_2 = \text{plim}\,\frac{X_1^2 + X_2^2 + \ldots + X_n^2}{X_1 + X_2 + \ldots + X_n},\ L_3 = \text{plim}(X_1 \cdot X_2 \cdot \ldots \cdot X_n)^{1/n}.$$

Hint: maybe there is a function that can transform the product L_3 into the sum? you are free to use any probability limit property.

3. Consider iid sequence X_1, X_2, ... of uniform on $[0; 10]$ random variables.

   (a) Find the probability $\mathbb{P}(|\max\{X_1, X_2, \ldots, X_n\} - 10| > \varepsilon)$.

   (b) Find the probability limit $\text{plim}\,\max\{X_1, X_2, \ldots, X_n\}$ by definition.

4. Joe Biden throws a die until six or five appears. For every throw he pays 0.1 dollars, but at the end he receives the result of the last throw in dollars.

   (a) What is the expected payoff of Joe?

   (b) Assume now that Joe can stop the game at every moment of time.
   What is the maximal expected payoff and the corresponding strategy?

5. Ilya Muromets stands before the first stone. There are three roads behind the stone. And every road ends with a new stone. And there are three new roads behind every new stone. And so on. Every road is guarded with one-third probability by a three headed dragon Zmei Gorynich. Yes, there are infinitely many Zmeis Gorynichs.

   (a) What is the probability that Ilya will never meet Zmei Gorynich if Ilya chooses a road at random?

   (b) What is the probability that Ilya will meet Zmei Gorynich after passing by even number of stones if Ilya chooses a road at random?

   (c) What is the probability that **there exists** at least one Eternal Peaceful Path without Zmei Gorynich?

Deadline: **2022-09-25, 21:00**.