[WOI A]

→ [модели рядов]
→ [тренир. и LSE.]

[глоб.]

① ⇒ белый шум
→ MA(1) процесс
② → стац.

[WOI]

опр (def) белый шум / white noise − (u_t)

$u_1, u_2, u_3, u_4, u_5, \ldots$ (u_t)

$E(u_t) = 0 \quad \forall t$

$Var(u_t) = \sigma^2 \quad \forall t$

$Cov(u_t, u_s) = 0$ при $\forall t \neq s$

опр (def) процесс (y_t) наз стац-м (stationary)

$E(y_t) = \mu \quad \forall t$

$Var(y_t) = \gamma_0 \quad \forall t$

$Cov(y_t, y_{t+k}) = \gamma_k \quad \forall t, k$

Упр. (u_t) − б. шум $\quad Var(u_t) = \sigma^2 \quad (\forall t)$

$y_t = u_t + 2u_{t-1} + 7 \quad (\forall t)$

а) $E(y_t)$, $Var(y_t)$, $Cov(y_t, y_s)$?

б) правда ли, что y_t − стац-ый?

$E(y_t) = E(u_t + 2u_{t-1} + 7) = 0 + 2 \cdot 0 + 7 = 7$

$Var(y_t) = Var(u_t + 2u_{t-1} + 7) =$

$= Var(u_t + 2u_{t-1}) = \sigma^2 + 4\sigma^2 + 0 = 5\sigma^2$

$cov = 0$

$Cov(y_t, y_{t+1}) = Cov(u_t + 2u_{t-1} + 7, u_{t+1} + 2u_t + 7) =$

$$\mathrm{Cov}(y_t, y_{t+1}) = \mathrm{Cov}(u_t, 2u_t) = 2\,\mathrm{Cov}(u_t, u_t) = 2\,\mathrm{Var}(u_t) = 2\cdot\sigma^2$$

$$\gamma_1 = \mathrm{Cov}(y_t, y_{t+1}) = 2\sigma^2$$

$$\gamma_2 = \mathrm{Cov}(y_t, y_{t+2}) = \mathrm{Cov}(u_t + 2u_{t-1} + 7,\ u_{t+2} + 2u_{t+1} + 7) = 0$$

нет u_j с один-ми индекс-ми

$$\gamma_3 = \mathrm{Cov}(y_t, y_{t+3}) \overset{?}{=} \mathrm{Cov}(u_t + 2u_{t-1} + 7,\ u_{t+3} + 2u_{t+2} + 7) = 0 \ \checkmark$$

$$\gamma_4 = \gamma_5 = \ldots = 0.$$

$$\begin{cases} \gamma_0 = \mathrm{Var}(y_t) = 5\sigma^2 \\ \gamma_1 = \mathrm{Cov}(y_t, y_{t+1}) = 2\sigma^2 \\ \gamma_2 = \gamma_3 = \gamma_4 = \ldots 0 \end{cases}$$

$E(y_t) = 7$

$$\mathrm{Cov}(y_t, y_s) = \begin{cases} 5\sigma^2, & \text{если } t = s \\ 2\sigma^2, & \text{если } |t-s| = 1 \\ 0, & \text{иначе } |t-s| > 1 \end{cases}$$

$t = s+1$ или $s = t+1$

б) (y_t) – стац-ый?

$E(y_t) = 7$

опр (def) процесс (y_t) наз стац-м (stationary)

$E(y_t) = \mu \quad \forall t$

$\mathrm{Var}(y_t) = \gamma_0 \quad \forall t$

$\mathrm{Cov}(y_t, y_{t+k}) = \gamma_k \quad \forall t, k$

в частности
$\mathrm{Cov}(y_2, y_7) = \mathrm{Cov}(y_3, y_8) = \mathrm{Cov}(y_{100}, y_{105})$
$\mathrm{Cov}(y_9, y_{11}) = \mathrm{Cov}(y_{20}, y_{22})$
...

yes, (y_t) – is stationary

Упр (u_t) – б.шум $\quad Var(u_t) = \sigma^2$

а) $a_t = u_t + 6t \qquad t \in \mathbb{Z}$

б) $b_t = u_1 + u_2 + u_3 + \dots + u_t + 6 \qquad t \geq 0$

общее: $E(y_t)$, $Var(y_t)$, $Cov(y_t, y_s)$, стац-н ли y_t?

$$E(a_t) = E(u_t + \underset{\text{non-random}}{6t}) = 6t + E(u_t) = 6t + 0 = \boxed{6t}$$

a_t – не б.шум, не стац-н

$$Cov(a_t, a_t) = Var(a_t) = Var(u_t + 6t) = Var(u_t) = \sigma^2$$

$$Cov(a_t, a_{t+1}) = Cov(u_t + 6t, u_{t+1} + 6(t+1)) = 0$$

нет соотн-ия между

$$Cov(a_t, a_{t+2}) = Cov(u_t + 6t, u_{t+2} + 6(t+2)) = 0$$

$$Cov(a_t, a_s) = \begin{cases} \sigma^2 & t = s \\ 0 & t \neq s \end{cases}$$

$$b_t = u_1 + u_2 + u_3 + \dots + u_t + 6$$

$$E(b_t) = E(u_1 + \dots + u_t + 6) = 6$$

$$Var(b_t) = Var(u_1 + u_2 + \dots + u_t + 6) = Var(u_1 + u_2 + \dots + u_t) = \underbrace{Var(u_1) + Var(u_2) + \dots + Var(u_t)}_{\sigma^2} +$$

$$+ 2Cov(u_1, u_2) + 2Cov(u_1, u_3) + \dots + 2Cov(u_{t-1}, u_t) =$$

0 0 0

$$= t \cdot \sigma^2 \Rightarrow (b_t) \qquad (b_t)_{t=0}^{\infty}$$

$Cov(b_t, b_s)$?

$Cov(b_t, b_{t+1}) = Cov(u_1 + u_2 + \dots + u_t + 6,\ u_1 + u_2 + \dots + u_{t+1} + 6)$

$= \underbrace{\sigma^2 + \sigma^2 + \dots + \sigma^2}_{t \text{ штук}} = t \cdot \sigma^2$

$Cov(b_t, b_{t+2}) = Cov(u_1 + u_2 + \dots + u_t + 6,$
$u_1 + u_2 + \dots + u_t + u_{t+1} + u_{t+2} + 6) =$

$= \underbrace{\sigma^2 + \dots + \sigma^2}_{t \text{ штук}} = t \cdot \sigma^2$

$Cov(b_t, b_{t+11}) \overset{?}{=} t \cdot \sigma^2$

$Cov(b_t, b_s) \overset{?}{=} \min(t, s) \cdot \sigma^2$

$Cov(y_7, y_{20}) = 7 \cdot \sigma^2$
$Cov(y_{30}, y_9) = 9 \cdot \sigma^2$

$\boxed{t, s \geq 0}$

$Cov(y_1, y_2) \neq Cov(y_{100}, y_{101})$

b_t - не стац.

если (y_t) - стац-ый, то

$\gamma_k = Cov(y_t, y_{t+k})$ наз-ся автоковар-ая ф-ция

$\rho_k = Corr(y_t, y_{t+k})$ автокорреляция

auto covariance fun-n
auto correlation fun-n

№6 ср10

$P(x_t = 0) = \frac{1}{2}$
$P(x_t = 1) = \frac{1}{2}$

$u_t \sim N(0;1)$

$x_1, x_2 \dots \quad u_1, u_2 \dots$ незав

$z_t = x_t \cdot (1 - x_{t-2}) \cdot u_t$

а) $Cov(z_t, z_s)$? (z_t) - stat?

б) $z_{100} = 2.3$

| №6 cap10

PI = predictive interval

$$P(x_t = 0) = \frac{1}{2} \qquad u_t \sim N(0;1)$$

$$P(x_t = 1) = \frac{1}{2}$$

$x_1, x_2 \ldots \quad u_1, u_2 \ldots$ незав

$$z_t = x_t \cdot (1 - x_{t-2}) \cdot u_t$$

a) $Cov(z_t, z_s)$? (z_t) – stat?

б) $P(z_{101} \in PI \mid z_{100} = 2.3) \geq 0{,}95$? shortest PI

c) $P(z_{102} \in PI \mid z_{100} = 2.3) \geq 0{,}95$? shortest PI

$$\boxed{E(z_t) = E(x_t \cdot (1 - x_{t-2}) \cdot u_t) = E(x_t) \cdot E(1 - x_{t-2}) \cdot E(u_t) = \frac{1}{2} \cdot \frac{1}{2} \cdot 0 = 0}$$

$$Var(z_t) = E(z_t^2) - 0^2 = E(x_t^2 (1 - x_{t-2})^2 \cdot u_t^2) =$$

$$= E(x_t^2) \cdot E((1 - x_{t-2})^2) \, E(u_t^2) =$$

$x_t^2 = x_t$!!

$$= \underbrace{E(x_t)}_{1/2} \cdot \underbrace{E(1 - x_{t-2})}_{1/2} \cdot \underbrace{E(u_t^2)}_{1} =$$

$$= \frac{1}{4}$$

$u_t \sim N(0;1)$

$E(u_t) = 0$

$Var(u_t) = E(u_t^2) = 1$

$$Cov(z_t, z_{t+1}) = Cov(x_t \cdot (1 - x_{t-2}) \cdot u_t,\ x_{t+1} \cdot (1 - x_{t-1}) \cdot u_{t+1})$$

$= 0$ (нет совп. инд)

$$Cov(z_t, z_{t+2}) = Cov(x_t (1 - x_{t-2}) \cdot u_t,\ x_{t+2} (1 - x_t) \cdot u_{t+2}) =$$

$$Cov(R, L) = E(R \cdot L) - E(R) \cdot E(L)$$

$$= E(x_t \cdot (1 - x_{t-2}) \cdot x_{t+2} \cdot (1 - x_t) \cdot u_t \cdot u_{t+2}) - \underbrace{E(z_t)}_{0} \cdot \underbrace{E(z_{t+2})}_{0}$$

$x_t \cdot (1 - x_t) = 0$ $\left[\begin{matrix} x_t \to 1 \\ x_t \to 0 \end{matrix}\right]$

$= 0$

$$\operatorname{Cov}(z_t, z_{t+3}) = \operatorname{Cov}\left(x_t\cdot(1-x_{t-2})\cdot u_t,\ x_{t+3}\cdot(1-x_{t+1})\cdot u_{t+3}\right)$$

$$= 0$$

$$\begin{array}{ccccccc}
u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \\
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\
z_1 & z_2 & z_3 & z_4 & z_5 & z_6 & z_7
\end{array}$$

x_4

$$\operatorname{Cov}(z_4, z_6) = 0$$

$$E(z_t) = 0.$$

$$\operatorname{Cov}(z_t, z_s) = \begin{cases} 1/4 & t = s \\ 0 & t \neq s \end{cases}$$

(z_t) – стац. ый процесс, б. шум