WOLd TSSP

→ (ACF), PACF

↙ theoretical ↘ sample

Теорет. ACF

$\rho_k = \text{corr}(y_t, y_{t+k})$

k — не зав-т от t.

! процесс явл-ся [слабо] стац-м

$E(y_t) = \mu$

$\text{Var}(y_t) = \gamma_0$

$\text{Cov}(y_t, y_s) = \gamma_{t-s}$

Пр. (u_t) - б. шум с $\text{Var}(u_t) = \sigma^2$

MA(2)

$y_t = u_t + 0.5u_{t-1} + 0.3u_{t-2} + \sqrt{3}$ [4-е]

(u_t) - б. шум

$E(u_t) = 0$

$\text{Cov}(u_t, u_s) = 0$

$t \neq s$

$\text{Var}(u_t) = \text{const}$

а) проверка, ст ли (y_t) - стац?

б) если да, то построение ACF?

$E(y_t) = \sqrt{3}$

$\text{Var}(y_t) = \sigma^2 + 0.5^2 \cdot \sigma^2 + 0.3^2 \cdot \sigma^2 + 0 = \sigma^2 \cdot 1.34 = \gamma_0$

$\text{Cov}(y_t, y_s)$?

$\text{Cov}(y_t, y_{t+1}) = \text{Cov}(u_t + 0.5u_{t-1} + 0.3u_{t-2},\ u_{t+1} + 0.5u_t + 0.3u_{t-1})$

$= 0.5 \cdot \sigma^2 + 0.5 \cdot 0.3 \cdot \sigma^2 = 0.65\sigma^2 = \gamma_1$

$\gamma_2 = \text{Cov}(y_t, y_{t+2}) = \text{Cov}(u_t + 0.5u_{t-1} + 0.3u_{t-2},\ u_{t+2} + 0.5u_{t+1} + 0.3u_t) =$

$= 0.3\sigma^2 = \gamma_2$

$\gamma_3 = \text{Cov}(y_t, y_{t+3}) = \{$ нет совп. индексов $\}$

$= 0$

[Ни одна γ_k не зав-т от t]

=> процесс (y_t) - стац-вен.

$$\rho_1 = Corr(y_t, y_{t+1}) = \frac{Cov(y_t, y_{t+1})}{\sqrt{Var(y_t)\,Var(y_{t+1})}} =$$

↑ стац-вен!

$$= \frac{Cov(y_t, y_{t+1})}{Var(y_t)} = \frac{\gamma_1}{\gamma_0} = \frac{0.65\,\sigma^2}{1.34\,\sigma^2} = \frac{0.65}{1.34} = \frac{65}{134}$$

$$\rho_2 = \dots = \frac{\gamma_2}{\gamma_0} = \frac{0.3\,\sigma^2}{1.34\,\sigma^2} = \frac{30}{134}$$

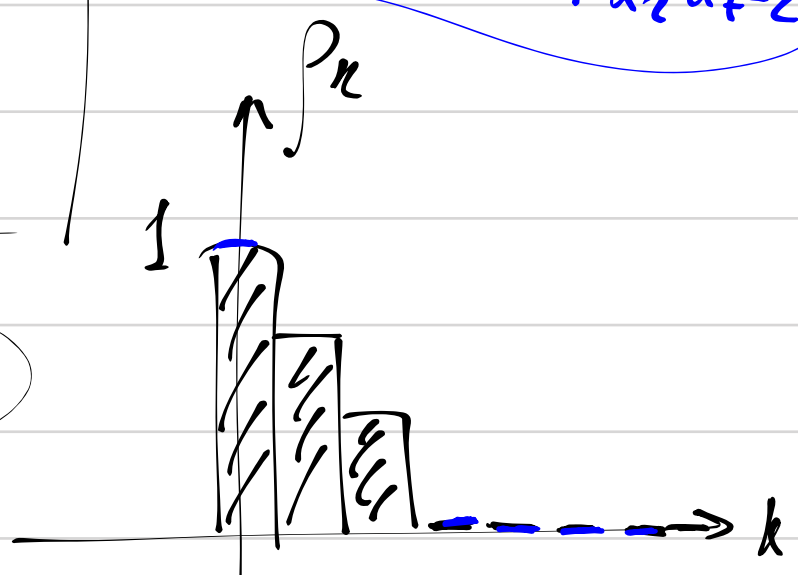
$$\rho_3 = \rho_4 = \dots 0$$

$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

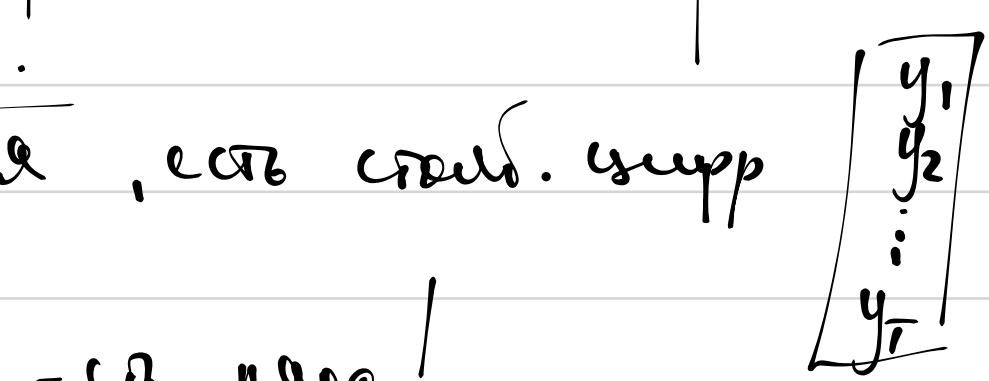
$$\rho_k = \begin{cases} 1 & k = 0 \\ 65/134 & k = 1 \\ 30/134 & k = 2 \\ 0 & k \geq 3 \end{cases}$$

$y_t = \beta_3 + u_t + \alpha_1 \cdot u_{t-1} + \alpha_2 u_{t-2}$

+ теорет.!



в реальности:



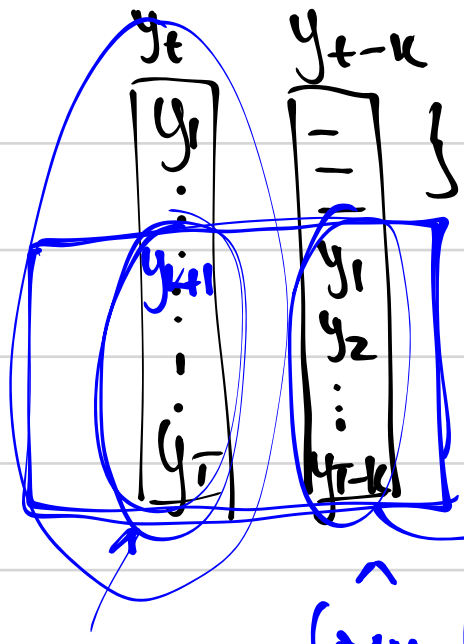
нет ур-ия, есть столб. вектор

! верим в стац-ть ряда !

выборочная ACF — sACF

$\hat{\rho}_k =$ (алгоритм!)

$= \widehat{Corr}(y_t, y_{t+k})$



$$\hat{corr}(y_t, y_{t-k}) = \frac{\sum (a_i - \bar{a}) \cdot (b_i - \bar{b})}{\sqrt{\sum (a_i - \bar{a})^2 \sum (b_i - \bar{b})^2}}$$

(*)

$$\hat{corr}(a, b)$$

$$\bar{a} = \frac{\sum_{i=k+1}^{T} y_i}{T-k}$$

$$\bar{b} = \frac{\sum_{i=1}^{T-k} y_i}{T-k}$$

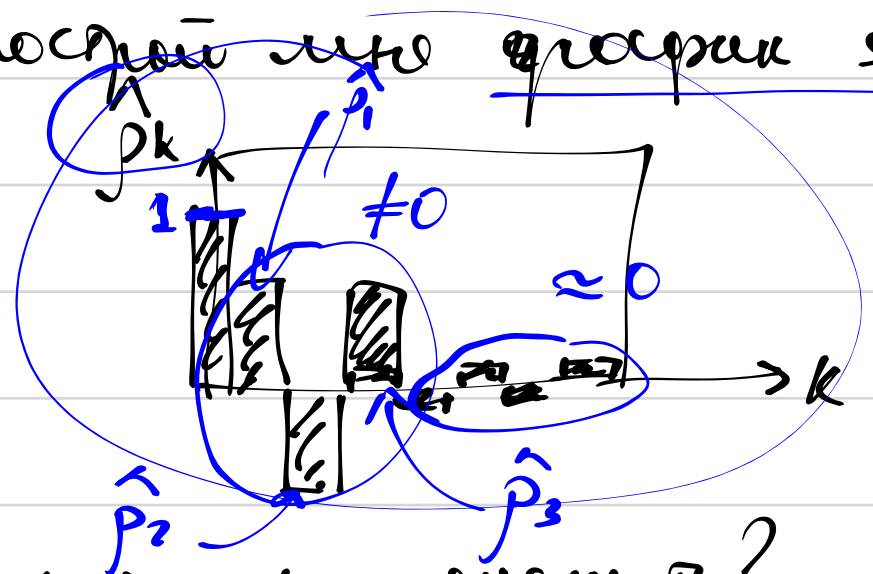
Вар-т:

$$\hat{corr}(y_t, y_{t-k}) = \frac{\sum (a_i - \bar{y})(b_i - \bar{y})}{\sqrt{\sum (a_i - \bar{y})^2 \sum (b_i - \bar{y})^2}}$$

Как-ое упр-ие.

$$\hat{\rho}_1 = \frac{\sum (y_i - \bar{y}) \cdot (y_{i+1} - \bar{y})}{\text{— // —}}$$

$y_1 \dots y_T$ → построить функцию sACF



sktime

Какую модель разумно оценить?

MA(1): $y_t = \mu + u_t + \alpha_1 \cdot u_{t-1}$

MA(2): $y_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2}$

☺ MA(3): $y_t = \mu + u_t + \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \alpha_3 u_{t-3}$

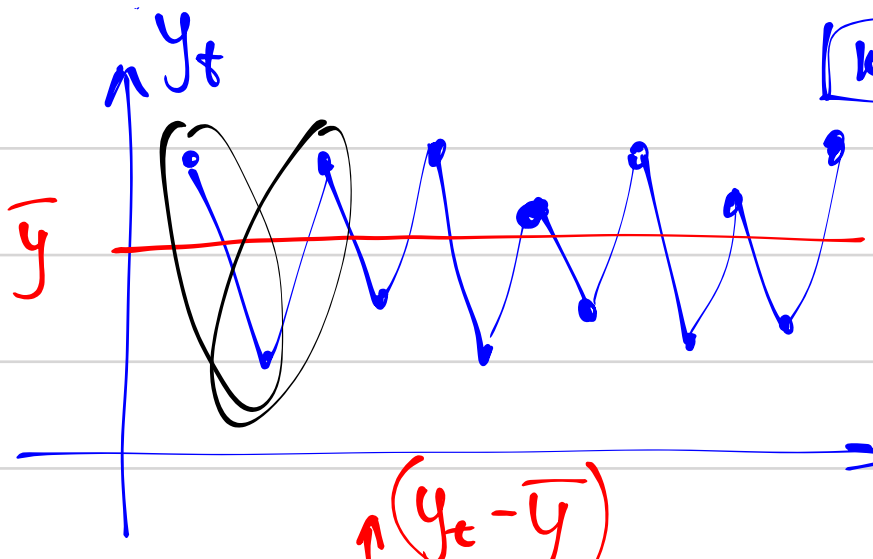
$\hat{\rho}_1$ сильно отл. от 0

$\hat{\rho}_2$ — // — 0

$\hat{\rho}_3$ — // — 0

$\hat{\rho}_4 \approx \hat{\rho}_5 \approx \hat{\rho}_6 \approx 0.$

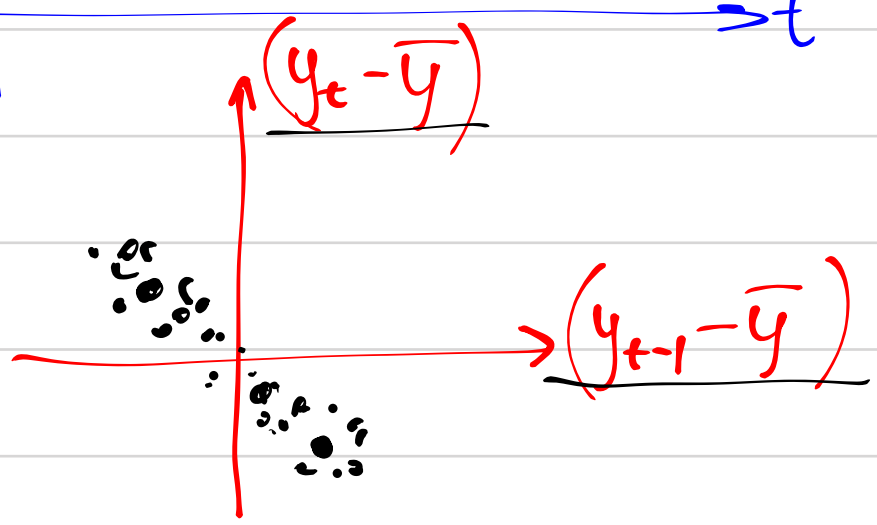
$\rho_k = corr(y_t, y_{t+k})$ $\quad$ $\hat{\rho}_k$ - оценка для ρ_k



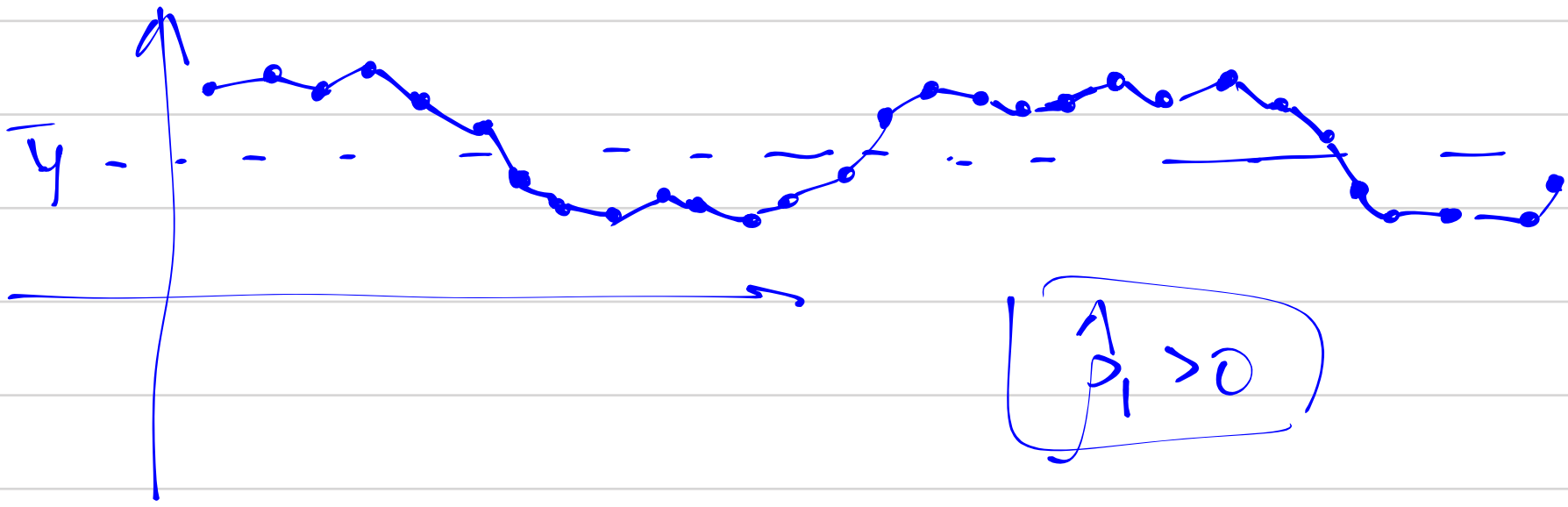
кол. упр.

$\hat{\rho}_1 > 0$ или $\hat{\rho}_1 < 0$?

или $\hat{\rho}_1 \approx 0$



$\hat{\rho}_1 < 0$



$\hat{\rho}_1 > 0$

Частная корр-ция. (partial correlation)

→ способ вычисления по опр-ию.

→ характ для стац-х процессов.

partial correlation

def $\rho_{corr}(R, L; M_1, M_2, M_3) =$

(R, L — между к-рыми счится; M_1, M_2, M_3 — от этих величин очистили R и L)

$= corr(\tilde{R}, \tilde{L})$, где

$\tilde{R} = R - (\alpha_1 M_1 + \alpha_2 M_2 + \alpha_3 M_3)$

т. что $\tilde{R}$ не корр с M_1, M_2, M_3!

$\tilde{L} = L - (\beta_1 M_1 + \beta_2 M_2 + \beta_3 M_3)$,

т. что $\tilde{L}$ не корр с M_1, M_2, M_3

Упр. $y_t = \sqrt{3} + u_t + 0{,}5u_{t-1} + 0{,}3u_{t-2}$ (u_t) - бел. шум

найдите теор-кую PACF $\varphi_{11}, \varphi_{22}, \varphi_{33}$

$\varphi_{11} = \rho_1 = Corr(y_t, y_{t+1}) \overset{?}{=} \frac{65}{134}$

$\varphi_{22} = pCorr(y_t, y_{t+2}; y_{t+1}) =$

$\varphi_{32} = Corr(y_t, y_{t+2}; y_{t+1}, y_{t+3})$

макс. ур. лаг. — между ними

$\varphi_{33} = pCorr(y_t, y_{t+3}; y_{t+1}, y_{t+2})$

$\vdots$

считаем φ_{22} !

$\varphi_{22} \neq \rho_2$

$\varphi_{22} = Corr(\tilde{y}_t, \tilde{y}_{t+2})$

$\tilde{y}_t = y_t - \alpha y_{t+1}$ т. чтобы $\tilde{y}_t$ некорр с y_{t+1}

α? $Cov(\tilde{y}_t, y_{t+1}) = 0$

$Cov(y_t - \alpha y_{t+1}, y_{t+1}) = 0$

$Cov(y_t, y_{t+1}) - \alpha \cdot Cov(y_{t+1}, y_{t+1}) = 0$

$\gamma_1 - \alpha \cdot \gamma_0 = 0$

$\alpha = \frac{\gamma_1}{\gamma_0} = \rho_1 = \frac{65}{134}$

$\tilde{y}_t = y_t - \frac{65}{134} \cdot y_{t+1}$

β $Cov(\tilde{y}_{t+2}, y_{t+1}) = 0$

y_t, y_{t+1}, y_{t+2}

$Cov(y_{t+2} - \beta y_{t+1}, y_{t+1}) = 0$

$Cov(y_{t+2}, y_{t+1}) - \beta Cov(y_{t+1}, y_{t+1}) = 0$ $\quad \gamma_1 - \beta \cdot \gamma_0 = 0$

$\tilde{y}_{t+2} = y_{t+2} - \frac{65}{134} \cdot y_{t+1}$

$\varphi_{22} = Corr(\tilde{y}_t, \tilde{y}_{t+2}) = Corr(y_t - \frac{65}{134} y_{t+1}, y_{t+2} - \frac{65}{134} y_{t+1})$

$$\varphi_{22} = \frac{Cov\left(y_t - \frac{65}{134}y_{t+1},\ y_{t+2} - \frac{65}{134}y_{t+1}\right)}{\sqrt{Var\left(y_t - \frac{65}{134}y_{t+1}\right)\cdot Var\left(y_{t+2} - \frac{65}{134}y_{t+1}\right)}} =$$

$\frac{65}{134}\cdot\gamma_1$; γ_2

стац-сть!

$$= \frac{\gamma_2 - \frac{65}{134}\gamma_1 - \frac{65}{134}\gamma_1 + \left(\frac{65}{134}\right)^2\gamma_0}{Var\left(y_t - \frac{65}{134}y_{t+1}\right)} =$$

$$= \frac{\gamma_2 - \frac{130}{134}\gamma_1 + \left(\frac{65}{134}\right)^2\gamma_0}{\gamma_0 + \left(\frac{65}{134}\right)^2\cdot\gamma_0 - 2\cdot\frac{65}{134}\cdot\gamma_1}$$

у стац-го процесса много ум-х точ-в.

$$Var\left(\frac{y_{t+2}}{n\to\infty} - 6y_t\right) = Var\left(6y_{t+2} - y_t\right)$$

$$= \gamma_0 + 6^2\gamma_0 - 2\cdot 6\cdot\gamma_1$$

$$Var\left(y_t - \frac{65}{134}y_{t+1}\right) = \underbrace{Var(y_t)}_{\gamma_0} + \left(\frac{65}{134}\right)^2\cdot\underbrace{Var(y_{t+1})}_{\gamma_0} - 2\cdot\frac{65}{134}\cdot Cov(y_t, y_{t+1})$$

$\gamma_k = Cov(y_t, y_{t+k})$

$\rho_k = Corr(y_t, y_{t+k})$

$\varphi_{kk} = pCorr(y_t, y_{t+k};\ y_{t+1}, y_{t+2}, y_{t+3} \dots y_{t+k-1})$

от каких случ. вел. оцениваем

Упр.

Дано: $X \sim N(0;1)$, $Y \sim N(0;1)$, $Z \sim N(0;1)$

$L = X + Z$, $R = Y + Z$

$$Corr(L, R) \overset{?}{=} \frac{Cov(X+Z, Y+Z)}{\sqrt{Var(X+Z)\,Var(Y+Z)}} = \frac{1}{\sqrt{2\cdot 2}} = \frac{1}{2}$$

$$pCorr(L, R; Z) \overset{?}{=} Corr(\tilde{L}, \tilde{R}) = Corr(X, Y) = 0$$

$\tilde{L} = L - \alpha\cdot Z = L - 1\cdot Z = X$

$\tilde{R} = R - \beta\cdot Z = R - 1\cdot Z = Y$