AI1110
PROBABILITY AND RANDOM VARIABLES
Assignment 2

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Question(12.13.6.11):In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.
Answer:-1.6852.
Solution:
Let us consider Random Variables X and Y.

Y=Amount he wins or loses for a particular outcome on die.

$$Y = \begin{cases} 1, & \text{If outcome on die is 6.} \\ -1, & \text{If outcome on die is not 6.} \end{cases}$$

$$\Pr(Y = 1) = \frac{1}{6} \tag{1}$$

$$\Pr(Y = -1) = \frac{5}{6} \tag{2}$$

X=Amount he wins or loses in atmost 3 die rolls.

$$X = \begin{cases} 1, & \text{If outcome on first die roll is 6.} \\ 0, & \text{If outcome on second die roll is 6.} \\ -1, & \text{If outcome on third die roll is 6.} \\ -3, & \text{If 6 doesn't occur in first 3 die rolls.} \end{cases}$$

$\Pr(X = 1)$=probability that 6 is outcome of first die roll and no further die is rolled.

$$\begin{aligned} \Pr(X = 1) &= \Pr(Y = 1) \\ &= \frac{1}{6} \end{aligned} \tag{3}$$

$\Pr(X = 0)$=probability that first die roll is not 6 and second die roll is 6,no further die is rolled.

$$\Pr(X = 0) = \Pr(Y = -1, Y = 1) \tag{4}$$

As 2 die rolls are independent

$$\begin{aligned} \Pr(Y = -1, Y = 1) &= \Pr(Y = -1)\Pr(Y = 1) \\ &= \frac{5}{6}.\frac{1}{6} \\ &= \frac{5}{36} \end{aligned}$$

$\Pr(X = -1)$=probability that 6 is not occured in first 2 die rolls and outcome of 3rd die roll is 6.

$$\Pr(X = -1) = \Pr(Y = -1, Y = -1, Y = 1) \tag{5}$$

As 3 die rolls are independent,above equation can be written as

$$\begin{aligned} &= \Pr(Y = -1)\Pr(Y = -1)\Pr(Y = 1) \\ &= \frac{5}{6}.\frac{5}{6}.\frac{1}{6} \\ &= \frac{25}{216} \end{aligned}$$

$\Pr(X = -3)$=probability that 6 is not occured in any of the 3 die rolls.

$$\Pr(X = -3) = \Pr(Y = -1, Y = -1, Y = -1) \tag{6}$$

As 3 die rolls are independent,above equation can be written as

$$\begin{aligned} &= \Pr(Y = -1)\Pr(Y = -1)\Pr(Y = -1) \\ &= \frac{5}{6}.\frac{5}{6}.\frac{5}{6} \\ &= \frac{125}{216} \end{aligned}$$

$$E(X) = \sum_{n=-\infty}^{n=\infty} n.\Pr(X = n) \tag{7}$$

$$= (1).\Pr(X = 1) + (0).\Pr(X = 0)$$
$$+ (-1).\Pr(X = -1) + (-3).\Pr(X = -3)$$
$$= (1).(\frac{1}{6}) + (0).(\frac{5}{36}) + (-1).(\frac{25}{216}) + (-3).(\frac{125}{216})$$
$$= \frac{-364}{216}$$
$$= -1.6851851851$$