AI1110
PROBABILITY AND RANDOM VARIABLES
Assignment 2

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Question(12.13.6.11):In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.
Answer:-1.6852.
Solution:

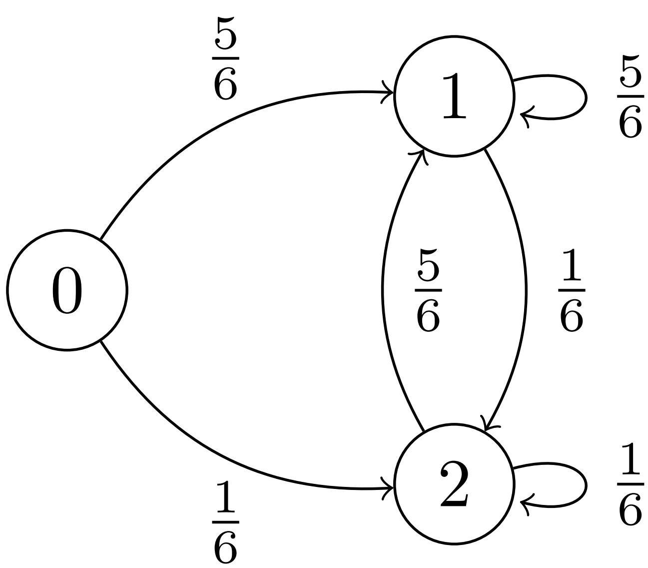

Fig. 0. Markov Chain Diagram

State 0 : Initial state
State 1 : occurence of {1,2,3,4,5} in die roll
State 2 : occurance of 6 in die roll
Transition matrix(P) of the above Markov chain is

$$\begin{bmatrix} 0 & 5/6 & 1/6 \\ 0 & 5/6 & 1/6 \\ 0 & 5/6 & 1/6 \end{bmatrix}$$

Let us consider a random variable X.

X=Amount he wins or loses in atmost 3 die rolls.

$$X = \begin{cases} 1, & \text{If outcome on first die roll is 6.} \\ 0, & \text{If outcome on second die roll is 6.} \\ -1, & \text{If outcome on third die roll is 6.} \\ -3, & \text{If 6 doesn't occur in first 3 die rolls.} \end{cases}$$

Let Y_i denote the states in markovs chain.
$\Pr(X = 1)$:= We go from state 0 to state 2.

$$\begin{aligned} \Pr(X = 1) &= \Pr(Y_1 = 2/Y_0 = 0) \\ &= P_{02} \end{aligned} \tag{1}$$

From Transition matrix its value is

$$\Pr(X = 1) = \frac{1}{6} \tag{2}$$

$\Pr(X = 0)$:= We go from state 0 to state 1 then to state 2.

$$\begin{aligned} \Pr(X = 0) &= \Pr(Y_1 = 1, Y_2 = 2/Y_0 = 0) \\ &= \Pr(Y_1 = 1/Y_0 = 0) . \Pr(Y_2 = 2/Y_1 = 1, Y_0 = 0) \\ &= \Pr(Y_1 = 1/Y_0 = 0) . \Pr(Y_2 = 2/Y_1 = 1) \\ &= P_{01}.P_{12} \end{aligned} \tag{3}$$

From Transition matrix its value is

$$\begin{aligned} \Pr(X = 0) &= \frac{5}{6}.\frac{1}{6} \\ &= \frac{5}{36} \end{aligned} \tag{4}$$

$\Pr(X = -1)$:= We pass through state 1 twice and then the state 2.

$$\Pr(X=-1) = \Pr(Y_1=1, Y_2=1, Y_3=2/Y_0=0) \tag{5}$$

$$= \Pr(Y_1=1/Y_0=0)\,.\Pr(Y_2=1/Y_1=1, Y_0=0)\,.$$
$$\Pr(Y_3=2/Y_2=1, Y_1=1, Y_0=0)$$

$$= \Pr(Y_1=1/Y_0=0)\,.\Pr(Y_2=1/Y_1=1)\,.$$
$$\Pr(Y_3=2/Y_2=1)$$

$$= P_{01}.P_{11}.P_{12}$$

From Transition matrix its value is

$$\Pr(X=-1) = \frac{5}{6}.\frac{5}{6}.\frac{1}{6}$$
$$= \frac{25}{216} \tag{6}$$

$\Pr(X=-3)$:= We pass through state 1 thrice.

$$\Pr(X=-3) = \Pr(Y_1=1, Y_2=1, Y_3=1/Y_0=0) \tag{7}$$

$$= \Pr(Y_1=1/Y_0=0)\,.\Pr(Y_2=1/Y_1=1, Y_0=0)\,.$$
$$\Pr(Y_3=1/Y_2=1, Y_1=1, Y_0=0)$$

$$= \Pr(Y_1=1/Y_0=0)\,.\Pr(Y_2=1/Y_1=1)\,.$$
$$\Pr(Y_3=1/Y_2=1)$$

$$= P_{01}.P_{11}.P_{11}$$

From Transition matrix its value is

$$\Pr(X=-3) = \frac{5}{6}.\frac{5}{6}.\frac{5}{6}$$
$$= \frac{125}{216} \tag{8}$$

Expected value of the amount he wins / loses is E(X)

$$E(X) = \sum_{n=-\infty}^{n=\infty} n.\Pr(X=n) \tag{9}$$
$$= (1).\Pr(X=1) + (0).\Pr(X=0)$$
$$+ (-1).\Pr(X=-1) + (-3).\Pr(X=-3)$$
$$= (1).(\frac{1}{6}) + (0).(\frac{5}{36}) + (-1).(\frac{25}{216}) + (-3).(\frac{125}{216})$$
$$= \frac{-364}{216}$$
$$= -1.6851851851$$