AI1110
PROBABILITY AND RANDOM VARIABLES
Assignment 2

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Question(12.13.6.11):In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.
Answer:-1.6852.
Solution:

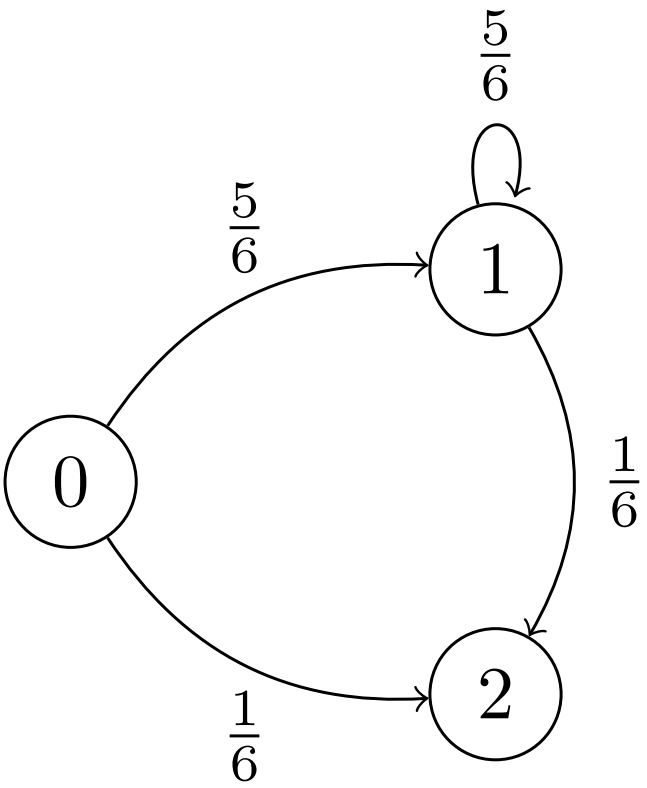

Fig. 0. Markov Chain Diagram

State 0 : Initial state
State 1 : loses a rupee
State 2 : gains a rupee

Let us consider a random variable X.

X=Amount he wins or loses in atmost 3 die rolls.

$$X = \begin{cases} 1, & \text{If outcome on first die roll is 6.} \\ 0, & \text{If outcome on second die roll is 6.} \\ -1, & \text{If outcome on third die roll is 6.} \\ -3, & \text{If 6 doesn't occur in first 3 die rolls.} \end{cases}$$

$\Pr(X = 1)$=We go from state 0 to state 2.

$$\Pr(X = 1) = \frac{1}{6} \tag{1}$$

$\Pr(X = 0)$=We go from state 0 to state 1 then to state 2.

$$\begin{aligned} \Pr(X = 1) &= \frac{5}{6}.\frac{1}{6} \\ &= \frac{5}{36} \end{aligned} \tag{2}$$

$\Pr(X = -1)$=We pass through state 1 twice and then the state 2.

$$\begin{aligned} \Pr(X = -1) &= \frac{5}{6}.\frac{5}{6}.\frac{1}{6} \\ &= \frac{25}{216} \end{aligned} \tag{3}$$

$\Pr(X = -3)$=We pass through state 1 thrice.

$$\begin{aligned} \Pr(X = -3) &= \frac{5}{6}.\frac{5}{6}.\frac{5}{6} \\ &= \frac{125}{216} \end{aligned} \tag{4}$$

Expected value of the amount he wins / loses is E(X)

$$E(X) = \sum_{n=-\infty}^{n=\infty} n.\Pr(X = n) \quad (5)$$

$$= (1).\Pr(X = 1) + (0).\Pr(X = 0)$$
$$+ (-1).\Pr(X = -1) + (-3).\Pr(X = -3)$$
$$= (1).(\frac{1}{6}) + (0).(\frac{5}{36}) + (-1).(\frac{25}{216}) + (-3).(\frac{125}{216})$$
$$= \frac{-364}{216}$$
$$= -1.6851851851$$