

AI1110

PROBABILITY AND RANDOM VARIABLES

Assignment 2

Nalavolu Chetana
CS22BTECH11042

Question(12.13.6.11):In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

Answer:-1.6852.

Solution:

Let us consider Random Variables X and Y_i .

Y_i =Amount he wins or loses for outcome on i th die roll.

$$Y_i = \begin{cases} 1, & \text{If outcome on die is 6.} \\ -1, & \text{If outcome on die is not 6.} \end{cases}$$

$$\Pr(Y_i = 1) = \frac{1}{6} \quad (1)$$

$$\Pr(Y_i = -1) = \frac{5}{6} \quad (2)$$

X =Amount he wins or loses in atmost 3 die rolls.

$$X = \begin{cases} 1, & \text{If outcome on first die roll is 6.} \\ 0, & \text{If outcome on second die roll is 6.} \\ -1, & \text{If outcome on third die roll is 6.} \\ -3, & \text{If 6 doesn't occur in first 3 die rolls.} \end{cases}$$

$\Pr(X = 1)$ =probability that 6 is outcome of first die roll and no further die is rolled.

$$\begin{aligned} \Pr(X = 1) &= \Pr(Y_1 = 1) \\ &= \frac{1}{6} \end{aligned} \quad (3)$$

$\Pr(X = 0)$ =probability that first die roll is not 6 and second die roll is 6,no further die is rolled.

$$\Pr(X = 0) = \Pr(Y_1 = -1, Y_2 = 1) \quad (4)$$

As 2 die rolls are independent

$$\begin{aligned} \Pr(Y_1 = -1, Y_2 = 1) &= \Pr(Y_1 = -1) \Pr(Y_2 = 1) \\ &= \frac{5}{6} \cdot \frac{1}{6} \\ &= \frac{5}{36} \end{aligned}$$

$\Pr(X = -1)$ =probability that 6 is not occurred in first 2 die rolls and outcome of 3rd die roll is 6.

$$\Pr(X = -1) = \Pr(Y_1 = -1, Y_2 = -1, Y_3 = 1) \quad (5)$$

As 3 die rolls are independent,above equation can be written as

$$\begin{aligned} &= \Pr(Y_1 = -1) \Pr(Y_2 = -1) \Pr(Y_3 = 1) \\ &= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \\ &= \frac{25}{216} \end{aligned}$$

$\Pr(X = -3)$ =probability that 6 is not occurred in any of the 3 die rolls.

$$\Pr(X = -3) = \Pr(Y_1 = -1, Y_2 = -1, Y_3 = -1) \quad (6)$$

As 3 die rolls are independent,above equation can be written as

$$\begin{aligned} &= \Pr(Y_1 = -1) \Pr(Y_2 = -1) \Pr(Y_3 = -1) \\ &= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \\ &= \frac{125}{216} \end{aligned}$$

Expected value of the amount he wins / loses is
 $E(X)$

$$\begin{aligned}
 E(X) &= \sum_{n=-\infty}^{n=\infty} n \cdot \Pr(X = n) & (7) \\
 &= (1) \cdot \Pr(X = 1) + (0) \cdot \Pr(X = 0) \\
 &\quad + (-1) \cdot \Pr(X = -1) + (-3) \cdot \Pr(X = -3) \\
 &= (1) \cdot \left(\frac{1}{6}\right) + (0) \cdot \left(\frac{5}{36}\right) + (-1) \cdot \left(\frac{25}{216}\right) + (-3) \cdot \left(\frac{125}{216}\right) \\
 &= \frac{-364}{216} \\
 &= -1.6851851851
 \end{aligned}$$