

# AI1110

## PROBABILITY AND RANDOM VARIABLES

### Assignment 2

Nalavolu Chetana  
CS22BTECH11042

Question(12.13.6.11): In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

Answer:-1.6852.

Solution:

Let us consider a random variable  $X$ .

$X$  = Amount he wins or loses in atmost 3 die rolls.

$$X = \begin{cases} 1, & \text{If outcome on first die roll is 6.} \\ 0, & \text{If outcome on second die roll is 6.} \\ -1, & \text{If outcome on third die roll is 6.} \\ -3, & \text{If 6 doesn't occur in first 3 die rolls.} \end{cases}$$

$\Pr(X = 1) :=$  We go from state 0 to state 2.

$$\Pr(X = 1) = \frac{1}{6} \quad (1)$$

$\Pr(X = 0) :=$  We go from state 0 to state 1 then to state 2.

$$\begin{aligned} \Pr(X = 0) &= \frac{5}{6} \cdot \frac{1}{6} \\ &= \frac{5}{36} \end{aligned} \quad (2)$$

$\Pr(X = -1) :=$  We pass through state 1 twice and then the state 2.

$$\begin{aligned} \Pr(X = -1) &= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \\ &= \frac{25}{216} \end{aligned} \quad (3)$$

$\Pr(X = -3) :=$  We pass through state 1 thrice.

$$\begin{aligned} \Pr(X = -3) &= \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{5}{6} \\ &= \frac{125}{216} \end{aligned} \quad (4)$$

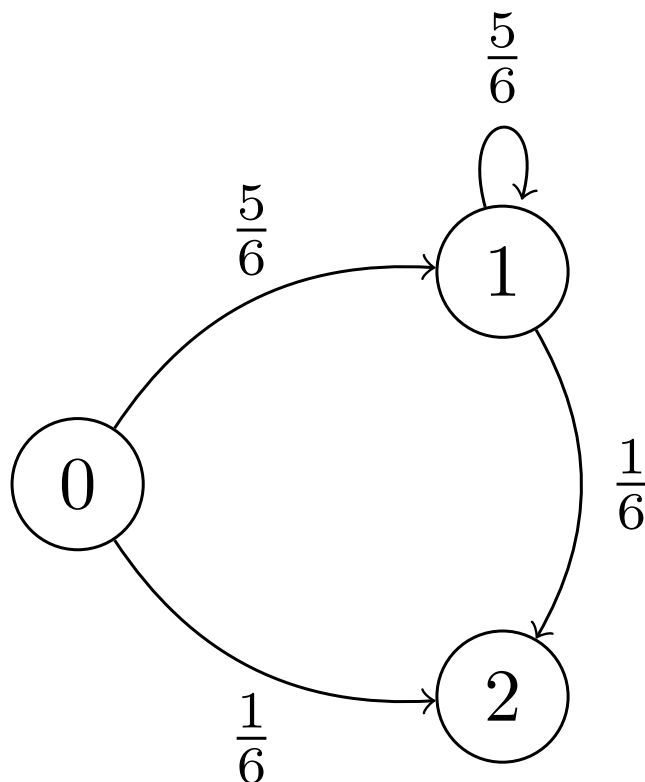


Fig. 0. Markov Chain Diagram

State 0 : Initial state

State 1 : loses a rupee

State 2 : gains a rupee

Expected value of the amount he wins / loses is  
 $E(X)$

$$\begin{aligned}
 E(X) &= \sum_{n=-\infty}^{n=\infty} n \cdot \Pr(X = n) & (5) \\
 &= (1) \cdot \Pr(X = 1) + (0) \cdot \Pr(X = 0) \\
 &\quad + (-1) \cdot \Pr(X = -1) + (-3) \cdot \Pr(X = -3) \\
 &= (1) \cdot \left(\frac{1}{6}\right) + (0) \cdot \left(\frac{5}{36}\right) + (-1) \cdot \left(\frac{25}{216}\right) + (-3) \cdot \left(\frac{125}{216}\right) \\
 &= \frac{-364}{216} \\
 &= -1.6851851851
 \end{aligned}$$