

# AI1110

## PROBABILITY AND RANDOM VARIABLES

### Assignment 1

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**Question(12.13.6.4):** Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

**Answer-**  $1 - \sum_{r=7}^{10} \binom{10}{r} (0.9)^r (0.1)^{10-r}$ .

**Solution:**

Let us consider a Binomial random variable X,

X = number of right-handed people among a random sample of 10 people.

$X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$X \sim \text{Bin}(10, p)$

Given that 90% of the people are right-handed.

Let **p** be the probability that the picked person is right-handed and **q** be the probability that the picked person is left-handed.

$$\Pr(X = k) = \binom{10}{k} p^k q^{10-k} \quad (1)$$

$$\Pr(X = k) = \binom{10}{k} (0.9)^k (0.1)^{10-k} \quad (2)$$

Where

$$p = 0.9$$

$$q = 0.1$$

**k** denotes the values that the random variable X can take.

Probability that at most 6 are right-handed among 10 is  $\Pr(X \leq 6)$

$$\Pr(X \leq 6) = 1 - \Pr(X \geq 7) \quad (3)$$

Probability of entire sample space is 1.

$$\begin{aligned} \Pr(X \leq 6) &= 1 - (\Pr(X = 7) + \Pr(X = 8) \\ &\quad + \Pr(X = 9) + \Pr(X = 10)) \end{aligned} \quad (4)$$

$$\begin{aligned} &= 1 - \left( \binom{10}{7} (0.9)^7 (0.1)^3 + \binom{10}{8} (0.9)^8 (0.1)^2 \right. \\ &\quad \left. + \binom{10}{9} (0.9)^9 (0.1)^1 + \binom{10}{10} (0.9)^{10} (0.1)^0 \right) \end{aligned} \quad (5)$$

Hence our required probability can be expressed as

$$\Pr(X \leq 6) = 1 - \sum_{r=7}^{10} \binom{10}{r} (0.9)^r (0.1)^{10-r}. \quad (6)$$