

AI1110

PROBABILITY AND RANDOM VARIABLES

Assignment 1

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Question(12.13.6.4): Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?

Answer-1- $\sum_{r=7}^{10} \binom{10}{r} (0.9)^r (0.1)^{10-r}$.

Solution:

Let us consider a random variable X,

X=number of right-handed people among a random sample of 10 people.

$$X=\{0,1,2,3,4,5,6,7,8,9,10\}$$

X is a Binomial random variable.

Given that 90% of the people are right-handed.

Let **p** be the probability that the picked person is right-handed and **q** be the probability that the picked person is left-handed.

$$p=0.9$$

$$q=0.1$$

As X is a Binomial random variable its probability distribution function can be given as following

$$\Pr(X = x) = \binom{10}{x} p^x q^{10-x}$$

$$\Pr(X = x) = \binom{10}{x} (0.9)^x (0.1)^{10-x}$$

Now our aim is to find the probability that atmost 6 are right-handed among 10. That is

$$\Pr(X \leq 6) = 1 - \Pr(X \geq 7)$$

As we know from axioms of probability that probability of entire sample space is 1.

$$\begin{aligned} \Pr(X \leq 6) &= 1 - (\Pr(X = 7) + \Pr(X = 8) \\ &\quad + \Pr(X = 9) + \Pr(X = 10)) \end{aligned}$$

$$\begin{aligned} &= 1 - \left(\binom{10}{7} (0.9)^7 (0.1)^3 + \binom{10}{8} (0.9)^8 (0.1)^2 \right. \\ &\quad \left. + \binom{10}{9} (0.9)^9 (0.1)^1 + \binom{10}{10} (0.9)^{10} (0.1)^0 \right) \end{aligned}$$

Hence our required probability can be expressed as

$$\Pr(X \leq 6) = 1 - \sum_{r=7}^{10} \binom{10}{r} (0.9)^r (0.1)^{10-r}.$$