

# Course Project 1

## Nondeterministic Finite Automata

CSE 30151 Spring 2018

Version of 2018/02/09

We've studied the theory of nondeterministic finite automata, and now it's time to implement them. The interesting challenge is that NFAs are nondeterministic, but real computers are deterministic – how do we simulate nondeterminism?

One option is backtracking: when two transitions are possible, try one, and if it fails, try the other. But this will lead to a  $O(2^n)$  time algorithm (where  $n$  is the input length). The theory provides another option: convert the NFA to an equivalent DFA. That gives a  $O(n)$  algorithm, but the conversion could take  $O(2^{|Q|})$  time and space (where  $|Q|$  is the number of states).

In this project, you'll implement a solution that runs in  $O(|\delta|n)$  time, where  $|\delta|$  is the number of transitions. You can write your implementation in C++ or Python (or another language with permission of the instructor).

### Getting started

You should have been given access on GitHub to a repository called `theory-project-team`, with `team` replaced by your team's name. Please clone this repository to wherever you plan to work on the project:

```
git clone https://github.com/ND-CSE-30151-SP18/theory-project-team
cd theory-project-team
```

If you're the first team member to do this, your repository is empty. In that case, run the commands:

```
git pull https://github.com/ND-CSE-30151-SP18/theory-project-skeleton
git push
```

If one of your teammates already did this, there's no need for you to repeat it. Whenever we make an update to `theory-project-skeleton`, we'll send out an announcement, and one of you will need to repeat the pull/push (resolving any merge conflicts if necessary) to get the update.

Now your directory should include the following files (among others):

```
bin.{linux,darwin}/
  nfa_path
  re_to_nfa
examples/
  sipser-n1.nfa
  sipser-n2.nfa
  sipser-n3.nfa
  sipser-n4.nfa
  cycle.nfa
tests/
  test-cp1.sh
cp1/
```

- The `bin.linux` and `bin.darwin` contain binaries for Linux and Mac, respectively. They contain reference implementations for the tools you will implement and tools used by the test scripts.
- The `examples` directory contains examples of NFAs that you will use for testing. See below for a description of the file format.
- The `tests` directory contains test scripts. The script `tests/test-cp1.sh` tests your code for correctness and speed. Your code needs to pass all tests in order to get full credit.
- Please place the programs that you write into the `cp1/` subdirectory.

## 1 NFAs

Design a data structure for representing a NFA  $M$ , and write functions to read and write NFAs. (For all projects, the names of functions and the way that they are called are just suggestions; if you prefer a different style, that's fine.)

`read_nfa(filename)`

- *filename*: Name of file containing definition of NFA  $M$
- Returns: The NFA  $M$

`write_nfa(M, filename)`

- $M$ : The NFA to write
- *filename*: Name of file to write to
- Effect: Writes definition of  $M$  to file

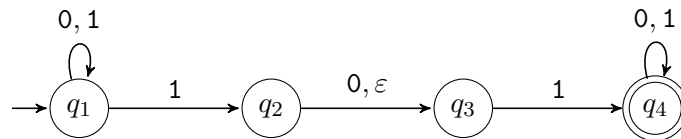
**NFA file format** The NFA definition should have the following format. It should begin with a four-line header:

1. A whitespace-separated list of states,  $Q$ .
2. A whitespace-separated list of input symbols,  $\Sigma$ . It should be disjoint from  $Q$ . Each symbol should be exactly one character long.
3. The start state,  $s \in Q$ .
4. A whitespace-separated list of accept states,  $F \subseteq Q$ .

The rest of the lines list the transitions, one transition per line. Each line has three fields, separated by whitespace:

1. The state  $q \in Q$  that the transition leaves from.
2. The symbol  $a \in \Sigma$  that the transition reads, or  $\&$  for the empty string.
3. The state  $r \in Q$  that the transition goes to.

For example, the following NFA ( $N_1$  in the book):



would be specified by the file (`examples/sipser-n1.nfa`):

```

q1 q2 q3 q4
0 1
q1
q4
q1 0 q1
q1 1 q1
q1 1 q2
q2 0 q3
q2 & q3
q3 1 q4
q4 0 q4
q4 1 q4

```

## 2 Matcher

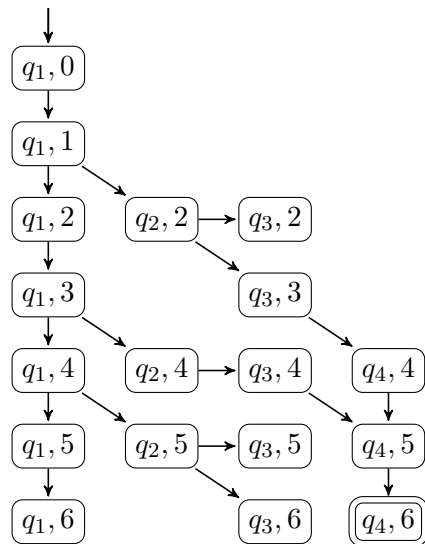
Write a function that tests whether a NFA  $M$  accepts a string  $w$ :

`match( $M$ ,  $w$ )`

- $M$ : An NFA to run
- $w$ : The string to run on
- Returns: A pair ( $flag$ ,  $path$ ), where
  - $flag$ : True if  $M$  accepts  $w$ ; false otherwise.
  - $path$ : List of the transitions on an accepting path. If there is more than one, an arbitrary path is returned.

This function is required to be **linear both in the length of  $w$  and the number of transitions in  $M$ .**

Here's how to do this. Define a *configuration* of  $M$  on input string  $w$  to be a pair  $(q, i)$ , where  $q \in Q$  and  $0 \leq i \leq |w|$ . These configurations can be thought of as nodes in a graph. For example, if the NFA is  $N_1$  above and  $w = 010110$ , then the graph of configurations would be:



This is similar to Sipser's Figure 1.29, but there are several differences here. The most important difference is that configuration  $(q_4, 5)$  appears only once with two incoming edges, instead of appearing twice. In general, each configuration appears at most once in the graph. As a result, the graph has at most  $|Q||w| + 1$  nodes and  $|\delta||w|$  edges.

Then, deciding whether  $N_1$  accepts  $w$  amounts to searching for a path from the start configuration (in this case,  $(q_1, 0)$ ) to an accept configuration (in this case,  $(q_4, 6)$ ). You can use any graph search algorithm; **breadth-first search** is probably the least hassle, but **depth-first search** corresponds to how the real Unix tools work. (One particular thing to watch out for is cycles of  $\varepsilon$ -transitions, as in `cycle.nfa`. Make sure your matcher doesn't hang when it encounters one.)

When you studied graph search algorithms, you may not have seen how to reconstruct the found path. Graph searches maintain a set that keeps track of which configurations have been visited (in your Data Structures notes, it was called **marked**). If you change this to a data structure that records, for each configuration, how you got to that configuration, then after the search finishes, you can use that information to reconstruct the path.

### 3 Putting it together

Package the above into a command-line tool called `nfa_path`:

`nfa_path nfile string`

- *nfile*: name of file defining an NFA  $M$
- *string*: string to run  $M$  on
- Output:
  - If  $M$  accepts *string*, prints **accept** followed by an accepting path
  - Otherwise, prints **reject**

The path should be printed with one transition per line, in the same format as the NFA file format. For example:

```
$ nfa_path examples/sipser-m1.nfa 11
accept
q1 1 q2
q2 & q3
q3 1 q4
```

Test your program by running `test-cp1.sh`. This script runs `nfa_path` on several NFAs and several test strings, and it also produces a graph of the running time of `nfa_path` on NFAs of various sizes. The sizes are chosen so that the graph should look roughly linear, like this:

```
n= 32  *
n= 45  *
```

```

n= 55    *
n= 64    *
n= 71    *
n= 78    *
n= 84    *
n= 90    *
n= 95    *
n=100    *

```

## Submission instructions

Your code should build and run on `studentnn.cse.nd.edu`. The automatic tester will clone your repository, run `make -C cp1`, and then run `tests/test-cp1.sh`. You're advised to try all of the above steps and ensure that all tests pass.

To submit your work, please push your repository to Github and then create a new release with tag version `cp1` (note that the tag version is not the same thing as the release title). If you are making a partial submission, then use tag version `cp1-1` for part 1, `cp1-2` for part 2, and so on.

## Rubric

Part 1	
data structure	3
read_nfa	3
write_nfa	3
Part 2 (match)	
correct algorithm	6
handling $\varepsilon$	3
reconstructing path	6
Part 3 (nfa_path)	
correctness	3
time complexity	3
Total	30