

Homework 6: Non-context-free languages

CSE 30151 Spring 2016

Due 2016/03/15

Instructions

Please note that you will **lose one point** if you don't follow these instructions.

- You can prepare your solutions however you like, but you must submit them as a single PDF file.
- Please name your PDF `netid-hw6.pdf`, where `netid` is replaced with your NetID, or `netid-hw6-1234.pdf`, where 1234 is replaced with the problems you are submitting.
- If you use the same name twice, only the most recent version will be graded!
- Submit your PDF file in Sakai. Don't forget to click the Submit (or Resubmit) button!

Problems

Each problem is worth 7 points. An additional one point is for legibility, and one point for following the submission instructions.

1. **Pumping lemma for CFLs.** Use the pumping lemma to show that the following languages are not context free:
 - (a) [Problem 2.30a] $\{0^n 1^n 0^n 1^n\}$
 - (b) [Problem 2.31] $\{w \in \{0, 1\}^* \mid w = w^R \text{ and } w \text{ has an equal number of 0s and 1s}\}$
 - (c) $\{a^i \mid i \text{ is prime}\}$

2. **The SCRAMBLE operation** [Problem 2.43]. If w and w' are strings over an alphabet Σ , define the relation $w \doteq w'$ to be true iff w' is a permutation of w , that is, they have the same number of each type of symbol, but possibly in a different order. If w is a string and L is a language, define

$$\begin{aligned} \text{SCRAMBLE}(w) &= \{w' \mid w' \doteq w\} \\ \text{SCRAMBLE}(L) &= \bigcup_{w \in L} \text{SCRAMBLE}(w). \end{aligned}$$

- (a) Show that if $\Sigma = \{0, 1\}$ and L is a regular language over Σ , then $\text{SCRAMBLE}(L)$ is context-free.
- (b) Let $\Sigma = \{a, b, c\}$. Show that there is a regular language over Σ such that $\text{SCRAMBLE}(L)$ is not context-free.
3. **Non-closure properties** [Exercise 2.2]

- (a) Use the languages

$$\begin{aligned} A &= \{a^m b^n c^n \mid m, n \geq 0\} \\ B &= \{a^n b^n c^m \mid m, n \geq 0\} \end{aligned}$$

to prove that context-free languages are *not* closed under intersection.

- (b) Use (a) and DeMorgan's law to prove that context-free languages are *not* closed under complementation.

4. **Intersection again**

- (a) [cf. Problem 2.36] Let $L = \{a^m b^n c^n d^n \mid m, n \geq 1\}$. Show that L satisfies the pumping lemma for context-free languages (that is, the pumping lemma fails to show that L is not context-free).
- (b) [Problem 2.18] Let C be any context-free language, and let R be any regular language. Prove that the language $C \cap R$ is context-free.
- (c) Show that L is not a regular language.