

Homework 9: NP-completeness

CSE 30151 Spring 2016

Due 2016/04/26

Instructions

Please note that you will **lose one point** if you don't follow these instructions.

- You can prepare your solutions however you like, but you must submit them as a single PDF file.
- Please name your PDF `netid-hw9.pdf`, where `netid` is replaced with your NetID, or `netid-hw9-1234.pdf`, where 1234 is replaced with the problems you are submitting.
- If you use the same name twice, only the most recent version will be graded!
- Submit your PDF file in Sakai. Don't forget to click the Submit (or Resubmit) button!

Problems

A *two-dimensional TM* (2dTM) has, in place of a tape, a sheet of graph paper that has a top and left edge but extends infinitely to the right and down. Initially the head is at the upper left corner. The transitions are as in a standard TM, except that there are four directions: L (left), R (right), U (up), and D (down).

1. Describe how to simulate a 2dTM using a TM that has one or more one-dimensional tapes, in three parts:
 - (a) Describe how a two-dimensional sheet with a head can be represented as one or more one-dimensional tapes with heads. You can use the construction we sketched in class or your own.
 - (b) Simulating reading and writing symbols is presumably trivial, but if not, give implementation-level descriptions.
 - (c) Give implementation-level descriptions of how to simulate moving left, right, up, and down.

2. Let n be the length of the input string. Choose a k and show that if a computation takes $t(n)$ steps in a 2dTM, it takes, in the worst case, $O((t(n))^k)$ steps in a standard TM (single one-way infinite tape). Your algorithm doesn't have to be optimal and your analysis doesn't have to be tight.
3. [Problem 7.39 in 2nd ed., 7.41 in 3rd ed.] In the proof of the Cook-Levin theorem, a window is a 2×3 rectangle of cells. Show why the proof would have failed if we had used 2×2 windows instead. Your answer should include:
 - (a) An example NTM N .
 - (b) Two configurations of N (call them upper and lower) such that all of the following hold:
 - The upper configuration is legal.
 - The lower configuration either is illegal or doesn't follow the upper configuration according to N 's rules.
 - Every window of the two configurations (when the upper is stacked on top of the lower) is legal according to N 's rules.
4. In class we saw a puzzle called TetraVex that is NP-complete.¹ In this problem, we'll prove the NP-completeness of a simpler version of the puzzle:
 - You are given a set of tiles, each with a north, south, east, and west label. You can make as many copies as you want of each tile. You can't rotate the tiles.
 - You are also given a rectangular frame with labels.
 - The object is to fill the frame with tiles such that all abutting labels match.

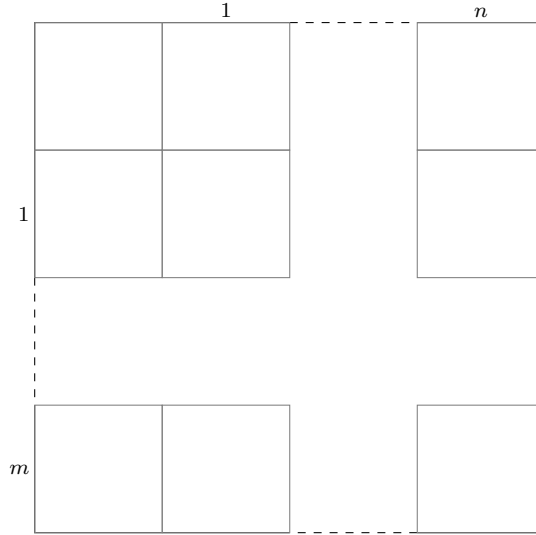
The proof is by reduction from 3SAT. Let x_1, \dots, x_n be a set of variables, and let ϕ be a formula in 3CNF,

$$\phi = (\phi_{11} \vee \phi_{12} \vee \phi_{13}) \wedge \cdots \wedge (\phi_{m1} \vee \phi_{m2} \vee \phi_{m3})$$

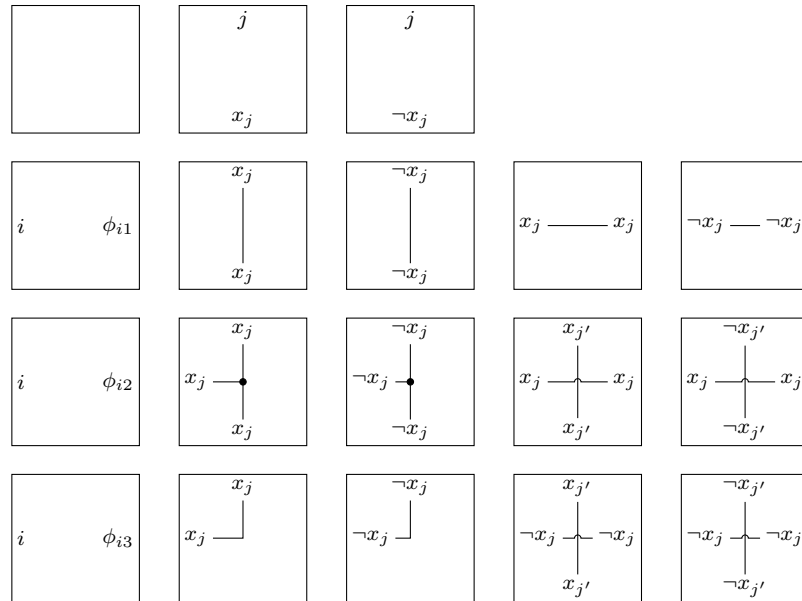
where each $\phi_{i1}, \phi_{i2}, \phi_{i3}$ is either x_j or $\neg x_j$ for some j .

¹<http://arxiv.org/pdf/0903.1147v1.pdf>

Define a function f that converts ϕ into an instance of the puzzle as follows. The frame is:



The tiles are, for all $i \in \{1, \dots, m\}$ and $j, j' \in \{1, \dots, n\}$:



Notes:

- Each $\phi_{i1}, \phi_{i2}, \phi_{i3}$ is equal to either x_j or $\neg x_j$ for some j .
- Blank edges only match other blank edges.
- The “wires” drawn in the interior of some tiles are merely suggestive; they don’t affect matching at all.

Prove the following statements. Your answers don’t have to be very formal, but try to make them clear.

- (a) The set of solvable puzzles is in NP.
- (b) f is computable in polynomial time.
- (c) If ϕ is satisfiable, then the corresponding puzzle $f(\phi)$ is solvable.
- (d) If the puzzle $f(\phi)$ is solvable, then ϕ is satisfiable.