Homework 5: CFGs and PDAs

CSE 30151 Spring 2016

Due 2016/03/01

Instructions

Please note that you will **lose one point** if you don't follow these instructions.

- You can prepare your solutions however you like, but you must submit them as a single PDF file.
- Please name your PDF as follows:
  - If you're making a complete submission, name your PDF file `netid-hw5.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name your PDF file `netid-hw5-1234.pdf`, where `1234` is replaced with the problems you are submitting at this time.
  - If you use the same name twice, only the most recent version will be graded!
- Submit your PDF file in Sakai. Don't forget to click the Submit (or Resubmit) button!

Problems

Each problem is worth 7 points. An additional one point is for legibility, and one point for following the submission instructions.

1. **Designing CFGs.** For each of the following languages, write a context-free grammar that generates it:
   (a) [Exercise 2.4e] $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid w = w^R\}$
   (b) $\{\mathtt{0}^m\mathtt{1}^n\mathtt{0}^n\mathtt{1}^m \mid m, n \geq 0\}$
   (c) [Exercise 2.6b] The complement of $\{\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$. (Use the fact, which we saw a while back in class, that this is equal to $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq n\} \cup \{w \mid w \text{ contains } \mathtt{10}\}$.)

2. **Designing a PDA** [Problem 2.23]. In this problem, you will show that the following language over $\Sigma = \{\mathtt{0}, \mathtt{1}\}$ is regular:

$$D = \{xy \mid x, y \in \Sigma^*, |x| = |y|, x \neq y\}$$

Do this in two steps:

(a) Show that another way of writing D is

$$D = \{uavwbz \mid u, v, w, z \in \Sigma^*, |u| = |v|, |w| = |z|; a, b \in \Sigma, a \neq b\}.$$

(b) Write a pushdown automaton for D.

3. **Arithmetic expressions.** Consider the grammar G_4 for arithmetic expressions:

$$E \to E + T \mid T$$
$$T \to T \times F \mid F$$
$$F \to (E) \mid \mathtt{a}$$

(a) Modify the grammar to allow an exponentiation operator $\uparrow$. It should have *higher precedence* than multiplication; that is, in the parse tree of the string $\mathtt{a} \times \mathtt{a} \uparrow \mathtt{a}$, there should be a subtree for $\mathtt{a} \uparrow \mathtt{a}$ and not for $\mathtt{a} \times \mathtt{a}$. It should be *right-associative*; that is, in the parse tree of the string $\mathtt{a} \uparrow \mathtt{a} \uparrow \mathtt{a}$, there should be a subtree for the second $\mathtt{a} \uparrow \mathtt{a}$ and not for the first.

(b) [Exercise 2.11] Convert the *original* grammar to a PDA using the construction in the proof of Lemma 2.21.

4. **PDA to CFG.** Convert the PDA in Example 2.18 (here modified to meet the requirements of the proof of Lemma 2.27) to a CFG: