Midterm Exam 2: Study Guide

CSE 30151 Spring 2016

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You will have the whole class period of 75 minutes. The exam will be open book and open paper notes. No computers, smartphones, or any other Turing-equivalent machines are allowed. Regrettably, I can't think of any way to allow the use of notes taken on an electronic tablet that is fair to all students.

The exam covers everything up to and including HW7. It does not cover any of these special topics: neural networks and finite automata, human language and context-free grammars, human intelligence and Turing machines. There will be six questions, worth 10 points each, for a total of 60 points (10% of your grade), on the following topics:

- Design: One of each of the following types of questions.
  - Write a pushdown automaton or context-free grammar (your choice) that recognizes a given language (like 2.4ad, 2.6ac, 2.7ac). If you write a PDA, either a state transition diagram or a table is acceptable; an informal description gets partial credit.
  - Write a Turing machine that decides a given language (like 3.8a). An implementation-level description is acceptable.
- Proofs: One of the following types of questions.
  - Show that a certain language is not regular (like 1.29ac or 1.46b), or
  - Show that a certain language is not context-free (like 2.30bc), or
  - Show that context-free languages are closed or not closed under a certain operation (like 2.38).
- You'll be given a definition of an extension of NFAs, PDAs or TMs (like Problem 3.10 or 3.14) and will be asked:
  - Write a machine of this type that recognizes a given language.
  - Show that a machine of this type can be converted into an equivalent Turing machine (implementation-level description). You'll be given a skeleton proof that you will fill in the missing parts of.
  - Show that a Turing machine can be converted into an equivalent machine of this type (only an informal description required). You'll be given a skeleton proof that you will fill in the missing parts of.

Queue automata

See Problem 3.14 for a description of a queue automaton. On the exam, you will be given a formal definition to refer to, although you won't necessarily need it: A *queue automaton* (QA) is a tuple $P = (Q, \Sigma, \Gamma, \delta, q_0, F)$, where

- Q, Σ, Γ, q_0, and F are as in a pushdown automaton
- δ is a transition function $\delta : Q \times \Sigma^? \times \Gamma^? \to Q \times \Gamma^?$, where $X^? = X \cup \{\varepsilon\}$.

The queue is initially empty. If the current state is q, with remaining input aw (where $a \in \Sigma^?$ and $w \in \Sigma^*$) and queue bx (where $b \in \Gamma^?$ and $x \in \Gamma^*$) and $\delta(q, a, b)$ contains (q', b'), then the QA can move to state q', with remaining input w and queue xb'.

Problems

1. Design a QA that recognizes the language $\{\mathtt{a}^n\mathtt{b}^n\mathtt{c}^n \mid n \geq 0\}$.
   (a) Give a brief informal description of your QA.
   (b) Write the formal description (state transition diagram or table) of your QA.

2. Show that any QA P can be converted into an equivalent TM M. You may use any of the TM extensions that were discussed in the book or in class. For each of the following, give a brief implementation-level description. Assume that w is the input string.
   (a) If P has read the first k input symbols and the stack is x, how would you represent this in M? How would you initialize M?
   (b) How would you simulate reading in an input symbol a?
   (c) How would you simulate pulling (dequeueing) a symbol b?
   (d) How would you simulate pushing (enqueueing) a symbol c?

3. Show that any TM M can be converted into an equivalent QA P. For each of the following, give a brief informal description.
   (a) Suppose that M's tape is $t_1 t_2 \cdots t_n \sqcup\sqcup\sqcup \cdots$, where each $t_i \in \Gamma$, and the head is at position h. How would you represent this using a queue? How would you initialize the queue?
   (b) Now consider a single one of M's transitions, $\delta(q, a) = (r, b, d)$, where $d \in \{\mathrm{L}, \mathrm{R}\}$. How would you simulate reading symbol a and writing symbol b?
   (c) How would you simulate moving the head left? Remember that the head cannot move past the left end of the tape.
   (d) How would you simulate moving the head right? Remember that the tape has blank symbols ($\sqcup$) extending infinitely to the right.