Homework 8: NP-Completeness

CSE 30151 Fall 2020

revised 2020/11/04
Due **Tuesday, 2020/11/10** at 5:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `*netid*-hw8.pdf`, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `*netid*-hw8-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems

1. This problem concerns details of the proof of the NP-completeness of *CLIQUE*, which is Theorem 7.32 in the book.
   (a) Convert the formula $\phi = (x \vee x \vee z) \wedge (\bar{x} \vee y \vee y) \wedge (\bar{y} \vee \bar{y} \vee \bar{z})$ into a graph $G = (V, E)$ and integer k, using the construction in the proof of Theorem 7.32 (so that ϕ is satisfiable iff G has a clique of size k).
   (b) For each satisfying truth assignment of ϕ,
      - Please write down the truth assignment.
      - How many subsets of V does it correspond to?
      - Please draw one of them.
      - Is it a clique of G?

   (c) Convert $\phi = (x \vee \bar{y} \vee \bar{y}) \wedge (\bar{x} \vee \bar{x} \vee \bar{x}) \wedge (y \vee y \vee y)$ into a graph G and integer k, and explain why ϕ is not satisfiable and G has no clique of size k.

2. In the *knapsack problem*, you are given a knapsack with maximum weight capacity W kilograms, and a set of k items,

$$S = \{(w_1, v_1), \ldots, (w_k, v_k)\}$$

where w_i is the weight (in kilograms) of item i and v_i is the value (in dollars) of item i. The decision version of the problem is: Is there a subset of the items with total weight at most W and total value at least V? More formally,

$$\mathit{KNAPSACK} = \left\{ \langle S, W, V \rangle \mid \exists T \subseteq S \text{ s.t. } \sum_{(w,v) \in T} w \leq W \text{ and } \sum_{(w,v) \in T} v \geq V \right\}.$$

Show that this problem is NP-complete.

3. A *regular expression with backreferences* (REB) over Σ is defined as follows:

   - $\emptyset$ and ε are REBs.
   - If $a \in \Sigma$, then a is a REB.
   - For any natural number i, $\backslash i$ is a REB (for example, $\backslash 1$, $\backslash 2$, etc.).
   - If α is a REB, then α^* and (α) are REBs.
   - If α and β are REBs, then $\alpha\beta$ and $\alpha \cup \beta$ are REBs.

A subexpression of the form (α) is called a *group*. The groups of a REB are numbered by their *left* parentheses, from left to right. A backreference $\backslash i$, which must come after group i, matches a substring equal to the first substring matched by group i. For example:

$$\alpha_{\text{copy}} = \underset{1}{(}\underset{2}{(}\mathtt{a} \cup \mathtt{b})^*)\backslash 1$$

The little numbers are not part of the REB; they indicate the numbering of the groups. This REB matches the language $\{ww \mid w \in \{\mathtt{a}, \mathtt{b}\}^*\}$.

Define the language

$$A_{\text{REB}} = \{\langle \alpha, w \rangle \mid \alpha \text{ matches } w\}.$$

What are the certificates of A_{REB}? Here is one suggestion. If α is a REB with g groups, and $w \in \Sigma^*$, a certificate that $\langle \alpha, w \rangle \in A_{\text{REB}}$ is a function $m : \{1, \ldots, g\} \to \Sigma^*$ such that $m(i)$ is the first substring of w matched by group i. For example, a certificate that $\langle \alpha_{\text{copy}}, \mathtt{aaabbb} \rangle \in A_{\text{REB}}$ is the function $m(1) = \mathtt{aaa}$, $m(2) = \mathtt{a}$.

You may assume that regular expression matching (without backreferences) can be done in time $O(|\alpha|^2|w|)$.

(a) Prove that A_{REB} is in NP. To make this simpler, you may use a more restricted definition of REB: if $\alpha = \beta \cup \gamma$, then neither β nor γ contain any groups, and if $\alpha = \beta^*$, then β does not contain any groups.

(b) Prove that A_{REB} is NP-hard, by reduction from 3SAT.

(c) What does your reduction convert the formula $\phi = (x \vee x \vee z) \wedge (\bar{x} \vee y \vee y) \wedge (\bar{y} \vee \bar{y} \vee \bar{z})$ to?

(d) Write down a certificate that verifies that your answer to 3c is in A_{REB}.