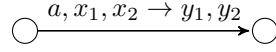


## 2-stack PDAs

Going beyond variants of TMs, there are also lots of automata, grammars, and other things that are equivalent to standard TMs. For example: a 2-stack PDA is a PDA that has two stacks instead of one. It has transitions that look like



which means, if:

- the 2-stack PDA is in state  $q$
- and the current input symbol is  $a$
- and the top symbol on the first stack is  $x_1$
- and the top symbol on the second stack is  $x_2$

then

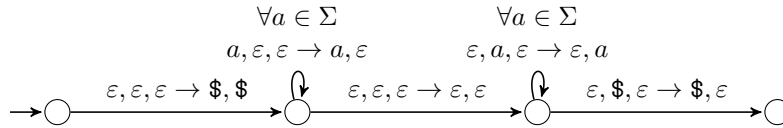
- consume  $a$
- pop  $x_1$  and push  $y_1$  on the first stack
- pop  $x_2$  and push  $y_2$  on the second stack
- go to state  $r$ .

We want to show that any Turing machine can be simulated by a 2-stack PDA.

First of all, the intuition is that if the TM is in state  $q$ , its tape is

$$t_1 t_2 \cdots t_{n-1} t_n \sqcup \cdots$$

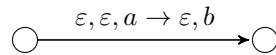
and its head is at position  $h$  (that is, over symbol  $t_h$ ), our 2-stack PDA will simulate this by storing  $t_{h-1} t_{h-2} \cdots t_2 t_1 \$$  on the first stack and  $t_h t_{h+1} \cdots t_{n-1} t_n \$$  on the second stack. This is done by the following 2-stack PDA fragment:



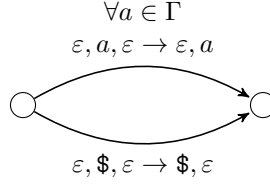
where “ $\forall a \in \Sigma$ ” means that there is a transition for each  $a$  in  $\Sigma$ .

We need to show, for each transition of the TM, how to convert it into 2-stack PDA transitions. To simplify this, let's think separately about how to simulate reading/writing and how to simulating moving left/right.

To simulate when the TM reads symbol  $a$  and writes symbol  $b$ :



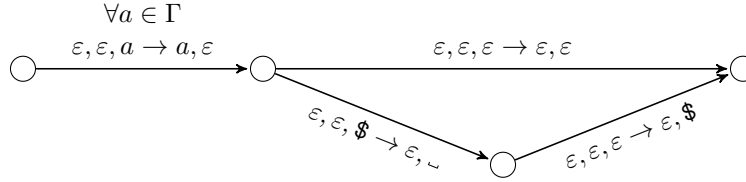
To simulate when the TM moves the head to the left,



The transition above transfers a tape symbol from the left stack to the right, simulating a move to the left.

The transition below is for the special case when the left stack is empty, which simulates when the head is already at the left end of the tape. In this case, we don't change anything.

To simulate when the TM moves the head to the right,



The first transition transfers a tape symbol from the right stack to the left, simulating a move to the right.

Then, the path below is for the special case where the right stack is empty, which means that the head is further right than it has gone before; in that case, we insert a blank.

By repeating the above translations for each transition of the TM, we obtain a 2-stack PDA that is equivalent to the TM.