Homework 4: CFGs and PDAs

CSE 30151 Fall 2020

Due Friday, 2020/09/18 at 5:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw4.pdf`, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw4-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems

Each problem is worth 10 points. For each of the following languages, show that it is context-free by doing all of the following:

(a) A context-free grammar. Please specify the start symbol if it is not S.

(b) A pushdown automaton. A state diagram is sufficient.

(c) A brief explanation of why your CFG and PDA work, including any proofs that the hints ask for.

It's okay to make use of the CFG-to-PDA conversion or PDA-to-CFG conversion.

1. $\{w \in \{\mathtt{0},\mathtt{1}\}^* \mid w$ has an equal number of $\mathtt{0}$s and $\mathtt{1}s\}$.
2. [Exercise 2.6b] The complement of $\{\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$.

   Hint: First prove that this is equal to $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq n\} \cup \overline{\mathtt{0}^*\mathtt{1}^*}$.
3. [Problem 2.23]. $D = \{xy \mid xy \in \{\mathtt{0},\mathtt{1}\}^*, |x| = |y|, x \neq y\}$. That is, strings of even length where the first and second halves are different.

   Hint 1: Observe that D is equal to

   $$E = \{w \in \{\mathtt{0},\mathtt{1}\}^* \mid \text{for some } n \text{ and } i,\ |w| = 2n \text{ and } w_i \neq w_{n+i}\}.$$

   Hint 2: Prove that E is equal to

   $$F = \{uavwbz \mid a, b \in \{\mathtt{0},\mathtt{1}\}, u, v, w, z \in \{\mathtt{0},\mathtt{1}\}^*, |u| = |v|, |w| = |z|, a \neq b\}.$$

   That is, strings that can be cut into two odd-length pieces (uav and wbz) that have different middle symbols (a and b).