# Homework 2: DFAs and NFAs

#### CSE 30151 Fall 2020

### Due 2020/08/28 at 5:00pm

## Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it netid-hw2.pdf, where netid is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it netid-hw2-123.pdf, where 123 is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

# Problems (10 points each)

1. **Designing finite automata** Define, for all k > 0,

 $D_k = \{w \in \{0, \dots, 9\}^* \mid w \text{ is the decimal representation of a multiple of } k\},$ 

where  $\varepsilon$  is considered to represent the number 0. For example, the strings  $\varepsilon$ , 0, 88, and 088 all belong to  $D_2$ .

- (a) Write a DFA for  $D_2$ .
- (b) Write a DFA for  $D_3$ .
- (c) Prove that for any k > 0,  $D_k$  is regular, by describing how to write the formal description of a DFA  $M = (Q, \{0, ..., 9\}, \delta, s, F)$  in terms of k. Hint: appending a digit d to a number x is equivalent to doing  $x \leftarrow 10x + d$ .

CSE 30151 Fall 2020 Homework 2

2. **Nondeterminism** Consider the following language:

 $L_2 = \{uv \mid u, v \in \{a, b\}^*, u \text{ contains an even number of a's, and}$   $v \text{ contains an even number of b's} \}$ 

Note that as long as there is *some* way of cutting a string into u and v so as to satisfy the constraints, it's in  $L_2$ . So ba  $\in L_2$ , because u = b has an even number (0) of a's and v = a has an even number (0) of b's.

- (a) Write an NFA  $N_2$  that recognizes  $L_2$ .
- (b) What is the accepting path for  $bab^n$  through  $N_2$ ? You can show the path for bab, babb, babb, ... until the pattern is clear, or describe the general case. Either way, please write a few words about what you observe.
- (c) Convert  $N_2$  to a DFA  $M_2$  using the subset construction (Theorem 1.39).
- (d) What is the accepting path for  $bab^n$  through  $M_2$ ? Again, you can show the path for bab, babb, babb,... until the pattern is clear, or describe the general case. Either way, please write a few words about what you observe.
- 3. **Regular/raluger** In the following, we'll use the language  $L_3$  as an example (but the results must be proved for all L):

 $L_3 = \{ \text{deed}, \text{deer}, \text{red}, \text{redder}, \text{reed} \}.$ 

(a) Recall that

$$L^R = \{ w \mid w^R \in L \}.$$

For example,  $L_3^R = \{ \text{deed}, \text{reed}, \text{der}, \text{redder}, \text{deer} \}$ . Prove that if L is regular, then  $L^R$  is also regular.

(b) Define

DOPPELGANGERS
$$(L) = \{ w \mid w \in L \text{ and } w \in L^R \}.$$

For example, DOPPELGANGERS $(L_3) = \{ deed, reed, deer, redder \}$ . Prove that if L is regular, then DOPPELGANGERS(L) is also regular.

(c) (Optional, not for credit) Define

$$HALF(L) = \{ w \mid ww^R \in L \}.$$

For example,  $HALF(L_3) = \{de, red\}$ . Prove that if L is regular, then HALF(L) is also regular.

## Appendix: NFA intersection

For Problem 3, it is convenient (though not essential) to use the fact that the intersection algorithm (Sipser, page 46, footnote 3) also works on NFAs. The idea is the same: the new NFA can be in state  $(r_1, r_2)$  after reading a string w iff the first NFA can be in state  $r_1$  after reading w and the second NFA can be in state  $r_2$  after reading w.

For completeness, here is the construction in more detail. Given two NFAs

Modified 2020/01/27

$$N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$$
  
 $N_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ 

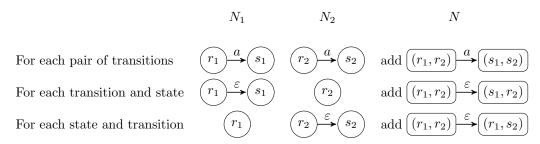
we can construct a new NFA N such that  $\mathcal{L}(N) = \mathcal{L}(N_1) \cap \mathcal{L}(N_2)$ . Let

$$N = (Q_1 \times Q_2, \Sigma, \delta, (q_1, q_2), F_1 \times F_2),$$

where  $\delta$  is defined as follows.

$$\delta((r_1, r_2), a) = \delta_1(r_1, a) \times \delta_2(r_2, a)$$
 if  $a \neq \varepsilon$   
$$\delta((r_1, r_2), \varepsilon) = (\delta_1(r_1, \varepsilon) \times \{r_2\}) \cup (\{r_1\} \times \delta_2(r_2, \varepsilon)).$$

Using state diagrams, we can express the construction of the transitions as follows:



The first case is similar to the DFA intersection construction: Both  $N_1$  and  $N_2$  read an a symbol. But the handling of  $\varepsilon$ -transitions is new: In the second case,  $N_1$  follows an  $\varepsilon$ -transition while  $N_2$  does nothing; in the third case,  $N_2$  follows an  $\varepsilon$ -transition while  $N_1$  does nothing.