Homework 2: DFAs and NFAs

CSE 30151 Spring 2018

Due 2018/02/01 at 10:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them in the library or using a smartphone to get them into PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it *`netid`*`-hw2.pdf`, where *`netid`* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it *`netid`*`-hw2-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems (10 points each)

1. **Designing finite automata** Define a natural number to include 0, and define the base-10 representation of a natural number to allow leading `0`'s as well as ε (representing 0). Write a DFA (both a formal description **and** a state diagram) for base-10 representations of natural numbers that are:

   (a) divisible by 2

   (b) divisible by 3

   (c) divisible by 4

   As an alternative to (c), you can do the following more challenging problem:

   (c′) Show, for any $k > 0$, how to construct a DFA for base-10 representations of natural numbers divisible by k. Your answer should show how to write a formal description $M = (Q, \{\texttt{0}, \ldots, \texttt{9}\}, \delta, s, F)$ in terms of k.

2. **Nondeterminism** Consider the following language:

$$L_2 = \{uv \mid u, v \in \{\texttt{a}, \texttt{b}\}^*, \text{ } u \text{ contains an even number of } \texttt{a}\text{'s, and } v \text{ contains an even number of } \texttt{b}\text{'s}\}$$

Note that as long as there is *some* way of cutting a string into u and v so as to satisfy the constraints, it's in L_2. So $\texttt{ba} \in L_2$, because $u = \texttt{b}$ has an even number (0) of a's and $v = \texttt{a}$ has an even number (0) of b's.

(a) Write an NFA N_2 that recognizes L_2.

(b) What is the accepting path for $\texttt{bab}^n$ through N_2? You can show the path for $n = 0, 1, 2, \ldots$ until the pattern is clear, or describe the general case. Either way, please write a few words about what you observe.

(c) Convert N_2 to a DFA M_2.

(d) What is the accepting path for $\texttt{bab}^n$ through M_2? Again, you can show the path for $n = 0, 1, 2, \ldots$ until the pattern is clear, or describe the general case. Either way, please write a few words about what you observe.

3. **Regular/raluger** In the following, we'll use the language L_3 as an example (but the results must be proved for all L):

$$L_3 = \{\texttt{deed}, \texttt{deer}, \texttt{red}, \texttt{redder}, \texttt{reed}\}.$$

(a) Recall that

$$L^R = \{w \mid w^R \in L\}.$$

For example, $L_3^R = \{\texttt{deed}, \texttt{reed}, \texttt{der}, \texttt{redder}, \texttt{deer}\}$. Show that if L is regular, then L^R is also regular. (That is, given a formal description or state diagram of a NFA for L, show how to construct a formal description or state diagram of a NFA for L^R.)

(b) Define

$$\text{DOPPELGANGERS}(L) = \{w \mid w \in L \text{ and } w \in L^R\}.$$

For example, $\text{DOPPELGANGERS}(L_3) = \{\texttt{deed}, \texttt{reed}, \texttt{deer}, \texttt{redder}\}$. Show that if L is regular, then $\text{DOPPELGANGERS}(L)$ is also regular.

(c) Define

$$\text{HALF}(L) = \{w \mid ww^R \in L\}.$$

For example, $\text{HALF}(L_3) = \{\texttt{de}, \texttt{red}\}$. Show that if L is regular, then $\text{HALF}(L)$ is also regular. Hint: The NFA for $\text{HALF}(L)$ differs from the NFA for $\text{DOPPELGANGERS}(L)$ only in the choice of accept states.