Homework 4: CFGs and PDAs

CSE 30151 Spring 2018

Due Thursday, 2018/03/01 at 10:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them in the library or using a smartphone to get them into PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw4.pdf`, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw4-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems

Each problem is worth 10 points. Wherever you are asked to write a CFG, don't forget to specify the start symbol if it is not S. Wherever you are asked to write a PDA, either a formal description or a state diagram is fine. It will be helpful both to you and the grader to preface a PDA with an *informal* description.

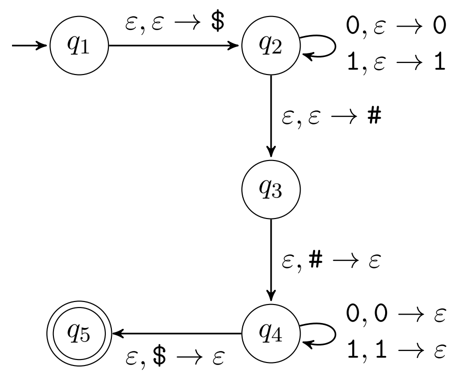
1. **Designing CFGs and PDAs.** For each of the following languages, write a context-free grammar that generates it *and* a pushdown automaton that recognizes it.

   (a) $\{\mathtt{0}^m\mathtt{1}^n\mathtt{0}^n\mathtt{1}^m \mid m, n \geq 0\}$

   (b) [Exercise 2.6b] The complement of $\{\mathtt{0}^n\mathtt{1}^n \mid n \geq 0\}$. Hint: First show that this is equal to $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq n\} \cup \{w \mid w \text{ contains } \mathtt{10}\}$.

2. **There and back again.** Consider the following CFG and PDA, which both recognize the language $\{ww^R \mid w \in \{\texttt{0},\texttt{1}\}^*\}$:

$$S \to \texttt{0}S\texttt{0}$$
$$S \to \texttt{1}S\texttt{1}$$
$$S \to \varepsilon$$



(The PDA is the same as M_3 in Example 2.18, but modified to meet the requirements of the proof of Lemma 2.27.)

(a) Convert the CFG to a PDA using the construction in the proof of Lemma 2.21. Briefly write down your observations about whether the resulting PDA looks like the one above.

(b) Convert the PDA to a CFG using the construction in the proof of Lemma 2.27. You don't need to include useless nonterminals (those that cannot be used in any complete derivation). Briefly write down your observations about whether the resulting CFG looks like the one above.

3. **Surprisingly context-free** [Problem 2.23]. Let $\Sigma = \{\texttt{0},\texttt{1}\}$. In Week 7, we will see that the language $\{xx \mid x \in \Sigma^*\}$ is *not* context-free. In this problem, you will prove that the following language *is* context-free:

$$D = \{xy \mid xy \in \Sigma^*, |x| = |y|, x \neq y\}.$$

That is, strings of even length where the first and second halves are different. We can prove this in three steps. Consider this language over Σ:

$$D' = \{uavwbz \mid uavwbz \in \Sigma^*, |u| = |v|, |w| = |z|, a \neq b\}.$$

That is, strings that can be cut into two odd-length pieces (uav and wbz) that have different middle symbols (a and b).

(a) Prove that $D \subseteq D'$. That is, for any $s = xy$ such that $|x| = |y|, x \neq y$, there is a way to write $s = uavwbz$ such that $|u| = |v|, |w| = |z|, a \neq b$.

(b) Prove that $D \supseteq D'$. That is, for any $s = uavwbz$ such that $|u| = |v|, |w| = |z|, a \neq b$, there is a way to write $s = xy$ such that $|x| = |y|, x \neq y$.

(c) Prove that D' (and therefore D) is context-free by writing either a context-free grammar or a pushdown automaton for D'.