Homework 5: Non-context-free languages

CSE 30151 Spring 2020

Due Friday, 2020/03/20 at 5:00pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them in the library or using a smartphone to get them into PDF.

- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:

  - If you're making a complete submission, name it *netid*-`hw5.pdf`, where *netid* is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it *netid*-`hw4-123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.

- Submit your PDF file(s) in Sakai. Don't forget to click the Submit button!

Problems

1. **Pumping lemma for CFLs**. For each of the following languages, use the pumping lemma to show that it is not context free.

   (a) [Problem 2.31]
   $$B = \{w \in \{0, 1\}^* \mid w = w^R \text{ and } w \text{ has an equal number of 0s and 1s}\}.$$

   (b) [cf. HW3, Q2b] The language over $\Sigma = \{0, 1, +, =\}$:
   $$ADD = \{x\texttt{=}y\texttt{+}z \mid x,y,z \text{ are binary natural numbers, and } x = y + z\}.$$

2. **The SCRAMBLE operation** [Problem 2.43]. If w and w' are strings over an alphabet Σ, define the relation $w \overset{\circ}{=} w'$ to be true iff w' is a permutation of

w, that is, they have the same number of each type of symbol, but possibly in a different order. If w is a string and L is a language, define

$$\text{SCRAMBLE}(w) = \{w' \mid w' \overset{\circ}{=} w\}$$
$$\text{SCRAMBLE}(L) = \bigcup_{w \in L} \text{SCRAMBLE}(w).$$

For example,

$$\text{SCRAMBLE}(0^*1^*) = \{0,1\}^*$$
$$\text{SCRAMBLE}((01)^*) = \{w \in \{0,1\}^* \mid w \text{ has an equal number of 0s and 1s}\}.$$

(a) Let $\Sigma = \{a, b\}$. Prove that for any regular language L over alphabet Σ, $\text{SCRAMBLE}(L)$ is context-free. (A hint or two will be posted on Piazza.)

(b) Let $\Sigma = \{a, b, c\}$. Prove that there exists a regular language L over alphabet Σ such that $\text{SCRAMBLE}(L)$ is *not* context-free.

3. **Non-closure properties**

   (a) [Exercise 2.2a] Use the languages

   $$A = \{a^m b^n c^n \mid m, n \geq 0\}$$
   $$B = \{a^n b^n c^m \mid m, n \geq 0\}$$

   to prove that context-free languages are *not* closed under intersection.

   (b) [Exercise 2.2b] Use (a) and DeMorgan's law to prove that context-free languages are *not* closed under complementation.