

Homework 2: DFAs and NFAs

Theory of Computing (CSE 30151), Spring 2024

Due 2024-02-02 5pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
 - If you're making a complete submission, name it *netid-hw2.pdf*, where *netid* is replaced with your NetID.
 - If you're submitting some problems now and want to submit other problems later, name it *netid-hw2-123.pdf*, where 123 is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Canvas.

Problems (10 points each)

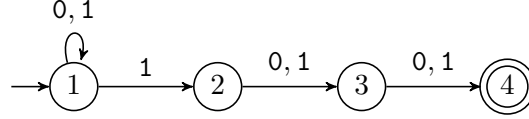
1. **Divisibility tests.** Define, for all $k > 0$,

$$D_k = \{w \in \{0, \dots, 9\}^* \mid w \text{ is the decimal representation of a multiple of } k\}$$

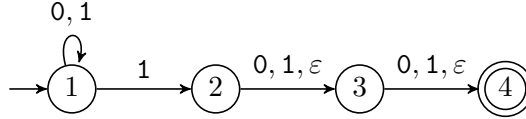
where ε is considered to represent the number 0. For example, the strings ε , 0, 88, and 088 all belong to D_2 , but 99 and 099 do not.

- (a) Prove that D_2 is regular by writing a DFA for D_2 .
 - (b) Prove that D_3 is regular by writing a DFA for D_3 .
 - (c) Prove that D_6 is regular. An explicit DFA is not necessary.
- (*) Optional alternative: You can get full credit for all of the above if you can prove that for any $k > 0$, D_k is regular, by describing how to write the formal description of a DFA $M = (Q, \{0, \dots, 9\}, \delta, s, F)$ in terms of k .

2. **Nondeterminism.** Consider the following NFA N_2 (same as in Figure 1.31), which accepts a string iff the third-to-last symbol is a 1:



- (a) Use the subset construction (Theorem 1.39) to convert N_2 to a DFA M . You may omit curly braces and commas when naming states; for example, instead of $\{1, 2, 3, 4\}$ you may write 1234. (Hint: the DFA should be equivalent to the one in Figure 1.32.)
- (b) Why are the states in Figure 1.32 named q_{abc} where $a, b, c \in \{0, 1\}$?
- (c) In Example 1.30, Sipser asks what happens if you modify N_2 into the following NFA – let's call it N'_2 :



Describe in English what language N'_2 recognizes.

- (d) Use the subset construction (Theorem 1.39) to convert N'_2 to a DFA M' .
3. **Procrustean closure properties.** Let Σ be an alphabet, and let $L_3 = \{\text{theory, of, computing}\}$ be an example language.

- (a) For any $w = w_1w_2 \cdots w_{n-1}w_n$, define

$$\text{STRETCH}(w_1w_2 \cdots w_n) = w_1w_1w_2w_2 \cdots w_{n-1}w_{n-1}w_nw_n.$$

This induces an operation on languages,

$$\text{STRETCH}(L) = \{\text{STRETCH}(w) \mid w \in L\}.$$

For example,

$$\text{STRETCH}(L_3) = \{\text{tthheeorrryy, ooff, ccoommppputtiinngg}\}.$$

Prove that if L is a regular language, then $\text{STRETCH}(L)$ is also regular.

- (b) For any $w = w_1w_2 \cdots w_{n-1}w_n$ with $n \geq 2$, define

$$\text{CHOP}(w_1w_2 \cdots w_{n-1}w_n) = w_2 \cdots w_{n-1}.$$

This induces an operation on languages,

$$\text{CHOP} = \{\text{CHOP}(w) \mid w \in L \text{ and } |w| \geq 2\}.$$

For example,

$$\text{CHOP}(L_3) = \{\text{heor, } \varepsilon, \text{omputin}\}.$$

Prove that if L is a regular language, then $\text{CHOP}(L)$ is also regular.