Homework 2: DFAs and NFAs

Theory of Computing (CSE 30151), Spring 2025

Due 2025-01-31 5pm

Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you're making a complete submission, name it `netid-hw2.pdf`, where `netid` is replaced with your NetID.
  - If you're submitting some problems now and want to submit other problems later, name it `netid-hw2-part123.pdf`, where `123` is replaced with the problem numbers you are submitting at this time.
- Submit your PDF file(s) in Canvas.

Problems (10 points each)

1. **Divisibility tests.** Define, for all $k > 0$,

$$D_k = \{w \in \{\mathtt{0}, \ldots, \mathtt{9}\}^* \mid w \text{ is the decimal representation of a multiple of } k\}$$

where ε is considered to represent the number 0. For example, the strings ε, `0`, `1234`, and `01234` all belong to D_2, but `99` and `099` do not.

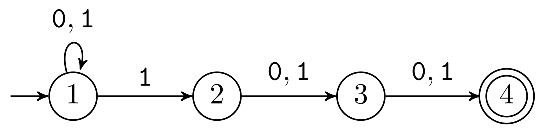
(a) Prove that D_2 is regular by writing a DFA for D_2.

(b) Prove that D_3 is regular by writing a DFA for D_3.

(c) Prove that D_6 is regular. An explicit DFA is not necessary.

(∗) Optional alternative: You can get full credit for all of the above if you can prove that for any $k > 0$, D_k is regular, by describing how to write the formal description of a DFA $M = (Q, \{\mathtt{0}, \ldots, \mathtt{9}\}, \delta, s, F)$ in terms of k.

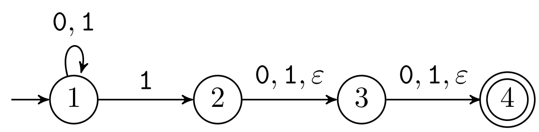
2. **Nondeterminism.** Consider the following NFA N_2 (same as in Figure 1.31), which accepts a string iff the third-to-last symbol is a `1`:



   (a) Use the subset construction (Theorem 1.39) to convert N_2 to a DFA M. You may omit curly braces and commas when naming states; for example, instead of $\{1, 2, 3, 4\}$ you may write 1234. (Hint: the DFA should be equivalent to the one in Figure 1.32.)

   (b) Why do you think the states in Figure 1.32 are named the way they are?

   (c) In Example 1.30, Sipser asks what happens if you modify N_2 into the following NFA – let's call it N_2':



   Use the subset construction (Theorem 1.39) to convert N_2' to a DFA M'.

3. **Procrustean closure properties.**

   (a) For any string $w = w_1 w_2 \cdots w_{n-1} w_n$, define

$$\text{STRETCH}(w_1 w_2 \cdots w_{n-1} w_n) = w_1 w_1 w_2 w_2 \cdots w_{n-1} w_{n-1} w_n w_n.$$

   For example,

$$\text{STRETCH}(\varepsilon) = \varepsilon$$
$$\text{STRETCH}(\texttt{abab}) = \texttt{aabbaabb}.$$

   This induces an operation on languages,

$$\text{STRETCH}(L) = \{\text{STRETCH}(w) \mid w \in L\}.$$

   For example,

$$\text{STRETCH}(\{\varepsilon, \texttt{abab}\}) = \{\varepsilon, \texttt{aabbaabb}\}.$$

   Prove that for any regular language L, STRETCH(L) is also regular.

(b) For any $w = w_1 w_2 \cdots w_{n-1} w_n$ with $n \geq 2$, define

$$\text{CHOP}(w_1 w_2 \cdots w_{n-1} w_n) = w_2 \cdots w_{n-1}.$$

For example,

$$\begin{aligned} \text{CHOP}(\texttt{ab}) &= \varepsilon \\ \text{CHOP}(\texttt{abab}) &= \texttt{ba} \\ \text{CHOP}(\texttt{bbaa}) &= \texttt{ba}. \end{aligned}$$

This induces an operation on languages,

$$\text{CHOP} = \{\text{CHOP}(w) \mid w \in L \text{ and } |w| \geq 2\}.$$

For example,

$$\begin{aligned} \text{CHOP}(\{\texttt{ab}, \texttt{abab}, \texttt{bbaa}\}) &= \{\varepsilon, \texttt{ba}\} \\ \text{CHOP}(\{\varepsilon, \texttt{a}\}) &= \emptyset. \end{aligned}$$

Prove that for any regular language L, $\text{CHOP}(L)$ is also regular.