

# Homework 4: Context-free languages

Theory of Computing (CSE 30151), Spring 2026

Due: 2026-02-20 5pm

## Instructions

- Create a PDF file (or files) containing your solutions. You can write your solutions by hand, but please scan them into a PDF.
- Please name your PDF file(s) as follows to ensure that the graders give you credit for all of your work:
  - If you’re making a complete submission, name it `netid-hw4.pdf`, where `netid` is replaced with your NetID.
  - If you’re submitting some problems now and want to submit other problems later, name it `netid-hw4-part123.pdf`, where 123 is replaced with the problem number(s) you are submitting at this time.
- Submit your PDF file(s) in Canvas.

## Problems (10 points each)

1. **Arithmetic expressions.** Consider the grammar  $G_4$  (page 105) for arithmetic expressions, with start symbol  $E$ :

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T * F \mid F \\ F &\rightarrow (E) \mid a \mid b \mid c \end{aligned}$$

- [cf. Exercise 2.1] Give derivations for the following strings. You may write them either as a sequence of rewrites ( $E \Rightarrow \dots$ ) or as a tree.
  - $a + b + c$
  - $a * b + c$
  - $a * (b + c)$

(b) Modify  $G_4$  to allow an exponentiation operator  $\uparrow$ .

- It should have *higher precedence* than multiplication; that is, in the derivation of the string  $a * b \uparrow c$ , there should be a nonterminal that rewrites to  $b \uparrow c$ , and there should not be a nonterminal that rewrites to  $a * b$ .
- It should be (unlike  $*$  and  $+$ ) *right-associative*; that is, in the derivation of the string  $a \uparrow b \uparrow c$ , there should be a nonterminal that rewrites to  $b \uparrow c$ , and there should not be a nonterminal that rewrites to  $a \uparrow b$ .

2. Prove that the following language (cf. page 80) is context-free by writing either a PDA *or* a CFG for it:

$$C = \{w \in \{0, 1\}^* \mid w \text{ has an equal number of } 0\text{s and } 1\text{s}\}.$$

Please include a brief explanation of why your PDA or CFG works.

3. [Exercise 2.6b] Prove that the following language is context-free by writing either a PDA *or* a CFG for it:

$$L_3 = \overline{\{0^n 1^n \mid n \geq 0\}}.$$

For example,  $000111 \notin L_3$ . Please include a brief explanation of why your PDA or CFG works.

Hint: First prove that this is equal to  $\{0^m 1^n \mid m \neq n\} \cup \overline{0^* 1^*}$ .