GSS Talk Abstract

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1 Topic

The topic of the talk is about the Hilbert Scheme of points in $\mathbb{C}^d$, $Hilb^n(\mathbb{C}^d)$. Very little is known for d large (here large means $d \geq 4$) and n arbitrary. When $d = 1, 2$ there is a complete description of the Hilbert scheme ($Hilb^n(\mathbb{C}) = S^n\mathbb{C}$, the n^{th} symmetric product, and $Hilb^n(\mathbb{C}^2)$ has dimension $2n$ and it is a resolution of singularities of $S^n\mathbb{C}^2$ [Fogarty]). It was recently shown that when $d = 3$, eventhough $Hilb^n(\mathbb{C}^3)$ is singular in general ($n \geq 3$), things are not too bad. It turns out that by generalizing a Theorem of Nakajima for the $d = 2$ case, we can realize $Hilb^n(\mathbb{C}^3)$ as a scheme-theoretic degeneracy locus of a regular function on a smooth quasi-projective variety. The main goal of the talk is to explain this result ($d = 3$ case).

Even though the word scheme appears in most of the above sentences (and in the title as well!), it will not be mentioned (not in a significant way at least) after the first 5 or 6 minutes of the talk. Knowledge of basic algebra (mainly Ring theory), linear algebra, (maybe basic Algebraic Geometry, like the notion of variety, and morphisms between them) will be enough to follow most of the talk. If time permits, I will survey some elements of (Geometric) Invariant Theory (GIT), since after all, the smooth quasi-projective variety that I claimed above is a GIT quotient!

2 References

For the above question ($d = 3$ case) the main reference is the paper Motivic degree zero Donaldson-Thomas Invariants by Kai Behrend, Jim Bryan and Balázs Szendrői. In particular, section 3.2 of the paper: The Hilbert scheme of $\mathbb{C}^3$ as critical locus.
Reference to the paper: Inventiones mathematicae , April 2013, Volume 192, Issue 1, pp 111-160

It can be found also in the arXiv, but the section numbers are different. In the arXiv version, the section would be 2.2.
The reference is: arXiv: 0909.5088v2