ORIENTATION WEEK RESEARCH DOCUMENT

Algebra

- **Katrina Barron**'s research focuses on vertex operator algebras and the algebraic and geometric foundations of conformal field theory. Conformal field theory (CFT), or more specifically, string theory, is an attempt at developing a physical theory that combines all fundamental interactions of particles, including gravity. Her research mainly involves the algebras (vertex operator algebras and related structures) that are governed by colliding particles in CFT modeled as vibrating loops or "strings". In addition to being a physical model for particle interactions, these vertex operator algebras have important and deep links to the theory of finite simple groups, number theory, topology, the theory of Lie algebras and Lie groups, and the corresponding symmetries of these and many other mathematical structures.
- **Matthew Dyer**'s research is focused on the study of Coxeter groups and their root systems, and their connections with and applications to associated geometric, algebraic and combinatorial structures. The finite Coxeter groups are precisely the finite groups generated by reflections in real Euclidean spaces, and include the finite dihedral and symmetric groups, and the symmetry groups of regular convex polytopes (such as the Platonic solids in dimension three). General Coxeter groups have realizations as discrete reflection groups and may be defined abstractly as certain free products, with amalgamation, of dihedral groups. In applications, Coxeter groups are often naturally associated with root systems, which in their most concrete form are sets of vectors specifying (e.g. normal to) the reflecting hyperplanes. A root system is said to be crystallographic if, roughly, it determines a reflection group defined on a free abelian group instead of just on a real vector space. Coxeter groups associated to crystallographic root systems include the finite and affine Weyl groups, and play a significant role in many areas of mathematics, notably in Lie theory (in describing the classification, structure and representations of semisimple Lie and algebraic groups and Lie algebras, Kac-Moody groups and Lie algebras, quantum groups, finite simple groups of Lie type etc) but also in singularity theory, geometric group theory, representation theory of finite-dimensional algebras, knot theory etc.
- **Sam Evens**: works on geometry and representation theory of reductive Lie groups and their Lie algebras. These groups include matrix groups, consisting of symmetries, and their Lie algebras are linearizations at the identity. Most recently, he has studied the cohomology rings of flag varieties and other homogeneous spaces, as well as integrable systems on Lie algebras. These methods use the geometry of Poisson structures in an essential way, as well as the DeConcini-Procesi compactifications of symmetric spaces, and ideas from algebraic geometry. He has also studied representation theory of reductive groups and its relationship with perverse sheaves and geometry of orbit closures.
- **Michael Gekhtman**'s research mainly focuses on algebraic structures arising in the study of exactly solvable classical and quantum mechanical systems. This includes Poisson brackets and invariant objects on Lie algebras, Lie groups and their homogeneous spaces and, in past several years, the theory of cluster algebras - a rather new field (founded in 2001 by Fomin and Zelevinsky) that proved useful and relevant in many areas of mathematics, including combinatorics, representation theory, hyperbolic geometry and mirror symmetry.
- **Andrei Jorza**'s research interests lie at the intersections of algebraic geometry, representation theory and number theory. For the past few decades number theory research has

been guided by the Langlands program which seeks to identify relationships between the analytic objects of representation theory of matrix groups and the arithmetic objects of Galois theory. Such deep relationships have been revealed using algebraic geometry as certain algebraic varieties contain information about both analysis and arithmetic. Andrei has mostly been dealing with degenerate situations where such algebraic varieties are not available. In such settings one can still use deformation theory to uncover deep arithmetic properties of analytic objects.

– **Juan Migliore** main research to date has been on liaison theory, a specialty within the broader area of algebraic geometry and commutative algebra. He began by studying the case of codimension two subschemes, and the high point of that early part of my work is probably the paper listed first below. More recently, He has been very much interested in questions in higher codimension, and in the more elusive case of Gorenstein liaison. While still quite interested in liaison theory, over the last decade his research has shifted somewhat. An important theme has been the study of Hilbert functions of various algebras, as well as their graded Betti numbers. In particular, He has studied the cases of level (possibly monomial) algebras, algebras defined by ideals of generic forms, and Gorenstein algebras. A related topic is the so-called Weak Lefschetz Property for artinian algebras.
– **Claudia Polini**[1] is particularly interested in the theory of linkage and residual intersections; blow-up algebras, such as Rees algebras, symmetric algebras, special fiber rings, and associated graded rings; objects associated to an ideal such as its integral closure, its core and its reductions; and Hilbert functions.
– **Claudiu Raicu**'s research interests lie at the confluence of algebraic geometry, commutative algebra, representation theory and combinatorics. He is particularly concerned with classical algebraic geometry topics that arise in a representation-theoretic context, such as the study of determinantal varieties and their generalizations, syzygies (or other cohomological invariants such as local cohomology, Ext modules) of algebraic varieties, or defining ideals for secant and tangential varieties.

Analysis and PDE's

– **Matthew Gurskey**'s research is in the area of geometric analysis. This is a broad field, and as the name suggests, it encompasses all problems which use the ideas and techniques of mathematical analysis (especially differential equations) to solve problems which are geometric in origin, or that can be formulated in geometric terms. One of the simplest examples is the study of soap bubbles. If you take a loop made of wire and dip it in soapy water, a soap bubble will span the loop. Using physical arguments one can show that this thin film describes a minimal surface: it is the shape that spans the loop with the least amount of area. With some basic differential geometry one can rephrase this fact as a condition on the curvature of this film (more precisely, the mean curvature). Then, if one makes some additional assumptions, it can be described purely as a differential equation. Understanding questions such as the existence of solutions to this equation, the smoothness of the solutions, etc., has had a profound influence on the development of the theory of nonlinear partial differential equations during the 20th century. And remarkably, the same equations describe fairly diverse phenomena, such as black holes in cosmology. In my current work I study problems in higher dimensions, and one is often lead to the study of systems of nonlinear equations. But like the example of soap bubbles, these problems always involves the interaction of geometry and analysis.
– **Brian Hall**'s research is in mathematical physics and involves Lie groups, functional analysis, probability, and geometry. Specifically, he studies generalizations of the Segal-Bargmann transform. The idea is as follows. In classical mechanics, one typically has a "configuration

[1]This summary has been obtained from her website and may be outdated

space," which is some manifold M. One then has the associated "phase space," which is the cotangent bundle of M. The cotangent bundle arises because Newton's equations are second order in time: a second-order equation on M becomes a first-order equation on the cotangent bundle of M. In the corresponding quantum system, one tries to construct a Hilbert space associated to the classical system. The simplest such space is the "position Hilbert space," which is a the space of square-integrable functions over M with respect to some measure. Alternatively, one can look for some nice complex structure on the cotangent bundle of M and then build a space of square-integrable holomorphic function on the cotangent bundle, again with respect to some (hopefully natural) measure. Such a space is called a (generalized) Segal-Bargmann space. A natural unitary map between the position Hilbert space and the Segal-Bargmann space is called a Segal-Bargmann transform. Associated to such a Segal-Bargmann transform is a collection of special quantum states called "coherent states."

– **Alex Himonas** works in the field of Partial Differential Equations (PDE) studying fundamental questions like existence and uniqueness of solution, stability, and regularity properties. Many of these PDE arise in mathematical physics and some model finance and economics situations. The techniques used are from harmonic analysis and analysis in general. In recent years I have been interested in the study of the initial and the initial-boundary problems for evolution equations. These include water wave equations like the Korteweg-de Vries (KdV) and the Camassa-Holm (CH) equations, which can be thought as one-dimensional models for the Euler equations of fluid dynamics, as well as the Boussinesq and the nonlinear Schrodinger (NLS) equations. For such equations, I have been studying the initial value problem in low (Sobolev) and high (analytic) regularity data spaces. Also, I have been studying the initial-boundary value problem with data in Sobolev spaces using the Unified Transform Method and a contraction mapping argument.
– **Gerard Misiolek** works in global analysis and partial differential equations with applications to fluid dynamics. He also works with incompressible Euler equations and well-posedness of nonlinear PDE, Riemannian geometry of diffeomorphism groups, nonlinear functional analysis, harmonic analysis, and infinite-dimensional hamiltonian systems.
– **Gabor Szekelyhidi** is interested in geometric analysis, which means studying geometric problems using analysis and partial differential equations. He is particularly interested in questions at the interface of complex algebraic geometry and Riemannian geometry. A famous such problem, which was resolved very recently is the existence of Kahler-Einstein metrics, and I am very interested in further developing these results, and finding applications. The ideas involved can be very algebraic, such as studying how algebraic varieties with certain properties can degenerate to other varieties, or very analytic, such as finding a priori estimates for a certain PDE, or usually something in between.

Coding Theory, Combinatorics and Number Theory

– **David Galvin** uses techniques from information theory, probability and combinatorics to study structural, enumerative and algorithmic aspects of graph homomorphisms and related models. A graph homomorphism is essentially a coloring of the vertices of a graph, with restrictions on which pairs of colors are allowed to appear on adjacent vertices. If the restriction is that adjacent vertices must receive distinct colors, we get the prototypical example, namely ordinary graph coloring, the realm, for example, of the famous 4-color conjecture/theorem. With more complicated restrictions, graph homomorphisms can be used to encode many other graph theory notions. In statistical physics, graph homomorphisms provide a natural language for the study of an important class of models, the hard-constraint spin systems, and they also arise naturally in the study of communication networks.
Broadly, the main questions I answer fall into three classes:

Structural: describing the typical appearance of a randomly chosen homomorphism, a question which relates directly to the important ones of Gibbs measures in statistical physics models and spatial unfairness in communication networks.

Algorithmic: studying how efficiently one may sample from the space of homomorphisms, a significant question from a theoretical computer science perspective.

Enumerative: counting or asymptotically estimating the number of elements in a space of homomorphisms, a source of many fascinating problems and a burgeoning research area in discrete mathematics.

- **Roxana Smarandache** is in coding theory. It focuses on theoretical questions that stem from practical applications (like improving the communication over different types of channels, for example in cell phone or computer networks). She mostly uses algebra, combinatorics, and probability in her research.
  Concrete topics about her research are: approximating the permanent of a matrix (unlike computing the determinant, computing the permanent is hard, in complexity), studying parameters of interest of certain graphs covers of a bipartite graph, using linear programming (modified simplex algorithm) to decode codes (and thus to transmit reliably over a channel), and designing and analyzing vector spaces (codes) with certain characteristics desired in practice.

Complex Analys/Dynamical System

- **Jeff Diller**'s main area of interest is complex (as in complex numbers) dynamical systems. The goals are dynamical: he starts with a (usually quite simple) map $f : X \to X$ from a space to itself and wants to understand the asymptotic behavior of orbits $p, f(p), f(f(p))$, $f(f(f(p)))$ of points p in X. In his case, the space X is always a complex manifold (e.g. the projective plane), and the map f is defined by polynomials. This allows him to use tools from complex analysis, potential theory and algebraic geometry in addition to more standard dynamical systems methods in order to understand the dynamics of the maps.
- **Richard Hind**'s research is in symplectic geometry. This studies mechanical systems, which are specified by Hamiltonian functions on a phase space. The goal is to describe quantitatively which dynamical systems can arise in this way. A famous example is Arnold's conjecture which gives lower bounds on the number of fixed points. He has studied the embedding question: if we start with a region in the phase space, how small a ball can it be mapped into? Using techniques from analysis, geometry and complex variables it is remarkable that we are sometimes able to give sharp bounds.
- **Mei-Chi Shaw**'s research interests are in several complex variables, partial differential equations and complex geometry. She is currently working on the closed-range property of the Cauchy-Riemann operator in complex manifolds. One of her goals is to understand how the presence of positive or negative curvature will influence solutions to the Cauchy-Riemann equations and function theory on complex manifolds.
- **Nancy Stanton**'s research interests are several complex variables, partial differential equations and differential geometry. The topics she has worked on include the spectral geometry of complex manifolds with boundaries and of CR manifolds and the local geometry of CR manifolds.

Logic

- **Peter Cholak** works in computability theory. In particular, he focuses on the relationship between computability and definability. A classical example is Posts result that the set of integers accepted by a Turing machine is a Σ^0_1 definable set in arithmetic. Another example is the result of Cholak and his coauthors that the question of whether two c.e. sets are in the same orbit in the structure of c.e. sets is Π^1_1 complete. Cholak has also spent a lot of time and energy exploring what is possible in the orbits of c.e. sets. The answer is that basically anything is possible. Some other examples of the relationship between computability and definability from Cholak and his co? authors include: Every computable two coloring of pairs of integers has a low$_2$ homogenous set. Then translated to reverse mathematics this implies the Ramsey Theorem for pairs and two colors is Π^1_1 conservative over RCA_0 plus induction for Σ^0_2 formulas. A recent result shows that every effectively prime models are also effectively atomic. The best way to learn more about Cholak mathematical interests is to explore his papers and talks which can be found on his personal website and on MathSciNet.

– **Julia Knight**'s work is on recursion-theoretic complexity of structures of familiar kinds—groups, fields, etc. She considers questions of the following kinds: When does a structure have a computable copy? How hard is it to describe a particular structure, up to isomorphism? Which of two kinds of structures is harder to classify? Many of the results on complexity are obtained using formulas of special forms. She and Uri Andrews have results on theories with very nice model theoretic properties, saying when all of the countable models have computable copies. She and Karen Lange have some purely algebraic results on lengths of roots of polynomials in fields of generalized power series. Computability normally deals with countable objects. There is some recent work on uncountable structures. Together with Noah Schweber and Greg Igusa, they have results comparing the computing power of various structures related to the ordered field of real numbers.
– **Annand Pillay** works in model theory, a branch of mathematical logic, as well as its connections with and applications to other parts of mathematics. Model theory is an abstract part of mathematics, analogous to algebra and category theory. It is concerned not only with abstract structures (e.g. groups, rings, linear orderings), but also with formal languages in which facts about these structures can be expressed. Among the connections and applications that he is currently working on are: functional transcendence, differential equations, topological dynamics, Lie groups, diophantine geometry, geometric group theory.
– **Sergei Starchenko**[2] is interested primarily in applications of logic specifically, model theory to real analytic geometry, geometric measure theory, asymptotic analysis and questions of differentiability and analyticity of real functions. His research involves studying sets and functions that are definable in "well-behaved" (e.g., o-minimal) first-order structures on the field of real numbers. Determining which structures should be regarded as well behaved is part of the job.

Topology and Geometry

– **Mark Behrens** studies algebraic topology. Specifically, He is interested in computational aspects of the field, such as the homotopy groups of spheres, and the structure within these that can be gleaned from certain moduli spaces arising in algebraic geometry.

The insights of Quillen and Morava, as developed computationally by Miller, Ravenel and Wilson, and conceptually through the nilpotence and periodicity theorems of Devinatz, Hopkins, and Smith, show that structurally the stable homotopy category is closely tied to the moduli space of commutative 1-dimensional formal groups. In particular, the stable homotopy groups of spheres (as well as those of any finite complex) are composed of periodic layers, reflecting the stratification of the moduli space of formal groups by height. The first periodic layer is well understood, using K theory. Much of his research has been devoted to understanding the second periodic layer using the theory of Topological Modular Forms (TMF). With Tyler Lawson, I have also begun to investigate how cohomology theories of Topological Automorphic Forms can be used to study the higher periodic layers. He is also interested in how stable homotopy can be used to understand unstable phenomena (e.g. unstable homotopy groups of spheres) via Goodwillie's calculus of functors.

[2]This summary has been obtained from his website and may be outdated

- **Karsten Grove** works in metric and differential geometry, topology and geometric analysis.
- **Liviu Nicolaescu** is currently investigating random Morse functions. On a compact Riemann manifold (M, g) we can define a random function to be a linear combination of eigenfunctions of the Laplacian, where the coefficients are independent random variables, and the eigenfunctions in this superposition correspond to eigenvalues below a certain threshold L. Intuitively, as we let $L \to \infty$, we sample the whole space $C^\infty(M)$. For large L this random function is almost surely Morse. Its sets of critical points and critical values can be viewed as random measures on M and respectively $\mathbb{R}$. We are interested in the large L statistics of these random measures.
- **Stephan Stolz**[3] is interested in geometric and algebraic topology, and differential geometry.

Missing research summaries

We have been unable to gather a research summary from these professors but you can always contact them to talk about their research: **Leonid Faybusovich, Alexander Hahn, Qing Han, Arthur Lim, Sonja Mapes, Anne Pilkington, Brian Smyth, Dennis Snow, and Laurence Taylor.**

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