

GSS Talk for November 10, 2008  
Speaker: Steven Broad

### **Umbilics, Lines of curvature and a conjecture of Carathéodory**

We will introduce umbilics and discuss the structure of the lines of curvature around generic isolated umbilics. We will also discuss a conjecture attributed to Carathéodory which states that a smooth, convex embedding of the 2-sphere into  $\mathbb{R}^3$  has at least two umbilics. Finally, we will discuss the deep connection between the Carathéodory conjecture and Loewner's conjecture about the index of the vector field  $\partial_{\bar{z}}^2 f$  for  $C^2$  functions  $f : \{|z| \leq 1\} \rightarrow \mathbb{R}$ .