WELL-POSEDNESS OF THE CAMASSA-HOLM EQUATION ON THE TORUS

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We consider the Cauchy problem for the Camassa Holm equation

$$\partial_t u + u\partial_x u + (1-\partial_x^2)^{-1}\partial_x \left[u^2 + (\partial_x u)^2\right] = 0, \quad x \in \mathbb{T}, \quad t \in \mathbb{R}, \tag{0.1}$$

$$u(x,0) = u_0(x), \tag{0.2}$$

and prove the following result.

Theorem 1. *If $u_0(x) \in H^s(\mathbb{T})$ for some $s > 3/2$, then there is a $T > 0$ depending only on $\|u_0\|_{H^s}$, such that there exists a unique function $u(x,t)$ solving the Cauchy problem (0.1)–(0.2) in the sense of distributions with $u \in C([0,T]; H^s)$. The solution u depends continuously on the initial data u_0 in the sense that the mapping of the initial data to the solution is continuous from the Sobolev space H^s to the space $C([0,T]; H^s)$. Furthermore, the lifespan (the maximal existence time) is greater than*

$$T \doteq \frac{1}{2c_s}\frac{1}{\|u_0\|_{H^s(\mathbb{T})}}, \tag{0.3}$$

where c_s is a constant depending only on s. Also, we have

$$\|u(t)\|_{H^s(\mathbb{T}))} \leq 2\|u_0\|_{H^s(\mathbb{T})}, \quad 0 \leq t \leq T. \tag{0.4}$$

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