CORRECTIONS des Exercices de Révisions de Logique Combinatoire

A). Algèbre de BOOLE :

I). Exercice 1 :

Simplifiez algébriquement les fonctions suivantes :

$$F1 = a.b.\overline{c} + \overline{a}.\overline{b}.c + a.\overline{b}.\overline{c} + a.\overline{b}.c$$
$$F1 = a.\overline{c}.(b + \overline{b}) + \overline{b}.c.(a + \overline{a})$$
$$F1 = a.\overline{c} + \overline{b}.c$$

$$F2 = a.b.c + \overline{a}.b.c + \overline{a}.\overline{b}.c + a.b.\overline{c}$$
$$F2 = a.b.(c + \overline{c}) + \overline{a}.c(b + \overline{b})$$
$$F2 = a.b + \overline{a}.c$$

$$F3 = \overline{a}.\overline{b}.c.\overline{d} + \overline{a}.b.\overline{c}.d + a.\overline{b}.c.\overline{d} + \overline{a}.\overline{b}.\overline{c}.\overline{d} + a.b.\overline{c}.d + \overline{a}.b.c.d + a.\overline{b}.\overline{c}.\overline{d}$$
$$F3 = \overline{b}.\overline{d}.(\overline{a}.c + a.\overline{c} + a.c + \overline{a}.\overline{c}) + a.b.\overline{c}.d + \overline{a}.b.d.(c + \overline{c})$$
$$F3 = \overline{b}.\overline{d} + a.b.\overline{c}.d + \overline{a}.b.d$$
$$F3 = \overline{b}.\overline{d} + b.d.(a.\overline{c} + \overline{a})$$
$$F3 = \overline{b}.\overline{d} + b.d.(\overline{c} + \overline{a})$$
$$F3 = \overline{b}.\overline{d} + \overline{a}.b.d + b.\overline{c}.d$$

$$F4 = \overline{a}.\overline{b}.\overline{c}.\overline{d} + \overline{a}.\overline{b}.\overline{c}.d + \overline{a}.\overline{b}.c.d + \overline{a}.\overline{b}.c.\overline{d} + a.b.\overline{c}.d + a.b.c.d +$$
$$a.\overline{b}.\overline{c}.d + a.\overline{b}.c.d + a.\overline{b}.c.\overline{d}$$
$$F4 = \overline{a}.\overline{b}.(\overline{c}.\overline{d} + \overline{c}.d + c.d + c.\overline{d}) + a.b.d.(\overline{c} + c) + a.\overline{b}.(\overline{c}.d + c.d + c.\overline{d})$$
$$F4 = \overline{a}.\overline{b} + a.b.d + a.\overline{b}.(d + c.\overline{d})$$
$$F4 = \overline{a}.\overline{b} + a.b.d + a.\overline{b}.d + a.\overline{b}.c$$
$$F4 = \overline{b}.(\overline{a}. + a.c) + a.d.(b + \overline{b})$$
$$F4 = \overline{a}.\overline{b} + a.d + \overline{b}.c$$

$$F5 = \overline{a}.\overline{b}.\overline{c}.\overline{d} + \overline{a}.\overline{b}.c.\overline{d} + a.b.\overline{c}.\overline{d} + a.b.c.\overline{d} + a.\overline{b}.\overline{c}.\overline{d} + a.\overline{b}.c.\overline{d}$$
$$F5 = \overline{a}.\overline{b}.\overline{d}.(\overline{c} + c) + a.b.\overline{d}.(\overline{c} + c) + a.\overline{b}.\overline{d}.(\overline{c} + c)$$
$$F5 = \overline{a}.\overline{b}.\overline{d} + a.b.\overline{d} + a.\overline{b}.\overline{d}$$
$$F5 = \overline{a}.\overline{b}.\overline{d} + a.\overline{d}.(b + \overline{b})$$

$$F5 = (\bar{a}.\bar{b} + a).\bar{d}$$
$$F5 = a.\bar{d} + \bar{b}.\bar{d}$$

$$F6 = (\bar{a}.b + a.b + a.\bar{b}).(c.\bar{d} + \bar{c}.\bar{d}) + \bar{c}.d.(\bar{a}.b + a.b)$$
$$F6 = ((\bar{a} + a).b + a.\bar{b}).((c + \bar{c}).\bar{d}) + \bar{c}.d.((\bar{a} + a).b)$$
$$F6 = (b + a.\bar{b}).(\bar{d}) + b.\bar{c}.d$$
$$F6 = b.(\bar{d} + \bar{c}.d) + a.\bar{d}$$
$$F6 = a.\bar{d} + b.\bar{d} + b.\bar{c}$$

$$F7 = \bar{a}.b.\bar{c}.\bar{d} + a.b.\bar{c}.\bar{d} + \bar{a}.\bar{b}.c.\bar{d} + a.\bar{b}.c.\bar{d} + \bar{a}.b.c.\bar{d} + \bar{a}.\bar{b}.\bar{c}.d + \bar{a}.\bar{b}.\bar{c}.d$$
$$F7 = (\bar{a} + a).b.\bar{c}.\bar{d} + (\bar{a} + a).\bar{b}.c.\bar{d} + \bar{a}.b.c.\bar{d} + \bar{a}.\bar{b}.\bar{c}.d$$
$$F7 = b.\bar{c}.\bar{d} + (\bar{b}. + \bar{a}.b).c.\bar{d} + \bar{a}.\bar{b}.\bar{c}.d$$
$$F7 = b.\bar{c}.\bar{d} + \bar{b}.c.\bar{d} + \bar{a}.c.\bar{d} + \bar{a}.\bar{b}.\bar{c}.d \text{ ou}$$
$$F7 = b.\bar{c}.\bar{d} + \bar{b}.c.\bar{d} + \bar{a}.b.\bar{d} + \bar{a}.\bar{b}.\bar{c}.d$$

$$F8 = \bar{a}.\bar{b}.\bar{c}.\bar{d} + \bar{a}.\bar{b}.c.\bar{d} + a.\bar{b}.c.\bar{d} + \bar{a}.b.c.\bar{d} + \bar{a}.\bar{b}.\bar{c}.d + a.\bar{b}.\bar{c}.d$$
$$F8 = \bar{a}.\bar{b}.\bar{d}.(\bar{c} + c) + a.\bar{b}.c.\bar{d} + \bar{a}.b.c.\bar{d} + (\bar{a} + a).\bar{b}.\bar{c}.d$$
$$F8 = \bar{a}.\bar{d}.(\bar{b} + b.c) + \bar{b}.\bar{c}.d + a.\bar{b}.c.\bar{d}$$
$$F8 = \bar{a}.\bar{b}.\bar{d} + (\bar{a} + a.\bar{b}).c.\bar{d} + \bar{b}.\bar{c}.d$$
$$F8 = \bar{a}.\bar{b}.\bar{d} + \bar{a}.c.\bar{d} + \bar{b}.c.\bar{d} + \bar{b}.\bar{c}.d \text{ ou}$$
$$F8 = \bar{a}.\bar{b}.\bar{c} + \bar{a}.c.\bar{d} + \bar{b}.c.\bar{d} + \bar{b}.\bar{c}.d$$

$$F9 = \bar{a}.\bar{b}.\bar{c}.\bar{d} + \bar{a}.b.\bar{c}.\bar{d} + a.b.\bar{c}.\bar{d} + a.\bar{b}.c.\bar{d} + \bar{a}.b.c.d + \bar{a}.\bar{b}.\bar{c}.d$$
$$F9 = \bar{a}.\bar{b}.\bar{c}.(\bar{d} + d) + (\bar{a} + a).b.\bar{c}.\bar{d} + a.\bar{b}.c.\bar{d} + \bar{a}.b.c.d$$
$$F9 = \bar{a}.\bar{b}.\bar{c} + b.\bar{c}.\bar{d} + a.\bar{b}.c.\bar{d} + \bar{a}.b.c.d$$

$$F10 = a.b + \bar{c}.\bar{d} + \bar{a}.\bar{b}.c.\bar{d} + \bar{a}.b.c.\bar{d}$$
$$F10 = a.b + \bar{c}.\bar{d} + \bar{a}.c.\bar{d}$$

$$F11 = a.b.c.d + a.b.c.\bar{d} + \bar{a}.b.c.d + \bar{a}.b.c.\bar{d} +$$
$$a.\bar{b}.c.d + a.\bar{b}.c.\bar{d} + a.\bar{b}.\bar{c}.d + a.\bar{b}.\bar{c}.\bar{d} + \bar{a}.\bar{b}.c.\bar{d}$$
$$F11 = a.b.c.(d + \bar{d}) + \bar{a}.b.c.(d + \bar{d}) + a.\bar{b}.(c.d + c.\bar{d} + \bar{c}.d + \bar{c}.\bar{d}) + \bar{a}.\bar{b}.c.\bar{d}$$

$$F11 = a.b.c + \overline{a}.b.c + a.\overline{b} + \overline{a}.\overline{b}.c.\overline{d}$$
$$F11 = \left(a + \overline{a}\right).b.c + \overline{b}.\left(a + \overline{a}.c.\overline{d}\right)$$
$$F11 = a.\overline{b} + c.\left(b + \overline{b}.\overline{d}\right)$$
$$F11 = a.\overline{b} + b.c + c.\overline{d}$$

$$F12 = \overline{a}.b.c.d + \overline{a}.\overline{b}.c.d + a.\overline{b}.\overline{c}.d + a.\overline{b}.\overline{c}.\overline{d} + a.b.c.d + a.b.c.\overline{d} + a.b.\overline{c}.d + a.b.\overline{c}.\overline{d}$$
$$F12 = \overline{a}.c.d.\left(b + \overline{b}\right) + a.\overline{c}.\left(\overline{b}.d + \overline{b}.\overline{d} + b.d + b.\overline{d}\right) + a.b.c.\left(d + \overline{d}\right)$$
$$F12 = \overline{a}.c.d + a.\left(\overline{c} + b.c\right)$$
$$F12 = \overline{a}.c.d + a.\overline{c} + a.b$$

$$F13 = a.b.c + a.\overline{b}.c + a.b.\overline{c} + a.\overline{b}.\overline{c} + \overline{a}.b.c + \overline{a}.b.\overline{c}$$
$$F13 = a.\left(b.c + \overline{b}.c + b.\overline{c} + \overline{b}.\overline{c}\right) + \overline{a}.b.\left(c + \overline{c}\right)$$
$$F13 = a + \overline{a}.b$$
$$F13 = a + b$$

$$F14 = (a + b + c).(a + \overline{b} + c).(a + \overline{b} + \overline{c}).(\overline{a} + b + \overline{c})$$
$$F14 = \left((a + c) + (b.\overline{b})\right).(a + \overline{b} + \overline{c}).(\overline{a} + b + \overline{c})$$
$$F14 = (a + c).(a + \overline{b} + \overline{c}).(\overline{a} + b + \overline{c})$$
$$F14 = (a + (c.(\overline{b} + \overline{c})).(\overline{a} + b + \overline{c})$$
$$F14 = (a + (c.\overline{b})).(\overline{a} + b + \overline{c})$$
$$F14 = (a + c).(a + \overline{b}).(\overline{a} + b + \overline{c})$$
$$F14 = (a.a + a.\overline{b} + a.c + \overline{b}.c).(\overline{a} + b + \overline{c})$$
$$F14 = \overline{a}.\overline{b}.c + a.b + a.b.c + a.\overline{c} + a.\overline{b}.\overline{c}$$
$$F14 = \overline{a}.\overline{b}.c + a.b + a.\overline{c}$$

$$F15 = (\overline{a} + b + c).(\overline{a} + \overline{b} + \overline{c}).(a + \overline{b} + c).(a + b + \overline{c})$$
$$F15 = (\overline{a} + ((b + c).(\overline{b} + \overline{c})).(a + (\overline{b} + c).(b + \overline{c}))$$
$$F15 = (\overline{a} + (b \oplus c).(a + (\overline{b \oplus c}))$$
$$F15 = \left(\overline{a \oplus (b \oplus c)}\right)$$

$$F16 = \overline{A + \overline{B.C} + \overline{C}.D}$$

$$F16 = \overline{A}.B.C.\overline{\overline{C}.D}$$
$$F16 = \overline{A}.B.C.\left(C + \overline{D}\right)$$
$$F16 = \overline{A}.B.C + \overline{A}.B.C.\overline{D}$$
$$F16 = \overline{A}.B.C\left(1 + \overline{D}\right)$$
$$F16 = \overline{A}.BC$$

$$F17 = \overline{A + \overline{B}.C + \overline{C.D + B}}$$
$$F17 = \overline{A}.\left(B + \overline{C}\right).\left(C.D + B\right)$$
$$F17 = \left(\overline{A}.B + \overline{A}.\overline{C}\right).\left(C.D + B\right)$$
$$F17 = \overline{A}.B.C.D + \overline{A}.\overline{C}.C.D + \overline{A}.B.B + \overline{A}.\overline{C}.B$$
$$F17 = \overline{A}.B.\left(C.D + 1 + \overline{C}\right)$$
$$F17 = \overline{A}.B$$

$$F18 = \overline{A.\overline{B} + \overline{C.D} + \overline{A}.B} + \overline{\overline{A.B} + \overline{B}.C}$$
$$F18 = \left(\overline{A} + B\right).C.D.\left(A + \overline{B}\right) + \left(\overline{A} + \overline{B}\right).\overline{B}.C$$
$$F18 = \overline{A}.\overline{B}.C.D + A.B.C.D + \overline{A}.\overline{B}.C + \overline{B}.\overline{B}.C$$
$$F18 = A.B.C.D + \left(\overline{A}.D + \overline{A} + 1\right).\overline{B}.C$$
$$F18 = \left(A.B.D + \overline{B}\right).C$$
$$F18 = A.C.D + \overline{B}.C$$

$$F19 = \overline{\overline{A + B} + \overline{\overline{A} + \overline{C}}} + \overline{A + C}$$
$$F19 = \left(A + B\right).\left(\overline{A} + \overline{C}\right) + \overline{A}.\overline{C}$$
$$F19 = \left(A + B\right).\left(\overline{A} + \overline{C}\right) + \overline{A}.\overline{C}$$
$$F19 = A.\overline{A} + A\overline{C} + \overline{A}.B + B.\overline{C} + \overline{A}.\overline{C}$$
$$F19 = \overline{A}.B + \left(A + B + \overline{A}\right).\overline{C}$$
$$F19 = \overline{A}.B + \overline{C}$$

$$F20 = \overline{\left(A \oplus B\right) + \left(\overline{\overline{A} + B}\right)}$$
$$F20 = \overline{\left(A \oplus B\right)}.\left(\overline{A} + B\right)$$
$$F20 = \left(\overline{A} + B\right).\left(A + \overline{B}\right).\left(\overline{A} + B\right)$$
$$F20 = \left(\overline{A} + B\right).\left(A + \overline{B}\right) = \overline{A \oplus B}$$