$$H(z^{-1}) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N}} = \frac{Y(z^{-1})}{X(z^{-1})}$$

$$\left(b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_M z^{-M}\right) X(z^{-1}) = \left(a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N}\right) Y(z^{-1})$$

$$b_0 X(z^{-1}) + b_1 z^{-1} X(z^{-1}) + b_2 z^{-2} X(z^{-1}) + \ldots + b_M z^{-M} X(z^{-1}) =$$
$$a_0 Y(z^{-1}) + a_1 z^{-1} Y(z^{-1}) + a_2 z^{-2} Y(z^{-1}) + \ldots + a_N z^{-N} Y(z^{-1})$$

$$b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_M\, x[n-M] =$$
$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] + \ldots + a_N\, y[n-N]$$

Haciendo $a_0 = 1$

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_M\, x[n-M] -$$
$$a_1 y[n-1] - a_2 y[n-2] - \ldots - a_N\, y[n-N]$$

Sin realimentacion:

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_M\, x[n-M]$$

$$y[n] = \sum_{k=0}^{M} b_k\, x[n-k] = b[n] * x[n]$$

Implica que:

$b[n] \Leftrightarrow h[n]$