$$H\left(z^{-1}\right)=\frac{\mathcal{Z}\{y[n]\}}{\mathcal{Z}\{x[n]\}}=\frac{Y\left(z^{-1}\right)}{X\left(z^{-1}\right)}$$

$$H\left(z^{-1}\right)=\frac{b_0+b_1z^{-1}+b_2z^{-2}+\ \ldots\ +b_Mz^{-M}}{a_0+a_1z^{-1}+a_2z^{-2}+\ \ldots\ +a_Nz^{-N}}=\frac{Y\left(z^{-1}\right)}{X\left(z^{-1}\right)}$$

$$\left(b_0 + b_1z^{-1} + b_2z^{-2} + \ldots + b_Mz^{-M}\right)X\left(z^{-1}\right)=\left(a_0 + a_1z^{-1} + a_2z^{-2} + \ldots + a_Nz^{-N}\right)Y\left(z^{-1}\right)$$

$$b_0X\left(z^{-1}\right) + b_1z^{-1}X\left(z^{-1}\right) + b_2z^{-2}X\left(z^{-1}\right) + \ldots + b_Mz^{-M}X\left(z^{-1}\right) =$$
$$a_0Y\left(z^{-1}\right) + a_1z^{-1}Y\left(z^{-1}\right) + a_2z^{-2}Y\left(z^{-1}\right) + \ldots + a_Nz^{-N}Y\left(z^{-1}\right)$$

$$b_0x[n] + b_1x[n-1] + b_2x[n-2] + \ldots + b_M\,x[n-M] =$$
$$a_0y[n] + a_1y[n-1] + a_2y[n-2] + \ldots + a_N\,y[n-N]$$

$$a_0y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + \ldots + b_M\,x[n-M] -$$
$$a_1y[n-1] - a_2y[n-2] - \ldots - a_N\,y[n-N]$$

Haciendo $a_0 = 1$

$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + \ldots + b_M\,x[n-M] -$$
$$a_1y[n-1] - a_2y[n-2] - \ldots - a_N\,y[n-N]$$

Sin realimentacion (FIR):

$$y[n] = b_0x[n] + b_1x[n-1] + b_2x[n-2] + \ldots + b_M\,x[n-M]$$

$$y[n] = \sum_{k=0}^{M} b_k\,x[n-k] = b[n] * x[n]$$

Implica que:

$b[n] \Leftrightarrow h[n]$

Con realimentacion (IIR):

$$y[n] = \sum_{k=0}^{M} b_k\,x[n-k] - \sum_{j=1}^{N} a_j\,y[n-j]$$