Control Barrier Functions for Safe Multi-Robot Navigation

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Introduction

- **Safety in robotics** varies by application domain
  - Manipulators, mobile platforms, drones, humanoids
- Safety-critical systems require formal, mathematically-grounded frameworks
- Control Barrier Functions (CBFs) provide systematic safety enforcement
- Here we present:
  - Theoretical foundations of CBFs
  - Practical implementation via quadratic programming (QP)
  - Application to robotic systems and human-robot collaboration

Control Barrier Functions: Definition

Consider nonlinear control-affine system:

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathcal{D} \subset \mathbb{R}^n, \quad u \in U$$

Safe set $\mathcal{S} \subset \mathcal{D}$: forward invariant region.

Definition of CBF

Function $h : \mathcal{D} \to \mathbb{R}$ is a **CBF** if:

$$\sup_{u \in U} \left[\frac{\partial h}{\partial x} f(x) + \frac{\partial h}{\partial x} g(x) u \right] \geq -\alpha(h(x))$$

where α is extended class $\mathcal{K}$.

CBF-Based Safety Filter

- CBF acts as **safety filter** on nominal controller
- Minimally invasive: modifies control only when necessary
- Implemented via Quadratic Programming (QP):

$$u^* = \arg\min_{u \in U} \|u - u_{\text{des}}\|^2$$

subject to:

$$L_f h(x) + L_g h(x) u \geq -\alpha(h(x))$$

Our Project: Multi-Robot CBF