I.Ellipse

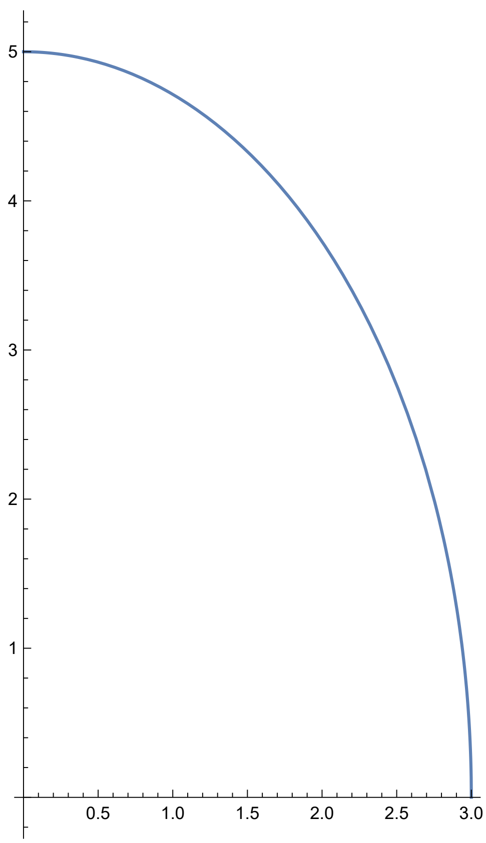
The graph of the equation $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$, with -a≤ x ≤a, -b≤ y≤ b is an ellipse.

iii. Find the volume of the solid of by rotating this ellipse about the x-axis. Hint: First find the volume of the solid of revolution of the first-quadrant portion, $y=b\left(1-\left(\frac{x}{a}\right)^2\right)^{\frac{1}{2}}$, 0≤ x ≤a, about the x-axis and multiply it by 2. Then, with a=3 and b=5, plot the solid of revolution about the x-axis using the built-in syntax of Mathematica.

We have a function $y=b\left(1-\left(\frac{x}{a}\right)^2\right)^{\frac{1}{2}}$, 0≤ x ≤a

In[4]:=
```
Plot[5 (1 - (x/3)^2)^(1/2), {x, 0, 3}, AspectRatio → Automatic]
```

Out[4]=



Solve the first quadrant of the ellipse and multiply it by 2

In[∘]:= **Solve**$\left[\int_0^a \pi * \left(b\left(1-\left(\frac{x}{a}\right)^2\right)^{\frac{1}{2}}\right)^2 dx\right]$

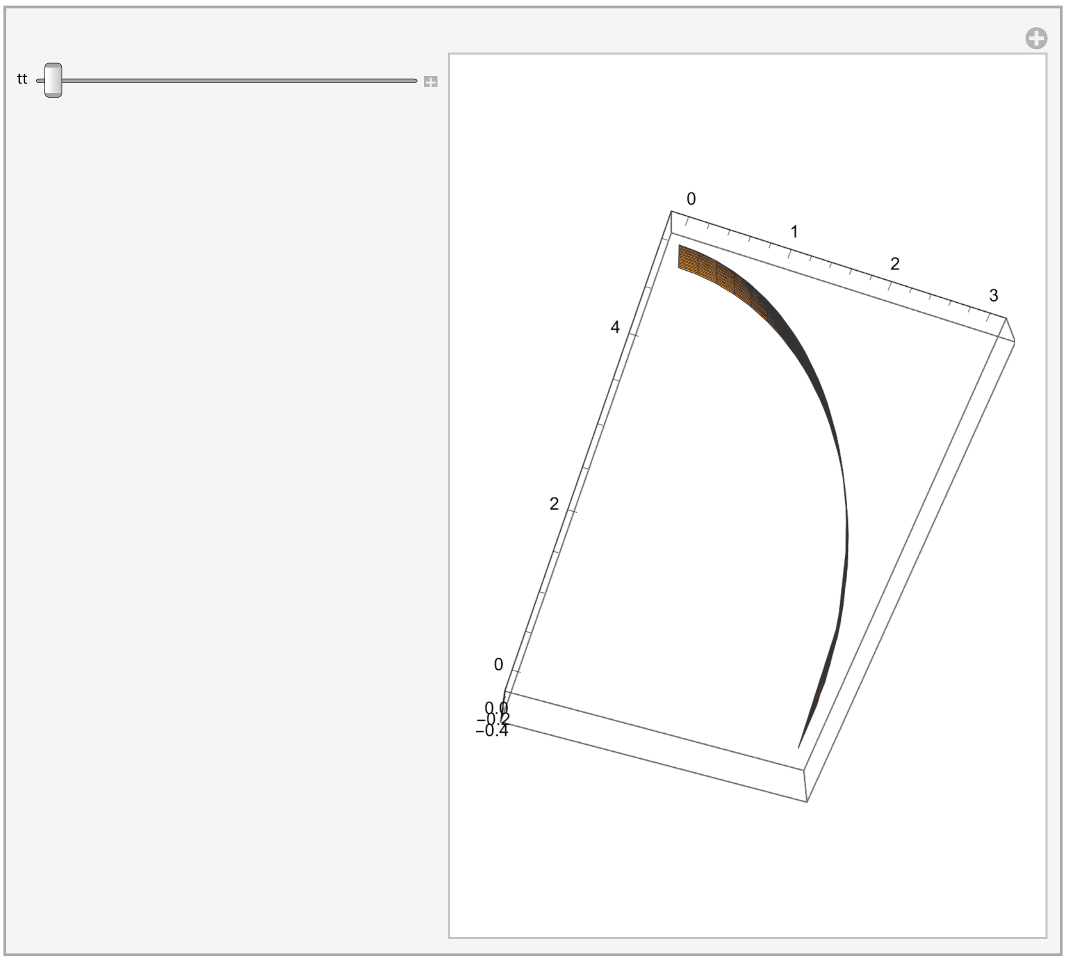
Out[∘]=

Solve$\left[\frac{2}{3}\, a\, b^2\, \pi\right]$

In[∘]:= **Manipulate**$\left[\textbf{RevolutionPlot3D}\left[\frac{5\sqrt{3^2-x^2}}{3},\right.\right.$

$\left.\left.\{x, 0, 10\}, \{t, 0, tt\}, \textbf{RevolutionAxis} \to \{1, 0, 0\}\right], \{tt, .1, 2\pi\}\right]$

Out[∘]=



Multiply it by 2 and plotting a and b

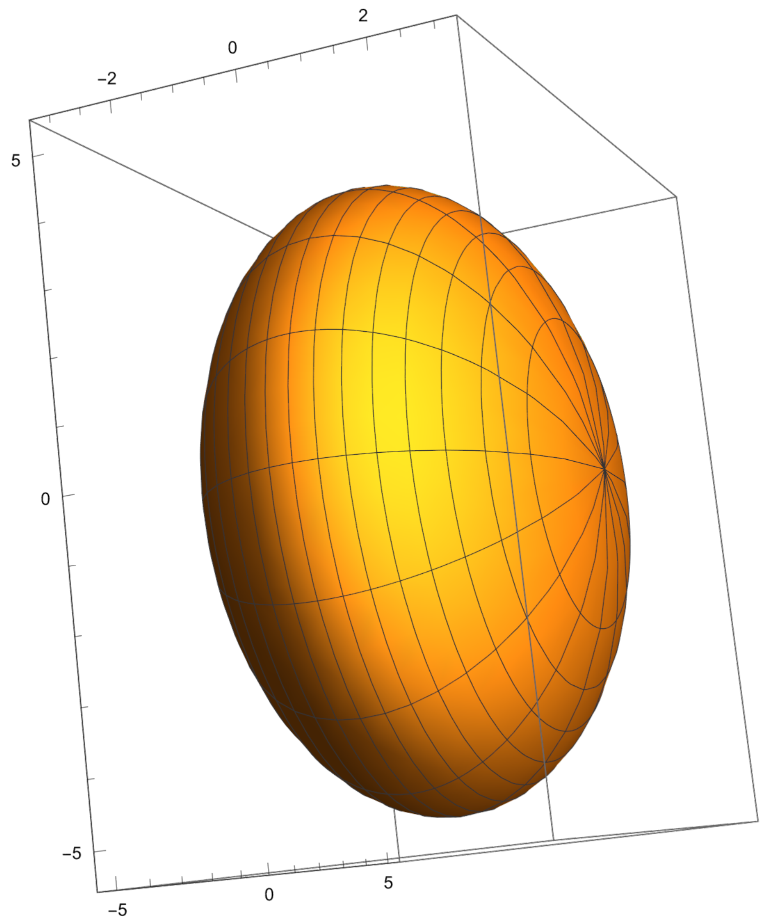
In[∘]:= **Solve**$\left[\left(\frac{2}{3}\times 3\times 5^2\,\pi\right) * 2\right]$

Out[∘]=

Solve$[100\,\pi]$

In[ ]:= RevolutionPlot3D$\left[\frac{5\sqrt{3^2 - x^2}}{3}\right.$, {x, -10, 10}, RevolutionAxis → {1, 0, 0}$\left.\right]$

Out[ ]=

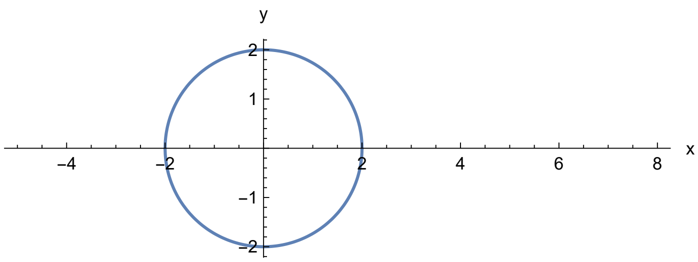


II.Torus

The disks $x^2+y^2 \leq a$ is revolved about the line x=b (b>a) to generate a solid shaped like a doughnut called a torus.
Plot the torus with a=2 and b=5.

In[ ]:=
```
Plot[y /. Solve[x^2 + y^2 == 2^2], {x, -5, 8},
 AxesLabel → {"x", "y"},
 PlotLegends → "Expressions",
 AspectRatio → Automatic
]
```

Out[ ]=



V=$\int_{-r}^{r}\pi\left(A\,(y)^2 - a\,(y)^2\right)dy$

$(x-R)^2$+y^2=r^2

x=R+- $\sqrt{r^2-y^2}$

A(y)=R+ $\sqrt{r^2-y^2}$

a(y)=R- $\sqrt{r^2-y^2}$

=>$\pi\int_{-r}^{r}\left(\left(R+\sqrt{r^2-y^2}\right)^2-\left(\left(R-\sqrt{r^2-y^2}\right)^2\right)dy\right.$

=$\pi\int_{-r}^{r}\left(4\,R\,\sqrt{r^2-y^2}\right)dy$

V=4πR$\int_{-r}^{r}\left(\sqrt{r^2-y^2}\right)dy$

=4πR($\frac{1}{2}\pi r^2$)

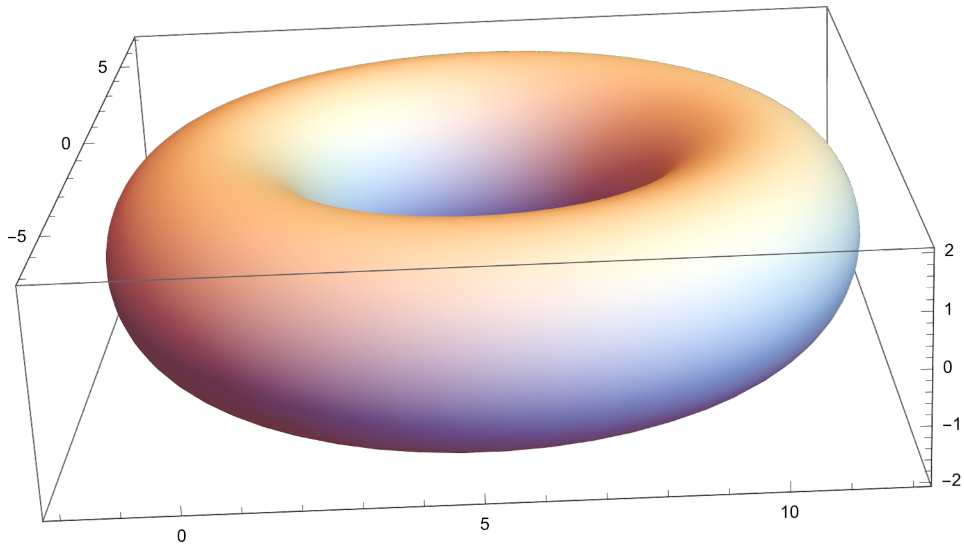
V=2$\pi^2 r^2$R

In[ ]:=
```
Solve[2 π^2 * 2^2 * 5]
```

Out[ ]=
```
Solve[40 π^2]
```

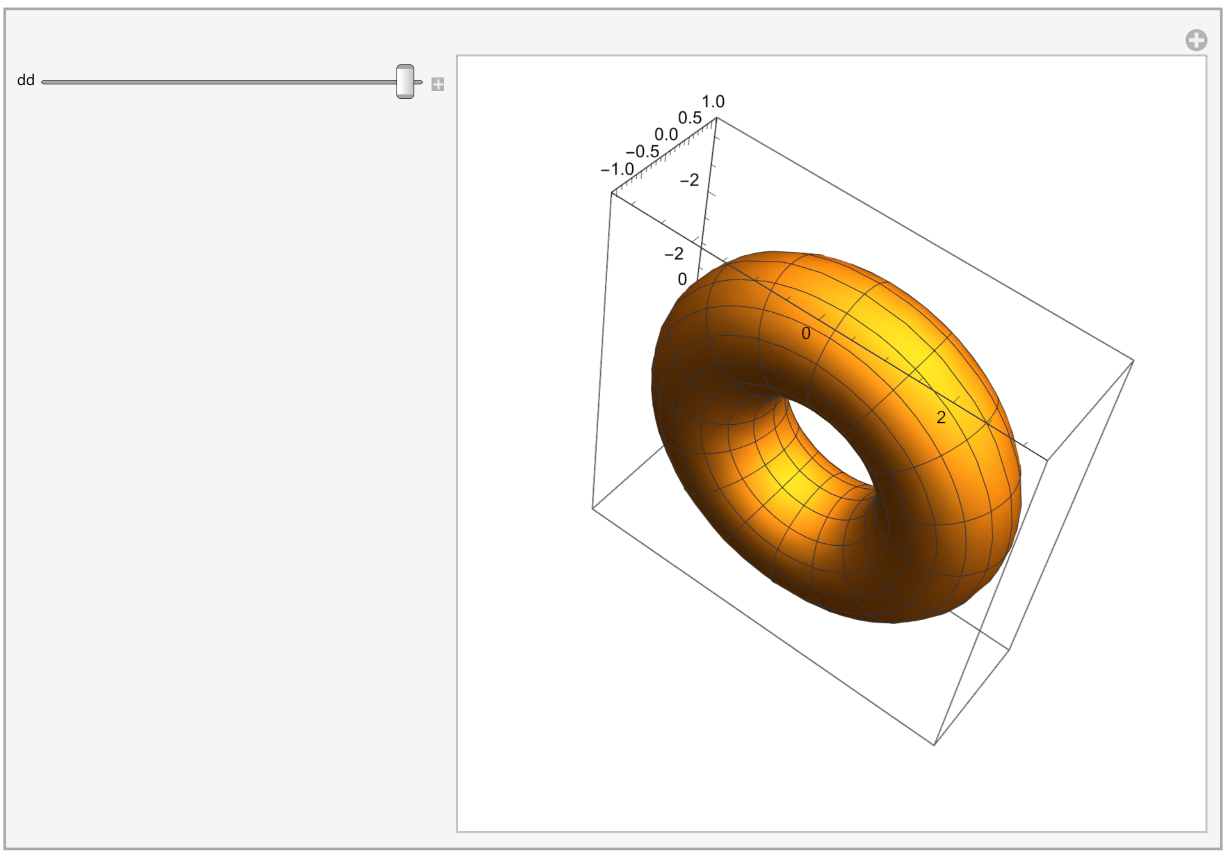
In[7]:=
```
Graphics3D[Torus[{5, 0, 0}, {3, 7}], Axes → True]
```

Out[7]=



In[•]:=
```
Manipulate[RevolutionPlot3D[{2 + Cos[t], Sin[t]}, {t, 0, 2 Pi}, {d, 0, dd}], {dd, .1, 2 π}]
```

Out[•]=



III. Asteroid

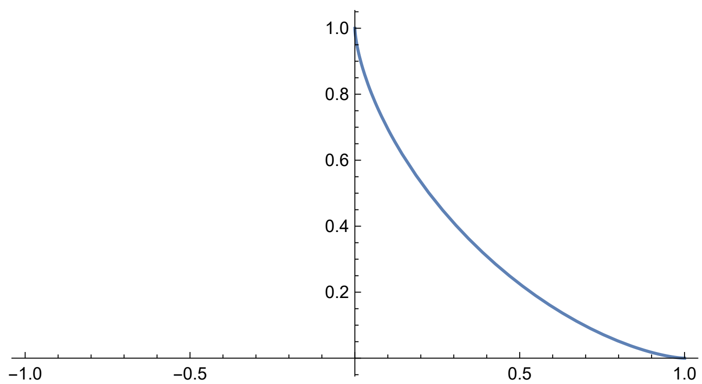
The graph of the equation $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$, with -1≤x≤1, -1≤y≤1 is called an “Asteroid” because of its starlike appearance.

iii. Find the volume of the solid by rotating this asteroid about the x-axis . Hint: First find the volume of the solid of revolution of the first quadrant portion. $y=\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}$, 0≤x≤1, about the x-axis and multiply it by . Then, plot the solid of revolution about the x-axis using the built-in syntax of Mathematica.

We have a function $y=\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}$, 0≤x≤1

In[∘]:= $\texttt{Plot}\left[\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}, \{x, -1, 1\}, \texttt{AspectRatio} \to \texttt{Automatic}\right]$

Out[∘]=



Solve the first quadrant of the asteroid

In[∘]:= $\texttt{Solve}\left[\int_0^1 \pi * \left(\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}\right)^2 dx\right]$

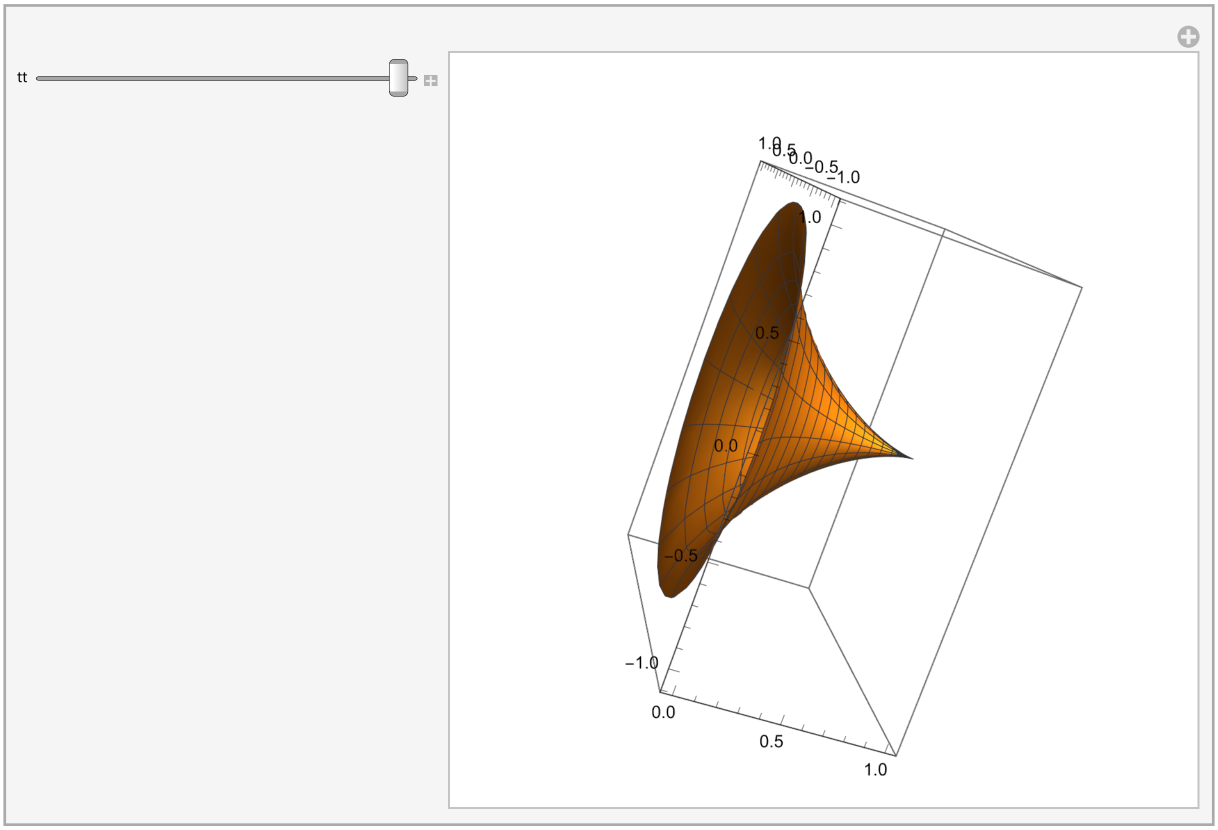
••• Solve: $\frac{16\pi}{105}$ is not a quantified system of equations and inequalities.

Out[∘]=

$\texttt{Solve}\left[\frac{16\pi}{105}\right]$

In[∘]:=
```
Manipulate[
 RevolutionPlot3D[(1 - x^(2/3))^(3/2), {x, 0, 1}, {t, 0, tt}, RevolutionAxis → {1, 0, 0}], {tt, .1, 2 π}]
```

Out[∘]=



Multiply it by 2

In[∘]:=
```
Solve[16 π/105 * 2]
```

Out[∘]=

$$\text{Solve}\left[\frac{32\,\pi}{105}\right]$$

In[∘]:=
```
Rev2 =
  RevolutionPlot3D[Sqrt[1 - 3 (-x)^(2/3) + 3 (-x)^(4/3) - (-x)^2], {x, -1, 1}, RevolutionAxis → {1, 0, 0}];
```

In[∘]:=
```
Rev1 = RevolutionPlot3D[(1 - x^(2/3))^(3/2), {x, -1, 1}, RevolutionAxis → {1, 0, 0}];
```

In[ ]:= **Show[Rev1, Rev2, PlotRange → All]**

Out[ ]=