Using Euler’s Method

Use Euler’s Method to calculate the first three approximations to the given initial value problem for the specified increment size. Calculate the exact solution and investigate the accuracy of your approximations.

16. y’=ye^x

We make a function to calculate the First Order Differential equation using Euler’s Method

```
In[31]:= EulersMethod[f_, {y0_, x0_, xn_, h_}] :=
  Module[{x = x0, y = y0, k1}, Reap[While[x < xn, k1 = h f[y, x];
      y = y + k1;
      x = x + h;
      Sow[{x, y}];]][[2, 1]]]
```

We then make a function in order to pass it through “EulersMethod” function

```
f[y_, t_] := y * ⅇ^t
```

We set the value of y(0)=2, x_0, y_0 and step size n

In[32]:=
```
Solution[n_] := EulersMethod[f, {2, 0, 1.5, n}];
```

We then find the exact value to the differential equation using mathematica built-in function called “DSolve”

In[4]:= Df[x_] = DSolve[{y'[x] == y[x] * e^x, y[0] == 2}, y[x], x]

Out[4]= {{y[x] → 2 e^{-1+e^x}}}

We then make a function to find the error between the exact value and Euler’s method

In[28]:= error[n_] = Abs[Solution[n] - 2 e^{-1+e^n}];

Tables of Euler’s Method and Exact Value

In[•]:=
```
TableForm[Solution[0.5], TableHeadings → {None, {"n step size", "Euler's Method"}}] //
 Magnify[#, 2] &
```

Out[•]=

n step size	Euler's Method
0.5	3.
1.	5.473081906
1.5	12.91177145

In[27]:=
```
TableForm[Table[{n, N[Df[n], 20], N[error[n], 20]}, {n, {0.5, 1, 1.5}}],
   TableHeadings → {None, {"n", "Exact value", "Error"}}] //
  N // Magnify[#, 2] &
```

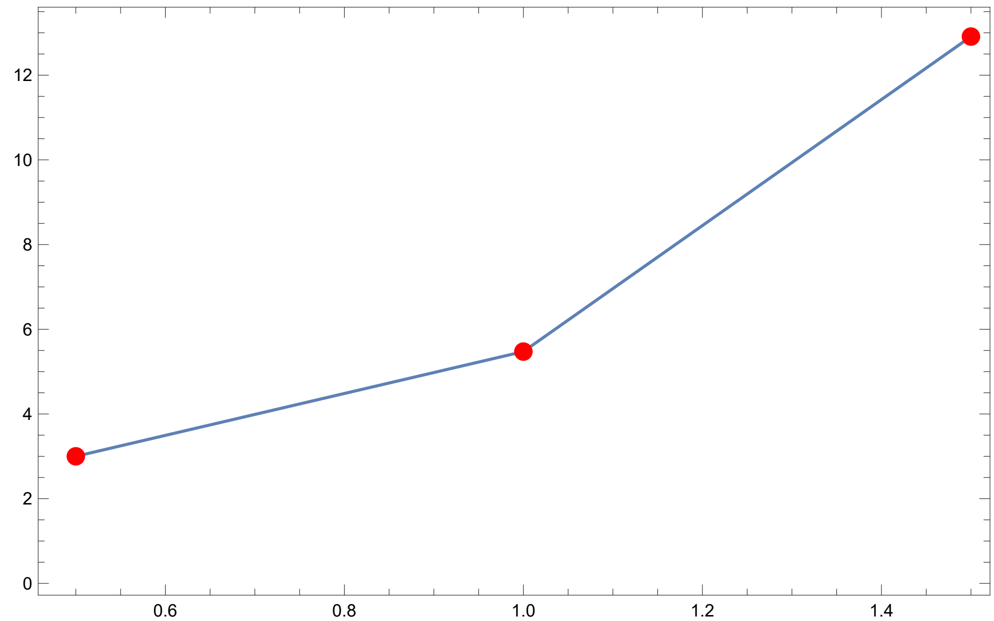
Out[27]=

n	Exact value	Error
0.5	y[0.5] → 3.826185873	0.8261858725
1.	y[1.] → 11.14988305	7.14988305
1.5	y[1.5] → 65.02919032	60.02919032

Graphical visualization of the functions

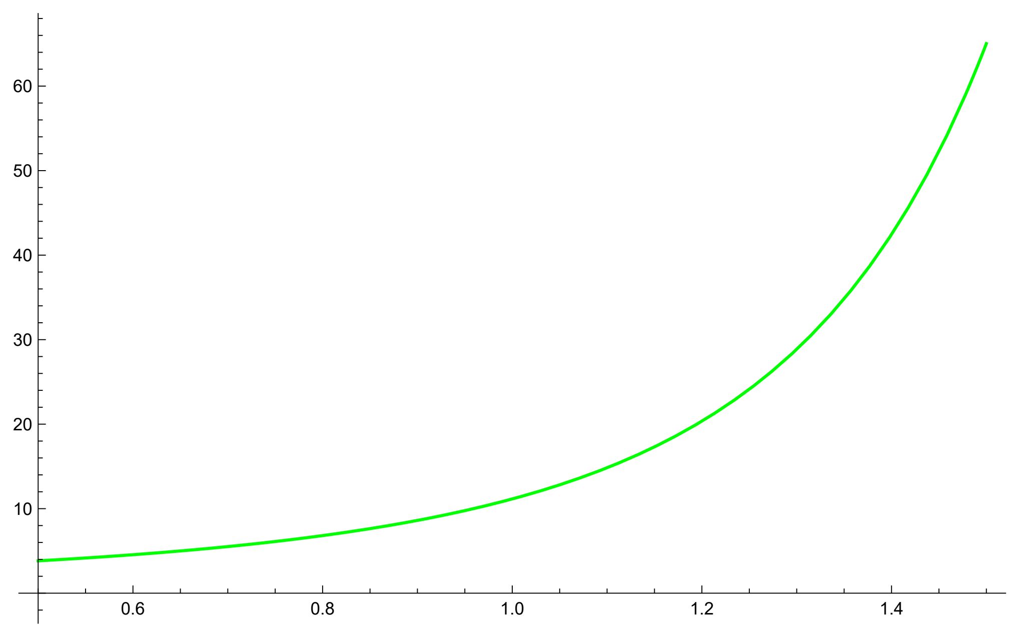
In[41]:=
```
Plot1 = ListLinePlot[Solution[0.5], Mesh → All, MeshStyle → Red, PlotRange → All,
   Frame → True, Epilog → {Red, PointSize[Medium], Point[Solution[0.5]]}]
```

Out[41]=



In[42]:= Plot2 = Plot$\left[2\, e^{-1+e^{x}}, \{x, 0.5, 1.5\}, \text{PlotStyle} \to \text{Green}\right]$

Out[42]=



In[46]:=
```
Legended[Show[Plot1, Plot2, PlotRange → All],
 SwatchLegend[{Green, Blue}, {"Exact", "Euler's Approximation"}]]
```

Out[46]=