# On the Calibration of Spectroscopic Data from High-purity Germanium Gamma-ray Detectors

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**Abstract**

To provide meaningful, interpretable results, gamma-ray spectrometers must be calibrated. Here, a procedure is outlined for calibrations of spectroscopic gamma-ray from high-purity germanium (HPGe) detectors. In particular, ordinary least squares regression is performed between electronic signals resulting from measurements of standard gamma-ray sources and their known energies. The calibration procedure is demonstrated using five common lab sources. The resulting relationship between channel number and energy shows to generalize well within the energy range of interest. The methods presented in this paper are broadly applicable to HPGe detectors, as well as other standard spectroscopic gamma-ray detectors.

Not just human-interpretable... energy relationship is meaningful for other algorithms as well

Good

## I. Introduction

One of the most important uses of gamma-ray detection instrumentation is to identify radionuclides of interest within an environment. Our ability to identify radionuclides of interest then requires a method to relate the output signals of our gamma-ray spectrometers to associated gamma-ray lines. Spectroscopic calibrations are a routine procedure performed on gamma-ray detectors to map electronic signals corresponding to measured gamma-ray events to operator-interpretable units (i.e., gamma-ray energy). In particular, by relating known inputs (i.e., specific gamma-ray source energies) to their respective outputs of our detection systems (i.e., voltages), we can quantitatively determine the mapping from electrical signal to human-interpretable measurements. By performing calibrations with known sources, we gain the ability to characterize gamma rays from additional sources.

In this work, we consider calibration procedures for high-purity germanium (HPGe) gamma-ray detectors. While there are significant differences between HPGe and other common gamma-ray detectors, namely scintillation detectors (e.g., NaI(Tl)), the methods described here can be followed as such detectors.

The remainder of this paper is outlined as follows. Section II describes the methods involved in performing calibrations. Section III shows results for performing spectroscopic calibrations on measured gamma-ray spectra. Section IV closes with a discussion of results.

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## II. Methods

### A. Spectroscopic Gamma-ray Data

The theory behind signal generation is HPGe detectors is described in detail in [1]. The spectroscopic information of a particular measured gamma-ray event $i$ is encoded as a voltage $V_i$. When this signal is passed to a multi-channel analyzer (MCA), the MCA increments the number of counts in bin $k$, where $k$ is such that $V_k \leq V_i \leq V_k + \Delta V$, where $\Delta V$ is the width of the histogram bin. All events recorded within a period $\Delta t$, referred to as the integration time, are aggregated to form a gamma-ray spectrum. The result of the measurement is a histogram $\frac{dN}{dE}$ with total number of counts $n_k$ in the the $k$th bin.

### B. Spectroscopic Calibration

To perform a calibration, we first choose a gamma-ray source with well-characterized energies. The source should be of high enough activity such that the resulting peaks can be easily determined. As a result, common lab sources typically have half-lives on the order of years to tens of years to remain useful for an extended period of time.

When performing the calibration, the source is placed in front of a gamma-ray detector and recorded for some integration time $\Delta t$. If the calibration is being performed using multiple sources, the source location should be the same for each source, and should be close enough to yield a high number of counts in a reasonable amount of time, but also should not be so close that it results in the detector having a high dead time. After recording the spectrum and identifying a particular source peak, we can then find the centroid of this peak within the histogram. For an accurate determination of the peak centroid, we can find the best fit of a unimodal function to the peak, and extract the centroid information. A common choice for a function to fit to the peak is a Gaussian function of the form

$$f(x; A, \mu, \sigma) = A \exp\left( -\frac{(x-\mu)^2}{2\sigma^2} \right) \tag{1}$$

where $A$ is the peak amplitude, $\mu$ is the centroid, and $\sigma^2$ is the variance.

After determining the peak centroid $C_i = \mu_i$ for each energy of interest, a regression is performed between $C_i$ and the associated energy $E_i$. While there may exist slight nonlinearities in the relationship between the MCA channel and energy, often times, a linear fit (i.e., first-order polynomial) between $C_i$ and $E_i$ is sufficient. In the case of a first-order polynomial fit, we wish to find coefficients $\mathbf{a} = \begin{bmatrix} a_0 & a_1 \end{bmatrix}^T$ such that

$$\begin{bmatrix} E_0 \\ E_1 \\ \vdots \\ E_n \end{bmatrix} = \begin{bmatrix} 1 & C_0 \\ 1 & C_1 \\ \vdots & \vdots \\ 1 & C_n \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \tag{2}$$

where $n$ is the number of gamma-ray lines used. This equation can then be inverted (e.g., using linear regression) to yield weights $\mathbf{a}$ that form a linear model. This linear transformation can then be used to find the gamma-ray energy corresponding to an arbitrary bin in the spectrum.

Nice background on form of the data to be analyzed

Is this an important consideration for energy calibration?

Good

Good

Typically we include a component to model background as well... linear model most often used

Good, consider citing

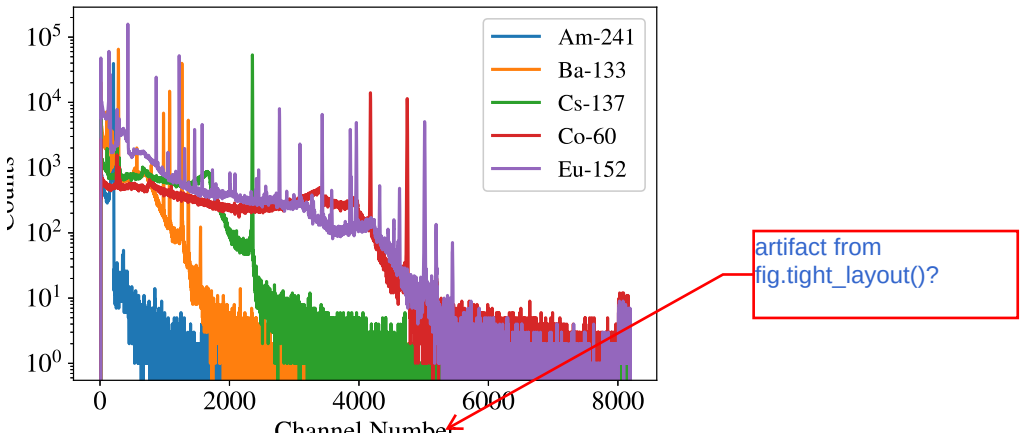


Fig. 1. Spectra of different sources captured by the HPGe detector used in the calibration procedure. The energies for a number of the dominant lines in these spectra are given in Table I.

TABLE I

GAMMA-RAY LINES USED IN THE CALIBRATION [2]

| Source | Energy (keV) |
|---|---|
| $^{241}$Am | 59.541 |
| $^{133}$Ba | 80.997 |
| | 356.017 |
| $^{60}$Co | 1173.237 |
| | 1332.501 |
| $^{137}$Cs | 661.657 |
| $^{152}$Eu | 121.781 |
| | 1408.006 |

Excellent table and reference to nuclear data

## C. Measurements

Measurements of various gamma-ray sources using a coaxial HPGe were performed and provided by by Dr. Ross Barnowski. The measurements were performed using a 13-bit resolution MCA, yielding 8192-bin spectra. The procedure for taking the measurements involved placing a source at a specific location and recording counts for a period of time, and repeating for each source. The spectra used in this analysis are shown in Fig. 1.

The sources used in this analysis, as well as their dominant gamma-ray lines, are given in Table I. Note that the sources used range from tens of keV to slightly over 1.4 MeV, and as a result, may not accurately describe behavior at higher energies in the range we are considering.

Good, highlighting limitations of given approach is important

TABLE II

MEASURED AND ESTIMATED SOURCE ENERGIES FOR $^{133}$BA

| Source | Energy (keV) | Estimated Energy (keV) | Percent Error |
|---|---|---|---|
| $^{133}$Ba | 80.997 | 80.684 | 0.1632 |
| | 356.017 | 356.110 | -0.0261 |

Note that this is likely due to a poor model fit due to the doublet peak in the 79-80 keV region - typically it is best to ignore both these peaks for calibration/evaluation purposes.

Though obvious in this case, consider a legend, in addition to the description in the figure caption

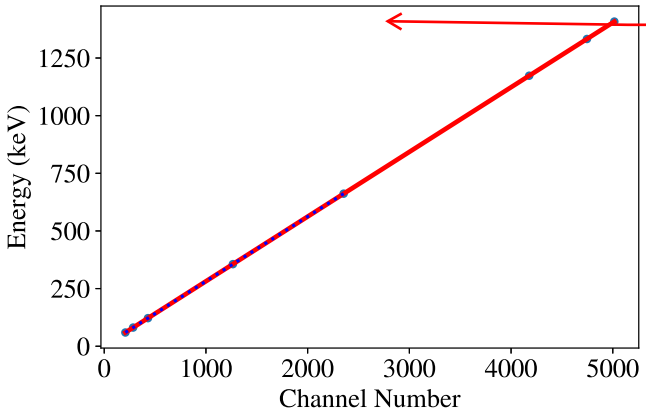


Fig. 2. Resulting linear fit of gamma-ray energies to channel number. The solid red line shows the fit using all data points, while the dashed blue results from the two-point fit.

Excellent attention paid to stated precision (given nuclear data values)

## III. RESULTS

In the simplest case, we can use two points to determine a linear relationship using ordinary least squares (OLS) regression. We used a two point fit with $^{137}$Cs and $^{241}$Am to give the model

$$E_k = 1.181 + 0.281k \tag{3}$$

Since these are derived analytically in the two point case, can include the uncertainty from the error propagation model

where $k$ is the bin number.

To evaluate the performance of this model, we estimate at which bins lines from $^{133}$Ba are expected to fall in, and compare this to what has been measured. Table II shows measured and estimated energies for $^{133}$Ba, as well as the percent error. We see that the two estimated energies are quite close to the actual energies.

In addition to the two points used here, it is also worth considering using additional data points, including some at higher energies. OLS was performed on several points to obtain the line below. The solid line in Figure 2 shows a fit using all data points, while the dashed line shows the two-point relationship.

The spectra shown in Fig. 1 could then be translated into energies using Equation (3). Calibrated spectra are shown in Fig. 3.

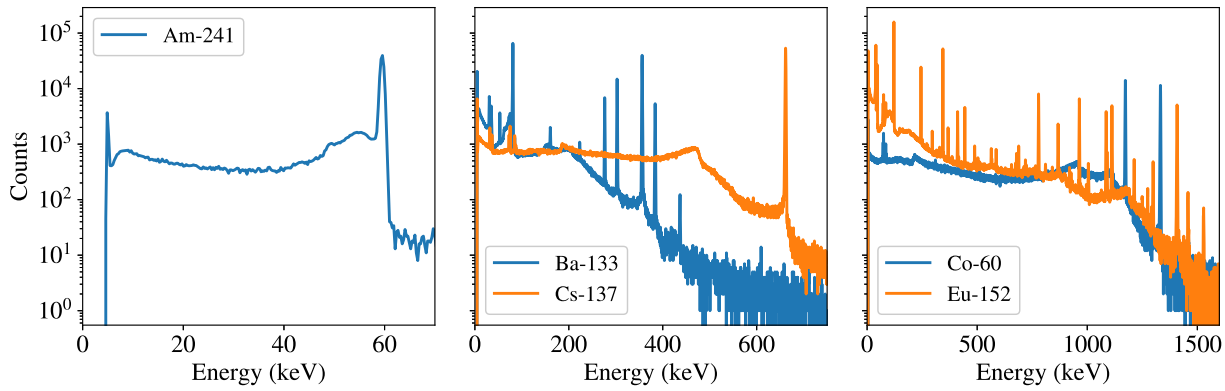


Fig. 3. Energy-calibrated spectra captured by the HPGe used in the calibration procedure. $^{241}$Am (left), $^{133}$Ba and $^{137}$Cs (center), and $^{60}$Co and $^{152}$Eu (right).

## IV. Discussion

The resulting fit from our linear regression can then be used for mapping MCA channels to gamma-ray energy. For routine lab meaurements and estimations, experts often time only use two energy measurements from a single source to determine a linear fit [1]. While a linear fit is generally sufficient, some specific applications may require nonlinear fitting. This can easily be accomplished by including higher-orders of the centroid $C_i$

$$\begin{bmatrix} E_0 \\ E_1 \\ \vdots \\ E_n \end{bmatrix} = \begin{bmatrix} 1 & C_0 & C_0^2 & \dots & C_0^m \\ 1 & C_1 & C_1^2 & \dots & C_1^m \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 1 & C_n & C_n^2 & \dots & C_n^m \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \tag{4}$$

As before, the weights $\mathbf{a}$ can be found using OLS.

Aside from using a higher-order model, one could also improve the fit used to determine the peaks. Here, a Gaussian model was assumed, and no linear offset or background subtraction was performed. To more accurately determine the peak centers, one could fit a Gaussian superimposed on a linear model. In our case, the simplified fit provided results sufficient for the purpose of this exercise, so only a single Gaussian peak was used.

Good, obviates my comment from above

## References

[1] G. Gilmore, *Practical gamma-ray spectrometry*. Wiley, 2011.

[2] "The lund/lbnl nuclear data search," http://nucleardata.nuclear.lu.se/toi/, accessed: 2018-01-31.