UNIT - IV

NETWORK SCHEDULING...

* **Introduction:-** Nework Scheduling is a technic used for planning and scheduling large projects in the fields of construction, maintanance, fabrication and purchasing of computer systems etc. It is a method of minimizing troublespots such as production, delays and interpretations by determining critical factors and co-ordinating various parts of the overall jobs.

There are two basic planning and control technics that utilize a network to complete a predetermined projects (or) schedule. This are critical path Method (CPM) and program evaluation review technic (PERT).

A project is define as a combination of inter related activities all of which much be executed in a certain order for its completion.

The work involved in a project can be divided into 3 phases corresponding to the management functions of planning, scheduling and controlling.

Planning:- This ~~page~~ phase involves setting the objectives of the project as well as the assumptions to be made. It is also involves the listing of tasks or jobs that must be performed in order to complete a perfect under consideration.

Scheduling:- The objective of scheduling is to give the earliest and the least allowable start and finish time of each activity as well as relationship with other activities in the project.

Controlling:- This ~~page~~ phase is exercised after the planning and scheduling. It involve s the making periodical progress reports, reviewing the progress analyzing the status of the project, making management decisions regarding updating, crashing and resource allocation etc.

* **Basic terms:-**

1. **Network:-** It is a graphic representation of logically and sequentionally connected activities and events in a project. Networks are called arrows and nodes, representing Arrow Diagrams.

2. **Activity:-** An activity is a task or item of work to be done. That consumes time, effort, money or other resources. It lies between two events called the preceeding and succeeding once. An activity is represented by an arrow

$O \xrightarrow{A} O$ Here, A is a activity.

The activities can be classified into the following 3 categories:-

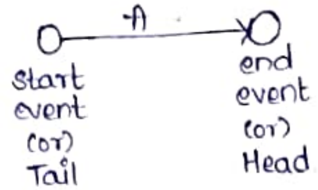
1. **Predecessor Activity:-** An activity which must be completed before one or more other activities start is known as Predecessor Activity.

2. **Successor Activity:-** An activity which started after one or more other activities are completed.

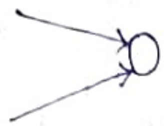
3. **Dummy Activity:-**

An activity which does not consume either any resource and time is known as Dummy Activity. A dummy activity is represented by dotted line in the network diagram.

Event:- The beginning and end points of an activity are called Events or nodes or connector. This is usually represented by circle in a network

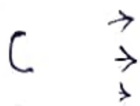


Merge Event :- When two or more activities comes from an event, it is known as Merge Event.



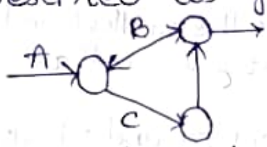
ves an event is

Burst Event :- W more th
known as Burst t.

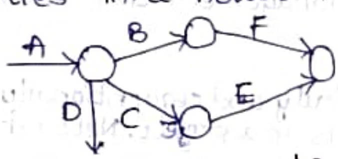


Merge and Burst t:- An activity may be merged and burst at the same time

Looping :- In a network diagram a looping error is also known as cycling error. Drawing an endless loop in a network is known as error of looping. A loop can be formed if an activity is represented as going back in time



Dangling :- To Disconnect an activity before completion of all the activities in a network diagram is known as Dangling



* Rules for Constructing Network :-

For the construction of a Network the following rules are followed :-

1. Each activity is represented by one and only an arrow.
2. No two activities can be identified by the same head and tail events.
3. Nodes are numbered to identified an activity uniquely. Tail node (starting point) should be lower than the head node (end point) of an activity.
4. Arrows should not cross each other.
5. Arrows should be straight and not curved or bended.
6. Every node must have atleast one activity preceeding it and atleast one activity following it. except for the node at the

beginning and at the end of the network.

Numbering the Event:-

After the network is drawn in a logical sequence every event is assigned a number. The number sequence must be such as so as to reflect the flow of the network. In numbering the events the following rules should be observed than this. rules is also known as "Fulkerson's Rule."

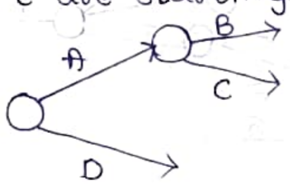
1. Event numbers should be unique.
2. Event numbering should be carried out on a sequential basis from left to right.
3. The initial event is numbered 0 or 1
4. The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow.
5. Gap should be left in the sequence of event numbering. to accomodate subsequence inclusion of activities if necessary.

1. Construct a Network for the project whose activities and precedence relation ships are given below.

Activity	Predecessors
A	—
B	A
C	A
D	—
E	D
F	B,C,E
G	F
H	D
I	G,H

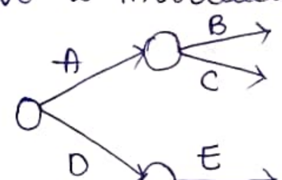
Sol. From the given constraints A and D are the starting activities and I the terminal activity.

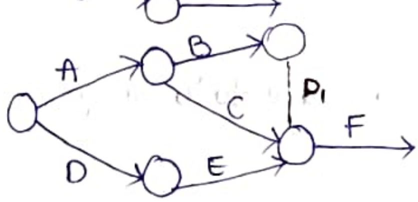
B and C are starting with the same event.

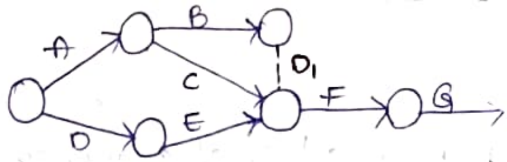


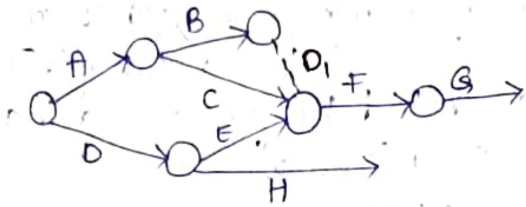
And are both the predecessors of the activity-F and also E has to be the predecessor of F

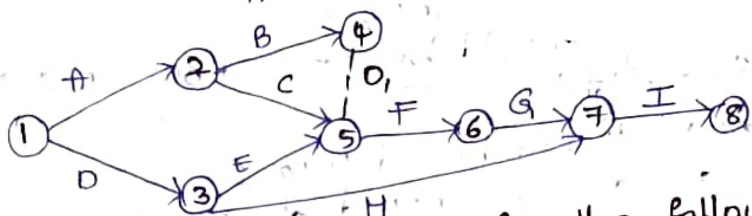
we have to introduce a dummy activity-D_1.









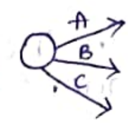


2. Construct a Network diagram for the following:

Activities	Predecessors
A	—
B	—
C	—
D	A
E	B
F	B
G	C
H	D
I	E
J	H, I
K	F, G

Sol.

①



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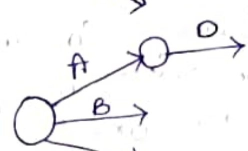

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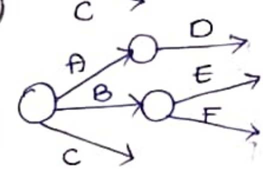

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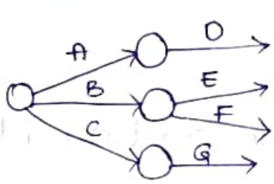

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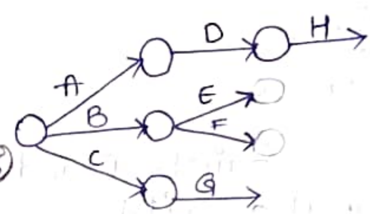

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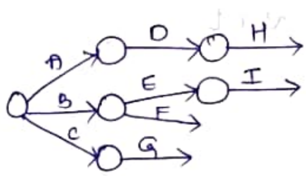

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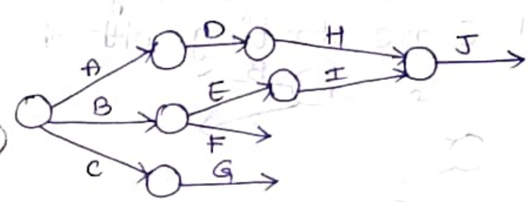

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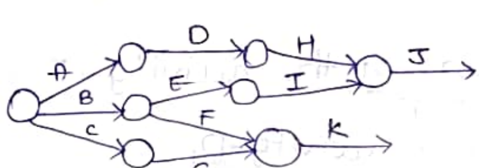

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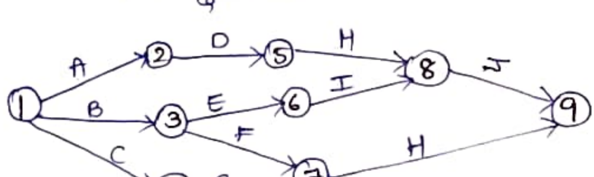
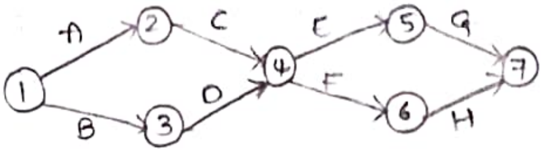


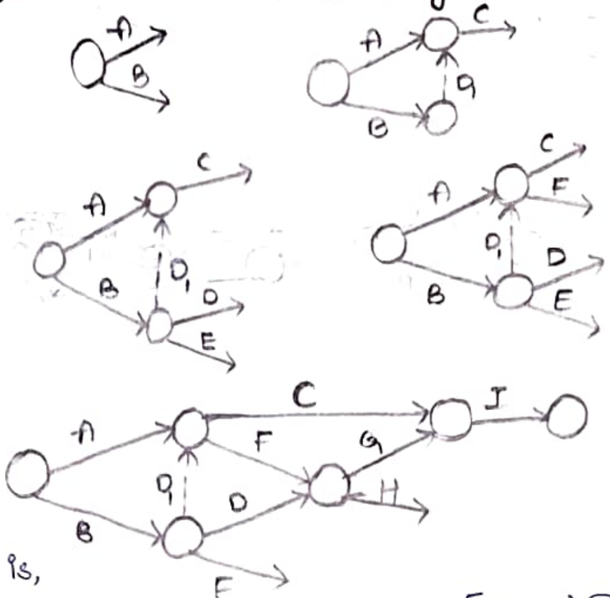
3. Construct a Network diagram for the following.

Activity	predecessors
A	–
B	–
C	A
D	B
E	C,D
F	C,D
G	E
H	F

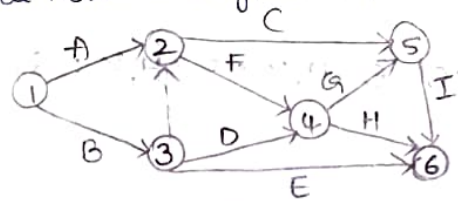


4. Construct a Network diagram for the following.

Activity	predecessors
A	–
B	–
C	A,B
D	B
E	B
F	A,B
G	F,D
H	F,D
I	C,G



∴ The Final Network Diagram is,



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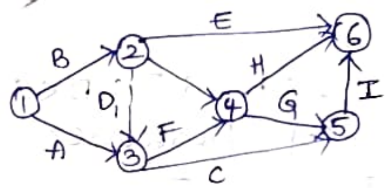


5. Construct a network diagram for the following situation.

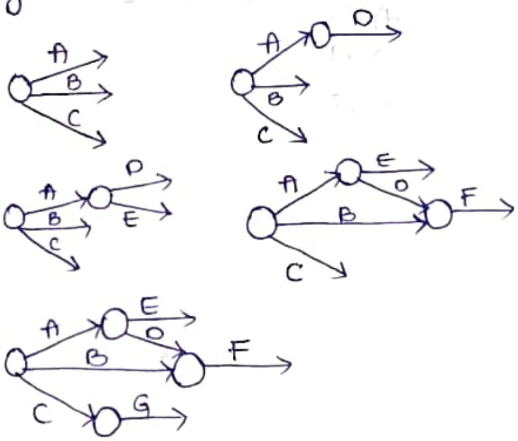
A<D, E ; B, D<F ; C<G ; B<H

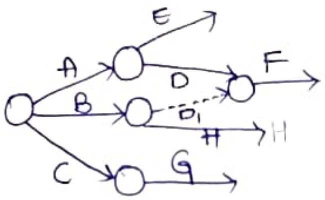
Sol. Given.

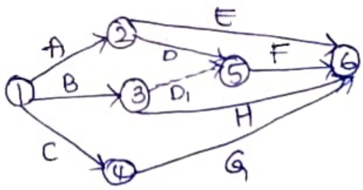
A<D which means that 'D' cannot be started untill 'A' is completed.
i.e, A is the preceeding activity to C.

The above constraints are given in table.

Activity	Predecessors
A	–
B	–
C	–
D	A
E	A
F	B, D
G	C
H	B



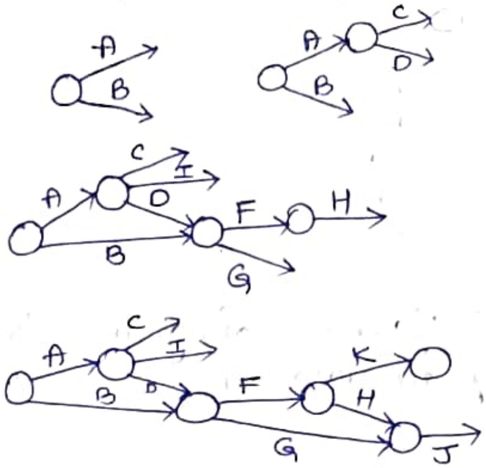


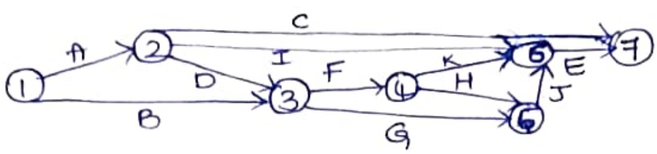


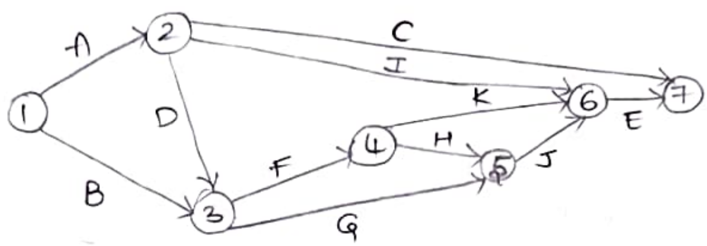
6. A < C, D, I ; B < G, F ; D < G, F ; F < H, K ; G, H < J ; I, J, K < E

Sol.

Activity	Predecessor
A	—
B	—
C	A
D	A
E	I, J, K
F	B, D
G	B, D
H	F
I	A
J	G, H
K	F



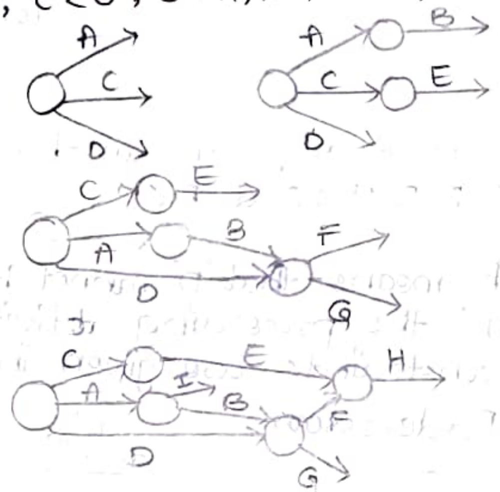


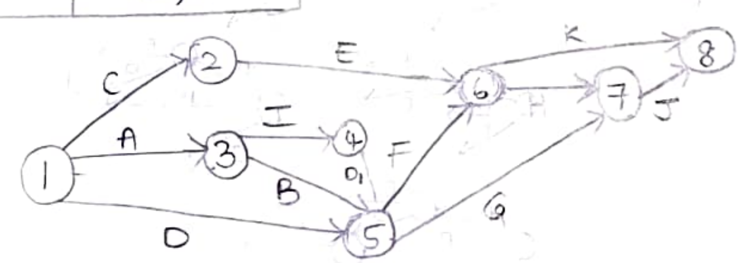


7. A < B, I ; B < G, F ; D < G, F ; C < E ; E < H, K ; F < H, K ; G, H < J

Sol.

Activity	Predecessor
A	—
B	A
C	—
D	—
E	C
F	B, D
G	B, D
H	E, F
I	A
J	G, H
K	E, F





* **Time analysis:-**

Once the network of a project is construction, the time analysis of the network becomes essential for planning various activities of the projects. Activity time is a forecast of the time an activity is expected to take from its starting point to its completion. we shall use the following notation for basic scheduling computations:

(i,j) = Activity (i,j) with tail event 'i' and head event 'j'.

T_{ij} = Estimated completion time of an activity (i,j).

ES_{ij} = Earliest starting time of an activity (i,j).

EF_{ij} = Earliest finishing time of an activity (i,j)

LS_{ij} = Latest starting time of an activity (i,j)

LF_{ij} = Latest finishing time of an activity (i,j).

Forward Pass Computations:- (for earliest event time):-

Before starting computations. the occurence time of the initial network event is fixed. The forward pass computation yields the earliest start and earliest finish time for each activity (i,j) and indirectly the earliest occurence time for each event namely 'E_i'. This consists the following steps.

① The computation begin from the start node and move towards the end node. Let, zero be the starting time for the project.

② Earliest start time $ES_{ij} = E_i$ is the earliest possible time when an activity can began, assuming that all the predecessor are also started at their earliest starting time. Earliest finish time of activity (i,j) is the earliest starting time + the activity time.

i.e, $(EF)_{ij} = (ES)_{ij} + t_{ij}$

③ Earliest event time for the event 'j' is the maximum of the earliest finish time of all the activities, ending at that event.

i.e, $E_j = Max(E_i + t_{ij})$

The computed 'E' values are put over the respective rectangles representing each event.

Backward Pass Computations (for Latest allowable time):-

The latest event time (L) indicates the time by which all activities entering into that event must be completed without delaying the completion of the project. These can be calculated by reversing the method of calculations used for the earliest event time. This is done in the following steps.

① For ending event assume $E = L$.

② Latest Finish time for activity (i,j) is the target time for completing the project.

i.e, $(LF)_{ij} = L_j$

③ Latest starting time of the activity (i,j) is equals to Lastest Completion time of (i,j) – the activity time

i.e, $(LS)_{ij} = (LF)_{ij} - t_{ij}$

$= L_j - t_{ij}$

④ Latest event time for event 'i' is the minimum of the Latest start time of all activities from the event. $L_i = \underset{j}{Min}(L_j - t_{ij})$

The computed 'L' values are put over the respective triangles representing each event.

* **Determination of Floats and Slack times:-**

→ Float is defined as the difference b/w the latest and earliest activity time.

→ Slack is defined as the difference b/w the latest and earliest event time.

Hence, the basic difference between the slack and float is that slack is used for events only ~~and~~ whereas float is used for activities. There are three kinds of floats:

1. **Total Float:** The total float of an activity (i,j) denoted by $(TF)_{ij}$ and it is calculated by the formulae

$(TF)_{ij}$ = Latest start – Earliest start for activity (i,j)

$(TF)_{ij} = (LS)_{ij} - (ES)_{ij}$

$= (L_j - E_i) - t_{ij}$

where, E_i and L_j are the earliest time and latest time for the tail event 'i' and head event 'j' and 't_{ij}' is the normal time for the activity (i,j).

2. **Free Float:** The free float for activity (i,j) is denoted by $(FF)_{ij}$ and it is calculated by the formulae

$(FF)_{ij}$ = Total Float – Head event slack

$= (E_j - E_i) - t_{ij}$

Head event slack $= L_j - E_j$

The free float can takes the values from 0 to upto total float. but, it cannot exceed total float.

3. **Independent Float**:- Independent float of an activity (i,j) is denoted by $(IF)_{ij}$ and it can be calculated by the formulae

$$(IF)_{ij} = \text{Free Float} - \text{Tail event slack}$$
$$= (E_j - L_i) - t_{ij}$$

where, Tail event slack $= (L_i - E_i)$

* **Critical Activity**:- The difference between the latest start time and earliest start time of an activity is called total float. Activity with zero total float are known as "Critical activities".

* **Critical Path**:- The sequence of critical activities in a network is called "critical path". It is the longest path in the network from the starting event to the ending event and defines the minimum time required to complete the project. In the network it is denoted by a double line and identifies all the critical activities of the project. Hence, for the activities (i,j) to lie on the critical path following conditions must be satisfied:

① $ES_i = LF_i$
② $ES_j = LF_j$
③ $ES_j - ES_i = LF_j - LF_i = t_{ij}$

* **Critical Path Method (CPM)**:-

The procedure of determining the critical path is as follows:

Step-1: List all the jobs and then draw an arrow (or) Network diagram. Each job is indicated by an arrow with the direction of the arrow show in the sequence of jobs. (The length of the arrows has no significance. The arrows are placed based on the predecessor, successor and concurrent relation within the job) optional

Step-2: Indicate the normal time (t_{ij}) for each activity (i,j) above the arrow, which is deterministic.

Step-3: Calculate the earliest start time and the earliest finish time for each event and write the earliest time 'E_i' for each event 'i' in the □, Also calculate the least finish and latest start time. From this we calculate the least time 'L_j' for each event 'j' and put it in the △.

Step-4: Tabulate the various times, namely, normal time, earliest time and Least time on the arrow diagram.

Step-5: Determine the total float for each activity by taking the difference between the earliest start and the ~~(least time)~~ latest start time.

Step-6: Identify the critical activities and connect them with the beginning and the ending events in the network diagram by double line arrows. This gives the critical path.

Step-7: Calculate the total project duration.

Problem:-

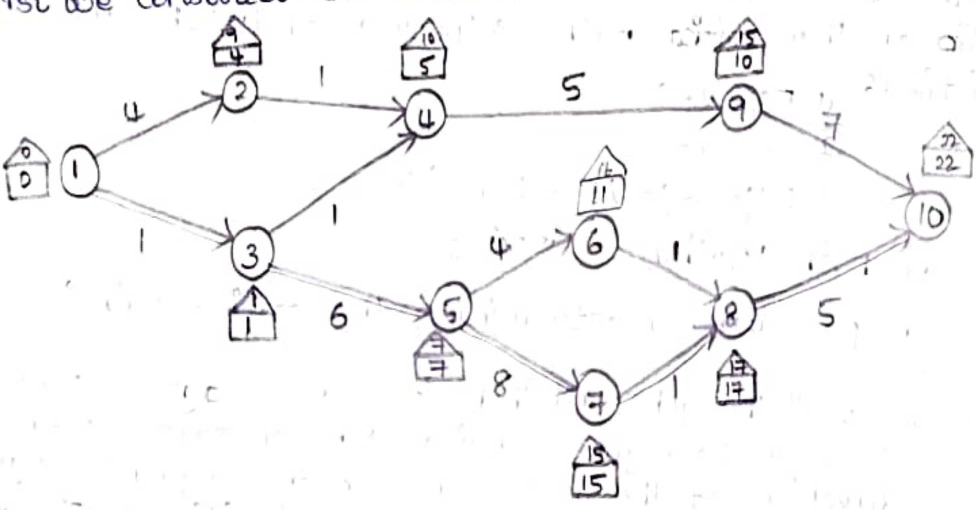
1. A project schedule as the following characteristics

Activity	Time
1-2	4
1-3	1
2-4	1
3-4	1
3-5	6
4-9	5
5-6	4
5-7	8
6-8	1
7-8	1
8-10	5
9-10	7

[From the above information you are required to
① Construct a Network diagram.
② Compute the earliest event time and latest event time.
③ Determine the critical path and the total project duration.
④ Compute the total and free float for each activity] Question

earliest latest time

Sol. First we construct the Network



Activity	Time	Earliest Start	Earliest Finish	Latest Start	Latest Finish	Total Float	Free Float
1-2 ⟶	4	0	4	5	9	5	5-5=0
1-3 ⟶	1	0	1	0	1	0	0
2-4 ⟶	1	4	5	1	10	5	5-5=0
3-4 ⟶	1	1	2	9	10	8	8-8=0
3-5 ⟶	6	1	7	1	7	0	0-0=0
4-9 ⟶	5	5	10	10	15	5	5-5=0
5-6 ⟶	4	7	11	12	16	5	5-5=0
5-7 ⟶	8	7	15	7	15	0	0-0=0
6-8 ⟶	1	11	12	16	17	5	5-5=0
7-8 ⟶	1	15	17	15	17	0	0-0=0
8-10 ⟶	5	17	22	17	22	0	0-0=0
9-10 ⟶	7	10	17	15	22	5	5-5=0

Forward Pass Calculation:- In this we estimate the earliest start and Finish time (ES)$_i$ the earliest start time for the event 'i' is given by $E_j = \max(ES_i + t_{ij})$

$E_1 = E_1 = 0$

$E_2 = E_2 = E_1 + t_{12} = 0 + 4 = 4$

$E_3 = E_1 + t_{13} = 0 + 1 = 1$

$E_4 = Max(E_2 + t_{24}, E_3 + t_{34}) = Max(4+1, 1+1) = Max(5, 2) = 5$

$E_5 = E_3 + t_{35} = 1 + 6 = 7$

$E_6 = E_5 + t_{56} = 7 + 4 = 11$

$E_7 = E_5 + t_{59} = 7 + 8 = 15$

$E_8 = Max(E_6 + t_{68}, E_7 + t_{78}) = Max(11+1, 15+2) = Max(12, 17) = 17$

$E_9 = E_4 + t_{49} = 5 + 5 = 10$

$E_{10} = Max(E_9 + t_{9,10}, E_8 + t_{8,10}) = Max(10+7, 17+5) = Max(17, 22) = 22$

Backward Pass Calculation :- In this we calculate the latest finish and the latest start time, the latest time 'L' for an event 'i' is given by $L_i = \min(LF_j - t_{ij})$ where LF_{ij} is the latest finish time for the event j, t_{ij} is the Normal time of the activity.

$$L_i = \min(LF_j - t_{ij})$$

$L_{10} = 22$, $L_9 = L_{10} - t_{9,10} = 22 - 7 = 15$

$L_8 = L_{10} - t_{8,10} = 22 - 5 = 17$, $L_7 = L_8 - t_{7,8} = 17 - 2 = 15$

$L_6 = L_8 - t_{6,8} = 17 - 1 = 16$

$L_5 = Min(L_6 - t_{5,6}, L_7 - t_{5,7}) = Min(16-4, 15-8) = Min(12, 7) = 7$

$L_4 = L_9 - t_{4,9} = 15 - 5 = 10$

$L_3 = Min(L_4 - t_{3,4}, L_5 - t_{3,5}) = Min(10-1, 7-6) = Min(9, 1) = 1$

$L_2 = L_4 - t_{2,4} = 10 - 1 = 9$

$L_1 = Min(L_2 - t_{1,2}, L_3 - t_{1,3}) = Min(9-4, 1-1) = Min(5, 0) = 0$

From the above table we observe that 1–3, 3–5, 5–7, 7–8, 8–10 of the critical path activities as their total float is 'zero'. Hence, we have the following critical path.

$1 \to 3 \to 5 \to 7 \to 8 \to 10$ with the total project duration are 22 days.

2. A small maintance project consist of the following Jobs, whose precedence relationships are given below.

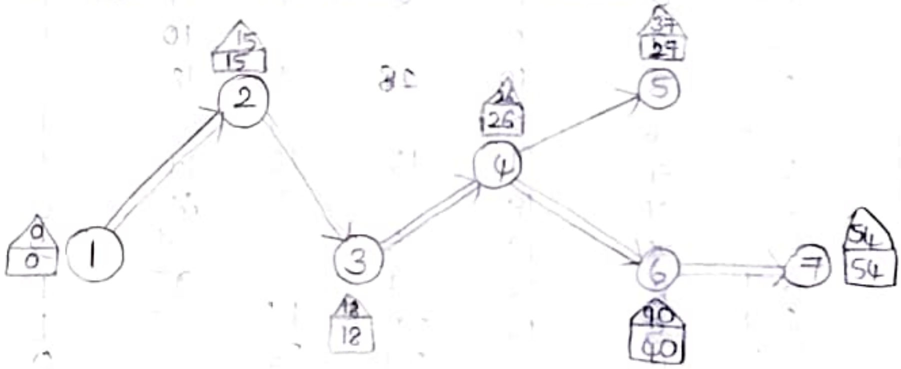
Jobs	Duration (in days)
1-2	15
1-3	15
2-3	3
2-5	5
3-4	8
3-6	12
4-5	1
4-6	14
5-6	3
6-7	14

① Draw an arrow diagram representing the project.

② Find the total float an each activity.

③ Find the critical path and project duration

Sol. First we construct the network.



The following table gives the critical path as well as the total float

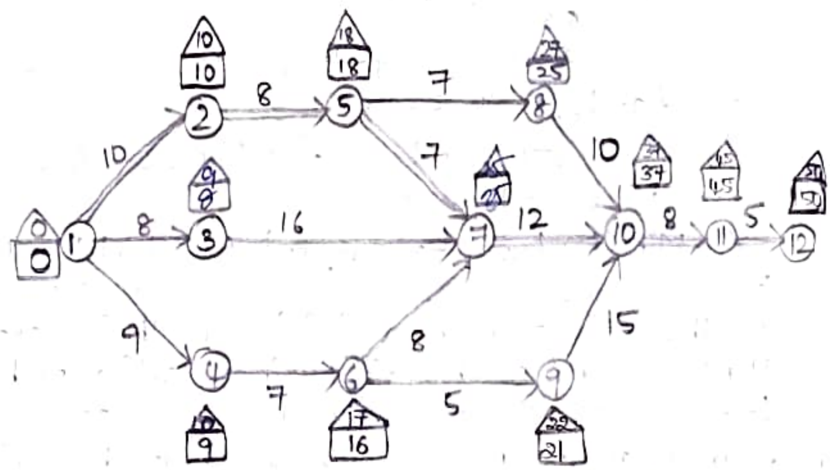
Jobs	Duration (in days)	Earliest S	Earliest F	Latest S	Latest F	Total Float L-E
1-2	15	0	15	0	15	0
1-3	15	0	15	3	18	3
2-3	3	15	18	15	18	0
2-5	5	15	20	32	37	17
3-4	8	18	26	18	26	0
3-6	12	18	30	28	40	10
4-5	1	26	27	36	37	10
4-6	14	26	40	26	40	0
5-6	3	27	30	37	40	10
6-7	14	40	54	40	54	0

From the above diagram table we observe that
1-2, 2-3, 3-4, 4-6, 6-7
Hence, we have following critical path $1 \to 2 \to 3 \to 4 \to 6 \to 7$
with the total project duration is 54 days.

3. A project schedule as the following characteristics

Jobs	duration
1-2	10
1-3	8
1-4	9
2-5	8
3-7	16
4-6	7
5-7	7
5-8	7
6-7	8
6-9	5
7-10	12
8-10	10
9-10	15
10-11	8
11-12	5

sol. First we construct the Network.



The following table gives the critical path as well as the total and free floats.

TF-(L-E)   FF-(L-E)

Job	duration	Earliest S	Earliest F(EFj)	Lastest S	Lastest F(LFj)	Total float	Free Float	Independent (FF-TES) Float
1-2	10	0	10	0	10	0	0-0=0	0
1-3	8	0	8	1	9	1	1-1=0	0
1-4	9	0	9	1	10	1	1-1=0	0
2-5	8	10	18	10	18	0	0-0=0	0
3-7	16	8	24	9	25	1	1-0=1	1-1=0
4-6	7	9	16	10	17	1	1-1=0	0-[illegible]=-1=0
5-7	7	18	25	18	25	0	0-0=0	0
5-8	7	18	25	20	27	2	2-2=0	0
6-7	8	16	24	17	25	1	1-0=1	1-1=0
6-9	5	16	21	17	22	1	1-1=0	0-1=-1=0
7-10	12	25	37	25	37	0	0-0=0	0
8-10	10	25	35	27	37	2	2-0=2	2-2=0
9-10	15	21	36	22	37	1	1-0=1	1-1=0
10-11	8	37	45	37	45	0	0-0=0	0
11-12	5	45	50	45	50	0	0-0=0	0

From the above table we observe that

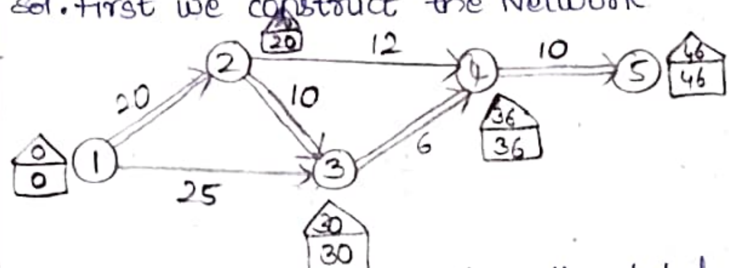
1-2, 2-5, 5-7, 7-10, 10-11, 11-12

Hence, we have following critical path 1→2→5→7→10→11→12 with the total project duration is 50 days

4.

Activity	Duration (Days)
1-2	20
1-3	25
2-3	10
2-4	12
3-4	6
4-5	10

Sol. First we construct the Network



The following table gives the critical path as well as the total and free floats.

Activity	Duration (Days)	Earliest S	Earliest F	Latest S	Latest F	Total Float	Free Float TF − HES
1-2	20	0	20	0	20	0	0−0=0
1-3	25	0	25	5	30	5	5−0=5
2-3	10	20	30	20	30	0	0−0=0
2-4	12	20	32	24	36	4	4−0=4
3-4	6	30	36	30	36	0	0−0=0
4-5	10	36	46	36	46	0	0−0=0

From the above diagram table we observe that,

1-2, 2-3, 3-4, 4-5

Hence, we have following critical path 1→2→3→4→5 with the total project duration is 46 days.

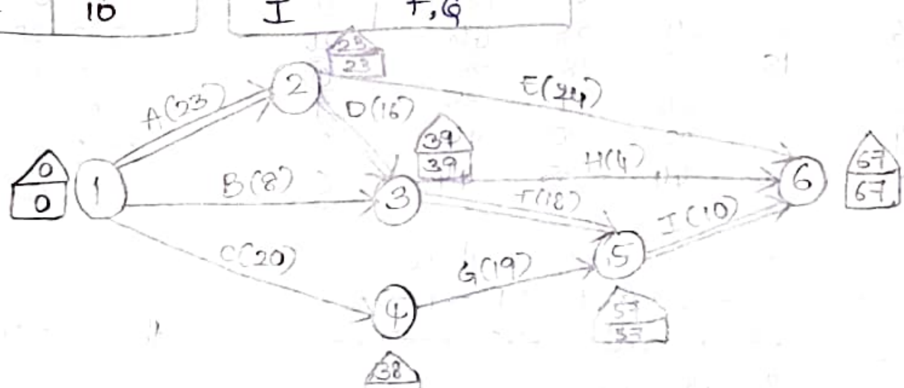
5. A project consists of a series of tasks labeled A, B, ---- H, I with the following constraints A<D, E; B, D<F; C<G; B<H; F, G<I; W<X, Y means X, Y cannot start until W is completed. You are required to construct a network using this notation also find the minimum time of completion of the project when the time of completion of each task as given as follows.

Task	Time (Days)
A	23
B	8
C	20
D	16
E	24
F	18
G	19
H	4
I	10

Activity	Preceeding Activity
A	–
B	–
C	–
D	A
E	A
F	B, D
G	C
H	B
I	F, G

To determine the minimum time of completion of the project and compute ES_{ij} and LF_{ij} for each of the activity (i, j) of the project.

The critical path calculation are as follows:-



From the above diagram we observe that

1-2, 2-3, 3-5, 5-6 (or) A, D, F, I

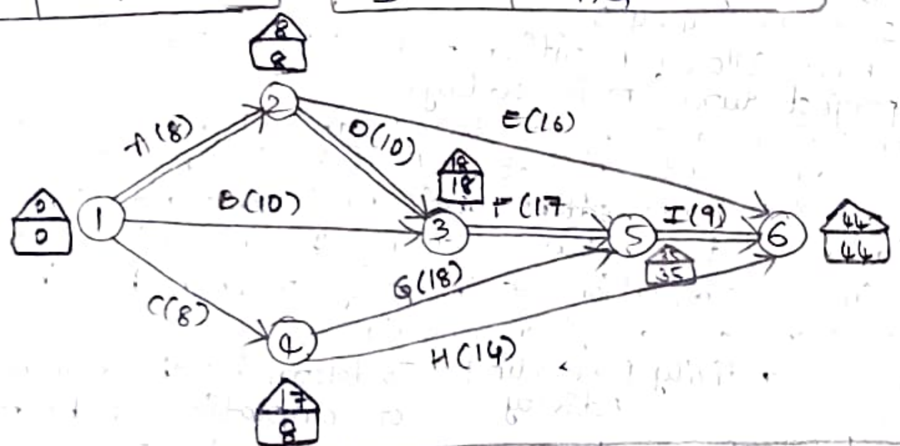
Hence we have critical path 1→2→3→5→6 with the total project duration is 67 days.

Jobs	Duration	Earliest S	Earliest F	Latest S	Latest F	Total Float	Free Float
A (1-2)	23	0	23	0	23	0	0-0=0
B (1-3)	8	0	8	31	39	31	31-0=31
C (1-4)	20	0	20	18	38	18	18-18=0
D (2-3)	16	23	39	23	39	0	0-0=0
E (2-6)	24	23	47	43	67	20	20-0=20
F (3-5)	18	39	57	39	57	0	0-0=0
G (4-5)	19	20	39	38	57	18	18-0=18
H (3-6)	4	39	43	63	67	24	24-0=24
I (5-6)	10	57	67	57	67	0	0-0=0

6. A<D ; A<E ; B<F ; D<F ; C<G ; C<H ; F<I ; G<I

Task	Time (Days)
A	8
B	10
C	8
D	10
E	16
F	17
G	18
H	14
I	9

Activity	Preceeding Activity
A	–
B	–
C	–
D	A
E	A
F	B, D
G	C
H	C
I	F, G



Task	Time	Earliest S	Earliest F	Latest S	Latest F	Total Float	Free Float
A (1-2)	8	0	8	0	8	0	0-0=0
B (1-3)	10	0	10	8	18	8	8-0=8
C (1-4)	8	0	8	9	17	9	9-9=0
D (2-3)	10	8	18	8	18	0	0-0=0
E (2-6)	16	8	24	28	44	20	20-0=20
F (3-5)	17	18	35	18	35	0	0-0=0
G (4-5)	18	8	26	17	35	9	9-0=9
H (4-6)	14	8	22	30	44	22	22-0=22
I (5-6)	9	35	44	35	44	0	0-0=0

From the above table we observe that
1-2, 2-3, 3-5, 5-6 or A → D → F → I
Hence, we have critical path 1→2→3→5→6 with the total project duration is 44 days.

* Program Evaluation and Review Technique (PERT):-

The activities are non-deterministic in nature, PERT was developed. Hence, PERT is a probabilistic method, where the activity time is represented by a probability distribution. This distribution of activity time is based on 3 different time estimates made for each activity which are as follows:

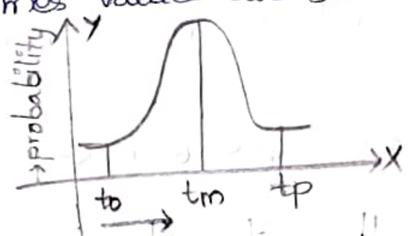
1. Optimistic time Estimate.
2. Most likely time Estimate.
3. Pessimistic time Estimate.

1. Optimistic time Estimate:- It is the smallest time taken to complete the activity if everything goes well. It is denoted by 't_o' (or) 'a'.

2. Most likely time Estimate:- It refer to the estimate of the normal time the activity would take this assumes normal delays. It is the mode of the probability distribution. It is denoted by 't_m' or 'm'.

3. Pessimistic Time Estimate:- It is the longest time that an activity would take, if everything goes wrong. It is denoted by 't_p' (or) 'b'.

These 3 times values are shown in the following figure.



From these 3 time estimates, we have to calculate the expected time of an activity. It is given by the weighted average of the 3 time estimates

$$t_e = \frac{t_o + 4t_m + t_p}{6} \quad \text{or} \quad t_e = \frac{a+4m+b}{6}$$

Variance of the Activity is given by $\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$ (or) $\sigma^2 = \left[\frac{b-a}{6}\right]^2$

The expected length is denoted by T_e or T_e of the entire project length of the critical path. That is the sum of the T_e's of all the activities along the critical path.

The main objective of the analysing through PERT is to find the completion for a particular event within the specified date T_s is given by $P(Z \le D)$ where

$$D = \frac{\text{Due date} - \text{Expected date of Completion}}{\sqrt{\text{Project Variance}}}$$

Here 'z' stands for standard normal variate.

PERT Procedure:-

Step-1:- Draw the project Network.

Step-2:- Compute the expected duration of each activity using the formulae, $t_e = \frac{t_o + 4t_m + t_p}{6}$

Also calculate the expected variance of each activity

$$\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2 \text{ (or) } \sigma^2 = \left[\frac{b-a}{6}\right]^2$$

Step-3:- Compute the earliest start, earliest finish, Latest start, Latest finish, total float for each activity.

Step-4:- Find the critical path and identify the critical activities

Step-5:- Compute the project length variance σ^2, which is the sum of the variances of all the critical activities and hence find the standard deviation of the project length 'σ'.

Step-6: Calculate the standard normal variate $\frac{T_s - T_e}{\sigma}$ where, T_s is the schedule time to complete the project, T_e is the normal expected project length duration, σ is the standard deviation of the project length. Using the normal curve we can estimate the probability of completing the project within a specified time.

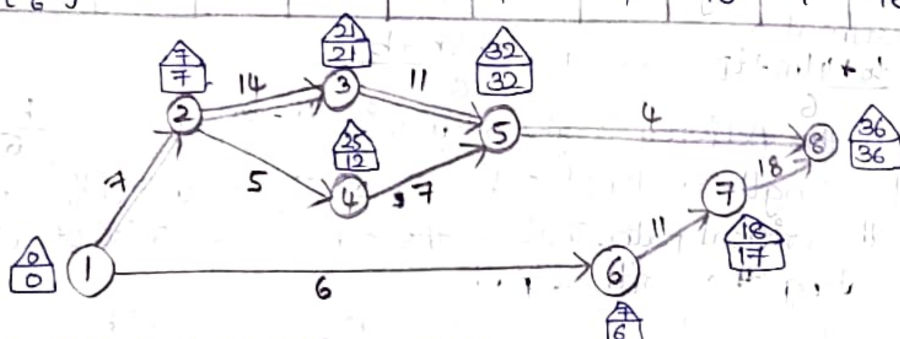
1. The following table shows the jobs of a network along with their time estimates.

Job	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
a(days)	1	2	2	2	7	5	5	3	8
m(days)	7	5	14	5	10	5	8	3	17
b(days)	13	14	26	8	19	17	29	9	32

Draw the project network and find the probability of the project completing in 40 days.

Sol. First we calculated the expected time and Standard deviation for each activity

Job	1-2	1-6	2-3	2-4	3-5	4-5	6-7	5-8	7-8
Expected time, $t_e = \frac{a+4m+b}{6}$	7	6	14	5	11	7	11	4	18
Variance, $\sigma^2 = \left[\frac{b-a}{6}\right]^2$	4	4	16	1	4	4	16	1	16



Expected Project duration = 36 days

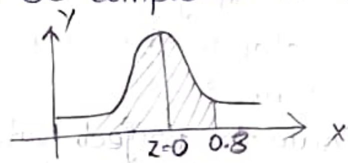
Critical Path 1-2-3-5-8

Project length variance, $\sigma^2 = 4+16+4+1 = 25$

$\therefore$ Standard deviation, $\sigma = 5$

The probability that the project will be completed in 40 days is given by $P(Z \leq D)$

$$D = \frac{T_s - T_e}{\sigma} = \frac{40-36}{5} = \frac{4}{5} = 0.8$$



Area under the normal curve for the region $Z \leq 0.8$

$\Rightarrow P(Z \leq 0.8) = 0.5 + P(0 < Z < 0.8) = 0.5 + 0.2881 = 0.7881$

$P(Z \leq 0.8) = 78.81\%$.

If the project is performed 100 times under the same condition there will be 78.81 occasions for this job to be completed in 40 days.

2. A small project is composed of 7 activities, whose time estimates are ~~ff~~ listed in the following table as follows:

Activity	1-2	1-3	2-4	2-5	3-5	4-6	5-6
Optimistic (weeks)	1	1	2	1	2	2	3
Most likely (weeks)	1	4	2	1	5	5	6
Pessimistic (weeks)	7	7	8	1	14	8	15

You are required to (i) Draw the project network

(ii). Find the expected duration and variance of each activity.

(iii). Calculate the earliest and latest occurence of each event and expected project length.

(iv). Calculate variance and standard deviation of project ~~of~~ length.

(v). What is the probability that the project will be completed

a. Four weeks earlier than expected..

b. not more than four weeks later than expected.

c. If the project due date is 19 weeks what is the probability of meeting the due date.

Activity	1-2	1-3	2-4	2-5	3-5	4-6	5-6
$t_e = \frac{a+4m+b}{6}$	2	4	3	1	6	5	7
$\sigma^2 = \left[\frac{t_b - t_a}{6}\right]^2$	1	1	1	0	4	1	4

Earliest and Latest occurence of each event :-

$E_1 = 0$

$E_2 = E_1 + t_{12} = 0 + 2 = 2$

$E_3 = E_1 + t_{13} = 0 + 4 = 4$

$E_4 = E_2 + t_{24} = 2 + 3 = 5$

$E_5 = Max(E_2 + t_{25}, E_3 + t_{35})$

$= Max(2+1, 4+6) = Max(3, 10) = 10$

$E_6 = Max(E_4 + t_{46}, E_5 + t_{56})$

$= Max(5+5, 10+7) = Max(10, 17) = 17$

$L_6 = 17$

$L_5 = L_6 - t_{56} = 17 - 7 = 10$

$L_4 = L_6 - t_{46} = 17 - 5 = 12$

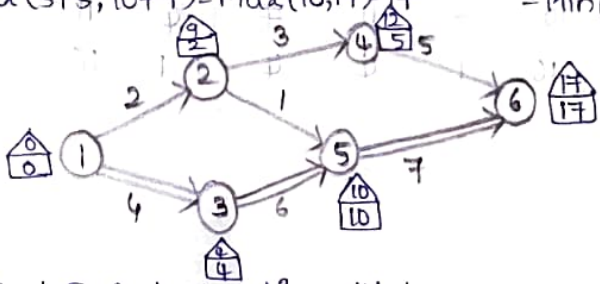
$L_3 = L_5 - t_{35} = 10 - 6 = 4$

$L_2 = Min(L_4 - t_{24}, L_5 - t_{25})$

$= Min(12-3, 10-1) = Min(9, 9) = 9$

$L_1 = Min(L_2 - t_{12}, L_3 - t_{13})$

$= Min(9-2, 4-4) = Min(7, 0) = 0$



Expected Project duration = 17 days

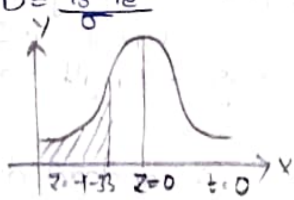
critical path 1-3-5-6

Project length variance, $\sigma^2 = 1 + 4 + 4 = 9$

Standard deviation, $\sigma = 3$

V.a) The probability of completing the project within 4 weeks earlier than expected is given by $P(Z \le D)$ wher $D = \frac{T_s - T_e}{\sigma}$

$D = \frac{17-4-17}{3} = \frac{13-17}{3} = \frac{-4}{3} = -1.33$



Area under the normal curve for the region

$Z \le -1.33$

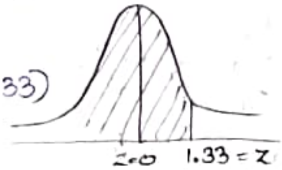
$P(Z \le D) = P(Z \le -1.33) = 0.5 - P(-1.33 < Z < 0)$

$= 0.5 - 0.4082 = 0.0918 = 9.18\%$

If the project is performed 100 times under the same condition there will be 9.18% or 9 occations for this job to be completed 4 weeks earlier than expected.

b). The probability of completing the project not more than

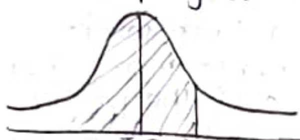
four weeks later than expected is given by $P(Z \le D)$
where, $D = \frac{T_s - T_e}{\sigma} = \frac{17+4-17}{3} = \frac{4}{3} = 1.33$



Area under the normal curve for the $P(Z \le 1.33)$
$P(Z \le 1.33) = 0.5 + P(0 < Z < 1.33) = 0.5 + 0.4082$
$= 0.9082 = 90.82\%$.
If the project is performed 100 times under the same condition there will be 90.82% occassions for this job to completed not more than 4 weeks later than expected.

c. The probability of meeting the due date. If the project due date is 19 weeks is given by $P(Z \le 0)$

$D = \frac{T_s - T_e}{\sigma} = \frac{19-17}{3} = \frac{2}{3} = 0.6667$



Area under the normal curve for the region $z \le 0.6667$ z=0 z=0.6667
$P(Z \le 0.6667) = 0.5 + P(0 < Z < 0.6667) = 0.5 + 0.2514 = 0.7514 = 75.14\%$.
If the project is performed 100 times under the same condition there will be 75.14% occassions for this job to be completed in 19 weeks from due data.

3.

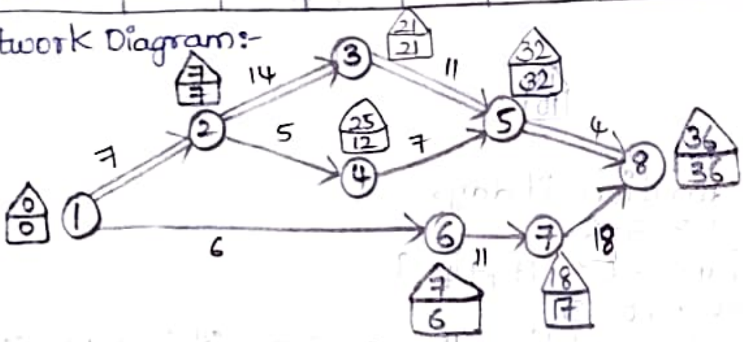
Job	1-2	1-6	2-3	2-4	3-5	4-5	5-8	6-7	7-8
a	3	2	6	2	5	3	1	3	4
m	6	5	12	5	11	6	4	9	19
b	15	14	30	8	17	15	7	27	28

(i). Draw the network.
(ii). Find the critical path
(iii). Find the probability of the project being completed in 31 days.

Sol.

Job	1-2	1-6	2-3	2-4	3-5	4-5	5-8	6-7	7-8
$t_e = \frac{a+4m+b}{6}$	7	6	14	5	11	7	4	11	18
$\sigma^2 = \left[\frac{b-a}{6}\right]^2$	4	4	16	1	4	4	1	16	16

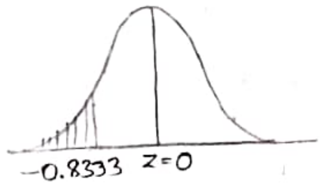
(i) Network Diagram:-



∴ Expected project duration = 36 days
(ii). Critical path 1-2-3-5-8
Project length variance, $\sigma^2 = 4+16+4+1 = 25$
$\sigma^2 = 25$

∴ Standard deviation, $SD = \sigma = 5$
(iii) The probability of project being completed within 31 days is given by, $P(Z \le D)$; $D = \frac{31-36}{5} = \frac{-\cancel{5}}{\cancel{5}} = -1$
Area under the normal curve for the given region $Z \le -1$
$P(Z \le -1) = 0.5 - P(-1 < Z < 0) = 0.5 - 0.3413 = 0.1587$
$P(Z \le -1) = 15.8\%$.

Conclusion:- If the project is performed too times under the same conditions then there will be 15.87% occassions when this job will be completed in 31 days

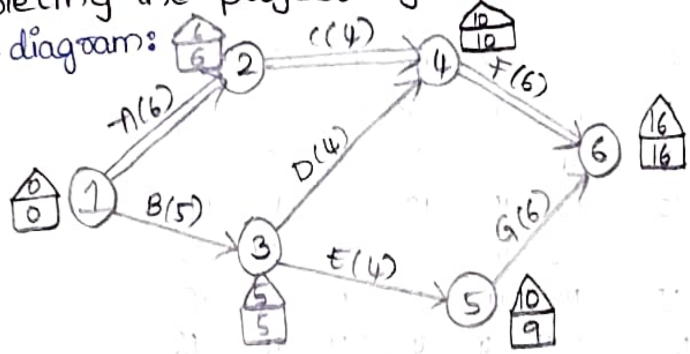


4. Consider the following project.

Activity	A	B	C	D	E	F	G
t_o	3	2	2	2	1	4	1
t_m	6	5	4	3	3	6	5
t_p	9	8	6	10	11	8	15
Predecessors	none	none	A	B	B	C,D	E

find the path and standard deviation also find the probability of completing the project by 18 weeks.

Sol. Network diagram:



∴ Expected project duration is 16 weeks

∴ Critical path is 1-2-4-6

i.e, A-C-F

Activity	t_o	t_m	t_p	$t_e = \frac{t_o+4t_m+t_p}{6}$	$\sigma^2 = \left[\frac{t_p-t_o}{6}\right]^2$
A	3	6	9	6	1
B	2	5	8	5	1
C	2	4	6	4	0.4444
D	2	3	10	4	1.778
E	1	3	11	4	2.7778
F	4	6	8	6	0.4444
G	1	5	15	6	5.4444

∴ The project length variance is $\sigma^2 = 1+0.4444+0.4444$ | $\sigma^2 = 1.8884$

∴ standard deviation, $\sigma = 1.3744$

∴ The probability of project being completed within 18 weeks is given by, $P(Z \le D)$

$$\therefore D = \frac{T_s - T_e}{\sigma} = \frac{18-16}{1.3744} = \frac{2}{1.3744} = 1.4552$$

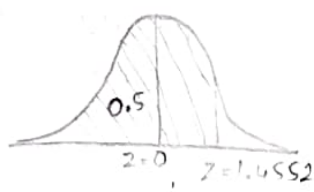
Area under the normal curve for given region $Z \le 1.4552$

i.e, $P(Z \le 1.4552) = 0.5 + P(0<Z<1.4552) = 0.5 + 0.4265 = 0.9265$

$= 92.65\%$

Conclusion:

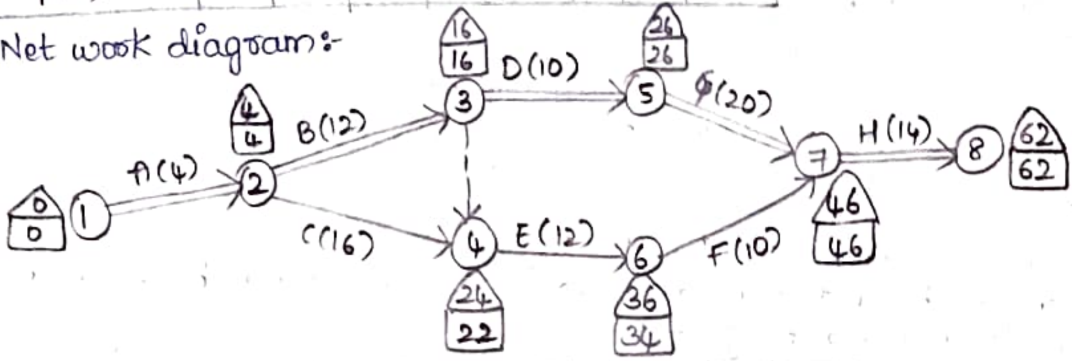
If the project is performed 100 times under the same conditions then there will be 92.65% occassions. When this job will be completed in 18 weeks.



5. What is the problem that the project will require atleast 75 days

Activity	A	B	C	D	E	F	G	H
predesser	—	A	A	B	C,B	E	D	F,G
to (a)	2	8	14	4	6	6	18	8
tm (m)	4	12	16	10	12	8	18	14
tp (b)	6	16	30	16	18	22	30	32

Sol. i. Net work diagram:-



Activity	A	B	C	D	E	F	G	H
predessor	—	A	A	B	C,B	E	D	F,G
a	2	8	14	4	6	6	18	8
m	4	12	16	10	12	8	18	14
b	6	16	30	16	18	22	30	32
te	4	12	18	10	12	10	20	16
σ^2	0.4444	1.7778	7.1111	4	4	7.1111	4	16

∴ Expected project duration = 62 weeks

ii. Critical path: 1-2-3-5-7-8 i.e, A-B-D-G-H

∴ The project length variance, $\sigma^2 = 0.4444 + 1.7778 + 4 + 4 + 16$

$\sigma^2 = 26.2222$

∴ standard deviation $\sigma = 5.1208$

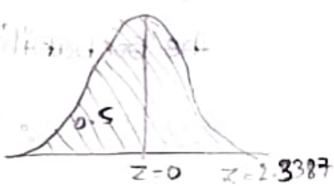
iii. The probability of project being completed within 75 days is given by: $P(Z \leq D)$; $D = \frac{75-62}{5.1208} = \frac{13}{5.1208} = 2.5387$

Area under the normal curve for the given region $z \leq 2.5387$

$P(Z \leq 2.5387) = 0.5 + P(0 < Z < 2.5387) = 0.5 + 0.4945 = 0.9945 = 99.45\%$

Conclusion:-

If the project is performed 100 times under the same conditions then there will be 99.45% occasions when this job will be completed in 75 days.



6.

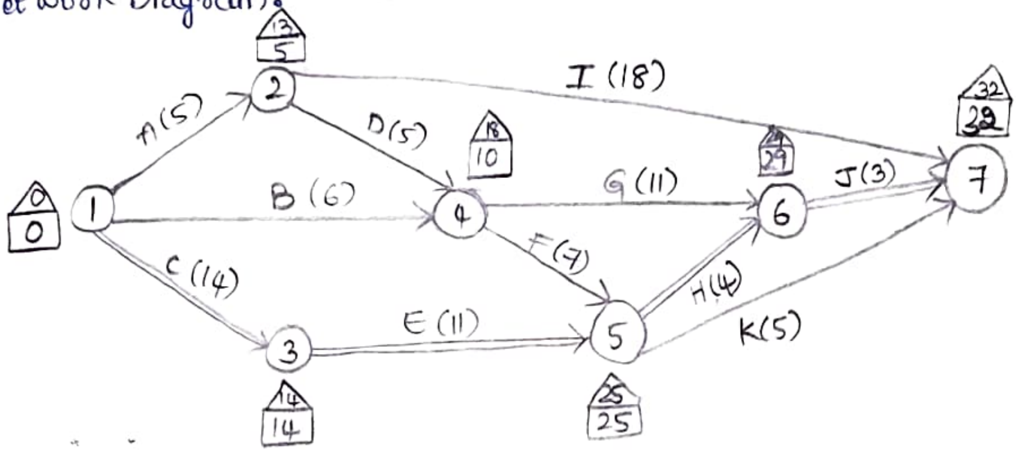
Activity	A	B	C	D	E	F	G	H	I	J	K
a	3	2	6	2	5	3	3	1	4	1	2
m	6	5	12	5	11	6	9	4	19	2	4
b	5	14	30	8	17	15	27	7	28	9	12

A, B and C can start simultaneous, A<D,I; B<G,F; D<G,F; C<E; E<H,K; F<H,K; G,H<J. What is the ~~problem~~ prob that the project will be completed 8 days later than expected

Activity	Predecessor	a	m	b	te	σ^2
A	–	3	6	5	5.3333	0.1111
B	–	2	5	14	6	4
C	–	6	12	30	14	16
D	A	2	5	8	5	1
E	C	5	11	17	11	4
F	B,D	3	6	15	7	4
G	B,D	3	9	27	11	16
H	E,F	1	4	7	4	1
I	A	4	19	28	108/6=18	16
J	G,H	1	2	9	3	1.7778
K	E,F	2	4	12	5	2.7778

62 93

Net work Diagram:-



∴ Expected project duration = 32 days

∴ critical path is 1–4–5–6–7 i.e, C–E–H–J

∴ The project Length variance $\sigma^2 = 16+4+1+1.7778$

$\sigma^2 = 22.7778$

∴ standard deviation : $\sigma = 4.7726$

(iii) The probability of project being completed 2 days later than expected is given by : $P(Z \leq D)$

$$\therefore D = \frac{32+2-32}{4.7726} = 0.4191$$

Area under the normal curve for the given region $Z \leq 0.4191$

$P(0 < Z < 0.4191) = 0.5 + P(0 < Z < 0.4191) = 0.5 + 0.1628 = 0.6628 = 66.28\%$

Conclusion:-

If the project is performed 100 times under the same conditions ~~the~~ then there will be 66.64% occassions when this job will be completed 2 days later than expected