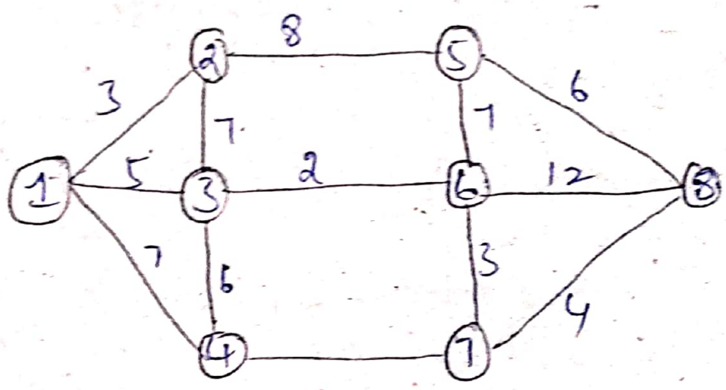
UNIT - III

Network Techniques

A network consist of a set of nodes and a set of arcs. Each node represents a location and each arc represents the connection between two different locations. The number on each arc represents the distance/time/cost/flow between the two locations.

Ex:



Types of Network techniques :- There are 3 types of Network techniques. They are

1. Shortest Path Model.
2. Minimum Spanning trees model
3. Maximal Flow Model.

1. **Shortest Path model** :- In transport organisation one of the objectives is to find the shortest path to a particular node from any of the other nodes in a road network for shipping carg

Determination of the shortest path using specialized procedures is known as shortest path model. The following methods are used to find the shortest path in a distance network.

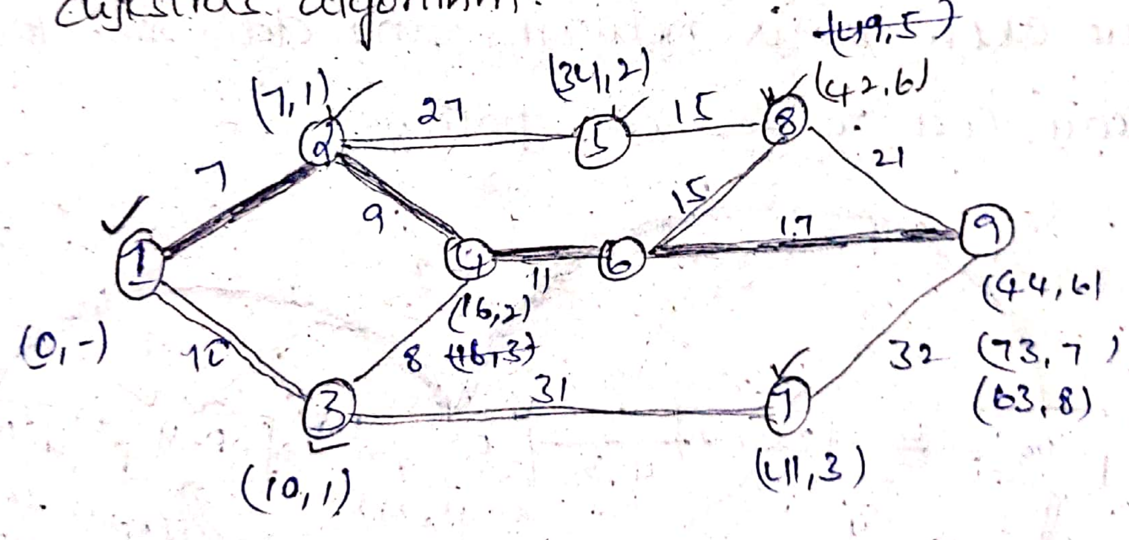
a) Systematic method

b) Dijkstra's algorithm

c) Floyd's algorithm.

b) Dijkstra's algorithm :-

→ Find the shortest path from node 1 to node 9 of the distance network shown below using dijkstra's algorithm.



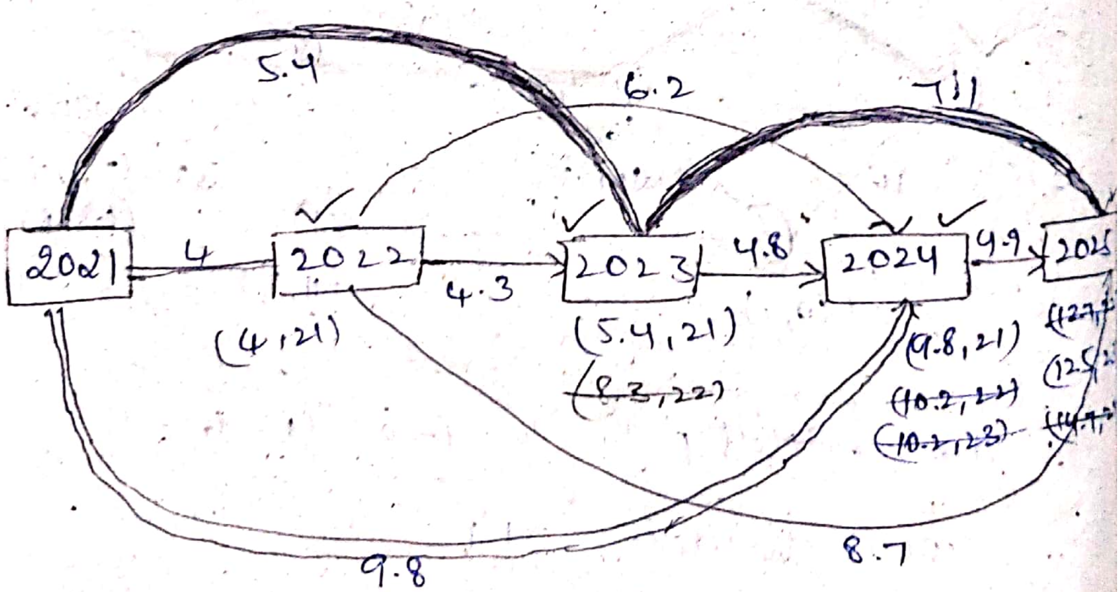
∴ The shortest path is 1 - 2 - 4 - 6 - 9.

Total distance = 44.

→ The following table gives replacement cost as a function of the year the car was purchased and the number of years it has been in use

Purchased at start of	Replacement cost (in 1000's) after using till end of year		
	1	2	3
2021	4.0	5.4	9.8
2022	4.3	6.2	8.7
2023	4.8	7.1	–
2024	4.9	–	–

At the start of each year a decision is made whether a car should be replaced. However a car must be in service for atleast one year and maximum of 3 years. Formulate the above as a network and determine the least cost replacement policy.



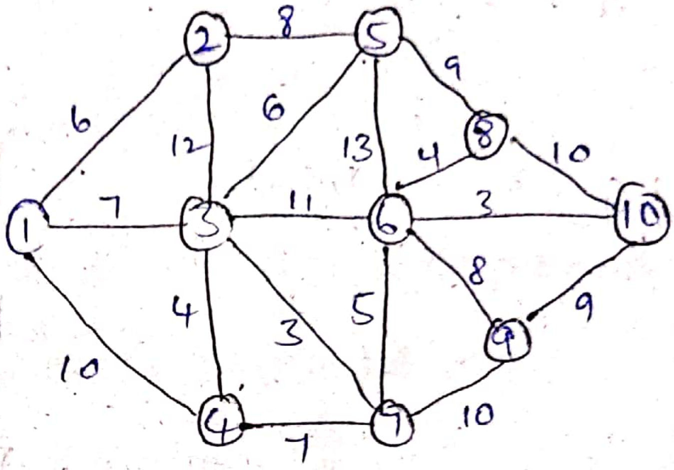
A car purchased in the year 2021 can be replaced in the year 2023 & 2025

The minimum cost of replacement = 12.5 lakhs

replacing

a) Systematic Method :-

→ Consider the road network shown below, where distance between different pairs of adjacent cities are summarized. Find the shortest path from city 1 to city 10. Using systematic method



Sol:-

1	2	3	4	5
1-2 6	2-1 6	3-1 7	4-3 4	~~5-3 6~~
1-3 7	~~2-3 12~~	~~3-2 12~~	4-7 7	5-2 8
1-4 10	2-5 8	3-7 3	~~4-1 10~~	5-8 9
		3-4 4		5-6 13
		3-5 6		
		3-6 11		

6	7	8	9	10
6-10 3	7-3 3	8-6 4	9-6 8	10-6 3
6-8 4	7-6 5	8-5 9	9-10 9	10-9 9
6-7 5	7-4 7	8-10 10	9-7 10	10-8 10
6-9 8	7-9 10			
~~6-3 11~~				
6-5 13				

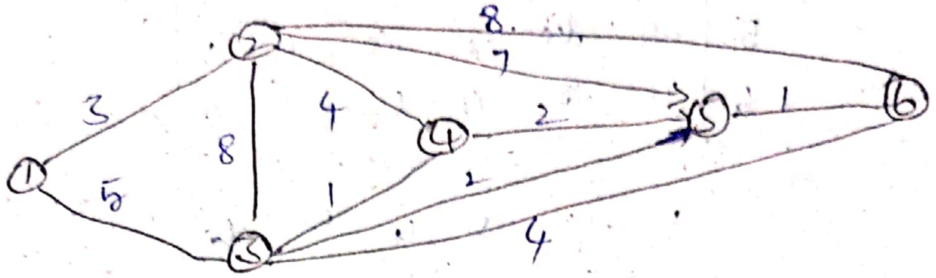
Iteration NO.	Nodes included	Nearest Nodes	Distance Calculation.
		$\boxed{1}$	0
1		$\boxed{2}$	0+6 =6
2	1	$\boxed{3}$	0+7 =7
3	1, 2	5	6+8 =14
4	1, 2, 3	$\boxed{4}$	0+10 =10
		5	6+8 =14
		$\boxed{7}$	7+3 =10
5	1, 2, 3, 7	$\boxed{4}$	0+10 =10
		5	6+8 =14
		4	7+4 =11
		6	10+5 =15
6	1, 2, 3, 7, 4	5	6+8 =14
		$\boxed{5}$	9+6 =13
		6	10+5 =15
7	3, 7, 5	6	7+11 =18
		$\boxed{6}$	10+5 =15
		8	13+9 =22
8	3, 7, 5, 6	9	10+10 =20
		8	13+9 =22
		$\boxed{10}$	15+3 =18

Shortest path is $1 \to 3 \to 7 \to 6 \to 10$

Shortest distance = 18Km.

Floyds Algorithm :-

Concider the dictance network shown below.



Find the shortest distance path and corresponding distan[ce] from the source need to the destinated node as indicated each of the cases.

i) 1–6 ii) 5–1 (iii) 5–2.

Sol:-

$$D = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 3 & 5 & \infty & \infty & \infty \\ 2 & 3 & 0 & 8 & 4 & 7 & 8 \\ 3 & 5 & 8 & 0 & 1 & 2 & 4 \\ 4 & \infty & 4 & 1 & 0 & 2 & \infty \\ 5 & \infty & \infty & 2 & \infty & 0 & 1 \\ 6 & \infty & 8 & 4 & \infty & 1 & 0 \end{array}$$

Calculate the distance matrix using the formula

$$D_{ij}^{k} = \min\left[D_{ij}^{k}, D_{ik}^{k-1} + R_{kj}^{k-1}\right]$$

$$D_1 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 3 & 5 & \infty & \infty & \infty \\ 2 & 3 & 0 & 8 & 4 & 7 & 8 \\ 3 & 5 & 8 & 0 & 1 & 2 & 4 \\ 4 & \infty & 4 & 1 & 0 & 2 & \infty \\ 5 & \infty & \infty & 2 & \infty & 0 & 1 \\ 6 & \infty & 8 & 4 & \infty & 1 & 0 \end{array}$$

$$D_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 7 & 10 & 11 \\ 3 & 0 & 8 & 4 & 7 & 8 \\ 5 & 8 & 0 & 1 & 2 & 4 \\ 7 & 8 & 1 & 0 & 2 & 12 \\ \infty & \infty & 2 & \infty & 0 & 1 \\ 11 & 8 & 4 & 12 & 1 & 0 \end{bmatrix} \end{array}$$

$$D_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 & 7 & 9 \\ 3 & 0 & 8 & 4 & 7 & 8 \\ 5 & 8 & 0 & 1 & 2 & 4 \\ 6 & 4 & 1 & 0 & 2 & 5 \\ 7 & 10 & 2 & 3 & 0 & 1 \\ 9 & 8 & 4 & 5 & 1 & 0 \end{bmatrix} \end{array}$$

$$D_4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 & 7 & 9 \\ 3 & 0 & 5 & 4 & 6 & 8 \\ 5 & 5 & 0 & 1 & 2 & 4 \\ 6 & 4 & 1 & 0 & 2 & 5 \\ 7 & 7 & 2 & 3 & 0 & 1 \\ 9 & 8 & 4 & 5 & 1 & 0 \end{bmatrix} \end{array}$$

$$D_5 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 3 & 5 & 6 & 7 & 8 \\ 6 & 0 & 5 & 4 & 6 & 8 \\ 5 & 5 & 0 & 1 & 2 & 3 \\ 6 & 4 & 1 & 0 & 2 & 3 \\ 7 & 7 & 2 & 3 & 0 & 1 \\ 8 & 8 & 3 & 4 & 1 & 0 \end{bmatrix} \end{array}$$

$$D_6 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 3 & 5 & 6 & 7 & 8 \\ 2 & 6 & 0 & 5 & 4 & 6 & 8 \\ 3 & 5 & 5 & 0 & 1 & 2 & 3 \\ 4 & 6 & 4 & 1 & 0 & 9 & 3 \\ 5 & 7 & 7 & 2 & 3 & 0 & 1 \\ 6 & 8 & 8 & 3 & 4 & 1 & 0 \end{array}$$

The shortest path from node 1 to node 6 is

$1 \to 3 \to 5 \to 6$, shortest distance is 8 km.

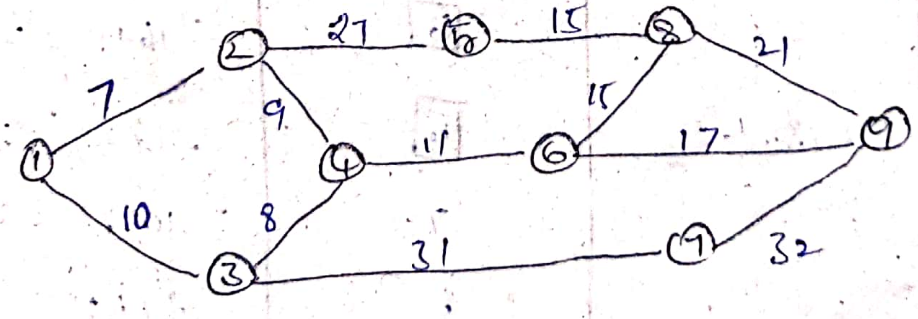
The shortest path from node 5 to 1 is

$5 \to 3 \to 1$, shortest distance is 7 km

The shortest path from node 5 to 2 is

$5 \to 3 \to 4 \to 2$, shortest distance is 7 km.

2. Findout the shortest path from node 1 to node 9 of the distance network shown below using systematic method.



Sol

1	2	3	4	5
~~1-2 7~~ ✓	~~2-1 7~~ ✓	~~3-4 8~~ ✓	~~4-3 8~~ ✓	~~5-8 15~~
~~1-3 10~~ ✓	~~2-4 9~~ ✓	3-1 10 ✓	~~4-2 9~~ ✓	~~5-2 27~~
	~~2-5 27~~ ✓	3-7 31 ✓	4-6 11 ✓	

6	7	8	9
~~6-4 11~~	~~7-3 31~~	~~8-5 15~~	~~9-6 17~~
6-8 11	7-9 32	8-6 15	~~9-8 21~~
~~6-9 17~~		8-9 21	~~9-7 32~~

Iteration no	nodes included	Nearest nodes	distance Calculation
1		$\boxed{1}$	0
2	1	$\boxed{2}$	0+7 =7
3	1, 2	$\boxed{3}$	0+10 =10
		4	7+9 =16
4	2, 3	$\boxed{4}$	7+9 =16
		4	10+8 =18
5	2, 3, 4	5	7+27 =34
		7	10+31 =41
		$\boxed{6}$	16+11 =27
6	2, 3, 6	$\boxed{5}$	7+27 =34
		7	10+31 =41
		8	27+15 =42
7	3, 6, 5	$\boxed{7}$	10+31 =41
		8	27+15 =42
		8	34+15 =49

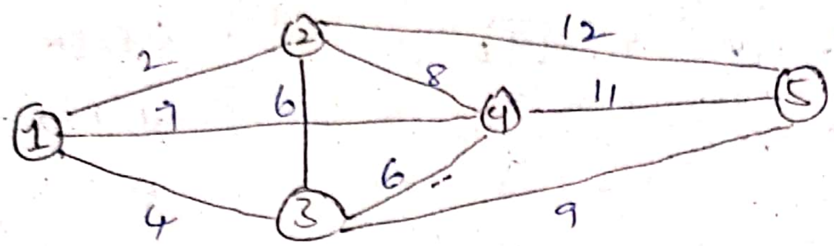
8	6, 5, 7	$\boxed{8}$	27 + 15 = 42
		8	34 + 15 = 49
		9	41 + 32 = 73
9	6, 7, 8	$\boxed{9}$	27 + 17 = 44
		9	41 + 32 = 73
		9	42 + 21 = 63

The shortest path from node 1 to node 9 is

$1 \rightarrow 2 \rightarrow 4 \rightarrow 6 \rightarrow 9.$

Shortest path = 44.

"Findout the shortest path from node 1 to node 5 using Floyd's algorithm.



	1	2	3	4	5
1	0	2	4	7	∞
2	2	~~0~~	6	8	12
D_0 = 3	4	6	0	6	9
4	7	8	6	0	11
5	∞	12	9	11	0

D_1 =

–	2	4	7	∞
2	–	6	8	12
4	6	–	6	9
7	8	6	–	11
∞	12	9	11	–

D_2 =

–	2	4	7	14
2	–	6	8	12
4	6	–	6	9
7	8	6	–	11
14	12	9	11	–

-	2	4	7	13
2	-	6	8	12
4	6	-	6	9
7	8	6	-	11
13	12	9	11	-

-	2	4	7	13
2	-	6	8	12
4	6	-	6	9
7	8	6	-	11
13	12	9	11	-

D_5 =

-	2	4	7	13
2	-	6	8	12
4	6	-	6	9
7	8	6	-	11
13	12	9	11	-

The shortest path from node 1 to node 5 is

1 - 3 - 5

The shortest distance is 13.

* Minimum Spanning Tree Problem :-

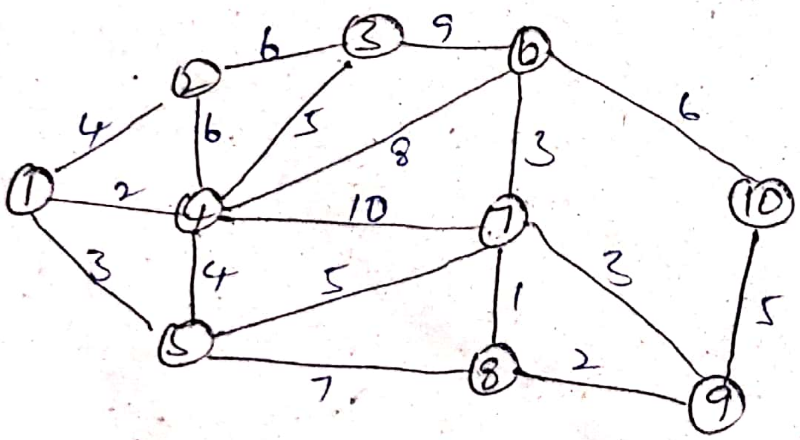
The objective of the minimum spanning tree problem is to connect the nodes of the network by a set of arcs such that the total length of all the arcs is minimized.

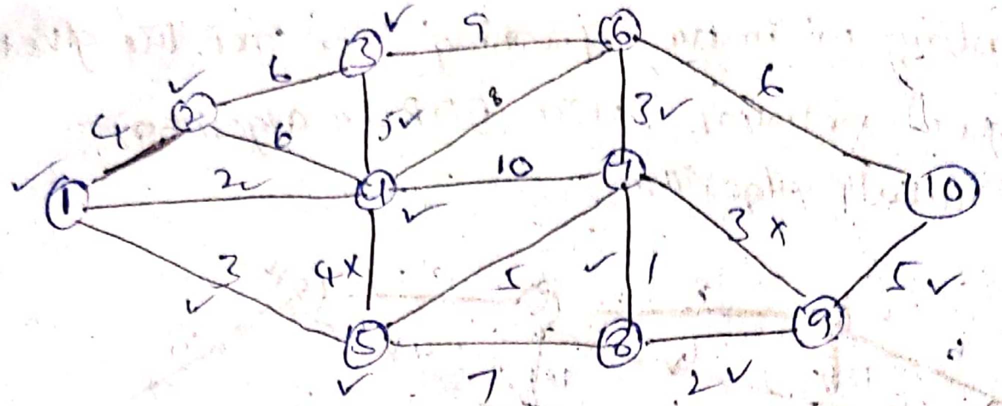
The following are the two methods to find solution for minimum spanning tree problems

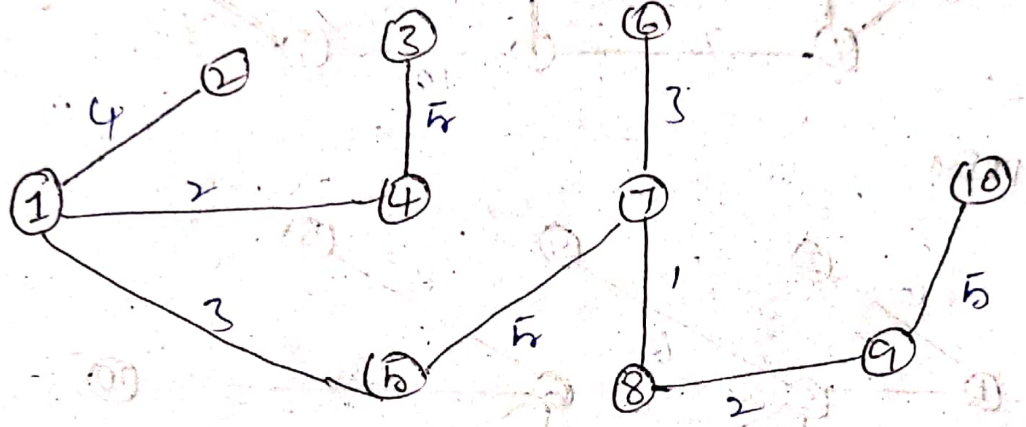
1. Prim (PRIM) Algorithm
2. Kruskal's Algorithm.

1. PRIM Algorithm

→ Consider the distance network given below, find the minimum spanning tree of this network using PRIM algorithm.



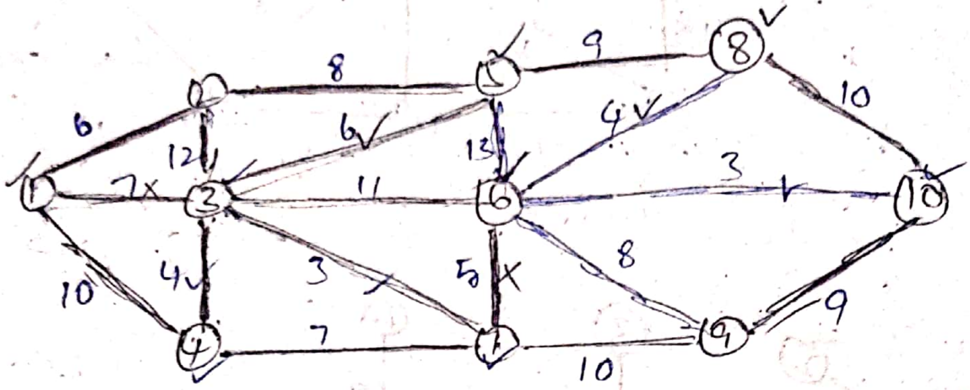




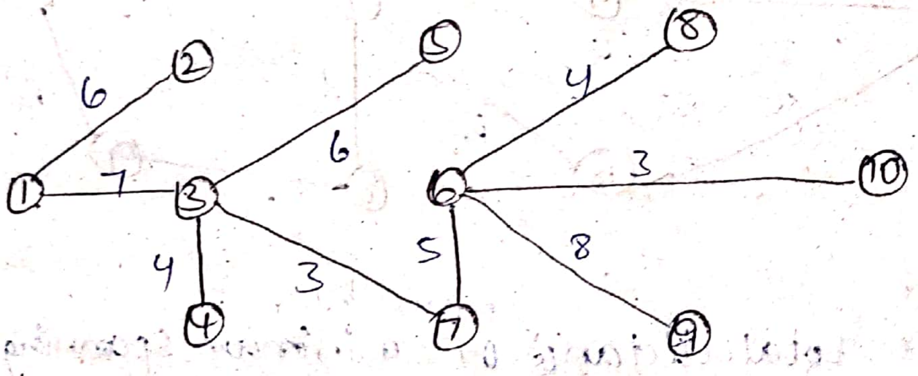
The total distance covered by ~~of~~ minimum spanning Tree ~~distance~~

$4+2+3+5+5+3+1+2+5 = 30$ km

Identify minimum spanning tree for the given distance network using 1) PRIM algorithm.
2) Kruskal's algorithm.

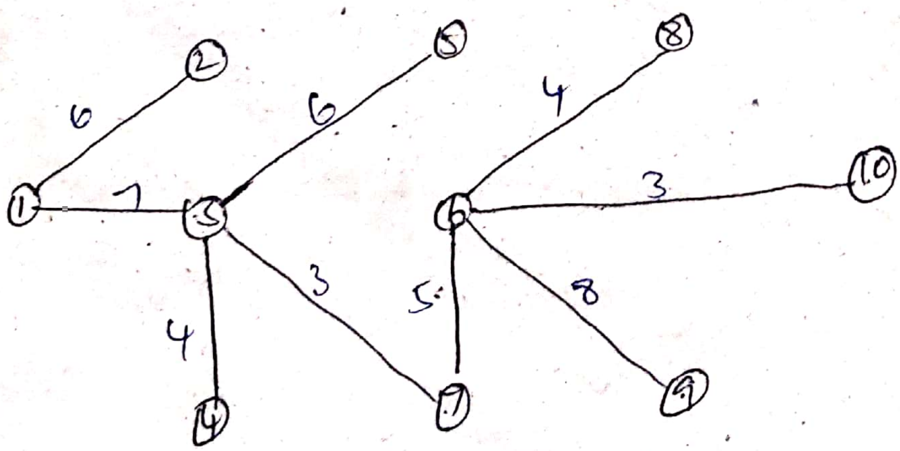


Sol: 1) PRIM



The total distance covered by minimum spanning tree $6+7+4+6+3+5+8+3+4 = 46$ Km.

2) Kruskal's Algorithm :-



* Maximal Flow Problem :-

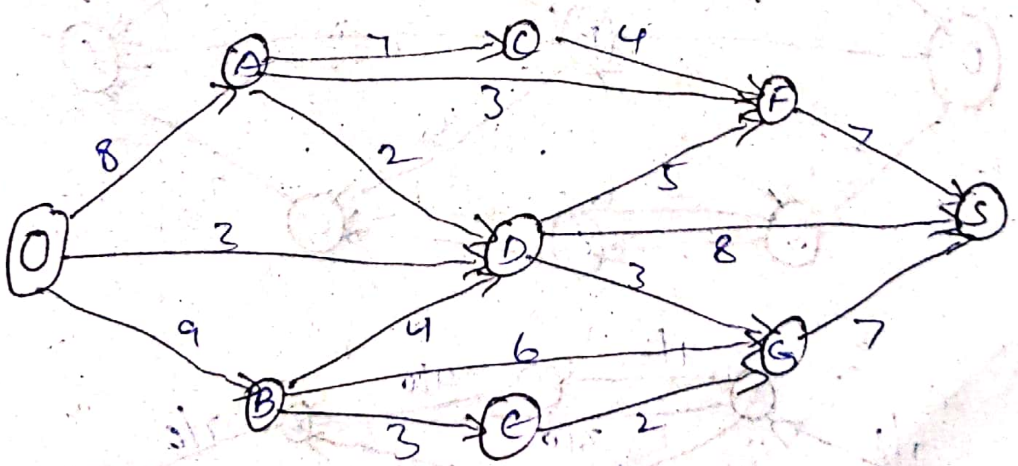
The objective of maximal flow problem is to determine the maximum flow of fluid from a given source node to a given destination node. The maximal flow problem can be solved by using two methods

1) Maximal Flow Algorithm

2) Linear Programming Modeling.

1) Maximal Flow Algorithm :-

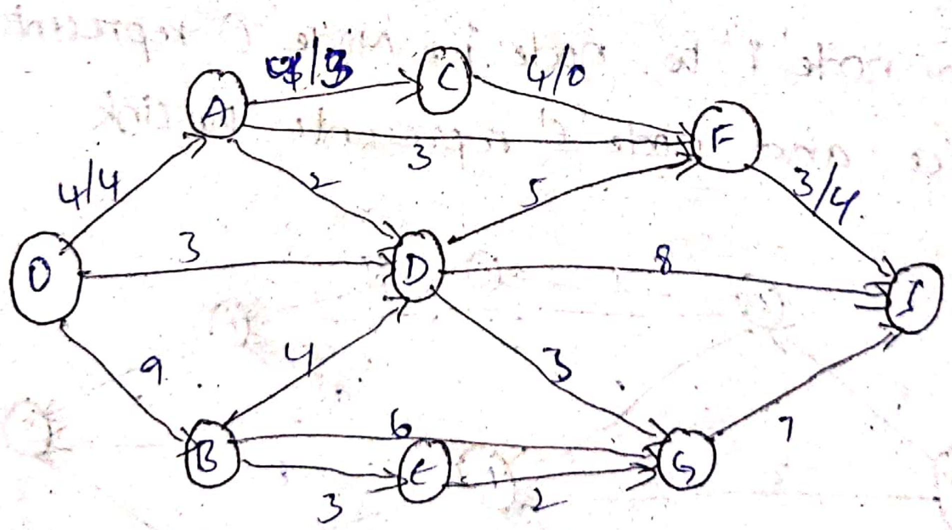
→ For the network shown below use the augmenting path algorithm to find the flow pattern giving the maximum flow from the source to the sink, given that the link capacity from node 'i' to node 'j'. Node 'O' represents the source and node 'S' represents the sink.



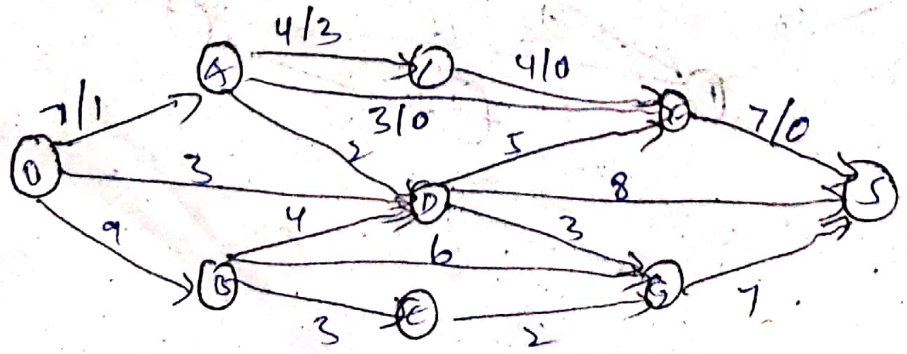
Sol:- Augmented Path | Flow

Augmented Path	Flow
1) O - A - C - F - S	4
2) O - A - F - S	3
3) O - A - D - F - S	0
4) O - A - D - S	1
5) O - A - D - G - S	0
6) O - D - F - S	0
7) O - D - S	3
8) O - D - G - S	0
9) O - B - D - F - S	0
10) O - B - D - S	4
11) O - B - D - G - S	0
12) O - B - G - S	5
13) O - B - E - G - S	0

1)

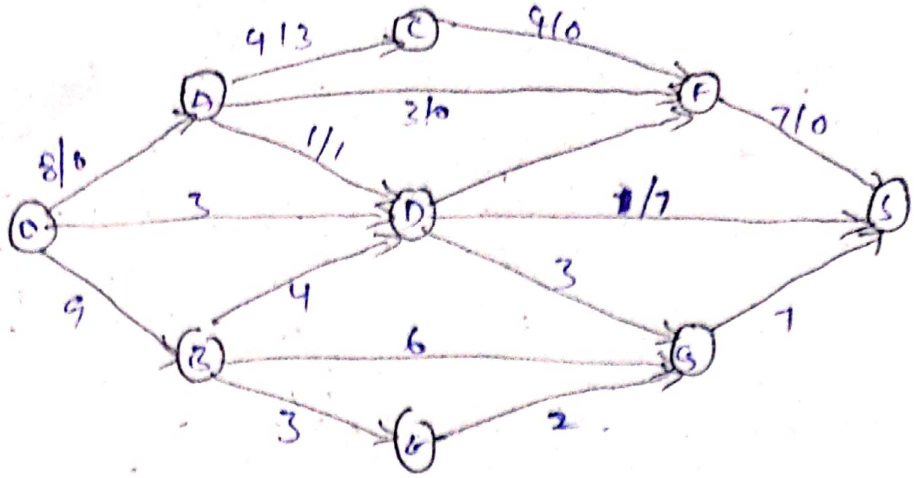


2)

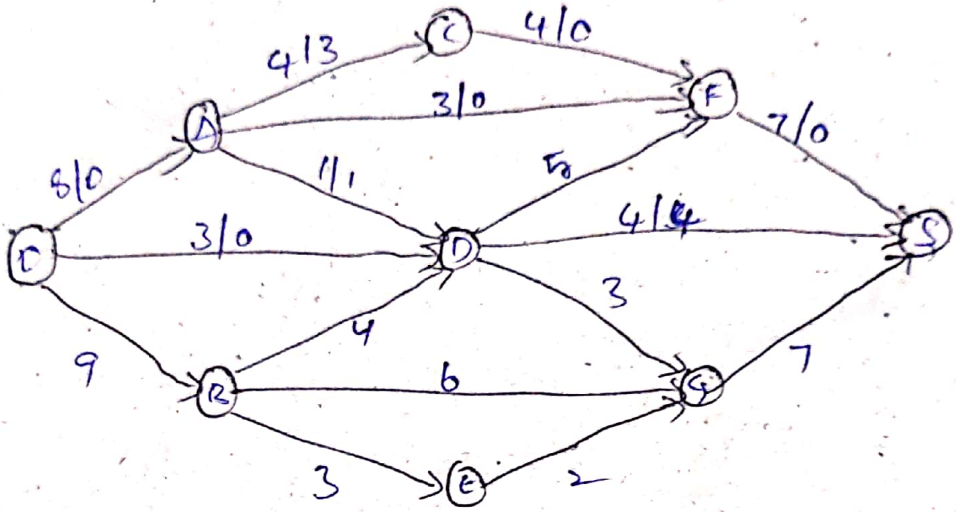


Scanned with CamScanner

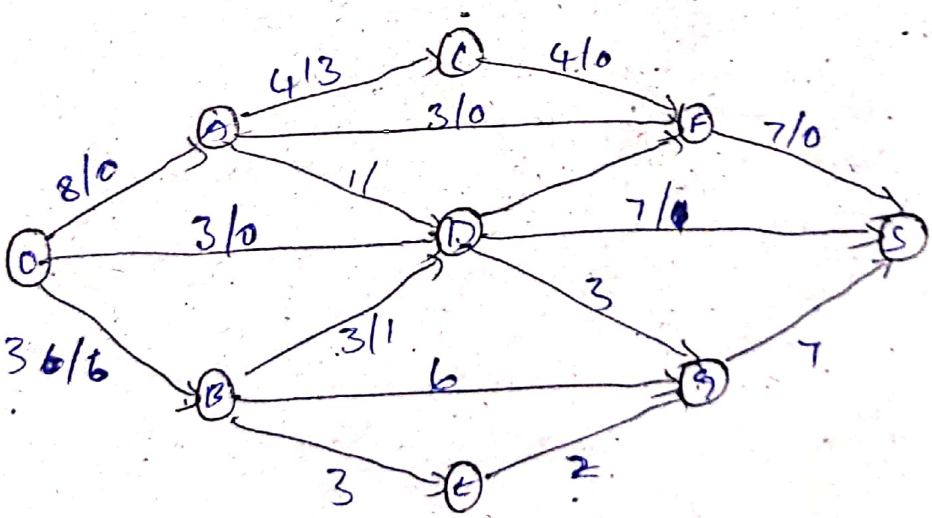
4)


4|3
9|0
3|0
1|1
7|0
8|0
3
4
3
9
6
7
3
2

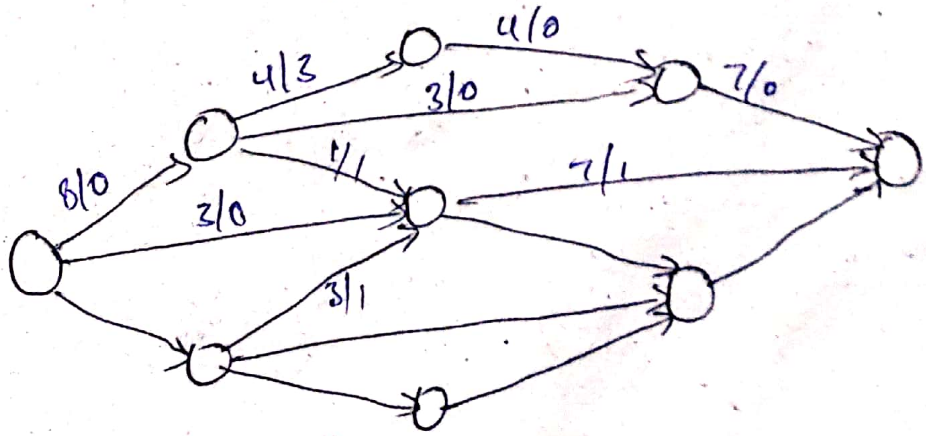
7


4|3
4|0
3|0
7|0
8|0
1|1
3|0
4
9
6
3
7
3
2

10


4|3
4|0
3|0
7|0
8|0
3|0
3
3|1
6
7
3
2

12


4|0
4|3
3|0
7|0
1|1
7|1
8|0
3|0
3|1