LINEAR PROGRAMMING PROBLEMS [LPP]

* **Linear Programming Problem:-**

Linear Programming problem deals with determining optimal allocation (or) optimization (maximization or minimization) of a function of variables known as Objective function. It is subject to a set of Linear equations and / or inequations known as constraints. Linear programming is a mathematical technique which involves the allocation of limited resources in an optimal manner on the basis of a given certain optimality.

The term Linear means that all the variables occur in the objective function and the constraints are the first degree in the problems under consideration and the term programming means the process of determining a particular course of action.

* **General Linear Programming Problem [GLPP] :-**

The General formulation of the LPP can be stated as follows:

Maximization (or) Minimization $Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$ and satisfying 'k' constraints $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = (\text{or}) \leq (\text{or}) \geq b_1$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = (\text{or}) \leq (\text{or}) \geq b_2$$

$$\vdots$$

$$a_{k1}x_1 + a_{k2}x_2 + \cdots + a_{kn}x_n = (\text{or}) \leq (\text{or}) \geq b_k$$

and finally satisfy the non Negativity restrictions (or) Constraints $x_1, x_2, \cdots, x_n > 0$

* **Matrix form of LLP :-**

The LPP can be expressed in the matrix as follows:

Maximum (or) Minimum $Z = CX$

Subject to the Constraints, s.to $AX \leq (\text{or}) = (\text{or}) \geq b$ and $X \geq 0$

where $C = [c_1\ c_2 \cdots c_n]$, $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kn} \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}$

$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

* **Formulation of LPP:-** The procedure for mathematical formulation of a LPP consist the following steps:-

Step-1:- To write down the decision variables of the problem.

Step-2:- To formulate the objective function to be optimized (Maximization or Minimization) as a linear function of the decision variables.

Step-3:- To formulate the other conditions of the problem such as resource limitations, market constraints, Inter-relationship between variables etc. as linear equation (or) Inequations in terms of the decision variables.

Step-4:- To write down the Non-negativity constraints.

1. A manufacturer produces two types of models M_1 and M_2 each model of the type M_1 requires 4 hours of grinding and 2 hours of polishing where as each model of the type M_2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has two grinders and 3 polishers. Each grinder works 40 hours a week, each polisher works 60 hours a week. Profit on M_1 is Rs. 3 and M_2 is Rs. 4. Whatever produced in a week is sold in the market. How should the manufacturer allocate the production capacity to the 2 types of the models so that he make the maximum profit in a week.

Sol. Step-1: Decision Variables. Let x_1, x_2 be the no. of units of M_1 and M_2

Step-2: Objective function.

Since the profit on both the models are given we have to maximize the profit.

maximum $z = 3x_1 + 4x_2$

Step-3: Constraints

There are two constraints, One for Grinding and the other for polishing. No. of hours available on each grinder is 40 hrs per week. There are two grinders hence the manufacturer does not have more than $2 \times 40 = 80$ hrs of grinding M_1 requires 4hrs of grinding and M_2 required 2hrs of grinding.

∴ The grinding constraint is given by $4x_1+2x_2 \le 80$
These are 3 Polishers the available polishers in a week is given by $3 \times 60 = 180$ hrs.
M_1 required 2 hrs of Polishing, M_2 requires 5 hrs of polishing
∴ The polishing constraint is given by $2x_1+5x_2 \le 180$
Step-4 :- Non-Negativity constraints $x_1, x_2 \ge 0$
∴ The LPP is maximum $z = 3x_1+4x_2$ s.to constraints $4x_1+2x_2 \le 80$
$2x_1+5x_2 \le 180$ and $x_1, x_2 \ge 0$.

2. A firm manufacturers two products A and B and sells them at a profit of Rs. 20 on Type-A and Rs. 30 on type-B each product is processed on two machines G and H. type-A requires 1 min of processing time on G and 2 Min on H. Type-B requires 1 min on G and 1 min on H. The machine G is available for not more than 400 min while machine H available for 10 hrs during any working day. ~~Fo~~ formulate the above problem as a Linear programming problem.

Step-1 :- Decision Variables.
Let x_1 and x_2 be the no. of units of A and B
Step-2 :- Objective function.
Since the profit is given then the objective function is maximization.

	Rs 20	Rs 30	
	A	B	
G	1	1	400m
H	2	1	10h 600

Maximum $z = 20x_1+30x_2$
Step-3 :- Constraints.
The time required for A on G is 1 min and for B on G is 1 min. the total availability of G is 400 min. ∴ the G constraint is given by $x_1+x_2 \le 400$
The time required for A on H is 2 min and for B on H is 1 min. the total availability of H is 10 hrs i.e, 600 min.
∴ The H constraint is given by $2x_1+x_2 \le 600$
Step-4 :- Non-Negativity constraints. $x_1, x_2 \ge 0$
∴ The LPP is maximum $z = 20x_1+30x_2$ s.to constraint
$x_1+x_2 \le 400$, $2x_1+x_2 \le 600$ and $x_1, x_2 \ge 0$

3. A company manufacturers two products A and B. These products are processed in the same machine. It takes 10 min to process (one) units of A and 2 min for each unit of B, and the machine operates for a maximum of 35 hrs in a week. Product A requires 1 kg and product B requires 0.5 kg of raw material for unit. The supply of raw material is 600 kg per week. Market constraint on product B is to be minimum of

800 units every week. Product A cost Rs.5 per unit and sold at Rs.10 per unit. Product B cost Rs.6 per unit and can be sold in the market at a unit prices of Rs.8. Formulate the above proble as a linear programming problem to get the maximum profit.

Sol. Step-1 :- Decision Variables.

Let x_1 and x_2 be the no. of units of product A and product B.

Step-2:- Objective function.

The cost of product-A is Rs.5 per unit and sold at Rs.10 per unit.

$\therefore$ The profit on A is $10-5 = Rs.5$.

The cost of product-B is Rs.6 per unit and sold at Rs.8 per unit.

$\therefore$ The profit on B is $8-6 = Rs.2$

$\therefore$ Maximum $z = 5x_1 + 2x_2$.

Step-3:- Constraints.

There are 3 constraints. One for machine, other for raw materials and the another for market demand constraint.

The processing time of product-A is 10 min and product-B is 2 min.

The total availability time of machine is 35 hrs $\times$ 60 = 2100 minutes.

$\therefore 10x_1 + 2x_2 \leq 2100$.

The product-A requires 1 kg and product-B requires 0.5 kg of raw material. The total availability of raw material is 600 kg per week.

$\therefore$ Raw material constraint is $x_1 + 0.5x_2 \leq 600$.

Market constraint on product B is minimum of 800 units every week.

$\therefore x_2 \geq 800$

Step-4:- Non-Negativity constraint $\Rightarrow x_1, x_2 \geq 0$

$\therefore$ The LPP is maximum $z = 5x_1 + 2x_2$ s.to constraint

$10x_1 + 2x_2 \leq 2100$, $x_1 + 0.5x_2 \leq 600$, $x_2 \geq 800$ and $x_1, x_2 \geq 0$

4. A person requires 10, 12 and 12 units chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 unit of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. If a liquid product sells for Rs.3 per jar

and the dry product sells ~~for~~ Rs. 2 per carton. How many of each should be purchased in order to minimize the cost and meet the requirments.

	A	B	C	
L	5	2	1	Sells Rs.3
D	1	2	4	Rs.2
	10	12	12	

Sol. Step-1 :- Decision Variables.

Let x_1, x_2 be the no. of units of Liquid and Dry products.

Step-2: Objective function.

Since, cost of the products are given we have to minimize the cost

Minimum $Z = 3x_1 + 2x_2$.

Step-3: Constraints.

There are three chemicals and their requirements are given. We have 3 constraints for these 3 chemicals.

$$\text{s.to} \Rightarrow 5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12$$
$$x_1 + 4x_2 \geq 12$$

Step-4: Non-Negative constraint $\Rightarrow x_1, x_2 \geq 0$

$\therefore$ The LPP is minimum $Z = 3x_1 + 2x_2$. s.to constraints $5x_1 + x_2 \geq 10$, $2x_1 + 2x_2 \geq 12$, $x_1 + 4x_2 \geq 12$ and $x_1, x_2 \geq 0$:

5. A company manufactures two products A and B. Each unit of B takes twice as long to produce (one) unit of A and if the company was to produce only A. it would have time to produce 2000 units per day. The availability of raw material is sufficient to produce 1500 units per day of both A and B combined. Product B requires a special ingredient only 600 units can be made per day. If A profit of Rs. 2 per unit and B a profit of Rs. 4 unit. Formulate the above problem as a linear programming problem to get the maximum profit.

Max Z

Constraints	A (x_1)	B (x_2)	
1	1	2	2000
2	1	1	1500
3		1	600

Sol. Step-1 :- Decision Variables.

Let x_1 and x_2 be the no. of units of A and B products.

Step-2 :- Objective function.

The profit is given then the objective function is maximum

Max $Z = 2x_1 + 4x_2$

Step-3 :- Constraints.

Since, the company produce at most 2000 units in a day and product-B requires twice as much time as that of product-A. The products restriction is given by, $x_1+2x_2 \leq 2000$.

The raw material is sufficient to produce 1500 units per day If both A and B are combined.

we have, $x_1+x_2 \leq 1500$

A special ingredient, for the product B we have $x_2 \leq 600$.

Step-4:- Non-Negativity Constraints $\Rightarrow x_1 \geq 0$ or $x_2 \geq 0$, (or) $x_1, x_2 \geq 0$

$\therefore$ The LPP is maximum $z = 2x_1+4x_2$ s.to constraints $x_1+2x_2 \leq 2000$, $x_1+x_2 \leq 1500$, $x_2 \leq 600$ and $x_1, x_2 \geq 0$

6. A Firm manufactures three products A, B and C. The profits are Rs.3, Rs.2 and Rs.4 respectively. The firm has two machines and given below is the required processing time in minutes for each machine on each product.

Machines	Products		
	A	B	C
M_1	4	3	5
M_2	3	2	4

machines M_1 and M_2 have 2000 and 2500 minutes respectively. The firm must manufactures 100 A's and 200 B's and 50's but not more than 150 A's. Set up an LPP to maximize the profit.

max z

	x_1 A	x_2 B	x_3 C	
M_1	4	3	5	20[illegible]
M_2	3	2	4	2[illegible]
Rs	3	2	4	

Sol. Step-1:- Decision variables.

Let x_1, x_2 and x_3 be the no. of units of A, B and C products.

Step-2:- Objective function.

Since, profit is given then the objective function is maximized.

$\therefore$ max $z = 3x_1+2x_2+4x_3$

Step-3:- Constraints.

The restrictions on machine M_1 and M_2 are given by 2000 and 2500 min respectively.

$4x_1+3x_2+5x_3 \leq 2000$

$3x_1+2x_2+4x_3 \leq 2500$

Also the firm manufacturers 100 A's, 200 B's and 50 C's. but not more than 150 A's

$$100 \leq x_1 \leq 150$$
$$200 \leq x_2 \geq 0$$
$$50 \leq x_3 \geq 0$$

Step-4:- Non-Negativity constraints. $x_1, x_2, x_3 \geq 0$

$\therefore$ The LPP is maximum $z = 3x_1 + 2x_2 + 4x_3$ S.to Constraints

$$100 \leq x_1 \leq 150,$$
$$200 \leq x_2 \geq 0,$$
$$50 \leq x_3 \geq 0$$
and $x_1, x_2, x_3 \geq 0$

7. A company produces two types of leather belts A and B. A is a superior quality and B is a inferior quality. The respective profits are Rs. 10 and Rs. 5 per belt. The supply of raw material is sufficient for making 850 belts per day. For belt A a special buckle is needed and 500 are available per day. There are 700 buckles for belt-B per day. Belt-A needs twice as much time as that required for belt-B and the company can produce 500 belts if all of them were of the type-A. Formulate the above problem as a LPP.

profit	10	5	
	x_1	x_2	
	A	B	
RM	1	1	850
	1		500
		1	700
	2	1	500

sol. Step-1: Decision variables.

Let x_1, x_2 be the no. of units of A and B belts.

Step-2: Objective function.

Since, profit is given then the Objective function is maximization

$\therefore \max z = 10x_1 + 5x_2$

Step-3: Constraints.

The raw material is sufficient for making 850 belts per day.

we have $x_1 + x_2 \leq 850$

There are 500 special buckles for belt A per day

$\therefore$ we have $x_1 \leq 500$

There are 700 buckles for belts B per day $\therefore$

we have $x_2 \leq 700$

Belt A needs twice as much time as that required for belt B and the company can produce

500 belts if all of them were of the type-A we have $2x_1+x_2 \leq 500$

Step-4 :- Non-Negativity constraints , $x_1, x_2 \geq 0$

$\therefore$ The LPP is maximum $z=10x_1+5x_2$ s.to constraints

$x_1+x_2 \leq 850$

$x_1 \leq 500$

$x_2 \leq 700$

$2x_1+x_2 \leq 500$

and $x_1, x_2 \geq 0$,

8. Egg contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and cost 12 paise for gram. milk contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and cost 20 paise for gram. The daily minimum requirement of vitamin-A and vitamin-B are 100 units and 120 units respectively. Formulate the above problem as a linear programming problem.

Sol. Step-1:- Decision Variables.

Let x_1 and x_2 be the no. of units of Eggs and Milk

Step-2:- Objective function.

Since the cost of the vitamins are given. we have to minimize the cost.

Minimum $z=12x_1+20x_2$.

Step-3:- Constraints.

There are 2 types of vitamins and there requirements are given,

we have 2 constraints for Vitamin A and Vitamin B

Egg contains 6 units, Milk contains 8 units of vitamin A per gram with daily minimum requirments of 100 units

$\therefore$ we have $6x_1+8x_2 \geq 100$

Egg contains 7 units, Milk contains 12 units of Vitamin B per gram with daily minimum requirements of 120 units

$\therefore$ we have $7x_1+12x_2 \geq 120$

Step-4:- Non-Negativity constraints. $x_1, x_2 \geq 0$

$\therefore$ The LPP is Minimum $z=12x_1+20x_2$ s.to constraints

$6x_1+8x_2 \geq 100$

$7x_1+12x_2 \geq 120$ and $x_1, x_2 \geq 0$

9. The standard weight of a special brick is 5kg and it contains two ingredients B_1 and B_2. B_1 cost Rs.5 per kg, B_2 cost Rs.8 per kg. Strength considerations that the brick contains not more than 4kg of B_1 and a minimum of 2kg of B_2. Since the demand for the product is likely to be related to the price of the brick. Formulate the above problem as a Linear Programming Problem.

Sol: Step-1:- Decision Variables.

Let x_1 and x_2 be the no. of ingredients of B_1 and B_2.

Step-2:- Objective function.

Since cost is given then the objective function is minimization.

$\therefore$ minimum $z = 5x_1 + 8x_2$

Step-3:- Constraint: The brick weight is equals to 5kg and it contains B1 and B_2 and is given by $x_1 + x_2 = 5$

The brick contains not more than 4kg of B_1

$\therefore x_1 \leq 4$

The brick contains minimum of 2kg of B_2

$\therefore x_2 \geq 2$

Step-4:- Non-Negativity Constraints

$\therefore x_1, x_2 \geq 0$

$\therefore$ The LPP is minimum $z = 5x_1 + 8x_2$ s.to constraints

$x_1 + x_2 = 5$

$x_1 \leq 4$

$x_2 \geq 2$

and $x_1, x_2 \geq 0$

* Feasible Solution:- The solution which satisfies the non-Negativity constraints of general linear programming problem (GLPP) is called a Feasible solution.

* Non-Feasible Solution:- The solution of process which doesn't satisfies the non-Negativity constraints of General Linear Programming Problem [GLPP] is called a Non-Feasible Solution.

* Optimum Solution:- A Feasible solution which optimizes (Maximization or Minimization) the objective function of GLPP is called Optimum solution.

* Solution to LPP:-

There are two methods to solve LPP. they are:

1. Graphic Method.
2. Simplex Method.

1. Graphic Method :-

Simple LPP with two decision variables can be easily solved by graphic Method. The steps involved in this method are as follows:

Step-1:- The problem should be converted into LPP.

Step-2:- The inequality constraints should be converted into equality constraints and determination of the variables in each equation by assuming the other variable should be zero.

Step-3:- In a Graphpaper we draw X-axis and Y-axis. On X-axis we take 'x_1' values and On Y-axis we take 'x_2' values

Step-4:- Represent the Linear Constraint equations in the graph paper as a straight line. To draw each straight line values of x_1 and x_2.

Step-5:- Mark the region, if the inequality constraint corresponding to the line is '$\leq$' then the region below the line lying in the first Quadrant is shaded. For the inequality constraint '$\geq$' sign the region above the line in the first Quadrant is shaded. The points lying in the common region will satisfy all the constraints simultaneously. The common region does obtained is called the feasible Region.

(or)

Identify the feasible region in the graph the region type for maximization type problems lies within the origin. It is the region from the origin from the straight lines crossing nearest to the origin. The region for the minimization type problems lies above the crossed lines of the graph.

Step-6:- The values of x_1 and x_2 at each co-ordinate can be find out.

Step-7:- The values of x_1 and x_2 at each co-ordinate should be applied in the equation of the objective function. The value of z for each co-ordinate should be calculated.

Step-8:- If the LPP is of maximization type then the highest value of z among the various values of z should be the optimum value, Values of x_1 and x_2 for the highest value of z should be considered as the optimum values of x_1 and x_2. If the LPP is ~~the~~ of minimization type then the lowest value of 'z' among the various values of z should be the optimum value, Values of x_1 and x_2 for the lowest values of z should be considered as the optimum values of x_1 and x_2.

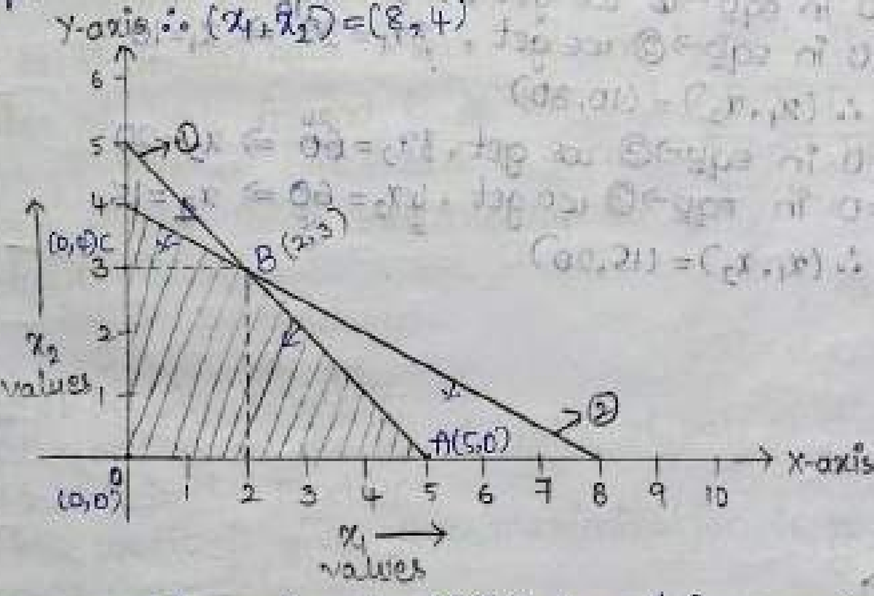
1. Solve the following LPP by Graphic method.

$$\text{Max } Z = 2x_1 + 5x_2$$
s.to $\Rightarrow x_1 + x_2 \le 5$
$$2x_1 + 4x_2 \le 16 \text{ and } x_1, x_2 \ge 0$$

Sol. By converting the inequalities into equalities equations

$$x_1 + x_2 = 5 \to ①$$
$$2x_1 + 4x_2 = 16 \to ②$$

put $x_1 = 0$ in eqn $\to ①$ we get, $x_2 = 5$
put $x_2 = 0$ in eqn $\to ①$ we get, $x_1 = 5$
$\therefore (x_1, x_2) = (5, 5)$
put $x_1 = 0$ in eqn $\to ②$ we get, $4x_2 = 16 \Rightarrow x_2 = 4$
put $x_2 = 0$ in eqn $\to ②$ we get, $2x_1 = 16 \Rightarrow x_1 = 8$
$\therefore (x_1, x_2) = (8, 4)$



The feasible region is OABC, by solving eqn $\to$ ① & ② we get point B.

$$① \times 2 \Rightarrow 2x_1 + 2x_2 = 10$$
$$② \Rightarrow 2x_1 + 4x_2 = 16$$
$$\overline{\quad -2x_2 = -6 \quad}$$
$$x_2 = 3$$

Corner points:
O = (0,0)
A = (5,0)
B = (2,3)
C = (0,4)

Substitute $x_2 = 3$ in eqn $\to$ ① we get,
$$x_1 = 5 - x_2 = 5 - 3 = 2$$
$$\therefore (x_1, x_2) = (2, 3)$$

Substitute the corner points in the objective function

Corner point	Value of $z = 2x_1 + 5x_2$
$O(0,0)$	0
$A(5,0)$	10
$B(2,3)$	19
max ✓ $C(0,4)$	$\boxed{20}$

$\therefore$ Optimum solution is Rs. 20 or Max $z = 20$; $x_1 = 0$ & $x_2 =$

2. Solve the following LPP by Graphic Method.

$$\text{Min } z = 20x_1 + 10x_2$$

s.to $x_1 + 2x_2 \leq 40$

$3x_1 + x_2 \geq 30$

$4x_1 + 3x_2 \geq 60$ and $x_1, x_2 \geq 0$

Sol. By converting inequations into equations

$x_1 + 2x_2 = 40 \rightarrow ①$

$3x_1 + x_2 = 30 \rightarrow ②$

$4x_1 + 3x_2 = 60 \rightarrow ③$

put $x_1 = 0$ in eqn→① we get, $2x_2 = 40 \Rightarrow x_2 = 20$

put $x_2 = 0$ in eqn→① we get, $x_1 = 40$

$\therefore (x_1, x_2) = (40, 20)$

put $x_1 = 0$ in eqn→② we get, $x_2 = 30$

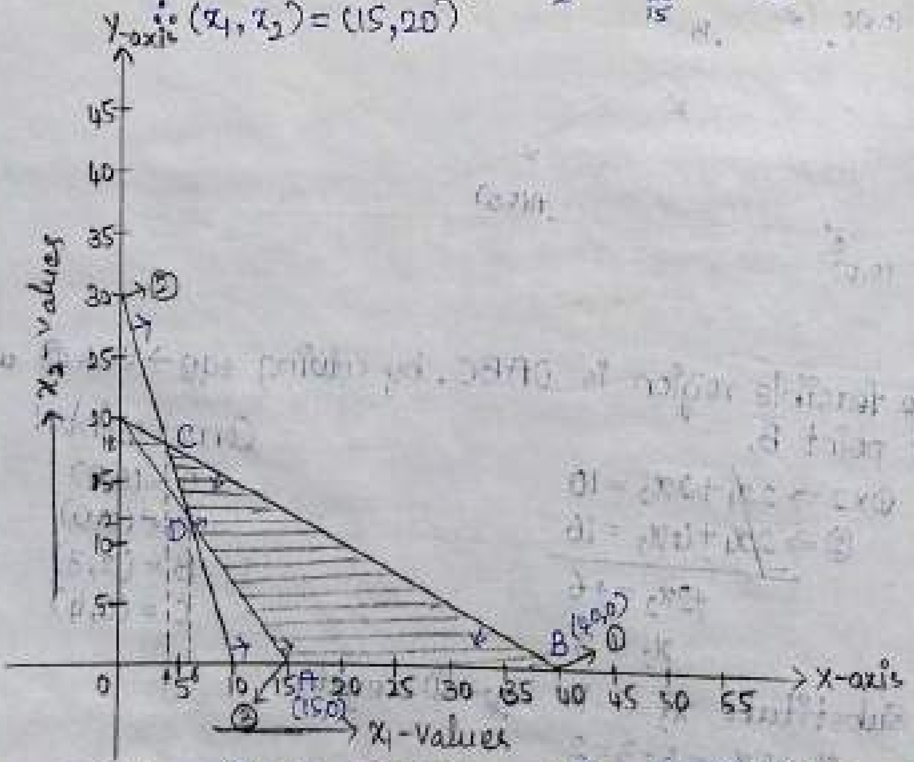
put $x_2 = 0$ in eqn→② we get, $3x_1 = 30 \Rightarrow x_1 = 10$

$\therefore (x_1, x_2) = (10, 30)$

put $x_1 = 0$ in eqn→③ we get, $3x_2 = 60 \Rightarrow x_2 = 20$

put $x_2 = 0$ in eqn→③ we get, $4x_2 = 60 \Rightarrow x_2 = 15$

$\therefore (x_1, x_2) = (15, 20)$



The feasible region is ABCD by solving ① and ② eqn we get point C

$① \times 3 \Rightarrow 3x_1 + 6x_2 = 120$
$② \Rightarrow 3x_1 + x_2 = 30$
$5x_2 = 90$
$x_2 = 18$

Sub x_2 in eqn → ① we get,
$\Rightarrow x_1 = 40 - 2(18)$
$x_1 = 40 - 36$
$x_1 = 4$ $\therefore C(x_1, x_2) = (4, 18)$

By solving eqn → ② and ③ we get,
$② \times 3 \Rightarrow 9x_1 + 3x_2 = 90$
$③ \Rightarrow 4x_1 + 3x_2 = 60$
$5x_1 = 30$
$x_1 = 6$

Sub x_1 in eqn → ② we get,
$x_2 = 30 - 3(6) = 30 - 18$
$x_2 = 12$
$\therefore D(x_1, x_2) = (6, 12)$

Substitute, the Corner points in the Objective function

Corner Point	Value of $Z = 20x_1 + 10x_2$
A(15,0)	300
B(40,0)	800
C(4,18)	80+180=260
min D(6,12)	$\boxed{120+120=240}$

$\therefore$ Optimum solution is Min $Z = 240$; $x_1 = 6$ and $x_2 = 12$

3. A company produces 2 types of hats. Every hat A requires twice as much labour time as the second hat B. If the company produces only hat B then it can produce a total of 500 hats a day. The market limits daily sales of hat A and B are 150 and 250 respectively. The profits on hat A and B are Rs.8 and Rs.5 respectively. Solve Graphically to get the optimum solution.

Sol. Step-1:- Decision Variables.
Let x_1 and x_2 be the no. of units of hats A and hat B

Step-2:- Objective Function.
Here profit is given the maximization is max $Z = 8x_1 + 5x_2$

Step-3:- Constraints.
Every Hat A requires twice as much labour time as the hat B. Total availability of hats per day is 500

$\therefore 2x_1 + x_2 \leq 500$

Market limit daily sales of hat A is 150. $\therefore x_1 \leq 150$
Market limit daily sales of hat B is 250. $\therefore x_2 \leq 250$

Step-4:- Non-Negativity Constraints. $x_1, x_2 \geq 0$

$\therefore$ LPP is max $(Z) = 8x_1 + 5x_2$ s. to constraint
$2x_1 + x_2 \leq 500,$
$x_1 \leq 150,$
$x_2 \leq 250$
and $x_1, x_2 \geq 0$

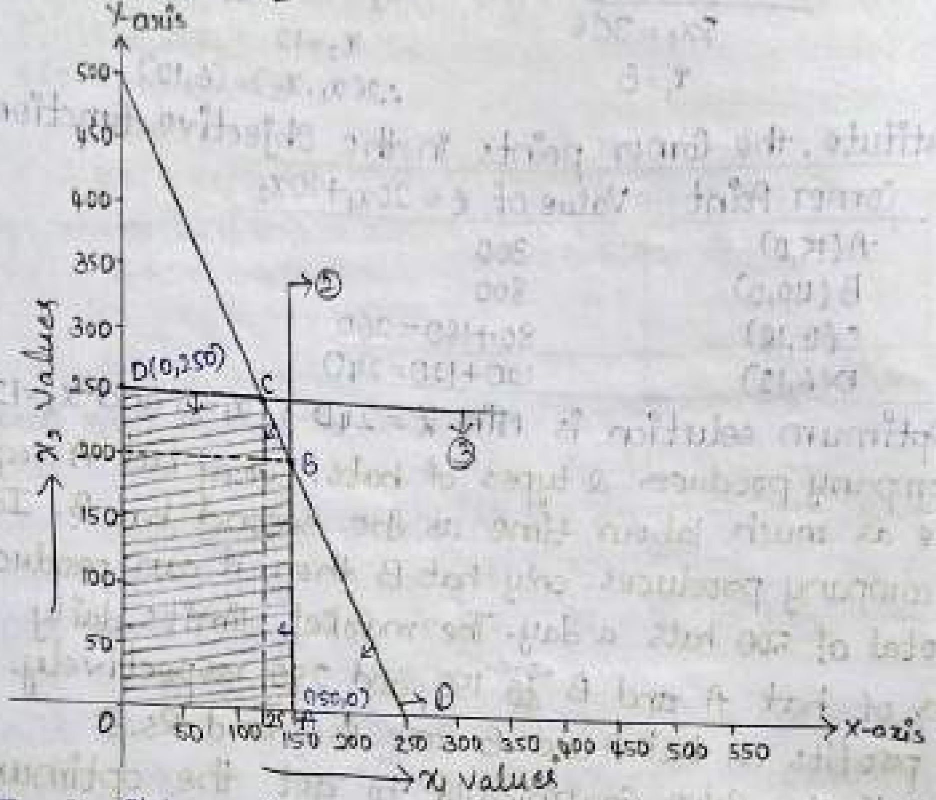
By converting inequations into equations

$$2x_1+x_1=500 \rightarrow ①$$
$$x_1=150 \rightarrow ②$$
$$x_2=250 \rightarrow ③$$

put $x_1=0$ in eqn $\rightarrow$ ① we get $\Rightarrow x_2=500$

put $x_2=0$ in eqn $\rightarrow$ ① we get $\Rightarrow x_1=250$

$\therefore (x_1, x_2)=(250, 500)$



The feasible region is 0ABCD by solving eqn $\rightarrow$ ① and ② we get point B

① $\Rightarrow 2x_1+x_2=500$

② $\Rightarrow x_1=150$

Sub x_1 in eqn $\rightarrow$ ① we get,

① $\Rightarrow 2(150)+x_2=500$

$\Rightarrow x_2=500-300$

$\Rightarrow x_2=200$

$\therefore B(x_1, x_2)=(150, 200)$

By solving eqn $\rightarrow$ ① and ③ we get point C

① $\Rightarrow 2x_1+x_2=500$

③ $\Rightarrow x_2=250$

Sub x_2 in eqn $\rightarrow$ ① we get,

① $\Rightarrow 2x_1+250=500$

$\Rightarrow \cancel{2}x_1 = \cancel{250}\ 125$

$\Rightarrow x_1=125$

$\therefore C(x_1, x_2)=(125, 250)$

Substituting the cornerpoints in the objective function,

	Corner Points	Values of $Z=8x_1+5x_2$
	O(0,0)	0
	A(150,0)	1200
max ✓	$\boxed{B(150,200)}$	$\boxed{2200}$
	C(125,250)	2000
	D(0,250)	1250

$\therefore$ Optimum solution is max $(Z)=2200$; $x_1=150$ & $x_2=200$.

4. Solve the following LPP by Graphic Method:-

Maximum $Z = 2x_1 + 3x_2$

s.to constraints $x_1 + x_2 \leq 400$

$2x_1 + x_2 \leq 600$ and

$x_1, x_2 \geq 0$

Sol. By converting inequations into equations

$x_1 + x_2 = 400 \rightarrow ①$

$2x_1 + x_2 = 600 \rightarrow ②$

put $x_1 = 0$ in equ $\rightarrow ①$ we get, $x_2 = 400$

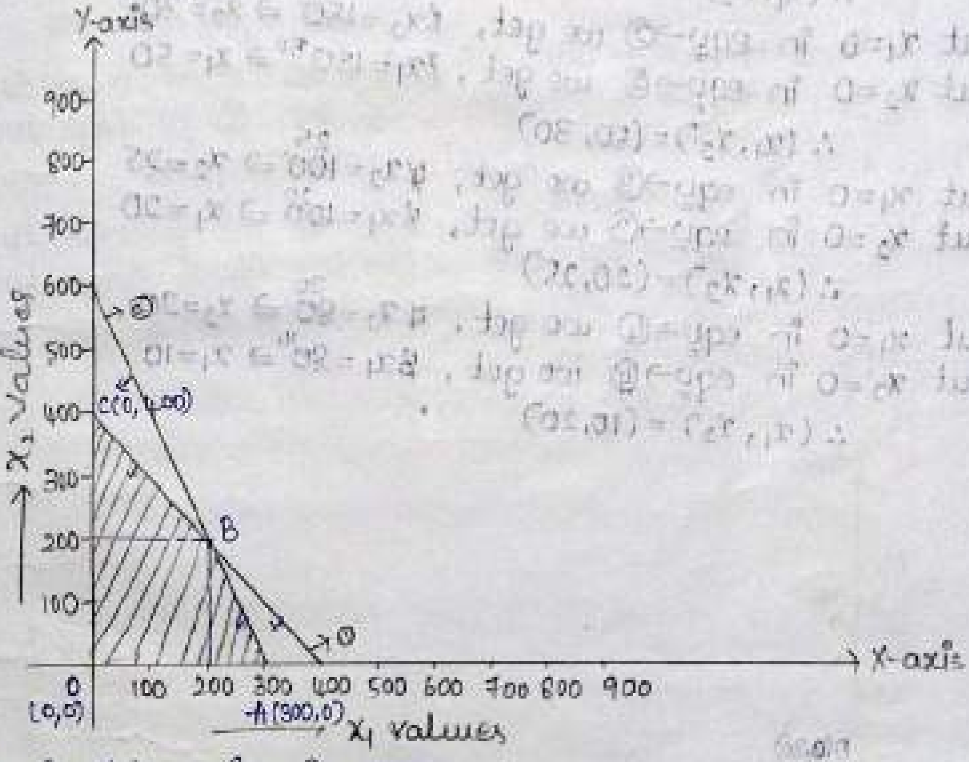
put $x_2 = 0$ in equ $\rightarrow ①$ we get, $x_1 = 400$

$\therefore (x_1, x_2) = (400, 400)$

put $x_1 = 0$ in equ $\rightarrow ②$ we get, $x_2 = 600$

put $x_2 = 0$ in equ $\rightarrow ②$ we get, $2x_1 = \cancel{600}^{300} \Rightarrow x_1 = 300$

$\therefore (x_1, x_2) = (300, 600)$



The feasible region is OABC

By solving equ $\rightarrow ①$ and equ $\rightarrow ②$ we get,

$① \Rightarrow x_1 + \cancel{x_2} = 400$

$② \Rightarrow \underline{2x_1 + \cancel{x_2} = 600}$

$+x_1 = +200$

$\boxed{x_1 = 200}$

sub $x_1 = 200$ in equ $\rightarrow ①$ we get,

$① \Rightarrow 200 + x_2 = 400$

$\boxed{x_2 = 200}$

$\therefore B(x_1, x_2) = (200, 200)$

Substituting the corner points in the objective function,

	Corner Points	Values of $Z = 2x_1 + 3x_2$
	O(0,0)	0
	A(300,0)	600
	B(200,200)	1000
max ✓	C(0,400)	1200.

$\therefore$ Optimum solution is max $(z) = 1200$; $x_1 = 0$, $x_2 = 400$

5. Solve the following LPP by Graphic method.

Max $z = 3x_1 + 4x_2$

s.to constraints. $5x_1 + 4x_2 \leq 200$

$3x_1 + 5x_2 \leq 150$

$5x_1 + 4x_2 \geq 100$

$8x_1 + 4x_2 \geq 80$ and $x_1, x_2 \geq 0$

Sol. By converting the inequalities into equations

$5x_1 + 4x_2 = 200 \rightarrow ①$

$3x_1 + 5x_2 = 150 \rightarrow ②$

$5x_1 + 4x_2 = 100 \rightarrow ③$

$8x_1 + 4x_2 = 80 \rightarrow ④$

put $x_1 = 0$ in equ $\rightarrow ①$ we get, $4x_2 = 200 \Rightarrow x_2 = 50$

put $x_2 = 0$ in equ $\rightarrow ①$ we get, $5x_1 = 200 \Rightarrow x_1 = 40$

$\therefore (x_1, x_2) = (40, 50)$

put $x_1 = 0$ in equ $\rightarrow ②$ we get, $5x_2 = 150 \Rightarrow x_2 = 30$

put $x_2 = 0$ in equ $\rightarrow ②$ we get, $3x_1 = 150 \Rightarrow x_1 = 50$

$\therefore (x_1, x_2) = (50, 30)$

put $x_1 = 0$ in equ $\rightarrow ③$ we get, $4x_2 = 100 \Rightarrow x_2 = 25$

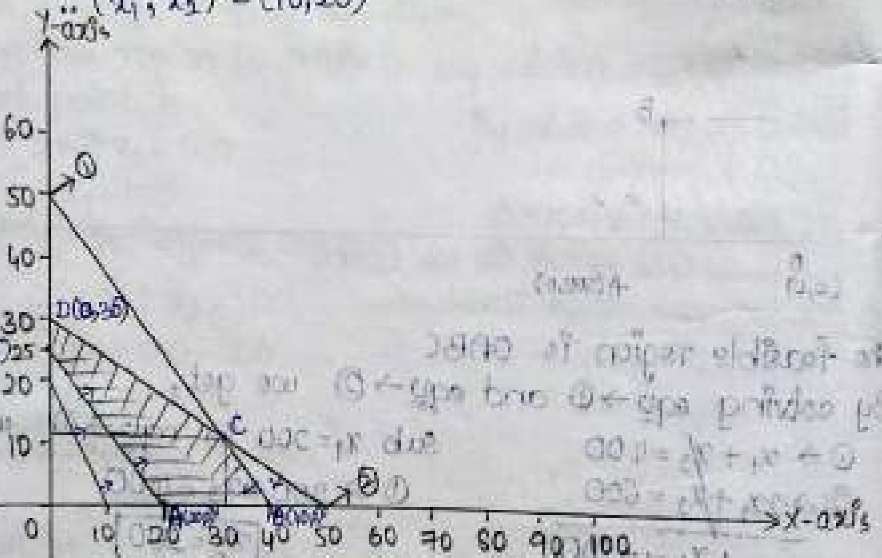
put $x_2 = 0$ in equ $\rightarrow ③$ we get, $5x_1 = 100 \Rightarrow x_1 = 20$

$\therefore (x_1, x_2) = (20, 25)$

put $x_1 = 0$ in equ $\rightarrow ④$ we get, $4x_2 = 80 \Rightarrow x_2 = 20$

put $x_2 = 0$ in equ $\rightarrow ④$ we get, $8x_1 = 80 \Rightarrow x_1 = 10$

$\therefore (x_1, x_2) = (10, 20)$



The feasible region is ABCDE,

By solving equ $\rightarrow ①$ and equ $\rightarrow ②$ we get C

$① \times 3 \Rightarrow 15x_1 + 12x_2 = 600$

$② \times 5 \Rightarrow 15x_1 + 25x_2 = 750$

$-13x_2 = -150$

$x_2 = \frac{150}{13}$

$\boxed{x_2 = 11.5385} \simeq 11.5$

Sub. $x_2 = 11.5385$ in equ $\rightarrow ①$

$① \Rightarrow 5x_1 + 4(11.5385) = 200$

$\Rightarrow 5x_1 = 200 - 46.1540$

$\Rightarrow 5x_1 = 153.8461$

$x_1 = 30.7692 \simeq 30.8$

$\boxed{x_1 = 30.8}$

$\therefore C(x_1, x_2) = (30.8, 11.5)$

Substituting the corner points in the Objective function,

Corner points	Values of $Z = 3x_1 + 4x_2$
$A(20,0)$	60
$B(40,0)$	120
Max ✓ $C(30.8, 11.5)$	$92.4 + 46 = 138.4$
$D = (0,30)$	120
$E(0,25)$	~~25~~ 100

$\therefore$ Optimum solution is $\max(Z) = 138.4$; $x_1 = 30.8$, $x_2 = 11.5$

6. Solve the following LPP by Graphic method.

Min $Z = 12x_1 + 20x_2$

S.to constraints $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$ and $x_1, x_2 \geq 0$

Sol. By converting the inequation into equation

$6x_1 + 8x_2 = 100 \rightarrow ①$

$7x_1 + 12x_2 = 120 \rightarrow ②$

put $x_1 = 0$ in ① we get, $8x_2 = 100 \Rightarrow x_2 = 12.5$

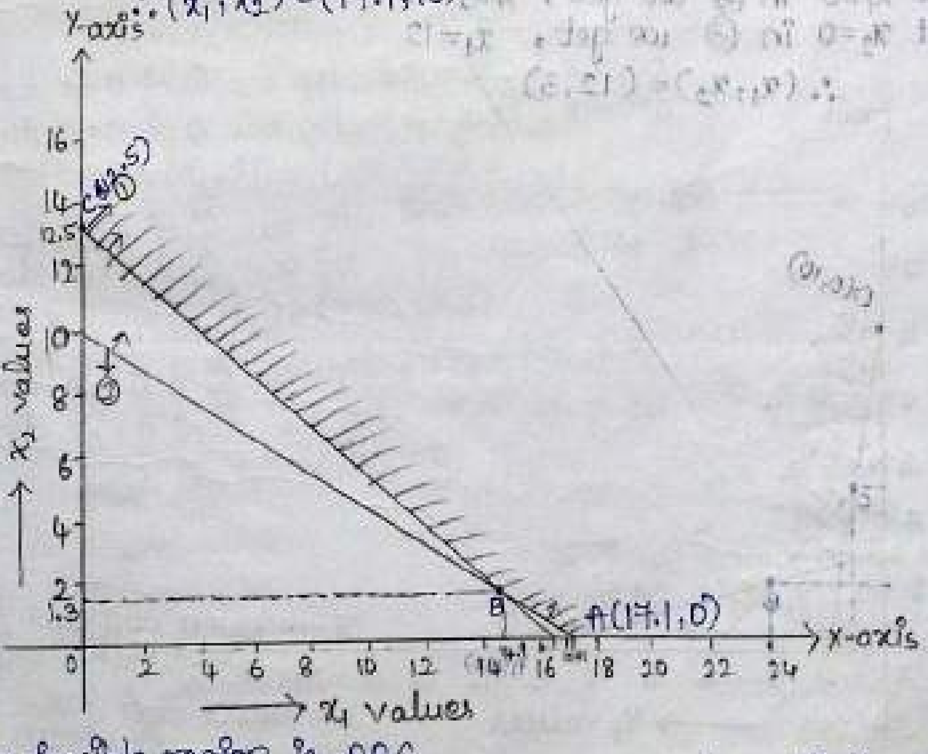
put $x_2 = 0$ in ① we get, $6x_1 = 100 \Rightarrow x_1 = 16.6$

$\therefore (x_1, x_2) = (16.6, 12.5)$

put $x_1 = 0$ in ② we get, $12x_2 = 120 \Rightarrow x_2 = 10$

put $x_2 = 0$ in ② we get, $7x_1 = 120 \Rightarrow x_1 = 17.1$

$\therefore (x_1, x_2) = (17.1, 10)$



The feasible region is ABC

By solving ① and ② eqn we get, B

$① \times 7 \Rightarrow 42x_1 + 56x_2 = 700$

$② \times 6 \Rightarrow 42x_1 + 72x_2 = 720$

$\overline{\quad -16x_2 = -20 \quad}$

$\therefore x_2 = \frac{20}{16} = 1.3$

Sub $x_2 = 1.3$ in eqn ①

$① \Rightarrow 6x_1 = 100 - 8(x_2)$

$\Rightarrow 6x_1 = 100 - (1.3)8$

$6x_1 = 100 - 10.4$

$6x_1 = 89.6$

$x_1 = \frac{89.6}{6} = 14.9$

$\therefore B(x_1, x_2) = (14.9, 1.3)$

Substitute the Corner points in the objective Function,

Corner points	Values $z=12x_1+20x_2$
A(17.1,0)	205.2
Min ✓ $\boxed{B(14.9,1.3)}$	$\boxed{178.8+26=204.8}$
C(0,12.5)	250

$\therefore$ Optimum solution min(z) = 804.8 ; $x_1=14.9$ and $x_2=1.3$

7. Solve the following LPP by Graphic Method.

Min $z=3x_1+2x_2$

S.to constraints $5x_1+x_2 \geq 10$

$x_1+x_2 \geq 6$

$x_1+4x_2 \geq 12$ and $x_1, x_2 \geq 0$

Sol. By converting inequations into equations

$5x_1+x_2=10 \rightarrow ①$

$x_1+x_2=6 \rightarrow ②$

$x_1+4x_2=12 \rightarrow ③$

put $x_1=0$ in ① we get, $x_2=10$

put $x_2=0$ in ① we get, $5x_1=10 \Rightarrow x_1=2$

$\therefore (x_1,x_2)=(2,10)$

put $x_1=0$ in ② we get, $x_2=6$

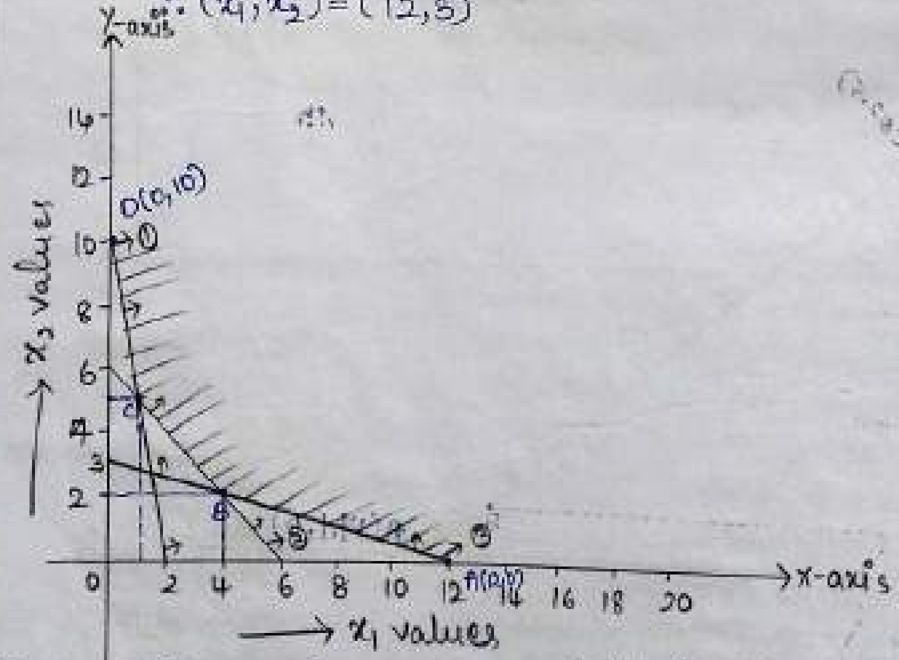
put $x_2=0$ in ② we get, $x_1=6$

$\therefore (x_1,x_2)=(6,6)$

put $x_1=0$ in ③ we get, $4x_2=12 \Rightarrow x_2=3$

put $x_2=0$ in ③ we get, $x_1=12$

$\therefore (x_1,x_2)=(12,3)$



The feasible region is ABCD

By solving ② and ③ eqn we get B

② $\Rightarrow x_1+x_2=6$

③ $\Rightarrow x_1+4x_2=12$

$-3x_2=-6$

$\boxed{x_2=2}$

Sub $x_2=2$ in ① eqn

① $\Rightarrow x_1+2=6$

$\boxed{x_1=4}$

$\therefore B(x_1,x_2)=(4,2)$

By solving equ → ① and ② we get, C.

① ⇒ $5x_1 + \cancel{x_2} = 10$
② ⇒ $\underline{-x_1 \pm \cancel{x_2} = -6}$
$\cancel{4}x_1 = \cancel{4}$
$\boxed{x_1 = 1}$

Sub. $x_1 = 1$ in equ → ② we get,
② ⇒ $1 + x_2 = 6$
$x_2 = 6-1$
$\boxed{x_2 = 5}$

∴ $C(x_1, x_2) = (1, 5)$

Substitute the corner points in the Objective function,

Corner points	Values of $z = 3x_1 + 2x_2$
A(12,0)	36
B(4,2)	12+4=16
min ✓ C(1,5)	$\boxed{3+10=13}$
D(0,10)	20

∴ Optimum solution min $(z) = 13$; $x_1 = 1$ and $x_2 = 5$

8. Solve the following LPP by Graphic Method.

Max $z = 100x_1 + 40x_2$

s.to constraints $5x_1 + 2x_2 \leq 1000$
$3x_1 + 2x_2 \leq 900$
$x_1 + 2x_2 \leq 500$ and $x_1, x_2 \geq 0$

Sol. By converting inequations into equations

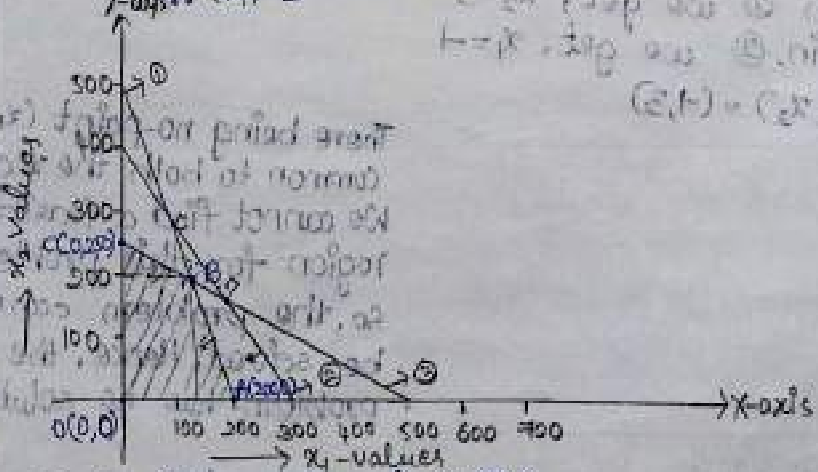
$5x_1 + 2x_2 = 1000$ → ①
$3x_1 + 2x_2 = 900$ → ②
$x_1 + 2x_2 = 500$ → ③

put $x_1 = 0$ in ① we get, $2x_2 = 1000 \Rightarrow x_2 = 500$
put $x_2 = 0$ in ① we get, $5x_1 = 1000 \Rightarrow x_1 = 200$
∴ $(x_1, x_2) = (200, 500)$

put $x_1 = 0$ in ② we get, $2x_2 = 900 \Rightarrow x_2 = 450$
put $x_2 = 0$ in ② we get, $3x_1 = 900 \Rightarrow x_1 = 300$
∴ $(x_1, x_2) = (300, 450)$

put $x_1 = 0$ in ③ we get, $2x_2 = 500 \Rightarrow x_2 = 250$
put $x_2 = 0$ in ③ we get, $x_1 = 500 \Rightarrow x_1 = 500$ (A B C)
∴ $(x_1, x_2) = (500, 250)$

A(x_1, x_2) — (200,000)
B(x_1, x_2)
C(x_1, x_2)



The feasible region is OABC

By solving ① and ③ equ we get 'B'

$① \Rightarrow 5x_1 + 2x_2 = 1000$
$③ \Rightarrow \underline{-x_1 \pm 2x_2 = -500}$
$4x_1 = 500$
$x_1 = \frac{500}{4}$
$\boxed{x_1 = 125}$

sub $x_1 = 125$ in ③ we get,
$③ \Rightarrow 125 + 2x_2 = 500$
$x_2 = \frac{500-125}{2} = \frac{375}{2}$
$\boxed{x_2 = 187.5}$

$\therefore B(x_1, x_2) = (125, 187.5)$

Substitute corner point in the objective function.

Corner Points	Values of $z = 10x_1 + 40x_2$
O(0,0)	0
✓ A(200,0)	20000
✓ B(125,187.5)	12500+7500 = 20000
C(0,250)	10000

max

$\therefore$ Maximum value occurs at two points A and B. since there are infinite no.of points on the line joining A and B. It gives the same maximum value of z. Thus, there are infinite no.of optimal solution for the given LPP.

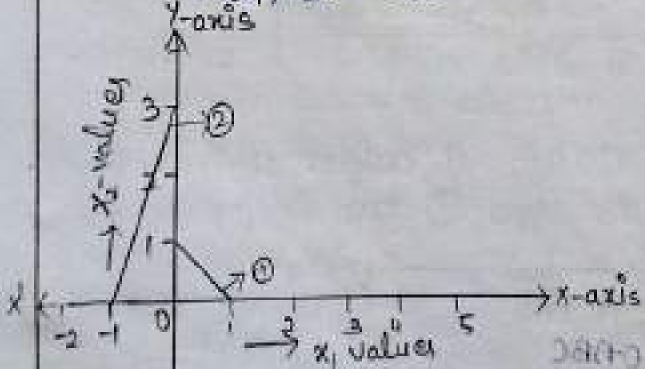
9. Solve the following LPP by Graphic method.
$\max z = x_1 + x_2$
s.to constraints $x_1 + x_2 \leq 1$
$-3x_1 + x_2 \geq 3$
and $x_1, x_2 \geq 0$

Sol. By converting inequations into equations,
$x_1 + x_2 = 1 \rightarrow ①$
$-3x_1 + x_2 = 3 \rightarrow ②$

put $x_1 = 0$ in ① we get, $x_2 = 1$
put $x_2 = 0$ in ① we get, $x_1 = 1$
$\therefore (x_1, x_2) = (1, 1)$

put $x_1 = 0$ in ② we get, $x_2 = 3$
put $x_2 = 0$ in ② we get, $x_1 = -1$
$\therefore (x_1, x_2) = (-1, 3)$

$-3x_1 = 3$
$x_1 = -1$



There being no-point (x_1, x_2) common to both the region we cannot find a feasible region for, this problem. so, the problem cannot be solved. Hence, the problem has no solution.

10. Solve the following LPP by Graphic method.
4M

$\max z = 3x_1 + 2x_2$

S.to constraints, $x_1 - x_2 \geq 1$

$x_1 + x_2 \geq 3$ and $x_1, x_2 \geq 0$

Sol. By converting inequations into equations

$x_1 - x_2 = 1 \rightarrow ①$

$x_1 + x_2 = 3 \rightarrow ②$

put $x_1 = 0$ in ① we get, $x_2 = -1$

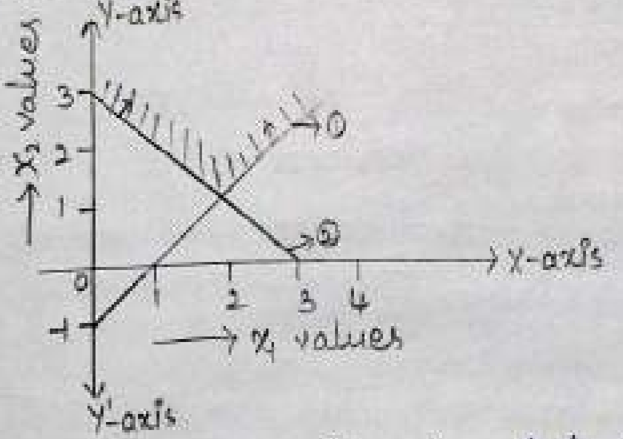
put $x_2 = 0$ in ① we get, $x_1 = 1$

$\therefore (x_1, x_2) = (1, -1)$

put $x_1 = 0$ in ② we get, $x_2 = 3$

put $x_2 = 0$ in ② we get, $x_1 = 3$

$\therefore (x_1, x_2) = (3, 3)$



The solution space is unbounded, the maximum value of z occurs at infinity. Hence the problem has unbounded solutions.

[obj function max → towards origin
obj function min → away from origin
constraints $\leq$ → towards origin
constraints $\geq$ → away from origin]

19/12/20

SIMPLEX METHOD

* **Canonical Form of LPP:-** The canonical form of LPP is as follows:

Maximum $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

s.to the constraints $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m$

and $x_1, x_2, \ldots x_n \ge 0$

The Characteristics of Canonical form:-

1. Objective function is maximization.
2. All the constraints are less than or equal to type.
3. All the variables are non-Negative.

* **Standard Form of LPP:-** The standard form of LPP is as follows:

Maximum $z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$

s.to the constraints $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$

and $x_1, x_2, \ldots x_n \ge 0$

The Characteristics of Standard form:-

1. The objective function is of maximization type.
2. All the constraints are expressed as equations.
3. Right hand side of the each constraint is non-negative
4. All the variables are non-Negative.

Basic Solution:-

Given a system of 'm' Linear equations with 'n' variables (m<n), Any solution that it is obtained by solving for 'm' variables keeping the remaining 'n-m' variables zero is called a Basic Solution. Such m variables are called Basic variables and the remaining are called Non-Basic variables.

Basic Feasible solution:-

A Basic Feasible solution is a basic solution which satisfies non-Negativity Restrictions i.e, all basic variables are non-Negative. The Basic feasible solutions is 2 types.

1. **Non-Degenerate**:- A Non-degenerate basic feasible solution is the basic solution that has exactly 'm' positive x_i i.e, none of the Basic variables are zero.

2. **De-generate**:- A Basic feasible solution is said to be Degenerate if one or more basic variables are zero.

* Unbounded Solution:-

If the value of the Objective function 'z' can be increased or decreased in-definitely such solutions are called unbounded solutions.

* Slack Variables:-

If the constraints of a general LPP be $\sum_{j=1}^{n} a_{ij} x_j \leq b_i \ (i=1,2---m)$ then the non-Negative variable 's_i' which are introducted to convert the inequalities ($\leq$ type) into the equations $\sum_{j=1}^{n} a_{ij} x_j + s_i = b_i$ are called slack Variables.

Slack Variables are also defined as the non-Negative variables that are added in the LHS of the constraint to convert the inequality [$\leq$] into equation.

* Surplus Variables:-

If the constraints of a general LPP be $\sum_{j=1}^{n} a_{ij} x_j \geq b_i \ (i=1,2,---m)$, then the Non-Negative variable 'S_i' which are introduced to convert the inequalities [$\geq$ type] into the equalities $\sum_{j=1}^{n} a_{ij} x_j - s_i = b_i$ are called Surplus Variables.

Surplus variables are defined as the Non-negative variables that are removed from the LHS of the constraint to the convert the inequality [$\geq$] in to an equation.

* Simplex Method:-

Algorithm:-

Step-1: Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then convert it into maximization by using the relation, $\min z = -\text{Max}(-z)$
$= -\text{Max } z^*$

Step-2: Check whether all b_i's $(i=1,2,---m)$ are non-Negative. If any one of b_i is negative then multiply the inequation of the constraint by '-1'. So, as to get all b_i's positive.

Step-3: Convert all the inequations into equations by introducing slack / Surplus variables in the constraints put, the cost of these variables equal to zero.

Step-4: Obtain Initial Basic Feasible Solution [IBFS] by using the relation $X_B = B^{-1}.b$ and put, it in the first column of the simplex table

Step-5: Compute the Net evalution '$z_j - c_j$' by using the relation, $(z_j - c_j) = C_B(a_j - c_j)$.

Examine the sign of $z_j - c_j$.

① If all $(z_j - c_j) \geq 0$, then the current basic feasible solution X_B is optimum.

② If atleast one $(z_j - c_j) < 0$ then go to step-6.

Step-6: To finding the entering variable

If there are more than one-negative $(z_j - c_j)$, choose the most negative of them. Let it be $(z_r - c_r)$ for some $j = r$, this gives the entering variable 'x_r' and it indicated by arrow ($\uparrow$) at the bottom of the r^{th} column. If there are more than one variable having the most negative $(z_j - c_j)$ then any one of them can be selected arbitrally as the entering variable

① If all $a_{ir} \leq 0$, then there is an unbounded solution to the given problem.

② If atleast one $a_{ir} > 0$, then the corresponding vector 'x_r' enters the basis.

Step-7: Compute the ratio $\left[\frac{X_{Bi}}{a_{ir}}\right]$, $a_{ir} > 0$ and choose the minimum of them. Let the minimum of these ratios be $\left[\frac{X_{Bi}}{a_{kr}}\right]$ then the vector 'x_k' to leave the basis called the 'Key row' and the element at the intersection of Key row and key column is called the Key element (or) Leading element/ Pivotal element.

Step-8: Convert the Leading element to unity by dividing itself and the other elements in the column to be converted into zero by using the relation,

* New equation = Old equation − New Pivot equation × (Corresponding column position)

Step-9 :- Go to step-5 and Repeat the procedure until either an optimum solution is obtained (or) there is an indication of unbounded solution.

1. Solving the LPP by using Simplex Method.

$\max Z = 2x_1 + 5x_2$

s.to constraints $x_1 + x_2 \le 5$

$2x_1 + 4x_2 \le 16$

and $x_1, x_2 \ge 0$

Sol. By introducing slack variables S_1 and S_2 to convert the inequations into equations

$x_1 + x_2 + S_1 = 5$

$2x_1 + 4x_2 + S_2 = 16$

∴ Objective function is $\max Z = 2x_1 + 5x_2 + 0S_1 + 0S_2$

The Initial Basic feasible Solution is, $X_B = B^{-1} b$

$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 16 \end{bmatrix}$

$\Rightarrow X_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 16 \end{bmatrix} = \begin{bmatrix} 5 \\ 16 \end{bmatrix}$

Simplex table :-

		C_j	2	5	0	0	
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	Ratio $(X_B/a_{ir}) = \frac{X_B}{x_2}$
0	S_1	5	1	1	1	0	$5/1 = 5$
0	S_2	16	2	4 PE	0	1	$16/4 = 4 \to$ OgV
	Z_j	0	0	0	0	0	
	$Z_j - C_j$		-2	-5	0	0	

OgV → Outgoing vector

EV → Entering Vector

		C_j	2	5	0	0
C_B	Y_B	X_B	x_1	x_2	S_1	S_2
0	S_1	1	$1/2$	0	1	$-1/4$
5	x_2	4	$1/2$	1	0	$1/4$
	Z_j	20	$5/2$	5	0	$5/4$
	$Z_j - C_j$		$1/2$	0	0	$5/4$

NE = OE − NFE (KR)

$5 - 4(1) = 5 - 4 = 1$

$1 - \frac{1}{2}(1) = 1 - \frac{1}{2} = \frac{1}{2}$

$1 - 1(1) = 0$

$1 - 0(1) = 1$

$0 - \frac{1}{4}(1) = -\frac{1}{4}$

All $(Z_j - C_j) \ge 0$, then the solution is optimum

$\max Z = 20$

$x_1 = 0, \; x_2 = 4$

8. Solve the LPP by using Simplex Method.

$\max z = 2x_1 + 3x_2$

s. to constraints $x_1 + x \leq 400$

$2x_1 + x_2 \leq 600$

and $x_1, x_2 \geq 0$

Sol. By introducing slack variables s_1 and s_2 to convert the inequations in to equations.

$x_1 + x_2 + s_1 = 400$

$2x_1 + x_2 + s_2 = 600$

$\therefore$ Objective function is $\max z = 2x_1 + 3x_2 + 0s_1 + 0s_2$

The Initial Basic Feasible Solution is, $x_B = B^{-1}.b$

$B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 400 \\ 600 \end{bmatrix}$

$\rightarrow x_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 400 \\ 600 \end{bmatrix} = \begin{bmatrix} 400 \\ 600 \end{bmatrix}$

Simplex table :-

		c_j	2	3	0	0	Ratio $(x_B/a_{ir}) = \frac{x_B}{x_2}$
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	
0	s_1	400	1	1 PE	1	0	$400/1 = 400 \rightarrow$ OgV (outgoing value) minimum value
0	s_2	600	2	1	0	1	$600/1 = 600$
	z_j	0	0	0	0	0	
	$z_j - c_j$		-2	-3	0	0	

EV (most negative value) entering value

		c_j	2	3	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
3	x_2	400	1	1	1	0
0	s_2	200	1	0	-1	1
	z_j	1200	3	3	3	0
	$z_j - c_j$		1	0	3	0

NE = OE − NPE(CP)

$600 - 400(1) = 200$

$2 - 1(1) = 1$

$1 - 1(1) = 0$

$0 - 1(1) = -1$

$1 - 0(1) = 1$

All $(z_j - c_j) \geq 0$, then the solution is optimum

$\therefore \max z = 1200$ and $x_1 = 0$ and $x_2 = 400$

3. Solve the LPP by using Simplex Method

$\max z = 3x_1 + 4x_2$

s. to constraints $4x_1 + 2x_2 \leq 80$

$2x_1 + 5x_2 \leq 180$

and $x_1, x_2 \geq 0$

Sol. By introducing slack variables s_1 and s_2 to convert the inequations into equations.

$$4x_1+2x_2+s_1=80$$
$$2x_1+5x_2+s_2=180$$

$\therefore$ Objective function is $\max Z = 3x_1+4x_2+0s_1+0s_2$

The Initial Basic Feasible solution is, $X_B = B^{-1}.b$

$B^{-1} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ and $b = \begin{bmatrix}80\\180\end{bmatrix}$

$\Rightarrow X_B = \begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}80\\180\end{bmatrix} = \begin{bmatrix}80\\180\end{bmatrix}$

Simplex Table:-

		C_j	3	4	0	0	Ratio $= \frac{x_B}{x_2}$ (x_B/a_{ir})
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	
0	s_1	80	4	2	1	0	$80/2 = 40$
0	s_2	180	2	5 PE	0	1	$180/5 = 36$ (ogv)
	Z_j	0	0	0	0	0	
	Z_j-C_j		-3	-4	0	0	

(out going value - minimum value)

EV (most negative value) (Entering value)

		C_j	3	4	0	0	Ratio $= \frac{x_B}{x_1}$ (x_B/a_{ir})
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	
0	s_1	8	16/5 PE	0	1	$-2/5$	$8/\frac{16}{5} = \frac{5}{2}$ (ogv)
4	x_2	$180/5 = 36$	2/5	1	0	1/5 (Divide by 5)	$36/\frac{2}{5} = 90$
	Z_j	144	8/5	4	0	4/5	
	Z_j-C_j		$-7/5$	0	0	4/5	

EV

NE $\Rightarrow$ OE $-$ NPE

$\Rightarrow 80-36(2) = 80-72 = 8$

$\Rightarrow 4-\frac{2}{5}(2) = \frac{20-4}{5}$

$\Rightarrow 2-1(2) = 0$

$\Rightarrow 1-0(2) = 1$

$\Rightarrow 0-\frac{1}{5}(2) = -2/5$

$Z_j-C_j = \frac{8}{5}-3 = -\frac{7}{5}$

$4-4=0$

$0-0=0$

		C_j	3	4	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
3	x_1	$8/\frac{16}{5} = \frac{5}{2}$	1	0	5/16	$-1/8$
4	x_2	35	0	1	$-1/8$	1/4
	Z_j	295/2	3	4	7/16	5/8
	Z_j-C_j		0	0	7/16	5/8

Divide by $\frac{16}{5}$

$-\frac{2/5}{16/5} = -1/8$

$36-\frac{5}{2}\left(\frac{2}{5}\right) = 36-1 = 35$

$\frac{2}{5}-1\left(\frac{2}{5}\right) = 0$

$1-0\left(\frac{2}{5}\right) = 1$

$0-\frac{5}{16}\left(\frac{2}{5}\right) = -\frac{1}{8}$

$\frac{1}{5}-\left(-\frac{1}{8}\right)\left(\frac{2}{5}\right) = \frac{1}{5}+\frac{1}{20} = \frac{4+1}{20} = \frac{5}{20} = \frac{1}{4}$

$\therefore$ All $(Z_j - C_j) \geq 0$, then the solution is optimum

$\therefore \max Z = \frac{295}{2}$ and $x_1 = 5/2$ and $x_2 = 35$

4. Solve the following LPP by using Simplex method

$\max Z = 3x_1+2x_2$

s.to Constraints $x_1+x_2 \leq 4$ and $x_1-x_2 \leq 2$ and $x_1, x_2 \geq 0$

Sol. By introducing slack variables s_1 and s_2 to convert inequati into equations.

$x_1+x_2+s_1=4$
$x_1-x_2+s_2=2$

$\therefore$ Objective Function is $\max z = 3x_1+2x_2+0s_1+0s_2$

The Initial Basic Feasible solution is, $x_B = B^{-1}.b$

$B^{-1} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$ and $b = \begin{bmatrix}4\\2\end{bmatrix}$ then, $x_B = \begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}4\\2\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$

Simplex Table:-

		c_j	3	2	0	0	Ratio $= \frac{x_B}{x_1}$
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	(x_B/a_{ir})
0	s_1	4	1	1	1	0	$4/1=4$
0	s_2	2	1 PE	-1	0	1	$2/1=2$ → ogv (min value)
	z_j	0	0	0	0	0	
	z_j-c_j		-3	-2	0	0	

↑ EV (most negative value)

NE = OE − NPE(CCP)
$4-2(1)=2$
$1-1(1)=0$
$1-(-1)(1)=2$
$1-0(1)=1$
$0-1(1)=-1$

		c_j	3	2	0	0	Ratio $= \frac{x_B}{x_2}$
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	(x_B/a_{ir})
0	s_1	2	0	2 PE	1	-1	$2/2=1$ → ogv (min value)
3	x_1	2	1	-1	0	1	$2/-1$ doesn't exist <0
	z_j	6	3	-3	0	3	
	z_j-c_j		0	-5	0	3	

↑ EV (most negative value)

		c_j	3	2	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
2	x_2	1	0	1	$1/2$	$-1/2$
3	x_1	3	1	0	$1/2$	$1/2$
	z_j	11	3	2	$\frac{2}{2}+\frac{3}{2}=\frac{5}{2}$	$1/2$
	z_j-c_j		0	0	$5/2$	$1/2$

→ divided by 2

NE = OE − NPE(CCP)
$2-1(-1)=3$
$1-0(-1)=1$
$-1-1(-1)=0$
$0-\frac{1}{2}(-1)=\frac{1}{2}$
$1-(-\frac{1}{2})(-1)=1-\frac{1}{2}=\frac{2-1}{2}=\frac{1}{2}$
$-\frac{2}{2}+\frac{3}{2}=\frac{1}{2}$

$\therefore$ All $(z_j-c_j) \geq 0$, then the solution is optimum

$\therefore \max z = 11$ and $x_1=3$, $x_2=1$

5. Solve the following LPP by using simplex Method

$\max z = 3x_1+2x_2$

s. to constraints $4x_1+3x_2 \leq 12$; $4x_1+x_2 \leq 8$; $4x_1-x_2 \leq 8$

and $x_1, x_2 \geq 0$

Sol. Given,

By introducing slack variables s_1, s_2 and s_3 to convert Inequations into equations

$4x_1+3x_2+s_1=12$
$4x_1+x_2+s_2=8$
$4x_1-x_2+s_3=8$

$\therefore$ Objective function is $\max z = 3x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3$

The Initial Basic Feasible solution is, $X_B = B^{-1} \cdot b$

$B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 12 \\ 8 \\ 8 \end{bmatrix}$ then, $X_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 8 \end{bmatrix}$

Simplex table:-

		c_j	3	2	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	$(X_B/a_{ir}) = \frac{X_B}{x_1}$
0	s_1	12	4	3	1	0	0	$12/4 = 3$
0	s_2	8	4 PE	1	0	1	0	$8/4 = 2 \rightarrow$ Ogv
0	s_3	8	4	4	0	0	1	$8/4 = 2$
	z_j	0	0	0	0	0	0	
	$z_j - c_j$		-3	-2	0	0	0	
			↑ EV					

NE = OE − NPE

$12 - 2(4) = 4$

$4 - 1(4) = 0$

$3 - 1/4(4) = 2$

$1 - 0(4) = 1$

$0 - 1/4(4) = -1$

$0 - 0(4) = 0$

		c_j	3	2	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	$(X_B/a_{ir}) = \frac{X_B}{x_2}$
0	s_1	4	0	2 PE	1	-1	0	$\frac{4}{2} = 2 \rightarrow$ Ogv (min value)
3	x_1	$\frac{8}{4} = 2$	1	$1/4$	0	$1/4$	0	$\frac{2}{1/4} = 8$
0	s_3	0	0	-2	0	4	1	0
	z_j	6	3	$3/4$	0	$3/4$	0	
	$z_j - c_j$		0	$-5/4$	0	$+3/4$	0	
				↑ EV (most negative value)				

NE = OE − NPE

$8 - 2(4) = 0$

$4 - 1(4) = 0$

$\frac{3}{4} - 2 = \frac{3-8}{4}$

		c_j	3	2	0	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
2	x_2	$\frac{4}{2} = 2$	0	1	$1/2$	$-1/2$	0
3	x_1	$3/2$	1	0	$-1/8$	$3/8$	0
0	s_3	$-1/2$	0	$-9/4$	$-1/8$	$-7/8$	1
	z_j	$17/2$	3	2	$5/8$	$1/8$	0
	$z_j - c_j$		0	0	$5/8$	$1/8$	0

NE = OE − NPE

$1 - 0(1/4) = 1$

$1/4 - 1(1/4) = 0$

$0 - 1/2(1/4) = -1/8$

$0 - 0(1/4) = 0$

$0 - 0(1/4) = 0$

$0 - 1/2(1/4) = -1/8$

$1 - 0(1/4) = 1$

$\therefore$ All $(z_j - c_j) \geq 0$, the optimum solution is,

$\text{Max } z = 17/2$ and $x_1 = \frac{3}{2}$ and $x_2 = 2$

6. Solve the following LPP by using Simplex Method.

$\text{Max } z = 3x_1 + 5x_2 + 4x_3$

s. to contraints $2x_1 + 3x_2 \leq 8$; $2x_2 + 5x_3 \leq 10$; $3x_1 + 2x_2 + 4x_3 \leq 15$

and $x_1, x_2, x_3 \geq 0$

Sol. By introducing slack variables s_1, s_2 and s_3 to convert inequations into equations;

$2x_1 + 3x_2 + s_1 = 8$

$2x_2 + 5x_3 + s_2 = 10$

$3x_1 + 2x_2 + 4x_3 = 15$

$\therefore$ Objective function is $\max z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$

The Initial Basic feasible Solution is, $X_B = B^{-1} b$

$$B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ b = \begin{bmatrix} 8 \\ 10 \\ 15 \end{bmatrix} \text{ then, } X_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 10 \\ 15 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 15 \end{bmatrix}$$

Simplex table:-

C_B	Y_B	c_j / X_B	3 / x_1	5 / x_2	4 / x_3	0 / s_1	0 / s_2	0 / s_3	Ratio (x_B/a_{ir})
0	s_1	8	2	3 PE	0	1	0	0	$8/3 = 2.6$ ← ogv
0	s_2	10	0	2	5	0	1	0	$10/2 = 5$
0	s_3	15	3	2	4	0	0	1	$15/2 = 7.5$
	z_j	0	0	0	0	0	0	0	
	$z_j - c_j$		-3	-5 ↑EV	-4	0	0	0	

NE = OE − NPE(CCP)
$10 - \frac{8}{3}(2) = \frac{14}{3}$
$0 - \frac{2}{3}(2) = -\frac{4}{3}$
$2 - 1(2) = 0$
$5 - 0(2) = 5$
$0 - \frac{1}{3}(2) = -\frac{2}{3}$
$1 - 0(2) = 1$
$0 - 0(2) = 0$

$15 - \frac{8}{3}(2) = \frac{29}{3}$
$3 - \frac{2}{3}(2) = \frac{5}{3}$
$2 - 1(2) = 0$
$4 - 0(2) = 4$
$0 - \frac{1}{3}(2) = -\frac{2}{3}$
$0 - 0(2) = 0$
$1 - 0(2) = 1$

C_B	Y_B	c_j / X_B	3 / x_1	5 / x_2	4 / x_3	0 / s_1	0 / s_2	0 / s_3	Ratio (x_B/a_{ir})
5	x_2	8/3	2/3	1	0	1/3	0	0	— ogv
0	s_2	14/3	-4/3	0	5 PE	-2/3	1	0	$\frac{14/3}{5} = \frac{14}{15}$
0	s_3	29/3	5/3	0	4 PE	-2/3	0	1	$\frac{29/3}{4} = \frac{29}{12}$ ogv
	z_j	40/3	10/3	5	0	5/3	0	0	
	$z_j - c_j$		1/3	0	-4 ↑EV	5/3	0	0	

$\Rightarrow 5\left(\frac{8}{3}\right) + 4\left(\frac{14}{15}\right)$
$\frac{40}{3} + \frac{56}{15}$
$\frac{200 + 56}{15} = \frac{256}{15}$

~~WRONG~~

~~C_B~~	~~Y_B~~	~~c_j / X_B~~	~~3 / x_1~~	~~5 / x_2~~	~~4 / x_3~~	~~0 / s_1~~	~~0 / s_2~~	~~0 / s_3~~	~~Ratio (x_B/a_{ir})~~
~~5~~	~~x_2~~	~~8/3~~	~~2/3~~		~~0~~	~~1/3 PE~~	~~0~~	~~0~~	~~8/3 / 1/3 = 8~~ ogv
~~0~~	~~s_2~~	~~-31/12~~	~~-31/12~~	~~0~~	~~0~~	~~7/6~~	~~1~~	~~-3/4~~	~~-31/14~~ X
~~4~~	~~x_3~~	~~$\frac{29/3}{4} = \frac{29}{12}$~~	~~5/12~~	~~0~~	~~1~~	~~-2/4~~	~~0~~	~~1/4~~	~~-29/12~~ X
	~~z_j~~	~~23~~	~~5~~	~~5~~	~~4~~	~~-1/3~~	~~0~~	~~4/4 = 1~~	
	~~$z_j - c_j$~~		~~2~~	~~0~~	~~0~~	~~-1/3~~ ↑EV	~~0~~	~~1~~	

NE = OE − NPE(CCP)
$\frac{8}{3} - \frac{29}{12}(0) = \frac{8}{3}$
$\frac{2}{3} - \frac{5}{12}(0) = \frac{2}{3}$
$1 - 0(0) = 1$
$0 - 1(0) = 0$
$\frac{1}{3} - \left(\frac{-2}{4}\right)(0) = \frac{1}{3}$
$0 - 0(0) = 0$
$0 - \frac{1}{4}(0) = 0$

$\frac{14}{3} - \frac{29}{12}(3) = \frac{-31}{12}$
$\frac{-4}{3} - \frac{5}{12}(3) = \frac{-31}{12}$
$0 - 0(3) = 0$
$3 - 1(3) = 0$
$\frac{-2}{3} - \left(\frac{-2}{4}\right)(3) = \frac{7}{6}$
$1 - 0(3) = 1$
$0 - \frac{1}{4}(3) = \frac{-3}{4}$

C_B	Y_B	c_j / X_B	3 / x_1	5 / x_2	4 / x_3	0 / s_1	0 / s_2	0 / s_3	Ratio (x_B/a_{ir}) $= \frac{x_B}{x_1}$
5	x_2	8/3	2/3	1	0	1/3	0	0	$\frac{8}{3} / \frac{2}{3} = 4$
4	x_3	14/15	-4/15	0	1	-2/15	1/5	0	—
0	s_3	89/15	41/15 PE	0	0	-2/15	-4/5	1	$\frac{89}{15} / \frac{41}{15} = \frac{89}{41} = 2.1707$ ↓ ogv
	z_j	256/15	34/15	5	4	17/15	4/5	0	
	$z_j - c_j$		-11/15 ↑EV	0	0	17/15	4/5	0	

NE = OE − NPE(CCP)
$\frac{29}{3} - \frac{14}{15}(4) = \frac{89}{15}$
$\frac{5}{3} - \left(\frac{-4}{15}\right)(4) = \frac{41}{15}$
$0 - 0(4) = 0$
$4 - 1(4) = 0$
$\frac{-2}{3} - \left(\frac{-2}{15}\right)(4) = \frac{-2}{15}$
$0 - \frac{1}{5}(4) = -4/5$
$1 - 0(4) = 1$

$\Rightarrow \frac{34}{15} - 3 = \frac{34 - 45}{15} = \frac{-11}{15}$

		C_j	3	5	4	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
5	x_2	50/41	0	1	0	15/41	8/41	10/41
4	x_3	62/41	0	0	1	-6/41	5/41	4/41
3	x_1	89/41	1	0	0	-2/41	-12/41	15/41
	Z_j	$\frac{765}{41}$	3	5	4	$\frac{165}{41}$	$\frac{12}{41}$	$\frac{3}{41}$
	Z_j-C_j		0	0	0	165/41	12/41	3/41

$\therefore$ All $(Z_j-C_j) \geq 0$ then, $\max Z = \frac{765}{41}$ and

$x_1 = \frac{89}{41}$, $x_2 = \frac{50}{41}$ and $x_3 = \frac{62}{41}$

7. Solve the following LPP by Simplex Method

$\text{Min } Z = x_1 - 3x_2 + 2x_3$

Subject to constraints, $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$ and $x_1, x_2, x_3 \geq 0$

Sol. By introducing slack variables s_1, s_2 and s_3 to convert inequations into equations

$\because \text{Min } Z = -\text{Max}(-Z)$

$= -\text{Max } Z^*$

$\Rightarrow \text{Max } Z^* = -\text{Min } Z$

$= -x_1 + 3x_2 - 2x_3$

then, $3x_1 - x_2 + 2x_3 + S_1 = 7$

$-2x_1 + 4x_2 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$\therefore$ the objective function is, $\text{Max } Z^* = -x_1 + 3x_2 - 2x_3 + 0S_1 + 0S_2 + 0S_3$

The Initial Basic feasible solution is, $X_B = B^{-1} \cdot b$

Simplex Table:

$X_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 7 \\ 12 \\ 10 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \\ 10 \end{bmatrix}$

		C_j	-1	3	-2	0	0	0	Ratio = $(X_B/a_{ij}) = \frac{X_B}{x_2}$
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
0	S_1	7	3	-1	2	1	0	0	—
0	S_2	12	-2	4 PE	0	0	1	0	12/4 = 3 → OgV
0	S_3	10	-4	3	8	0	0	1	10/3 = 3.3
	Z_j	0	0	0	0	0	0	0	
	Z_j-C_j		1	-3	2	0	0	0	

↑ EV

		C_j	-1	3	-2	0	0	0	Ratio $(x_B/a_{ir}) = \frac{x_B}{x_1}$
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	
0	S_1	10	5/2	0	2	1	1/4	0	$10/5/2 = \frac{20}{5} = 4$
3	x_2	3	-1/2	1	0	0	1/4	0	—
0	S_3	1	-5/2	0	8	0	-3/4	1	—
	Z_j	9	-3/2	3	0	0	3/4	0	
	$Z_j - C_j$		-1/2	0	2	0	3/4	0	

↑ EV

$7-3(1) = 10$
$3-(1/2)(-1) = \frac{6-1}{2} = \frac{5}{2}$
$-1-1(-1)-0$
$10-3(3) = 1$
$-4-(-1/2)3$
$= -4+3/2 = \frac{-8+3}{2} = \frac{-5}{2}$

		C_j	-1	3	-2	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
-1	x_1	4	1	0	4/5	2/5	1/10	0
3	x_2	5	0	1	2/5	1/5	3/10	0
0	S_3	11	0	0	10	1	-1/2	1
	Z_j	11	-1	3	2/5	1/5	4/5	0
	$Z_j - C_j$		0	0	12/5	1/5	4/5	0

$\therefore$ All $(Z_j - C_j) \geq 0$ then max $Z^* = 11$ and $x_1 = 4, x_2 = 5$ and $x_3 = 0$

8. Solve the following LPP by Simplex Method.

Max $Z = x_1 + x_2 + 3x_3$

s.to constraints $3x_1 + 2x_2 + x_3 \leq 2$

$2x_1 + x_2 + 2x_3 \leq 2$ and $x_1, x_2, x_3 \geq 0$

Sol. By introducing slack (value) variables S_1 and S_2 to convert inequations into equations.

$3x_1 + 2x_2 + x_3 + S_1 = 2$

$2x_1 + x_2 + 2x_3 + S_2 = 2$

$\therefore$ the objective function is, max $Z = x_1 + x_2 + x_3 + 0S_1 + 0S_2$

The Initial Basic Feasible solution is, $X_B = B^{-1} \cdot b$

$X_B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

Simplex Table:-

		C_j	1	1	3	0	0	Ratio $(x_B/a_{ir}) = \frac{x_B}{x_3}$
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	
0	S_1	2	3	2	1	1	0	$2/1 = 2$
0	S_2	2	2	1	2	0	1	$2/2 = 1$ → LV
	Z_j	0	0	0	0	0	0	
	$Z_j - C_j$		-1	-1	-3	0	0	

↑ EV

	C_j		1	1	3	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2
0	S_1	1	2	3/2	0	1	-1/2
3	x_3	$\frac{2}{2}=1$	1	1/2	1	0	1/2
	z_j	3	3	3/2	3	0	3/2
	z_j-C_j		2	1/2	0	0	3/2

$2-1(1)=1$
$3-1(1)=2$
$2-\frac{1}{2}(1)=\frac{3}{2}$
$1-1(1)=0$
$1-0(1)=1$
$0-\frac{1}{2}(0)=-\frac{1}{2}$

$\frac{3}{2}-1=\frac{3-2}{2}$

$\therefore$ All $(z_j-C_j)\geq 0$ then max $z=3$

$x_1=0$ and $x_2=0$ and $x_3=1$

9. Solve the following LPP by Simplex Method.

Max $z=x_1+2x_2+x_3$

S. to constraints, $2x_1+x_2-x_3\geq -2$

$-2x_1+x_2-5x_3\leq 6$

$4x_1+x_2+x_3\leq 6$ and $x_1,x_2,x_3\geq 0$

Sol. Since, one b_i is negative then we convert it into non-negative by multiplying the inequation by '-1'

$-2x_1-x_2+x_3\leq 2$

By introducing slack variables S_1, S_2 and S_3 we convert the inequations into equations.

$-2x_1-x_2+x_3+S_1=2$

$-2x_1+x_2-5x_3+S_2=6$

$4x_1+x_2+x_3+S_3=6$

$\therefore$ The objective function is max $z=x_1+2x_2+x_3+0S_1+0S_2+0S_3$

The Initial Basic feasible solution is $X_B=B^{-1}.b$

$$X_B=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}2\\6\\6\end{bmatrix}=\begin{bmatrix}2\\6\\6\end{bmatrix}$$

Simplex table :-

	C_j		1	2	1	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	$(X_B/a_{ir})=\frac{X_B}{x_2}$
0	S_1	2	-2	-1	1	1	0	0	-
0	S_2	6	-2	1	-5	0	1	0	$6/1=6$ ← LV
0	S_3	6	4	1	1	0	0	1	$6/1=6$
	z_j	0	0	0	0	0	0	0	
	z_j-C_j		-1	-2 ↑EV	-1	0	0	0	

		c_j	1	2	1	0	0	0	Ratio $(x_B/a_{ir}) = \frac{x_B}{x_3}$
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	8	-4	0	-4	1	1	0	–
2	x_2	6	-2	1	-5	0	1	0	–
0	s_3	0	6	0	6 PE	0	-1	1	0
	z_j	12	-4	2	-10	0	2	0	
	$z_j - c_j$		-5	0	-11	0	2	0	

↑ EV

$2-6(-1)(-1)=0$
$-2-(-5)(-1)=-4$
$-1-(1)(-1)=0$
$1-(-5)(-1)=-4$
$b-b(-1)=1$
$0-0(-1)=0$

$6-6(1)=0$
$0+2(1)=6$
$1-1(1)=0$
$1+5(1)=6$
$0-0(1)=0$
$0-1(1)=-1$
$1-0(1)=1$

		c_j	1	2	1	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
0	S_1	8	0	0	0	1	1/3	2/3
2	x_2	6	3	1	0	0	1/6	5/6
1	x_3	0	1	0	1	0	-1/6	1/6
	z_j	12	7	2	1	0	1/6	3/2
	$z_j - c_j$		6	0	0	0	1/6	3/2

$8-0(-4)=8$
$-4-1(-4)=0$
$0-0(-4)=0$
$-4-1(-4)=0$
$1-0(-4)=1$
$1+1/6(-4)=1/3$
$0-1/6(-4)=2/3$

$6-0(-5)=6$
$-2-1(-5)=3$
$1-0(-5)=1$
$-5-1(-5)=0$
$0-0(-5)=0$
$1+1/6(-5)=1/6$
$0-1/6(-5)=5/6$

$\therefore$ All $(z_j - c_j) \geq 0$ then max $z = 12$

$x_1 = 0$ and $x_2 = 6$ and $x_3 = 0$.

10. Solve the following LPP by simplex method.

Max $z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

s. to constraints, $2x_1 + x_2 + 5x_3 + 6x_4 \leq 20$

$3x_1 + x_2 + 3x_3 + 25x_4 \leq 24$

$7x_1 + x_4 \leq 70$ and $x_1, x_2, x_3, x_4 \geq 0$

Sol. By introducing slack variables s_1, s_2 and s_3 to convert inequations into equations.

$2x_1 + x_2 + 5x_3 + 6x_4 + s_1 = 20$

$3x_1 + x_2 + 3x_3 + 25x_4 + s_2 = 24$

$7x_1 + x_4 + s_3 = 70$

$\therefore$ The objective function is max $z = 15x_1 + 6x_2 + 9x_3 + 2x_4 + 0s_1 + 0s_2 + 0s_3$

The Initial Basic Feasible solution is $x_B = B^{-1}b$

$$x_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 20 \\ 24 \\ 70 \end{bmatrix} = \begin{bmatrix} 20 \\ 24 \\ 70 \end{bmatrix}$$

Simplex Table:

		c_j	15	6	9	2	0	0	0	Ratio $(x_B/a_{ir}) = \frac{x_B}{x_1}$
C_B	Y_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	
0	s_1	20	2	1	5	6	1	0	0	20/2 = 10
0	s_2	24	3 PE	1	3	25	0	1	0	24/3 = 8 ← OBV
0	s_3	70	7	0	0	1	0	0	1	70/7 = 10
	z_j	0	0	0	0	0	0	0	0	
	$z_j - c_j$		-15	-6	-9	-2	0	0	0	

↑ EV

		C_j	15	6	9	2	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3	$(x_B/a_{ir}) = \frac{x_B}{x_2}$
0	s_1	4	0	$1/3$	3	$-32/3$	1	$-2/3$	0	$4/(1/3) = 12$ ← LV
15	x_1	$\frac{24}{3}=8$	1	$1/3$	1	$25/3$	0	$1/3$	0	$8/(1/3) = 24$
0	s_2	14	0	$-7/3$	-7	$-172/3$	0	$\frac{7}{3}$	1	–
	Z_j	120	15	5	15	125	0	5	0	
	$Z_j - C_j$		0	-1	6	123	0	5	0	

↑EV

$20?-8(2) = 4$
$2-1(2) = 0$
$1-1/3(2) = \frac{3-2}{3} = \frac{1}{3}$
$5-1(2) = 3$
$6-25/3(2) = \frac{18-50}{3} = \frac{-32}{3}$
$1-0(2) = 1$
$0-1/3(2) = -\frac{2}{3}$
$0-0(2) = 0$

$70-8(7) = 70-56 = 14$
$7-1(7) = 0$
$0-1/3(7) = -7/3$
$0-1(7) = -7$
$1-25/3(7) = 1-\frac{175}{3} = \frac{3-175}{3} = \frac{-172}{3}$
$0-0(7) = 0$
$0-1/3(7) = -7/3$
$1-0(7) = 1$

		C_j	15	6	9	2	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	x_4	s_1	s_2	s_3
6	x_2	12	0	1	9	-32	3	-2	0
15	x_1	4	1	0	-2	19	-1	1	0
0	s_2	42	0	0	14	132	7	-7	1
	Z_j	132	15	6	24	93	3	3	0
	$Z_j - C_j$		0	0	15	91	3	3	0

$8-\cancel{12}^{4}(1/\cancel{3}) = 4$
$1-0(1/3) = 1$
$1/3-1(1/3) = 0$
$1-\cancel{9}^{3}(1/\cancel{3}) = -2$
$25/3-(-32)(1/3) = \frac{25+32}{3} = \frac{57}{3} = 19$
$0-(3)(1/3) = -1$
$1/3-(-2)(1/3) = \frac{3}{3} = 1$
$0-0(1/3) = 0$

$14-\cancel{12}^{4}(-\frac{7}{\cancel{3}}) = 14+28 = 42$
$0-0(-7/3) = 0$
$-\frac{7}{3}-1(-7/3) = 0$
$-7-\cancel{9}^{3}(-7/\cancel{3}) = -7+21 = 14$
$\frac{-172}{3}-(-32)(-7/3) = \frac{396}{3} = 132$
$0-\cancel{3}(-7/\cancel{3}) = 7$
$-7/3-(-2)(-7/3) = -7$
$1-0(-7/3) = 1$

∴ All $(Z_j - C_j) \geq 0$ then max $Z = 132$ & $x_1 = 4$, $x_2 = 12$, $x_3 = 0$ and $x_4 = 0$

11. Solve the following LPP by using Simplex method

max $Z = 3x_1 + 2x_2 + 5x_3$

s. to constraints, $x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 260$

$x_1 + 4x_2 \leq 420$

and x_1, x_2 and $x_3 \geq 0$

Sol. By introducing slack variables s_1, s_2 and s_3 to convert inequations into equations.

$$x_1 + 2x_2 + x_3 + s_1 = 430$$
$$3x_2 + 2x_3 + s_2 = 260$$
$$x_1 + 4x_2 + s_3 = 420$$

∴ The objective function is Max $z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$

The Initial Basic feasible solution is, $X_B = B^{-1} \cdot b$

$$X_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 430 \\ 260 \\ 420 \end{bmatrix} = \begin{bmatrix} 430 \\ 260 \\ 420 \end{bmatrix}$$

Simplex Table:-

		c_j	3	2	5	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	$(x_B/a_{ir}) = \frac{x_B}{x_3}$
0	s_1	430	1	2	1	1	0	0	$\frac{430}{1} = 430$
0	s_2	260	3	0	2 PE	0	1	0	$\frac{260}{2} = 130$ ← ogv
0	s_3	420	1	4	0	0	0	1	—
	z_j	0	0	0	0	0	0	0	
	$z_j - c_j$		-3	-2	-5	0	0	0	
					↑EV				

		c_j	3	2	5	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	$(x_B/a_{ir}) \frac{x_B}{x_2}$
0	s_1	300	$-1/2$	2	0	1	$-1/2$	0	$\frac{300}{2} = 150$ ogv
5	x_3	$\frac{260}{2} = 130$	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	0	
0	s_3	420	1	4 PE	0	0	0	1	$\frac{420}{4} = 105$ ← ogv
	z_j	650	$15/2$	0	5	0	$5/2$	0	
	$z_j - c_j$		$9/2$	-2	0	0	$5/2$	0	
				↑EV					

		c_j	3	2	5	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
0	s_1	90	-1	0	0	1	$-1/2$	$-1/2$
5	x_3	130	$3/2$	0	1	0	$1/2$	0
2	x_2	$\frac{420}{4} = 105$	$1/4$	1	0	0	0	$1/4$
	z_j	860	8	2	5	0	$5/2$	$1/2$
	$z_j - c_j$		5	0	0	0	$5/2$	$1/2$

NE = OE − NPE(CCP)
$430 - 130(1) = 300$
$1 - 3/2(1) = -1/2$
$2 - 0(1) = 2$
$3 - 1(1) = 0$
$1 - 0(1) = 1$
$0 - 1/2(1) = -1/2$
$0 - 0(1) = 0$

$420 - 130(0)$
$1 - 3/2(0)$
$4 - 0(0)$
$0 - 1(0)$
$0 - 0(0)$
$0 - 1/2(0)$
$1 - 0(0)$

$\frac{15}{2} - 3 = \frac{15-6}{2} = \frac{9}{2}$
$-\frac{1}{2} + \frac{5}{2} = \frac{4}{2}$ $-3/2 + \frac{5}{2}$

~~$420 - 20(4) = 340$~~
~~$1 - (\frac{-1}{4})(4) = 8$~~
~~$4 - (1/4)(4) = 0$~~
~~$0 - (0)(4) = 0$~~
~~$0 - (1/2)(4) = -2$~~
~~$0 - (-1/4)(4) = +3$~~
~~$1 - (0)(4) = 1$~~

$300 - 105(2) = 90$
$-\frac{1}{2} - 1/4(2) = \frac{-2}{2} = -1$
$2 - 1(2) = 0$
$0 - 0(2) = 0$
$1 - 0(2) = 1$
$-\frac{1}{2} - 0(2) = -1/2$
$0 - 1/4(2) = -1/2$

∴ All $(z_j - c_j) \geq 0$ then max $z = 860$ and

$x_1 = 0$, $x_2 = 105$ and $x_3 = 130$

Q. Solve the following LPP by using simplex method.

Max $z = x_1 + 6x_2 + 5x_3$

s.to constraints $3x_1 + 6x_2 + 3x_3 \leq 22$,

$x_1 + 2x_2 + 3x_3 \leq 14$,

$3x_1 + 2x_2 \leq 14$ and $x_1, x_2, x_3 \geq 0$

Sol: By introducing slack variables s_1, s_2 and s_3 to convert inequations into equations.

$3x_1 + 6x_2 + 3x_3 + s_1 = 22$

$x_1 + 2x_2 + 3x_3 + s_2 = 14$

$3x_1 + 2x_2 + s_3 = 14$

∴ The optimum objective function is, max $Z = x_1 + 4x_2 + 5x_3 + 0s_1 + \dots$

The Initial Basic Feasible solution is, $X_B = B^{-1} \cdot b$

$$x_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 22 \\ 14 \\ 14 \end{bmatrix} = \begin{bmatrix} 22 \\ 14 \\ 14 \end{bmatrix}$$

Simplex Table:-

		C_j	1	4	5	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	$(X_B/a_{ir}) = \frac{X_B}{x_3}$
0	S_1	22	3	6	3	1	0	0	$22/3 = 7.3$
0	S_2	14	1	2	3 PE	0	1	0	$14/2 = 7$ ← ogv
0	S_3	14	3	2	0	0	0	1	$14/2 = 7$ ← ogv
	Z_j	0	0	0	0	0	0	0	
	$Z_j - C_j$		-1	-4	-5	0	0	0	
					↑EV				

		C_j	1	4	5	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	$[X_B/a_{ir}] = \frac{X_B}{x_2}$
0	S_1	8	2	4 PE	0	1	-1	0	$8/4 = 2$ ← ogv
5	x_3	14/3	1/3	2/3	1	0	1/3	0	$\frac{14}{3} / \frac{2}{3} = 7$
0	S_3	14	3	2	0	0	0	1	$14/2 = 7$
	Z_j	70/3	5/3	10/3	5	0	5/3	0	
	$Z_j - C_j$		2/3	-2/3	0	0	5/3	0	
				↑EV					

		C_j	1	4	5	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
4	x_2	$\frac{8}{4} = 2$	1/2	1	0	1/4	-1/4	0
5	x_3	10/3	0	0	1	-1/6	1/2	0
0	S_3	10	2	0	0	-1/2	1/2	1
	Z_j	74/3	2	4	5	1/6	3/2	0
	$Z_j - C_j$		1	0	0	1/6	3/2	0

∴ All $(z_j - c_j) \geq 0$ then Max $Z = \frac{74}{3}$ and

$x_1 = 0$, $x_2 = 2$ and $x_3 = 10/3$

Q. A firm has available 240, 370 and 180 kgs of wood, plastic and steel respectively. The firm produces two products A and B each unit of A requires 1, 3 and 2 kgs of wood, plastic and steel respectively. The corresponding requirement of each unit of B are 3, 4 and 1 respectively. If A sells for Rs.4 and B for Rs.6, determine how many units of A and B should be produced in order to obtain the maximum gross income. use the simplex method.

Sol. Step-1:- Decision Variables:

Let x_1, x_2 be the no. of units of ~~A and B~~ A and B products.

Step-2:- Objective function:-

since, here the gross income should be maximum on both the products so, we have to maximize the gross income.

$\therefore \max z = 4x_1 + 6x_2$

Step-3:- Constraints:-

→ Both the products A and B required 1 and 3 kg of wood respectively and 240 kgs of wood is available in the firm $\therefore x_1 + 3x_2 \leq 240$

→ Both the products A and B required 3 and 4 kg of plastic respectively and 370 kgs of plastic is available in the firm $\therefore 3x_1 + 4x_2 \leq 370$

→ Both the products A and B required 2 and 1 kg of steel respectively and 180 kgs of steel is available in the firm $\therefore 2x_1 + x_2 \leq 180$

Step-4:- Non-Negativity Constraints $\Rightarrow x_1, x_2 \geq 0$

$\therefore$ The LPP $\max z = 4x_1 + 6x_2$ and s.to constraints $x_1 + 3x_2 \leq 240$, $3x_1 + 4x_2 \leq 370$ and $2x_1 + x_2 \leq 180$, $x_1, x_2 \geq 0$

<u>Simplex</u> <u>Method</u>:- By introducing slack variables s_1, s_2 and s_3 to convert inequations in to equations.

$x_1 + 3x_2 + s_1 = 240$, $3x_1 + 4x_2 + s_2 = 370$ and $2x_1 + x_2 + s_3 = 180$.

$\therefore$ The objective function is, $\max z = 4x_1 + 6x_2 + 0s_1 + 0s_2 + 0s_3$

The Initial Basic feasible solution is, $X_B = B^{-1}.b = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}240\\370\\180\end{bmatrix} = \begin{bmatrix}240\\370\\180\end{bmatrix}$

<u>Simplex</u> <u>Table</u>:-

C_j			4	6	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	$[X_B/a_{ir}] = X_B/x_2$
0	s_1	240	1	3	1	0	0	$240/3 = 80$ ← OV
0	s_2	370	3	4	0	1	0	$370/4 = 92.5$
0	s_3	180	2	1	0	0	1	$180/1 = 180$
	z_j	0	0	0	0	0	0	
	$z_j - c_j$		-4	-6	0	0	0	
				↑EV				

C_j			4	6	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3	$[X_B/a_{ir}] = X_B/x_1$
6	x_2	80	1/3	1	1/3	0	0	$80/\frac{1}{3} = 240$
0	s_2	50	5/3	0	-4/3	1	0	$50 \times \frac{3}{5} = 30$ OV
0	s_3	100	5/3	0	-1/3	0	1	$100 \times \frac{3}{5} = 60$
	z_j	480	2	6	2	0	0	
	$z_j - c_j$		-2	0	2	0	0	
			↑EV					

$1 - \frac{1}{3}(4)$ $\frac{3-4}{3} = \frac{-1}{3}$

$\frac{1}{3}$ $-3/4 / 5/3$ $\frac{-9}{20}$

C_j			4	6	0	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	s_3
6	x_2	70	0	1	3/5	-1/5	0
4	x_1	$50/\frac{5}{3} = 30$	1	0	-4/5	3/5	0
0	s_3	50	0	0	1	-1	1
	z_j	540	4	6	2/5	6/5	0
	$z_j - c_j$		0	0	2/5	6/5	0

420
120
―――
540

$\frac{1}{3} - \frac{1}{3}(1)$ $\frac{-3}{4}$ | $5/3$

$\frac{1-4}{3} = \frac{-3}{4}$

$\therefore$ All $(z_j - c_j) \geq 0$ then $\max z = 540$ and $x_1 = 30$ & $x_2 = 70$

Degeneracy :-

The phenomena of obtaining a degenerate basic feasible solution in a LPP is known as degeneracy.

Degeneracy in LPP may araise

① At the initial stage

② At any subsequent Iteration stage.

In case of the Initial stage (1) atleast one of the basic variables should be zero in the Initial Basic feasible solution. where as in case of (2) at any iteration of the simplex method more than one variable is eligible to leave the Basis and Hence the next simplex iteration produces a degenerate solution in which atleast one basic variable is zero.

Methods to Resolve Degeneracy:-

Step-1:- First, find out the rows for which the minimum non-negative ratio is the same (tie). Suppose there is a tie b/w 1st and 3rd row.

Step-2:- Now, Rearrange the columns of the usual simplex tables. So that, the columns forming the original unit matrix come first in proper order.

Step-3:- Find the minimum of the ratio. Element of the first column of the unit matrix divided by corresponding elements of key column. Only for the tied rows. i.e, for the 1st and 3rd rows.

① If the 3rd row has the minimum ratio. then, this row will be the key row and the key element can be determined by intersecting the key row with key column.

② If this minimum is also not unique, then go to the next step.

Step-4:- Now, find the Minimum of the ratio, only for the tied rows, If this minimum ratio is unique for the first row then this row will be the key row for determining the key element by intersecting with key column.

$$\frac{\text{Elements of the second column in the unit matrix}}{\text{Corresponding elements of key column}}$$

If this minimum is also not unit then go the next step.

Step-5:- Find the minimum ratio the above step is repeated till the minimum ratio obtained. So, as to resolve the degeneracy. After the resolution of this type simplex method is applied to obtain the optimum solution.

$$\frac{\text{Elements of the } 3^{rd} \text{ column of the unit matrix}}{\text{Corresponding elements of key column.}}$$

1. Solve the following LPP by Simplex Method.

$\max z = 3x_1 + 9x_2$

s. to constraints $x_1 + 4x_2 \le 8$

$x_1 + 2x_2 \le 4$ and $x_1, x_2 \ge 0$.

Sol By introducing slack variables s_1 and s_2 to convert inequations into equations.

$x_1 + 4x_2 + s_1 = 8$

$x_1 + 2x_2 + s_2 = 4$

$\therefore$ The objective function is, $\max z = 3x_1 + 9x_2 + 0s_1 + 0s_2$

The Intial Basic Feasible solution is, $X_B = B^{-1}b = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}8\\4\end{bmatrix} = \begin{bmatrix}8\\4\end{bmatrix}$

Simplex Table:-

C_B	Y_B	X_B / c_j	3 (x_1)	9 (x_2)	0 (s_1)	0 (s_2)	Ratio (x_B/a_{ir})
0	s_1	8	1	4	1	0	$8/4 = 2$ } tie
0	s_2	4	1	2	0	1	$4/2 = 2$ } tie
	z_j	0	0	0	0	0	
	$z_j - c_j$		-3	-9 ↑EV	0	0	

Since, the minimum ratio is not unique, the slack variables s_1 and s_2 leave the basis. This is an indication of degeneracy. To resolve this we rearrange the columns in such a way that the identity matrix first.

C_B	Y_B	X_B / c_j	0 (s_1)	0 (s_2)	3 (x_1)	9 (x_2)	Ratio (s_1/x_r)
0	s_1	8	1	0	1	4	$1/4 = 0.25$
0	s_2	4	0	1	1	2 KE	$0/2 = 0$ ← LV
	z_j	0	0	0	0	0	
	$z_j - c_j$		0	0	-3	-9 ↑EV	

C_B	Y_B	X_B / c_j	0 (s_1)	0 (s_2)	3 (x_1)	9 (x_2)
0	s_1	0	1	-2	-1	0
9	x_2	2	0	$1/2$	$1/2$	1
	z_j	18	0	$9/2$	$9/2$	9
	$z_j - c_j$		0	$9/2$	$3/2$	0

$\therefore$ All $(z_j - c_j) \ge 0$ then,

$\max z = 18$

$x_1 = 0$ and $x_2 = 2$

Q. Solve the following LPP by simplex method.

$\max z = 2x_1 + x_2$

s. to constraints $4x_1 + 3x_2 \le 12$

$4x_1 + x_2 \le 8$

$4x_1 - x_2 \le 8$ and $x_1, x_2 \ge 0$

Sol. By introducing slack variables s_1, s_2 and s_3 to convert inequation into equations.

$4x_1 + 3x_2 + s_1 = 12$, $4x_1 + x_2 + s_2 = 8$ and $4x_1 - x_2 + s_3 = 8$

∴ The objective function is, $\max z = 2x_1 + x_2 + 0s_1 + 0s_2 + 0s_3$

The Initial Basic feasible solutions is, $x_B = B^{-1}.b$

$$x_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 8 \\ 8 \end{bmatrix}$$

Simplex Table :-

C_B	Y_B	X_B (c_j)	x_1 (2)	x_2 (1)	s_1 (0)	s_2 (0)	s_3 (0)	Ratio x_B/a_{ir}
0	s_1	12	1	3	1	0	0	12
0	s_2	8	4	1	0	1	0	$8/4 = 2$ } tie
0	s_3	8	4	-1	0	0	1	$8/4 = 2$ } tie
	z_j	0	0	0	0	0	0	
	$z_j - c_j$		-2 ↑EV	-1	0	0	0	

∴ Since there is a tie the minimum ratio in the 2 & 3rd row. It is an indication of degeneracy. To resolve this we rearrange the columns in such a way the unit matrix comes first.

C_B	Y_B	X_B (c_j)	s_1 (0)	s_2 (0)	s_3 (0)	x_1 (2)	x_2 (1)	Ratio (s_1/x_1)	Ratio (s_2/x_1)
0	s_1	12	1	0	0	1	3	$-$	$-$
0	s_2	8	0	1	0	4	1	0 } tie	$1/4 = 0.25$
0	s_3	8	0	0	1	4 KE	-1	0 } tie	$\frac{0}{4} = 0$ ← DV
	z_j	0	0	0	0	0	0		
	$z_j - c_j$		0	0	0	-2 ↑EV	-1		

C_B	Y_B	X_B (c_j)	s_1 (0)	s_2 (0)	s_3 (0)	x_1 (2)	x_2 (1)	Ratio (x_B/x_2)
0	s_1	4	1	0	-1	0	4	$\frac{4}{4} = 1$
0	s_2	0	0	1	-1	0	2 KE	0 ← DV
2	x_1	2	0	0	$1/4$	1	$-1/4$	$-$
	z_j	4	0	0	$1/2$	2	$-1/2$	
	$z_j - c_j$		0	0	$1/2$	0	$-3/2$ ↑EV	

		C_j	0	0	0	2	1	Ratio
C_B	Y_B	X_B	S_1	S_2	S_3	x_1	x_2	(X_B/S_3)
0	S_1	4	1	-2	1 PE	0	0	$4/1=4 \to$ ogv
1	x_2	0	0	1/2	-1/2	0	1	–
2	x_1	2	0	-1/8	1/8	1	0	16
	Z_j	4	0	3/4	-1/4	2	1	
	Z_j-C_j		0	3/4	-1/4	0	0	

↑EV

$\frac{1}{2}+\frac{1}{4}=\frac{2+1}{4}=\frac{3}{4}$

		C_j	0	0	0	2	1
C_B	Y_B	X_B	S_1	S_2	S_3	x_1	x_2
0	S_3	4	1	-2	1	0	0
1	x_2	2	1/2	-1/2	0	0	1
2	x_1	3/2	-1/8	3/8	0	1	0
	Z_j	5	1/4	1/4	0	2	1
	Z_j-C_j		1/4	1/4	0	0	0

$\frac{1}{2}-\frac{1}{4}=\frac{2-1}{4}=\frac{1}{4}$

$\frac{-1}{2}+\frac{3}{8}=\frac{-4+3}{8}=\frac{-1}{8}$

$\frac{-1}{2}+\frac{3}{4}=\frac{-2+3}{4}=\frac{1}{4}$

$\therefore$ All $(Z_j-C_j)\geq 0$ then, max $Z=5$ and $x_1=\frac{3}{2}$ and $x_2=2$

3. Solve the following LPP by simplex method.

$\max Z = x_1+2x_2+x_3$

s. to constraints $2x_1+x_2-x_3\leq 2$

$-2x_1+x_2-5x_3\geq -6$

$4x_1+x_2+x_3\leq 6$ and $x_1,x_2,x_3\geq 0$

Ans: $\boxed{x_1=0,\ x_2=4,\ x_3=2,\ Z=10}$

Sol. Here, 1 bi is negative then we have to convert it by multiply with '-1' into non-negative.

$2x_1-x_2+5x_3\leq 6$

By introducing slack variable S_1, S_2 and S_3 to convert ~~it into~~ inequations into equations.

$2x_1+x_2-x_3+S_1=2$

$2x_1-x_2+5x_3+S_2=6$

$4x_1+x_2+x_3+S_3=6$

$\therefore$ The objective function is, $\max Z = x_1+2x_2+x_3+0S_1+0S_2+0S_3$

The Initial Basic Feasible Solution is, $X_B=B^{-1}.b=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}2\\6\\6\end{bmatrix}$

$\Rightarrow X_B=\begin{bmatrix}2\\6\\6\end{bmatrix}$

Simplex Table:-

		C_j	1	2	1	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	(X_B/x_2)
0	S_1	2	2	1 PE	-1	1	0	0	2 ← ogv
0	S_2	6	2	-1	+5	0	1	0	–
0	S_3	6	4	1	1	0	0	1	6
	Z_j	0	0	0	0	0	0	0	
	Z_j-C_j		-1	-2	-1	0	0	0	

↑EV

C_B	Y_B	X_B	x_1 (1)	x_2 (2)	x_3 (1)	S_1 (0)	S_2 (0)	S_3 (0)	Ratio (x_B/x_3)
2	x_2	2	2	1	-1	1	0	0	–
0	S_2	8	4	0	4	1	1	0	$2 = 8/4$
0	S_3	4	2	0	2	-1	0	1	$2 = 4/2$
	Z_j	4	4	2	-2	2	0	0	
	$Z_j - C_j$		3	0	-3	2	0	0	

} tie

↑EV

Since, there is a tie in the minimum ratio in the 2^{nd} & 3^{rd} rows. It is an indication of degeneracy. To resolve this we arrange the columns in such a way the S_1, S_2 & S_3 matrix comes first.

C_B	Y_B	X_B	S_1 (0)	S_2 (0)	S_3 (0)	x_1 (1)	x_2 (2)	x_3 (1)	Ratio S_1/x_3
2	x_2	2	1	0	0	2	1	-1	–
0	S_2	8	1	1	0	4	0	4 PE	$1/4 = 0.25$ ← LV
0	S_3	4	-1	0	1	2	0	2	–
	Z_j	4	2	0	0	4	2	-2	
	$Z_j - C_j$		2	0	0	3	0	-3	

↑EV

C_B	Y_B	X_B	S_1 (0)	S_2 (0)	S_3 (0)	x_1 (1)	x_2 (2)	x_3 (1)
2	x_2	4	5/4	1/4	0	3	1	0
1	x_3	$\frac{8}{4} = 2$	1/4	1/4	0	1	0	1
0	S_3	0	-3/2	-1/2	1	0	0	0
	Z_j	10	11/4	3/4	0	7	2	1
	$Z_j - C_j$		11/4	3/4	0	6	0	0

∴ All $(Z_j - C_j) \geq 0$ then, max $Z = 10$ and $x_1 = 0, x_2 = 4$ and $x_3 = 2$

4. max $Z = 2x_1 + x_2$

S. to constraints $x_1 - x_2 \leq 10$

$2x_1 - x_2 \leq 40$ and $x_1, x_2 \geq 0$

Sol. By introducing slack variables S_1, S_2 to convert inequations in to equations

$$x_1 - x_2 + S_1 = 10$$
$$2x_1 - x_2 + S_2 = 40$$

∴ The objective function is, max $Z = 2x_1 + x_2 + 0S_1 + 0S_2$

The Initial Basic feasible solution is, $X_B = B^{-1}.b = \begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}10\\40\end{bmatrix}$

$$X_B = \begin{bmatrix}10\\40\end{bmatrix}$$

Simplex Table :-

		C_j	2	1	0	0	Ratio X_B/x_1
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	
0	S_1	10	1 PE	-1	1	0	$10/1 = 10$ ← OGV
0	S_2	40	2	-1	0	1	$\frac{40}{2} = 20$
	Z_j	0	0	0	0	0	
	$Z_j - C_j$		-2	-1	0	0	
			↑EV				

$40 - 10(2) = 20$
$2 - 1(2) = 0$
$-1 + 1(2) = 1$
$0 - 1(2) = -2$
$1 - 0(2) = 1$

		C_j	2	1	0	0	Ratio (X_B/x_2)
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	
2	x_1	10	1	-1	1	0	–
0	S_2	20	0	1 PE	-2	1	$\frac{20}{1}$ ← OGV
	Z_j	20	2	-2	2	0	
	$Z_j - C_j$		0	-3	2	0	
				↑EV			

$10 - 20(-1) = 30$
$1 - 0(-1) = 1$
$-1 - 1(-1) = 0$
$1 + 2(-1) = -1$
$0 - 1(-1) = 1$

$-2 - 2 \quad -4$

		C_j	2	1	0	0	Ratio (X_B/S_1)
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	
2	x_1	30	1	0	-1	1	–
1	x_2	20	0	1	-2	1	–
	Z_j	80	2	1	-4	3	
	$Z_j - C_j$		0	0	-4	3	
					↑EV		

$\therefore$ Since $(Z_j - C_j) < 0$ but all the values in the key column are negative there is an indication of unbounded solution.

ARTIFICIAL VARIABLE TECHNIQUE

* **Definition:-** Linear programming problem in which constraints may also have '$\geq$' or '$=$' signs after ensuring that all b_i's ≥ 0 are considered. In such cases basis matrix cannot be obtained as an 'identity' matrix in the starting simplex table. Therefore we introduce a new type of variable called the Artificial variable.

To solve such LPP there are 2 methods:-

1. Big-M Method (or) Method of Penalities.
2. Two-phase simplex Method.

1. Big-M Method (or) Method of Penalities:-

The following steps are involved in solving an LPP by using Big-M method:

Step-1: Express the problem in the standard form.

Step-2: Add non-Negative Artificial variables to the left side of each of the equations corresponding to constraints of the type '$\geq$' or '$=$'. assign a very large penality [$-M$ for maximization and M for minimization] in the objective function.

Step-3: Solve the modified LPP by using simplex method until any one of the cases may arises.

Case-I: If no artificial variable appears in the in the Basis and the optimality conditions are satisfied. then, the current solution is an "Optimal Basic Feasible solution."

Case-II: If atleast one artificial variable appears in the Basis with zero value or at zero level and optimality conditions are satisfied then, the current solution is "Degenerate"

Case-III: If atleast one artificial variable appears in the Basis at positive level and the optimality conditions are satisfied. But, doesn't optimize the objective function. i.e, The original has no feasible solution. Since, it contains a very large penality 'M' and is called "pseudo-optimal solution."

Note:- While appling simplex method whenever an artificial variable leaves the basis we drop the artificial variable and we omit all the entries corres-ponding to its column from the simplex table.

1. Solve the following LPP by using Big-M method.

$$\max z = 3x_1 + 2x_2$$

s.to constraints, $2x_1 + x_2 \leq 2$

$$3x_1 + 4x_2 \geq 12$$

and $x_1, x_2 \geq 0$

Sol. By introducing slack variable $s_1 \geq 0$, surplus variable $s_2 \geq 0$, artificial variable $A_1 \geq 0$ to convert the inequations into equations.

$$2x_1 + x_2 + s_1 = 2$$

$$3x_1 + 4x_2 - s_2 + A_1 = 12$$

$\therefore$ The modified objective function, $\max z = 3x_1 + 2x_2 + 0s_1 + 0s_2 - MA_1$

Initial Basic feasible solution, $X_B = B^{-1}b = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}2\\12\end{bmatrix} = \begin{bmatrix}2\\12\end{bmatrix}$

Simplex Table:-

		C_j	3	2	0	0	-M	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	A_1	(X_B/x_2)
0	s_1	2	2	1 KE	1	0	0	$2/1 = 2$ ← LV
-M	A_1	12	3	4	0	-1	1	$\frac{12}{4} = 3$
	z_j	-12M	-3M	-4M	0	M	-M	
	$z_j - c_j$		-3M-3	-4M-2	0	M	0	

↑EV

$12 - 2(4) = 4$
$3 - 2(4) = -5$
$4 - 1(4) = 0$
$0 - 1(4) = -4$
$-1 - 0(4) = -1$
$1 - 0(4) = 1$

		C_j	3	2	0	0	-M
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	A_1
2	x_2	2	2	1	1	0	0
-M	A_1	4	-5	0	-4	-1	1
	z_j	-4M+4	5M+4	2	4M+2	M	-M
	$z_j - c_j$		5M+1	0	4M+2	M	0

Since, All $(z_j - c_j) \geq 0$ & one artificial variable appears in the basis at positive level. The given LPP doesn't possess any feasible solution. But, the LPP possess a pseudo optimum solution.

2. Solve the following LPP by using Big-M method.

$$\max z = 2x_1 + x_2 + x_3$$

s.to constraints $x_1 + x_2 + 2x_3 \leq 5$

$$2x_1 + 3x_2 + 4x_3 = 12$$

and $x_1, x_2, x_3 \geq 0$

Sol. By introducing slack variables $s_1 \geq 0$ and Artificial Variable $A_1 \geq 0$ to convert inequations into equations.

$$x_1+x_2+2x_3+s_1=5$$
$$2x_1+3x_2+4x_3+A_1=12$$

∴ The modified Objective function, $\max z = 2x_1+x_2+x_3+0s_1-MA_1$

The Initial Basic Feasible solution, $x_B = B^{-1}.b = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}5\\12\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}$

Simplex table :-

C_B	Y_B	X_B	x_1 (2)	x_2 (1)	x_3 (1)	s_1 (0)	A_1 (-M)	Ratio (x_B/x_3)
0	s_1	5	1	1	2 PE	1	0	$5/2=2.5 \rightarrow$ OgV
-M	A_1	12	2	3	4	0	1	$12/4=3$
	z_j	-12M	-2M	-3M	-4M	0	-M	
	z_j-c_j		-2M-2	-3M-1	-4M-1 ↑EV	0	0	

$12-\frac{5}{2}(4)=2$
$2-\frac{1}{2}(4)=0$
$3-\frac{1}{2}(4)=1$
$4-1(4)=0$
$0-\frac{1}{2}(4)=-2$
$1-0(4)=1$

C_B	Y_B	X_B	x_1 (2)	x_2 (1)	x_3 (1)	s_1 (0)	A_1 (-M)	Ratio (x_B/x_2)
1	x_3	5/2	1/2	1/2	1	1/2	0	5
-M	A_1	2	0	1 PE	0	-2	1	$2 \leftarrow$ ogV
	z_j	$-2M+\frac{5}{2}$	1/2	$-M+\frac{1}{2}$	1	$2M+\frac{1}{2}$	-M	
	z_j-c_j		$-\frac{3}{2}$	$-M-\frac{1}{2}$ ↑EV	0	$2M+\frac{1}{2}$	0	

$\frac{5}{2}-2(\frac{1}{2})=\frac{3}{2}$
$\frac{1}{2}-0(\frac{1}{2})=\frac{1}{2}$
$\frac{1}{2}-1(\frac{1}{2})=0$
$1-0(\frac{1}{2})=1$
$\frac{1}{2}-2(\frac{1}{2})=-\frac{1}{2}$

C_B	Y_B	X_B	x_1 (2)	x_2 (1)	x_3 (1)	s_1 (0)	Ratio (x_B/x_1)
1	x_3	3/2	1/2 PE	0	1	3/2	$3 \leftarrow$ ogV
1	x_2	2	0	1	0	-2	–
	z_j	7/2	1/2	1	1	-1/2	
	z_j-c_j		-3/2 ↑EV	0	0	-1/2	

C_B	Y_B	X_B	x_1 (2)	x_2 (1)	x_3 (1)	s_1 (0)
2	x_1	3	1	0	2	3
1	x_2	2	0	1	0	-2
	z_j	8	2	1	4	4
	z_j-c_j		0	0	3	4

∴ All $(z_j-c_j) \geq 0$ then, $\max z = 8$ and $x_1=3, x_2=2$ and $x_3=0$

∴ The solution is an optimal Basic feasible solution.

3. Solve the following LPP by using Big-M method.

$\min z = 4x_1+x_2$

s. to constraints $3x_1+x_2=3$

$4x_1+3x_2 \geq 6$

$x_1+2x_2 \leq 3$ and $x_1, x_2 \geq 0$

Sol.

$$\text{Min } z = -\text{Max}(-z)$$
$$= -\text{Max } z^*$$
$$\Rightarrow \text{Max } z^* = -\text{Min } z = -(4x_1 + x_2)$$
$$= -4x_1 - x_2$$

then,

By introducing Artificial variable, $A_1 \geq 0$ and surplus variable $s_1 \geq 0$ and $A_2 \geq 0$ and slack variable $s_2 \geq 0$

$$3x_1 + x_2 + A_1 = 3$$
$$4x_1 + 3x_2 - s_1 + A_2 = 6$$
$$x_1 + 2x_2 + s_2 = 3$$

∴ The modified Objective function is,

$$\max z^* = -4x_1 - x_2 - MA_1 - 0s_1 - MA_2 + 0s_2$$

The Initial Basic Feasible solution is, $x_B = B^{-1} \cdot b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$

$$x_B = \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix}$$

Simplex Table :-

we neglect s_1 column in row

C_B	Y_B	X_B	x_1	x_2	A_1	s_1	A_2	s_2	Ratio (x_B/x_1)
		C_j	-4	-1	$-M$	0	$-M$	0	
$-M$	A_1	3	3 PE	1	1	0	0	0	$3/3 = 1$ ← ogv
$-M$	A_2	6	4	3	0	-1	1	0	$6/4 = 3/2 = 1.5$
0	s_2	3	1	2	0	0	0	1	$3/1 = 3$
	Z_j	$-3M-6M = -9M$	$-3M-4M = -7M$	$-M-3M = -4M$	$-M$	M	$-M$	0	
	$Z_j - C_j$		$-3M-4M+4 = -7M+4$	$-M-3M+1 = -4M+1$	0	M	0	0	

↑EV

C_B	Y_B	X_B	x_1	x_2	s_1	A_2	s_2	Ratio (x_B/x_2)
		C_j	-4	-1	0	$-M$	0	
-4	x_1	$3/3 = 1$	1	$1/3$	0	0	0	$1/1/3 = 3$
$-M$	A_2	2	0	$5/3$ PE	-1	1	0	$2/5/3 = \frac{6}{5} = 1.2$ ← ogv
0	s_2	2	0	$5/3$	0	0	1	$2/5/3 = \frac{6}{5} = 1.2$
	Z_j	$-2M-4$	-4	$\frac{-4-5M}{3}$	M	$-M$	0	
	$Z_j - C_j$		0	$\frac{-5M+1}{3}$	M	0	0	

↑EV

$0 - \frac{1}{3}$

$-1 - 0(4)$

C_B	Y_B	X_B	x_1	x_2	s_1	s_2	Ratio (x_B/s_1)
		C_j	-4	-1	0	0	
-4	x_1	$3/5$	1	0	$+1/5$	0	$3/5/1/5 = 3$
-1	x_2	$6/5$	0	1	$-3/5$	0	—
0	s_2	0	0	0	1 PE	1	0 ← ogv
	Z_j	$-18/5$	-4	-1	$-1/5$	0	
	$Z_j - C_j$		0	0	$-1/5$	0	

↑EV

$0 + \frac{2}{5}(\frac{1}{3})$

$0 + \frac{2}{5}(\frac{1}{3})$

$0 + \frac{3}{5}(1/3)$

$0 + \frac{2}{5}(5/3)$

$\frac{1}{5/3} = \frac{3}{5}$

$\frac{3}{5} / \frac{-1}{5} = -3$

$\frac{4+1}{5}$

~~∴ All ($z_j - c_j$) ≥ 0 then max $z^* = \frac{-18}{5}$ and $x_1 = \frac{3}{5}$ and $x_2 = \frac{6}{5}$~~
~~∴ The solution is an Basic Optimum Feasible solution.~~

		c_j	-4	-1	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
-4	x_1	3/5	1	0	0	-1/5
-1	x_2	6/5	0	1	0	3/5
0	s_1	0	0	0	1	1
	z_j	-18/5	-4	-1	0	1/5
	$z_j - c_j$		0	0	0	1/5

∴ All ($z_j - c_j$) ≥ 0 then

$\max z^* = \frac{-18}{5}$

∴ $\min z = 18/5$

$x_1 = \frac{3}{5}$ and $x_2 = \frac{6}{5}$

∴ The Solution is an Basic Optimum Feasible Solution.

4. Solve the following LPP by Big-M method

$\min z = 12x_1 + 20x_2$

s. to constraints $6x_1 + 8x_2 \geq 100$

$7x_1 + 12x_2 \geq 120$, and $x_1, x_2 \geq 0$

Sol. $\min z = -\max(-z)$

$= -\max z^*$

$\max z^* = -\min z$

$\max z^* = -(12x_1 + 20x_2) = -12x_1 - 20x_2$

then,

By introducing surplus variables $s_1 \geq 0$ and $s_2 \geq 0$ and Artificial variables $A_1 \geq 0$ & $A_2 \geq 0$

$6x_1 + 8x_2 - s_1 + A_1 = 100$

$7x_1 + 12x_2 - s_2 + A_2 = 120$

∴ The modified Objective function is,

$\max z^* = -12x_1 - 20x_2 - 0s_1 - MA_1 - 0s_2 - MA_2$

The Initial Basic Feasible Solution is, $X_B = B^{-1} b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 120 \end{bmatrix}$

$X_B = \begin{bmatrix} 100 \\ 120 \end{bmatrix}$

Simplex Table:-

		c_j	-12	-20	0	-M	0	-M	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	A_1	s_2	A_2	(X_B/x_2)
-M	A_1	100	6	8	-1	1	0	0	$\frac{100}{8} = 22.2$
-M	A_2	120	7	12 PE	0	0	-1	1	$\frac{120}{12} = 10$ ← ogv
	z_j	-220M	-13M	-20M	M	-M	M	-M	
	$z_j - c_j$		-13M+12	-20M+20	M	0	M	0	

↑ EV

$\frac{17.1}{\frac{120}{7}}$

		c_j	-12	-20	0	-M	0	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	A_1	s_2	(X_B/x_1)
-M	A_1	20	4/3 PE	0	-1	1	2/3	$\frac{20}{4/3} = 15$ ← ogv
-20	x_2	10	7/12	1	0	0	-1/12	$\frac{10}{7/12} = \frac{120}{7} = 17.1$
	z_j	-20M-200	$\frac{-4M-35}{3}$	-20	M	-M	$\frac{-2M+5}{3}$	
	$z_j - c_j$		$\frac{-4M+1}{3}$	0	M	0	$\frac{-2M+5}{3}$	

↑ EV

$100 - 10(8) = 20$

$6 - 7/12(8) = \frac{18-14}{3} = \frac{4}{3}$

$8 - 1(8) = 0$

$-1 - 0(8) = -1$

$1 - 0(8) = 1$

$0 + 1/12(8) = 2/3$

$\frac{-4M}{3} - \frac{140}{12}$ $\frac{12 \times 3}{36}$

$\frac{-2M}{3} + \frac{20}{3}$ =

C_B	Y_B	X_B	x_1	x_2	S_1	S_2
		C_j	-12	-20	0	0
-12	x_1	15	1	0	$-3/4$	$1/2$
-20	x_2	$\frac{15}{12} = \frac{5}{4}$	0	1	$7/16$	$-3/8$
	z_j	-205	-12	-20	$1/4$	$3/2$
	$z_j - c_j$		0	0	$1/4$	$3/2$

$\therefore$ All $(z_j - c_j) \geq 0$ then $\max z^* = -205$

$\therefore \min z = 205$

$x_1 = 15$ and $x_2 = \frac{5}{4}$

$\therefore$ The solution is an Basic optimum Feasible solution.

5. Solve the following LPP by using Big-M method.

~~$\max z = 2x_1 + x_2 + 3x_3$~~ $\quad \max z = x_1 + 2x_2 + 3x_3 - x_4$

s. to constraints ~~$x_1 + x_2 + 2x_3 \leq 5$~~ $\quad x_1 + 2x_2 + 3x_3 = 15$

~~$2x_1 + 3x_2 + 4x_3 = 12$~~ $\quad 2x_1 + x_2 + 5x_3 = 20$

~~and $x_1, x_2, x_3 \geq 0$~~ $\quad x_1 + 2x_2 + x_3 + x_4 = 10$

and $x_1, x_2, x_3, x_4 \geq 0$

Sol. By introducing $A_1, A_2, A_3 \geq 0$ then, $x_1 + 2x_2 + 3x_3 + A_1 = 15$

$2x_1 + x_2 + 5x_3 + A_2 = 20$

$x_1 + 2x_2 + x_3 + x_4 + A_3 = 10$

$\therefore$ The modified objective function is, $\max z = x_1 + 2x_2 + 3x_3 - x_4 \ldots$

$\therefore$ The Initial Basic Feasible solution is,

$$X_B = B^{-1} \cdot b = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 15 \\ 20 \\ 10 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \\ 10 \end{bmatrix}$$

Simplex table :-

C_B	Y_B	X_B	x_1	x_2	x_3	x_4	A_1	A_2	A_3	Ratio (X_B/x_3)
		C_j	1	2	3	-1	-M	-M	-M	
-M	A_1	15	1	2	3	0	1	0	0	$15/3 = 5$
-M	A_2	20	2	1	5 PE	0	0	1	0	$20/5 = 4$ ← OGV
-M	A_3	10	1	2	1	1	0	0	1	$10/1 = 10$
	z_j	-45M	-4M	-5M	-9M	-M	-M	-M	-M	
	$z_j - c_j$		-4M-1	-5M-2	-9M-3	-M+1	0	0	0	

↑ IEV

C_B	Y_B	X_B	x_1	x_2	x_3	x_4	A_1	A_3	Ratio (X_B/x_2)
		C_j	1	2	3	-1	-M	-M	
-M	A_1	3	$-1/5$	$7/5$ PE	0	0	1	0	$3/\frac{7}{5} = \frac{15}{7} = 2.1$ ← OGV
3	x_3	4	$2/5$	$1/5$	1	0	0	0	$4/\frac{1}{5} = 20$
-M	A_3	6	$3/5$	$9/5$	0	1	0	1	$30/9 = 3.3$
	z_j	$-9M+12$	$\frac{-2M+6}{5}$	$\frac{-16M+3}{5}$	3	-M	-M	-M	
	$z_j - c_j$		$\frac{-2M+1}{5}$	$\frac{-16M-7}{5}$	0	-M+1	0	0	

↑ IEV

C_B	Y_B	X_B	x_1 (1)	x_2 (2)	x_3 (3)	x_4 (-1)	A_3 (-M)	Ratio (x_B/x_4)
2	x_2	15/7	-1/7	1	0	0	0	—
3	x_3	25/7	3/7	0	1	0	0	—
-M	A_3	15/7	6/7	0	0	1 PE	1	15/7 = 2.14 ← ogv
	z_j	$\frac{-15M+105}{7}$	$\frac{-6M+1}{7}$	2	3	-M	-M	
	z_j-c_j		$\frac{-6M+0}{7}$	0	0	-M+1	0	

↑EV

$4-\frac{15}{7}(1/8)-\frac{28-3}{7}=\frac{25}{7}$

$6-\frac{15}{7}$

$\frac{3}{5}+\frac{18}{35}\cdot\frac{24}{35}$

$\frac{9}{5}$ $\frac{30}{7}+\frac{75}{7}$

$\frac{-2+9-6M}{7}$

C_B	Y_B	X_B	x_1 (1)	x_2 (2)	x_3 (3)	x_4 (-1)	Ratio (x_B/x_1)
2	x_2	15/7	-1/7	1	0	0	—
3	x_3	25/7	3/7	0	1	0	25/3 = 8.3
-1	x_4	15/7	6/7 PE	0	0	1	15/6 = 2.5 ← ogv
	z_j	90/7	1/7	2	3	-1	
	z_j-c_j		-6/7	0	0	0	

↑EV

$\frac{30+75-15}{7}=\frac{90}{7}$

$\frac{-2+9-6}{7}=\frac{1}{7}$

$\frac{1}{7}-1=\frac{-6}{7}$

$\frac{25}{7}-\frac{5}{2}\left(\frac{3}{7}\right)=\frac{50-15}{14}=\frac{35}{14}=\frac{5}{2}$

C_B	Y_B	X_B	x_1 (1)	x_2 (2)	x_3 (3)	x_4 (-1)
2	x_2	5/2	0	1	0	1/6
3	x_3	5/2	0	0	1	-1/2
1	x_1	5/2	1	0	0	7/6
	z_j	15	1	2	3	0
	z_j-c_j		0	0	0	1

$\therefore$ All $(z_j-c_j)\geq 0$ then $\max z = 15$ and $x_1=5/2,\ x_2=5/2,\ x_3=5/2$ and $x_4=0$

$\therefore$ The solution is an basic optimum feasible solution.

6. Use penality method to solve the following LPP

$$\max z = 2x_1+3x_2+5x_3$$

s. to constraints, $3x_1+10x_2+5x_3\leq 15$

$33x_1-10x_2+9x_3\leq 33$

$x_1+2x_2+x_3\geq 4$

and $x_1, x_2, x_3\geq 0$

Sol. By introducing slack variables $s_1\geq 0$ and $s_2\geq 0$ and surplus variables $s_3\geq 0$ (~~and $s_4\geq 0$~~) and artificial variables $A_1\geq 0$ (~~and $A_2\geq 0$~~)

$$3x_1+10x_2+5x_3+s_1=15$$

$$33x_1-10x_2+9x_3+s_2=33$$

$$x_1+2x_2+x_3-s_3+A_1=4$$

∴ The modified objective function is, $\max z = 2x_1 + 3x_2 + 5x_3 + 0s_1 + 0s_2 \ldots$

The Initial Basic feasible solution is, $X_B = B^{-1} b$

$$X_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 15 \\ 33 \\ 4 \end{bmatrix} = \begin{bmatrix} 15 \\ 33 \\ 4 \end{bmatrix}$$

Simplex Table :-

C_B	Y_B	X_B \ C_j	2	3	5	0	0	0	−M	Ratio
			x_1	x_2	x_3	s_1	s_2	s_3	A_1	(X_B/x_2)
0	s_1	15	3	10 PE	5	1	0	0	0	$15/10 = 3/2 = 1.5 \leftarrow$ ogv
0	s_2	33	33	−10	9	0	1	0	0	−
−M	A_1	4	1	2	1	0	0	−1	1	$4/2 = 2$
	z_j	−4M	−M	−2M	−M	0	0	+M	−M	
	$z_j - c_j$		−M−2	−2M−3	−M−5	0	0	M	0	

↑EV

C_B	Y_B	X_B \ C_j	2	3	5	0	0	0	−M	Ratio
			x_1	x_2	x_3	s_1	s_2	s_3	A_1	(X_B/x_1)
3	x_2	3/2	$\frac{3}{10}$	1	$\frac{5}{10} = \frac{1}{2}$	1/10	0	0	0	$\frac{3/2}{3/10} = 5$
0	s_2	48	36 PE	0	14	1	1	0	0	$\frac{48}{36} = \frac{4}{3} = 1.33 \leftarrow$ ogv
−M	A_1	1	2/5	0	0	−1/5	0	−1	1	$\frac{1}{2/5} = \frac{5}{2} = 2.5$
	z_j	$\frac{9-2M}{2}$	$\frac{9-4M}{10}$	3	3/2	$\frac{3+2M}{10}$	0	M	−M	
	$z_j - c_j$		$\frac{-4M-11}{10}$	0	−7/2	$\frac{3+2M}{10}$	0	M	0	

↑EV

C_B	Y_B	X_B \ C_j	2	3	5	0	0	0	−M
			x_1	x_2	x_3	s_1	s_2	s_3	A_1
3	x_2	11/10	0	1	23/60	11/120	−1/120	0	0
2	x_1	4/3	1	0	$\frac{14}{36} = \frac{7}{18}$	1/36	1/36	0	0
−M	A_1	7/15	0	0	−7/45	−19/90	−1/90	−1	1
	z_j	$\frac{-14M+169}{30}$	2	3	$\frac{28M+347}{180}$	$\frac{76M+119}{360}$	$\frac{4M+1}{360}$	M	−M
	$z_j - c_j$		0	0	$\frac{28M-553}{180}$	$\frac{119+76M}{360}$	$\frac{4M+1}{360}$	M	0

∴ There is an artificial variable left in the basis at positive level. It has no feasible solution. Since, it contains a very large penality 'M' and is called "pseudo-optimal solution".

7. Solve the following LPP by using Big-M method:

$$\max z = 2x_1 + 4x_2 + x_3$$

s. to contraints
$$x_1 - 2x_2 - x_3 \le 5$$
$$2x_1 - x_2 + 2x_3 = 2$$
$$-x_1 + 2x_2 + 2x_3 \ge 1$$
and $x_1, x_2, x_3 \ge 0$

Sol. By introducing slack variable, $s_1 \geq 0$ and artificial variable $A_1 \geq 0$ and surplus variable $s_2 \geq 0$ and $A_2 \geq 0$

$$x_1 - 2x_2 - x_3 + s_1 = 5$$
$$2x_1 - x_2 + 2x_3 + A_1 = 2$$
$$-x_1 + 2x_2 + 2x_3 - s_2 + A_2 = 1$$

∴ The objective function is, $\max z = 2x_1 + 4x_2 + x_3 + 0s_1 - MA_1 - 0s_2 - MA_2$

The Initial Basic feasible solution is, $x_B = B^{-1} \cdot b$

$$x_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

Simplex Table:-

C_B	Y_B	X_B / C_j	2	4	1	0	-M	0	-M	Ratio
			x_1	x_2	x_3	s_1	A_1	s_2	A_2	(X_B/x_3)
0	s_1	5	1	-2	-1	1	0	0	0	–
-M	A_1	2	2	-1	2	0	1	0	0	$2/2 = 1$
-M	A_2	1	-1	2	2 PE	0	0	-1	1	$1/2 = 0.5$ ← ogv
	z_j	-3M	-M	-M	-4M	0	-M	M	-M	
	$z_j - c_j$		-M-2	-M-4	-4M-1 ↑EV	0	0	M	0	

C_B	Y_B	X_B / C_j	2	4	1	0	-M	0	Ratio
			x_1	x_2	x_3	s_1	A_1	s_2	(X_B/x_1)
0	s_1	11/2	1/2	-1	0	1	0	-1/2	$\frac{11}{2}/\frac{1}{2} = 11$
-M	A_1	1	3 PE	-3	0	0	1	1	$1/3 = 0.3$ ← ogv
1	x_3	1/2	-1/2	1	1	0	0	-1/2	–
	z_j	$\frac{-2M+1}{2}$	$\frac{-3M+1}{2}$	3M+1	1	0	-M	$\frac{-2M+1}{2}$	
	$z_j - c_j$	$\frac{-2M-1}{2}$	$\frac{-6M-5}{2}$ ↑EV	3M-3	0	0	0	$\frac{-2M+1}{2}$	

C_B	Y_B	X_B / C_j	2	4	1	0	0	Ratio
			x_1	x_2	x_3	s_1	s_2	(X_B/x_2)
0	s_1	16/3	0	-1/2	0	1	-2/3	–
2	x_1	1/3	1	-1	0	0	-1/3	
1	x_3	2/3	0	1/2 PE	1	0	-1/3	$\frac{2}{3}/\frac{1}{2} = \frac{4}{3}$ ← ogv
	z_j	4/3	2	-3/2	1	0	1/3	
	$z_j - c_j$		0	-11/2 ↑EV	0	0	1/3	

		C_j	2	4	1	0	0	Ratio (X_B/S_2)
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	
0	S_1	6	0	0	1	1	-1	–
2	x_1	$5/3$	1	0	2	0	$-4/3$	–
4	x_2	$4/3$	0	1	2	0	$-2/3$	–
	Z_j	$26/3$	2	4	12	0	$-10/3$	
	$Z_j - C_j$		0	0	11	0	$-10/3$	

↑EV

Since, one $(Z_j - C_j) < 0$, but all the values in the key-column are negative. There is an Indication of Unbounded solution.

* Two-phase Simplex Method :-

The two-phase simplex method is another method to solve the given LPP. Involving some artifical variable. The solution is obtained in two phases.

Phase-I :-

In this phase we construct an Auxilary LPP leading to a final simplex Table containing a basic feasible solution to the original problem.

Step-1 :- Assign a cost '-1' to each artificial variable and a cost '0' to all other variables and get a new objective function.

$$Z^* = -A_1 - A_2 - \dots$$

Step-2 :- Write down the auxilary LPP in which the objective function is to be maximized s.to the given set of constraints.

Step-3 :- Solve the auxilary LPP by simplex method until either of the following 3-cases arise.

Case-I :- Max $Z^* < 0$ and atleast one artificial variable appears in the optimum basis at positive level.

Case-II :- Max $Z^* = 0$ and atleast one artificial variable appears in the optimum basis at zero level.

Case-III :- Max $Z^* = 0$ and no artificial variable appears in the Basis.

In case-I, given LPP does not poses any feasible solution, whereas in cases-II & III we go to phase-II.

Phase-II:- Use the optimum Basic feasible solution of phase-I as a starting solution for the Original LPP. Assign the actual costs to the variable in the objective function and a zero cost to every artificial variable in the basis at zero level. Delete the artificial variable column from the table which is eliminated from the basis in phase-I

Apply simplex method to the modified simplex table obtained at the end of phase-I till an optimum basic feasible solution is obtained (or) There is an indication of unbound.

1. Solve the Given LPP by using Two Phase Simplex Method.

$$\text{Max } Z = 5x_1 + 3x_2$$

s.to constraints $2x_1 + x_2 \leq 1$

$$x_1 + 4x_2 \geq 6$$

and $x_1, x_2 \geq 0$

Sol. By introducing slack variable $s_1 \geq 0$ and surplus, artificial variables $s_2 \geq 0$ and $A_1 \geq 0$ to convert inequations into equations.

$$2x_1 + x_2 + s_1 = 1$$

$$x_1 + 4x_2 - s_2 + A_1 = 6$$

$\therefore$ The Initial Basic feasible solution is given by

$$s_1 = 1, A_1 = 6$$

Phase-I:-

Assign the cost '-1' for the artifical variables and cost '0' for other variables.

The objective function of auxilary variable is,

$$\text{Max } Z^* = 0x_1 + 0x_2 + 0s_1 + 0s_2 - A_1$$

$$= -A_1$$

s. to constraints, $2x_1 + x_2 + s_1 = 1$

$x_1 + 4x_2 - s_2 + A_1 = 6$

Simplex Table:-

		C_j	0	0	0	0	-1	Ratio
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	A_1	(X_B/x_2)
0	S_1	1	2	1 (p.e)	1	0	0	1 $\longrightarrow$ OGV
-1	A_1	6	1	4	0	-1	1	6/4 = 1.5
	Z_j	-6	-1	-4	0	1	-1	
	$Z_j - C_j$		-1	-4	0	1	0	

$\uparrow$ EV

		c_j	0	0	0	0	-1
C_B	Y_B	X_B	x_1	x_2	S_1	S_2	A_1
0	x_2	1	2	1	1	0	0
-1	A_1	2	-7	0	-4	-1	1
	z_j	-2	7	0	4	1	-1
	z_j-c_j		7	0	4	1	0

∴ Since, All $(z_j-c_j)\geq 0$ the optimum solution is obtained the given auxilary LPP. but max $z^* < 0$ and one artificial variable appears in the basis at positive level. The original LPP doesn't posses any feasible solution.

2. Max $z = 2x_1+x_2+x_3$

s.to constraints, $4x_1+6x_2+3x_3\leq 8$

$3x_1-6x_2-4x_3\leq 1$

$2x_1+3x_2-5x_3\geq 4$ and $x_1,x_2,x_3\geq 0$

Solve the above LPP by Two-phase simplex method.

Sol. By introducing slack variables $S_1\geq 0$ and $S_2\geq 0$ and surplus and artificial variables $S_3\geq 0$ and $A_1\geq 0$

$4x_1+6x_2+3x_3+S_1=8$

$3x_1-6x_2-4x_3+S_2=1$

$2x_1+3x_2-5x_3-S_3+A_1=4$

∴ The Initial Basic feasible solution is,

$S_1=8$, $S_2=1$ and $A_1=4$

Phase-I:-

Assign the cost '-1' for the artificial variable and cost '0' for the other variables.

∴ The objective function to auxilary variable is,

$\text{Max } z^* = 0x_1+0x_2+0x_3+0S_1+0S_2+0S_3-A_1$

$= -A_1$

s.to constraints. $4x_1+6x_2+3x_3+S_1=8$

$3x_1-6x_2-4x_3+S_2=1$

$2x_1+3x_2-5x_3-S_3+A_1=4$

Simplex Table:-

		c_j	0	0	0	0	0	0	-1	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	A_1	(X_B/x_2)
0	S_1	8	4	6	3	1	0	0	0	$8/6=1.3$
0	S_2	1	3	-6	-4	0	1	0	0	-
-1	A_1	4	2	3 PE	-5	0	0	-1	1	$4/3=1.3$ → LV
	z_j	-4	-2	-3	5	0	0	1	-1	
	z_j-c_j		-2	-3	5	0	0	1	0	
				↑EV						

		c_j	0	0	0	0	0	0	-1
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	A_1
0	s_1	0	0	0	13	1	0	2	-2
0	s_2	9	7	0	-14	0	1	-2	2
0	x_2	4/3	2/3	1	-5/3	0	0	-1/3	1/3
	z_j	0	0	0	0	0	0	0	0
	$z_j - c_j$		0	0	0	0	0	0	1

∴ Since, All $(z_j - c_j) \geq 0$ the optimum solution is obtained to the given auxilary LPP. But $\max z^* = 0$ and no artificial variable appears in the basis. then, we go to phase-II

Phase-II :- Consider the final simplex table of phase-I, Consider the actual cost associated with the original values, delete the artificial variable column as it is eliminated in phase-I

$\max z = 2x_1 + x_2 + x_3$

		c_j	2	1	1	0	0	0	Ratio (X_B/x_3)
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	
0	s_1	0	0	0	13 PE	1	0	2	$\frac{0}{13} = 0 \rightarrow$ lgv
0	s_2	9	7	0	-14	0	1	-2	–
1	x_2	4/3	2/3	1	-5/3	0	0	-1/3	–
	z_j	4/3	2/3	1	-5/3	0	0	-1/3	
	$z_j - c_j$		-4/3	0	-8/3	0	0	-1/3	
					↑EV				

		c_j	2	1	1	0	0	0	Ratio (X_B/x_1)
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3	
1	x_3	0	0	0	1	1/13	0	2/13	–
0	s_2	9	7 PE	0	0	14/13	1	2/13	$\frac{9}{7} = 1.2 \rightarrow$ lgv
1	x_2	4/3	2/3	1	0	5/39	0	-1/13	$\frac{4}{3} / \frac{2}{3} = 2$
	z_j	4/3	2/3	1	1	8/39	0	1/13	
	$z_j - c_j$		-4/3	0	0	8/39	0	1/13	
			↑EV						

		c_j	2	1	1	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	s_2	s_3
1	x_3	0	0	0	1	1/13	0	2/13
2	x_1	9/7	1	0	0	2/13	1/7	-2/91
1	x_2	10/21	0	1	0	1/39	-2/21	-25/273
	z_j	64/21	2	1	1	16/39	4/21	29/273
	$z_j - c_j$		0	0	0	16/39	4/21	29/273

∴ All $(z_j - c_j) \geq 0$, $\text{Max } z = \frac{64}{21}$ and $x_1 = \frac{9}{7}$, $x_2 = \frac{10}{21}$ and $x_3 = 0$

3. Solve the LPP by two-phase method. -

$\max z = -4x_1 - 3x_2 - 9x_3$

s.to constraints $2x_1 + 4x_2 + 6x_3 \geq 15$

$6x_1 + x_2 + 6x_3 \geq 12$ and $x_1, x_2, x_3 \geq 0$

Sol: By introducing surplus and Artificial variables $s_1 \geq 0$, $A_1 \geq 0$ and $s_2 \geq 0$, $A_2 \geq 0$ to convert inequations into equations

$2x_1 + 4x_2 + 6x_3 - s_1 + A_1 = 15$

$6x_1 + x_2 + 6x_3 - s_2 + A_2 = 12$

∴ The Initial Basic Feasible solution is, $A_1 = 15$ and $A_2 = 12$

Phase-I :-

Assign the cost '-1' to the artificial variable and cost '0' to the other variables.

∴ The objective function of auxilary variable is,

$\max z^* = 0x_1 + 0x_2 + 0x_3 - 0s_1 - A_1 - 0s_2 - A_2$

$= -A_1 - A_2$

Simplex Table :-

C_B	Y_B	X_B \ C_j	0	0	0	0	-1	0	-1	Ratio
			x_1	x_2	x_3	s_1	A_1	s_2	A_2	(X_B/x_3)
-1	A_1	15	2	4	6	-1	1	0	0	$15/6 = 2.5$
-1	A_2	12	6	1	6 PE	0	0	-1	1	$12/6 = 2$ ← Dv
	z_j	-27	-8	-5	-12	1	-1	1	-1	
	$z_j - c_j$		-8	-5	-12	1	0	1	0	

↑EV

C_B	Y_B	X_B \ C_j	0	0	0	0	-1	0	-1	Ratio
			x_1	x_2	x_3	s_1	A_1	s_2	A_2	(X_B/x_2)
-1	A_1	3	-4	3 PE	0	-1	1	1	-1	$3/3 = 1$ → Dv
0	x_3	2	1	1/6	1	0	0	-1/6	1/6	$2/(1/6) = 12$
	z_j	-3	4	-3	0	1	-1	-1	1	
	$z_j - c_j$		4	-3	0	1	0	-1	2	

↑EV

C_B	Y_B	X_B \ C_j	0	0	0	0	-1	0	-1
			x_1	x_2	x_3	s_1	A_1	s_2	A_2
0	x_2	1	-4/3	1	0	-1/3	1/3	1/3	-1/3
0	x_3	11/6	11/9	0	1	1/18	-1/18	-2/9	2/9
	z_j	0	0	0	0	0	0	0	0
	$z_j - c_j$		0	0	0	0	1	0	1

$\therefore$ Since, All $(z_j - c_j) \geq 0$ then, the optimum solution is obtained to the given auxiliary LPP. But $\max z^* = 0$ and no artificial variable appears in the basis. we go to Phase-II

Phase-II:-

Consider, the final simplex table of phase-I, Consider the actual cost associated with the original values, delete the artificial variable column as it is eliminated in phase-I.

		c_j	-4	-3	-9	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	(x_B/x_1)
-3	x_2	1	$-4/3$	1	0	$-1/3$	$1/3$	$-$
-9	x_3	$11/6$	$(11/9)^{PE}$	0	1	$1/18$	$-2/9$	$\frac{11}{6} / \frac{11}{9} = \frac{3}{2} = 1.5 \rightarrow$ ogv
	z_j	$\frac{-117}{6}$	-7	-3	-9	$1/2$	1	
	$z_j - c_j$		-3	0	0	$1/2$	1	

$\uparrow$ EV

		c_j	-4	-3	-9	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2
-3	x_2	3	0	1	$8/11$	$-3/11$	$1/11$
-4	x_1	$3/2$	1	0	$6/11$	$1/22$	$-2/11$
	z_j	-15	-4	-3	$-\frac{72}{11}$	$7/11$	$5/11$
	$z_j - c_j$		0	0	$27/11$	$7/11$	$5/11$

$\therefore$ All $(z_j - c_j) \geq 0$ then $\max z = -15$ and $x_1 = \frac{3}{2}$ & $x_2 = 3$ & $x_3 = 0$

4. Solve the LPP by using two-phase method.

$\max z = +5x_1 - 4x_2 + 3x_3$

s. to constraints $2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 - 3x_2 + 6x_3 \leq 50$ and x_1, x_2 and, $x_3 \geq 0$

A. By introducing Artificial variable $A_1 \geq 0$ and slack variables $s_1 \geq 0$ and $s_2 \geq 0$, to convert inequations into equations.

$2x_1 + x_2 - 6x_3 + A_1 = 20$

$6x_1 + 5x_2 + 10x_3 + s_1 = 76$

$8x_1 - 3x_2 + 6x_3 + s_2 = 50$

$\therefore$ The Initial Basic Feasible solution is, $A_1 = 20$, $s_1 = 76$ & $s_2 = 50$

Phase-I:-

Assign the cost '-1' to the artificial variable and cost '0' the other variables.

$\therefore$ The objective function of auxiliary variable is,

$\max z^* = 0x_1 + 0x_2 + 0x_3 - A_1 + 0s_1 + 0s_2 = -A_1$

s. to constraints $2x_1+x_2-6x_3+A_1=20$

$6x_1+5x_2+10x_3+S_1=76$

$8x_1-3x_2+6x_3+S_2=50$

Simplex - Table :-

	C_j		0	0	0	-1	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	A_1	S_1	S_2	(X_B/x_1)
-1	A_1	20	2	1	-6	1	0	0	$20/2=10$
0	S_1	76	6	5	10	0	1	0	$76/6=12.6$
0	S_2	50	8 KE	-3	6	0	0	1	$50/8=\frac{25}{4}=6.2$ ← OGV
	Z_j	-20	-2	-1	6	-1	0	0	
	Z_j-C_j		-2	-1	6	0	0	0	

↑EV

	C_j		0	0	0	-1	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	A_1	S_1	S_2	(X_B/x_2)
-1	A_1	15/2	0	7/4 KE	-15/2	1	0	-1/4	$30/7=4.2$ ← OGV
0	S_1	~~77/2~~	0	29/4	1/2	0	1	-3/4	$81/2 / 29/4 = \frac{162}{29} = 7.7$
0	x_1	25/4	1	-3/8	3/4	0	0	1/8	
	Z_j	-15/2	0	-7/4	15/2	-1	0	1/4	
	Z_j-C_j		0	-7/4	15/2	0	0	1/4	

↑EV

	C_j		0	0	0	-1	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	A_1	S_1	S_2
0	x_2	30/7	0	1	-30/7	4/7	0	-1/7
0	S_1	52/7	0	0	256/7	-29/7	1	4/7
0	x_1	55/7	1	0	-6/7	3/14	0	1/14
	Z_j	0	0	0	0	0	0	0
	Z_j-C_j	0	0	0	0	1	0	0

∴ Since, All $(Z_j-C_j)\geq 0$ then, the optimum sol. is obtained to the given auxiliary LPP. But max $Z^*=0$ and no artificial variable appears in the basis. We go to phase-II

PHASE - II :-

Consider, the final simplex tables of phase-I. Consider the cost associated with the original values, delete the artificial variable column as it is eliminated in phase-I.

	C_j		5	-4	3	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2
-4	x_2	30/7	0	1	-30/7	0	-1/7
0	S_1	52/7	0	0	256/7	1	4/7
5	x_1	55/7	1	0	-6/7	0	1/14
	Z_j	155/7	5	-4	90/7	0	13/14
	Z_j-C_j		0	0	69/7	0	13/14

∴ All $(Z_j-C_j)\geq 0$ then

$\text{Max } z = \frac{155}{7}$

$x_1 = \frac{55}{7}$

$x_2 = \frac{30}{7}$

and $x_3 = 0$

5. Solve the LPP by using Two-phase Method.

Maximize $z = 5x_1 - 2x_2 + 3x_3$

s.to constraints $2x_1 + 2x_2 - x_3 \geq 2$

$3x_1 - 4x_2 \leq 3$

$x_2 + 3x_3 \leq 5$ and $x_1, x_2, x_3 \geq 0$

Sol. By introducing surplus and artificial variables $s_1 \geq 0$ and $A_1 \geq 0$ and slack variables $s_2 \geq 0$ and $s_3 \geq 0$

$2x_1 + 2x_2 - x_3 - s_1 + A_1 = 2$

$3x_1 - 4x_2 + s_2 = 3$

$x_2 + 3x_3 + s_3 = 5$

$\therefore$ The Initial Basic Feasible solution is $A_1 = 2$, $s_2 = 3$ and $s_3 = 5$

Phase-I:-

Assign the cost '-1' to the artificial variable and cost '0' to the other variables.

$\therefore$ The objective function of auxilary variable is,

$$\text{Max } z^* = 0x_1 + 0x_2 + 0x_3 - 0s_1 - A_1 + 0s_2 + 0s_3$$
$$= -A_1$$

s.to constraints

$2x_1 + 2x_2 - x_3 - s_1 + A_1 = 2$

$3x_1 - 4x_2 + s_2 = 3$

$x_2 + 3x_3 + s_3 = 5$

Simplex Table:-

		c_j	0	0	0	0	-1	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	A_1	s_2	s_3	(X_B/x_2)
-1	A_1	2	2	2 PE	-1	-1	1	0	0	1 → LV
0	s_2	3	3	-4	0	0	0	1	0	–
0	s_3	5	0	1	3	0	0	0	1	5
	z_j	-2	-2	-2	1	1	-1	0	0	
	$z_j - c_j$		-2	-2	1	1	0	0	0	

↑EV

		c_j	0	0	0	0	-1	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	s_1	A_1	s_2	s_3
0	x_2	1	1	1	-1/2	-1/2	1/2	0	0
0	s_2	7	7	0	-2	-2	2	1	0
0	s_3	4	-1	0	7/2	1/2	-1/2	0	1
	z_j	0	0	0	0	0	0	0	0
	$z_j - c_j$		0	0	0	0	1	0	0

$\therefore$ Since, All $(z_j - c_j) \geq 0$ then, the optimum solution is obtained to the given auxilary LPP, But max $z^* = 0$ and no artificial variable appears in the Basis we go to phase-II

Phase-II:-

Consider the final simplex table of phase-I, Consider the actual cost associated with the original values, delete the artificial variable column as it is elimated from basis in phase-I.

		C_j	5	-2	3	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	(X_B/x_1)
-2	x_2	1	1	1	-1/2	-1/2	0	0	1
0	S_2	7	7 PE	0	-2	-2	1	0	1 → ogv
0	S_3	4	-1	0	9/2	1/2	0	1	
	z_j	-2	-2	-2	1	1	0	0	
	z_j-c_j		-7	0	-2	1	0	0	

↑EV

		C_j	5	-2	3	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	(X_B/x_3)
-2	x_2	0	0	1	-3/14	-3/14	-1/7	0	—
5	x_1	1	1	0	-2/7	-2/7	1/7	0	—
0	S_3	5	0	0	45/14 PE	3/14	1/7	1	$5/\frac{45}{14} = \frac{14}{9}$ → ogv
	z_j	5	5	-2	-1	-1	1	0	
	z_j-c_j		0	0	-4	-1	1	0	

↑EV

		C_j	5	-2	3	0	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3	(X_B/S_1)
-2	x_2	1/3	0	1	0	-1/15	-2/15	1/15	—
5	x_1	13/9	1	0	0	-4/15	7/45	4/45	—
3	x_3	14/9	0	0	1	1/15 PE	2/45	14/45	$\frac{14}{9}/\frac{1}{15} = \frac{70}{3}$ → ogv
	z_j	61/9	5	-2	3	-11/15	53/45	56/45	
	z_j-c_j		0	0	0	-11/15	53/45	56/45	

↑EV

		C_j	5	-2	3	0	0	0
C_B	Y_B	X_B	x_1	x_2	x_3	S_1	S_2	S_3
-2	x_2	5	0	1	3	0	0	1
5	x_1	23/3	1	0	4	0	1/3	4/3
0	S_1	70/3	0	0	15	1	2/3	14/3
	z_j	85/3	5	-2	14	0	5/3	14/3
	z_j-c_j		0	0	11	0	5/3	14/3

$\therefore$ All $(z_j-c_j) \geq 0$ and the optimum solution is obtained and no artificial variable is held in the Basis

$\therefore \max z = \frac{85}{3}$, and $x_1 = \frac{23}{3}$, $x_2 = 5$ and $x_3 = 0$

6. Solve the following LPP by Two-phase simplex Method..

minimize $z = -2x_1 - x_2$

s.to constraints $x_1 + x_2 \geq 2$

$x_1 + x_2 \leq 4$ and $x_1, x_2 \geq 0$

Sol. $\min z = -\max(-z)$

$= -\operatorname{Max}(z)^*$

$\operatorname{Max} z^* = -\min z = -(-2x_1 - x_2) = 2x_1 + x_2$

By Introducing surplus and artificial variables $s_1 \geq 0$ and $A_1 \geq 0$ and slack variable $s_2 \geq 0$

$$x_1 + x_2 - s_1 + A_1 = 2$$
$$x_1 + x_2 + s_2 = 4$$

∴ The Initial Basic Feasible Solution is, $A_1 = 2$ and $s_2 = 4$

Phase-I :-

Assign the cost '-1' to the artificial variable and cost '0' to the other variables.

∴ The objective function of auxilary variable is,

$$\max z^* = 0x_1 + 0x_2 - 0s_1 - A_1 + 0s_2$$
$$= -A_1$$

s. to constraints $x_1 + x_2 - s_1 + A_1 = 2$

$x_1 + x_2 + s_2 = 4$

Simplex Table :-

C_B	Y_B	X_B \ c_j	0 x_1	0 x_2	0 s_1	-1 A_1	0 s_2	Ratio (X_B/x_2)
-1	A_1	2	1	1 PE	-1	1	0	2/1 = 2 → OGV
0	s_2	4	1	1	0	0	1	4/1 = 4
	z_j	-2	-1	-1	1	-1	0	
	$z_j - c_j$		-1	-1	1	0	0	

↑EV

4 − 2(1) = 2
1 − 1 = 0
1 − 1 = 0
0 + 1 = 1
0 − 1 = −1
1 − 0 = 1

C_B	Y_B	X_B \ c_j	0 x_1	0 x_2	0 s_1	-1 A_1	0 s_2
0	x_2	2	1	1	-1	1	0
0	s_2	2	0	0	1	-1	1
	z_j	0	0	0	0	0	0
	$z_j - c_j$		0	0	0	1	0

∴ since, All $(z_j - c_j) \geq 0$ then the optimum solution is obtained to the given auxilary LPP. But max $z^* = 0$ and no artificial variable appears in the Basic we go to phase-II

Phase-II :-

Consider, the final simplex table of phase-I, Consider the actual cost associated with the original values, delete the artificial column A_1 is eliminated in phase-I

C_B	Y_B	X_B \ c_j	2 x_1	1 x_2	0 s_1	0 s_2	Ratio (X_B/x_1)
1	x_2	2	1 PE	1	-1	0	2/1 = 2 → OGV
0	s_2	2	0	0	1	1	—
	z_j	2	1	1	-1	0	
	$z_j - c_j$		-1	0	-1	0	

↑EV

		C_j	2	1	0	0	Ratio
C_B	Y_B	X_B	x_1	x_2	s_1	s_2	(X_B/s_1)
2	x_1	2	1	1	-1	0	–
0	s_2	2	0	0	1^{PE}	1	$2 \longrightarrow$ OgV
	Z_j	4	2	2	-2	0	
	$Z_j - C_j$		0	1	-2	0	

↑EV

		C_j	2	1	0	0
C_B	Y_B	X_B	x_1	x_2	s_1	s_2
2	x_1	4	1	1	0	1
0	s_1	2	0	0	1	1
	Z_j	8	2	2	0	2
	$Z_j - C_j$		0	1	0	2

$2+2=4$
$1+0=1$
$1+0=1$
$-1+1=0$
$0+1=1$

∴ All $(Z_j - C_j) \geq 0$ than max $Z^* = 8$

∴ min $Z = -8$ and $x_1 = 4$, $x_2 = 0$