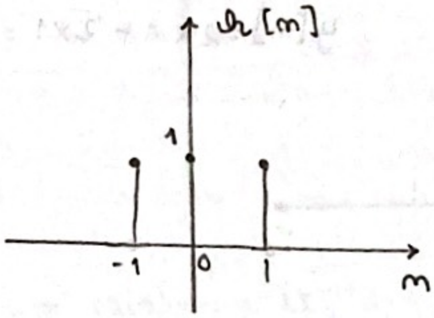
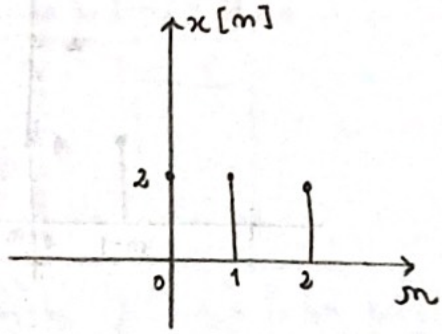
① Calcule a resposta impulsional do sistema cujo $h[n]$ e $x[n]$ são:

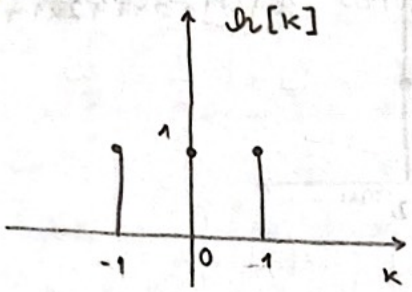




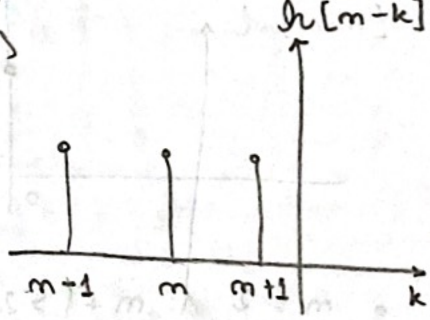
resposta impulsional do sistema:

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k]$$

Transformando h:

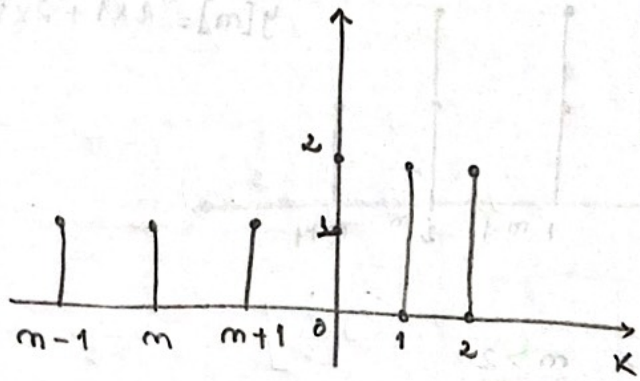


falta o gráfico do h[-k] →



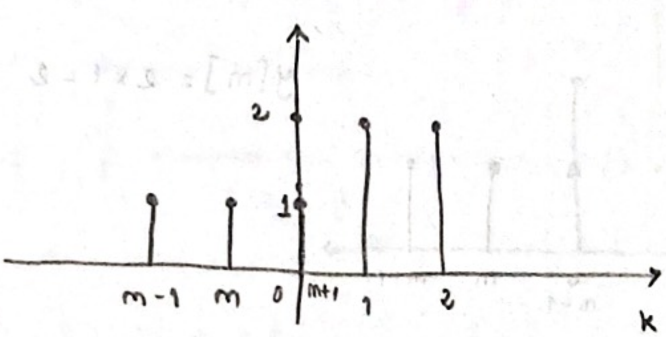
Logo, aplicando a convolução:

- $n+1 < 0 \Leftrightarrow n < -1$



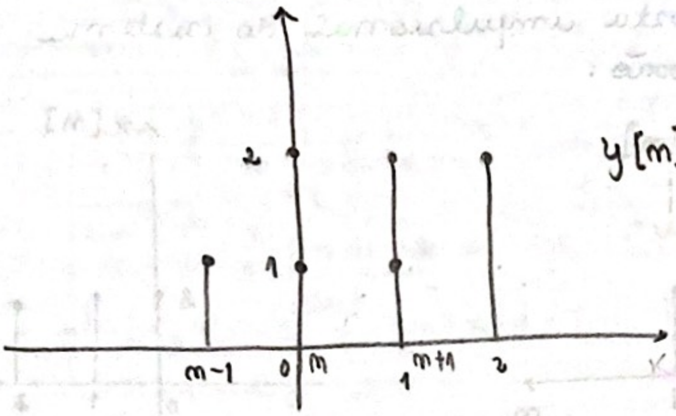
$y[n] = 0$

- $n+1 = 0 \Leftrightarrow n = -1$



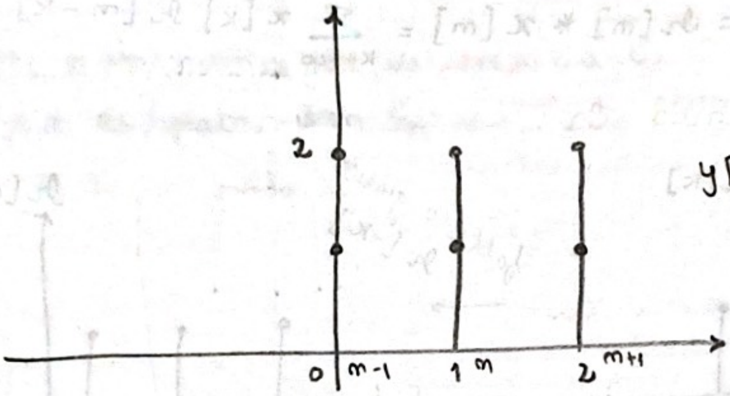
$y[n] = 1 \times 2 = 2$

• $m+1=1 \Leftrightarrow \underline{m=0} \quad \wedge \quad m-1<0 \Leftrightarrow \underline{m<1}$



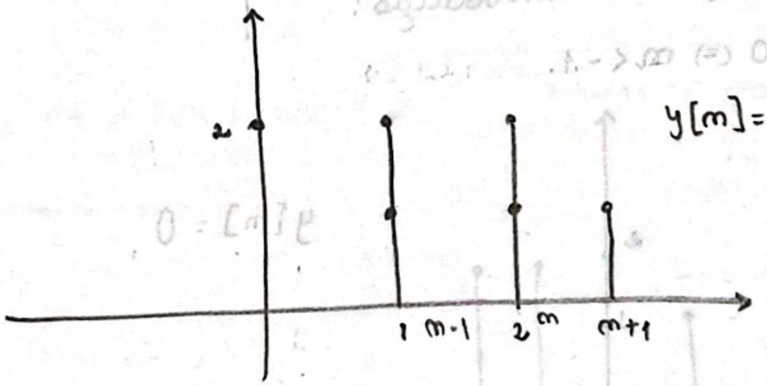
$y[m] = 2\times 1 + 2\times 1 = 4$

• $\underline{m=1}$



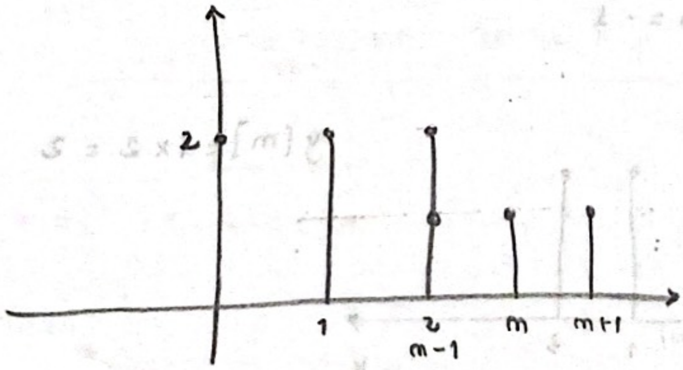
$y[m] = 2\times 1 + 2\times 1 + 2\times 1 = 6$

• $\underline{m=2} \wedge m+1>2 \Leftrightarrow \underline{m>1}$



$y[m] = 2\times 1 + 2\times 1 = 4$

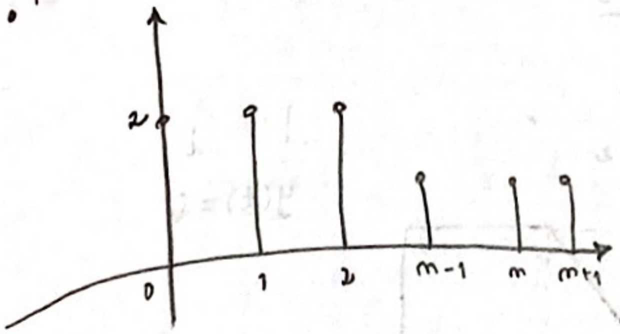
• $m-1=2 \Leftrightarrow \underline{m=3} \quad \wedge \quad \underline{m>2}$



$y[m] = 2\times 1 = 2$

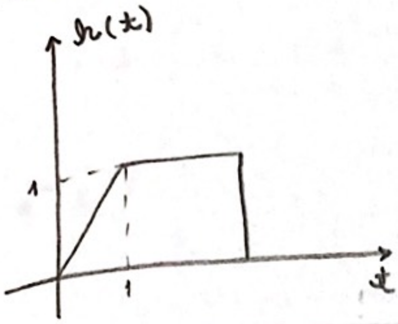
NAL

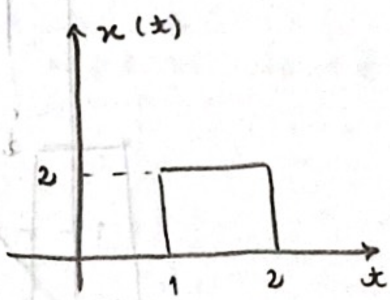
• $n-1 > 2 \iff \underline{n > 3}$



$y[n] = 0$

② Um sistema LIT tem a resposta impulsional $h(t)$. Determine e esboce o sinal de saída quando é aplicado ao sistema o sinal $x(t) = 2[u(t-1) - u(t-2)]$



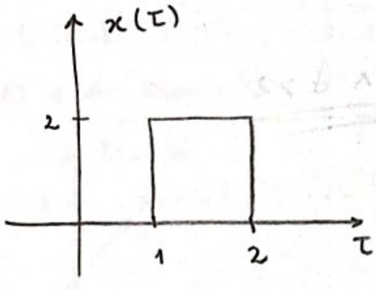


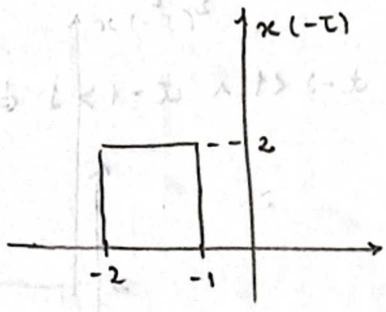
Para calcular o sinal de saída:

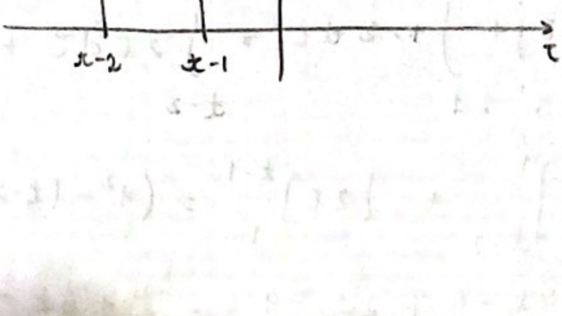
pela propriedade da comutatividade

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Aplicando a transformação em x:

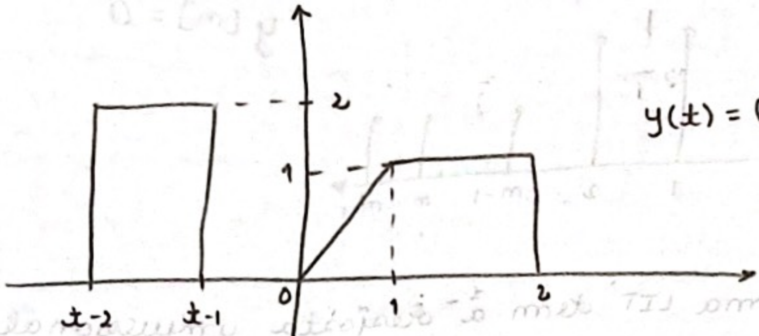






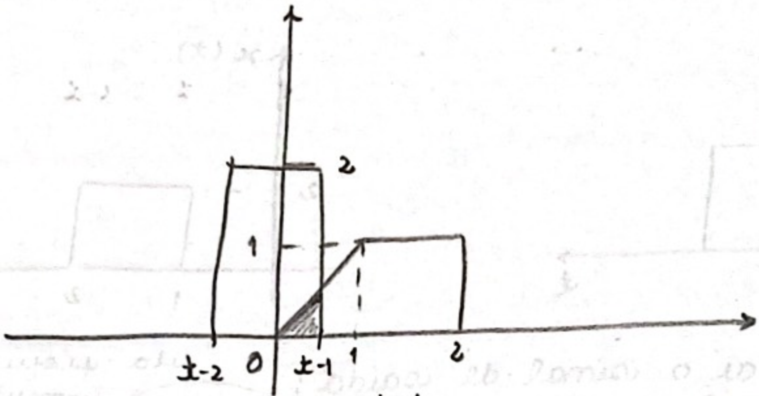
Aplicando a convolução vamos ter que:

- $t-1<0 \Leftrightarrow t<+1$



$y(t) = 0$

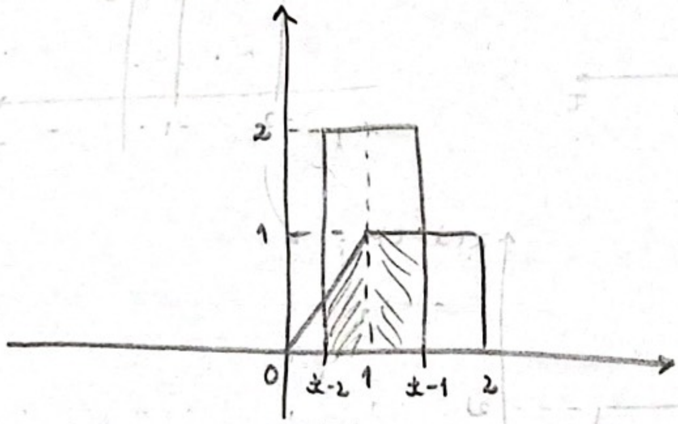
- $t-2<0 \wedge t-1>0 \Leftrightarrow t<2 \wedge t>1$



$$y(t) = \int_0^{t-1} \tau \cdot 2 \, d\tau = \int_0^{t-1} 2\tau \, d\tau = 2\left[\frac{1}{2}\tau^2\right]_0^{t-1} =$$

$$= 2 \cdot \frac{1}{2}\left[(t-1)^2 - 0^2\right] = (t-1)^2 = t^2 - 2t + 1$$

- $t-2<1 \wedge t-1>1 \Leftrightarrow t<3 \wedge t>2$

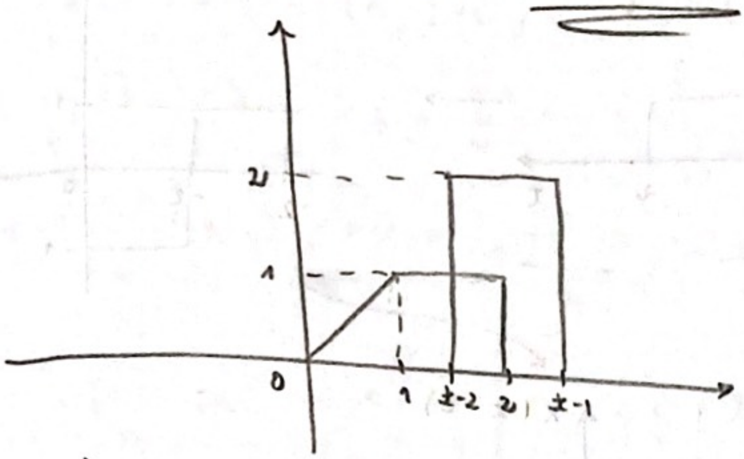


$$y(t) = \int_{t-2}^{1} \tau \cdot 2 \, d\tau + \int_{1}^{t-1} 1 \cdot 2 \, d\tau = \int_{t-2}^{1} 2\tau \, d\tau + \int_{1}^{t-1} 2 \, d\tau =$$

$$= 2\left[\frac{1}{2}\tau^2\right]_{t-2}^{1} + \left[2\tau\right]_1^{t-1} = \left(1^2 - (t-2)^2\right) + 2\left((t-1)-1\right) =$$

$$= 1 - t^2 + 4t - 4 + 2t - 4 = -t^2 + 6t - 7$$

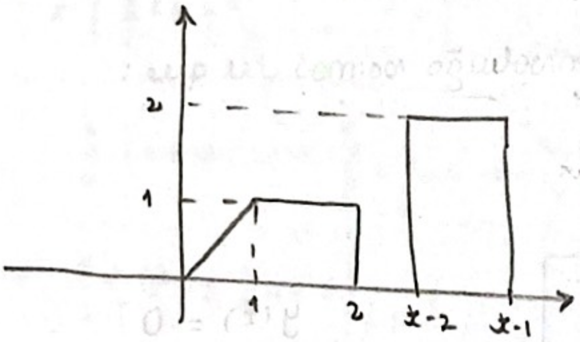
• $t-2<2 \wedge t-1>2 \Leftrightarrow \underline{t<4 \wedge t>3}$



$$y(t) = \int_{t-2}^{2} 1 \cdot 2 \, d\tau = \int_{t-2}^{2} 2 \, d\tau = \left[2\tau\right]_{t-2}^{2} =$$

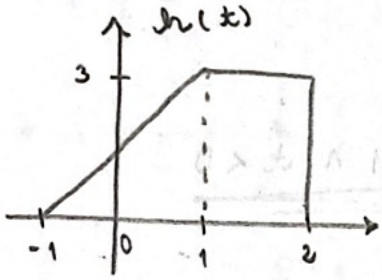
$$= 2\left(2-(t-2)\right) = 2\left(2-t+2\right) = -2t+8$$

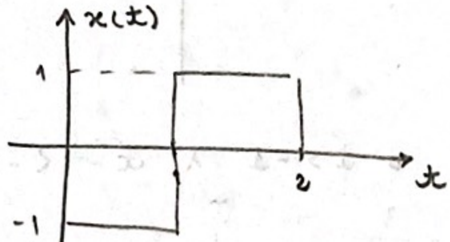
• $t-2>2 \Leftrightarrow \underline{t>4}$



$y(t) = 0$

③ Calcule a resposta do sistema cuja entrada é $x(t)$ e a resposta impulsional é $h(t)$.





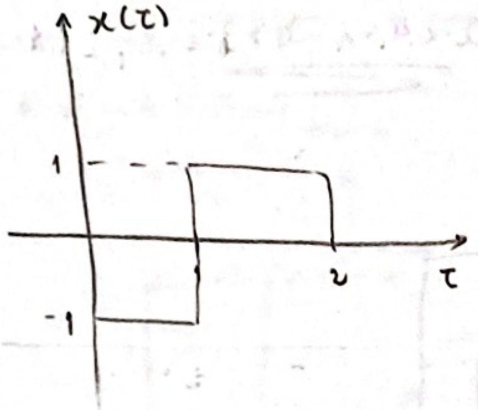
Para se obter a resposta do sistema:

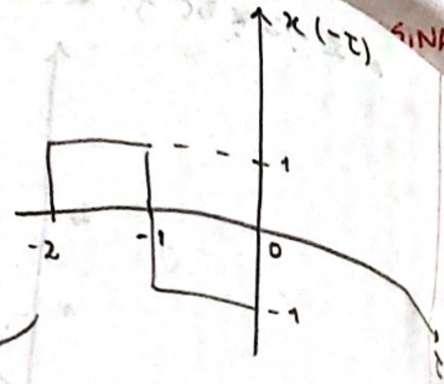
$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

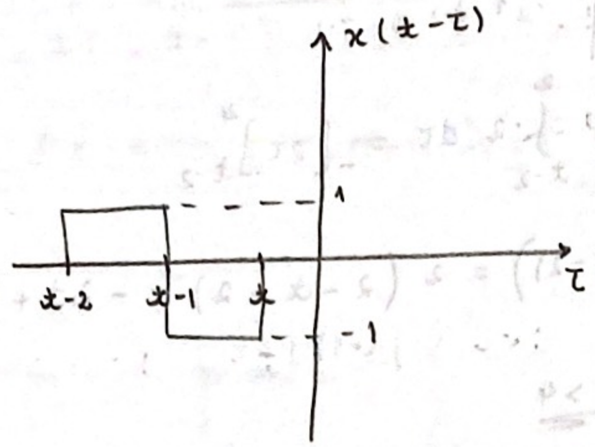
pela propriedade comutativa

Aplicando a transformação a x, vamos ter que:

→

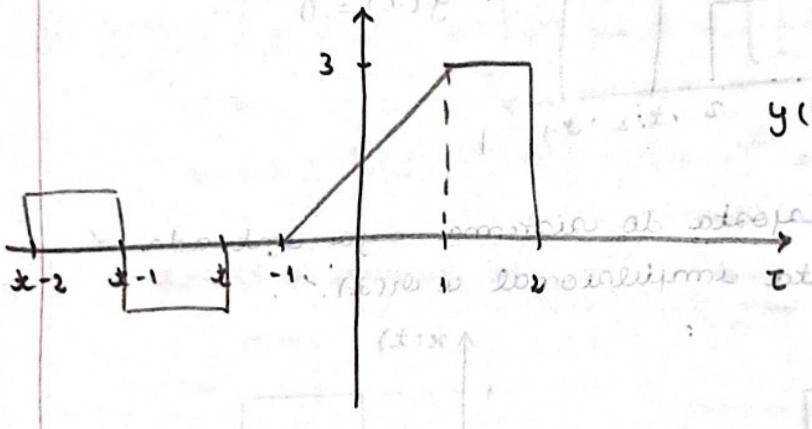






Logo, aplicando a convolução vamos ter que:

- $t < -1$



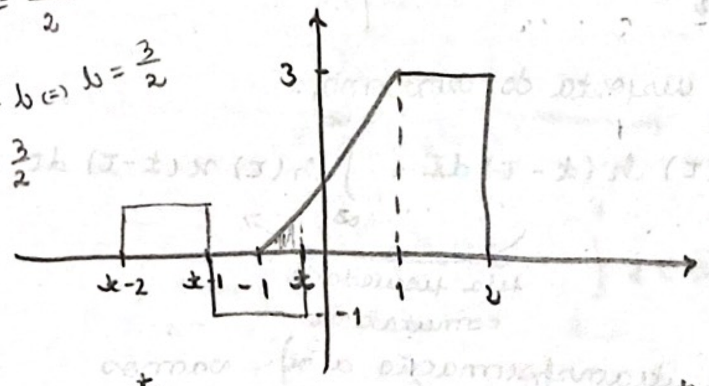
$y(t) = 0$

- $t > -1 \wedge t-1 < -1 \Leftrightarrow t > -1 \wedge t < 0$

$\frac{3-0}{1-(-1)} = \frac{3}{2}$

$0 = \frac{3}{2}(-1) + b \Leftrightarrow b = \frac{3}{2}$

$y = \frac{3}{2}x + \frac{3}{2}$

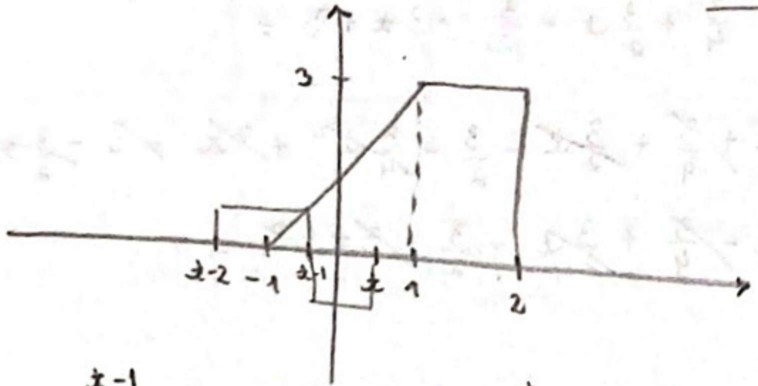


$$y(t) = \int_{-1}^{t} \left(\frac{3}{2}\tau + \frac{3}{2}\right)(-1)\, d\tau = (-1)\left(\int_{-1}^{t} \frac{3}{2}\tau\, d\tau + \int_{-1}^{t} \frac{3}{2}\, d\tau\right) =$$

$$= (-1)\left(\frac{3}{2}\left[\frac{1}{2}\tau^2\right]_{-1}^{t} + \left[\frac{3}{2}\tau\right]_{-1}^{t}\right) = (-1)\left(\frac{3}{4}\left(t^2-1\right)+\frac{3}{2}(t+1)\right) =$$

$$= (-1)\left(\frac{3}{4}t^2 \overbrace{- \frac{3}{4} + \frac{3}{2}t + \frac{3}{2}}^{=\frac{3}{4}}\right) = -\frac{3}{4}t^2 - \frac{3}{2}t - \frac{3}{4}$$

- $t-2 < -1 \wedge t-1 > -1 \wedge t > 0 \iff \underline{t < 1 \wedge t > 0}$



$$y(t) = \int_{-1}^{t-1} (1)\cdot\left(\frac{3}{2}\tau + \frac{3}{2}\right)d\tau + \int_{t-1}^{t}(-1)\left(\frac{3}{2}\tau+\frac{3}{2}\right)d\tau =$$

$$= \left(\frac{3}{2}\left[\frac{1}{2}\tau^2\right]_{-1}^{t-1} + \left[\frac{3}{2}\tau\right]_{-1}^{t-1}\right) + (-1)\left(\frac{3}{2}\left[\frac{1}{2}\tau^2\right]_{t-1}^{t} + \left[\frac{3}{2}\tau\right]_{t-1}^{t}\right) =$$

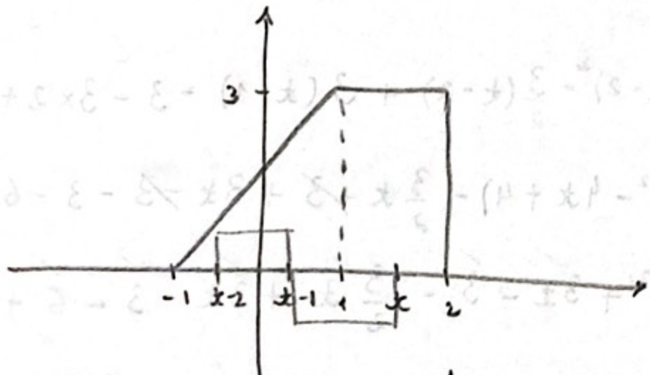
$$= \frac{3}{4}\left((t-1)^2-(-1)^2\right) + \frac{3}{2}(t\cancel{-1}\cancel{+1}) + (-1)\left(\frac{3}{4}\left(t^2-(t-1)^2\right)+\frac{3}{2}(\cancel{t}\cancel{-t}+1)\right) =$$

$$= \frac{3}{4}\left(t^2-2t+1+1\right) + \frac{3}{2}t + (-1)\left(\frac{3}{4}\left(\cancel{t^2}-\cancel{t^2}+2t-1\right)+\frac{3}{2}\right) =$$

$$= \frac{3}{4}t^2 - \cancel{\frac{3}{2}t} + \cancel{\frac{3}{2}} + \cancel{\frac{3}{2}t} - \frac{3}{2}t + \frac{3}{4} - \cancel{\frac{3}{2}} =$$

$$= \frac{3}{4}t^2 - \frac{3}{2}t + \frac{3}{4}$$

- $t-2 > -1 \wedge t-1 > 0 \wedge t < 2 \iff \underline{t > 1 \wedge t < 2}$



$$y(t) = \int_{t-2}^{t-1} 1\cdot\left(\frac{3}{2}\tau+\frac{3}{2}\right)d\tau + \int_{t-1}^{1}(-1)\cdot\left(\frac{3}{2}\tau+\frac{3}{2}\right)d\tau + \int_{1}^{t}(-1)\cdot 3\, d\tau =$$

$$= \left[\frac{3}{4}\tau^2+\frac{3}{2}\tau\right]_{t-2}^{t-1} + \left[-\frac{3}{4}\tau^2-\frac{3}{2}\tau\right]_{t-1}^{1} + \left[-3\tau\right]_{1}^{t} =$$

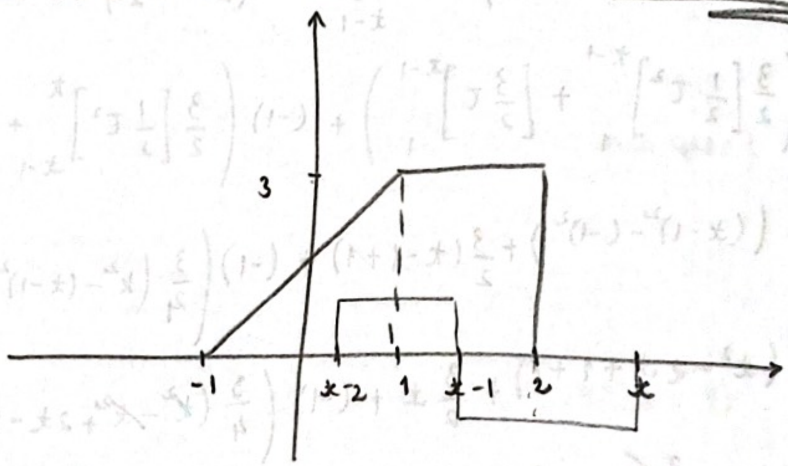
$$= \frac{3}{4}(t-1)^2 + \frac{3}{2}(t-1) - \frac{3}{4}(t-2)^2 - \frac{3}{2}(t-2) - \frac{3}{4} - \frac{3}{2} + \frac{3}{4}(t-1) + \frac{3}{2}(t-1) - 3t + 3 =$$

$$= \frac{3}{4}(t^2 - 2t + 1) + \frac{3}{2}t - \frac{3}{2} - \frac{3}{4}(t^2 - 4t + 4) - \frac{3}{2}t + 3 - \frac{3}{4} - \frac{3}{2} + \frac{3}{4}t - \frac{3}{4} + \frac{3}{2}t - \frac{3}{2} - 3t + 3 =$$

$$= \cancel{\frac{3}{4}t^2} - \frac{3}{2}t + \cancel{\frac{3}{4}} + \cancel{\frac{3}{2}t} - \frac{3}{2} - \cancel{\frac{3}{4}t^2} + \cancel{3t} - \cancel{3} - \frac{3}{2}t + 3 - \frac{3}{4} - \frac{3}{2} + \frac{3}{4}t - \cancel{\frac{3}{4}} + \cancel{\frac{3}{2}t} - \frac{3}{2} - \cancel{3t} + \cancel{3} =$$

$$= \frac{3}{4}t - \frac{9}{4}$$

- $t-2>0 \wedge t-1<2 \wedge t>2 \Leftrightarrow \underline{t>2 \wedge t<3}$



$$y(t) = \int_{t-2}^{1} (1)\cdot\left(\frac{3}{2}\tau + \frac{3}{2}\right) d\tau + \int_{1}^{t-1} 1\cdot 3\, d\tau + \int_{t-1}^{2} (-1)\cdot 3\, d\tau =$$

$$= \left[\frac{3}{4}\tau^2 + \frac{3}{2}\tau\right]_{t-2}^{1} + \left[3\tau\right]_{1}^{t-1} + \left[-3\tau\right]_{t-1}^{2} =$$

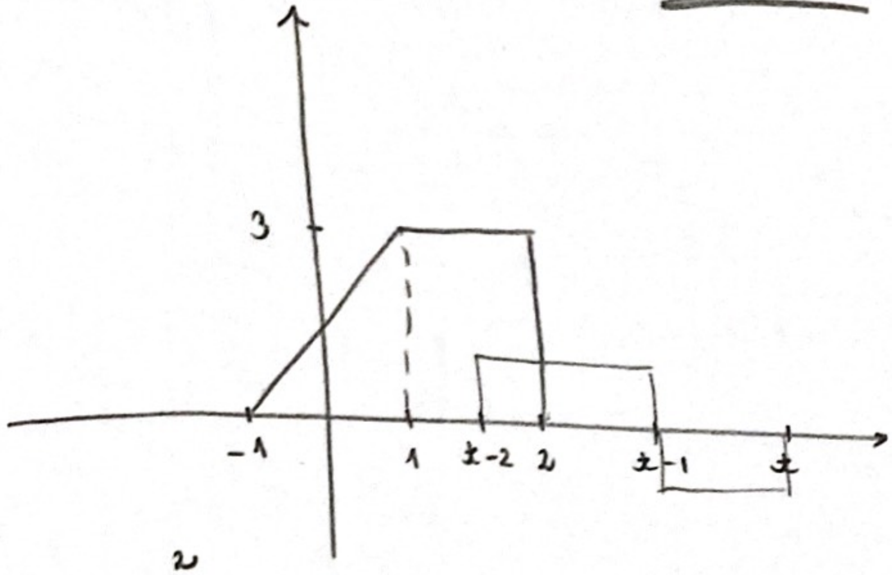
$$= \frac{3}{4} + \frac{3}{2} - \frac{3}{4}(t-2)^2 - \frac{3}{2}(t-2) + 3(t-1) - 3 - 3\times 2 + 3(t-1) =$$

$$= \frac{3}{4} + \frac{3}{2} - \frac{3}{4}(t^2 - 4t + 4) - \frac{3}{2}t + \cancel{3} + 3t - \cancel{3} - 3 - 6 + 3t - 3 =$$

$$= \frac{3}{4} + \frac{3}{2} - \frac{3}{4}t^2 + 3t - 3 - \frac{3}{2}t + 3t - 3 - 6 + 3t - 3 =$$

$$= -\frac{3}{4}t^2 + \frac{15}{2} - \frac{51}{4}$$

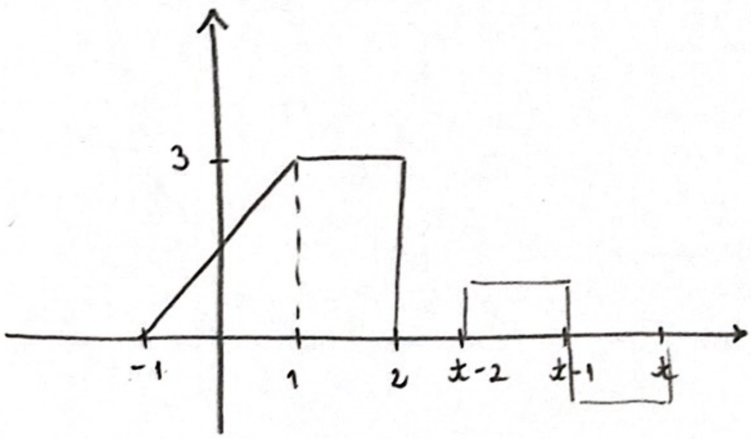
- $t-2<2 \wedge t-1>2 \iff \underline{t<4 \wedge t>3}$



$$y(t) = \int_{t-2}^{2} (1)\cdot 3 \, d\tau = [3\tau]_{t-2}^{2} = 3\cdot 2 - 3(t-2) =$$

$$= \cancel{6} - 3t - \cancel{6} = -3t$$

- $t-2>2 \iff \underline{t>4}$



$y(t) = 0$