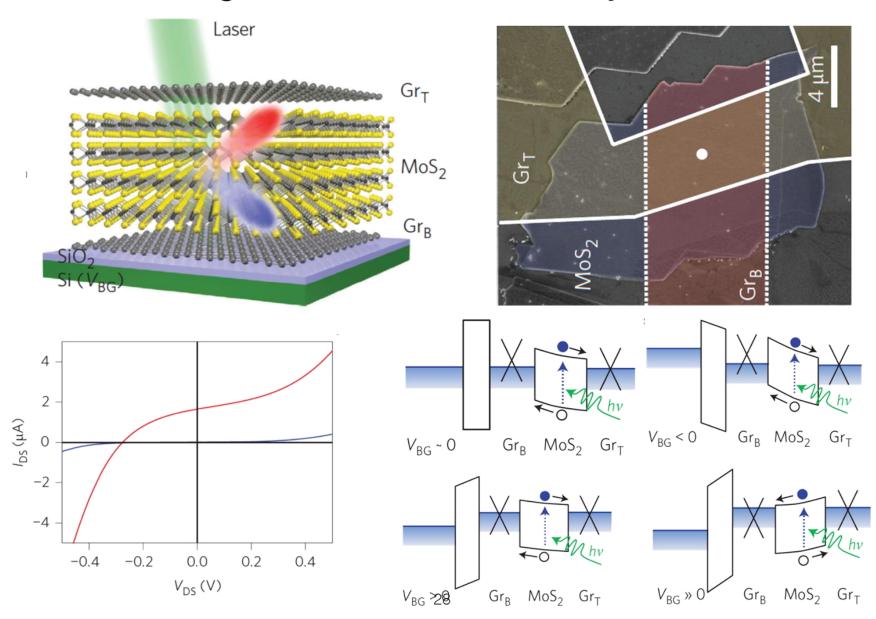
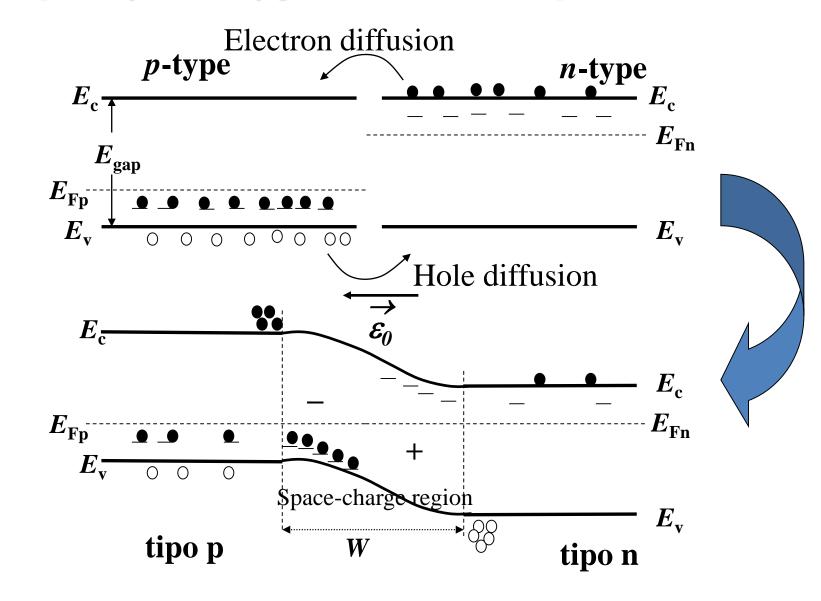
A photodetector made of 2D materials

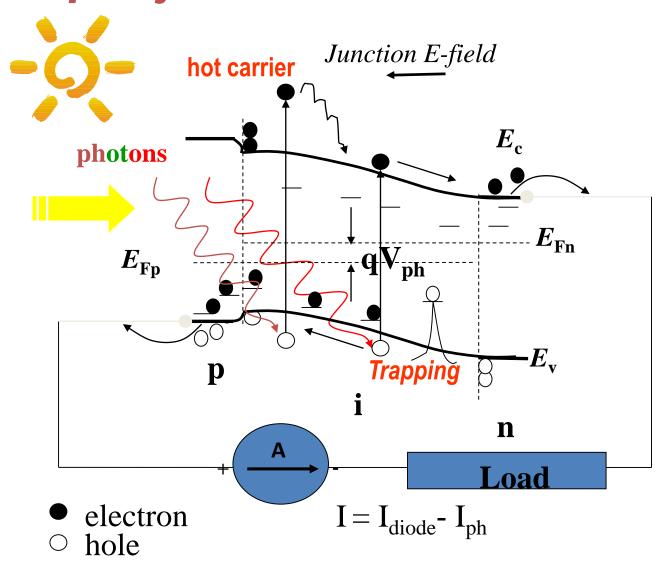
Woo Jong Yu et al., from University of California



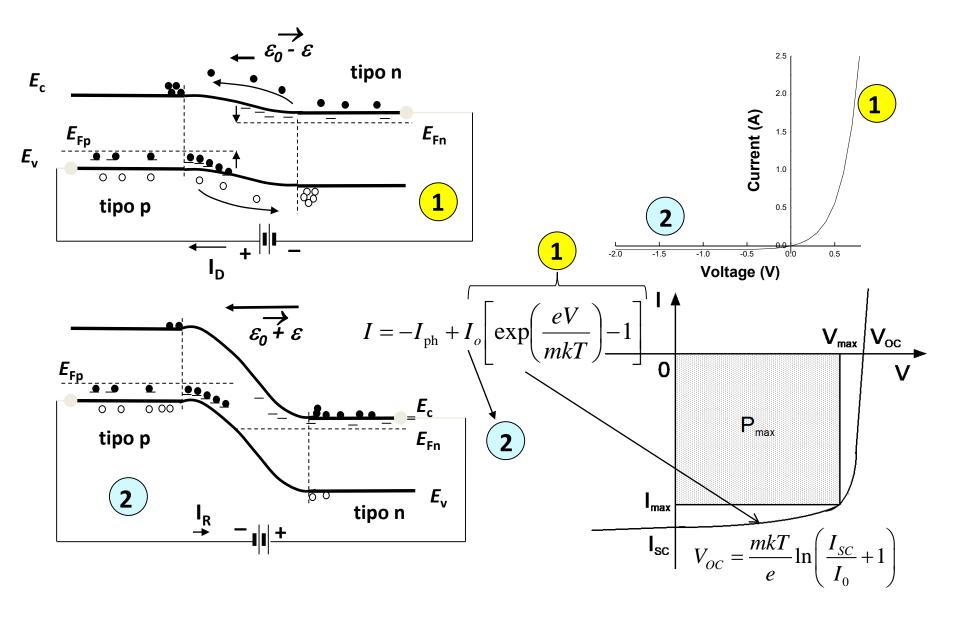
The p-n (homo) junction in equilibrium

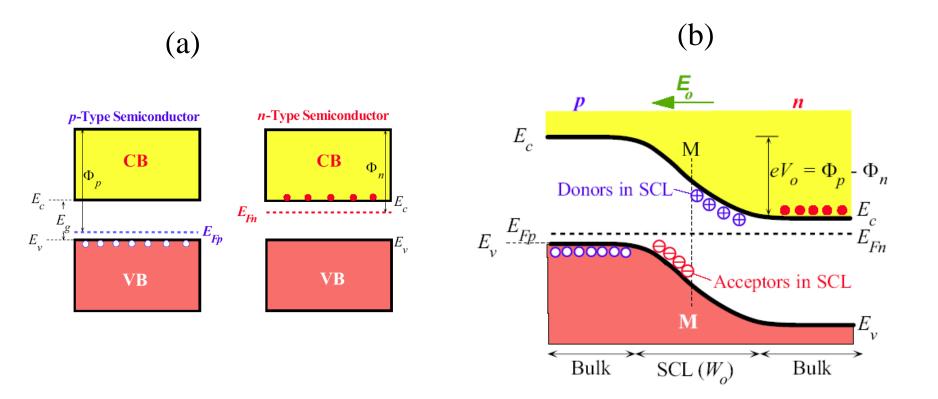


The *p-n* junction under illumination

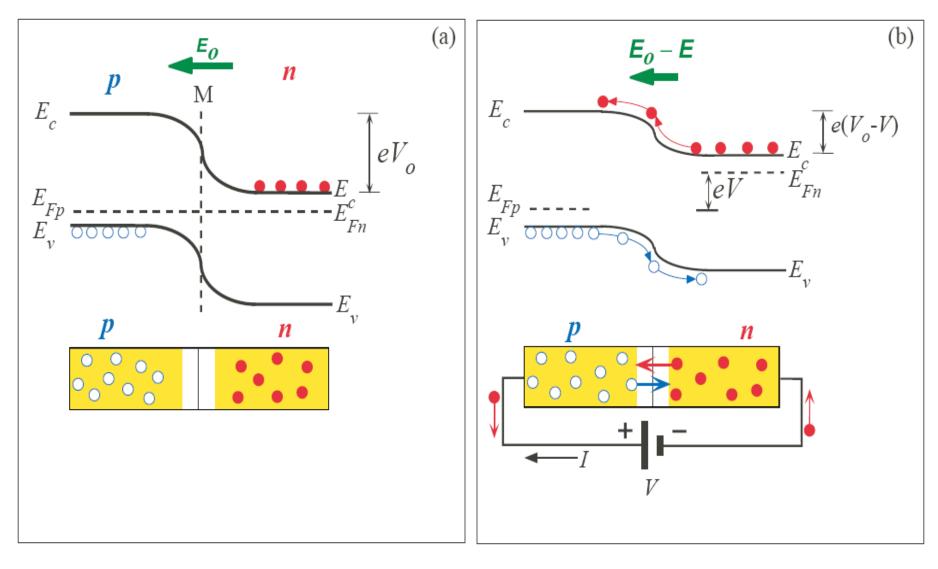


I-V characteristic of p-n junction (dark & light)

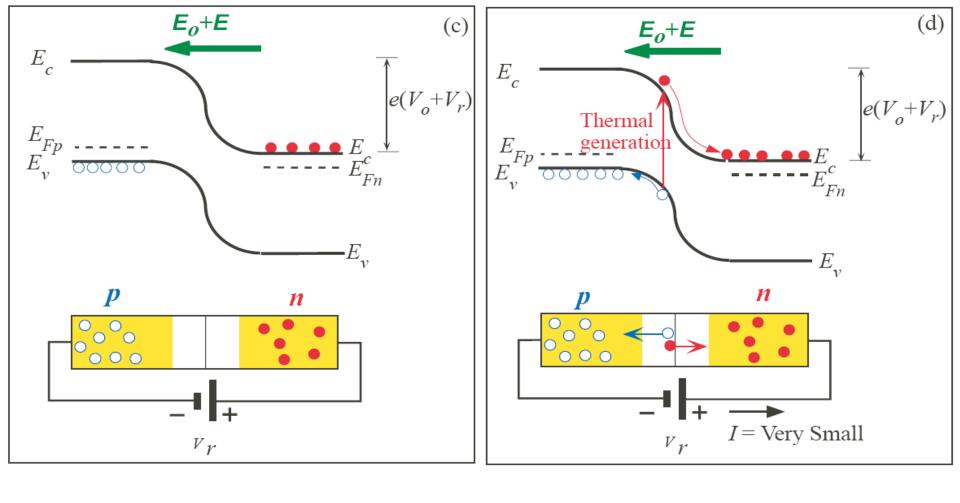




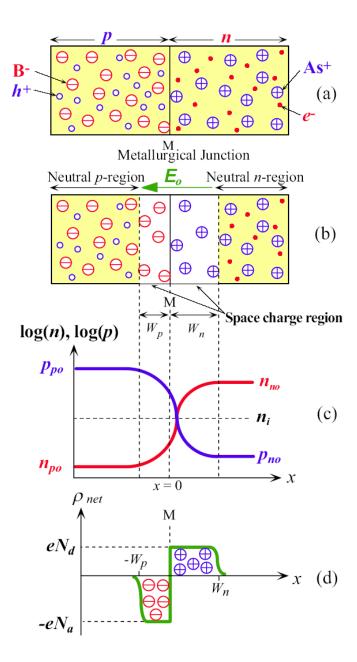
- (a) Two isolated p and n-type semiconductors (same material).
- (b) A pn junction band diagram when the two semiconductors are in contact. The Fermi level must be uniform in equilibrium. The metallurgical junction is at M. The region around M contains the space charge layer (SCL). On the *n*-side of M, SCL has the exposed positively charged donors whereas on the p-side it has the exposed negatively charged acceptors.



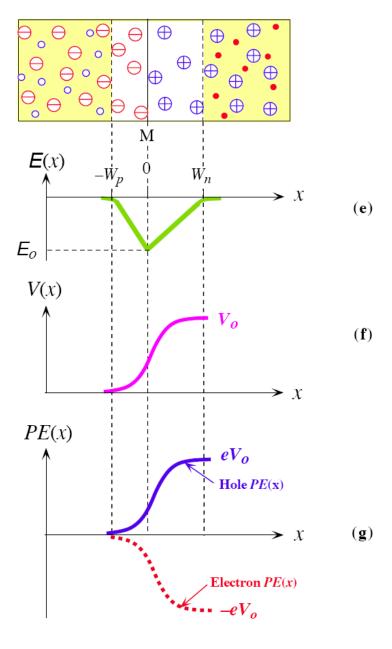
Energy band diagrams for a pn junction under (a) open circuit and (b) forward bias



Energy band diagrams for a pn junction under (c) reverse bias conditions; and (d) thermal generation of electron hole pairs in the depletion region results in a small reverse current.



Properties of the pn junction



Ideal pn Junction

Acceptor concentration Donor concentration

Depletion Widths

Field (E) and net space charge density

Net space charge density

$$\frac{d\mathcal{E}}{dx} = \frac{\rho_{\text{net}}^{/}(x)}{\varepsilon_{\text{net}}}$$

Field in depletion region

Permittivity of the medium

$$\mathcal{E}(x) = \frac{1}{\varepsilon} \int_{-W_p}^{x} \rho_{\text{net}}(x) dx$$
Electric Field

Ideal pn Junction

Built-in field

$$\mathcal{E}_{o} = -\frac{eN_{d}W_{n}}{\mathcal{E}} \qquad \text{where } \varepsilon = \varepsilon_{o} \ \varepsilon_{r}$$

Built-in voltage

$$V_{o} = -\frac{1}{2}E_{0}W_{0} = \frac{eN_{a}N_{d}W_{0}^{2}}{2\varepsilon(N_{a} + N_{d})}$$

Depletion region width

$$W_o = \left[\frac{2\varepsilon (N_a + N_d)V_o}{eN_a N_d} \right]^{1/2}$$

where $W_o = W_n + W_p$ is the total width of the depletion region under a zero applied voltage

Boltzmann statistics for electrons and holes

$$\frac{n_2}{n_1} = \exp\left(-\frac{(E_2 - E_1)}{kT}\right)$$

Minority carrier concentrations in equilibrium

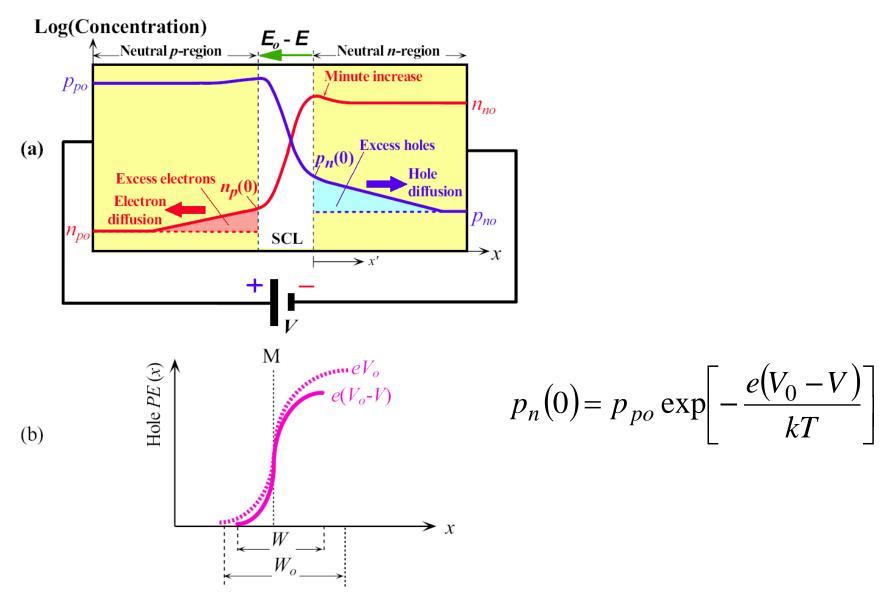
$$p_{no}(0) = p_{po} \exp\left(-\frac{eV_0}{kT}\right) \implies V_0 = \frac{kT}{e} \ln\left(\frac{p_{po}}{p_{no}}\right)$$

$$n_{po}(0) = n_{no} \exp\left(-\frac{eV_0}{kT}\right) \implies V_0 = \frac{kT}{e} \ln\left(\frac{n_{no}}{n_{po}}\right)$$

Law of mass action

$$p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_D}$$

$$p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_D} \qquad V_o = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right)$$



Forward biased *pn* junction and the injection of minority carriers.

- (a) Carrier concentration profiles across the device under forward bias.
- (b) The hole potential energy with and without an applied bias. *W* is the width of the SCL with forward bias.

Law of the Junction: Minority Carrier Concentrations and Voltage

$$p_n(0) = p_{no} \exp\left(\frac{eV}{kT}\right)$$

$$n_p(0) = n_{po} \exp\left(\frac{eV}{kT}\right)$$

 $p_n(0)$ is the hole concentration just outside the depletion region on the n-side

 $n_p(0)$ is the electron concentration just outside the depletion region on the p-side

Definition of Particle Flux

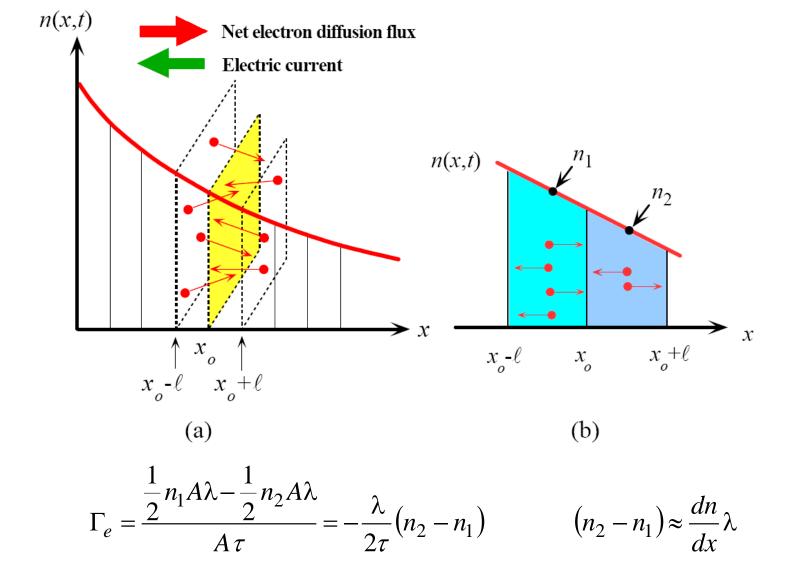
$$\Gamma = rac{\Delta N}{A \Delta t}$$

 Γ = particle flux, ΔN = number of particles crossing A in a time interval Δt , A = area, Δt = time interval

Definition of Current Density

$$J = q\Gamma$$

J = electric current density, q = charge of the particle, Γ = particle flux



- (a) Arbitrary electron concentration n(x, t) profile in a semiconductor. There is a net diffusion (flux) of electrons from higher to lower concentrations.
- (b) Expanded view of two adjacent sections at x_0 . There are more electrons crossing x_0 coming From the left $(x_0-\lambda)$ than coming from the right $(x_0+\lambda)$

Fick's First Law

$$\Gamma_e = -\frac{\lambda^2}{2\tau} \left(\frac{dn}{dx} \right) \qquad \Gamma_e = -D_e \frac{dn}{dx}$$

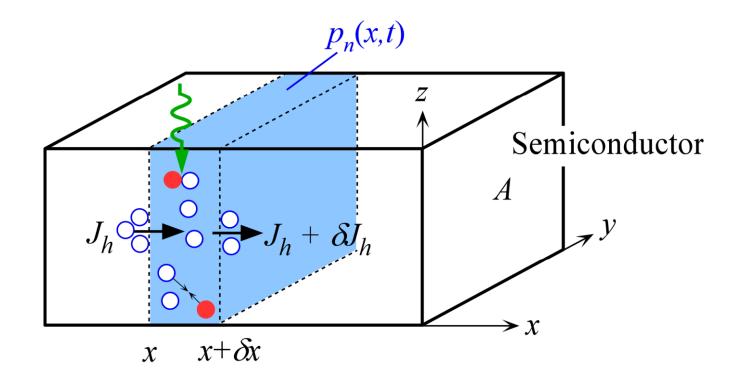
 Γ_e = electron flux, D_e = diffusion coefficient of electrons, dn/dx = electron concentration gradient

Electron Diffusion Current Density

$$J_{D,e} = -e\Gamma_e = eD_e \frac{dn}{dx}$$

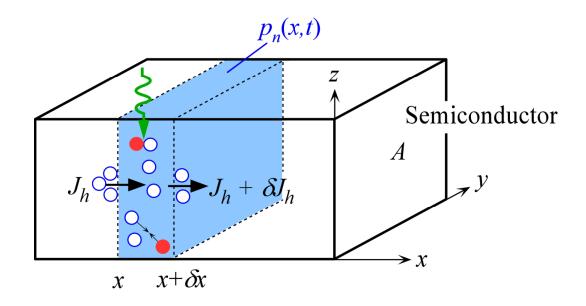
 $J_{D,e}$ = electric current density due to electron diffusion, Γ_e = electron flux, e = electronic charge, D_e = diffusion coefficient of electrons, dn/dx = electron concentration gradient

Continuity Equation



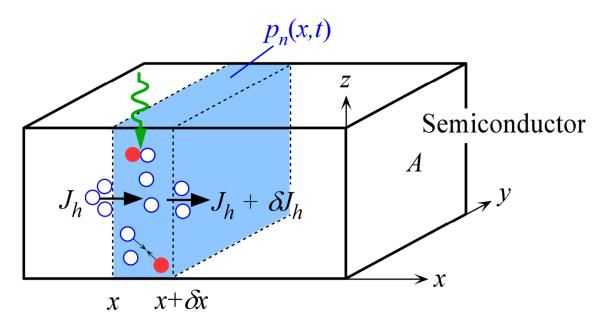
Consider an elemental volume $A\delta x$ in which the instantaneous hole concentration is p(x, t). The electric current flow and hole drift are in the same direction.

Continuity Equation



$$\frac{1}{A\delta x} \left(\frac{-A \delta J_h}{e} \right) = \begin{bmatrix} \text{Rate of increase in hole concentration due to the change in } J_h. \end{bmatrix}$$

Continuity Equation



The *net* rate of increase in the hole concentration p_n in $A\delta x$

- \blacksquare Rate of increase due to decrease in J_h
 - Rate of photogeneration
 - Rate of recombination

Continuity Equation for Holes

$$\frac{\partial p_n}{\partial t} = -\frac{1}{e} \left(\frac{\partial J_h}{\partial x} \right) - \frac{p_n - p_{no}}{\tau_h} + G_{ph}$$

 p_n = hole concentration in an n-type semiconductor, p_{no} = equilibrium minority carrier (hole concentration in an n-type semiconductor) concentration, J_h = hole current due to drift and diffusion, τ_h = hole recombination time (lifetime), $G_{\rm ph}$ = photogeneration rate at x at time t, x = position, t = time

OR

$$\frac{\partial \Delta p_n}{\partial t} = -\frac{1}{e} \left(\frac{\partial J_h}{\partial x} \right) - \frac{\Delta p_n}{\tau_h} + G_{\text{ph}}$$

 $\Delta p_n = p_n - p_{no}$ is the excess hole concentration

Steady-State Continuity Equation for Holes

No *t* dependence $\therefore \partial/\partial t = 0$

$$\frac{1}{e} \left(\frac{\partial J_h}{\partial x} \right) = -\frac{p_n - p_{no}}{\tau_h}$$

 J_h = hole current due to drift and diffusion, p_n = hole concentration in an n-type semiconductor, p_{no} = equilibrium minority carrier (hole concentration in an n-type semiconductor) concentration, τ_h = hole recombination time (lifetime)

Steady-State Continuity Equation with $\mathcal{E} = 0$

$$\frac{d^2 \Delta p_n}{dx^2} = \frac{\Delta p_n}{L_h^2}$$

 $\Delta p_n = p_n - p_{no}$ is the excess hole concentration, $L_h =$ diffusion length of the holes

$$L_h = \sqrt{(D_h \tau_h)}$$

Hole Diffusion Current Density

$$J_{D,h} = e\Gamma_h = -eD_h \frac{dp}{dx}$$

 $J_{D,h}$ = electric current density due to hole diffusion, e = electronic charge, Γ_h = hole flux, D_h = diffusion coefficient of holes, dp/dx = hole concentration gradient

Total Electron Current Due to Drift and Diffusion

$$J_e = en\mu_e \mathcal{E}_x + eD_e \frac{dn}{dx}$$

 J_e = electron current due to drift and diffusion, n = electron concentration, μ_e = electron drift mobility, \mathcal{E}_x = electric field in the x direction, D_e = diffusion coefficient of electrons, dn/dx = electron concentration gradient

Total Hole Current Due to Drift and Diffusion

$$J_h = ep\mu_h \mathcal{E}_x - eD_h \frac{dp}{dx}$$

 J_h = hole current due to drift and diffusion, p = hole concentration, μ_h = hole drift mobility, \mathcal{E}_x = electric field in the x direction, D_h = diffusion coefficient of holes, dp/dx = hole concentration gradient

Einstein Relation

$$\frac{D_e}{\mu_e} = \frac{kT}{e}$$
 $\frac{D_h}{\mu_h} = \frac{kT}{e}$

 D_e = diffusion coefficient of electrons, μ_e = electron drift, D_h = diffusion coefficient of the holes, μ_h = hole drift mobility

Excess minority carrier concentration profile

$$\frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{L_h^2} \implies \Delta p_n(x') = \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

where L_h is the hole diffusion length, defined by $L_h = \sqrt{D_h \tau_h}$ in which τ_h is the mean hole recombination lifetime (minority carrier lifetime in the *n*-region.

$$L_h \sim 1 - 100 \, \mu \text{m}$$
, $\tau_h \sim 1 \, \text{ns} - 10 \, \mu \text{s}$

Excess minority carrier concentration

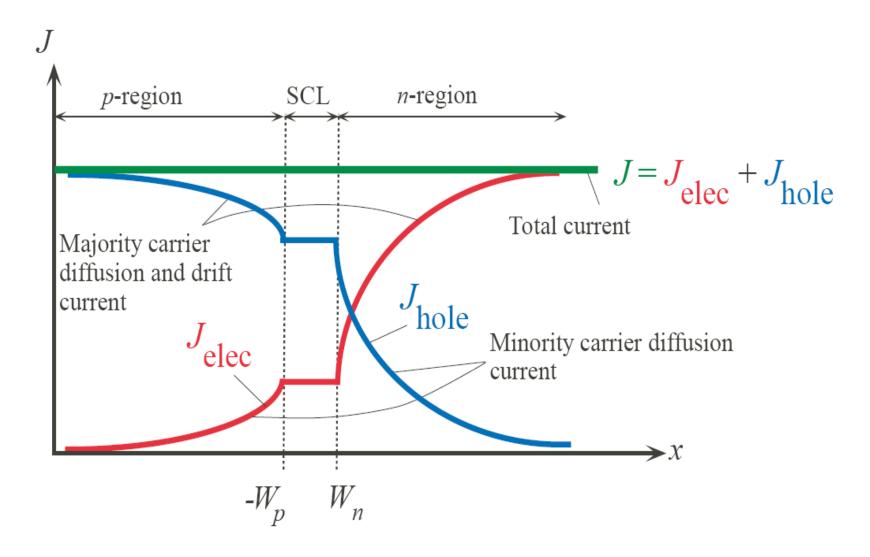
$$\Delta p_n(x') = p_n(x') - p_{no}$$

Forward Bias: Diffusion Current Density

$$\Delta p_n(x') = \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

$$J_{D,hole} = -eD_h \frac{dp_n(x')}{dx'} = -eD_h \frac{d\Delta p_n(x')}{dx'}$$

$$J_{D,hole} = \left(\frac{eD_h}{L_h}\right) \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$



The total current anywhere in the device is constant. Just outside the depletion region it is due to the diffusion of minority carriers.

Minority carrier diffusion current at x' = 0

$$J_{D,hole} = \left(\frac{eD_h}{L_h}\right) \Delta p_n(0)$$

$$\Delta p_n(0) = p_n(0) - p_{no} = p_{no} \left[\exp\left(\frac{eV}{kT}\right) - 1 \right]$$

$$p_{no} = \frac{n_i^2}{n_{no}} = \frac{n_i^2}{N_D}$$

Hole diffusion current in *n*-side in the neutral region

$$J_{D,\text{hole}} = \left(\frac{eD_h n_i^2}{L_h N_d}\right) \left[\exp\left(\frac{eV}{kT} - 1\right)\right]$$

There is a similar expression for the electron diffusion current density $J_{D,\mathrm{elec}}$ in the p-region.

Ideal diode (Shockley) equation

$$J = J_{so} \left[\exp \left(\frac{eV}{kT} \right) - 1 \right]$$

Reverse saturation current

$$J_{so} = \left[\left(\frac{eD_h}{L_h N_d} \right) + \left(\frac{eD_e}{L_e N_a} \right) \right] n_i^2$$

Intrinsic concentration

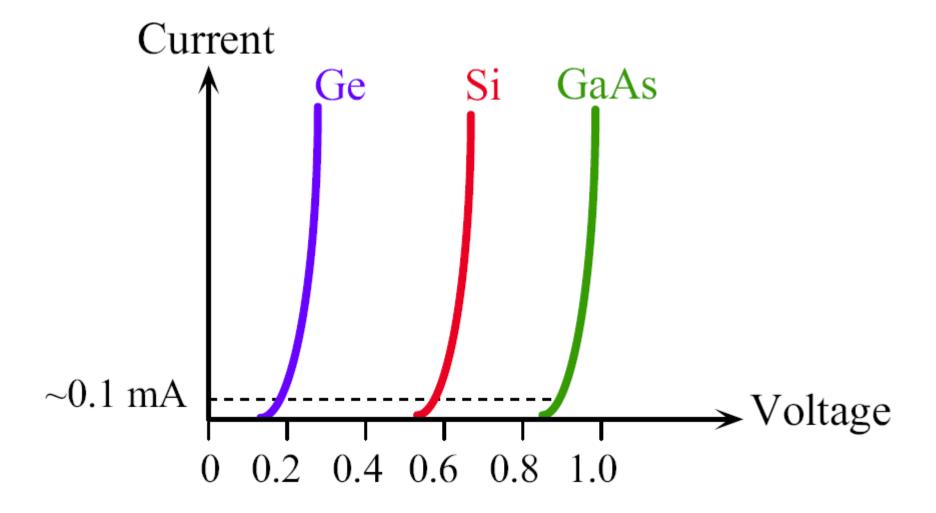
$$n_i^2 = (N_c N_v) \exp\left(-\frac{eV_g}{kT}\right)$$

where $V_g = E_g / e$ is the bandgap energy expressed in volts

 $V_g = 0.67 \text{ V for Ge}$, 1.1 V for Si, and 1.42 V for GaAs

$$J = J_{so} \exp\left(\frac{eV}{kT}\right) = C \exp\left(\frac{e(V - V_g)}{kT}\right) \qquad V > kT/e$$

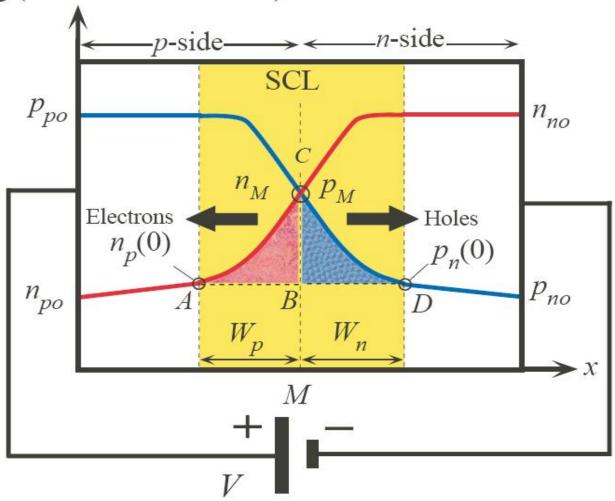
We can plot I vs. V for Ge, Si and Ge



Schematic sketch of the I-V characteristics of Ge, Si, and GaAs pn junctions.

Forward Bias: Recombination and Total Current

Log (carrier concentration)



Forward biased pn junction and the injection of carriers and their recombination in the SCL

Forward bias: Recombination Current

$$\boldsymbol{J}_{\text{recom}} = \frac{eABC}{\tau_e} + \frac{eBCD}{\tau_h}$$

$$ABC \approx \frac{1}{2}W_p n_M \qquad BCD \approx \frac{1}{2}W_n p_M$$

$$\frac{p_M}{p_{po}} = \exp\left(-\frac{e(V_0 - V)}{2kT}\right)$$

$$p_{M} = p_{po} \exp\left(-\frac{eV_{0}}{2kT}\right) \exp\left(\frac{eV}{2kT}\right) = n_{i} \exp\left(\frac{eV}{2kT}\right)$$

Forward Bias: Recombination and Total Current

Recombination Current

$$J_{\text{recom}} = J_{ro} \left[\exp(eV/2kT) - 1 \right] \text{ where } J_{ro} = \frac{en_i}{2} \left(\frac{W_p}{\tau_e} + \frac{W_n}{\tau_h} \right)$$

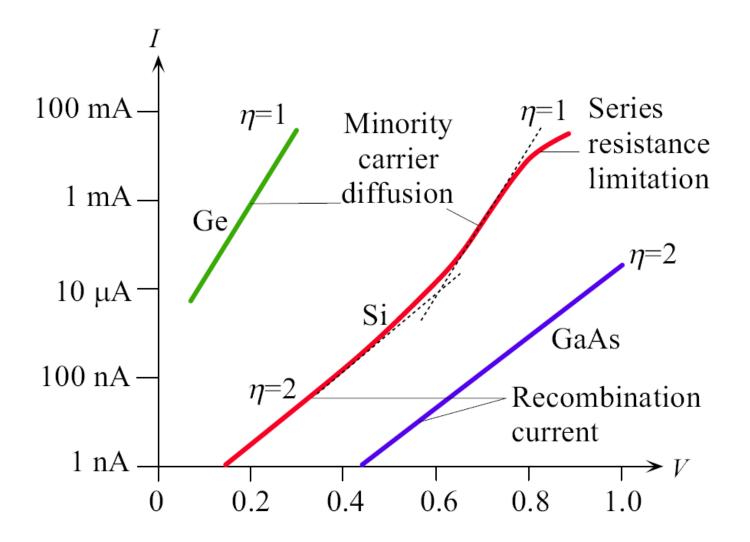
Total diode current = diffusion + recombination

$$J = J_{so} \exp\left(\frac{eV}{kT}\right) + J_{ro} \exp\left(\frac{eV}{2kT}\right) \qquad V > \frac{kT}{e}$$

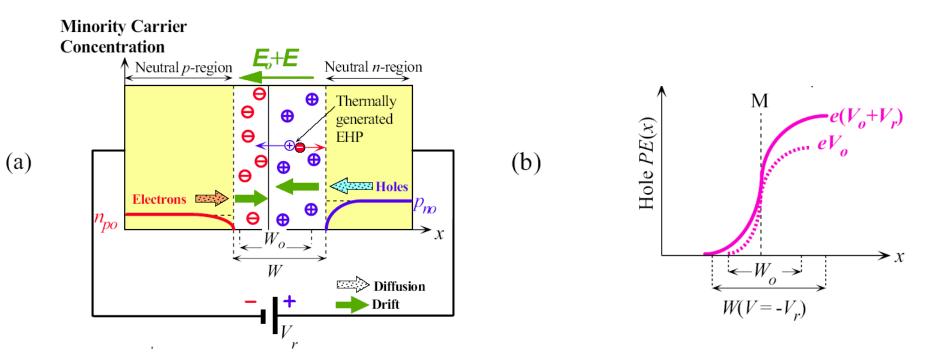
The diode equation

$$J = J_o \exp\left(\frac{eV}{\eta kT}\right) \qquad V > \frac{kT}{e}$$

Where J_o is a new constant and η is an ideality factor



Schematic sketch of typical *I-V* characteristics of Ge, Si and GaAs pn junctions as log(I) vs. V. The slope indicates $e/(\Box kT)$



Reverse biased pn junction. (a) Minority carrier profiles and the origin of the reverse current. (b) Hole PE across the junction under reverse bias

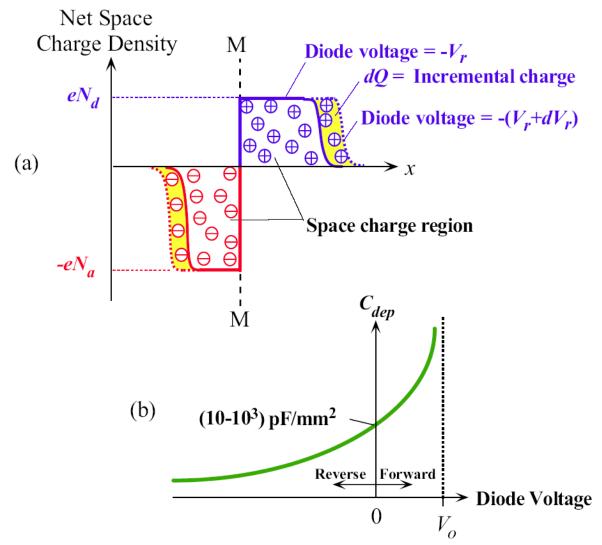
Reverse Bias

Total Reverse Current

Thermal generation in depletion region

$$J_{\text{rev}} = \left(\frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a}\right) n_i^2 + \frac{eWn_i}{\tau_g}$$
Mean thermal generation time

Diffusion current in neutral regions the Shockley reverse current



The depletion region behaves like a capacitor.

- (a) The charge in the depletion region depends on the applied voltage just as in a capacitor
- (b) The incremental capacitances of the depletion region increases with forward bias and decreases with reverse bias. Its value is typically in the range of picofarads per mm²

Depletion layer capacitance of the pn junction

Depletion region width

$$W = \left[\frac{2\varepsilon (N_a + N_d)(V_o - V)}{eN_a N_d} \right]^{1/2}$$

where, for forward bias, V is positive, which reduces V_o , and, for reverse bias, V is negative, so V_o is increased.

Definition of depletion layer capacitance C_{dep}

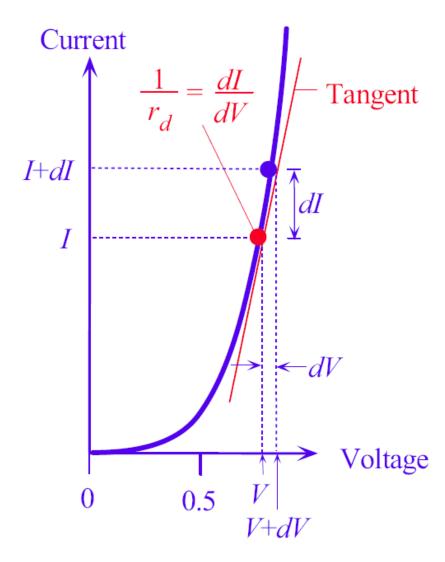
$$C_{\text{dep}} = \left| \frac{dQ}{dV} \right|$$

where the amount of charge on any one side of the depletion layer is $|Q| = eN_dW_nA = eN_aW_nA$ and $W = W_n + W_n$

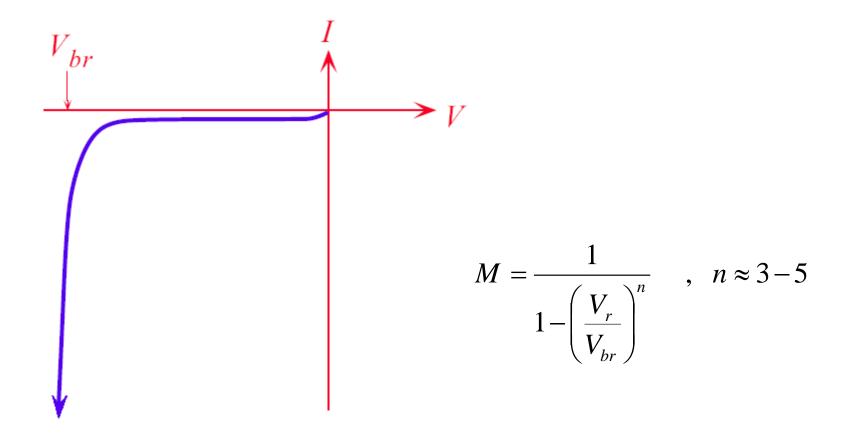
Depletion Capacitance

$$C_{\text{dep}} = \frac{\varepsilon A}{W} = \frac{A}{(V_o - V)^{1/2}} \left[\frac{e\varepsilon(N_a N_d)}{2(N_a + N_d)} \right]^{1/2}$$

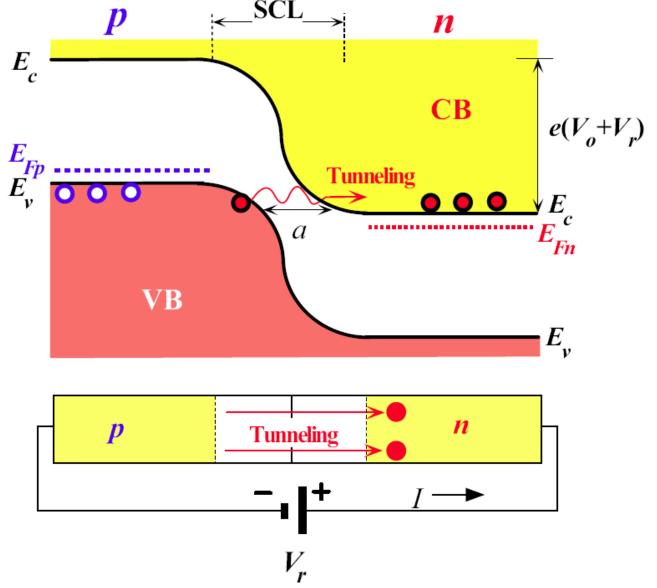
Varactor diodes (varicaps) are based on this effect, providing tuning circuits with voltage-dependent capacitors



The dynamic resistance of the diode is defined as dV/dI which is the inverse of the tangent at I.



Reverse *I-V* characteristics of a *pn* junction.



Zener breakdown involves electrons tunneling from the VB of p side to the CB of n-side when the reverse bias reduces E_c to line up with E_v .

Intrinsic carrier mobility

semiconductor	E _g (eV)	u _e (cm ² V ⁻¹ s ⁻¹)	u _h (cm ² V ⁻¹ s ⁻¹)
c-Si	1.10	1350	450
Ge	0.66	3900	1900
GaAs	1.42	8500	400
CIGS	1.04	1000 (crystal)	~150-300
a-Si:H	1.85	1	0.003
nc-Si:H	1.10	20-100	0.05
OPV	1.6-2.0	0.001	0.001
graphene	0	1500-40 000 200 000 (suspended)	1500-40 000 200 000 (suspended)

Thin films

Minority carrier diffusion length: $L = \sqrt{D\tau}$

Einstein relation: $D = \frac{kT}{e}\mu$