Problema - 1

a)

$$\tilde{\vec{E}} = E_0 \, \hat{m} \, e^{i(\vec{k}\cdot\vec{r} - \omega t)} \qquad \vec{K} = \frac{\omega}{c}\hat{k}$$

$$\tilde{\vec{B}} = B_0 \, (\hat{k}\times\hat{m}) \, e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$\nabla\times\vec{E} = -\dot{\vec{B}} \implies +i\omega \vec{B}_0 = i K (\hat{k}\times\vec{E}_0)$$

$$B_0 = \frac{k}{\omega}|\hat{K}\times\vec{E}_0| = \frac{1}{c}E_0$$

Logo:

$$\vec{k} = \frac{\omega}{c}\frac{1}{\sqrt{3}}[\hat{x}+\hat{y}+\hat{z}] \qquad \hat{K} = \frac{1}{\sqrt{3}}[\hat{x}+\hat{y}+\hat{z}]$$

$$\hat{m} \in \text{plano } xz \implies \hat{m} = \alpha\hat{x} + \beta\hat{z} \qquad (\sqrt{\alpha^2+\beta^2} = 1)$$

$$\hat{m}\cdot\hat{k} = 0 \implies \alpha = -\beta = \frac{1}{\sqrt{2}}$$

$$\hat{m} = \frac{1}{\sqrt{2}}[\hat{x} - \hat{z}]$$

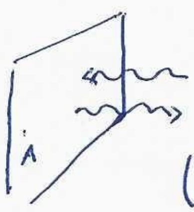
$$\hat{k}\times\hat{m} = \frac{1}{\sqrt{6}}[-\hat{x} + 2\hat{y} - \hat{z}]$$

$$\mathbb{R}\,\tilde{\vec{E}}(\vec{r},t) = \vec{E}(\vec{r},t) = E_0\left[\frac{\hat{x}-\hat{z}}{\sqrt{2}}\right]\cos\left[\frac{\omega}{\sqrt{3}\,c}(x+y+z) - \omega t\right]$$

$$\mathbb{R}\,\tilde{\vec{B}}(\vec{r},t) = \vec{B}(\vec{r},t) = \frac{E_0}{c}\left[\frac{-\hat{x}+2\hat{y}-\hat{z}}{\sqrt{6}}\right]\cos\left[\frac{\omega}{\sqrt{3}\,c}(x+y+z) - \omega t\right]$$

b)

$$I = \langle S\rangle = \frac{1}{2}c\,\varepsilon_0 E_0^2 = c\langle u\rangle \qquad \left(\vec{S} = \frac{1}{\mu_0}[\vec{E}\times\vec{B}]\right)$$



$$p = \frac{1}{A}\frac{\Delta p}{\Delta t}\cdot 2 = \frac{1}{A}\,2\,\frac{\frac{1}{2c}\varepsilon_0 E_0^2\cdot A\cdot c\,\Delta t}{\Delta t} = 2\,\frac{1}{2}\varepsilon_0 E_0^2 = \frac{I\,2}{c}$$

(Dado que se trata de um perfeito reflector devemos multiplicar este valor por 2)

Logo $\quad p = \frac{1000 \ W/m^2 \cdot 2}{3 \times 10^8 \ m\,s^{-1}} \simeq 0{,}6 \times 10^{-5} \ \frac{J}{m^3}$

$$\left(\frac{J}{m^3} \equiv \frac{N}{m^2} \right)$$

Problema - 2

$$\tilde{k} = k + i\eta \quad \begin{cases} k = \omega \sqrt{\frac{\varepsilon\mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} + 1 \right]^{1/2} \\ \eta = \omega \sqrt{\frac{\varepsilon\mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1 \right]^{1/2} \end{cases}$$

$$\vec{\tilde{E}} = \tilde{E}_0 \, e^{-\eta z} \, e^{i(kz - \omega t)}$$

$$\frac{\sigma}{\varepsilon\omega} \sim \frac{10^7}{10^{-11} \cdot 10^{10}} \sim 10^8 \gg 1 \Rightarrow k \simeq \eta \sim \sqrt{\frac{\omega\sigma\mu}{2}}$$

$(\mu \sim \mu_0)$ $\qquad k \sim \omega \sim \sqrt{\frac{2\pi \cdot 10^{10} \cdot 4\pi \cdot 10^{-7} \cdot 10^7}{2}} = 2\pi \cdot 10^5$

a) $\quad I \propto E^2 \propto e^{-2\eta d}$ (diminuição da intensidade depois de percorrer a distância d)

d: $\quad I(d) = 0{,}1 \Rightarrow d = \frac{\ln(0{,}1)}{-4\pi \cdot 10^{-5}}$

b) $\quad 2\pi \cdot 10^5 \simeq k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{2\pi \cdot 10^5} = 10\,\mu m$

Problema - 3

$a = 2\ cm$

$b = 1\ cm$

$$k = \sqrt{\left(\frac{\omega}{c}\right)^2 - \pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]}$$

$$= \frac{1}{c}\sqrt{\omega^2 - \pi^2 c^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]}$$

$$= \frac{1}{c}\sqrt{\omega^2 - \omega_{mn}^2}$$

O modo TE ω_{mn} é excitado se $\omega > \omega_{mn} = \pi c\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$\omega = 2\pi \cdot 1{,}2 \cdot 10^{11} = 7{,}536 \times 10^{11}$

$\omega_{10} = \pi c\sqrt{\frac{1}{a^2}} = 3{,}14 \cdot 3 \cdot 10^8 \frac{1}{2 \times 10^{-2}} \approx 4{,}7 \cdot 10^{10}$ o.k.

$\omega_{20} = \pi c\sqrt{\frac{4}{a^2}} = 2\,\omega_{10} = 9{,}4 \times 10^{10}$ o.k.

$\omega_{30} = 3\,\omega_{10} = 1{,}41 \times 10^{11}$ o.k.

$\omega_{40} = 4\,\omega_{10} = 1{,}88 \times 10^{11}$

(...)

$\vdots$

$\omega_{15\,0} = 15\,\omega_{10} = 7 \times 10^{11}$ o.k.

$\omega_{01} = 9{,}4 \times 10^{10}$ o.k.

$\vdots$

$\omega_{08} = 8 \times \omega_{01} \approx 7{,}5 \times 10^{11}$ o.k.

$\omega_{11} = 9{,}4 \times 10^{10}\sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{1}\right)^2}$ o.k.

etc.

(...)

Nota: bastaria indicar o critério, não sendo necessário especificar exaustivamente os modos excitados.

Problema 4

(a) i) $\nabla \cdot \vec{B} = 0$

ii) $\nabla \times \vec{E} + \dot{\vec{B}} = 0$

i) $\Rightarrow$ $\vec{B} = \nabla \times \vec{A}$; substituindo em ii) $\nabla \times [\vec{E} + \dot{\vec{A}}] = 0 \Rightarrow$

$\Rightarrow \vec{E} + \dot{\vec{A}} = -\nabla \phi$; Logo $\vec{E} = -\nabla \phi - \dot{\vec{A}}$

$\vec{B} = \nabla \times \vec{A}$

(b)

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \iff \nabla \cdot [-\nabla \phi - \dot{\vec{A}}] = \frac{\rho}{\varepsilon_0}$$

$$\boxed{-\nabla^2 \phi - \nabla \cdot \dot{\vec{A}} = \frac{\rho}{\varepsilon_0}}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \dot{\vec{E}}$$

$$\nabla \times (\nabla \times \vec{A}) = \mu_0 \vec{J} - \varepsilon_0 \mu_0 [\nabla \dot{\phi} + \ddot{\vec{A}}]$$

$$\nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J} - \varepsilon_0 \mu_0 [\nabla \dot{\phi} + \ddot{\vec{A}}]$$

$$\boxed{(\nabla^2 \vec{A} - \mu_0 \varepsilon_0 \ddot{\vec{A}}) - \nabla (\nabla \cdot \vec{A} + \varepsilon_0 \mu_0 \dot{\phi}) = -\mu_0 \vec{J}}$$

c) Os potenciais $\vec{A}$ e ϕ não são bem definidos visto que

$\vec{A}' \rightarrow \vec{A} + \vec{\alpha}$ $\quad \phi' = \phi + \beta$ podem ficar podemos gerar exactamente o mesmo campo desde que

$$\vec{\alpha} = \nabla\lambda \quad \text{e} \quad \beta = -\frac{\partial\lambda}{\partial t} + K(t) = -\frac{\partial}{\partial t}\underbrace{\left(\lambda - \int K(t)\,dt\right)}_{\lambda'}$$

$$\vec{\alpha} = \nabla\lambda' \qquad \beta = -\frac{\partial\lambda'}{\partial t}$$

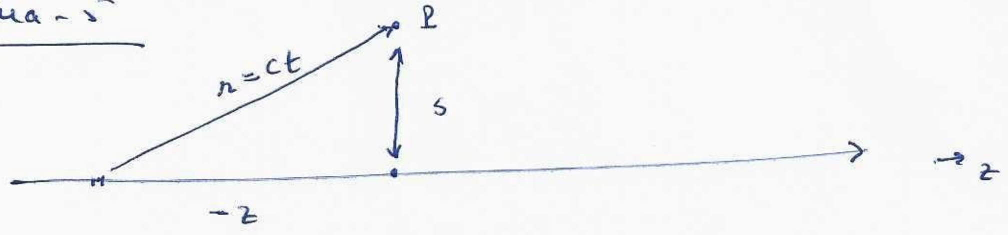
Isto é:

$$\vec{A}' = \vec{A} + \nabla\lambda'$$

$$\phi' = \phi - \frac{\partial\lambda'}{\partial t}$$

geram o mesmo campo.

Problema - 5



$$\vec{A}(s,t) = \frac{\mu_0}{4\pi}\hat{z}\int_{-\infty}^{+\infty}\frac{I(t_R)}{r}\,dz$$

$$t_R = \frac{r}{c} \Rightarrow$$

$$\vec{A}(s,t) = \frac{\mu_0 I_0}{4\pi}\hat{z}\int_{-\sqrt{c^2t^2-s^2}}^{+\sqrt{c^2t^2-s^2}}\frac{dz}{\sqrt{s^2+z^2}} =$$

$$= \frac{\mu_0 I_0}{2\pi}\hat{z}\int_0^{\sqrt{c^2t^2-s^2}}\frac{dz}{\sqrt{s^2+z^2}}$$

$$= \frac{\mu_0 I_0}{2\pi} \hat{z} \ln \left[z + \sqrt{s^2 + z^2} \right]_0^{\sqrt{c^2t^2 - s^2}} =$$

$$= \frac{\mu_0 I_0}{2\pi} \hat{z} \ln \left[\frac{\sqrt{c^2t^2 - s^2} + ct}{s} \right]$$

$$\vec{E} = - \frac{\partial \vec{A}}{\partial t} = \frac{-\mu_0 I c}{2\pi \sqrt{c^2t^2 - s^2}} \hat{z}$$

$$\vec{B} = - \frac{\partial A_z}{\partial s} \hat{\phi} = \frac{\mu_0 I_0}{2\pi s} \frac{ct}{\sqrt{c^2t^2 - s^2}} \hat{\phi}$$

b) $\lim_{t \to \infty} \vec{E} = 0$

$\lim_{t \to \infty} \vec{B} = \frac{\mu_0 I_0}{2\pi s} \hat{\phi}$ (Resultado obtido em regime estático)