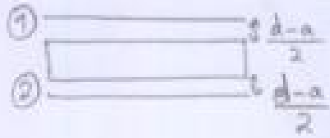
2º Teste

①



$A_1 = A_2 \equiv A$

$\lambda = \frac{d-a}{2}$

geral: $C = \frac{\varepsilon_0 A}{\lambda}$

$C_1 = \frac{\varepsilon_0 A_1}{\frac{d-a}{2}}$ $\quad C_2 = \frac{\varepsilon_0 A_2}{\frac{d-a}{2}}$

Em série: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} =$

$$= \frac{d-a}{2\varepsilon_0 A} + \frac{d-a}{2\varepsilon_0 A} = = \frac{d-a}{\varepsilon_0 A}$$

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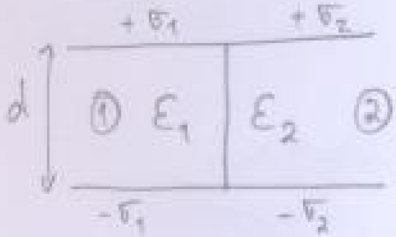
②



paralelo: $C_{eq} = C_1 + C_2$

a) $C = \frac{\varepsilon_0 A}{\lambda}$ $\quad \varepsilon \equiv k\varepsilon_0$ $\quad A_1 = A_2 \equiv A$

$C_1 = \frac{\varepsilon_1 A_1}{d}$ $\quad C_2 = \frac{\varepsilon_2 A_2}{d}$

$$C_{eq} = \frac{\varepsilon_1 A}{d} + \frac{\varepsilon_2 A}{d} = \frac{A}{d}(\varepsilon_1 + \varepsilon_2) = \frac{\varepsilon_0 A}{d}(k_1 + k_2)$$

b) Pela lei de Gauss:

$$\int_S \vec{D}\cdot d\vec{a} = Q_{livre}$$

Considerando que cada um é um dielectrico linear:

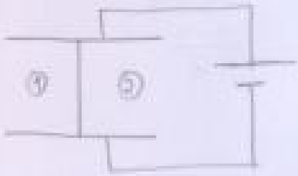
$D = \varepsilon E = k\varepsilon_0 E$

$Q = \sigma A$

① $\int_S \varepsilon_1 E\, da = Q_{livre} \Leftrightarrow \varepsilon_1 E A = Q_{livre} \Leftrightarrow E = \frac{Q_{livre}}{\varepsilon_1 A} = \frac{\sigma_1 \cancel{A}}{\varepsilon_1 \cancel{A}} = \frac{\sigma_1}{\varepsilon_1}$

② $\int_S \varepsilon_2 E\, da = Q_{livre} \Leftrightarrow \varepsilon_2 E A = Q_{livre} \Leftrightarrow E = \frac{Q_{livre}}{\varepsilon_2 A} = \frac{\sigma_2 \cancel{A}}{\varepsilon_2 \cancel{A}} = \frac{\sigma_2}{\varepsilon_2}$

$$V = -\int E\,ds \quad \begin{cases} V_1 = +\dfrac{\sigma_1}{\varepsilon_1} d \\ V_2 = +\dfrac{\sigma_2}{\varepsilon_2} d \end{cases}$$



$$V_1 = V_2 \Leftrightarrow \frac{\sigma_1}{\varepsilon_1}\cancel{d} = \frac{\sigma_2}{\varepsilon_2}\cancel{d} \Leftrightarrow \frac{\sigma_1}{\varepsilon_1} = \frac{\sigma_2}{\varepsilon_2} \Leftrightarrow \sigma_2 = \frac{\varepsilon_2 \sigma_1}{\varepsilon_1}$$

$$C = \frac{Q}{V} = \frac{A\sigma_1 + A\sigma_2}{V} = \frac{A\cancel{\sigma_1} + A\varepsilon_2 \cancel{\sigma_1}/\varepsilon_1}{\frac{\cancel{\sigma_1} d}{\varepsilon_1}} =$$

$$= \frac{A\varepsilon_1 + A\varepsilon_2}{d} = \frac{A}{d}(\varepsilon_1 + \varepsilon_2) = \frac{\varepsilon_0 A}{d}(k_1 + k_2)$$

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③ Num caso geral: $\vec{F} = I \int_{\Gamma} d\vec{l} \times \vec{B}$

$$= -I \int_{\Gamma} \vec{B} \times d\vec{l}$$

como $\vec{B}$ é const.

$$= -I\vec{B} \times \int_{\Gamma_{\text{fechado}}} d\vec{l}$$

$$= -I\vec{B} \times \vec{0} = 0$$

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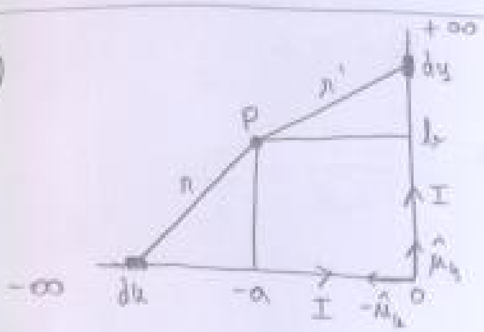
④



$$\begin{cases} r^2 = b^2 + (x-(-a))^2 = b^2 + (x+a)^2 \\ r'^2 = (-a)^2 + (y-b)^2 = a^2 + (y-b)^2 \end{cases}$$

$$\begin{cases} d\vec{l} \times \hat{r} = |d\vec{l}| \times |\hat{r}| = dx \times 1 \times \sin\theta\,\hat{z} \\ \qquad = dx \sin\theta\,\hat{z} \\ d\vec{l} \times \hat{r} = |d\vec{l}| \times |\hat{r}| = dy \times 1 \times \sin\theta\,\hat{z} \\ \qquad = dy \times \sin\theta\,\hat{z} \end{cases}$$

Lei de Biot-Savart:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$\begin{cases} \sin\theta = \dfrac{b}{r} \\ \sin'\theta = -\dfrac{a}{r} \end{cases}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi}\left[\int_{-\infty}^{0} \frac{d\vec{l}\times\hat{r}}{r^2} + \int_{0}^{+\infty}\frac{d\vec{l}\times\hat{r}}{r'^2}\right] =$$

$-\hat{u}_x$

$$= \frac{\mu_0 I}{4\pi}\left[-\int_{-\infty}^{0}\frac{dx\sin\theta}{r^2}\hat{z} + \int_{0}^{+\infty}\frac{dy\sin'\theta}{r'^2}\hat{z}\right] =$$

$$= \frac{\mu_0 I}{4\pi}\left[-\int_{-\infty}^{0}\frac{b}{(b^2+(x+a)^2)^{3/2}}dx\,\hat{z} - \int_{0}^{+\infty}\frac{a}{(a^2+(y-b)^2)^{3/2}}dy\,\hat{z}\right] =$$

$(*)$

$$= \frac{\mu_0 I}{4\pi}\left[-\int_{-\infty}^{a}\frac{b\,dx'}{(b^2+x'^2)^{3/2}}\hat{z} - \int_{-b}^{+\infty}\frac{a\,dy'}{(a^2+y'^2)^{3/2}}\hat{z}\right] =$$

C.A.: $(*)$

mudança de variável:

$x+a=x' \rightarrow d(x+a)=dx'$ $\Leftrightarrow$ $dx'=dx$

quando $x=0$

$x'=a$

$y-b=y' \rightarrow dy'=d(y-b)$

$dy'=dy$

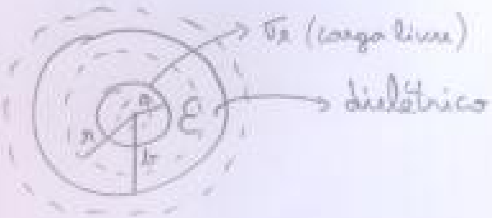
quando $y=0$

$y'=-b$

$$= \frac{\mu_0 I}{4\pi}\left[-\cancel{b}\left|\frac{x'}{b^{\cancel{2}}(b^2+x'^2)^{1/2}}\right|_{-\infty}^{a}\hat{z} - \cancel{a}\left|\frac{y'}{a^{\cancel{2}}(a^2+y'^2)^{1/2}}\right|_{-b}^{+\infty}\hat{z}\right] =$$

$$= \frac{\mu_0 I}{4\pi}\left[-\left|\frac{x'}{b(b^2+x'^2)^{1/2}}\right|_{-\infty}^{a} - \left|\frac{y'}{a(a^2+y'^2)^{1/2}}\right|_{-b}^{+\infty}\right]\hat{z}$$

⑤



a) $\varepsilon = k\varepsilon_0$

<u>$r < a$</u> $\rightarrow$ Como se trata de um metal, como tal comporta-se como um condutor, portanto o campo $\vec{E}$ no interior do mesmo, é nulo, pois a carga eléctrica está toda distribuída à superfície.

<u>$a < r < b$</u>

Lei de Gauss:

$$\int k\vec{E}\,\hat{n}\,d\vec{a} = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow \varepsilon E \cdot 2\pi r^2 = q_{int} \Leftrightarrow$$

$$dq = \sigma_l\, da \Leftrightarrow q = \sigma_l \int_0^{\pi}\int_0^{2\pi} r'^2 \sin\theta\, d\theta\, d\phi = \sigma_l\, r'^2 4\pi$$

$$q_{int} = \sigma_l\, r'^2 4\pi \quad (r' = a)$$
$$= \sigma_l\, a^2 4\pi$$

$$\Leftrightarrow E = \frac{\sigma_l a^2 4\pi}{\varepsilon 2\pi r^2} = \frac{\sigma_l a^2 2}{\varepsilon r^2} = \frac{2\sigma_l a^2}{k\varepsilon_0 r^2}$$

<u>$r > b$</u>

$$\int \vec{E}\,\hat{n}\,d\vec{a} = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow E \cdot 2\pi r^2 = \frac{q_{int}}{\varepsilon_0} \Leftrightarrow \qquad q_{int} = \sigma_l\, r'^2 4\pi \quad (r' = b)$$
$$= \sigma_l\, b^2 4\pi$$

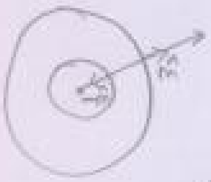
$$\Leftrightarrow E = \frac{\sigma_l b^2 4\pi}{2\pi r^2 \varepsilon_0} = \frac{2\sigma_l b^2}{\varepsilon_0 r^2}$$

b) $P = (k-1)\varepsilon_0 E_D \qquad E_D = \frac{2\sigma_l a^2}{k\varepsilon_0 r^2}$

$$= k\varepsilon_0 E_D - E_D\varepsilon_0$$
$$= \cancel{k}\cancel{\varepsilon_0}\left(\frac{2\sigma_l a^2}{\cancel{k\varepsilon_0} r^2}\right) - \cancel{\varepsilon_0}\left(\frac{2\sigma_l a^2}{k\cancel{\varepsilon_0} r^2}\right) = \frac{2\sigma_l a^2}{r^2}\left(1 - \frac{1}{k}\right)$$

$\sigma = \hat{n} \cdot \vec{P}$

$$\begin{cases} \sigma_{ext} = \hat{n} \cdot \vec{P} \Big|_{r=b} \\ \sigma_{int} = -\hat{n} \cdot \vec{P} \Big|_{r=a} \end{cases}$$



$\vec{P} = P \cdot \hat{n}$

$$\begin{cases} \sigma_{ext} = P \Big|_{r=b} = \dfrac{2\,\sigma_\ell\, a^2}{b^2}\left(1 - \dfrac{1}{k}\right) \\ \sigma_{int} = -P \Big|_{r=a} = \dfrac{2\,\sigma_\ell\, \cancel{a^2}}{\cancel{a^2}}\left(1 - \dfrac{1}{k}\right) = 2\sigma_\ell\left(1 - \dfrac{1}{k}\right) \end{cases}$$