Teste 2 - Cálculo para Ciências

Correção 19/01/2022

(1) $sh^2(2x) + ch^2(2x) = \left(\frac{e^{2x}-e^{-2x}}{2}\right)^2 + \left(\frac{e^{2x}+e^{-2x}}{2}\right)^2$

$= \frac{e^{4x} - 2\cancel{e}^0 + e^{-4x} + e^{4x} + 2\cancel{e}^0 + e^{-4x}}{4}$

$= \frac{e^{4x}+e^{-4x}}{2} = \cancel{ch(2x)}\ ch(4x)$

(2) a) $\frac{x^2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} = \frac{A(x+1)^2 + B(x-1)(x+1) + C(x-1)}{(x-1)(x+1)^2}$

$\Longleftrightarrow x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x=1 \Longrightarrow 1 = 4A \Longleftrightarrow A = 1/4$

$x=-1 \Longrightarrow 1 = -2C \Longleftrightarrow C = -1/2$

$x=0 \Longrightarrow 0 = \frac{1}{4} - B + \frac{1}{2} \Longleftrightarrow B = \frac{3}{4}$

$\int \frac{x^2}{(x-1)(x+1)^2}\,dx = \frac{1}{4}\int \frac{1}{x-1}\,dx + \frac{3}{4}\int \frac{1}{x+1}\,dx - \frac{1}{2}\int (x+1)^{-2}\,dx$

$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| - \frac{1}{2}\frac{(x+1)^{-1}}{-1} + C,\ C\in\mathbb{R}$

b) $\int x^3(x^2+5)^6\,dx = \int \underset{f}{x^2} \cdot \underbrace{x(x^2+5)^6}_{g'}\,dx$

$f = x^2 \qquad f' = 2x$

$g' = x(x^2+5)^6 \qquad g = \frac{1}{2}\frac{(x^2+5)^7}{7}$

$= \frac{1}{14}x^2(x^2+5)^7 - \frac{1}{7}\int x(x^2+5)^7\,dx$

$= \frac{1}{14}x^2(x^2+5)^7 - \frac{1}{14}\frac{(x^2+5)^8}{8} + C,\ C\in\mathbb{R}$

c) $\displaystyle\int \frac{x}{\sqrt{x^2-1}+\sqrt[4]{x^2-1}}\,dx =$

C.A.

$y^4 = x^2-1$

$4y^3 dy = 2x\,dx \Leftrightarrow x\,dx = 2y^3 dy$

(2)

$$\begin{array}{l|l} y^2+0y+0 & y+1 \\ \cline{2-2} -y^2-y & y-1 \\ \hline \quad -y+0 & \\ \quad +y+1 & \\ \hline \qquad 1 & \end{array}$$

$y^2 = (y+1)(y-1)+1$

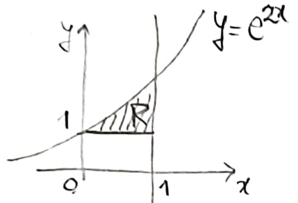
$\displaystyle = \int \frac{1}{y^2+y}\,2y^3 dy = 2\int \frac{y^2}{y+1}\,dy =$

$\displaystyle = 2\left(\int (y-1)\,dy + \int \frac{1}{y+1}\,dy\right)$

$= 2y^2 - 2y + 2\ln|y+1| + C$

$= 2\sqrt{x^2-1} - 2\sqrt[4]{x^2-1} + 2\ln\left|\sqrt[4]{x^2-1}+1\right| + C,\ C\in\mathbb{R}$

3 a)



$\displaystyle \text{Área } (R) = \int_0^1 (e^{2x}-1)\,dx = \left[\frac{e^{2x}}{2} - x\right]_0^1 = \left(\frac{e}{2}-1\right) - \frac{1}{2}$

b) não sei

4 não sei

5 $\displaystyle \int_0^{\pi} x f(\operatorname{sen} x)\,dx = \int_{\pi}^{0} (\pi-y) f(\operatorname{sen}(\pi-y))\,(-dy) = \int_0^{\pi} (\pi-y) f(\operatorname{sen} y)\,dy$

$x = \pi - y \qquad x=0 \Rightarrow y=\pi$

$dx = -dy \qquad x=\pi \Rightarrow y=0$

$\{ \operatorname{sen}(\pi-y) = \operatorname{sen}\pi \cos(-y) + \cos\pi \operatorname{sen}(-y) = \operatorname{sen} y$

$\displaystyle = \int_0^{\pi} (\pi - x) f(\operatorname{sen} x)\,dx$

Então

$\displaystyle 2\int_0^{\pi} x f(\operatorname{sen} x)\,dx = \pi \int_0^{\pi} f(\operatorname{sen} x)\,dx$

ou, equivalentemente,

$\displaystyle \int_0^{\pi} x f(\operatorname{sen} x)\,dx = \frac{\pi}{2}\int_0^{\pi} f(\operatorname{sen} x)\,dx$