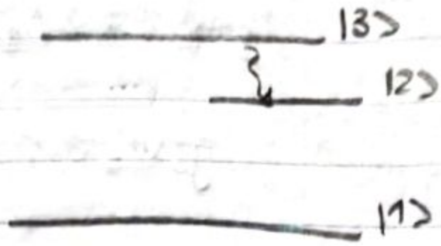
13.

a)



$$\frac{dN_3}{dt} = 0$$

$$\frac{dN_2}{dt} = PN_1 - \frac{\sigma(\nu)\, I_\nu}{h\nu}(N_2 - N_1) - \Gamma_{21} N_2$$

$$\frac{dN_1}{dt} = -PN_1 + \sigma(\nu)\frac{I_\nu}{h\nu}(N_2 - N_1) + \Gamma_{21} N_2$$

No estado estacionário $\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0$

$$\begin{cases} 0 = PN_1 - \Gamma_{21} N_2 \Leftrightarrow N_2 = \dfrac{P}{\Gamma_{21}} N_1 \\ 0 = -PN_1 + \Gamma_{21} N_2 \end{cases}$$

$$N_1 + N_2 = N_T \Leftrightarrow \left(1 + \frac{P}{\Gamma_{21}}\right) N_1 = N_T \Leftrightarrow$$

$$N_T = \frac{\Gamma_{21} + P}{\Gamma_{21}} N_1 \Leftrightarrow N_1 = \frac{\Gamma_{21}}{\Gamma_{21} + P} N_T$$

$$N_2 = \frac{P}{\Gamma_{21} + P} N_T$$

$$N_2 - N_1 = \frac{\Gamma_{21} - P}{\Gamma_{21} + P} N_T$$

b) $\sigma_0 = 10^{-12}\,cm^2$; $N_t = 3\times10^9/cm^3$; $P = 2\times\Gamma_{21}$, $R_1 = 1$; $R_2 = 0{,}99$

se $l = 0{,}2\,cm$

$N_2 - N_1 = \frac{1}{3} N_T \Rightarrow g(\nu) = \sigma(\nu)(N_2 - N_1) = 0{,}001\,cm^{-1}$

$g_{limiar} = -\frac{1}{2l}\ln(r_1 r_2) = 0{,}025$. Como $g(\nu) < g_{limiar}$ o laser não vai emitir

9.

$L = 1\,m$

$T_2 = 100\,\mu s$

Não homogénio → devido a colisões]
Lorentziana

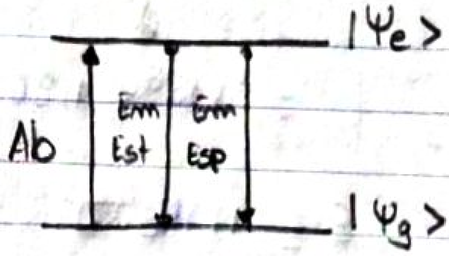
?

$\gamma_0(\omega) \xrightarrow{peak} \gamma_0(\omega_0) = 2\gamma_T$

3. Pergunta A

1.

$$\frac{dN_e}{dt} = -\sigma(\nu)\frac{I_\nu}{h\nu}(N_e - N_g) - A_{21}N_e$$



$$\frac{dN_g}{dt} = \sigma(\nu)\frac{I_\nu}{h\nu}(N_e - N_g) + A_{21}N_e$$

No estado estacionário $\frac{dN_e}{dt} = \frac{dN_g}{dt} = 0$

$$0 = -\sigma(\nu)\,\Phi_\nu (N_e - N_g) - A_{21}N_e$$

$$\Leftrightarrow (A_{21} + \sigma(\nu)\,\Phi_\nu)N_e = \sigma(\nu)\,\Phi_\nu N_g$$

$$N_e = \frac{\sigma(\nu)\,\Phi_\nu}{A_{21} + \sigma(\nu)\,\Phi_\nu} N_g$$

Não sei se está certa!

6.

Não homogéneo: Dopler ⇒ Gaussiana

Homogénea: Colisões ⇒ Lorentziana

Para um único modo longitudinal será melhor o meio homogéneo

1.

$\lambda = 1\ \mu m$

$d = 5mm \quad L = 10\,cm$

$N_{+} = 5 \times 10^{18}\ /cm^3$

$P(t) = P_0\, e^{-t^2/\tau}\ ,\ \tau = 100\ ms$

$E = N \times V \times h\nu$

$= NV \dfrac{c}{\lambda}$

$V = \pi\,(2{,}5 \times 10^{-1})^2 \times 10$

$= 1{,}96\ cm^3$

$= 5 \times 10^{18} \times 1{,}96 \times 6{,}626 \times 10^{-34} \times \dfrac{3 \times 10^8}{1 \times 10^{-6}} \left[\dfrac{1}{\cancel{cm^3}} \cdot \cancel{cm^3} \times Js \times \dfrac{cm}{s} \cdot \dfrac{1}{cm}\right]$

$E = 1{,}95\ J$

$E = \int P(t)\,dt = \int P_0\, e^{-t^2/\tau}\,dt = P_0\, \tau \sqrt{\pi} = 1{,}95\,J$

$P_0 = \dfrac{1{,}95}{100 \times 10^{-9} \times \sqrt{\pi}} = 11\ MW$

2.

$\lambda = 589{,}2\ nm$

$I = 100\ mW/cm^2$

$\tau = 16{,}25\ ms\ \ (3\,P_{3/2})$

$g_1 = g_2$

a)

Sabemos que $\sigma(\nu) = \dfrac{\lambda^2}{8\pi n^2} A_{21}\, S(\nu)$

Emissão Espontânea

$A_{21} = \dfrac{1}{\tau} = 61{,}5 \times 10^{6}\ s^{-1}$

$n = 1$

→ Alargamento natural em ressonância => $\delta\nu = \frac{A_{21}}{4\pi}$
$\hookrightarrow \nu = \nu_0$

$$L(\nu_0) = \frac{\delta\nu_0/\pi}{\delta\nu_0^2} = \frac{1}{\pi\delta\nu_0} = \frac{4}{A_{21}}$$

Logo, $\sigma(\nu) = \frac{(589,2\times10^{-9})^2}{8\pi} \times 4 = 5,53\times10^{-14}\ m^2$

→ A_ν aumenta, o $L(\nu)$ diminui, $\sigma(\nu)$ diminui

b)

Quando $\nu = \nu_0$

↳ Emissão Estimulada

$$B_{21} = \frac{A_{21}\, c^3}{8\pi n^3 \nu_{21}^3\, h} = \frac{A_{21}\, \cancel{c^3}\, \lambda^3}{8\pi n^3 \cancel{c^3}\, h} = 7,55\times10^{20}\ s^{-1}$$

Como $g_1 = g_2$ então $B_{12} = 7,55\times10^{20}\ s^{-1}$

↳ Aborção

c)

→ O mesmo de sempre

$$N_1 = \frac{T_{21}}{T_{21}+P} N_T \qquad N_2 = \frac{P}{T_{21}+P} N_T$$

$P = B_{12}$ e $A_{21} = T_{21}$ $\qquad \frac{N_2}{N_1} = \frac{B_{12}}{A_{21}}$

d)

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \exp\left[\frac{-h\nu}{k_B T}\right] \Leftrightarrow \ln\left(\frac{N_2}{N_1}\right) = \frac{-h\nu}{k_B T}$$

$\frac{g_2}{g_1} = 1$

??

$$T = \frac{-h\nu}{k_B \ln(N_2/N_1)} = \cdot\ 810\ K$$