Problem 7.31 A fat wire, radius a, carries a constant current I, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor, as shown in Fig. 7.43. Find the magnetic field in the gap, at a distance $s < a$ from the axis.

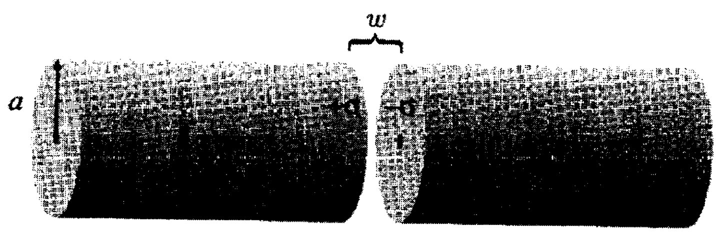


Figure 7.43

Problem 7.32 The preceding problem was an artificial model for the charging capacitor, designed to avoid complications associated with the current spreading out over the surface of the plates. For a more realistic model, imagine *thin* wires that connect to the centers of the plates (Fig. 7.44a). Again, the current I is constant, the radius of the capacitor is a, and the separation of the plates is $w \ll a$. Assume that the current flows out over the plates in such a way that the surface charge is uniform, at any given time, and is zero at $t = 0$.

(a) Find the electric field between the plates, as a function of t.

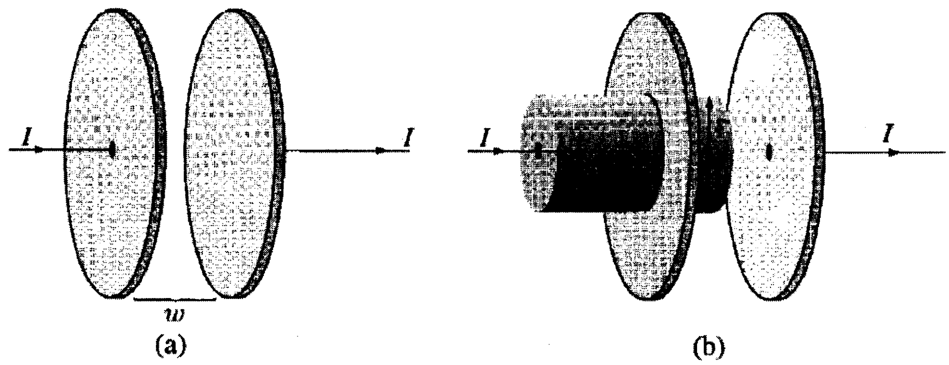


Figure 7.44

(b) Find the displacement current through a circle of radius s in the plane midway between the plates. Using this circle as your "Amperian loop," and the flat surface that spans it, find the magnetic field at a distance s from the axis.

(c) Repeat part (b), but this time use the cylindrical surface in Fig. 7.44b, which extends to the left through the plate and terminates outside the capacitor. Notice that the displacement current through this surface is zero, and there are two contributions to I_{enc}.[14]

Problem 7.33 Refer to Prob. 7.16, to which the correct answer was

$$\mathbf{E}(s,t) = \frac{\mu_0 I_0 \omega}{2\pi} \sin(\omega t) \ln\left(\frac{a}{s}\right) \hat{\mathbf{z}}.$$

(a) Find the displacement current density $\mathbf{J}_d$.

(b) Integrate it to get the total displacement current,

$$I_d = \int \mathbf{J}_d \cdot d\mathbf{a}.$$

(c) Compare I_d and I. (What's their ratio?) If the outer cylinder were, say, 2 mm in diameter, how high would the frequency have to be, for I_d to be 1% of I? [This problem is designed to indicate why Faraday never discovered displacement currents, and why it is ordinarily safe to ignore them unless the frequency is extremely high.]

Problem 7.34 Suppose

$$\mathbf{E}(\mathbf{r},t) = -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\theta(vt \quad r)\hat{\mathbf{r}}; \quad \mathbf{B}(\mathbf{r},t) = 0$$

(the theta function is defined in Prob. 1.45b). Show that these fields satisfy all of Maxwell's equations, and determine ρ and $\mathbf{J}$. Describe the physical situation that gives rise to these fields.

Problem 7.35 Assuming that "Coulomb's law" for magnetic charges (q_m) reads

$$\mathbf{F} = \frac{\mu_0}{4\pi}\frac{q_{m_1} q_{m_2}}{\imath^2}\hat{\boldsymbol{\imath}}, \tag{7.45}$$

work out the force law for a monopole q_m moving with velocity $\mathbf{v}$ through electric and magnetic fields $\mathbf{E}$ and $\mathbf{B}$. [For interesting commentary, see W. Rindler, *Am. J. Phys.* **57**, 993 (1989).]

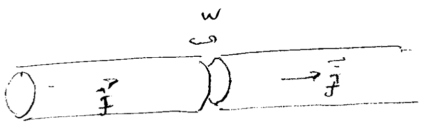
Problem 7.36 Suppose a magnetic monopole q_m passes through a resistanceless loop of wire with self-inductance L. What current is induced in the loop? [This is one of the methods used to search for monopoles in the laboratory; see B. Cabrera, *Phys. Rev. Lett.* **48**, 1378 (1982).]

Problem Show that $\rho_b = -\nabla \cdot \mathbf{P}$ and $\mathbf{J}_b = \nabla \times \mathbf{M}$

Problem 7.37 Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\epsilon = 81\epsilon_0$, permeability $\mu = \mu_0$, and resistivity $\rho = 0.23\ \Omega\cdot\text{m}$. What is the ratio of conduction current to displacement current? [*Hint:* consider a parallel-plate capacitor immersed in sea water and driven by a voltage $V_0 \cos(2\pi\nu t)$.]

23/Sep/2013

7.31



Campo magnético no hiato:

$$\vec{j}_D = \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \vec{j} = \epsilon_0 \frac{d}{dt}\left(\frac{\sigma}{\epsilon_0}\right) = \frac{I}{\pi a^2}$$

$$\oint \vec{B}\cdot d\vec{l} = 2\pi s\, B(s) = \mu_0 \frac{I}{\pi a^2}\cdot \pi s^2$$

$$B(s) = \frac{\mu_0 I s}{a^2\, 2\pi}$$

7.32 a) carga flui nas placas do condensador de tal forma que $\sigma = \text{const.}$ em cada instante.

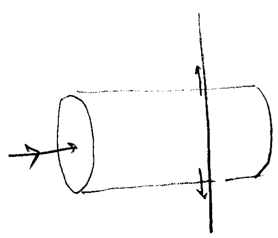
$$E = \frac{\sigma}{\epsilon_0} = \frac{Q(t)}{\epsilon_0 \pi a^2} = \frac{I t}{\epsilon_0 \pi a^2}$$

(visto que, por hipótese, $I = \text{const}$)

b) $$I_d = j_D \pi s^2 = \epsilon_0 \frac{dE}{dt} \pi s^2 = \epsilon_0 \frac{I}{\epsilon_0 \pi a^2} \pi s^2 = I \frac{s^2}{a^2}$$

$$\mu_0 I_d = \mu_0 I \frac{s^2}{a^2} = B\, 2\pi s \rightarrow B = \frac{\mu_0 I s}{2\pi a^2}$$

c)



Para a superfície da figura b, a carga flui para o exterior na placa esquerda do condensador. Seja $I(s)$ a carga total que atravessa o círculo de raio s na mesma placa.

~~A carga~~ A densidade de carga na armadura é = carga que entra − a que sai. :

$$\sigma(t) = \frac{(I - I(s))\, t}{\pi s^2}$$

Dado que σ é uniforme (por hipótese), σ é independente de s.

Logo $I - I(s) = \beta s^2$. Por outro lado, $I(a) = 0$, pelo que

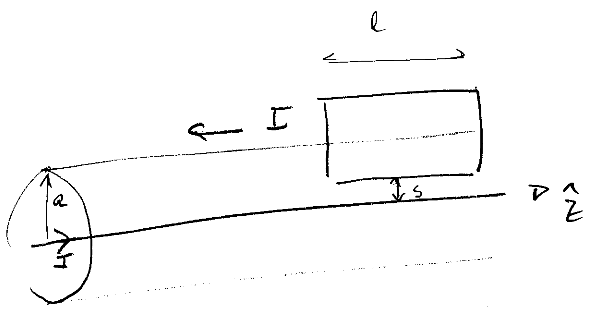
$\beta a^2 = I \Rightarrow \beta = \frac{I}{a^2}$. Logo:

$$I(s) = I\left(1 - \frac{s^2}{a^2}\right)$$

$$B\, 2\pi s = \mu_0 I_{\text{enc}} = \mu_0 [I - I(s)] = \mu_0 \frac{s^2}{a^2} I$$

$$B = \frac{\mu_0 I s}{2\pi a^2}$$

7.33



co-axial cable.

current flows as shown

$I = I_0 \cos \omega t$

a) Direcção do campo eléctrico in.

$$\nabla \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$|\vec{B}|$ fora do cabo co-axial é nulo $\Rightarrow$ $\vec{E}$ também é nulo.

$$\int_{\Sigma} \nabla \wedge \vec{E} \cdot d\vec{\Sigma} = -\frac{d\phi}{dt}$$

$$\oint \vec{E} \cdot d\vec{l} = El = -\frac{\partial \phi}{\partial t} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{\Sigma} = -\frac{\partial}{\partial t} \int_s^a \frac{\mu I}{2\pi s'} l\, ds'$$

$$\vec{E} = -\frac{\mu_0}{2\pi} \frac{dI}{dt} \ln\left(\frac{a}{s}\right) \hat{z}$$

$$\boxed{\vec{E} = \frac{\mu_0 I_0 \omega}{2\pi} \sin(\omega t) \ln \frac{a}{s}\, \hat{z}.}$$

...ndad Corrent de deslocamento?

(a) $\vec{J}_D = \varepsilon_0 \frac{\partial}{\partial t}\vec{E} = \varepsilon_0 \frac{\mu_0 I_0 \omega^2}{2\pi} \cos\omega t \ \ln\left(\frac{a}{s}\right)\hat{z}$

$= \frac{\mu_0 \varepsilon_0 \omega^2 I}{2\pi} \ln\left(\frac{a}{s}\right)\hat{z}$

(b) $I_d = \int \vec{J}_d \cdot d\vec{a} = \frac{\mu_0 \varepsilon_0 \omega^2 I}{2\pi} \int_0^a \ln\left(\frac{a}{s}\right) 2\pi s \, ds =$

$= \mu_0 \varepsilon_0 \omega^2 I \int_0^a \cancel{\ln\left(\frac{a}{s}\right)} \left[s \ln a - s \ln s \right] ds$

$= \mu_0 \varepsilon_0 \omega^2 I \left[\ln a \frac{s^2}{2} - \frac{s^2}{2}\ln s + \frac{s^2}{4} \right]_0^a = \frac{\mu_0 \varepsilon_0 \omega^2 I a^2}{4}$

c) $\frac{I_d}{I} = \frac{\mu_0 \varepsilon_0 \omega^2 a^2}{4} = \frac{1}{c^2} \frac{\omega^2 a^2}{4}$!!

7.34

$\vec{E}(\vec{r},t) = -\frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \theta(vt - r)\hat{r}$ $\qquad \vec{B}(\vec{r},t) = 0$

$\theta \equiv$ Heavyside

Maxwell?

$\nabla \cdot \vec{E} = -\frac{q}{4\pi\varepsilon_0} \nabla \cdot \left[\frac{\hat{r}}{r^2} \theta(vt - r) \right]$

…s: $\nabla \cdot (f\vec{A}) = f\,(\nabla\cdot\vec{A}) + \vec{A}\cdot\nabla f$

$$\nabla\cdot\vec{E} = -\frac{q}{4\pi\varepsilon_0}\,\Theta(vt-r)\,\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) - \frac{q}{4\pi\varepsilon_0}\,\frac{\hat{r}}{r^2}\cdot\nabla\left[\Theta(vt-r)\right]$$

$$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = \nabla\cdot\left(\frac{\vec{r}}{r^3}\right) \equiv 0 \quad \text{(em coordenadas cartesianas)}$$

Mas $\frac{\hat{r}}{r^2}$ é singular em $r=0$!

~~Usemos coordenadas esféricas~~

~~$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) =$~~

Mas, o fluxo de $\frac{\hat{r}}{r^2}$ através de uma superfície esférica de raio R é:

$$\iint_\Sigma \frac{\hat{r}}{r^2}\cdot R^2 \sin\theta\, d\theta\, d\phi\, \hat{r} = 4\pi \neq 0 \implies$$

$\implies \nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 0$ em todo o lado excepto na origem

$$\iiint_V \nabla\cdot\left(\frac{\hat{r}}{r^2}\right) dV \equiv 4\pi \implies \nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 4\pi\,\delta(\vec{r})$$

Então:

$$\nabla\cdot\vec{E} = -\frac{q}{\cancel{4\pi}\varepsilon_0}\,\Theta(vt-r)\cdot\cancel{4\pi}\,\delta(\vec{r}) - \frac{q}{4\pi\varepsilon_0}\,\frac{\hat{r}}{r^2}\cdot\underbrace{\frac{\partial}{\partial r}\left(\Theta(vt-r)\right)}_{-\delta(vt-r)}\hat{r}$$

$$= -\frac{q}{\cancel{4\pi}\varepsilon_0}\,\delta(\vec{r})\,\theta(t) + \frac{q}{4\pi\varepsilon_0}\,\frac{1}{r^2}\,\delta(vt-r)$$

$\nabla \cdot \vec{B} = 0$ $\nabla \wedge \vec{E} = 0$ (o campo é radial)

$$\nabla \wedge \vec{B} = 0 = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow$$

$$\Rightarrow \vec{j} = -\varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{q}{4\pi\varepsilon_0} \cdot \varepsilon_0 \frac{1}{r^2} v\, \delta(vt - r)\, \hat{r}$$

Interpretar.