Correção do 2º teste

Cálculo para Ciências

04/01/2023

1. $$\operatorname{ch}^2 x = \left(\frac{e^x+e^{-x}}{2}\right)^2 = \frac{e^{2x}+2e^xe^{-x}+e^{-2x}}{4} = \frac{1}{2}\left(\frac{e^{2x}+e^{-2x}}{2}+1\right)$$
$$= \frac{\operatorname{ch}(2x)+1}{2}, \quad \forall x\in\mathbb{R}$$

2. a) $$\int \frac{e^x}{(2e^x-4)^{3/2}}\,dx = \frac{1}{2}\int 2e^x(2e^x-4)^{-3/2}\,dx = \frac{1}{2}\,\frac{(2e^x-4)^{-1/2}}{-1/2}+C$$
$$= \frac{-1}{(2e^x-4)^{1/2}} + C, \ C\in\mathbb{R}$$

b) $$\int \underset{f'}{\underset{\shortparallel}{x^3}}\,\underset{g}{\underset{\shortparallel}{\ln(x+1)}}\,dx = \frac{1}{4}x^4\ln(x+1) - \frac{1}{4}\int\frac{x^4}{x+1}\,dx = (*)$$

$$\left.\begin{array}{ll} f'=x^3 & f=\frac{1}{4}x^4 \\ g=\ln(x+1) & g'=\frac{1}{x+1}\end{array}\right\}$$

$$\begin{array}{l|l} x^4 & x+1 \\ \hline -x^4-x^3 & x^3-x^2+x-1 \\ \quad -x^3 & \\ \quad x^3+x^2 & \\ \qquad x^2 & \\ \qquad -x^2-x & \\ \qquad\quad -x & \\ \qquad\quad x+1 & \\ \qquad\qquad 1 & \end{array}$$

Logo
$$x^4 = (x+1)(x^3-x^2+x-1)+1$$

$$(*) = \frac{1}{4}x^4\ln(x+1) - \frac{1}{4}\int(x^3-x^2+x-1)\,dx - \frac{1}{4}\int\frac{1}{x+1}\,dx$$
$$= \frac{1}{4}x^4\ln(x+1) - \frac{1}{4}\left(\frac{x^4}{4}-\frac{x^3}{3}+\frac{x^2}{2}-x\right) - \frac{1}{4}\ln(x+1) + C, \ C\in\mathbb{R}$$

Alternativamente à divisão de polinómios, podíamos verificar que

$$\frac{x^4}{x+1} = \frac{x^4-1}{x+1} + \frac{1}{x+1} = \frac{(x^2-1)(x^2+1)}{x+1} + \frac{1}{x+1} = \frac{(x-1)\cancel{(x+1)}(x^2+1)}{\cancel{x+1}} + \frac{1}{x+1}$$

c) $$\int x^3(1+2x^2)^6\,dx = \int \underset{f}{\underset{\shortparallel}{x^2}}\cdot\underset{g'}{\underset{\shortparallel}{x(1+2x^2)^6}}\,dx = \frac{x^2}{28}(1+2x^2)^7 - \frac{1}{14}\int x(1+2x^2)^7\,dx$$

$$f=x^2 \qquad f'=2x \qquad g'=x(1+2x^2)^6 \qquad g=\frac{1}{4}\frac{(1+2x^2)^7}{7}$$

$$= \frac{x^2}{28}(1+2x^2)^7 - \frac{1}{14}\cdot\frac{1}{4}\frac{(1+2x^2)^8}{8} + C, \ C\in\mathbb{R}$$

d) $\dfrac{x^2}{(x-1)(x+1)^2} = \dfrac{A}{x-1} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2} = \dfrac{A(x+1)^2 + B(x-1)(x+1) + C(x-1)}{(x-1)(x+1)^2}$ (2)

Então

$x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x=-1 \Rightarrow 1 = -2C \Leftrightarrow C = -1/2 \qquad x=1 \Rightarrow 1 = 4A \Leftrightarrow A = 1/4$

$x=0 \Rightarrow 0 = A - B - C \Leftrightarrow B = A - C = 1/4 + 1/2 = \frac{3}{4}$

Então

$$\int \frac{x^2}{2(x-1)(x+1)^2}\,dx = \frac{1}{2}\int \frac{\frac{1}{4}}{x-1}\,dx + \frac{1}{2}\int \frac{\frac{3}{4}}{x+1}\,dx + \frac{1}{2}\int -\frac{1}{2}(x+1)^{-2}dx$$

$$= \frac{1}{8}\ln|x-1| + \frac{3}{8}\ln|x+1| - \frac{1}{4}\frac{(x+1)^{-1}}{-1} + C$$

$$= \frac{1}{8}\ln|x-1| + \frac{3}{8}\ln|x+1 + \frac{1}{4(x+1)} + C,\ C \in \mathbb{R}$$

e) $\displaystyle\int \frac{x^3}{\sqrt[4]{1+x^2}}\,dx = \int \frac{(u^4-1)^{3/2}}{u}\frac{2u^3}{(u^4-1)^{1/2}}\,du = 2\int (u^4-1)\,u^2\,du$

$x = \sqrt{u^4-1} \Leftrightarrow u^4 = x^2+1 \Leftrightarrow u = \sqrt[4]{x^2+1}$

$dx = \frac{1}{2}(u^4-1)^{-1/2}\,4u^3\,du$

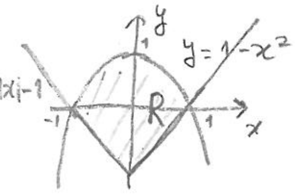
$$= 2\frac{u^7}{7} - 2\frac{u^3}{3} + C = \frac{2}{7}(x^2+1)^{7/4} - \frac{2}{3}(x^2+1)^{3/4} + C,\ C \in \mathbb{R}$$

f) $\displaystyle\int_0^{\pi/4} \frac{\cos x}{\operatorname{sen}^2 x + 1}\,dx = \int_0^{\sqrt{2}/2} \frac{1}{t^2+1}\,dt = \left[\operatorname{arctg} t\right]_0^{\sqrt{2}/2} = \operatorname{arctg}\left(\frac{\sqrt{2}}{2}\right) - \underbrace{\operatorname{arctg} 0}_{=0}$

$\operatorname{sen} x = t \qquad x=0 \Rightarrow t=0$

$\cos x\,dx = dt \qquad x = \pi/4 \Rightarrow t = \sqrt{2}/2$

(3) a)



b) $\text{Área}(R) = \displaystyle\int_{-1}^{0}\left((1-x^2)-(-x-1)\right)dx + \int_0^1\left((1-x^2)-(x-1)\right)dx$

$$= \int_{-1}^{0}(2-x^2+x)\,dx + \int_0^1 (2-x^2-x)\,dx = \left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{0} + \left[2x - \frac{x^3}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \left(0 - \left(-2 - \frac{1}{3} + \frac{1}{2}\right)\right) + \left(\left(2 - \frac{1}{3} - \frac{1}{2}\right) - 0\right) = \ldots$$

(4) a) $\displaystyle\int \operatorname{sen}^n x\,dx = \int \underbrace{\operatorname{sen} x}_{=f'} \cdot \underbrace{\operatorname{sen}^{n-1} x}_{=g} = -\cos x \operatorname{sen}^{n-1} x + (n-1)\int \operatorname{sen}^{n-2} x \cos^2 x$

$f' = \operatorname{sen} x \qquad f = -\cos x \qquad g = \operatorname{sen}^{n-1} x \qquad g' = (n-1)\operatorname{sen}^{n-2} x \cos x$

b) $I_n = \displaystyle\int \operatorname{sen}^n x\,dx = -\cos x \operatorname{sen}^{n-1} x + (n-1)\int \operatorname{sen}^{n-2} x(1 - \operatorname{sen}^2 x)\,dx$

$$= -\cos x \operatorname{sen}^{n-1} x + (n-1) I_{n-2} - (n-1) I_n$$

Então

$n I_n = -\cos x \operatorname{sen}^{n-1} x + (n-1) I_{n-2}$

e $I_n = -\frac{1}{n}\cos x \operatorname{sen}^{n-1} x + \frac{n-1}{n} I_{n-2}$