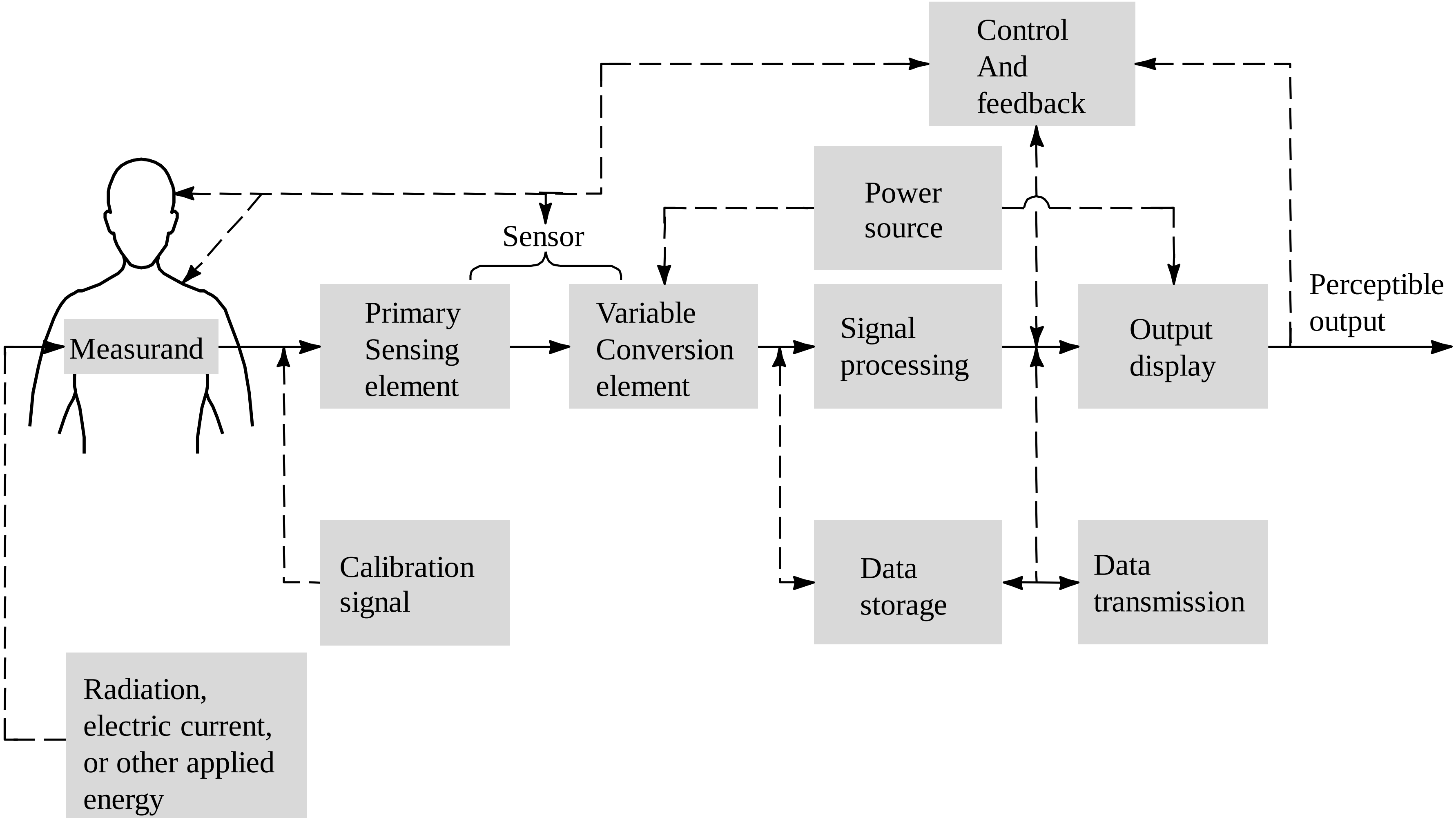

Figure 1.1 Generalized instrumentation system The sensor converts energy or information from the measurand to another form (usually electric). This signal is the processed and displayed so that humans can perceive the information. Elements and connections shown by dashed lines are optional for some applications.

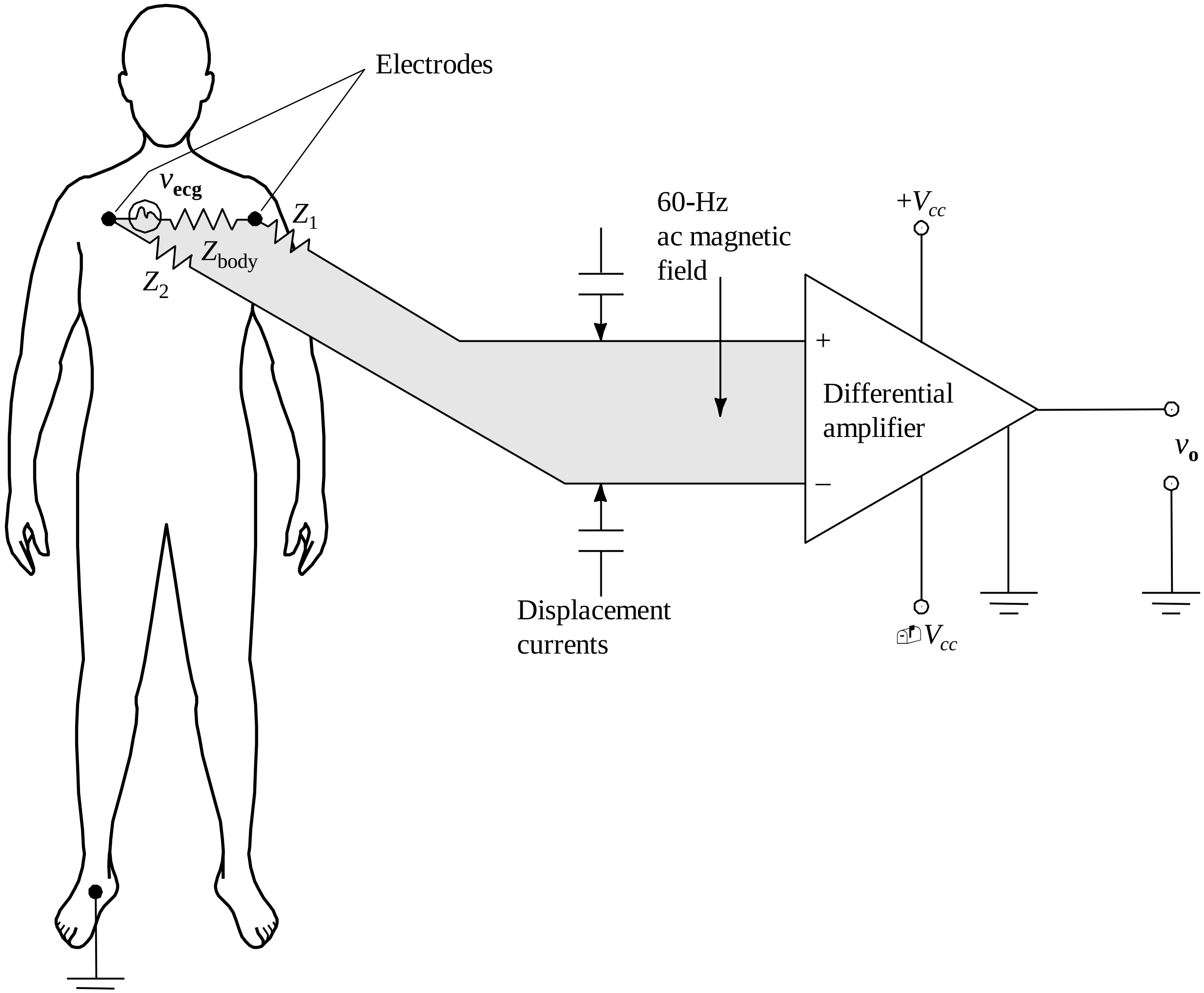

Figure 1.2 Simplified electrocardiographic recording system Two possible interfering inputs are stray magnetic fields and capacitively coupled noise. Orientation of patient cables and changes in electrode-skin impedance are two possible modifying inputs. Z_1 and Z_2 represent the electrode-skin interface impedances.

$$(x_d - H_f y)G_d = y \qquad (1.1)$$

$$x_d G_d = y(1 + H_f G_d) \qquad (1.2)$$

$$y = \frac{G_d}{1 + H_h G_d} x_d \qquad (1.3)$$

$$\overline{X} = \frac{\sum X_i}{n} \qquad (1.4)$$

$$GM = \sqrt[n]{X_1 X_2 X_3 \cdots X_n} \qquad (1.5)$$

$$s = \sqrt{\frac{\sum \left(X_i - \overline{X}\right)^2}{n-1}} \qquad (1.6)$$

$$CV = \left(\frac{s}{\overline{X}}\right)(100\%) \qquad (1.7)$$

$$r = \frac{\sum \left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sqrt{\sum \left(X_i - \overline{X}\right)^2} \sqrt{\sum \left(Y_i - \overline{Y}\right)^2}} \qquad (1.8)$$

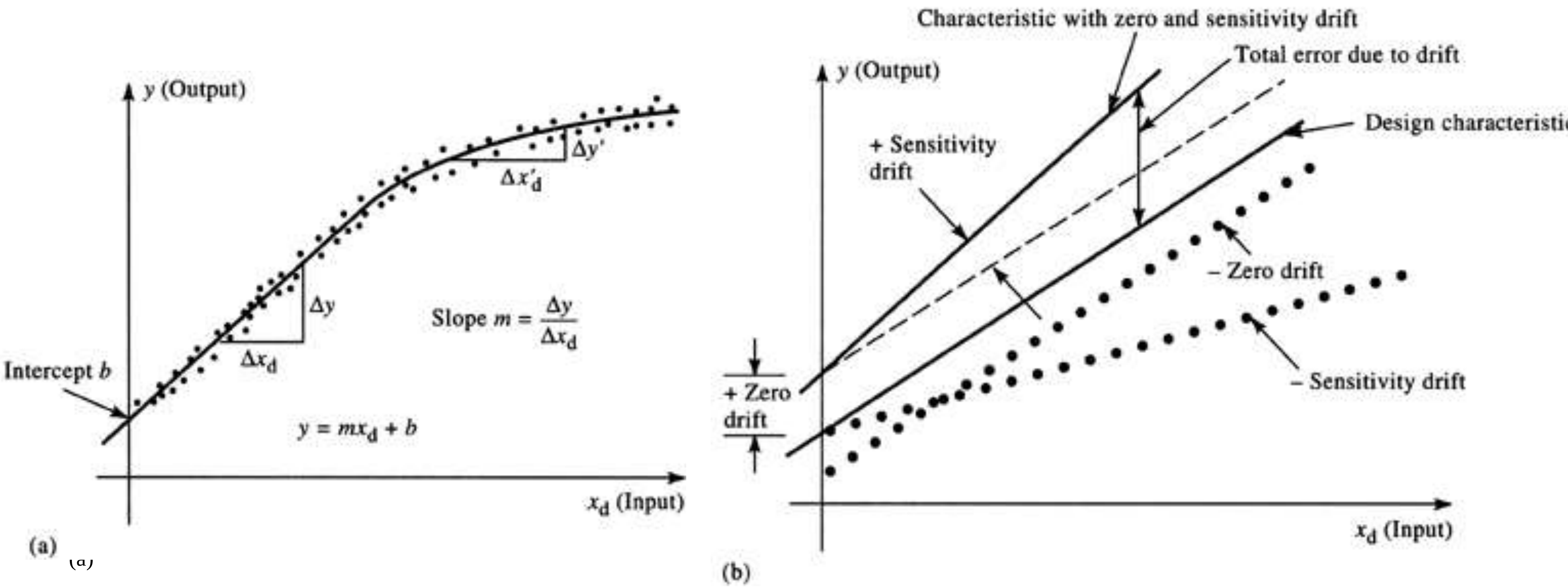

Figure 1.3 (a) Static-sensitivity curve that relates desired input x_d to output y. Static sensitivity may be constant for only a limited range of inputs. (b) Static sensitivity: zero drift and sensitivity drift. Dotted lines indicate that zero drift and sensitivity drift can be negative. [Part (b) modified from *Measurement Systems: Application and Design*, by E. O. Doebelin. Copyright © 1990 by McGraw-Hill, Inc. Used with permission of McGraw-Hill Book Co.]

$$m = \frac{n\sum x_{\mathrm{d}}y - \left(\sum x_{\mathrm{d}}\right)\left(\sum y\right)}{n\sum x_{\mathrm{d}}^2 - \left(\sum x_{\mathrm{d}}\right)^2} \qquad (1.9)$$

$$b = \frac{\left(\sum y\right)\left(\sum x_{\mathrm{d}}^2\right) - \left(\sum x_{\mathrm{d}}y\right)\left(\sum x_{\mathrm{d}}\right)}{n\sum x_{\mathrm{d}}^2 - \left(\sum x_{\mathrm{d}}\right)^2} \qquad (1.10)$$

$$y = mx_{\mathrm{d}} + b \qquad (1.11)$$

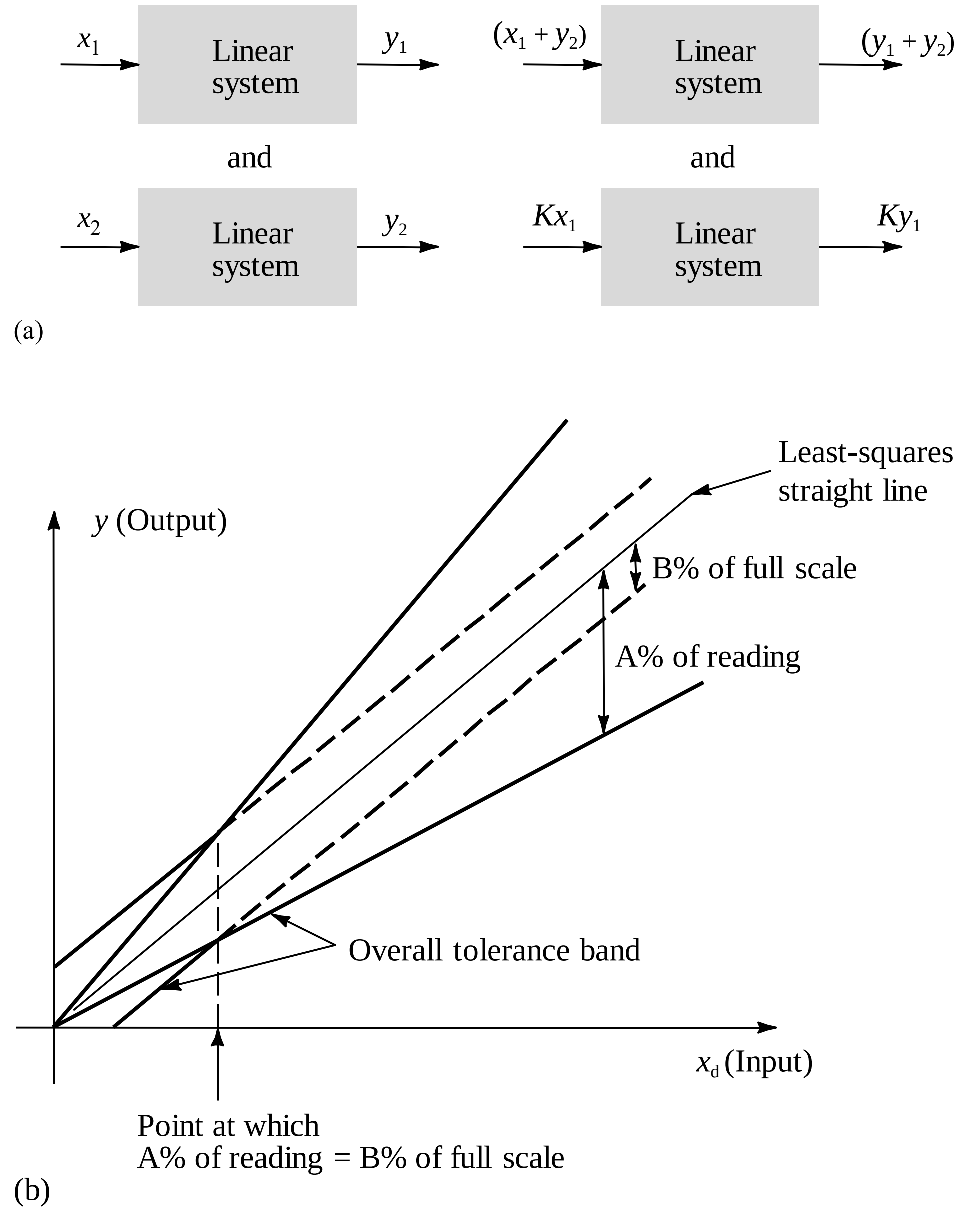

Figure 1.4 (a) Basic definition of linearity for a system or element. The same linear system or element is shown four times for different inputs. (b) A graphical illustration of independent nonlinearity equals ±A% of the reading, or ±B% of full scale, whichever is greater (that is, whichever permits the larger error). [Part (b) modified from *Measurement Systems: Application and Design*, by E. O. Doebelin. Copyright © 1990 by McGraw-Hill, Inc. Used with permission of McGraw-Hill Book Co.]

$$Z_x = \frac{X_{\mathrm{d1}}}{X_{\mathrm{d2}}} = \frac{\text{effort variable}}{\text{flow variable}} \tag{1.12}$$

$$P = X_{\mathrm{d1}} \cdot X_{\mathrm{d2}} = \frac{X_{\mathrm{d1}}^2}{Z_x} = Z_x X_{\mathrm{d2}}^2 \tag{1.13}$$

$$a_n \frac{d^n y}{dt^n} + \cdots + a_1 \frac{dy}{dt} + a_0 y(t) = b_m \frac{d^m x}{dt^m} + \cdots + b_1 \frac{dx}{dt} + b_0 x(t) \tag{1.14}$$

$$\left(a_n D^n + \cdots + a_1 D + a_0\right) y(t) = \left(b_m D^m + \cdots + b_1 D + b_0\right) x(t) \tag{1.15}$$

$$\frac{y(D)}{x(D)} = \frac{b_m D^m + \cdots + b_1 D + b_0}{a_n D^n + \cdots + a_1 D + a_0} \tag{1.16}$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{b_m (j\omega)^m + \cdots + b_1 (j\omega) + b_0}{a_n (j\omega)^n + \cdots + a_1 (j\omega) + a_0} \tag{1.17}$$

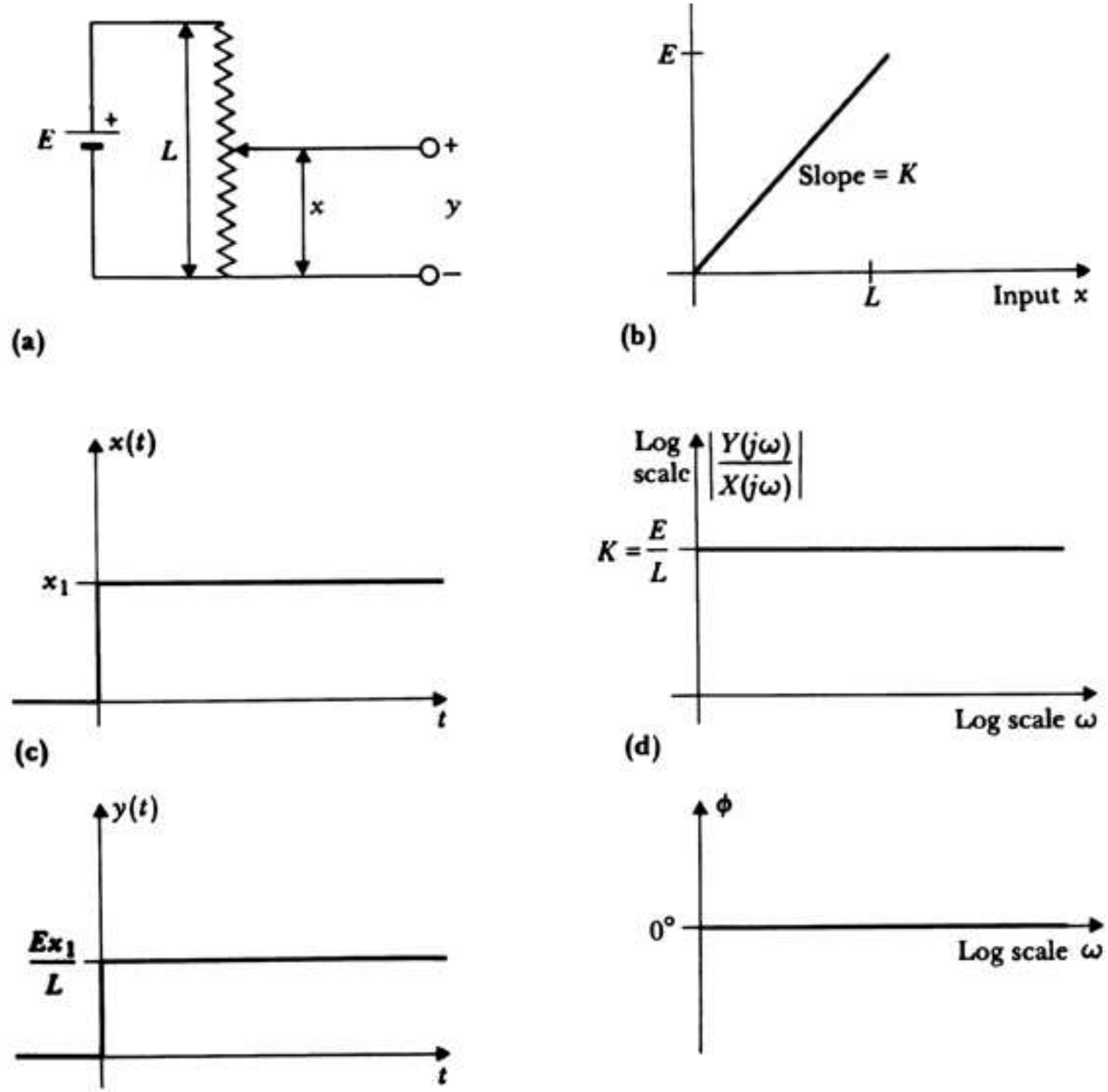

Figure 1.5 (a) A linear potentiometer, an example of a zero-order system. (b) Linear static characteristic for this system. (c) Step response is proportional to input. (d) Sinusoidal frequency response is constant with zero phase shift.

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$$a_0 y(t) = b_0 x(t) \tag{1.18}$$

$$\frac{y(D)}{x(D)} = \frac{Y(j\omega)}{X(j\varpi)} = \frac{b_0}{a_0} = K = \text{static sensitivity} \tag{1.19}$$

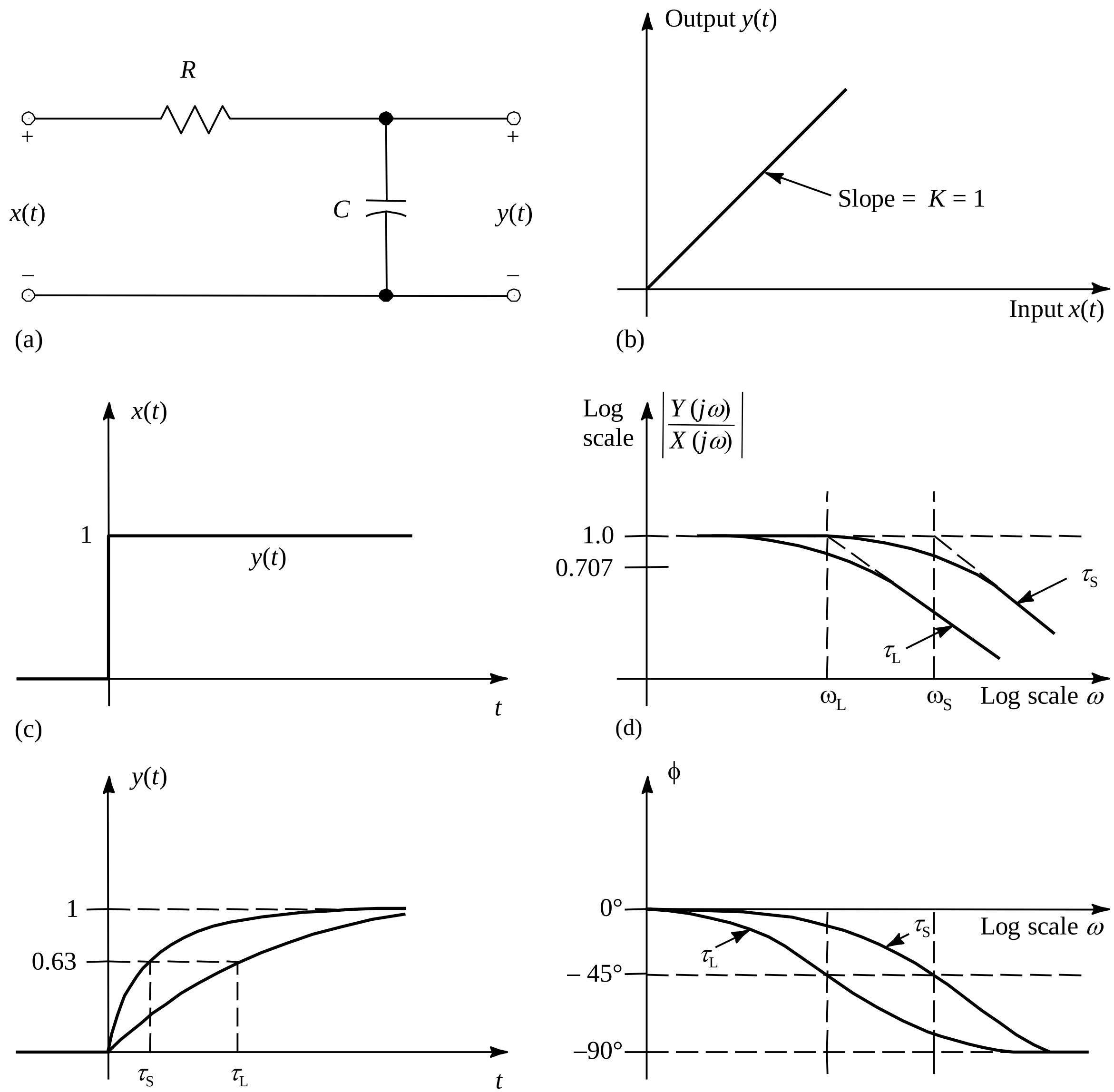


Figure 1.6 (a) A low-pass *RC* filter, an example of a first-order instrument. (b) Static sensitivity for constant inputs. (c) Step response for larger time constants (τ_L) and small time constants (τ_S). (d) Sinusoidal frequency response for large and small time constants.

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t) \tag{1.20}$$

$$(\tau D + 1) y(t) = Kx(t) \tag{1.21}$$

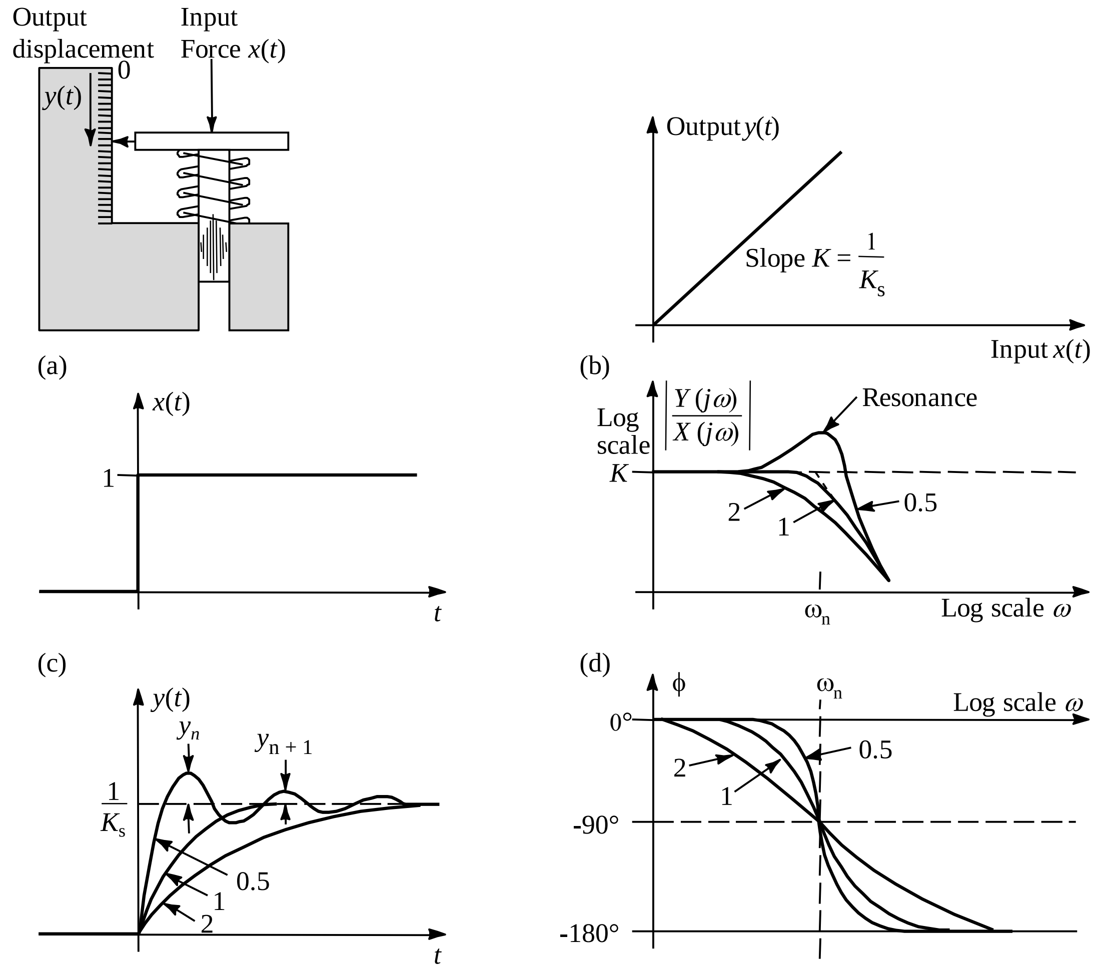
$$\frac{y(D)}{x(D)} = \frac{K}{1 + \tau D} \tag{1.22}$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{1 + j\omega\tau} = \frac{K}{\sqrt{1 + \omega^2 \tau^2}} \tag{1.23}$$

$$\phi = \arctan(-\omega\tau/1)$$

$$y(t) = K\left(1 - e^{-t/\tau}\right) \tag{1.24}$$

Figure 1.7 (a) Force-measuring spring scale, an example of a second-order instrument. (b) Static sensitivity. (c) Step response for overdamped case $\zeta = 2$, critically damped case $\zeta = 1$, underdamped case $\zeta = 0.5$. (d) Sinusoidal steady-state frequency response, $\zeta = 2$, $\zeta = 1$, $\zeta = 0.5$. [Part (a) modified from *Measurement Systems: Application and Design*, by E. O. Doebelin. Copyright © 1990 by McGraw-Hill, Inc. Used with permission of McGraw-Hill Book Co.]



$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t) \tag{1.25}$$

$$\left[\frac{D^2}{\omega_n^2} + \frac{2\zeta D}{\omega_n} + 1\right] y(t) = Kx(t) \tag{1.26}$$

where

$$K = \frac{b_0}{a_0} = \text{static sensitivit y, output units defined by input units}$$

$$\omega_n = \sqrt{\frac{a_0}{a_2}} = \text{undamped natural frequency, rad/s}$$

$$\zeta = \frac{a_1}{2\sqrt{a_0 a_2}} = \text{damping ratio, dimensionl ess}$$

$$\frac{y(D)}{x(D)} = \frac{K}{\dfrac{D^2}{\omega_n^2} + \dfrac{2\zeta D}{\omega_n} + 1} \tag{1.27}$$

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{K}{(j\omega/\omega_n)^2 + (2\zeta j\omega/\omega_n) + 1} = \frac{K}{\sqrt{\left[1 - (\omega/\omega_n)^2\right]^2 + 4\zeta^2\omega^2/\omega_n^2}} \tag{1.28}$$

$$\phi = \arctan \frac{2\zeta}{\omega/\omega_n - \omega_n/\omega}$$

$$x(t) - B\frac{dy(t)}{dt} - K_s y(t) = M\frac{d^2 y(t)}{dt^2} \quad (1.29)$$

$$K = 1/K_s \quad (1.30)$$

$$\omega_n = \sqrt{\frac{K_s}{M}} \quad (1.31)$$

$$\zeta = \frac{B}{2\sqrt{K_m M}} \quad (1.32)$$

Overdamped,

$\zeta > 1:$

$$y(t) = -\frac{\zeta + \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} Ke^{\left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n t} + \frac{\zeta - \sqrt{\zeta^2 - 1}}{2\sqrt{\zeta^2 - 1}} - Ke^{\left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n t} + K \quad (1.33)$$

Critically damped,

$\zeta = 1:$

$$y(t) = -(1 + \omega_n t)Ke^{-\omega_n t} + K \quad (1.34)$$

Underdamped,

$\zeta < 1:$

$$y(t) = -\frac{e^{-\zeta\omega_n t}}{\sqrt{1 - \zeta^2}} K \sin\left(\sqrt{1 - \zeta^2}\,\omega_n t + \phi\right) + K \quad (1.35)$$

$$\phi = \arcsin \sqrt{1 - \zeta^2}$$

$$t_{n=1} = \frac{7\pi / 2 - \phi}{\omega_n \sqrt{1 - \zeta^2}} \qquad \text{and} \qquad t_n = \frac{3\pi / 2 - \phi}{\omega_n \sqrt{1 - \zeta^2}} \tag{1.36}$$

$$\frac{y_n}{y_{n+1}} = \frac{\left(\dfrac{K}{\sqrt{1-\zeta^2}}\right)\left(\exp\left\{-\zeta\omega_n\left[\dfrac{(3\pi/2-\phi)}{\omega_n\sqrt{1-\zeta^2}}\right]\right\}\right)}{\left(\dfrac{K}{\sqrt{1-\zeta^2}}\right)\left(\exp\left\{-\zeta\omega_n\left[\dfrac{(7\pi/2-\phi)}{\omega_n\sqrt{1-\zeta^2}}\right]\right\}\right)} \tag{1.37}$$

$$= \exp\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

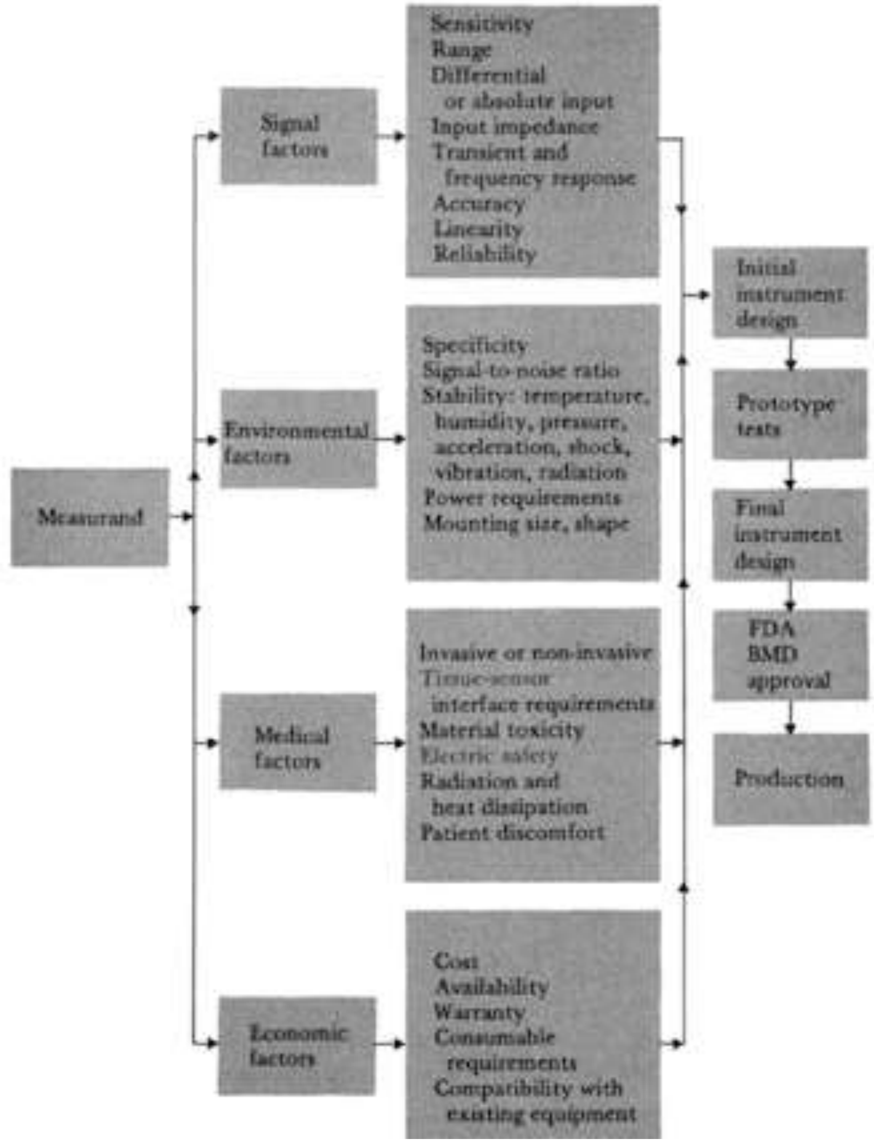
$$\ln\left(\frac{y_n}{y_{n+1}}\right) = \Lambda = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\zeta = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}} \tag{1.38}$$

$$y(t) = Kx(t - \tau_{\mathrm{d}}), \qquad t > \tau_d \tag{1.39}$$

$$\frac{Y(j\omega)}{X(j\omega)} = Ke^{-j\omega\omega_{\mathrm{d}}} \tag{1.40}$$

Figure 1.8 Design process for medical instruments Choice and design of instruments are affected by signal factors, and also by environmental, medical, and economic factors. (Revised from *Transducers for Biomedical Measurements: Application and Design*, by R. S. C. Cobbold. Copyright © 1974, John Wiley and Sons, Inc. Used by permission of John Wiley and Sons, Inc.)



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