**coord. cilind.**

$$\nabla \times \vec{F} = \frac{1}{r}\begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_r & rF_\phi & F_z \end{vmatrix} \qquad \boxed{\begin{matrix}\hat{\phi}\times\hat{z} = \hat{r} \\ \hat{r}\times\hat{\phi} = \hat{z} \\ \hat{z}\times\hat{r} = \hat{\phi}\end{matrix}}$$

$$\nabla \cdot \vec{F} = \frac{1}{r}\cdot\frac{\partial F_r}{\partial r} + \frac{1}{r}\cdot\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \qquad \boxed{\theta = \frac{\pi}{2}}$$

$$\nabla^2 \vec{F} = \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}F_r\right) + \frac{1}{r^2}\cdot\frac{\partial^2 F_r}{\partial r^2} + \frac{\partial^2 F_z}{\partial z^2}$$

**coord. esf.**

$$\nabla \times \vec{F} = \frac{1}{r^2 \operatorname{sen}\phi}\begin{vmatrix} \hat{r} & r\operatorname{sen}\phi\,\hat{\theta} & \hat{\phi} \\ \partial r & \partial\theta & \partial\phi \\ F_r & rF_\theta & r\operatorname{sen}\theta . F_\phi \end{vmatrix}$$

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\cdot\frac{\partial (r^2 F_r)}{\partial r} + \frac{1}{r\operatorname{sen}\theta}\cdot\frac{\partial(\sin\theta F_\phi)}{\partial\phi} + \frac{1}{r\sin\theta}\cdot\frac{\partial F_\theta}{\partial\theta}$$

$$\vec{r} = r\cos\phi\operatorname{sen}\theta\,\hat{x} + r\operatorname{sen}\phi\operatorname{sen}\theta\,\hat{y} + r\cos\theta\,\hat{z}$$

$$\boxed{\begin{matrix}\int_S (\nabla\times\vec{A})\cdot d\vec{a} = \oint_C \vec{A}\cdot d\vec{l} \\ \int_V (\nabla\cdot\vec{A})\, d\vec{v} = \oint_S \vec{A}\cdot d\vec{a}\end{matrix}}$$

$$Q = \int \rho\, d\vec{v} \,/\, \int \sigma\, d\vec{a} \,/\, \int \lambda\, d\vec{l}$$

$$\frac{1}{c^2} = \varepsilon_0 \cdot \mu_0$$

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$$\nabla\cdot\vec{E} = \frac{\rho_l}{\varepsilon_0} \quad\Big|\quad \nabla\cdot\vec{B} = 0 \quad\Big|\quad \nabla\times\vec{E} = -\frac{\partial}{\partial t}\vec{B} \quad\Big|\quad \nabla\times\vec{B} = \mu_0\cdot\vec{j}_l + \mu_0\cdot\varepsilon_0\cdot\underbrace{\frac{\partial\vec{E}}{\partial t}}_{\vec{j}_d}$$

(Gerais)

situação estática $\rightarrow \nabla\times\vec{E} = 0$

vácuo $\Rightarrow \vec{j}_l = 0$, $\rho_l = 0$, $u_{mec} = 0$

$$\boxed{I = \int \vec{j}\cdot d\vec{a}}$$

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$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \Rightarrow \vec{E} = -\nabla\vec{V}$$

$$\int \frac{1}{s}\, ds = \ln(s) \qquad \int \frac{1}{s^2}\, ds = -\frac{1}{s} \qquad V = -\int_0^a \vec{E}\cdot d\vec{l}$$

$$\vec{B} = \nabla\times\vec{A} \qquad \nabla\left(\frac{1}{r}\right) = \frac{\hat{r}}{r^2} \rightarrow \nabla\left(\frac{\hat{r}}{r^2}\right) = 4\pi\,\delta(\vec{r})$$

**Cond. plano**

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A}\cdot\frac{1}{\varepsilon_0}$$

$$Q = \frac{\partial I}{\partial t}$$

**coaxial** $\rightarrow \vec{B} = \hat{\phi}$ (entre cond.)

↳ $\int d\vec{a} = \int_0^{2\pi} d\theta \int_0^s r\, dr$

↳ $\vec{E} = \vec{s}$

↳ $V(r)$

**Solenóide** (dentro)

↳ $\vec{B} = \mu_0 . n . I_s(t) . \hat{z}$

↳ $\vec{E} = \phi$

**esfera**

↳ $\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R^2}\,\hat{r}$

**loop** $\rightarrow \frac{\partial W}{\partial t} = -\varepsilon . I$

energia dispendida p/ tempo

$$\varepsilon = -L . \frac{\partial I}{\partial t} = \oint_C E . d\vec{l}$$

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**meio material**

$\sigma_b = \vec{P}\cdot\hat{n}$ — dens. sup. cargas ligadas

$\rho_b = -\nabla\cdot\vec{P}$ — dens. vol. cargas ligadas

caso dinâmico $\rightarrow \vec{j}_P = \frac{\partial\vec{P}}{\partial t}$

$$\boxed{\frac{\partial\rho_b}{\partial t} + \nabla\cdot\vec{j}_P = 0}$$

$\vec{K}_b = \vec{M}\times\hat{n}$ — dens. sup. correntes

$\vec{j}_b = \nabla\times\vec{M}$ — dens. vol. correntes

$$\vec{j} = \vec{j}_l + \vec{j}_b + \vec{j}_P$$

$$\rho = \rho_l + \rho_b$$

eletricamente e magneticamente polarizável

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$\nabla\cdot\vec{D} = \rho_l$ , $\vec{D} = \varepsilon_0 . \vec{E} + \vec{P}$ (desloc. elétrico)

$\nabla\times\vec{H} = \vec{j}_l + \frac{\partial\vec{D}}{\partial t}$ , $\vec{H} = \mu_0^{-1} . \vec{B} - \vec{M}$ (excitação magnética)

**meio linear, neutral, isotrópico:**

$\vec{P} = \varepsilon_0 . \chi_e . \vec{E}$ → $\boxed{\chi_e = \varepsilon_r - 1}$

$\vec{M} = \chi_m . \vec{H}$

$\vec{D} = \underbrace{\varepsilon_0\,[1+\chi_e]}_{\varepsilon} . \vec{E}$

$\vec{B} = \underbrace{\mu_0\,[1+\chi_m]}_{\mu} . \vec{H}$

$$\boxed{\begin{matrix}\rho . \vec{v} = \vec{j} \\ q = \rho\, d\vec{r}\end{matrix}}$$

- $\vec{P}$ uniforme $\rightarrow \boxed{\nabla\cdot\vec{P} = 0} \rightarrow \rho_b = 0$
- simetria $\rightarrow \boxed{\nabla\times\vec{P} = 0} \rightarrow$ basta $\nabla\cdot\vec{D} = \rho_l$
- $\vec{M}$ uniforme $\rightarrow \boxed{\nabla\cdot\vec{M} = 0}$ e $\boxed{\nabla\times\vec{M} = 0}$
- No metal, as cargas acumulam-se na superfície.

**simetria esférica**

↓

meio dielétrico polariza-se radialmente

**fio e⁻ com ΔV**

$\vec{E} = \frac{V}{l} . \hat{z}$ (direção fio)

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**energia sist. dielétrico**: $\Delta W = \Delta\left[\frac{1}{2}.\varepsilon.E^2\right] = \Delta\left[\frac{1}{2}\vec{D}.\vec{E}\right] \Rightarrow W = \frac{1}{2}\int_V \vec{D}.\vec{E}\, d\vec{v} = \frac{\varepsilon_0}{2}\int \vec{E}^2\, d\vec{v}$

$\Delta W$: meio simples; $u_e$: Δ dens. vol. energia elétrica

trabalho necessário para produzir o sistema ⇔ energia do sistema

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial t}\int_V \underbrace{\frac{1}{2}\left(\varepsilon . E^2 + \frac{1}{\mu}.B^2\right)}_{u_{em}} d\vec{v} - \int_S \underbrace{\frac{1}{\mu}\left[\vec{E}\times\vec{B}\right]}_{\vec{S}} . \hat{n}\, d\vec{a}$$

$\frac{\partial W}{\partial t}$: trabalho mecânico; $u_{em}$: variação da energia eletro. em dt

$\vec{S}$ → densidade do fluxo de energia / tempo

energia/tempo: $\boxed{P = \int_S \vec{S} . d\vec{a}}$ → transversal

$$\boxed{\begin{matrix}\frac{\partial}{\partial t}u_{tot} = -\nabla.\vec{S} \\ u_{tot} = u_{em} + u_{mec}\end{matrix}}$$

$$\boxed{f_i^s = T_{ij} . n_j}$$

↳ força/área que atua na superfície S

→ dens. vol. momento linear

$$\vec{P}_{em} = \mu_0 . \varepsilon_0 . \vec{S}$$

$$T_{ij} = \varepsilon\left[E_iE_j - \tfrac{1}{2}\delta_{ij}E^2\right] + \tfrac{1}{\mu}\left[B_iB_j - \tfrac{1}{2}\delta_{ij}B^2\right]$$

↳ tensor Maxwell

$\vec{f} = \nabla.\overset{\leftrightarrow}{T} - \varepsilon.\mu.\frac{\partial\vec{S}}{\partial t}$ → dens. vol. da força eletromag.

$\vec{F} = \int_V \vec{f}\, d\vec{v}$ → força total atua no sistema

momento mecânico total das partículas

$$\vec{P}_{mec} = \int \vec{F} . dt$$

$$\vec{P}_{em} = \mu.\varepsilon.\int_V \vec{S}\, d\vec{v}$$

momento linear (dos campos)

$$\boxed{\begin{matrix}\vec{M}_a = \vec{r}\times[Q_a.\vec{E}_a] \\ \vec{L}_a = \int \vec{M}_a(t) . dt\end{matrix}}$$

↳ gerado por perder I

$\vec{l}_{em} = \vec{r}\times\vec{P}_{em} \Rightarrow \boxed{\vec{L}_{em} = \vec{l}_{em}\times V}$

↳ dens. vol. momento angular

$$\boxed{\frac{\partial p_{mec}}{\partial t} + \frac{\partial p_{em}}{\partial t} = \int_V \nabla.\overset{\leftrightarrow}{T}\, d\vec{v}}$$

⇒ conservação local do mom. linear