1 a) $\int (3x^2-2x^5)\,dx = x^3 - \frac{1}{3}x^6 + C,\ C\in\mathbb{R}$

b) $\int (\sqrt{x}+2)^2\,dx = \int (x+4\sqrt{x}+4)\,dx = \frac{x^2}{2} + 4\,\frac{x^{3/2}}{3/2} + 4x + C,\ C\in\mathbb{R}$

c) $\int (2x+10)^{20}\,dx = \frac{1}{2}\int 2(2x+10)^{20}\,dx = \frac{1}{2}\,\frac{(2x+10)^{21}}{21} + C,\ C\in\mathbb{R}$

d) $\int x^2 e^{x^3}\,dx = \frac{1}{3}\int 3x^2 e^{x^3}\,dx = \frac{1}{3}\,e^{x^3} + C,\ C\in\mathbb{R}$

e) $\int x^4(x^5+10)^9\,dx = \frac{1}{5}\int 5x^4(x^5+10)^9\,dx = \frac{1}{5}\,\frac{(x^5+10)^{10}}{10} + C,\ C\in\mathbb{R}$

f) $\int \frac{2x+1}{x^2+x+3}\,dx = \ln(x^2+x+3) + C,\ C\in\mathbb{R}$

g) $\int \sqrt{2x+1}\,dx = \frac{1}{2}\int 2(2x+1)^{1/2}\,dx = \frac{1}{2}\,\frac{(2x+1)^{3/2}}{3/2} + C,\ C\in\mathbb{R}$

h) $\int \frac{x}{3-x^2}\,dx = \frac{1}{2}\int \frac{2x}{3-x^2}\,dx = \frac{1}{2}\ln|3-x^2| + C,\ C\in\mathbb{R}$

i) $\int \frac{1}{4-3x}\,dx = -\frac{1}{3}\int \frac{-3}{4-3x}\,dx = -\frac{1}{3}\ln|4-3x| + C,\ C\in\mathbb{R}$

j) $\int \frac{1}{e^{3x}}\,dx = -\frac{1}{3}\int (-3)\,e^{-3x}\,dx = -\frac{1}{3}\,e^{-3x} + C,\ C\in\mathbb{R}$

k) $\int \frac{-7}{\sqrt{1-5x}}\,dx = -7\cdot\left(-\frac{1}{5}\right)\int (-5)(1-5x)^{-1/2}\,dx = \frac{7}{5}\,\frac{(1-5x)^{1/2}}{1/2} + C,\ C\in\mathbb{R}$

l) $\int \frac{\sqrt{1+3\ln x}}{x}\,dx = \int \frac{1}{x}(1+3\ln x)^{1/2}\,dx = \frac{(1+3\ln x)^{3/2}}{3/2} + C,\ C\in\mathbb{R}$

m) $\int x\,\mathrm{sen}(x^2)\,dx = \frac{1}{2}\int 2x\,\mathrm{sen}(x^2)\,dx = -\frac{1}{2}\cos(x^2) + C,\ C\in\mathbb{R}$

n) $\int \frac{1}{x(\ln^2 x+1)}\,dx = \int \frac{\frac{1}{x}}{(\ln x)^2+1}\,dx = \mathrm{arctg}(\ln x) + C,\ C\in\mathbb{R}$

o) $\int \left(\frac{2}{x}-3\right)^2 \frac{1}{x^2}\,dx = -\frac{1}{2}\int \left(\frac{2}{x}-3\right)^2\left(-\frac{2}{x^2}\right)dx = -\frac{1}{2}\,\frac{\left(\frac{2}{x}-3\right)^3}{3} + C,\ C\in\mathbb{R}$

p) $\int \mathrm{sen}(\pi-2x)\,dx = -\frac{1}{2}\int -2\,\mathrm{sen}(\pi-2x)\,dx = -\frac{1}{2}\left(-\cos(\pi-2x)\right) + C,\ C\in\mathbb{R}$

q) $\int \mathrm{th}\,x\,dx = \int \frac{\mathrm{sh}\,x}{\mathrm{ch}\,x}\,dx = \ln(\mathrm{ch}\,x) + C,\ C\in\mathbb{R}$

r) $\int \mathrm{sen}\,x\cos x = \frac{\mathrm{sen}^2 x}{2} + C,\ C\in\mathbb{R}$

s) $\int \mathrm{sen}(2x)\cos x\,dx = \int 2\,\mathrm{sen}\,x\cos^2 x\,dx = -2\int -\mathrm{sen}\,x\cos^2 x\,dx = -2\,\frac{\cos^3 x}{3} + C,\ C\in\mathbb{R}$

t) $\int \mathrm{sen}^2 x\,dx = \int \frac{1-\cos(2x)}{2}\,dx = \frac{1}{2}x - \frac{\mathrm{sen}(2x)}{4} + C,\ C\in\mathbb{R}$

u) $\int \mathrm{sen}^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)dx = \int \left(\mathrm{sen}\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\right)^2 dx = \int \left(\frac{1}{2}\,\mathrm{sen}\left(2\,\frac{x}{2}\right)\right)^2 dx$

$= \frac{1}{4}\int \mathrm{sen}^2 x\,dx = \frac{1}{4}\left(\frac{1}{2}x - \frac{\mathrm{sen}(2x)}{4}\right) + C,\ C\in\mathbb{R}$

v) $\int \cos^3 x\,dx = \int \cos x\,(1-\operatorname{sen}^2 x)\,dx = \int \cos x - \int \cos x \operatorname{sen}^2 x\,dx$
$= \operatorname{sen} x - \frac{\operatorname{sen}^3 x}{3} + C,\ C\in\mathbb{R}$

w) $\int \frac{x}{x^2-1}\,dx = \frac{1}{2}\int \frac{2x}{x^2-1}\,dx = \frac{1}{2}\ln|x^2-1| + C,\ C\in\mathbb{R}$

x) $\int \frac{x}{\sqrt{x^2-1}} = \frac{1}{2}\int 2x\,(x^2-1)^{-1/2}\,dx = \frac{1}{2}\frac{(x^2-1)^{1/2}}{1/2} + C,\ C\in\mathbb{R}$

y) $\int \frac{1}{x}\operatorname{sen}(\ln x)\,dx = -\cos(\ln x) + C,\ C\in\mathbb{R}$

z) $\int \frac{-3}{x(\ln x)^3}\,dx = -3\int \frac{1}{x}(\ln x)^{-3}\,dx = -3\frac{(\ln x)^{-2}}{-2} + C,\ C\in\mathbb{R}$

② a) $\int 1.\ \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx = x\ln x - x + C,\ C\in\mathbb{R}$

$f'=1 \quad f=x$
$g=\ln x \quad g'=1/x$

b) $\int x\operatorname{sen}(2x)\,dx = -\frac{1}{2}x\cos(2x) + \frac{1}{2}\int \cos(2x)\,dx = -\frac{1}{2}x\cos(2x) + \frac{1}{4}\operatorname{sen}(2x) + C,\ C\in\mathbb{R}$

$f=x \quad f'=1$
$g'=\operatorname{sen}(2x) \quad g=-\frac{\cos(2x)}{2}$

c) $\int 1.\ \operatorname{arctg} x\,dx = x\operatorname{arctg} x - \int \frac{x}{1+x^2}\,dx = x\operatorname{arctg} x - \frac{1}{2}\ln(1+x^2) + C,\ C\in\mathbb{R}$

$f'=1 \quad f=x$
$g=\operatorname{arctg} x \quad g'=\frac{1}{1+x^2}$

d) $\int x\cos x\,dx = x\operatorname{sen} x - \int \operatorname{sen} x\,dx = x\operatorname{sen} x + \cos x + C,\ C\in\mathbb{R}$

$f=x \quad f'=1$
$g'=\cos x \quad g=\operatorname{sen} x$

e) $\int 1.\ \ln(1-x)\,dx = x\ln x + \int \frac{x}{1-x}\,dx = x\ln x + \int \frac{x-1+1}{1-x}\,dx$

$f'=1 \quad f=x$
$g=\ln(1-x) \quad g'=\frac{-1}{1-x}$

$= x\ln x - \int 1\,dx + \int \frac{1}{1-x}\,dx = x\ln x - x - \ln|1-x| + C,\ C\in\mathbb{R}$

f) $\int x\ln x\,dx = \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C,\ C\in\mathbb{R}$

$f'=x \quad f=\frac{x^2}{2}$
$g=\ln x \quad g'=1/x$

g) $\int x^2\operatorname{sen} x\,dx = -x^2\cos x + \int 2x\cos x\,dx = -x^2\cos x + 2\left(2x\operatorname{sen} x - \int 2\operatorname{sen} x\,dx\right)$

$f=x^2 \quad f'=2x$
$g'=\operatorname{sen} x \quad g=-\cos x$

$f=2x \quad f'=2$
$g'=\cos x \quad g=\operatorname{sen} x$

$= -x^2\cos x + 4x\operatorname{sen} x + 4\cos x + C,\ C\in\mathbb{R}$

h) $\int x \operatorname{sen} x \cos x \, dx = \frac{1}{2} x \operatorname{sen}^2 x - \frac{1}{2} \underbrace{\int \operatorname{sen}^2 x \, dx}_{\text{Calculado em 1-t)}}$

$f = x \quad f' = 1$

$g' = \operatorname{sen} x \cos x \quad g = \frac{\operatorname{sen}^2 x}{2}$

i) $\int 1. \ln^2 x = x \ln^2 x - 2\int x \ln x \frac{1}{x} \, dx = x \ln^2 x - \underbrace{2\int \ln x \, dx}_{\text{calculado em 2-a)}}$

$f' = 1 \quad f = x$

$g = \ln^2 x \quad g' = 2 \ln x \cdot \frac{1}{x}$

j) $\int e^x \cos x \, dx = e^x \cos x + \int e^x \operatorname{sen} x \, dx = e^x \cos x + e^x \operatorname{sen} x - \int e^x \cos x \, dx$

$f' = e^x \quad f = e^x$ $\quad\Bigg\{$ $f' = e^x \quad f = e^x$

$g = \cos x \quad g' = -\operatorname{sen} x$ $\quad$ $g = \operatorname{sen} x \quad g' = \cos x$

Então $\quad 2\int e^x \cos x \, dx = e^x (\cos x + \operatorname{sen} x) + C$

$\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \operatorname{sen} x) + C, \; C \in \mathbb{R}$

k) $\int 1. \operatorname{arcsen} x = x \operatorname{arcsen} x - \frac{1}{2} \int 2x (1-x^2)^{-1/2} dx = x \operatorname{arcsen} x - \frac{1}{2} \frac{(1-x^2)^{1/2}}{1/2} + C, \; C \in \mathbb{R}$

$f' = 1 \quad f = x$

$g = \operatorname{arcsen} x \quad g' = \frac{1}{\sqrt{1-x^2}}$

l) $\int \underbrace{e^{\operatorname{sen} x} \cos x}_{f'} \cdot \underbrace{\operatorname{sen} x}_{g} = \operatorname{sen} x \, e^{\operatorname{sen} x} - \int e^{\operatorname{sen} x} \cos x \, dx = \operatorname{sen} x \, e^{\operatorname{sen} x} - e^{\operatorname{sen} x} + C, \; C \in \mathbb{R}$

$f' = e^{\operatorname{sen} x} \cos x \quad f = e^{\operatorname{sen} x}$

$g = \operatorname{sen} x \quad g' = \cos x$

m) $\int x^{-1/2} \operatorname{arcsen}(x^{1/2}) \, dx = 2\sqrt{x} \operatorname{arcsen} x - \int \cancel{2\sqrt{x}} \frac{1}{\cancel{2\sqrt{x}} \sqrt{1-x}} \, dx$

$f' = x^{-1/2} \quad f = \frac{x^{1/2}}{1/2}$

$g = \operatorname{arcsen}(x^{1/2}) \quad g' = \frac{(x^{1/2})'}{\sqrt{1-(x^{1/2})^2}} = \frac{\frac{1}{2} x^{-1/2}}{\sqrt{1-x}}$

$= 2\sqrt{x} \operatorname{arcsen} x - \int (1-x)^{-1/2} dx = 2\sqrt{x} \operatorname{arcsen} x + \frac{(1-x)^{1/2}}{1/2} + C,$

$C \in \mathbb{R}$

n) $\int x \operatorname{arctg} x \, dx = \frac{x^2}{2} \operatorname{arctg} x + \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx$

$f' = x \quad f = \frac{x^2}{2}$

$g = \operatorname{arctg} x \quad g' = \frac{1}{1+x^2}$

$= \frac{x^2}{2} \operatorname{arctg} x + \frac{1}{2} \int \frac{x^2+1}{1+x^2} \, dx - \frac{1}{2} \int \frac{1}{1+x^2} \, dx$

$= \frac{x^2}{2} \operatorname{arctg} x + \frac{1}{2} x - \frac{1}{2} \operatorname{arctg} x + C, \; C \in \mathbb{R}$

o) $\int x^2 \ln x \, dx = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 \, dx = \frac{x^3}{3} \ln x - \frac{1}{9} x^3 + C, \; C \in \mathbb{R}$

$f' = x^2 \quad f = x^3/3$

$g = \ln x \quad g' = 1/x$

p) $\int \operatorname{sen}(\ln x)\,dx = \int \underset{f}{\overset{\shortparallel}{x}}\ \underbrace{\operatorname{sen}(\ln x)\frac{1}{x}}_{g'}\,dx = -x\cos(\ln x) + \int \cos(\ln x)\,dx$

$f = x \qquad f' = 1$

$g' = \operatorname{sen}(\ln x)\frac{1}{x} \qquad g = -\cos(\ln x)$

$= -x\cos(\ln x) + \int \underset{f}{\overset{\shortparallel}{x}}\ \underbrace{\cos(\ln x)\frac{1}{x}}_{g'}\,dx$

$f = x \qquad f' = 1$

$g' = \cos(\ln x)\frac{1}{x} \qquad g = \operatorname{sen}(\ln x)$

$= -x\cos(\ln x) + \left(x\operatorname{sen}(\ln x) - \int \operatorname{sen}(\ln x)\,dx\right)$

Então

$2\int \operatorname{sen}(\ln x)\,dx = x\left(\operatorname{sen}(\ln x) - \cos(\ln x)\right) + C$

e $\int \operatorname{sen}(\ln x)\,dx = \frac{1}{2}\left(\operatorname{sen}(\ln x) - \cos(\ln x)\right) + C, \ C \in \mathbb{R}$

q) $\int \operatorname{ch} x \operatorname{sen}(3x)\,dx = \operatorname{sh} x \operatorname{sen}(3x) - 3\int \operatorname{sh} x \cos(3x)\,dx =$

$f' = \operatorname{ch} x \qquad f = \operatorname{sh} x$

$g = \operatorname{sen}(3x) \qquad g' = 3\cos(3x)$

$\left\{ \begin{array}{ll} f' = \operatorname{sh} x & f = \operatorname{ch} x \\ g = \cos(3x) & g' = -3\operatorname{sen}(3x) \end{array} \right.$

$= \operatorname{sh} x \operatorname{sen}(3x) - 3\left(\operatorname{ch} x \cos(3x) + 3\int \operatorname{ch} x \operatorname{sen}(3x)\,dx\right)$

Então

$10\int \operatorname{ch} x \operatorname{sen}(3x) = \operatorname{sh} x \operatorname{sen}(3x) - 3\operatorname{ch} x \cos(3x) + C$

e $\int \operatorname{ch} x \operatorname{sen}(3x) = \frac{1}{10}\left(\operatorname{sh} x \operatorname{sen}(3x) - 3\operatorname{ch} x\cos(3x)\right) + C, \ C \in \mathbb{R}$

r) $\int x^3 e^{x^2} = \int x^2 \left(x e^{x^2}\right) dx = \frac{1}{2}x^2 e^{x^2} - \int \cancel{2}x \cdot \frac{1}{\cancel{2}} e^{x^2}\,dx$

$f = x^2 \qquad f' = 2x$

$g' = x e^{x^2} \qquad g = \frac{1}{2}e^{x^2}$

$= \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C, \ C \in \mathbb{R}$

3- a) $\boxed{\text{Substituição} \quad x+3 = y^3} \Leftrightarrow y = (x+3)^{1/3}$

$dx = 3y^2\,dy$

$\int x(x+3)^{1/3}\,dx = \int (y^3-3)\,y \cdot 3y^2\,dy = \int (3y^6 - 9y^3)\,dy = \frac{3}{7}y^7 - \frac{9}{4}y^4 + C$

$= \frac{3}{7}(x+3)^{7/3} - \frac{9}{4}(x+3)^{4/3} + C, \ C \in \mathbb{R}$

b) Ver na pg seguinte

c) $\boxed{\text{Substituição} \quad 2-3x = y^2} \Leftrightarrow x = \frac{1}{3}(2-y^2) \qquad -3dx = 2y\,dy$

$\int \frac{x}{\sqrt{2-3x}}\,dx = \int \frac{\frac{1}{3}(2-y^2)}{\cancel{y}}\left(-\frac{2}{3}\cancel{y}\right)dy = -\frac{2}{9}\int (2-y^2)\,dy = -\frac{4}{9}y + \frac{2}{9}\frac{y^3}{3} + C =$

$$= -\frac{4}{9}(2-3x)^{1/2} + \frac{2}{27}(2-3x)^{3/2} + C,\ C\in\mathbb{R}$$

d) $\boxed{\text{Substituição } \sqrt{x}=y}$ $\Longleftrightarrow x = y^2 \qquad dx = 2y\,dy$

$$\int \frac{\operatorname{sen}\sqrt{x}}{\sqrt{x}}\,dx = \int \frac{\operatorname{sen} y}{\cancel{y}}\,2\cancel{y}\,dy = -2\cos y + C = -2\cos\sqrt{x} + C,\ C\in\mathbb{R}$$

e) $\boxed{\text{Substituição } e^x=y}$ $\Longleftrightarrow x = \ln y \qquad dx = \frac{1}{y}\,dy$

$$\int \frac{e^{2x}}{3+e^x}\,dx = \int \frac{y^2}{3+y}\,\frac{1}{y}\,dy = \int \frac{y+3-3}{3+y}\,dy = \int 1\,dy - 3\int \frac{1}{3+y}\,dy$$

$$= y - 3\ln|3+y| + C = e^x - 3\ln(3+e^x) + C,\ C\in\mathbb{R}$$

f) $\boxed{\text{Substituição } x=\operatorname{sen} t}$ $\Longleftrightarrow t = \operatorname{arcsen} x \qquad dx = \cos t\,dt$

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\operatorname{sen}^2 t}{\sqrt{1-\operatorname{sen}^2 t}}\cos t\,dt = \int \frac{\operatorname{sen}^2 t\,\cancel{\cos t}}{\cancel{\cos t}}\,dt$$

$$= \int \frac{1-\cos(2t)}{2}\,dt = \frac{t}{2} - \frac{\operatorname{sen}(2t)}{4} + C = \frac{1}{2}\operatorname{arcsen} x - \frac{1}{4}\operatorname{sen}(2\operatorname{arcsen} x) + C,\ C\in\mathbb{R}$$

g) $\boxed{\text{Substituição } x=y^6}$ $\Longleftrightarrow y = x^{1/6} \qquad dx = 6y^5\,dy$

$$\int \frac{\sqrt{x}}{x-\sqrt[3]{x}}\,dx = \int \frac{y^3}{y^6-y^2}\,6y^5\,dy = 6\int \frac{y^6}{y^4-1}\,dy =$$ esqueci-me de acabar, continue em (*)

h) $\boxed{\text{Substituição } x=\operatorname{sh} y}$ $\Longleftrightarrow y = \operatorname{argsh} x \qquad dx = \operatorname{ch} y\,dy$

$$\int \sqrt{1+x^2}\,dx = \int \sqrt{1+\operatorname{sh}^2 y}\,\operatorname{ch} y\,dy = \int \operatorname{ch}^2 y\,dy = \int \frac{1+\operatorname{ch}(2y)}{2}\,dy$$

$$= \frac{y}{2} + \frac{\operatorname{sh}(2y)}{4} + C = \frac{1}{2}\operatorname{argsh} x + \frac{1}{4}\operatorname{sh}(2\operatorname{argsh} x) + C,\ C\in\mathbb{R}$$

b) $\boxed{\text{Substituição } \operatorname{sen} x=t}$ $\Longleftrightarrow x = \operatorname{arcsen} t \qquad dx = \frac{1}{\sqrt{1-t^2}}\,dt$

$$\int \frac{1}{\operatorname{sen} x}\,dx = \int \frac{1}{t\sqrt{1-t^2}}\,dt = \operatorname{argsech} t + C = \operatorname{argsech}(\operatorname{arcsen} x) + C,\ C\in\mathbb{R}$$

(*) Continuação de (3-g)

$$\frac{y^6}{y^4-1} = \frac{y^2(y^4-1)+y^2}{y^4-1} = y^2 + \frac{y^2}{y^4-1}$$

$y^4-1 = (y^2-1)(y^2+1)$
$\quad = (y-1)(y+1)(y^2+1)$

Zeros de y^4-1:
- -1 c/ multiplicidade 1
- 1 c/ " 1
- i " " 1
- $-i$ " " 1

$$\frac{y^2}{y^4-1} = \frac{A}{y-1} + \frac{B}{y+1} + \frac{Cy+D}{y^2+1}$$

$$= \frac{A(y+1)(y^2+1)+B(y-1)(y^2+1)+(Cy+D)(y-1)(y+1)}{y^4-1}$$

$\Longrightarrow y^2 = A(y+1)(y^2+1) + B(y-1)(y^2+1) + (Cx+D)(y-1)(y+1)$

$y=-1 \Longrightarrow 1 = -4B \Longleftrightarrow B = -1/4$

$y=1 \Longrightarrow 1 = 4A \Longleftrightarrow A = 1/4$

$y=i \Longrightarrow -1 = (Ci+D)(i-1)(i+1) \Longleftrightarrow -1 = (Ci+D)(i^2-1)$

$\Longleftrightarrow -1 = -2Ci - 2D \Longleftrightarrow \begin{cases} -2C = 0 \\ -2D = -1 \end{cases} \begin{cases} C = 0 \\ D = 1/2 \end{cases}$

Então

$$\int \frac{y^6}{y^4-1}\,dy = \int y^2 dy + \frac{1}{4}\int \frac{dy}{y-1} - \frac{1}{4}\int \frac{dy}{y+1} + \frac{1}{2}\int \frac{dy}{y^2+1}$$
$$= \frac{y^3}{3} + \frac{1}{4}\ln|y-1| - \frac{1}{4}\ln|y+1| + \frac{1}{2}\,\mathrm{arctg}\,y + C,\ C\in\mathbb{R}$$

$$\int \frac{\sqrt{x}}{x-\sqrt[3]{x}}\,dx = 2y^3 + \frac{3}{2}\ln|y-1| - \frac{3}{2}\ln|y+1| + 3\,\mathrm{arctg}\,y + C$$
$$= 2x^{1/2} + \frac{3}{2}\ln|x^{1/6}-1| - \frac{3}{2}\ln|x^{1/6}+1| + 3\,\mathrm{arctg}(x^{1/6}) + C,\ C\in\mathbb{R}$$

5 a) $\boxed{\text{Substituição } \sqrt{x}=y}$ $\Leftrightarrow x=y^2$ $\quad dx=2y\,dy$

$$\int \frac{1}{(2+\sqrt{x})^7\sqrt{x}}\,dx = \int \frac{1}{(2+y)^7\cancel{y}}\,2\cancel{y}\,dy = 2\int (2+y)^{-7}dy = 2\,\frac{(2+y)^{-6}}{-6} + C$$
$$= \frac{-1}{3(2+\sqrt{x})^6} + C,\ C\in\mathbb{R}$$

b) $\int \mathrm{tg}^2 x\,dx = \int (\sec^2 x - 1)\,dx = \mathrm{tg}\,x - x + C,\ C\in\mathbb{R}$

c) $\boxed{\text{Substituição } \mathrm{arcsen}(3x)=y}$ $\Leftrightarrow 3x = \mathrm{sen}\,y$ $\quad dx = \frac{1}{3}\cos y\,dy$

$$\int \frac{x+(\mathrm{arcsen}(3x))^2}{\sqrt{1-9x^2}}\,dx = \int \frac{\frac{1}{3}\,\mathrm{sen}\,y + y^2}{\cancel{\sqrt{1-\mathrm{sen}^2 y}}}\cdot\frac{1}{3}\cancel{\cos y}\,dy$$
$$= \frac{1}{9}\int \mathrm{sen}\,y\,dy + \frac{1}{3}\int y^2 dy = -\frac{1}{9}\cos y + \frac{1}{9}y^3 + C = -\frac{1}{9}\cos(\mathrm{arcsen}(3x)) + \frac{1}{9}(\mathrm{arcsen}(3x))^3 + C,\ C\in\mathbb{R}$$

d) $\boxed{\text{Substituição } 1-x^2=y^2}$ $\Leftrightarrow y=\sqrt{1-x^2}$ $\quad -\cancel{2}x\,dx = \cancel{2}y\,dy$

$$\int \frac{e^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}\,x\,dx = \int \frac{e^y}{\cancel{y}}(-\cancel{y}\,dy) = -e^y + C = -e^{\sqrt{1-x^2}} + C,\ C\in\mathbb{R}$$

e) $$\int \frac{1}{\cos^2 x\,\mathrm{sen}^2 x}\,dx = \int \frac{\cos^2 x + \mathrm{sen}^2 x}{\cos^2 x\,\mathrm{sen}^2 x}\,dx = \int \frac{1}{\mathrm{sen}^2 x}\,dx + \int \frac{1}{\cos^2 x}\,dx$$
$$= \int \mathrm{cosec}^2 x\,dx + \int \sec^2 x\,dx = -\mathrm{cotg}\,x + \mathrm{tg}\,x + C,\ C\in\mathbb{R}$$

f) $$\int \cos^2 x\,\mathrm{sen}^2 x\,dx = \int \left(\frac{1}{2}\,\mathrm{sen}(2x)\right)^2 dx = \frac{1}{4}\int \mathrm{sen}^2(2x)\,dx$$
$$= \frac{1}{4}\int \frac{1-\cos(4x)}{2}\,dx = \frac{1}{8}x - \frac{\mathrm{sen}(4x)}{32} + C,\ C\in\mathbb{R}$$

g) $\boxed{\text{Substituição } e^x=y}$ $\Leftrightarrow x=\ln y$ $\quad dx=\frac{1}{y}\,dy$

$$\int \frac{1}{1+e^x}\,dx = \int \frac{1}{1+y}\,\frac{1}{y}\,dy = -\int \frac{1}{1+y}\,dy + \int \frac{1}{y}\,dy = -\ln|1+y| + \ln|y| + C$$
$$= -\ln(1+e^x) + \underbrace{\ln(e^x)}_{=x} + C,\ C\in\mathbb{R}$$

$$\frac{1}{(1+y)y} = \frac{A}{1+y} + \frac{B}{y} \Leftrightarrow 1 = Ay + B(1+y) \qquad y=0 \Rightarrow 1=B$$
$$y=-1 \Rightarrow 1=-A$$

h) $\boxed{\text{Substituição } x = 2\cos y}$ $\qquad dx = -2\,\text{sen}\, y\, dy$

$$\int \frac{1}{x^2\sqrt{4-x^2}}\,dx = \int \frac{1}{4\cos^2 y\,\sqrt{4-4\cos^2 y}}\,(-2\,\cancel{\text{sen}\, y})\,dy$$

$$= -\frac{1}{2}\int \frac{1}{\cos^2 y}\,dy = -\frac{1}{2}\int \sec^2 y\,dy = -\frac{1}{2}\,\text{tg}\, y + C = -\frac{1}{2}\,\text{tg}\left(\arccos\frac{x}{2}\right) + C,\ C\in\mathbb{R}$$

6 $f(x) = x^2\,\text{sen}\, x$

Calculemos as primitivas de f:

$$F_C(x) = \int x^2\,\text{sen}\, x\,dx = -x^2\cos x + 2\int x\cos x = -x^2\cos x + 2\left(x\,\text{sen}\, x - \int \text{sen}\, x\,dx\right)$$

$f = x^2 \quad f' = 2x \qquad\qquad f = x \quad f' = 1$

$g' = \text{sen}\, x \quad g = -\cos x \qquad\qquad g' = \cos x \quad g = \text{sen}\, x$

$$= -x^2\cos x + 2x\,\text{sen}\, x + 2\cos x + C,\quad C\in\mathbb{R}$$

Queremos a primitiva de f que passa em $(\frac{\pi}{2}, \pi)$, isto é,

$$\pi = F_C\left(\frac{\pi}{2}\right) = -\frac{\pi^2}{4}\underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} + \pi\underbrace{\text{sen}\left(\frac{\pi}{2}\right)}_{=1} + 2\underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0} + C \Leftrightarrow \pi = \pi + C \Leftrightarrow C = 0$$

Então a primitiva pretendida é $F_0(x) = -x^2\cos x + 2x\,\text{sen}\, x + 2\cos x$

7 a) $f''(x) = 4x - 1 \Rightarrow f'(x) = 2x^2 - x + C_1 \Rightarrow f(x) = \frac{2x^3}{3} - \frac{x^2}{2} + C_1 x + C_2$, $C_1, C_2 \in \mathbb{R}$

$f'(2) = -2 \Rightarrow -2 = 8 - 2 + C_1 \Rightarrow C_1 = -8$

$f(1) = 3 \Rightarrow 3 = \frac{2}{3} - \frac{1}{2} - 8 + C_2 \Rightarrow C_2 = 11 - \frac{1}{6}$

Então $f(x) = \frac{2x^3}{3} - \frac{x^2}{2} - 12x + \frac{65}{6}$, $x\in\mathbb{R}$

b) $f''(x) = \frac{1}{2}\,\text{sen}(2x) \Rightarrow f'(x) = -\frac{1}{4}\cos(2x) + C_1 \Rightarrow f(x) = -\frac{1}{8}\,\text{sen}(2x) + C_1 x + C_2$

$f(0) = 0 \Rightarrow C_2 = 0 \qquad f'(0) = 1 \Rightarrow 1 = -\frac{1}{4} + C_1 \Rightarrow C_1 = \frac{5}{4}$

Então $f(x) = -\frac{1}{8}\,\text{sen}(2x) + \frac{5}{4}x$, $x\in\mathbb{R}$

8 a) $\int_0^1 e^{\pi x}\,dx = \left[\frac{1}{\pi}e^{\pi x}\right]_0^1 = \frac{1}{\pi}e^{\pi} - \frac{1}{\pi}$

b) $\int_{-\pi/2}^{\pi/2} |\text{sen}\, x|\,dx = \int_{-\pi/2}^{0} -\text{sen}\, x\,dx + \int_0^{\pi/2}\text{sen}\, x\,dx = \left[\cos x\right]_{-\pi/2}^{0} + \left[-\cos x\right]_0^{\pi/2}$

$$= (1 - 0) + (-0 + 1) = 2$$

c) $\int_{-3}^{5} |x-1|\,dx = \int_{-3}^{0}(1-x)\,dx + \int_0^5 (x-1)\,dx = \left[x - \frac{x^2}{2}\right]_{-3}^{0} + \left[\frac{x^2}{2} - x\right]_0^5$

$$= \left(0 - \left(-3 - \frac{9}{2}\right)\right) + \left((16 - 5) - 0\right) = \frac{15}{2} + 11$$

d) $\int_0^2 |(x-1)(3x-2)|\,dx$

$= \int_0^{2/3}(x^2-5x+2)\,dx + \int_{2/3}^{1}(-x^2+5x-2)\,dx +$

$+ \int_1^2 (x^2-5x+2)\,dx = \cdots$

C.A.

$(x-1)(3x-2)\geq 0 \iff 3x^2-2x-3x+2\geq 0$

$3x^2-5x+2=0 \iff x=\frac{5\pm\sqrt{25-24}}{6}$

$\iff x=1 \lor x=\frac{2}{3}$

Então $(x-1)(3x-2)\geq 0 \iff x\notin \left]\frac{2}{3},1\right[$

e) $\int_0^3 \sqrt{9-x^2}\,dx = \int_0^{\pi/2}\sqrt{9-9\,\mathrm{sen}^2 y}\;3\cos y\,dy = 9\int_0^{\pi/2}\cos^2 y\,dy = 9\int_0^{\pi/2}\frac{1+\cos(2y)}{2}\,dy$

$\boxed{\text{Substituição } x=3\,\mathrm{sen}\,y}$

$dx=3\cos y\,dy$

$x=0 \Rightarrow y=0\,,\quad x=3\Rightarrow y=\pi/2$

$= \left[\frac{9}{2}y+\frac{9}{4}\mathrm{sen}(2y)\right]_0^{\pi/2}$

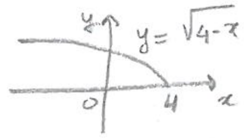
$=\left(\frac{9\pi}{4}+0\right)-(0+0)=\frac{9\pi}{4}$

f) $\int_{-5}^{0} 2x\sqrt{4-x}\,dx = (*)$

Substituição $4-x=y^2$

$-dx=2y\,dy$

notem que $x\in[-5,0]\Rightarrow 4-x\geq 0$ e a mudança de variável é injetiva neste intervalo



$x=-5\Rightarrow y=3$

$x=0\Rightarrow y=2$

$\boxed{\text{Convenção } \int_b^a f(x)\,dx = -\int_a^b f(x)\,dx}$

$(*)=\int_3^2 2(4-y^2)\,y\,(-2y\,dy)$

$=4\int_2^3(4-y)y^2\,dy = 4\left[\frac{4y^3}{3}-\frac{y^4}{4}\right]_2^3 = \left(36-\frac{81}{4}\right)-\left(\frac{32}{3}-4\right)=\cdots$

g) $\int_{3/4}^{4/3}\frac{1}{x^2\sqrt{1+x^2}}\,dx = \int_{\mathrm{argsh}(3/4)}^{\mathrm{argsh}(4/3)}\frac{1}{\mathrm{sh}^2 y\,\cancel{\mathrm{ch}\,y}}\,\cancel{\mathrm{ch}\,y}\,dy = \int_{\mathrm{argsh}(3/4)}^{\mathrm{argsh}(4/3)}\mathrm{cosech}^2 y\,dy =$

$\boxed{\text{Substituição } x=\mathrm{sh}\,y}$ $\quad dx=\mathrm{ch}\,y\,dy$

$x=3/4\Rightarrow y=\mathrm{argsh}\left(\frac{3}{4}\right)\qquad x=\frac{4}{3}\Rightarrow y=\mathrm{argsh}\left(\frac{4}{3}\right)$

$=\left[-\coth y\right]_{\mathrm{argsh}(3/4)}^{\mathrm{argsh}(4/3)} = -\coth(\mathrm{argsh}(4/3))+\coth(\mathrm{argsh}(3/4))$

h) $\int_0^1 1\cdot\ln(x^2+1) = \left[x\ln(x^2+1)\right]_0^1 - 2\int_0^1\frac{x^2+1-1}{x^2+1} = (\ln 2-0)-\left[2x\right]_0^1+\left[2\,\mathrm{arctg}\,x\right]_0^1$

$=\ln 2-2+2\left(\frac{\pi}{4}-0\right)=\ln 2-2+\frac{\pi}{2}$

$f'=1\qquad f=x$

$g=\ln(x^2+1)\qquad g'=\frac{2x}{x^2+1}$

i) $\int_0^2 x^2\,(xe^{x^2})\,dx = \left[\frac{1}{2}x^2e^{x^2}\right]_0^2-\int_0^2 xe^{x^2}\,dx = (2e^4-0)-\left[\frac{1}{2}e^{x^2}\right]_0^2 = 2e^4-\frac{1}{2}e^4+\frac{1}{2}$

$f=x^2\qquad f'=2x$

$g'=xe^{x^2}\qquad g=\frac{1}{2}e^{x^2}$

j) $\int_0^{\pi} x\,\mathrm{sen}\,x\,dx = \left[-x\cos x\right]_0^{\pi}+\int_0^{\pi}\cos x\,dx = (\pi-0)+\left[\mathrm{sen}\,x\right]_0^{\pi}=\pi$

$f=x\qquad f'=1$

$g'=\mathrm{sen}\,x\qquad g=-\cos x$

k) $\int_0^{\sqrt{2}/2} 1.\,\text{arcsen}\,x\,dx = \left[x\,\text{arcsen}\,x\right]_0^{\sqrt{2}/2} - \int_0^{\sqrt{2}/2} x(1-x^2)^{-1/2}dx$

$f'=1 \quad f=x$

$g=\text{arcsen}\,x \quad g'=\dfrac{1}{\sqrt{1-x^2}}$

$$= \left(\frac{\sqrt{2}}{2}\cdot\frac{\pi}{4} - 0\right) + \left[\cancel{\tfrac{1}{2}}\,\frac{(1-x^2)^{1/2}}{\cancel{1/2}}\right]_0^{\sqrt{2}/2} = \frac{\sqrt{2}\pi}{8} + \left(\left(\frac{1}{2}\right)^{1/2} - 1^{1/2}\right)$$

$$= \frac{\sqrt{2}\pi}{8} + \frac{1}{\sqrt{2}} - 1$$

l) $\int_{-3}^{2}\sqrt{|x|}\,dx = \int_{-3}^{0}(-x)^{1/2}dx + \int_0^2 x^{1/2}dx = \left[-\dfrac{(-x)^{3/2}}{3/2}\right]_{-3}^{0} + \left[\dfrac{x^{3/2}}{3/2}\right]_0^2$

$$= \left[-\frac{2}{3}\sqrt{|x|^3}\right]_{-3}^{0} + \left[\frac{2}{3}\sqrt{x^3}\right]_0^2 = \left(-0+\frac{2}{3}\sqrt{27}\right) + \left(\frac{2}{3}\sqrt{8}-0\right)$$

m) $\int_0^2 f(x)dx = \int_0^1 x^2dx + \int_1^2(2-x)dx = \left[\dfrac{x^3}{3}\right]_0^1 + \left[2x-\dfrac{x^2}{2}\right]_1^2 = \left(\dfrac{1}{3}-0\right) + \left((4-2)-(2-1/2)\right)$

$$= \frac{1}{3} + 2 - \frac{3}{2}$$

n) $\int_0^1 g(x)dx = \int_0^{1/2} x\,dx + \int_{1/2}^{1} -x\,dx = \left[\dfrac{x^2}{2}\right]_0^{1/2} + \left[-\dfrac{x^2}{2}\right]_{1/2}^{1} = \left(\dfrac{1}{8}-0\right) + \left(-\dfrac{1}{2}+\dfrac{1}{8}\right) = \dfrac{1}{4}-\dfrac{1}{2} = -\dfrac{1}{4}$

9 a) $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx = \int_a^0 f(-y)(-dy) + \int_0^a f(x)dx = (*)$

Fazendo a mudança de variável $y=-x$ no 1º integral do 2º membro

$(x=-a \Rightarrow y=a,\ x=0 \Rightarrow y=0,\ dy=-dx$

$(*) = \int_0^a f(-y)dy + \int_0^a f(x)dx = \int_0^a f(y)dy + \int_0^a f(x)dx = 2\int_0^a f(x)dx$

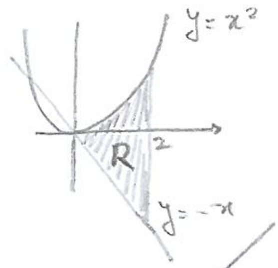
f é par

b) $\int_{-a}^{a} f(x)dx = \int_0^a f(-y)dy + \int_0^a f(x)dx = \int_0^a -f(y)dy + \int_0^a f(x)dx = 0$

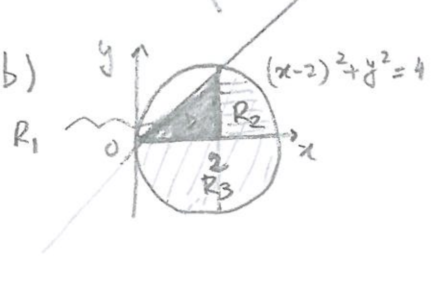
como na alínea a) f é ímpar

Exercícios 10, 11, 12 e 13 excluídos

14 a) Área $(R) = \int_0^2 (x^2-(-x))\,dx$



b)



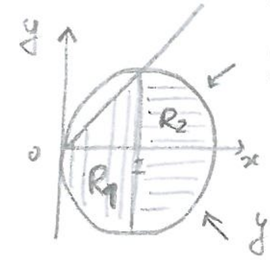
Área (R) = Área (R_1) + Área (R_2) + Área (R_3)

$$= \frac{2\times 2}{2} + \frac{1}{4}\pi 2^2 + \frac{1}{2}\pi 2^2$$

$$= 2 + \pi + 2\pi = 2 + 3\pi$$

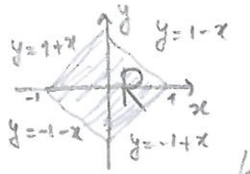
04

$(x-2)^2 + y^2 = 4 \iff y = \pm\sqrt{4-(x-2)^2}$



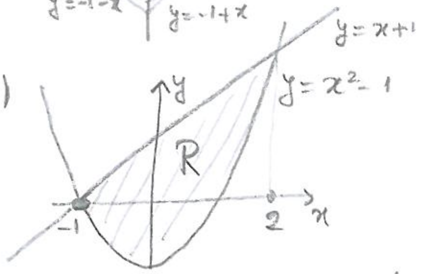
$$\text{Área}(R) = \text{Área}(R_1) + \text{Área}(R_2)$$
$$= \int_0^2 \left(x - \left(-\sqrt{4-(x-2)^2}\right)\right)dx +$$
$$+ \int_2^4 \left(\sqrt{4-(x-2)^2} - \left(-\sqrt{4-(x-2)^2}\right)\right)dx = \cdots$$

c)



$$\text{Área}(R) = \int_{-1}^{0} (1+x-(-1-x))\,dx + \int_0^1 (1-x-(-1-x))\,dx$$

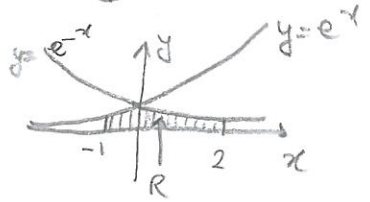
d)



$$\begin{cases} y = x+1 \\ y = x^2-1 \end{cases} \begin{cases} x^2-1-x-1=0 \\ - \end{cases} \begin{cases} x^2-x-2=0 \\ - \end{cases} \begin{cases} x = \frac{1\pm\sqrt{9}}{2} = \left\langle \begin{matrix} 2 \\ -1 \end{matrix} \right. \end{cases}$$

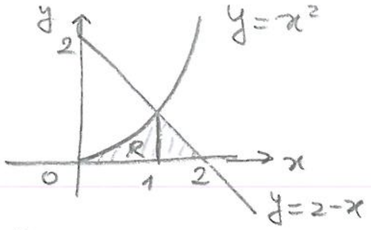
$$\text{Área}(R) = \int_{-1}^{2} (x+1-(x^2-1))\,dx = \cdots$$

e)



$$\text{Área}(R) = \int_{-1}^{0} (e^x - 0)\,dx + \int_0^2 (e^{-x} - 0)\,dx$$

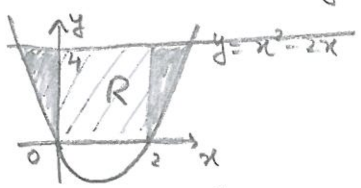
f)



$$\begin{cases} y = x^2 \\ y = 2-x \end{cases} \begin{cases} - \\ x^2+x-2=0 \end{cases} \begin{cases} x = \frac{-1\pm\sqrt{1+8}}{2} = \left\langle \begin{matrix} 1 \\ -2 \end{matrix} \right. \end{cases}$$

$$\text{Área}(R) = \int_0^1 (x^2-0)\,dx + \int_1^2 (2-x-0)\,dx$$

g)



$$\begin{cases} y = x^2-2x \\ y = 4 \end{cases} \begin{cases} x^2-2x-4=0 \\ - \end{cases} \begin{cases} x = \frac{2\pm\sqrt{4+16}}{2} = \frac{2\pm 2\sqrt{5}}{2} = 1\pm\sqrt{5} \end{cases}$$

$$\text{Área}(R) = \int_{1-\sqrt{5}}^{0} (4-(x^2-2x))\,dx + \int_0^2 (4-0)\,dx + \int_2^{1+\sqrt{5}} (4-(x^2-2x))\,dx$$

14-h) Excluído