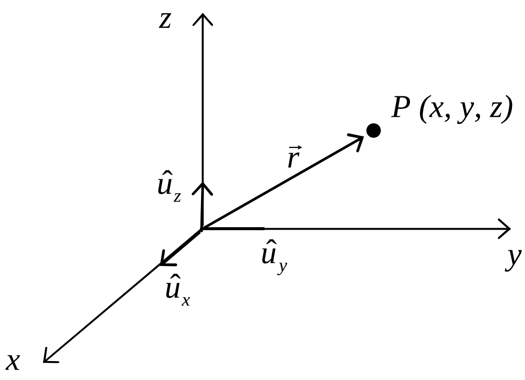
# COORDENADAS CARTESIANAS

**Coordenadas cartesianas** $(x, y, z)$




**vector de posição**

$\vec{r} = x\hat{u}_x + y\hat{u}_y + z\hat{u}_z$

**elemento de comprimento**

$d\vec{l} = dx\hat{u}_x + dy\hat{u}_y + dz\hat{u}_z$

**elementos de superfície**

$ds_x = dydz$ (superfície perpendicular a $\hat{u}_x$)

$ds_y = dxdz$

$ds_z = dxdy$

**elemento de volume**

$dv = dxdydz$

**gradiente**

$$grad\, V = \nabla V = \frac{\partial V}{\partial x}\hat{u}_x + \frac{\partial V}{\partial y}\hat{u}_y + \frac{\partial V}{\partial z}\hat{u}_z$$

**divergência**

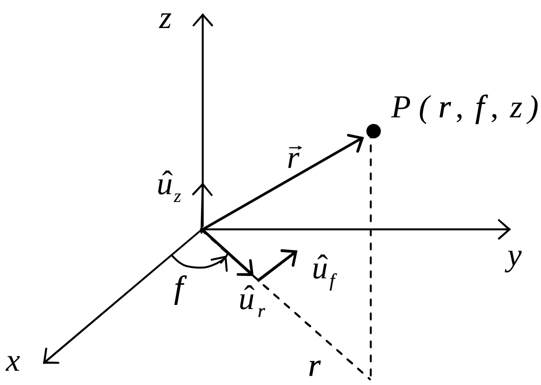
$$div\ \vec{A} = \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

**rotacional**

$$rot\ \vec{A} = \nabla \times \vec{A} = \begin{vmatrix} \hat{u}_x & \hat{u}_y & \hat{u}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{u}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{u}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{u}_z$$

## COORDENADAS CILÍNDRICAS

**Coordenadas cilíndricas** $( r , f , z )$




$x = r \cos f$

$y = r \sin f$

**versores**

$\hat{u}_r = \cos f \, \hat{u}_x + \sin f \, \hat{u}_y$

$\hat{u}_f = -\sin f \, \hat{u}_x + \cos f \, \hat{u}_y$

**vector de posição**

$\vec{r} = r \, \hat{u}_r + z \, \hat{u}_z$

**elemento de comprimento**

$d\vec{l} = dr \, \hat{u}_r + r \, df \, \hat{u}_f + dz \, \hat{u}_z$

**elementos de superfície**

$ds_r = r\, df\, dz$

$ds_f = dr\, dz$

$ds_z = r\, dr\, df$

**elemento de volume**

$dv = r\, dr\, df\, dz$

**gradiente**

$$grad\, V = \nabla V = \frac{\partial V}{\partial r}\hat{u}_r + \frac{1}{r}\frac{\partial V}{\partial f}\hat{u}_f + \frac{\partial V}{\partial z}\hat{u}_z$$

**divergência**

$$div\ \vec{A} = \nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial}{\partial r}\left(rA_r\right) + \frac{1}{r}\frac{\partial A_f}{\partial f} + \frac{\partial A_z}{\partial z}$$

**rotacional**

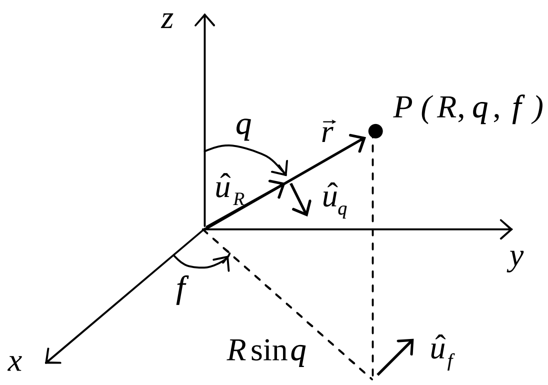
$$rot\ \vec{A} = \nabla \times \vec{A} = \frac{1}{r}\begin{vmatrix} \hat{u}_r & r\,\hat{u}_f & \hat{u}_z \\ \dfrac{\partial}{\partial r} & \dfrac{\partial}{\partial f} & \dfrac{\partial}{\partial z} \\ A_r & r\,A_f & A_z \end{vmatrix} =$$

$$= \left(\frac{1}{r}\frac{\partial A_z}{\partial f} - \frac{\partial A_f}{\partial z}\right)\hat{u}_r + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\hat{u}_f + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(rA_f\right) - \frac{\partial A_r}{\partial f}\right]\hat{u}_z$$

**NOTA:** $0 \le f \le 2p$

# COORDENADAS ESFÉRICAS

**Coordenadas esféricas** $( R, q , f )$




$x = R \sin q \cos f$

$y = R \sin q \sin f$

$z = R \cos q$

**versores**

$\hat{u}_R = \sin q \cos f \, \hat{u}_x + \sin q \sin f \, \hat{u}_y + \cos q \, \hat{u}_z$

$\hat{u}_q = \cos q \cos f \, \hat{u}_x + \cos q \sin f \, \hat{u}_y - \sin q \, \hat{u}_z$

$\hat{u}_f = - \sin f \, \hat{u}_x + \cos f \, \hat{u}_y$

**vector de posição**

$\vec{r} = R \hat{u}_R$

**elemento de comprimento**

$$d\vec{l} = dR\,\hat{u}_R + R\,dq\,\hat{u}_q + R\sin q\,df\,\hat{u}_f$$

**elementos de superfície**

$$ds_R = R^2 \sin q\,dq\,df$$

$$ds_q = R\sin q\,dR\,df$$

$$ds_f = R\,dR\,dq$$

**elemento de volume**

$$dv = R^2 \sin q\,dR\,dq\,df$$

**gradiente**

$$grad\,V = \nabla V = \frac{\partial V}{\partial R}\hat{u}_R + \frac{1}{R}\frac{\partial V}{\partial q}\hat{u}_q + \frac{1}{R\sin q}\frac{\partial V}{\partial f}\hat{u}_f$$

**divergência**

$$div\,\vec{A} = \nabla\cdot\vec{A} = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2 A_R\right) + \frac{1}{R\sin q}\frac{\partial}{\partial q}\left(\sin q\;A_q\right) + \frac{1}{R\sin q}\frac{\partial A_i}{\partial f}$$

**rotacional**

$$rot\,\vec{A} = \nabla\times\vec{A} = \frac{1}{R^2\sin q}\begin{vmatrix} \hat{u}_R & R\,\hat{u}_q & R\sin q\,\hat{u}_f \\ \dfrac{\partial}{\partial R} & \dfrac{\partial}{\partial q} & \dfrac{\partial}{\partial f} \\ A_R & R\,A_q & R\sin q\;A_f \end{vmatrix} = \frac{1}{R\sin q}\left[\frac{\partial}{\partial q}\left(\sin q\;A_f\right) - \frac{\partial A_q}{\partial f}\right]\hat{u}_R +$$

$$+\frac{1}{R}\left[\frac{1}{\sin q}\frac{\partial A_R}{\partial f} - \frac{\partial}{\partial R}\left(R\,A_f\right)\right]\hat{u}_q + \frac{1}{R}\left[\frac{\partial}{\partial R}\left(R\,A_q\right) - \frac{\partial A_R}{\partial q}\right]\hat{u}_f$$

**NOTA:** $0 \le q \le p$

$0 \le f \le 2p$