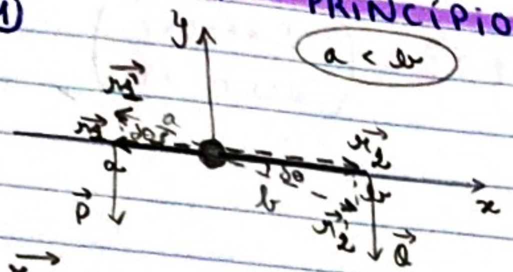
MECÂNICA ANALÍTICA E ONDAS

SÉRIE 1 - PRINCÍPIO DOS TRABALHOS VIRTUAIS

①



$$\sum_i \vec{F_i} \cdot \delta \vec{r_i} = 0$$

$$\vec{F_1} = -P\vec{e_y} = \begin{pmatrix} 0 \\ -P \end{pmatrix}$$

$$\vec{r_1} = -a\,\vec{e_x} = \begin{pmatrix} -a \\ 0 \end{pmatrix} \qquad \vec{r_{1'}} = -a\cos(\delta\theta)\vec{e_x} + a\sin(\delta\theta)\vec{e_y} =$$

$$= \begin{pmatrix} -a\cos(\delta\theta) \\ a\sin(\delta\theta) \end{pmatrix}$$

$$\delta\vec{r_1} = \vec{r_{1'}} - \vec{r_1} = \begin{pmatrix} -a\cos(\delta\theta) \\ a\sin(\delta\theta) \end{pmatrix} - \begin{pmatrix} -a \\ 0 \end{pmatrix} =$$

$$= a(1-\underbrace{\cos(\delta\theta)}_{=1})\vec{e_x} + (a\underbrace{\sin(\delta\theta)}_{=\delta\theta})\vec{e_y} =$$

$$= a\,\delta\theta\,\vec{e_y} = \begin{pmatrix} 0 \\ a\,\delta\theta \end{pmatrix}$$

$$\vec{F_1}\cdot\delta\vec{r_1} = -P a\,\delta\theta$$

$$\vec{F_2} = -Q\vec{e_y} = \begin{pmatrix} 0 \\ -Q \end{pmatrix} \qquad \vec{r_2} = b\,\vec{e_x} = \begin{pmatrix} b \\ 0 \end{pmatrix}$$

$$\vec{r_{2'}} = b\cos(\delta\theta)\vec{e_x} - b\sin(\delta\theta)\vec{e_y} =$$

$$= b\,\vec{e_x} - b\,\delta\theta\,\vec{e_y} = \begin{pmatrix} b \\ -b\,\delta\theta \end{pmatrix}$$

$$\delta\vec{r_2} = \vec{r_{2'}} - \vec{r_2} = \begin{pmatrix} b \\ -b\,\delta\theta \end{pmatrix} - \begin{pmatrix} b \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -b\,\delta\theta \end{pmatrix}$$

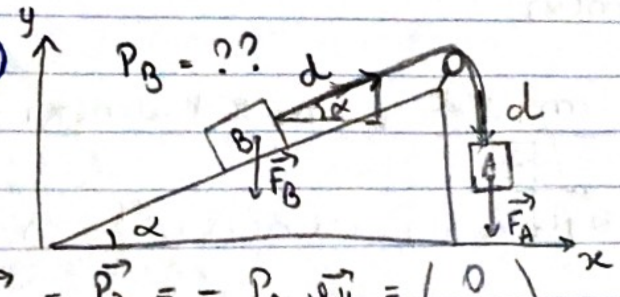
$$\vec{F_2}\cdot\delta\vec{r_2} = \begin{pmatrix} 0 \\ -Q \end{pmatrix}\cdot\begin{pmatrix} 0 \\ -b\,\delta\theta \end{pmatrix} = Q b\,\delta\theta$$

Logo,

$$\vec{F_1}\cdot\delta\vec{r_1} + \vec{F_2}\cdot\delta\vec{r_2} = 0 \Leftrightarrow$$

$$\Leftrightarrow -Pa\cancel{\delta\theta} + Qb\cancel{\delta\theta} = 0 \Leftrightarrow \underbrace{Pa = Qb}_{\text{condição de equilíbrio.}}$$

②



$$\sum_i \vec{F_i}\cdot\delta\vec{r_i} = 0$$

$$\vec{F_A} = \vec{P_1} = -P_1\vec{e_y} = \begin{pmatrix} 0 \\ -P_1 \end{pmatrix}$$

$$\vec{F_B} = \vec{P_2} = -P_2\vec{e_y} = \begin{pmatrix} 0 \\ -P_2 \end{pmatrix}$$

→

$$\delta\vec{r_1} = -d\,\vec{e_y} = \begin{pmatrix} 0 \\ -d \end{pmatrix}$$

$$\delta\vec{r_2} = d\cos(\alpha)\,\vec{e_x} + d\sin(\alpha)\,\vec{e_y} = \begin{pmatrix} d\cos(\alpha) \\ d\sin(\alpha) \end{pmatrix}$$

$$\vec{F_A}\cdot\delta\vec{r_1} = P_1 d$$

$$\vec{F_B}\cdot\delta\vec{r_2} = -P_2 d \sin(\alpha)$$

Logo,

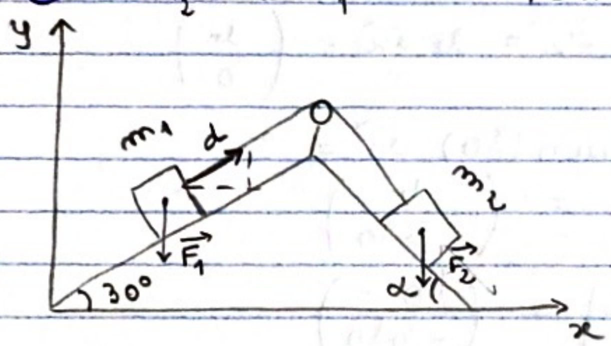
$$\vec{F_A}\cdot\delta\vec{r_1} + \vec{F_B}\cdot\delta\vec{r_2} = 0 \Leftrightarrow$$

$$\Leftrightarrow P_A \cancel{d} - P_B \cancel{d}\sin(\alpha) = 0 \Leftrightarrow P_A = P_B \sin(\alpha) \Leftrightarrow$$

$$\Leftrightarrow \underbrace{P_B = \frac{P_A}{\sin(\alpha)}}_{}$$

P_B para o sistema estar em equilíbrio

③ $m_2 = 2m_1$ $\qquad P = mg$



$$\sum_i \vec{F_i}\cdot\delta\vec{r_i} = 0$$

$$\vec{F_1} = \vec{P_1} = -m_1 g\,\vec{e_y} = \begin{pmatrix} 0 \\ -m_1 g \end{pmatrix}$$

$$\vec{F_2} = \vec{P_2} = -m_2 g\,\vec{e_y} = \begin{pmatrix} 0 \\ -m_2 g \end{pmatrix}$$

$$\delta\vec{r_1} = d\cos(30^\circ)\,\vec{e_x} + d\sin(30^\circ)\,\vec{e_y} =$$

$$= \frac{\sqrt{3}d}{2}\,\vec{e_x} + \frac{d}{2}\,\vec{e_y} = \begin{pmatrix} \frac{\sqrt{3}}{2}d \\ \frac{d}{2} \end{pmatrix}$$

$$\delta\vec{r_2} = d\cos(\alpha)\,\vec{e_x} - d\sin(\alpha)\,\vec{e_y} = \begin{pmatrix} d\cos(\alpha) \\ -d\sin(\alpha) \end{pmatrix}$$

$$\vec{F_1}\cdot\delta\vec{r_1} = -\frac{m_1 g d}{2}$$

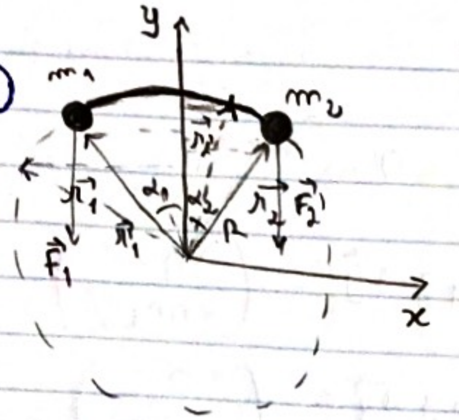
$$\vec{F_2}\cdot\delta\vec{r_2} = +m_2 g d \sin(\alpha)$$

$$\vec{F_1}\cdot\delta\vec{r_1} + \vec{F_2}\cdot\delta\vec{r_2} = 0 \Leftrightarrow \frac{-m_1 \cancel{g}\cancel{d}}{2} + m_2 \cancel{g}\cancel{d}\sin(\alpha) = 0 \Leftrightarrow$$

$$\Leftrightarrow -\frac{\cancel{m_1}}{2} + 2\cancel{m_1}\sin(\alpha) = 0 \Leftrightarrow \sin(\alpha) = \frac{1}{4} \Leftrightarrow$$

$\Leftrightarrow \alpha \simeq 0,253$ rad $\longrightarrow$ para o sistema estar em equilíbrio

4



$$\vec{F_1} = \vec{P_1} = -m_1 g \, \vec{e_y} = \begin{pmatrix} 0 \\ -m_1 g \end{pmatrix}$$

$$\vec{F_2} = \vec{P_2} = -m_2 g \, \vec{e_y} = \begin{pmatrix} 0 \\ -m_2 g \end{pmatrix}$$

$$\alpha_1' = \alpha_1 + \delta\alpha$$
$$\alpha_2' = \alpha_2 - \delta\alpha$$

$$\vec{r_2} = R\sin(\alpha_2)\,\vec{e_x} + R\cos(\alpha_2)\,\vec{e_y} = \begin{pmatrix} R\sin(\alpha_2) \\ R\cos(\alpha_2) \end{pmatrix}$$

$$\vec{r_1} = -R\sin(\alpha_1)\,\vec{e_x} + R\cos(\alpha_1)\,\vec{e_y} = \begin{pmatrix} -R\sin(\alpha_1) \\ R\cos(\alpha_1) \end{pmatrix}$$

$$\vec{r_1'} = -R\sin(\alpha_1')\,\vec{e_x} + R\cos(\alpha_1')\,\vec{e_y} =$$
$$= -R\sin(\alpha_1 + \delta\alpha)\,\vec{e_x} + R\cos(\alpha_1+\delta\alpha)\,\vec{e_y} =$$
$$= -R(\sin(\alpha_1) + \cos(\alpha_1)\delta\alpha)\,\vec{e_x} + R(\cos(\alpha_1) - \sin(\alpha_1)\delta\alpha)\,\vec{e_y} =$$
$$= \begin{pmatrix} -R(\sin(\alpha_1) + \cos(\alpha_1)\delta\alpha) \\ R(\cos(\alpha_1) - \sin(\alpha_1)\delta\alpha) \end{pmatrix}$$

$$\delta\vec{r_1} = \vec{r_1'} - \vec{r_1} = -R\cos(\alpha_1)\delta\alpha\,\vec{e_x} - R\sin(\alpha_1)\delta\alpha\,\vec{e_y}$$

$$\vec{r_2'} = R\sin(\alpha_2')\,\vec{e_x} + R\cos(\alpha_2')\,\vec{e_y} =$$
$$= R\sin(\alpha_2 - \delta\alpha)\,\vec{e_x} + R\cos(\alpha_2 - \delta\alpha)\,\vec{e_y} =$$
$$= R(+\sin(\alpha_2) - \cos(\alpha_2)\delta\alpha)\,\vec{e_x} + R(\cos(\alpha_2) + \sin(\alpha_2)\delta\alpha)\,\vec{e_y} =$$
$$= \begin{pmatrix} R(\sin(\alpha_2) - \cos(\alpha_2)\delta\alpha) \\ R(\cos(\alpha_2) + \sin(\alpha_2)\delta\alpha) \end{pmatrix}$$

$$\delta\vec{r_2} = -R\cos(\alpha_2)\delta\alpha\,\vec{e_x} + R\sin(\alpha_2)\delta\alpha\,\vec{e_y}$$

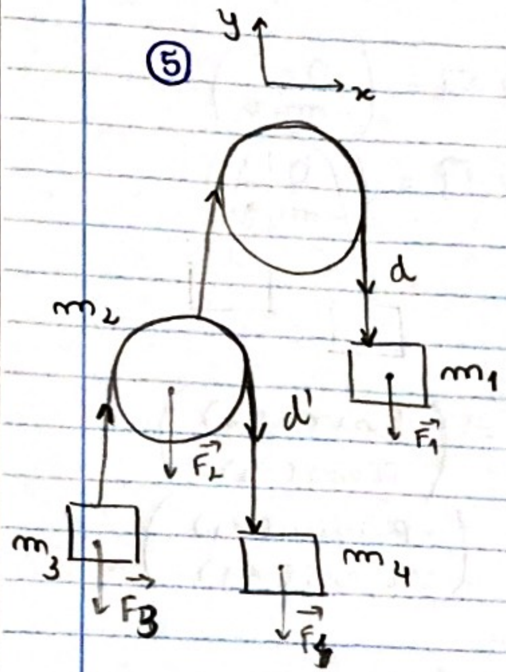
$$\vec{F_1}\cdot\delta\vec{r_1} = m_1 g R\sin(\alpha_1)\delta\alpha$$
$$\vec{F_2}\cdot\delta\vec{r_2} = -m_2 g R\sin(\alpha_2)\delta\alpha$$

$$\vec{F_1}\cdot\delta\vec{r_1} + \vec{F_2}\cdot\delta\vec{r_2} = 0 \Leftrightarrow m_1 \cancel{g}\cancel{R}\sin(\alpha_1)\cancel{\delta\alpha} = m_2\cancel{g}\cancel{R}\sin(\alpha_2)\cancel{\delta\alpha} \Leftrightarrow$$

$$\Leftrightarrow \underbrace{m_1\sin(\alpha_1) = m_2\sin(\alpha_2)}$$

condição para o sistema estar em equilíbrio.

⑤



$$\sum_i \vec{F_i} \cdot \delta \vec{r_i} = 0$$

$$\vec{F_1} = \vec{P_1} = -m_1 g \vec{e_y} = \begin{pmatrix} 0 \\ -m_1 g \end{pmatrix}$$

$$\vec{F_2} = \vec{P_2} = -m_2 g \vec{e_y} = \begin{pmatrix} 0 \\ -m_2 g \end{pmatrix}$$

$$\vec{F_3} = \vec{P_3} = -m_3 g \vec{e_y} = \begin{pmatrix} 0 \\ -m_3 g \end{pmatrix}$$

$$\vec{F_4} = \vec{P_4} = -m_4 g \vec{e_y} = \begin{pmatrix} 0 \\ -m_4 g \end{pmatrix}$$

$$\delta \vec{r_1} = -d \vec{e_y} = \begin{pmatrix} 0 \\ -d \end{pmatrix} \qquad \delta \vec{r_2} = d \vec{e_y} = \begin{pmatrix} 0 \\ d \end{pmatrix}$$

$$\delta \vec{r_3} = (d + d') \vec{e_y} = \begin{pmatrix} 0 \\ d + d' \end{pmatrix}$$

$$\delta \vec{r_4} = (d - d') \vec{e_y} = \begin{pmatrix} 0 \\ d - d' \end{pmatrix}$$

$$\vec{F_1} \cdot \delta \vec{r_1} + \vec{F_2} \cdot \delta \vec{r_2} + \vec{F_3} \cdot \delta \vec{r_3} + \vec{F_4} \cdot \delta \vec{r_4} = 0 \Leftrightarrow$$

$$\Leftrightarrow m_1 \cancel{g} d - m_2 \cancel{g} d - m_3 \cancel{g} (d + d') - m_4 \cancel{g} (d - d') = 0 \Leftrightarrow$$

$$\Leftrightarrow d (m_1 - m_2 - m_3 - m_4) \quad d' (-m_3 + m_4) = 0$$

Logo,

$$\left.\begin{aligned} m_1 &= m_2 + m_3 + m_4 \\ m_3 &= m_4 \end{aligned}\right\}$$ para o sistema estar em equilíbrio.