2º TESTE DE CÁLCULO B

MIEPOL 11/01/2012

CORREÇÃO

Exercício 1

$$\frac{2x}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$$

$$= \frac{(Ax+B)(x-1)^2 + C(x^2+1)(x-1) + D(x^2+1)}{(x^2+1)(x-1)^2}$$

$$\Leftrightarrow \quad 2x = (Ax+B)(x-1)^2 + C(x^2+1)(x-1) + D(x^2+1)$$

$\boxed{x=1}$ $\quad 2 = 2D \Leftrightarrow D = 1$

$\boxed{x=i}$ $\quad 2i = (Ai+B)(i-1)^2 \Leftrightarrow 2i = (Ai+B)(i^2-2i+1)$

$\Leftrightarrow 2i = (Ai+B)(-2i) \Leftrightarrow Ai+B = -1 \Leftrightarrow B = -1,\ A = 0$

$\boxed{x=0}$ $\quad 0 = -1 - C + 1 \Leftrightarrow C = 0$

Então

$$\int \frac{2x}{(x^2+1)(x-1)^2}\,dx = \int \frac{-1}{x^2+1}\,dx + \int \frac{1}{(x-1)^2}\,dx$$

$$= -\operatorname{arctg} x + \frac{(x-1)^{-1}}{-1} + C$$

$$= -\operatorname{arctg} x - \frac{1}{x-1} + C, \quad C \in \mathbb{R}$$

Exercício 2

$x = 3\,sh\,t$

$dx = 3\,ch\,t\,dt$

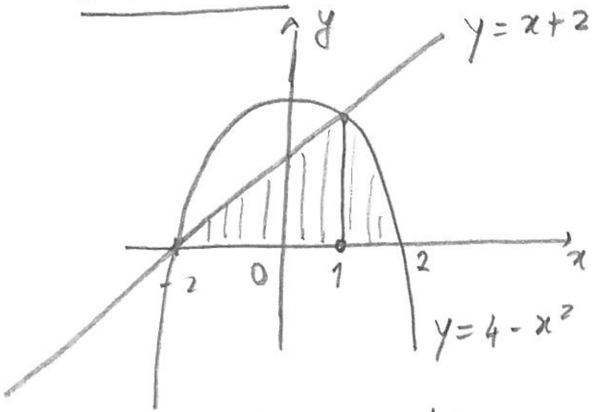
$$\int \frac{x^3}{\sqrt{9+x^2}}\,dx = \int \frac{27\,sh^3 t}{\sqrt{9(1+sh^2 t)}}\,3\,ch\,t\,dt$$

$$= 27 \int \frac{sh^3 t}{\cancel{3\,ch\,t}}\,\cancel{3\,ch\,t}\,dt =$$

$$= 27\int sh\,t\,(ch^2 t - 1)\,dt = 27\int sh\,t\,ch^2 t\,dt - 27\int sh\,t\,dt$$ ②

$$= 27\frac{ch^3 t}{3} - 27ch\,t + C = 9ch^3 t - 27ch\,t + C,\ C\in\mathbb{R}$$

Exercício 3



$$4 - x^2 = x + 2 \Leftrightarrow x^2 + x - 2 = 0$$

$$\Leftrightarrow x = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$\Leftrightarrow x = -2 \ \vee\ x = 1$$

$$\text{Área}(R) = \int_{-2}^{1}\big((x+2)-0\big)\,dx + \int_{1}^{2}\big((4-x^2)-0\big)\,dx$$

$$= \left[\frac{x^2}{2} + 2x\right]_{-2}^{1} + \left[4x - \frac{x^3}{3}\right]_{1}^{2}$$

$$= \left[\frac{5}{2} - (-2)\right] + \left[\frac{16}{3} - \frac{11}{3}\right] = \frac{37}{15}$$

Exercício 4

$$\int_0^1 \frac{x}{\sqrt{1+x^2} - \sqrt{(1+x^2)^3}}\,dx = \int_1^{\sqrt{2}} \frac{t\,dt}{t - t^3}$$

$$1 + x^2 = t^2 \qquad x = 0 \Rightarrow t^2 = 1 \Rightarrow t = 1$$

$$\cancel{2}x\,dx = \cancel{2}t\,dt \qquad x = 1 \Rightarrow t^2 = 2 \Rightarrow t = \sqrt{2}$$

$$= \int_1^{\sqrt{2}} \frac{1}{1-t^2}\,dt = \text{argcoth}\,t\Big]_1^{\sqrt{2}} = \text{argcoth}\sqrt{2} - \text{argcotg}\,1$$

Exercício 5

$$f: \mathbb{R}_0^+ \longrightarrow \mathbb{R}$$
$$t \longmapsto \frac{1}{1+t^3}$$

é contínua, pois é o quociente da função constante igual a 1 pela função polinomial $1+t^2$.

Então, pelo 1º Teorema Fundamental do Cálculo,

$F(x) = \int_0^x \frac{1}{1+t^3}\,dt$ é derivável e $F'(x) = \frac{1}{1+x^3}$.

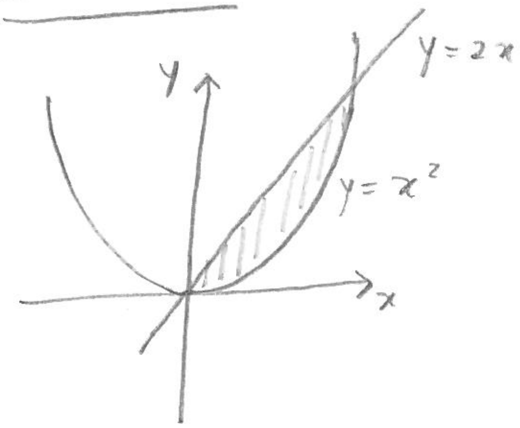
Como $G(x) = F(x^3)$, G é derivável, pois é a composta de duas funções deriváveis.

$$G'(x) = F'(x^3)(x^3)' = F'(x^3)\,3x^2$$
$$= \frac{1}{1+(x^3)^3} \cdot 3x^2 = \frac{3x^2}{1+x^9}$$

Exercício 6

$$L = \int_0^4 \sqrt{1+(ch'x)^2}\,dx = \int_0^4 \sqrt{1+sh^2x}\,dx$$
$$= \int_0^4 ch x\,dx = sh x \Big]_0^4 = sh 4 - sh 0 = sh 4$$

Exercício 7



$2x = x^2 \Leftrightarrow x(x-2) = 0$
$\Leftrightarrow x = 0 \ \vee\ x = 2$

$$V = \int_0^2 \pi\left((2x)^2 - (x^2)^2\right)dx = \pi\int_0^2 4x^2dx - \pi\int_0^2 x^4dx$$
$$= \pi\ \frac{4x^3}{3}\Big]_0^2 - \frac{\pi x^5}{5}\Big]_0^2 = \frac{32\pi}{3} - \frac{32\pi}{5} = \frac{64}{15}\pi$$