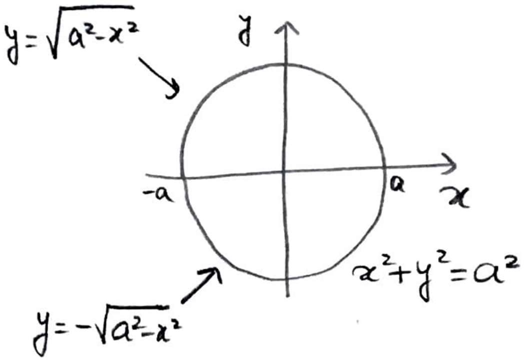
Cálculo da área de um círculo de raio a



$$R=\{(x,y)\in\mathbb{R}^2 : x^2+y^2\leq a^2\}$$
$$R^+=R\cap\{(x,y)\in\mathbb{R}^2 ; y\geq 0\}$$

$$\text{Área}\,(R) = 2\text{Área}\,(R^+) = 2\int_{-a}^{a}\sqrt{a^2-x^2}\,dx$$

- Cálculo de uma primitiva de $f(x)=\sqrt{a^2-x^2}$

$$\int\sqrt{a^2-x^2}\,dx = \int\sqrt{a^2-a^2\cos^2 t}\,(-a\,\text{sen}\,t)\,dt$$

$x = a\cos t$
$dx = -a\,\text{sen}\,t\,dt$
$\Leftrightarrow t=\arccos\left(\frac{x}{a}\right)$

$$= -a^2\int \text{sen}^2 t\,dt = -a^2\int\frac{1-\cos(2t)}{2}\,dt$$
$$= -\frac{a^2}{2}t + \frac{a^2}{4}\,\text{sen}(2t)+C$$
$$= \underbrace{-\frac{a^2}{2}\arccos\left(\frac{x}{a}\right)+\frac{a^2}{4}\,\text{sen}\left(2\arccos\left(\frac{x}{a}\right)\right)}_{F(x)} + \underset{\overset{\shortparallel}{0}}{C}$$

- $\text{Área}\,(R^+) = \int_{-a}\sqrt{a^2-x^2}\,dx = F(a)-F(-a)$

$$F(a) = -\frac{a^2}{2}\arccos(1)+\frac{a^2}{4}\,\text{sen}\,(2\arccos(1)) = -\frac{a^2}{2}\cdot 0+\frac{a^2}{4}\,\text{sen}\,0 = 0$$
$$F(-a) = -\frac{a^2}{2}\arccos(-1)+\frac{a^2}{4}\,\text{sen}\,(2\arccos(-1))$$
$$= -\frac{a^2}{2}\pi+\frac{a^2}{4}\,\text{sen}\,(2\pi) = -\frac{a^2\pi}{2}$$

Então

$$\text{Área}\,(R^+) = 0-\left(-\frac{a^2\pi}{2}\right) = \frac{\pi a^2}{2}$$

e $\text{Área}\,(R) = 2\,\text{Área}\,(R^+) = \pi a^2$