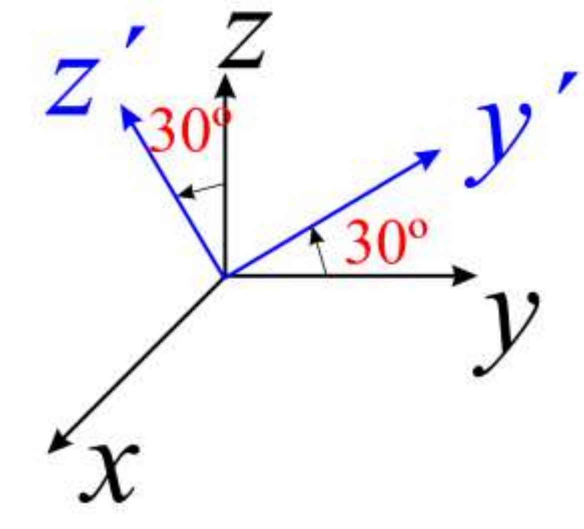
1- Dois referenciais S e S' estão coincidentes no instante $t = 0$. O sistema S' roda de 30 gaus em torno do eixo do x. Qual a matriz que representa a transformação?



$$\left[\hat{R}_x\left(\frac{\pi}{6}\right)\right]^{-1} = \left[\hat{R}_x\left(\frac{\pi}{6}\right)\right]^{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{pmatrix}$$

$$\begin{aligned} \mathbf{e}'_x &= \mathbf{e}_x \\ \mathbf{e}'_y &= \mathbf{e}_y \cos(\pi/6) + \mathbf{e}_z \sin(\pi/6) \\ \mathbf{e}'_z &= -\mathbf{e}_y \sin(\pi/6) + \mathbf{e}_z \cos(\pi/6) \end{aligned}$$

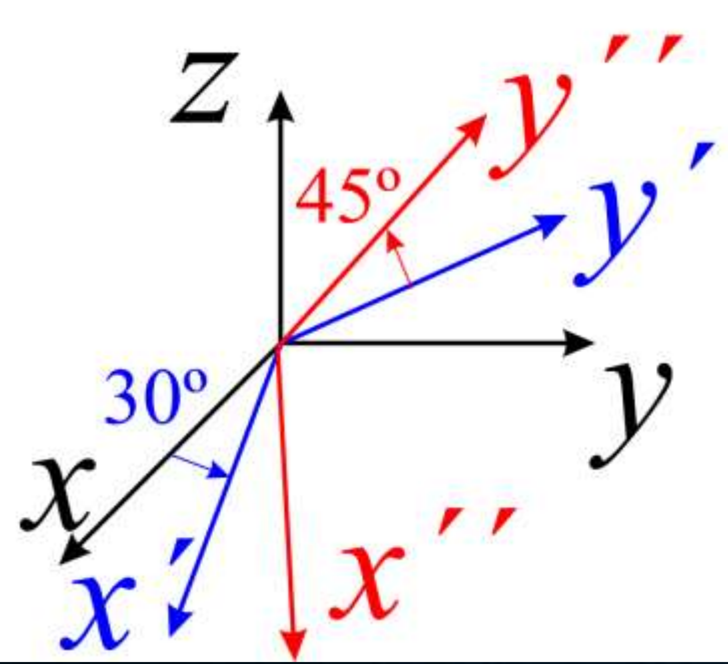
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \hat{R}_x\left(\frac{\pi}{6}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos(x', x) & \cos(x', y) & \cos(x', z) \\ \cos(y', x) & \cos(y', y) & \cos(y', z) \\ \cos(z', x) & \cos(z', y) & \cos(z', z) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

$$= \begin{pmatrix} \cos(0) & \cos(\pi/2) & \cos(\pi/2) \\ \cos(\pi/2) & \cos(\pi/6) & \cos(\pi/3) \\ \cos(\pi/2) & \cos(\pi/2 + \pi/6) & \cos(\pi/6) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}/2 & 1/2 \\ 0 & -1/2 & \sqrt{3}/2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

2- Um referencial S' inicialmente coincidente com um referencial S roda 30 graus em torno do eixo do z e roda em seguida em torno deste mesmo eixo de um ângulo de 45 graus.

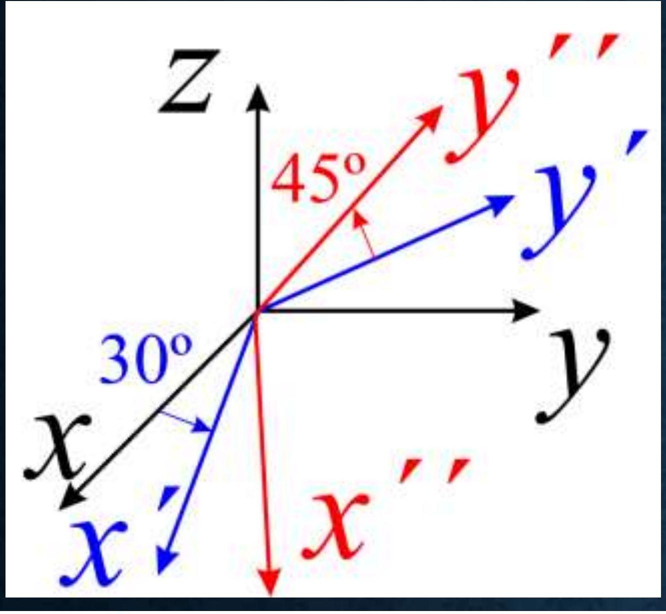
(a) Obtenha a matriz da transformação correspondente à rotação total.

(b) Obtenha a matriz correspondente à transformação que se efectua por ordem inversa.



$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \hat{R}_z\left(\frac{\pi}{6}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

$$= \begin{pmatrix} \cos(x', x) & \cos(x', y) & \cos(x', z) \\ \cos(y', x) & \cos(y', y) & \cos(y', z) \\ \cos(z', x) & \cos(z', y) & \cos(z', z) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$



$$\hat{R}_z^{(total)} = \hat{R}_z\left(\frac{\pi}{4}\right)\hat{R}_z\left(\frac{\pi}{6}\right)$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \hat{R}_z\left(\frac{\pi}{6}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

$$= \begin{pmatrix} \cos(x',x) & \cos(x',y) & \cos(x',z) \\ \cos(y',x) & \cos(y',y) & \cos(y',z) \\ \cos(z',x) & \cos(z',y) & \cos(z',z) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

$$= \begin{pmatrix} \cos(\pi/6) & \cos(\pi/3) & \cos(\pi/2) \\ \cos(\pi/2+\pi/6) & \cos(\pi/6) & \cos(\pi/2) \\ \cos(\pi/2) & \cos(\pi/2) & \cos(0) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \hat{R}_z\left(\frac{\pi}{4}\right)\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos(x'',x') & \cos(x'',y') & \cos(x'',z') \\ \cos(y'',x') & \cos(y'',y') & \cos(y'',z') \\ \cos(z'',x') & \cos(z'',y') & \cos(z'',z') \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} =$$

$$= \begin{pmatrix} \cos(\pi/4) & \cos(\pi/4) & \cos(\pi/2) \\ \cos(\pi/2+\pi/4) & \cos(\pi/4) & \cos(\pi/2) \\ \cos(\pi/2) & \cos(\pi/2) & \cos(0) \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$\hat{R}_z^{(total)} = \hat{R}_z\left(\frac{\pi}{4}\right)\hat{R}_z\left(\frac{\pi}{6}\right)$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \hat{R}_z\left(\frac{\pi}{4}\right)\hat{R}_z\left(\frac{\pi}{6}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

$$= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$$

$$= \begin{pmatrix} \left(\sqrt{3}-1\right)/\left(2\sqrt{2}\right) & \left(\sqrt{3}+1\right)/\left(2\sqrt{2}\right) & 0 \\ -\left(\sqrt{3}+1\right)/\left(2\sqrt{2}\right) & \left(\sqrt{3}-1\right)/\left(2\sqrt{2}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

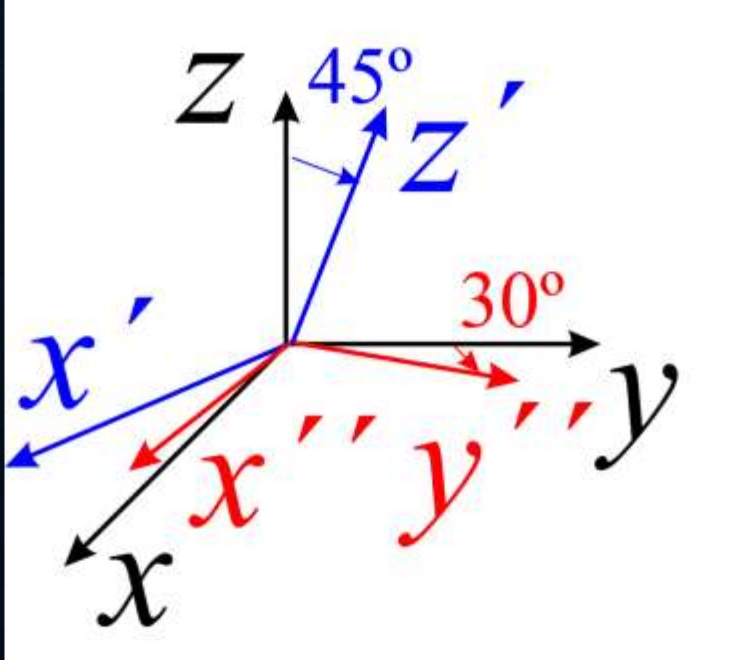
$$\hat{R}_z\left(\frac{\pi}{6}\right)\hat{R}_z\left(\frac{\pi}{4}\right) = \begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \left(\sqrt{3}-1\right)/\left(2\sqrt{2}\right) & \left(\sqrt{3}+1\right)/\left(2\sqrt{2}\right) & 0 \\ -\left(\sqrt{3}+1\right)/\left(2\sqrt{2}\right) & \left(\sqrt{3}-1\right)/\left(2\sqrt{2}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3- Considere dois referenciais S e S' inicialmente coincidentes. O referencial S' roda 45 graus em torno do eixo do y e roda em seguida em torno do novo eixo do z obtido pela primeira rotação de um ângulo de 30 graus.

(a) Obtenha a matriz da transformação correspondente à rotação total.

(b) Obtenha a matriz da transformação correspondente à rotação total por ordem inversa.

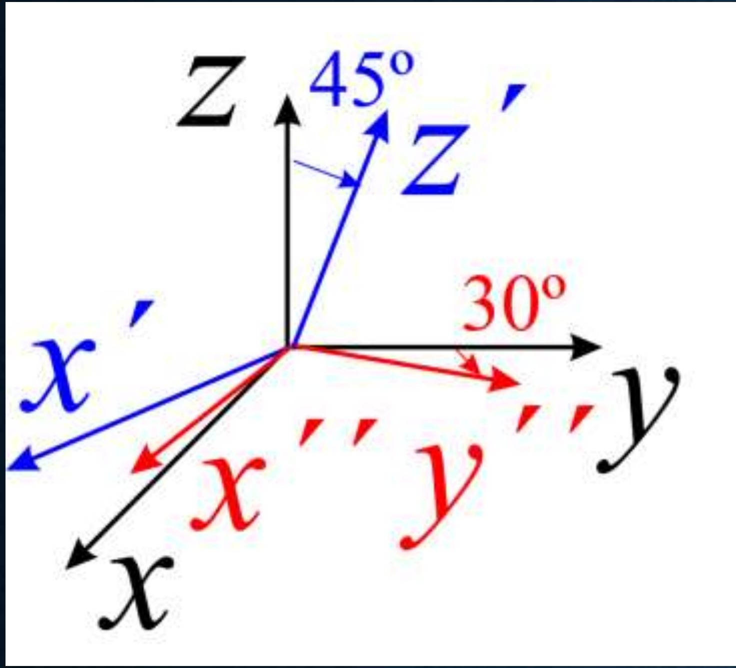


$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \hat{R}_y\left(\frac{\pi}{4}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos(x',x) & \cos(x',y) & \cos(x',z) \\ \cos(y',x) & \cos(y',y) & \cos(y',z) \\ \cos(z',x) & \cos(z',y) & \cos(z',z) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \hat{R}_{z'}\left(\frac{\pi}{6}\right)\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos(x'',x') & \cos(x'',y') & \cos(x'',z') \\ \cos(y'',x') & \cos(y'',y') & \cos(y'',z') \\ \cos(z'',x') & \cos(z'',y') & \cos(z'',z') \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$\hat{R}_z^{(total)} = \hat{R}_{z'}\left(\frac{\pi}{6}\right)\hat{R}_y\left(\frac{\pi}{4}\right)$$

$$\hat{R}_z^{(total)} = \hat{R}_{y'}\left(\frac{\pi}{4}\right)\hat{R}_z\left(\frac{\pi}{6}\right)$$



$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \hat{R}_y\left(\frac{\pi}{4}\right)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos(x',x) & \cos(x',y) & \cos(x',z) \\ \cos(y',x) & \cos(y',y) & \cos(y',z) \\ \cos(z',x) & \cos(z',y) & \cos(z',z) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\pi/4) & \cos(\pi/2) & \cos(\pi/4) \\ \cos(\pi/2) & \cos(0) & \cos(\pi/2) \\ \cos(\pi/2+\pi/4) & \cos(\pi/2) & \cos(\pi/4) \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \hat{R}_{z'}\left(\frac{\pi}{6}\right)\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos(x'',x') & \cos(x'',y') & \cos(x'',z') \\ \cos(y'',x') & \cos(y'',y') & \cos(y'',z') \\ \cos(z'',x') & \cos(z'',y') & \cos(z'',z') \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\pi/6) & \cos(\pi/3) & \cos(\pi/2) \\ \cos(\pi/2+\pi/6) & \cos(\pi/6) & \cos(\pi/2) \\ \cos(\pi/2) & \cos(\pi/2) & \cos(0) \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

$$\hat{R}_z^{(total)} = \hat{R}_{z'}\left(\frac{\pi}{6}\right)\hat{R}_y\left(\frac{\pi}{4}\right) =$$

$$= \begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} \sqrt{3}/\left(2\sqrt{2}\right) & 1/2 & \sqrt{3}/\left(2\sqrt{2}\right) \\ -1/\left(2\sqrt{2}\right) & \sqrt{3}/2 & -1/\left(2\sqrt{2}\right) \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix}$$

$$\hat{R}_z^{(total)} = \hat{R}_{y'}\left(\frac{\pi}{4}\right)\hat{R}_z\left(\frac{\pi}{6}\right) =$$

$$= \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \sqrt{3}/2 & 1/2 & 0 \\ -1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \sqrt{3}/\left(2\sqrt{2}\right) & 1/\left(2\sqrt{2}\right) & 1/\sqrt{2} \\ -1/2 & \sqrt{3}/2 & 0 \\ -\sqrt{3}/\left(2\sqrt{2}\right) & -1/\left(2\sqrt{2}\right) & 1/\sqrt{2} \end{pmatrix}$$

4- Considere a seguinte matriz A correspondente a uma transformação linear,

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

(a) Calcule os valores próprios e os vetores próprios desta matriz e no caso destes últimos forneça as suas componentes cartesianas depois de normalizados (módulo um).

(b) Representa esta matriz uma rotação?

$$\hat{A} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$V = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$\mathrm{Det}\left(\hat{A} - \hat{I}\lambda\right) =$$

$$= \begin{vmatrix} -\lambda & 1 & -1 \\ 2 & -1-\lambda & -1 \\ 1 & -1 & -1-\lambda \end{vmatrix} = 0$$

$$\hat{A}V = \lambda V$$

$$\left(\hat{A} - \hat{I}\lambda\right) V = 0$$

$$\operatorname{Det}\left(\hat{A}-\hat{I}\lambda\right)=\begin{vmatrix}-\lambda & 1 & -1\\ 2 & -1-\lambda & -1\\ 1 & -1 & -1-\lambda\end{vmatrix}=0$$

$$-\lambda\left[(1+\lambda)^2-1\right]-\left[-2(1+\lambda)+1\right]-\left[-2+(1+\lambda)\right]=0$$

$$\lambda(1+\lambda)^2-\cancel{\lambda}-2(1+\lambda)+\cancel{1}+\cancel{\lambda}-\cancel{1}=0$$

$$(1+\lambda)\left[\lambda(1+\lambda)-2\right]=0$$

$$(1+\lambda)\left[\lambda^2+\lambda-2\right]=0$$

$$(\lambda+1)(\lambda-1)(\lambda+2)=0$$

$$\lambda_1=-1$$

$$\lambda_2=1$$

$$\lambda_3=-2$$

$$\hat{A} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\left(\hat{A} - \hat{I}\lambda_1\right) V_1 = 0$$

$$\lambda_1 = -1$$
$$\lambda_2 = 1$$
$$\lambda_3 = -2$$

$$\left[\begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$\times 2$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$-$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$-$$

$$\begin{pmatrix} 2 & 2 & -2 \\ 2 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & -2 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$\begin{aligned} -2Y_1 + Z_1 &= 0 \\ Y_1 &= Z_1/2 \end{aligned}$$

$$\begin{aligned} X_1 + Y_1 - Z_1 &= 0 \\ X_1 + Z_1/2 - Z_1 &= 0 \end{aligned}$$

$$X_1 = Z_1/2$$

$$V_1 = Z_1 \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix}$$

$$\begin{aligned} |V_1|^2 = Z_1^2\,(1/4 + 1/4 + 1) &= 1 \\ Z_1^2\,(3/2) &= 1 \\ Z_1 &= \pm\sqrt{2/3} \end{aligned}$$

$$\boxed{V_1 = \pm \begin{pmatrix} \sqrt{1/6} \\ \sqrt{1/6} \\ \sqrt{4/6} \end{pmatrix}}$$

$$\hat{A} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\left(\hat{A} - \hat{I}\lambda_2\right) V_2 = 0$$

$$\lambda_1 = -1$$
$$\lambda_2 = 1$$
$$\lambda_3 = -2$$

$$\left[\begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$\times 2$

$$\begin{pmatrix} -1 & 1 & -1 \\ 2 & -2 & -1 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$+$

$$\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & -3 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$+$

$$\begin{pmatrix} -2 & 2 & -2 \\ 2 & -2 & -1 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & -3 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 1 & -1 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$$\begin{aligned} -3Z_2 &= 0 \\ Z_2 &= 0 \end{aligned}$$

$$\begin{aligned} -X_2 + Y_2 - 1 \cdot 0 &= 0 \\ X_2 &= Y_2 \end{aligned}$$

$$V_2 = Y_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$|V_2|^2 = Y_2^2 (1 + 1 + 0) = 1$$

$$\begin{aligned} Y_2^2 \cdot 2 &= 1 \\ Y_2 &= \pm 1/\sqrt{2} \end{aligned}$$

$$\boxed{V_2 = \pm \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}}$$

$$\hat{A} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\left(\hat{A} - \hat{I}\lambda_3\right) V_3 = 0$$

$$\lambda_1 = -1$$
$$\lambda_2 = 1$$
$$\lambda_3 = -2$$

$$\left[\begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} = 0$$

$-$

$$\begin{pmatrix} 2 & 1 & -1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} = 0$$

$\times 1/2$

$$\begin{pmatrix} 2 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} = 0$$

$-$

$$\begin{pmatrix} 1 & 1/2 & -1/2 \\ 0 & 0 & 0 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & -3/2 & 3/2 \end{pmatrix} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & -3/2 & 3/2 \end{pmatrix} \begin{pmatrix} X_3 \\ Y_3 \\ Z_3 \end{pmatrix} = 0$$

$$X_3 + Y_3 - Z_3 = 0$$
$$X_3 = 0$$

$$-(3/2)\,Y_3 + (3/2)\,Z_3 = 0$$
$$Y_3 = Z_3$$

$$V_3 = Z_3 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$|V_3|^2 = Z_3^2\,(0 + 1 + 1) = 1$$
$$2Z_3^2 = 1$$
$$Z_3 = \pm 1/\sqrt{2}$$

$$V_3 = \pm \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

(b) Representa esta matriz uma rotação?

$$\hat{A} = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \qquad \hat{A}^T = \begin{pmatrix} 0 & 2 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix}$$

$$\hat{A}\hat{A}^T = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & -1 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 2 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 4 \\ 0 & 4 & 3 \end{pmatrix} \neq \hat{I}$$

5- Considere a transformação linear que transforma cada ponto de coordenadas (x, y, z) num ponto de coordenadas $(y, x, -z)$.

(a) Qual a matriz correspondente a esta transformação?

(b) Quais as retas que se mantêm invariantes sob esta transformação linear?

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} y \\ x \\ -z \end{pmatrix}$$

$$\begin{pmatrix} y \\ x \\ -z \end{pmatrix} = \hat{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\left(\hat{A} - \hat{I}\lambda\right) V = 0$$

$$\mathrm{Det}\left(\hat{A} - \hat{I}\lambda\right) = \begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix} = 0$$

$$\mathrm{Det}\left(\hat{A}-\hat{I}\lambda\right)=\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix}=0$$

$$\lambda_1=-1$$
$$\lambda_2=1$$

$$-\lambda\left[\lambda\left(1+\lambda\right)\right]-\left[-\left(1+\lambda\right)\right]=0$$
$$\left(\lambda^2-1\right)\left(1+\lambda\right)=0$$
$$\left(\lambda-1\right)\left(\lambda+1\right)^2=0$$

$$\left(\hat{A}-\hat{I}\lambda_1\right)V_1=0$$

$$\left[\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}+\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix}=0$$

$$\left[\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} X_1 \\ Y_1 \\ Z_1 \end{pmatrix} = 0$$

$$X_1 + Y_1 = 0$$

$$X_1 = -Y_1$$

$$V_1 = \begin{pmatrix} -Y_1 \\ Y_1 \\ Z_1 \end{pmatrix}$$

$$\left(\hat{A} - \hat{I}\lambda_2\right) V_2 = 0$$

$$\left[\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right] \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$$V_2 = Y_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{pmatrix} \begin{pmatrix} X_2 \\ Y_2 \\ Z_2 \end{pmatrix} = 0$$

$$\begin{aligned} Z_2 &= 0 \\ -X_2 + Y_2 &= 0 \\ X_2 &= Y_2 \end{aligned}$$

$$\begin{aligned} |V_2|^2 = 2Y_2^2 &= 1 \\ Y_2^2 &= 1/2 \\ Y_2 &= \pm 1/\sqrt{2} \end{aligned}$$

$$V_2 = \pm \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$