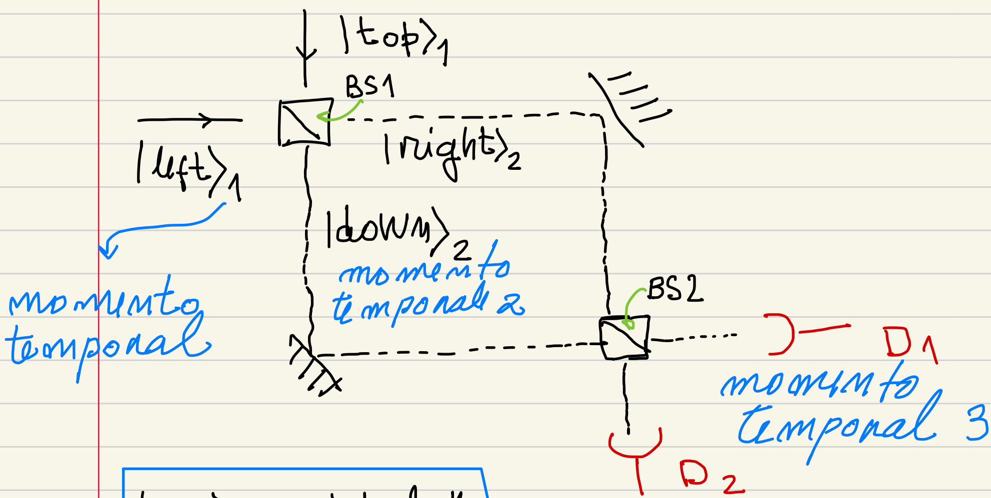
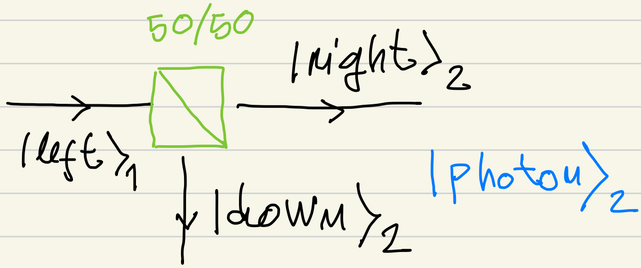
2.) Fotões e interferência



$$\boxed{|\ldots\rangle \rightarrow \text{" ket "}}$$

Olhemos para o BS1



- o fotão não decide o caminho com base numa moeda.

$$|photon\rangle_2 = \frac{1}{\sqrt{2}}\,|right\rangle_2 + \frac{1}{\sqrt{2}}\,|down\rangle_2$$

↳ sobreposição quântica

$|right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; $|down\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

produto interno do $|right\rangle$:

$$(1,0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

produto interno de $|down\rangle$

$$(0\ 1)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$$

$\langle right| = (1,0)$; $\langle down| = (0,1)$

$\boxed{\langle \cdots | = \text{"bra"}}$

Produto inter. do $|right\rangle$ por si mesmo é:

$$(1,0)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \langle right | right \rangle = 1$$

Produto interno do $|down\rangle$ por si mesmo é:

$$(0,1)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \langle down | down \rangle = 1$$

$$\boxed{\underbrace{\langle \cdots | \cdots \rangle}_{\text{"braket"}}}$$

$$\langle Right|down\rangle = (1,0)\begin{pmatrix}0\\1\end{pmatrix} = 0$$

$$\langle down|Right\rangle = (0,1)\begin{pmatrix}1\\0\end{pmatrix} = 0$$

Os estados $|down\rangle$ e $|right\rangle$ são ortogonais:

— // —

$$\underbrace{|photon\rangle}_{\text{ket}}{}_2 = \frac{1}{\sqrt{2}}|right\rangle + \frac{1}{\sqrt{2}}|down\rangle$$

$$\underbrace{{}_2\langle photon|}_{\text{bra}} = \frac{1}{\sqrt{2}}\langle right| + \frac{1}{\sqrt{2}}\langle down|$$

braket: ${}_2\langle photon|photon\rangle_2 =$

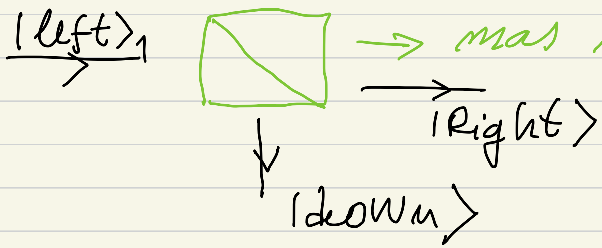
$$\left(\frac{1}{\sqrt{2}}\langle right| + \frac{1}{\sqrt{2}}\langle down|\right)\underbrace{\left(\frac{1}{\sqrt{2}}|right\rangle + \frac{1}{\sqrt{2}}|down\rangle\right)}$$

$$= \frac{1}{2}\langle right|right\rangle + \frac{1}{2}\langle right|down\rangle + \frac{1}{2}\langle down|right\rangle + \frac{1}{2}\langle down|down\rangle$$

$$= \frac{1}{2} + 0 + 0 + \frac{1}{2} = 1.$$

$$\boxed{\langle right|photon\rangle_2 = \frac{1}{\sqrt{2}}}$$
amplitude de probabilidade.

situação mais geral:



$\rightarrow$ mas não é 50/50

$$|photon\rangle_2 = a\,|Right\rangle + b\,|down\rangle$$

$${}_2\langle photon| = a^*\langle right| + b^*\langle down|$$

complexo conjugado

$${}_2\langle photon|photon\rangle_2 =$$

$$\left(a^*\langle right| + b^*\langle down| \right)\left(a|right\rangle + b|down\rangle \right)$$

$$= a^*a\cdot 1 + a^*b\cdot 0 + b^*a\cdot 0 + b^*b\cdot 1$$

$$= a^*a + b^*b = |a|^2 + |b|^2 = 1$$

$$\boxed{\begin{aligned} &z = x+iy\,;\ z^* = x-iy \\ &z^*z = (x-iy)(x+iy) \\ &= x^2+y^2 = |z|^2 \end{aligned}}$$

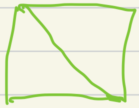
temos que ter

$$\boxed{|a|^2+|b|^2 = 1}$$

$\downarrow$

CONDIÇÃO DE NORMALIZAÇÃO.

⇒ O divisor de feixe como uma matriz:

$|\overrightarrow{left}\rangle$  = matriz

$$U_{BS}|in\rangle_1 = |out\rangle_2$$

$$|in\rangle_1 = |left\rangle$$

$$U_{BS}|left\rangle = a\,|right\rangle_2 + b\,|down\rangle_2$$

Se U_{BS} representa um divisor de feixe 50/50: $a = \frac{1}{\sqrt{2}}$, $b = \frac{1}{\sqrt{2}}$

$$U_{BS}|left\rangle_1 = \frac{1}{\sqrt{2}}|right\rangle_2 + \frac{1}{\sqrt{2}}|down\rangle_2$$

$$U_{BS}\begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$$

vetor

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

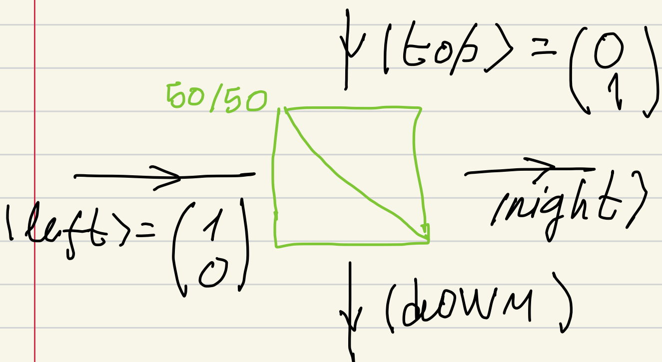
origina outro vetor

$$U_{BS} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}$$

$$\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Leftrightarrow$$

$$\begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{matrix} u_{11} = 1/\sqrt{2} \\ u_{21} = 1/\sqrt{2} \end{matrix}$$

$$U_{BS} = \begin{pmatrix} 1/\sqrt{2} & u_{12} \\ 1/\sqrt{2} & u_{22} \end{pmatrix}$$



$\langle left | top \rangle = 0$

$$U_{BS} |top\rangle = \underbrace{a}_{\frac{1}{\sqrt{2}}} |right\rangle + \underbrace{b}_{-\frac{1}{\sqrt{2}}} |down\rangle$$

$a = \pm 1/\sqrt{2}$

$b = \pm 1/\sqrt{2}$

importante

$$U_{BS}|top\rangle = \frac{1}{\sqrt{2}}|right\rangle - \frac{1}{\sqrt{2}}|down\rangle \equiv |\psi_a\rangle$$

$$U_{BS}|left\rangle = \frac{1}{\sqrt{2}}|right\rangle + \frac{1}{\sqrt{2}}|down\rangle \equiv |\psi_b\rangle$$

$$\langle\psi_a|\psi_b\rangle = \left(\frac{1}{\sqrt{2}}\langle right| - \frac{1}{\sqrt{2}}\langle down|\right)\left(\frac{1}{\sqrt{2}}|right\rangle + \frac{1}{\sqrt{2}}|down\rangle\right)$$

$$= \frac{1}{2}\underbrace{\langle right|right\rangle}_{1} + \frac{1}{2}\underbrace{\langle right|down\rangle}_{0}$$

$$- \frac{1}{2}\underbrace{\langle down|right\rangle}_{0} - \frac{1}{2}\underbrace{\langle down|down\rangle}_{1}$$

$$= \frac{1}{2} - \frac{1}{2} = 0$$

$|\psi_a\rangle$ e $|\psi_b\rangle$ são ortogonais.

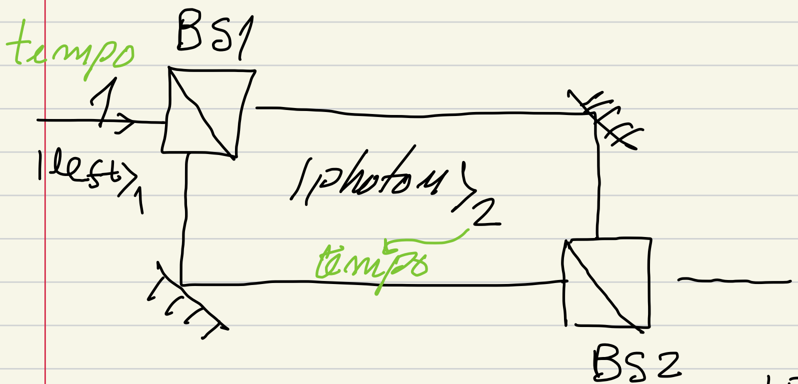
Cálculos dos demais elementos de matriz do U_{BS}

$$\begin{pmatrix} 1/\sqrt{2} & u_{12} \\ 1/\sqrt{2} & u_{22} \end{pmatrix} |top\rangle = U_{BS}\begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

$$= \frac{1}{\sqrt{2}}|right\rangle - \frac{1}{\sqrt{2}}|down\rangle \; \} |\psi_a\rangle$$

$$\begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

$$U_{BS} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad (50/50)$$

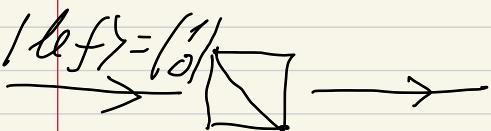


$|photon\rangle_3$

tempo

$$BS1 = BS2 = 50/50$$

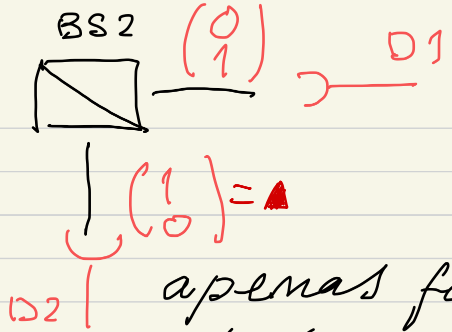
$$|photon\rangle_3 = U_{BS2}\, |photon_b\rangle_2$$



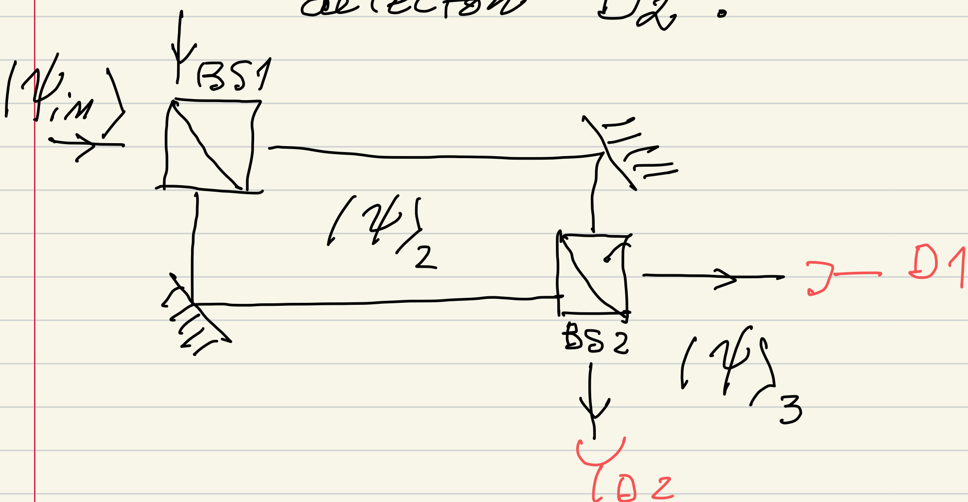
$$|\psi_b\rangle_2 = \frac{1}{\sqrt{2}}|right\rangle + \frac{1}{\sqrt{2}}|down\rangle$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$U_{BS2}|\psi_b\rangle_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



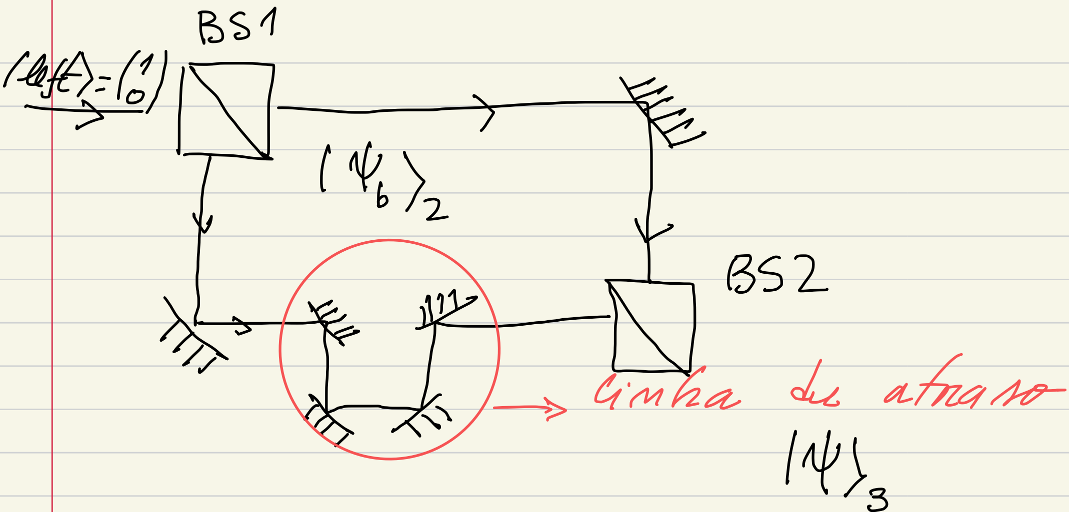
apenas faz "click" o detector D_2.



$$|\psi\rangle_3 = U_{BS2}\,|\psi\rangle_2 = \underbrace{U_{BS2}\,U_{BS1}}_{\text{matriz que descreve o interferômetro}}|\psi_{in}\rangle$$

no tempo 3

no tempo 2



Matriz do interferómetro:

$$|\psi\rangle_3 = U_{BS2}\, U_\phi\, U_{BS1}\, |left\rangle$$

$$= \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}}_{U_{BS2}} \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix} \underbrace{\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}}_{U_{BS1}} \underbrace{\begin{pmatrix}1\\ 0\end{pmatrix}}_{|left\rangle}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}a_{11}+a_{12}\\ a_{21}+a_{22}\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}a_{11}+a_{12}+a_{21}+a_{22}\\ a_{11}+a_{12}-a_{21}-a_{22}\end{pmatrix} = \underbrace{\begin{pmatrix}0\\ 1\end{pmatrix}}_{\text{FAZ CLICK D1}}$$

$a_{12} = a_{21} = 0$

$$= \frac{1}{2}\begin{pmatrix} a_{11} + a_{22} \\ a_{11} - a_{22} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow a_{11} = 1 \text{ e } a_{22} = -1$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \text{faz "click" o detector } D_1$$

$$U_\phi = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \underline{e}^{i\pi} \end{pmatrix}$$

U_π

Em geral: $$\boxed{U_\phi = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}}$$

matriz de fase.

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→ A MATRIZ DE FASE E CÁLCULO DE PROBABILIDADES

U_{BS} e U_ϕ são matrizes unitárias

$$U_{BS} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} ; \quad U_\phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$$

$$U_{BS}^{\dagger} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

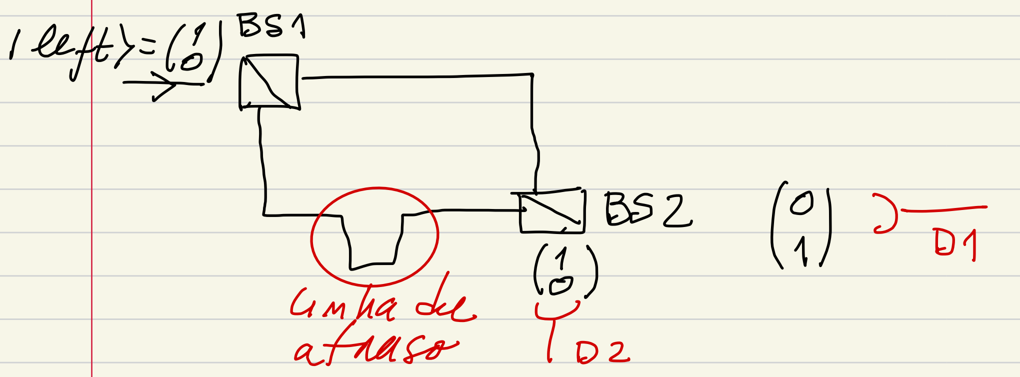
$$U_{BS}U_{BS}^{\dagger} = \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix} = \mathbb{1}$$

$$U_{\phi} = \begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix} \longrightarrow U_{\phi}^{\dagger} = \begin{bmatrix}1 & 0\\ 0 & e^{-i\phi}\end{bmatrix}$$

$$U_{\phi}\cdot U_{\phi}^{\dagger} = \begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & e^{-i\phi}\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \mathbb{1}$$

A evolução temporal de um sistema é feita com matrizes unitárias

$$U^{-1} = U^{\dagger} : \; U^{\dagger}U = UU^{\dagger} = \mathbb{1}$$



O interferômetro é descrito por uma matriz:

$$U_{M-Z} = U_{BS2}\, U_{\phi}\, U_{BS1} \qquad : 50/50$$

$$U_{M-Z} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & e^{i\phi}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ e^{i\phi} & -e^{+i\phi}\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1+e^{i\phi} & 1-e^{i\phi}\\ 1-e^{i\phi} & 1+e^{i\phi}\end{bmatrix}$$

$$U_{M-Z}|left\rangle = \frac{1}{2}\begin{bmatrix}1+e^{i\phi} & 1-e^{i\phi}\\ 1-e^{i\phi} & 1+e^{i\phi}\end{bmatrix}\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$|final\rangle_3 = \frac{1}{2}\begin{bmatrix}1+e^{i\phi}\\ 1-e^{i\phi}\end{bmatrix} =$$

$$= \underbrace{\frac{1}{2}(1+e^{i\phi})}_{C_2}\underbrace{\begin{pmatrix}1\\ 0\end{pmatrix}}_{D2} + \underbrace{\frac{1}{2}(1-e^{i\phi})}_{C_1}\underbrace{\begin{pmatrix}0\\ 1\end{pmatrix}}_{D1}$$

<u>Postulado</u>: C_2 e C_1 chamam-se amplitudes de probabilidade e as probabilidades são módulo ao quadrado das amplitudes.

$$Prob(D_2) = C_2 C_2^* = |C_2|^2$$

$$Prob(D_1) = C_1 C_1^* = |C_1|^2$$

$$\mathrm{prob}(D_2) = c_2 c_2^* = \frac{1}{2}\left(1+e^{i\phi}\right)\frac{1}{2}\left(1+e^{-i\phi}\right)$$

$$= \frac{1}{4}\left(1+1+e^{-i\phi}+e^{i\phi}\right)$$

$$= \frac{1}{4}\left(2+2\cos\phi\right)$$

$$= \frac{1}{2}\left(1+\cos\phi\right)$$

$$= \frac{1}{2}\left(\underbrace{1+\cos^2\frac{\phi}{2}-\sin^2}_{\cos^2\phi/2}\frac{\phi}{2}\right) = \underbrace{\cos^2\frac{\phi}{2}}_{0<\cdots<1}$$

$$\frac{e^{ix}+e^{-ix}}{2} = \cos(x)$$

$$\cos\phi = \cos^2\frac{\phi}{2} - \sin^2\frac{\phi}{2}$$

$$\mathrm{prob}(D_1) = \overbrace{\frac{1}{2}\left(1-e^{i\phi}\right)}^{c_1}\frac{1}{2}\left(1-e^{-i\phi}\right) =$$

$$= \frac{1}{4}\left(2-2\cos\phi\right) = \sin^2\frac{\phi}{2}$$

$$\mathrm{prob}(D_1) + \mathrm{prob}(D_1) = \sin^2\frac{\phi}{2} + \cos^2\frac{\phi}{2} = 1.$$

$$U_{BS} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = H$$

$\hookrightarrow$ gate (porta) de Hadamard

U_ϕ ; $\phi = \pi/4$: $\quad T = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$

$$T \cdot T = T_{\pi/2} = U_{\pi/2}$$

Cálculo geral das probabilidades

$$|\tilde{\psi}\rangle = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}.$$

É necessário normalizar o estado:

$$\langle\tilde{\psi}|\tilde{\psi}\rangle = \text{norma } \tilde{\psi} = 1$$

$$\langle\tilde{\psi}|\tilde{\psi}\rangle = (a^*, b^*)\begin{pmatrix}a\\b\end{pmatrix} = |a|^2 + |b|^2$$

$$|\psi\rangle = \frac{|\tilde{\psi}\rangle}{\underbrace{\sqrt{\langle\tilde{\psi}|\tilde{\psi}\rangle}}_{\text{número real}}}$$

$$\langle\psi|\psi\rangle = \frac{\langle\tilde{\psi}|\tilde{\psi}\rangle}{\sqrt{\langle\tilde{\psi}|\tilde{\psi}\rangle}\cdot\sqrt{\langle\tilde{\psi}|\tilde{\psi}\rangle}} = \frac{\langle\tilde{\psi}|\tilde{\psi}\rangle}{\langle\tilde{\psi}|\tilde{\psi}\rangle} = 1$$

Se $|\psi\rangle$ estiver normalizado:

$$|\psi\rangle = c_1 \underbrace{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}_{|down\rangle} + c_2 \underbrace{\begin{pmatrix} 0 \\ 1 \end{pmatrix}}_{|right\rangle}$$

$$Prob(|down\rangle) = |c_1|^2$$

$$Prob(|right\rangle) = |c_2|^2$$

$$|\psi\rangle = c_1 |down\rangle + c_2 |right\rangle$$

$$\langle down|\psi\rangle = c_1 \overbrace{\langle down|down\rangle}^{1} + c_2 \underbrace{\langle down|right\rangle}_{0} = c_1$$

$c_1 = \langle down|\psi\rangle$ (amplitude de probabilidade)

$$Prob(|down\rangle) = |c_1|^2$$

$$= \langle down|\psi\rangle \cdot \langle down|\psi\rangle^*$$

$$\langle down|\psi\rangle \langle\psi|down\rangle$$

$$= |\langle down|\psi\rangle|^2.$$