



1 *Signals, Systems and other supporting functions*

$$\begin{aligned}
 E_{\infty} &= \int_{-\infty}^{+\infty} |x(t)|^2 dt & E_{\infty} &= \sum_{n=-\infty}^{+\infty} |x[n]|^2 & P_{\infty} &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 dt \\
 P_{\infty} &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2 & x(t) &= x(t+T), T \in \mathbb{R} & x[n] &= x[n+N], N \in \mathbb{Z} \\
 x(t) &= x_p(t) + x_i(t) & x_p(t) &= \frac{x(t) + x(-t)}{2} & x_i(t) &= \frac{x(t) - x(-t)}{2} \\
 u[n] &= \sum_{k=-\infty}^n \delta[k] & \delta[n] &= u[n] - u[n-1] & x(t-t_0)\delta(t) &= x(t_0)\delta(t-t_0) \\
 u(t) &= \int_{-\infty}^t \delta(\tau) d\tau & \delta(t) &= \frac{d}{dt} u(t) & \sum_{k=0}^{N-1} \alpha^k &= \frac{1-\alpha^N}{1-\alpha} \\
 \sum_{k=1}^N k &= \frac{N(N+1)}{2} & \text{sinc}(x) &= \frac{\sin(\pi x)}{\pi x} & & \\
 A_{ik} &= \frac{1}{(\sigma_i - k)!} \left[\frac{d^{\sigma_i - k}}{d u^{\sigma_i - k}} [(v - \rho_i)^{\sigma_i} G(v)] \right]_{v=\rho_i} & B_{ik} &= \frac{1}{(\sigma_i - k)!} (-\rho_i)^{\sigma_i - k} \left[\frac{d^{\sigma_i - k}}{d u^{\sigma_i - k}} [(1 - \rho_i^{-1} v)^{\sigma_i} G(v)] \right]_{v=\rho_i}
 \end{aligned}$$

2 *LIT Systems*

$$\begin{aligned}
 x[n] &= \sum_{k=-\infty}^{+\infty} x[k] \delta[n-k] & y[n] &= \sum_{k=-\infty}^{+\infty} x[k] h[n-k] & y(t) &= \int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau
 \end{aligned}$$

3 *CFS — Continuous Fourier Series*

$$\begin{aligned}
 x(t) &= \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} & a_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \\
 Ax(t) + By(t) &\xrightarrow{CFS} Aa_k + Bb_k & x(t-t_0) &\xrightarrow{CFS} a_k e^{-jk\omega_0 t_0} \\
 e^{-jl\omega_0 t} x(t) &\xrightarrow{CFS} a_{k-l} & x(-t) &\xrightarrow{CFS} a_{-k} \\
 x^*(t) &\xrightarrow{CFS} a_{-k}^* & x^*(-t) &\xrightarrow{CFS} a_k^* \\
 x(t)y(t) &\xrightarrow{CFS} \sum_{l=-\infty}^{+\infty} a_l b_{k-l} & \frac{d}{dt} x(t) &\xrightarrow{CFS} jk\omega_0 a_k \\
 x_i(t) &\xrightarrow{CFS} j \text{imag}\{a_k\} & x_p(t) &\xrightarrow{CFS} \text{real}\{a_k\} \\
 \text{real}\{x(t)\} &\xrightarrow{CFS} a_{kp} = \frac{1}{2}[a_k + a_{-k}^*] & j \text{imag}\{x(t)\} &\xrightarrow{CFS} a_{ki} = \frac{1}{2}[a_k - a_{-k}^*] \\
 \int_{-\infty}^t x(t) dt &\xrightarrow{CFS} \frac{1}{jk\omega_0} a_k & \frac{1}{T} \int_T |x(t)|^2 dt &= \sum_{k=-\infty}^{+\infty} |a_k|^2
 \end{aligned}$$



Sinal periódico: $x(t) = x(t + T)$ onde, $x(t) = \begin{cases} 1 & , \quad |t| \leq T_1 \\ 0 & , \quad |t| > T_1 \end{cases} \xrightarrow{CFS} a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$

4 DFS — *Discrete Fourier Series*

$$\begin{aligned}
 x[n] &= \sum_{k=0}^{N-1} a_k e^{jk\Omega_0 n} & a_k &= \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n} \\
 Ax[n] + By[n] &\xrightarrow{DFS} Aa_k + Bb_k & x[n - n_0] &\xrightarrow{DFS} a_k e^{-jk\Omega_0 n_0} \\
 x[n] e^{jn\Omega_0 l} &\xrightarrow{DFS} a_{k-l} & x^*[n] &\xrightarrow{DFS} a_{-k}^* \\
 x[-n] &\xrightarrow{DFS} a_{-k} & x[n]y[n] &\xrightarrow{DFS} \sum_{l=\langle N \rangle} a_l b_{k-l} \\
 \sum_{r=\langle N \rangle} x[r]y[n-r] &\xrightarrow{DFS} Na_k b_k & x[n] - x[n-1] &\xrightarrow{DFS} (1 - e^{-jk\Omega_0})a_k \\
 \sum_{k=-\infty}^n x[k] &\xrightarrow{DFS} \frac{1}{(1 - e^{-jk\Omega_0})} a_k & \text{real}\{x[n]\} &\xrightarrow{DFS} a_{kp} = \frac{1}{2}[a_k + a_{-k}^*] \\
 j \text{imag}\{x[n]\} &\xrightarrow{DFS} a_{ki} = \frac{1}{2}[a_k - a_{-k}^*] & x_p[n] &\xrightarrow{DFS} \text{real}\{a_k\} \\
 x_i[n] &\xrightarrow{DFS} j \text{imag}\{a_k\} & \frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 &= \sum_{k=\langle N \rangle} |a_k|^2
 \end{aligned}$$

5 CTFT — *Continuous-time Fourier Transform*

$$\begin{aligned}
 x(t) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega & X(j\omega) &= \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \\
 Ax(t) + By(t) &\xrightarrow{CTFT} AX(j\omega) + BY(j\omega) & x(t - t_0) &\xrightarrow{CTFT} e^{-j\omega t_0} X(j\omega) \\
 x^*(t) &\xrightarrow{CTFT} X^*(-j\omega) & x(t) \in \mathbb{R} &\xrightarrow{CTFT} X(j\omega) = X^*(-j\omega) \\
 \text{par}\{x(t)\} &\xrightarrow{CTFT} \text{real}\{X(j\omega)\} & \text{impar}\{x(t)\} &\xrightarrow{CTFT} j \text{imag}\{X(j\omega)\} \\
 \frac{d}{dt} x(t) &\xrightarrow{CTFT} j\omega X(j\omega) & \int_{-\infty}^t x(t) dt &\xrightarrow{CTFT} \frac{1}{j\omega} X(j\omega) + \pi X(0) \delta(\omega) \\
 x(\alpha t) &\xrightarrow{CTFT} \frac{1}{|\alpha|} X\left(\frac{j\omega}{\alpha}\right) & x(-t) &\xrightarrow{CTFT} X(-j\omega) \\
 X(t) &\xrightarrow{CTFT} 2\pi x(-j\omega) & tx(t) &\xrightarrow{CTFT} j \frac{d}{d\omega} X(j\omega) \\
 x(t)y(t) &\xrightarrow{CTFT} \frac{1}{2\pi} X(j\omega) * Y(j\omega) & x(t) * y(t) &\xrightarrow{CTFT} X(j\omega)Y(j\omega)
 \end{aligned}$$



6 CTFT — Pares de Transformadas

$$\begin{array}{llll}
 \sum_{k=-\infty}^{+\infty} a_k e^{j\omega_0 t} & \xrightarrow{CTFT} & 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0) & e^{j\omega_0 t} \xrightarrow{CTFT} 2\pi \delta(\omega - \omega_0) \\
 \cos(\omega_0 t) & \xrightarrow{CTFT} & \pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] & \sin(\omega_0 t) \xrightarrow{CTFT} \frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] \\
 x(t) = 1 & \xrightarrow{CTFT} & 2\pi \delta(\omega) & \delta(t) \xrightarrow{CTFT} 1 \\
 u(t) & \xrightarrow{CTFT} & \frac{1}{j\omega} + \pi \delta(\omega) & \sum_{n=-\infty}^{+\infty} \delta(t - nT) \xrightarrow{CTFT} \frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right) \\
 \frac{\sin(Wt)}{\pi t} & \xrightarrow{CTFT} & X(j\omega) = \begin{cases} 1 & , \quad |\omega| \leq W \\ 0 & , \quad |\omega| > W \end{cases} & \delta(t - t_0) \xrightarrow{CTFT} e^{-j\omega t_0} \\
 e^{-\alpha t} u(t), \quad \Re\{\alpha\} > 0 & \xrightarrow{CTFT} & \frac{1}{\alpha + j\omega} & \\
 te^{-\alpha t} u(t), \quad \Re\{\alpha\} > 0 & \xrightarrow{CTFT} & \frac{1}{(\alpha + j\omega)^2} & \\
 \frac{t^{n-1}}{(n-1)!} e^{-\alpha t} u(t), \quad \Re\{\alpha\} > 0 & \xrightarrow{CTFT} & \frac{1}{(\alpha + j\omega)^n} & \\
 \text{Sinal periódico: } x(t) = x(t+T) \quad \text{onde, } x(t) = \begin{cases} 1 & , \quad |t| \leq T_1 \\ 0 & , \quad |t| > T_1 \end{cases} & \xrightarrow{CTFT} & \sum_{k=-\infty}^{+\infty} \frac{2 \sin(k\omega_0 T_1)}{k} \delta(\omega - k\omega_0) & \\
 x(t) = \begin{cases} 1 & , \quad |t| \leq T_1 \\ 0 & , \quad |t| > T_1 \end{cases} & \xrightarrow{CTFT} & \frac{2 \sin(\omega T_1)}{\omega} = 2T_1 \text{sinc}(\omega T_1 / \pi) & \\
 \int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega & & &
 \end{array}$$

7 DTFT — Discrete-time Fourier Transform

$$\begin{array}{llll}
 x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega & & X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\Omega n} & \\
 Ax[n] + By[n] & \xrightarrow{DTFT} & AX(\Omega) + BY(\Omega) & x[n - n_0] \xrightarrow{DTFT} e^{-j\Omega n_0} X(\Omega) \\
 x^*[n] & \xrightarrow{DTFT} & X^*(-\Omega) & x[-n] \xrightarrow{DTFT} X(-\Omega) \\
 x[n] - x[n-1] & \xrightarrow{DTFT} & (1 - e^{-j\Omega})X(\Omega) & \sum_{k=-\infty}^n x[k] \xrightarrow{DTFT} \frac{1}{1 - e^{-j\Omega}} X(j\omega) + \pi X(0) \sum_{k=-\infty}^{+\infty} \delta(\Omega - 2\pi k) \\
 nx[n] & \xrightarrow{DTFT} & j \frac{d}{d\Omega} X(\Omega) & \\
 x[n]y[n] & \xrightarrow{DTFT} & \frac{1}{2\pi} X(\Omega) * Y(\Omega) & x[n] * y[n] \xrightarrow{DTFT} X(\Omega)Y(\Omega)
 \end{array}$$



8 DTFT — Pares de Transformadas

$$\begin{aligned}
 x[n] = \sum_{n < N} a_k e^{j\Omega_0 n} & \xrightarrow{DTFT} X(\Omega) = 2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\Omega - k\Omega_0) \\
 e^{j\Omega_0 n} & \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} 2\pi \delta(\Omega - \Omega_0 - 2\pi l) \\
 \cos(\Omega_0 n) & \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} \pi [\delta(\Omega - \Omega_0 - 2\pi l) + \delta(\Omega + \Omega_0 - 2\pi l)] \\
 \sin(\Omega_0 n) & \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} \frac{\pi}{j} [\delta(\Omega - \Omega_0 - 2\pi l) - \delta(\Omega + \Omega_0 - 2\pi l)] \\
 x[n] = 1 & \xrightarrow{DTFT} \sum_{l=-\infty}^{+\infty} 2\pi \delta(\Omega - 2\pi l) \\
 \delta[n] & \xrightarrow{DTFT} 1 \\
 u[n] & \xrightarrow{DTFT} \frac{1}{1 - e^{-j\Omega}} + \sum_{l=-\infty}^{+\infty} \pi \delta(\Omega - 2\pi l) \\
 \sum_{n=-\infty}^{+\infty} \delta[n - nN] & \xrightarrow{DTFT} \frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta(\Omega - \frac{2\pi k}{N}) \\
 \frac{\sin(Wn)}{\pi n} & \xrightarrow{DTFT} X(j\Omega) = \begin{cases} 1 & , \quad 0 \leq |\Omega| \leq W \\ 0 & , \quad W < |\Omega| \leq \pi \end{cases} \\
 \delta[n - n_0] & \xrightarrow{DTFT} e^{-j\Omega n_0} \\
 \alpha^n u[n], |\alpha| < 1 & \xrightarrow{DTFT} \frac{1}{1 - \alpha e^{-j\Omega}} \\
 (n+1)\alpha^n u[n], |\alpha| < 1 & \xrightarrow{DTFT} \frac{1}{(1 - \alpha e^{-j\Omega})^2} \\
 \frac{(n+r-1)!}{n!(r-1)!} \alpha^n u[n], |\alpha| < 1 & \xrightarrow{DTFT} \frac{1}{(1 - \alpha e^{-j\Omega})^r}
 \end{aligned}$$

9 Z — Z-Transform

$$\begin{aligned}
 X(z) &= \sum_{n=-\infty}^{+\infty} x[n] z^{-n} \quad , \quad ROC : R \\
 \alpha x_1[n] + \beta x_2[n] & \xrightarrow{Z} \alpha X_1(z) + \beta X_2(z) \quad , \quad ROC : (R_1 \cap R_2) \\
 x[n - n_0] & \xrightarrow{Z} z^{-n_0} X(z) \quad , \quad ROC : R \pm \{z = 0\} \\
 z_0^n x[n] & \xrightarrow{Z} X\left(\frac{z}{z_0}\right) \quad , \quad ROC : |z_0| \cdot R \\
 e^{j\omega_0 n} x[n] & \xrightarrow{Z} X(e^{-j\omega_0} z) \quad , \quad ROC : R \\
 x[-n] & \xrightarrow{Z} X(z^{-1}) \quad , \quad ROC : R^{-1} \\
 x_{(k)}[n] = \begin{cases} x[r] & , \quad n = rk \\ 0 & , \quad n \neq rk \end{cases} & \xrightarrow{Z} X(z^k) \quad , \quad ROC : R^{1/k}
 \end{aligned}$$



$$\begin{aligned}
 x^*[n] &\xrightarrow{Z} X^*(z^*) \quad , \quad ROC : R \\
 x_1[n] * x_2[n] &\xrightarrow{Z} X_1(z) \cdot X_2(z) \quad , \quad ROC : (R_1 \cap R_2) \\
 x[n] - x[n-1] &\xrightarrow{Z} (1 - z^{-1}) X(z) \quad , \quad ROC : (R \cap |z| > 0) \\
 \sum_{k=-\infty}^n x[k] &\xrightarrow{Z} \frac{1}{1 - z^{-1}} X(z) \quad , \quad ROC : (R \cap |z| > 1) \\
 nx[n] &\xrightarrow{Z} -z \frac{dX(z)}{dz} \quad , \quad ROC : R
 \end{aligned}$$

10 Z — Pares de Transformadas

$$\begin{aligned}
 \delta[n] &\xrightarrow{Z} 1 \quad , \quad \forall z \\
 u[n] &\xrightarrow{Z} \frac{1}{1 - z^{-1}} \quad , \quad |z| > 1 \\
 -u[-n-1] &\xrightarrow{Z} \frac{1}{1 - z^{-1}} \quad , \quad |z| < 1 \\
 \alpha^n u[n] &\xrightarrow{Z} \frac{1}{1 - \alpha z^{-1}} \quad , \quad |z| > |\alpha| \\
 -\alpha^n u[-n-1] &\xrightarrow{Z} \frac{1}{1 - \alpha z^{-1}} \quad , \quad |z| < |\alpha| \\
 n\alpha^n u[n] &\xrightarrow{Z} \frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2} \quad , \quad |z| > |\alpha| \\
 -n\alpha^n u[-n-1] &\xrightarrow{Z} \frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2} \quad , \quad |z| < |\alpha|
 \end{aligned}$$

11 Filtro IIR

$$\begin{aligned}
 y[n] &= \sum_{k=1}^{N-1} b_k y[n-k] + \sum_{k=0}^{N-1} a_k x[n-k] \quad s_k = \omega_c e^{j\frac{\pi}{2N}(2k+N+1)}, \quad k = 0, \dots, 2N-1 \\
 H_s(s) &= \sum_{k=0}^{N-1} \frac{A_k}{s - s_k} & H_z(z) &= T \sum_{k=0}^{N-1} \frac{A_k}{1 - e^{s_k T} z^{-1}} \\
 s &= \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}} & z &= \frac{2 + sT}{2 - sT} \\
 \omega &= \frac{2}{T} \operatorname{tg} \frac{\Omega}{2} & \Omega &= 2 \operatorname{arctg} \frac{\omega T}{2} \\
 H_b(s)H_b(-s) &= \frac{1}{1 + \left(\frac{s}{j\omega_c}\right)^{2N}} & |H_b(j\omega)|^2 &= \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2N}}
 \end{aligned}$$



12 Filtro FIR

$$\text{Janela retangular} \quad w[n] = \begin{cases} 1 & , \quad 0 \leq n \leq M \\ 0 & , \quad \text{outro } n \end{cases}$$

$$\text{Janela Barlett} \quad w[n] = \begin{cases} \frac{2n}{M} & , \quad 0 \leq n \leq M/2 \\ 2 - \frac{2n}{M} & , \quad M/2 < n \leq M \\ 0 & , \quad \text{outro } n \end{cases}$$

$$\text{Janela Hanning} \quad w[n] = \begin{cases} 0.5 - 0.5 \cos(2\pi n/M) & , \quad 0 \leq n \leq M \\ 0 & , \quad \text{outro } n \end{cases}$$

$$\text{Janela Hamming} \quad w[n] = \begin{cases} 0.54 - 0.46 \cos(2\pi n/M) & , \quad 0 \leq n \leq M \\ 0 & , \quad \text{outro } n \end{cases}$$

$$\text{Janela de Kaiser} \quad w[n] = \begin{cases} \frac{I_0(\beta(1-[(n-\alpha)/\alpha]^2)^{1/2})}{I_0(\beta)} & , \quad 0 \leq n \leq M \\ 0 & , \quad \text{outro } n \end{cases}$$

$$\text{onde } \alpha = M/2$$

$$A = -20 \log(\delta), \quad \text{onde } \delta \text{ é a atenuação do filtro}$$

$$\beta = \begin{cases} 0.1102(A - 8.7) & , \quad A > 50 \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21) & , \quad 21 \leq A \leq 50 \\ 0.0 & , \quad A < 21 \end{cases}$$

$$\Delta\Omega = \Omega_s - \Omega_p$$

$$M = \frac{A - 8}{2.285\Delta\Omega}$$

$$\text{Resposta impulsional do filtro ideal} \quad h[n] = \frac{\sin(\Omega_c n)}{\pi n}$$

13 DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-kn}$$

$$\text{onde } W_N = e^{-j2\pi/N}$$



COMPARISON OF COMMONLY USED WINDOWS

Window Type	Peak Sidelobe Amplitude (Relative)	Approximate Width of Mainlobe	Peak Approximation Error $20 \log_{10} \delta$ (dB)	Equivalent Kaiser Window β	Transition Width of Equivalent Kaiser Window
Rectangular	-13	$4\pi/(M + 1)$	-21	0	$1.81\pi/M$
Bartlett	-25	$8\pi/M$	-25	1.33	$2.37\pi/M$
Hanning	-31	$8\pi/M$	-44	3.86	$5.01\pi/M$
Hamming	-41	$8\pi/M$	-53	4.86	$6.27\pi/M$
Blackman	-57	$12\pi/M$	-74	7.04	$9.19\pi/M$