Propriedades logarítmicas:

$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$

$\ln(x!) = x\ln(x) - x$

$\ln(x^y) = y\ln(x)$

Função de partição:

N_i partículas indistinguíveis:

$\mathcal{Z} = \frac{Z^{N_i}}{N_i!}$

N_i partículas distinguíveis:

$\mathcal{Z} = Z^{N_i}$

Apenas uma partícula:

$\mathcal{Z} = Z$

Sistema paramagnético:

$\varepsilon_\pm = \mp \mu H$

$\bar{\mu} = \sum_m \mu_m p_m = \mu \tanh(\beta\mu H)$

$\beta = \frac{1}{k_B T}$ $\qquad U = -\frac{\partial \ln(Z)}{\partial\beta}$

$R = N_A k_B$ $\qquad P = \frac{1}{\beta}\frac{\partial \ln(Z)}{\partial V}$

$\boxed{C_V = \left(\frac{\partial U}{\partial T}\right)_V}$ $\qquad \frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_V$

$\boxed{S = k_B \ln(\mathcal{Z}) + \frac{U}{T}}$

Macroestado: $S = k_B \sum_i p_i \ln(p_i)$

$Z = \sum_i e^{-\beta\varepsilon_i}$ $\qquad m_i = N p_i$

$p_i = \frac{e^{-\beta\varepsilon_i}}{Z}$ $\qquad \bar{\varepsilon} = \sum p_i \varepsilon_i$

Microestado:

$S = k_B \ln(\Omega)$, $\Omega = \frac{N!}{m_1! m_2! \dots}$

$F = -k_B T \ln(\mathcal{Z})$ $\qquad m = n M_c$, $N = m N_A$ $\xrightarrow{N=1}$ $\Rightarrow m = \frac{M_c}{N_A}$

Distribuição microcanónica: $\frac{1}{\Omega} = p_i$

" canónica: $p_i = \frac{e^{-\beta\varepsilon_i}}{Z}$

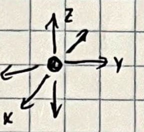
Gás monoatómico:

$Z = \frac{V}{h^3}\left(\frac{2\pi m}{\beta}\right)^{3/2}$

$U = \frac{3}{2} N k_B T$

$C_V = \frac{3}{2} N k_B$

$C_P = \frac{5}{2} N k_B$



Apenas movi. de translação

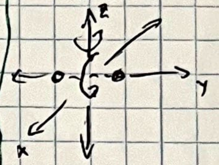
Gás diatómico:

$Z = \frac{V}{h^3}\left(\frac{2\pi m}{\beta}\right)^{5/2}$

$U = \frac{5}{2} N k_B T$

$C_V = \frac{5}{2} N k_B$

$C_P = \frac{7}{2} N k_B$



Não há vibração

Velocidades: $\langle \vec{v}_x \rangle^2 = 0$ $\qquad \langle v_x^2 \rangle = \frac{k_B T}{m}$

$v_{mp} = \frac{2 k_B T}{m}$ $\qquad \langle v^2 \rangle = \frac{3 k_B T}{m}$ $\qquad \langle v \rangle^2 = \frac{8 k_B T}{\pi m}$

$f(v)\,dv = 4\pi \frac{N}{V} v^2 e^{-\frac{\beta m v^2}{2}} dv$

$f(\vec{v})\,d\vec{v} = \frac{N}{V}\left(\frac{\beta m}{2\pi}\right)^{3/2} e^{-\frac{\beta m v^2}{2}} d\vec{v}$

Lei de Wien:

$\lambda_M T = 2{,}898 \times 10^{-3}\ \mathrm{m\,K}$

Lei STEFAN:

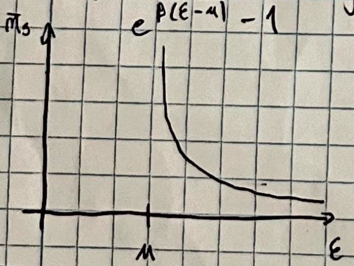
$\varepsilon(T) = \sigma T^4$, $\sigma = 5{,}67 \times 10^{-8}\ \mathrm{W/m^2 K^4}$

Bosões - Estatística de Bose-Einstein (Fotões)

$\mu = \varepsilon_B$

- Partículas independentes
- É possível várias partículas no mesmo estado (Não obedece ao princípio de exclusão de Pauli)

$\bar{m}(\varepsilon)_B = \frac{1}{e^{\beta(\varepsilon-\mu)} - 1}$ $\qquad g(\vec{u})\,d\vec{u} = \frac{V}{(2\pi)^3} d\vec{u}$



$g(\vec{u})\,d\vec{u} = \boxed{\frac{V}{(2\pi)^3} u^2 \sin\theta\, du\, d\theta\, d\phi}$

$\frac{V}{2\pi^2} u^2 du$

$\boxed{\varepsilon = \frac{\hbar^2 u^2}{2m}} \Rightarrow u = \sqrt{\frac{2m}{\hbar^2}}\sqrt{\varepsilon}$

$u^2 du = \frac{2m}{\hbar^2}\varepsilon \cdot \sqrt{\frac{2m}{\hbar^2}} \cdot \frac{1}{2\sqrt{\varepsilon}} d\varepsilon =$

$= \left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot \frac{\sqrt{\varepsilon}}{2} d\varepsilon$

$= \underbrace{\frac{V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon}\, d\varepsilon}_{g(\varepsilon)}$

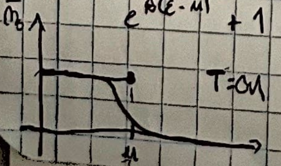
Fermiões - Estatística de Fermi-Dirac (Electrões)

$\mu = \varepsilon_F$

- Partículas independentes
- Apenas uma partícula em cada estado (Princípio da Exclusão de Pauli)

$\varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$

$\bar{m}(\varepsilon)_F = \frac{1}{e^{\beta(\varepsilon-\mu)} + 1}$ $\qquad g(\vec{u})\,d\vec{u} = \frac{V}{(2\pi)^3} d\vec{u}$ (Meta)



$g(\varepsilon)\,d\varepsilon = \frac{V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{\varepsilon}\, d\varepsilon$

$N = \int_0^{\varepsilon_F} 2 \cdot 1 \frac{V}{(2\pi)^2} \cdot \left(\frac{2m}{\hbar^2}\right)^{3/2} (\varepsilon)^{\frac{1}{2}} d\varepsilon$

$= \frac{2V}{(2\pi)^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \frac{\varepsilon_F^{3/2}}{3} \cdot 2 \Rightarrow \varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{\frac{2}{3}}$