Problema - 1

a) Na aproximação indicada (desprezando efeito de borda):

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{z}$$ (Sendo $\hat{z}$ o versor // eixo cilindro)

$$\sigma = \frac{It}{\pi a^2}$$

$$\boxed{\vec{E} = \frac{It}{\pi a^2 \varepsilon_0}\hat{z}}$$

A densidade de corrente de deslocamento no hiato é:

$$\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{I}{\pi a^2}\hat{z}$$

A lei de Ampère-Maxwell impõe então (contorno de raio s no hiato)

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \int \vec{J}_d \cdot \hat{n}\, d\Sigma$$

$$2\pi \cancel{s}\, B(s) = \mu_0 \frac{I}{\cancel{\pi} a^2} \cancel{\pi} s^{\cancel{2}}$$

$$\boxed{\vec{B}(s) = \frac{\mu_0 I}{2\pi a^2} s\,\hat{\phi}}$$

b) $$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{I^2 t}{\pi^2 a^2 \varepsilon_0\, 2\, a^2} s\,(\hat{z}\times\hat{\phi}) = \frac{I^2 t\, s}{2\pi^2 a^4 \varepsilon_0}(-\hat{s})$$

$$u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2 = \frac{I^2 t^2}{\pi^2 a^4 \varepsilon_0^{\cancel{2}}}\cdot\frac{1}{2}\cancel{\varepsilon_0} + \frac{1}{2}\frac{I^2 \mu_0 s^2}{4\pi^2 a^4} =$$

$$= \frac{\mu_0 I^2}{2\pi^2 a^4}\left[c^2 t^2 + \left(\frac{s}{2}\right)^2\right]$$

c) U_{em} no gap:

$$d^3r = w\, 2\pi s\, ds$$

$$U_{em} = w\int_0^a u\, 2\pi s\, ds = 2\pi w \frac{\mu_0 I^2}{2\pi^2 a^4}\int_0^a \left[c^2t^2 + \left(\frac{s}{2}\right)^2\right] s\, ds$$

$$= \frac{\mu_0 w I^2 a^2}{2\pi a^4}\left[c^2t^2 - \frac{a^2}{16}\right]$$

Problema - 2

a) Os campos no interior da esfera são:

$$\vec{E} = -\frac{\vec{P}}{3\varepsilon_0} = -\frac{P}{3\varepsilon_0}(-\hat{y} + \hat{z})$$

$$\vec{B} = -\frac{2}{3}\mu_0 \frac{1}{\sqrt{2}} M (\hat{z} + \hat{x})$$

$$\vec{p}_{ele} = \vec{D}\times\vec{B} = \varepsilon_0(\vec{E}\times\vec{B}) = \frac{P}{3}\cdot\frac{2}{3}\mu_0\frac{1}{\sqrt{2}} M \underbrace{(-\hat{y}+\hat{z})\times(\hat{z}+\hat{x})}$$

(densidade volúmica de momento linear)

$$-\hat{y}\times\hat{z} - \hat{y}\times\hat{x} + \hat{z}\times\hat{x}$$

$$-\hat{x} + \hat{z} + \hat{y}$$

$$\vec{p}_{em} = \frac{PM\mu_0 2}{9\sqrt{2}}\left[-\hat{x} + \hat{z} + \hat{y}\right]$$

(densidade volúmica de momento linear electromagnético no interior da esfera)

$$\vec{P}_{em} = \frac{4}{3}\pi R^3\, \vec{p}_{em}$$ = (momento linear no interior da esfera)

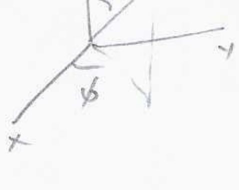
b) A desmagnetização da esfera (sem a realização de trabalho mecânico sobre ela: por exemplo, aquecendo uma esfera com uma magnetização espontânea uniforme com a indução) destrói o momento linear armazenado nos campos. A conservação do momento linear implica o surgimento de um momento mecânico equivalente.

Quantitativamente, é necessário analisar o momento linear total (dentro e fora da esfera). Fora da esfera os campos são os correspondentes a um perfeito dipolo eléctrico ($\vec{p} = \frac{4}{3}\pi R^3 \vec{P}$) e a um perfeito dipolo magnético.

Problema -3 (ver Griffiths, pág. 353; ou aulas)

Consideremos uma superfície envolvendo a esfera que consiste no disco equatorial e na "tampa" de raio R (hemisférica).

Na tampa: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \hat{r}$; (No interior da esfera $E=0$ e plano equatorial)



$$\hat{r} = \sin\theta \cos\phi \, \hat{x} + \sin\theta \sin\phi \, \hat{y} + \cos\theta \, \hat{z}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2} \left[\sin\theta \cos\phi \, \hat{x} + \sin\theta \sin\phi \, \hat{y} + \cos\theta \, \hat{z} \right]$$

A força $\parallel \hat{z}$ (por simetria) $\Rightarrow$ T_{zx}, T_{zy}, T_{zz} são os únicos elementos relevantes

$$T_{zx} = \varepsilon_0 E_x E_z = \varepsilon_0 \left(\frac{Q}{4\pi \varepsilon_0 R^2} \right)^2 \sin\theta \cos\theta \cos\phi$$

$$T_{zy} = \varepsilon_0 E_z E_y = \varepsilon_0 \left(\frac{Q}{4\pi \varepsilon_0 R^2} \right)^2 \sin\theta \cos\theta \sin\phi$$

$$T_{zz} = \frac{\varepsilon_0}{2} \left(E_z^2 - E_x^2 - E_y^2 \right) = \frac{\varepsilon_0}{2} \left(\frac{Q}{4\pi \varepsilon_0 R^2} \right)^2 \left(\cos^2\theta - \sin^2\theta \right)$$

$$df_z = T_{zx}\, d\Sigma_x + T_{zy}\, d\Sigma_y + T_{zz}\, d\Sigma_z = \frac{\varepsilon_0}{2} \left(\frac{Q}{4\pi \varepsilon_0 R^2} \right)^2 \cdot$$

$$\cdot \sin\theta \cos\theta \, d\theta \, d\phi$$

$$\left(d\vec{\Sigma} = R^2 \sin\theta \, d\theta \, d\phi \, \hat{r} \right)$$

$$\left(F_z \right)_{\text{Tamp}} = \frac{\varepsilon_0}{2} \left(\frac{Q}{4\pi \varepsilon_0 R^2} \right)^2 2\pi \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta$$

$$= \frac{1}{4\pi \varepsilon_0} \frac{Q^2}{8 R^2}$$

Parte - II

Problema - 4

a) $$\vec{E}(z,t) = \vec{E}_0 \, e^{i\delta_E} \, e^{i(\tilde{k}z - \omega t)}$$

$$\vec{B}(z,t) = \vec{B}_0 \, e^{i\delta_B} \, e^{i(\tilde{k}z - \omega t)}$$

$$(\tilde{k} = k + i\eta)$$

Dado que o meio é electricamente neutro

$\nabla \cdot \vec{E} = \nabla \cdot \vec{B} = 0 \Rightarrow \vec{B}_0$ e $\vec{E}_0$ são perpendicula[res] a $\hat{z}$

(ondas são transversais).

escolhemos $\vec{E}_0 \parallel \hat{x}$ $\quad (\vec{k} \parallel \hat{z})$

$$\vec{E}(z,t) = E_0 \hat{x} \, e^{i\delta_E} \, e^{-\eta z} \, e^{i(kz - \omega t)}$$

Então:

$$\nabla \times \vec{E} = -\dot{\vec{B}} \Rightarrow i(\tilde{\vec{k}} \times \vec{E}_0) = -i\omega \tilde{\vec{B}}_0 \Rightarrow \vec{B}_0 \parallel \hat{y} \; e$$

$$\tilde{\vec{k}} = |\vec{k}| \, e^{i \arctg\left(\frac{\eta}{k}\right)}$$

$$E_0 e^{i\delta_E} \, K e^{i\phi} = \omega B_0 e^{i\delta_B}$$

$$B_0 = \frac{K e^{i\phi}}{\omega} E_0 \, e^{i(\delta_E - \delta_B)} \Rightarrow \begin{cases} B_0 = \dfrac{K}{\omega} E_0 \\ \delta_B - \delta_E = \phi \end{cases}$$

$K = |\tilde{k}|$

b) $\delta_B - \delta_E = \operatorname{arctg}\left(\frac{\eta}{\kappa}\right)$

$$\eta \sim \kappa \sim \omega \sqrt{\frac{\cancel{\varepsilon}\mu}{2}} \cdot \sqrt{\frac{\sigma}{\cancel{\varepsilon}\omega}} = \sqrt{\frac{\sigma\mu\omega}{2}}$$

Logo $\delta_B - \delta_E = \operatorname{arctg}(1) = \frac{\pi}{4}$

c) $\kappa_1 = \frac{2\pi}{\lambda} = \sqrt{\frac{\sigma\mu\omega}{2}} \rightarrow \lambda = \frac{2\pi\sqrt{2}}{\sqrt{\sigma\mu\omega}} \sim \frac{1}{\sqrt{10^7\, 10^{-7}\, 10^6}}\ m$

$$v_f = \frac{\omega}{\kappa_1} = \frac{\omega\sqrt{2}}{\sqrt{\sigma\mu\omega}} = \sqrt{\frac{2\omega}{\sigma\mu_0}} \sim \sqrt{\frac{10^6}{10^7 \cdot 10^{-7}}}\ m \cdot s^{-1}$$

Probleme - 5

a) $\vec{E} = -\nabla\phi - \frac{\partial \vec{A}}{\partial t} \rightarrow \vec{E} = -\frac{\partial \vec{A}}{\partial t}$

$\vec{B} = \nabla \times \vec{A} \rightarrow \vec{B} = -\frac{\partial A_z}{\partial x}\hat{y}$

$$\frac{\partial \vec{A}}{\partial t} = \frac{\kappa\mu_0}{2\cancel{c}}[ct - |x|] \cdot \cancel{c}\,\hat{z} \quad \text{se } |x| < ct$$

$$= 0 \quad \text{se } |x| > ct$$

$$\vec{E} = \begin{cases} -\frac{\kappa\mu_0}{2}[ct - |x|]\,\hat{z} & ; x < ct \\ 0 & x > ct \end{cases}$$

$$\frac{\partial A_z}{\partial x} = \frac{\partial A_z}{\partial |x|} \cdot \frac{\partial |x|}{\partial x} = -\frac{k\mu_0}{2c}\left[ct - |x|\right] \cdot \left(\substack{+\\-}\right)$$

$+ \rightarrow x > 0$

$- \quad x < 0$

$$= \begin{cases} -\dfrac{k\mu_0}{2c}\left[ct - x\right] & \text{se } x > 0 \\ +\dfrac{k\mu_0}{2c}\left[ct + x\right] & \text{se } x < 0 \end{cases}$$

$$\vec{B} = \begin{cases} \hat{y}\,\dfrac{k\mu_0}{2c}\left[ct - x\right] \; ; x > 0 \\ -\dfrac{k\mu_0}{2c}\left[ct + x\right]\hat{y} \; ; x < 0 \end{cases}$$

b) $\nabla \cdot \vec{E} = \nabla \cdot \vec{B} = 0 \Rightarrow \boxed{\rho = 0}$

$$\nabla \times \vec{E} = \mp \frac{\mu_0 k}{2}\,\hat{y} \qquad \frac{\partial \vec{E}}{\partial t} = -\frac{\mu_0 k c}{2}\,\hat{z}$$

$$\nabla \times \vec{B} = -\frac{\mu_0 k}{2c}\,\hat{z} \qquad \frac{\partial \vec{B}}{\partial t} = \pm \frac{\mu_0 k}{2}\,\hat{y}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} \Rightarrow \boxed{\vec{J} = 0}$$

Mas:

$(*)$ $\quad \dfrac{B_1^{\parallel}}{\mu_1} - \dfrac{1}{\mu_2} B_2^{\parallel} = \vec{K}_f \times \hat{n}$ (cond. fronteira imposta pelas eq. Maxwell)

$\vec{B}$ tem uma descontinuidade em $x = 0 \Rightarrow$ uma corrente superficial no plano YZ. $(\hat{n} = \hat{x}$

$$X=0 \;\rightarrow\; (*) \Rightarrow Kt = \vec{K} \times \hat{x} \;\Rightarrow\; \boxed{\vec{K} = kt\,\hat{z}}$$

densidad sup. de corriente $\parallel \hat{z}$

Problema - 6

a) $$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\,\hat{z}\cdot 2\int_0^{\sqrt{c^2t^2-s^2}} \frac{k\left(t - \frac{\sqrt{s^2+z^2}}{c}\right)}{\sqrt{s^2+z^2}}\,dz =$$

$$= \frac{\mu_0 k}{2\pi}\,\hat{z}\left[t\int_0^{\sqrt{c^2t^2-s^2}} \frac{dz}{\sqrt{s^2+z^2}} - \frac{1}{c}\int_0^{\sqrt{c^2t^2-s^2}} dz \right]$$

$$= \frac{\mu_0 k}{2\pi}\,\hat{z}\left[t\ln\left[\frac{ct - \sqrt{c^2t^2-s^2}}{s}\right] - \frac{1}{c}\sqrt{c^2t^2-s^2}\right]$$

b) $$\vec{E} = -\frac{\partial \vec{A}}{\partial t} = -\frac{\mu_0 k}{2\pi}\,\hat{z}\left\{ \ln\left[\frac{ct-\sqrt{c^2t^2-s^2}}{s}\right] + t\left(\frac{s}{ct-\sqrt{c^2t^2-s^2}}\right)\cdot\right.$$

$$\left.\cdot\left(\frac{1}{s}\right)\left(c + \frac{1}{2}\frac{2c^2t}{\sqrt{c^2t^2-s^2}}\right) - \frac{1}{2c}\frac{2c^2t}{\sqrt{c^2t^2-s^2}} \right\}$$

$$\vec{E} = -\frac{\mu_0 k}{2\pi} \ln\left[\frac{ct - \sqrt{c^2t^2 - s^2}}{s}\right]\hat{z}$$