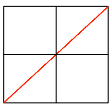
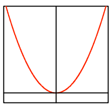
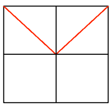
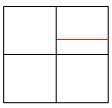
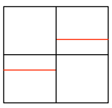
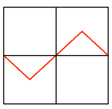
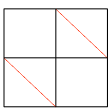
Algumas séries de Taylor

função = série de Taylor-Maclaurin	disco de convergência
$f(x) = \sum_{n=0}^{\infty} c_n\, z^n \qquad \text{where } c_n = \frac{f^{(n)}(0)}{n!}$	$\lvert z\rvert < R = (\limsup \lvert c_n\rvert^{1/n})^{-1}$
$(1-z)^{-1} = \sum_{n=0}^{\infty} z^n = 1 + z + z^2 + z^3 + \dots$	$\lvert z\rvert < 1$
$e^z = \sum_{n=0}^{\infty} \frac{1}{n!}\, z^n = 1 + z + \frac{1}{2}z^2 + \frac{1}{6}z^3 + \dots$	$\lvert z\rvert < \infty$
$\cosh(z) = \sum_{n=0}^{\infty} \frac{1}{(2n)!}\, z^{2n} = 1 + \frac{1}{2}z^2 + \frac{1}{24}z^4 + \dots$	$\lvert z\rvert < \infty$
$\sinh(z) = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}\, z^{2n+1} = z + \frac{1}{6}z^3 + \frac{1}{120}z^5 + \dots$	$\lvert z\rvert < \infty$
$\cos(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, z^{2n} = 1 - \frac{1}{2}z^2 + \frac{1}{24}z^4 + \dots$	$\lvert z\rvert < \infty$
$\sin(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}\, z^{2n+1} = z - \frac{1}{6}z^3 + \frac{1}{120}z^5 + \dots$	$\lvert z\rvert < \infty$
$\log(1+z) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\, z^n = z - \frac{1}{2}z^2 + \frac{1}{3}z^3 - \frac{1}{4}z^4 + \dots$	$\lvert z\rvert < 1$
$(1+z)^{\alpha} = 1 + \sum_{n=1}^{\infty} \binom{\alpha}{n}\, z^n = 1 + \alpha z + \frac{\alpha(\alpha-1)}{2}z^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6}z^3 + \dots$	$\lvert z\rvert < 1$
$\arcsin(z) = \sum_{n=0}^{\infty} \frac{(2n)!}{4^n\,(n!)^2\,(2n+1)}\, z^{2n+1}$	$\lvert z\rvert < 1$

Séries de Fourier em $[-\pi,\pi]$

	função $\sim$ série de Fourier ($x\in[-\pi,\pi]$)	coefficientes de Fourier ($n\in\mathbb{Z}$)
complexa	$f(x)\sim\sum_{-\infty}^{\infty} c_n e^{inx}$	$c_n := \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-inx} f(x)\,dx$
real	$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n\cos(nx)+b_n\sin(nx)\right)$	$a_n := \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$ $b_n := \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$

Algumas séries de Fourier em $[-\pi,\pi]$

função em $[-\pi,\pi]$	$\sim$ série de Fourier
 x	$\sim 2\left(\sin(x)-\frac{1}{2}\sin(2x)+\frac{1}{3}\sin(3x)-\frac{1}{4}\sin(4x)+\dots\right)$
 x^2	$\sim \frac{\pi^2}{3}-4\left(\cos(x)-\frac{1}{4}\cos(2x)+\frac{1}{9}\cos(3x)-\frac{1}{16}\cos(4x)+\dots\right)$
 $\lvert x\rvert$	$\sim \frac{\pi}{2}-\frac{4}{\pi}\left(\cos(x)+\frac{1}{9}\cos(3x)+\frac{1}{25}\cos(5x)+\frac{1}{49}\cos(7x)+\dots\right)$
 $\Theta(x) := \begin{cases} 1 & \text{se } 0\le x<\pi \\ 0 & \text{se } -\pi\le x<0 \end{cases}$	$\sim \frac{1}{2}+\frac{2}{\pi}\left(\sin(x)+\frac{1}{3}\sin(3x)+\frac{1}{5}\sin(5x)+\frac{1}{7}\sin(7x)+\dots\right)$
 $2\Theta(x)-1 := \begin{cases} 1 & \text{se } 0\le x<\pi \\ -1 & \text{se } -\pi\le x<0 \end{cases}$	$\sim \frac{4}{\pi}\left(\sin(x)+\frac{1}{3}\sin(3x)+\frac{1}{5}\sin(5x)+\frac{1}{7}\sin(7x)+\dots\right)$
 $Z(x) := \begin{cases} \pi-x & \text{se } \frac{\pi}{2}\le x<\pi \\ x & \text{se } -\frac{\pi}{2}\le x<\frac{\pi}{2} \\ -\pi-x & \text{se } -\pi\le x<-\frac{\pi}{2} \end{cases}$	$\sim \frac{4}{\pi}\left(\sin(x)-\frac{1}{9}\sin(3x)+\frac{1}{25}\sin(5x)-\frac{1}{49}\sin(7x)+\dots\right)$
 $S(x) := \begin{cases} \pi-x & \text{se } 0\le x<\pi \\ -\pi-x & \text{se } -\pi\le x<0 \end{cases}$	$\sim 2\left(\sin(x)+\frac{1}{2}\sin(2x)+\frac{1}{3}\sin(3x)+\frac{1}{4}\sin(4x)+\dots\right)$

Transformadas de Fourier (unitária, frequência)

	(espaço do *tempo* t)	(espaço da *frequência* ξ)
	$f(t) = \int_{-\infty}^{\infty} e^{2\pi i \xi t} \widehat{f}(\xi)\, d\xi$	$\widehat{f}(\xi) = \int_{-\infty}^{\infty} e^{-2\pi i \xi t} f(t)\, dt$
(linearidade ($\lambda, \mu \in \mathbb{C}$))	$\lambda f(t) + \mu g(t)$	$\lambda \widehat{f}(\xi) + \mu \widehat{g}(\xi)$
(conjugação)	$\overline{f(t)}$	$\overline{\widehat{f}(-\xi)}$
(homotetia ($\lambda \neq 0$))	$f(t/\lambda)$	$\lambda\, \widehat{\ }(\lambda\xi)$
(translação/modulação)	$f(t-a)$	$e^{-2\pi i a \xi} \widehat{f}(\xi)$
(modulação/translação)	$e^{2\pi i b t} f(t)$	$\widehat{f}(\xi - b)$
(derivação/multiplicação)	$\frac{\partial}{\partial t} f\,(t)$	$2\pi i \xi\, \widehat{f}(\xi)$
(multiplicação/derivação)	$-2\pi i t\, f\,(t)$	$\frac{\partial}{\partial \xi} \widehat{f}\,(\xi)$
(convolução/produto)	$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau) g(\tau)\, d\tau$	$\widehat{f}(\xi)\, \widehat{g}(\xi)$
(produto/convolução)	$f(t)\, g(t)$	$(\widehat{f} * \widehat{g})(\xi) = \int_{-\infty}^{\infty} \widehat{f}(\xi - \eta)\, \widehat{g}(\eta)\, d\eta$
(energia)	$E = \Vert f \Vert^2 = \int_{-\infty}^{\infty} \vert f(t) \vert^2\, dt$	$E = \Vert \widehat{f} \Vert^2 = \int_{-\infty}^{\infty} \vert \widehat{f}(\xi) \vert^2\, d\xi$
(pulso/sinc)	$\mathrm{rect}(t) := \begin{cases} 1 & \text{se } \vert t \vert < 1/2 \\ 0 & \text{se } \vert t \vert \geq 1/2 \end{cases}$	$\mathrm{sinc}(\xi) := \frac{\sin(\pi \xi)}{\pi \xi}$
(sinc/pulso)	$\mathrm{sinc}(t) := \frac{\sin(\pi t)}{\pi t}$	$\mathrm{rect}(\xi) := \begin{cases} 1 & \text{se } \vert \xi \vert < 1/2 \\ 0 & \text{se } \vert \xi \vert \geq 1/2 \end{cases}$
(Gaussiana)	$e^{-\pi t^2}$	$e^{-\pi \xi^2}$
(secante hiperbólica)	$\mathrm{sech}(\pi t) := \frac{1}{\cosh(\pi t)}$	$\mathrm{sech}(\pi \xi)$
(exponencial em t, $a > 0$)	$e^{-a\vert t \vert}$	$\frac{2a}{4\pi^2 \xi^2 + a^2}$

Transformadas de Fourier (não-unitária, frequência angular)

	(espaço do *tempo* t)	(espaço da *frequência angular* $\omega = 2\pi\xi$)
	$f(t) = \int_{-\infty}^{\infty} e^{i\omega t} F(\omega)\, \frac{d\omega}{2\pi}$	$F(\omega) = \int_{-\infty}^{\infty} e^{-i\omega t} f(t)\, dt$
(linearidade ($\lambda, \mu \in \mathbb{C}$))	$\lambda f(t) + \mu g(t)$	$\lambda F(\omega) + \mu G(\omega)$
(conjugação)	$\overline{f(t)}$	$\overline{F(-\omega)}$
(homotetia ($\lambda \neq 0$))	$f(t/\lambda)$	$\lambda\, F(\lambda\omega)$
(translação/modulação)	$f(t-a)$	$e^{-ia\omega} F(\omega)$
(modulação/translação)	$e^{ibt} f(t)$	$F(\omega - b)$
(derivação/multiplicação)	$\frac{\partial}{\partial t} f(t)$	$i\omega\, F(\omega)$
(multiplicação/derivação)	$-it\, f(t)$	$\frac{\partial}{\partial \omega} F(\omega)$
(convolução/produto)	$(f * g)(t) = \int_{-\infty}^{\infty} f(t-\tau) g(\tau)\, d\tau$	$F(\omega)\, G(\omega)$
(produto/convolução)	$f(t)\, g(t)$	$\frac{1}{2\pi}(F * G)(\omega) = \int_{-\infty}^{\infty} F(\omega - \nu)\, G(\nu)\, \frac{d\nu}{2\pi}$
(energia)	$E = \lVert f \rVert^2 = \int_{-\infty}^{\infty} \lvert f(t) \rvert^2\, dt$	$E = \lVert F \rVert^2 / 2\pi = \int_{-\infty}^{\infty} \lvert F(\omega) \rvert^2\, \frac{d\omega}{2\pi}$
(pulso/sinc)	$\mathrm{rect}(t) := \begin{cases} 1 & \text{se } \lvert t \rvert < 1/2 \\ 0 & \text{se } \lvert t \rvert \geq 1/2 \end{cases}$	$\mathrm{sinc}(\omega/2\pi) = \frac{\sin(\omega/2)}{\omega/2}$
(sinc/pulso)	$\mathrm{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$	$\mathrm{rect}(\omega/2\pi) = \begin{cases} 1 & \text{se } \lvert \omega \rvert < \pi \\ 0 & \text{se } \lvert \omega \rvert \geq \pi \end{cases}$
(Gaussiana)	$e^{-\pi t^2}$	$e^{-\omega^2/4\pi}$
(secante hiperbólica)	$\mathrm{sech}(\pi t) := \frac{1}{\cosh(\pi t)}$	$\mathrm{sech}(\omega/2)$
(exponencial em t, $a > 0$)	$e^{-a\lvert t \rvert}$	$\frac{2a}{\omega^2 + a^2}$
(exponencial em ω, $b > 0$)	$\frac{b/\pi}{t^2 + b^2}$	$e^{-b\lvert \omega \rvert}$