

TP - 22/02/2022

1.

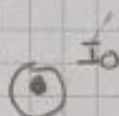
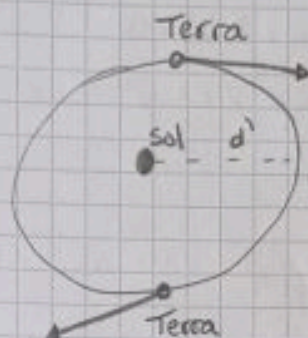
$$z = \frac{3+i}{1+2i} - i e^{i\frac{\pi}{3}} = \frac{(3+i)(1-2i)}{(1+2i)(1-2i)} - i \cos\left(\frac{\pi}{3}\right) - i^2 \sin\left(\frac{\pi}{3}\right)$$

$$= \frac{3+i-6i+2}{1+4} - \frac{i}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{5-5i}{5} - \frac{i}{2} + \frac{\sqrt{3}}{2} = 1 + \frac{\sqrt{3}}{2} - i - \frac{i}{2} = 1 + \frac{\sqrt{3}}{2} - \frac{3i}{2}$$

2.

$$d' = 1,5 \times 10^{11} \text{ m}$$



40 órbitas completas de Io

$T = 425$ horas (1 órbita)

Comparar o tempo de 40 órbitas para quando a Terra se aproxima de Io e se afasta dele.

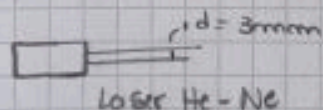
$$T_{40} = 40 \times 425 = \frac{17000 \text{ horas}}{24 \text{ h}} = 70,8 \text{ dias}$$

$$\Delta d = R \Delta \theta = 1,5 \times 10^{11} \cdot 2\pi \left(\frac{70,8}{365,25} \right) \approx 1,8 \times 10^{11} \text{ m}$$

$$c = \frac{\Delta d}{\Delta t} = \frac{2 \times (1,8 \times 10^{11})}{24 \times 60 \times 60 \times \frac{\text{min}}{\text{h}}} = 2,8 \times 10^8 \text{ m/s}$$

3.

$$\lambda = 632,8 \text{ nm} \quad R = \hat{x}$$



$$\vec{E}_0 = 900 \hat{z} \text{ V/cm}$$

Lei de Faraday

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{E} = 900 \hat{z} e^{i(kx - \omega t + \phi)}$$

$$\vec{B} = \vec{B}_0 e^{i(kx - \omega t + \phi)}$$

$$i\vec{k} \times \vec{E} = (i\omega \vec{B})$$

$$\frac{\vec{k}}{w} \times \vec{E} = \vec{B}$$

$$|\vec{B}_0| = \frac{|\vec{E}_0|}{c} (-\hat{y})$$

$$|\vec{B}_0| = \frac{900}{3 \times 10^8} (-\hat{y}) = -3 \times 10^{-6} \hat{y} \text{ Tesla}$$

①

$\vec{p} = ?$ e N° de fótons por segundo

$$\text{Fluxo de energia} = \frac{\text{Energia}}{\text{área} \times \text{tempo}} = \frac{\text{Potência}}{\text{Área}}$$

$$\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) = I$$

$$I = \frac{1}{2} \epsilon_0 c |E_0|^2 \Rightarrow$$

$$P = I \cdot A = \frac{1}{2} \epsilon_0 c (900)^2 \cdot \pi (1,5 \times 10^{-3})$$

$$= 7,6 \text{ mW}$$

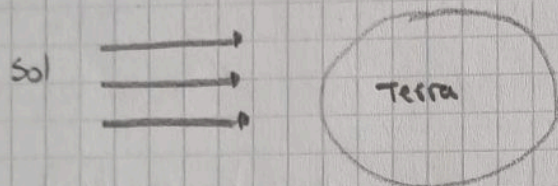
$$E_{\text{fóton}} = \hbar \omega = \hbar f = \frac{hc}{\lambda}$$

$$h = 6,63 \times 10^{-34} \text{ J} \cdot \text{s}$$

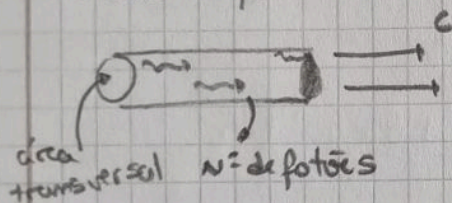
$$c = 3 \times 10^8 \text{ m/s}$$

$$P = \frac{E}{\Delta t} = \frac{7,6 \times 10^{-3}}{\frac{hc}{\lambda}} = 2,4 \times 10^{16} \text{ fótons/s}$$

4. $I = 1350 \text{ W/m}^2$ (no topo da atmosfera)



Modelo de fótons



$$I = \frac{\text{Energia}}{\text{Área} \cdot \text{tempo}} = \frac{N \hbar \omega}{A \Delta t}$$

$$u_{\text{en}} = \frac{N \hbar \omega}{A \Delta t} \rightarrow \text{densidade de energia}$$

$$\Delta p = N \hbar \vec{k}$$

↓
Transferência de momento

$$\vec{p}_{\text{fóton}} = \hbar \vec{k}$$

$$\frac{\Delta p}{\Delta t} = F = \frac{N \hbar k}{\Delta t}$$

$$\frac{\text{Força}}{\text{Área}} = \frac{P}{A} = \frac{N \hbar k}{A \Delta t} = \frac{N \hbar \omega}{A \Delta t c} = u_{\text{en}} = \frac{I}{c}$$

↓
pressão radiativa

Pressão refletida é o dobro da absorvida

$$P = \frac{1350}{3 \times 10^8}$$

→ Na aproximação de que todos os fótons são absorvidos dos termos que

$$F_{\text{radiativa}} = \frac{I}{c} \pi R_T^2$$

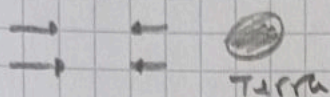
$$= 5,7 \times 10^8 \text{ N}$$

$$R_T = 6,37 \times 10^6 \text{ m}$$

$$F_{\text{Grav}} = - \frac{G \pi \rho_{\text{sol}} \pi \rho_{\text{terra}} r^2}{R_{\text{Terra-sol}}^2}$$

$$G = 6,7 \times 10^{-11} \text{ N m}^2 / \text{kg}^2; \rho_{\text{sol}} = 2 \times 10^{30} \text{ kg}; \rho_T = 6 \times 10^{24} \text{ kg}; R_{T-S} = 1,5 \times 10^{11} \text{ m}$$

$$|F_{\text{Grav}}| = 3,6 \times 10^{22} \text{ N}$$



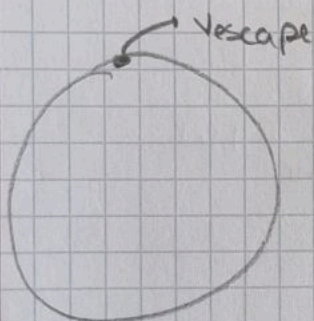
$$\text{Força} = P \cdot A = 2,8 \times 10^{-5} \text{ N}$$

$$R_{\text{Sol-Terra}} = 42,167 \text{ km}$$

$$F_{\text{grav}} = 790 \text{ N}$$

$$\Delta p = F \cdot \Delta t = 2,8 \times 10^{-5} \left[\frac{11}{2} \times 10^7 \text{ s} \right] = 440 \text{ kg m/s}$$

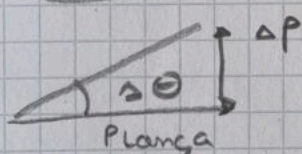
desvio do movimento durante 6 meses devido à pressão radiativa



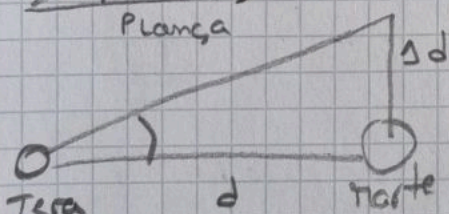
$$\frac{1}{2} m v_{\text{esc}}^2 = \frac{G M_T m}{R_{\text{Terra}}}$$

$$v_{\text{esc}} = \sqrt{2 \frac{G M_T}{R_{\text{Terra}}}}$$

$$P_{\text{lançamento}} = \pi_{\text{sol}} v_{\text{esc}} = 3,9 \times 10^2 \text{ kg m/s}$$



$$\Delta \theta \approx \frac{\Delta p}{P_{\text{lança}}} \approx 1,1 \times 10^{-5}$$



$$\frac{\Delta d}{d} = \Delta \theta \quad \Delta d = \Delta \theta \cdot d = 2,5 \times 10^6 \text{ m}$$

$$d = 2,3 \times 10^{11} \text{ m}$$

Faz uma grande diferença na chegada a Marte

1. $\vec{E}(\vec{r}, t) = 2000(\hat{x} + \hat{y}) \operatorname{Re} \left\{ e^{i(kz - \omega t + \varphi)} \right\} \left| \frac{V}{m} \right|$

a) amplitude complexa? E_0

$$\vec{E}(\vec{r}, t) = \operatorname{Re} \left\{ \vec{E}_0 e^{i(kz - \omega t)} \right\}$$

$$\vec{E}_0 = 2000 \left[\frac{\hat{x} + \hat{y}}{\sqrt{2}} \right] e^{0,3i} \cdot \sqrt{2} \frac{V}{m}$$

$$k = 1,307 \times 10^7 \text{ m}^{-1} = \frac{2\pi}{\lambda}$$

$$\omega = 2,22 \times 10^{15} \text{ R/s}$$

$$\varphi = 0,3$$

b) Faraday: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$i\vec{k} \times \vec{E} = i\omega \vec{B}$$

$$\vec{B} = \frac{\vec{k}}{\omega} \times \vec{E}$$

$$= \frac{k}{\omega} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & k_0 \\ E_x & E_y & 0 \end{vmatrix}$$

$$= \frac{k}{\omega} \sqrt{2} \cdot 2000 \left[\frac{-\hat{x} + \hat{y}}{\sqrt{2}} \right] \operatorname{Re} \left\{ e^{i(kz - \omega t + \varphi)} \right\}$$

$$= 13,3 \times 10^{-6} \text{ T} \approx 0,27 \text{ B}_{\text{Terre}}$$

$$v_{\text{fase}} = \frac{\omega}{|\vec{k}|} = 2,12 \times 10^8 \text{ m/s}$$

$$v_{\text{grupo}} = \frac{d\omega}{dk}$$

$$v_{\text{fase}} = \frac{c}{n} \quad n = \frac{c}{v_{\text{fase}}} = 1,41$$

$$B_{\text{Terre}} = 50 \times 10^{-6} \text{ T}$$

c) $\nabla \cdot \vec{E} = 0$

$$i\vec{k} \cdot \vec{E} = 0$$

d) $I = \left| \langle \vec{S} \rangle \right| = \left\langle \frac{1}{\mu_0} \vec{E} \times \vec{B} \right\rangle$

$$= \left\langle \frac{1}{\mu_0} \frac{m}{c} |\vec{E}|^2 \right\rangle$$

como:

$$|\vec{B}| = \frac{k}{\omega} |\vec{E}|$$

$$= \frac{m}{c} |\vec{E}|$$

$$= \frac{1}{2} \epsilon_0 c^2 \frac{m}{c} |E|^2$$

$$= \frac{1}{2} \epsilon_0 c m |E|^2$$

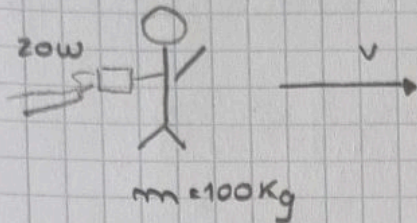
$$= 5 \text{ kW/cm}^2$$

Nota

$$\frac{1}{\mu_0 \epsilon_0} = c^2$$

$$\epsilon_0 = 8,85 \times 10^{-12} \frac{\text{As}}{\text{Vm}}$$

2.



$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

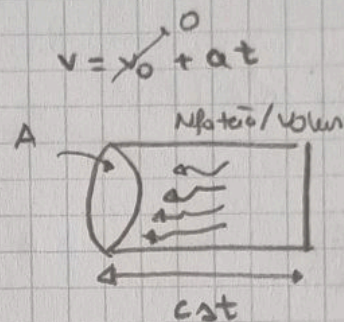
$$\vec{a} = \frac{\vec{F}}{m} = \frac{1}{m} \frac{\Delta \vec{p}}{\Delta t}$$

$$\vec{p} = \hbar \vec{k} \xrightarrow{1 \text{ fotão}} \frac{\hbar}{i} (\vec{i} \vec{k}) = \hbar \vec{k}$$

$$H = \frac{\hbar^2}{2m} \nabla^2 = \frac{p^2}{2m} = \frac{1}{2} m v^2$$

$$\frac{\Delta p}{\Delta t} = \frac{N(A c \Delta t) \hbar k}{\Delta t} = \frac{20 \text{ W}}{\text{Potência}} \frac{\text{K}}{\text{W}}$$

$$= \frac{\text{Potência}}{c}$$



$$\text{Potência} = \frac{\Delta \text{Energia}}{\Delta t} = \frac{N(A c \Delta t) \hbar \omega}{\Delta t}$$

$$\vec{a} = \frac{1}{100 \text{ Kg}} \frac{20 \text{ W}}{3 \times 10^8 \text{ m/s}} = 6,67 \times 10^{-10} \text{ m/s}^2$$

$$v = a t \quad \hookrightarrow \quad 10 = 6,67 \times 10^{-10} t \quad \hookrightarrow \quad \Delta t = 1,50 \times 10^{10} \text{ s} \approx 473 \text{ anos}$$

3.

$$f = 2,45 \text{ GHz}$$

a)

$$f = \frac{1}{T} \quad \hookrightarrow \quad T = 4,08 \times 10^{-10} \text{ s}$$

$$\lambda = \frac{c}{f} = 12,2 \text{ cm}$$

$$\int e^{-i\omega t} dt = \frac{1}{-i\omega} e^{-i\omega t}$$

$$v_m = \frac{e E_0}{im\omega} e^{-i\omega t}$$

$$v_{max} = \left| \frac{e E_0}{m\omega} \right| = 0,01 \text{ C}$$

$$|E_0| = \frac{0,01 \text{ C}}{e} m\omega$$

$$J = \frac{1}{2} E_0 C \left(\frac{0,01 \text{ C}}{e} m\omega \right)^2$$

$$= 5 \times 10^{18} \text{ W/m}^2$$

$$\epsilon_0 = 8,85 \times 10^{-12} \frac{\text{As}}{\text{Vm}}$$

$$m = 9,1 \times 10^{-31} \text{ Kg}$$

$$C = 3 \times 10^8 \text{ m/s}$$

$$e = 1,6 \times 10^{-19} \text{ C}$$

→ Força magnética é muito pequena porque a velocidade é muito pequena, então é desprezível.

$$5. \quad x(t) = -\frac{e E_0}{2m\omega} \frac{1}{\omega - \omega_0 + i\gamma} e^{-i\omega t}$$

$$\rightarrow \frac{(\omega - \omega_0) - i\gamma}{(\omega - \omega_0)^2 + \gamma^2}$$

$$\gamma = 0,01 \omega_0 \text{ (valor elevado)}$$

$$\lambda = 532 \text{ nm}$$

$$\tan \phi = \frac{-\gamma}{(\omega - \omega_0)} = \frac{\gamma}{\omega_0 - \omega}$$

$$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}} = \frac{\gamma}{\omega_0 - \omega}$$

$$\sqrt{3} \gamma = \omega_0 - \omega \rightarrow \omega = \omega_0 - \sqrt{3} \gamma$$

$$= \omega_0 - \sqrt{3} \times 0,01 \omega_0 = \omega_0 \frac{(1 - 0,01\sqrt{3})}{0,98}$$

dados

$$J = 10^5 \text{ W/m}^2$$

h

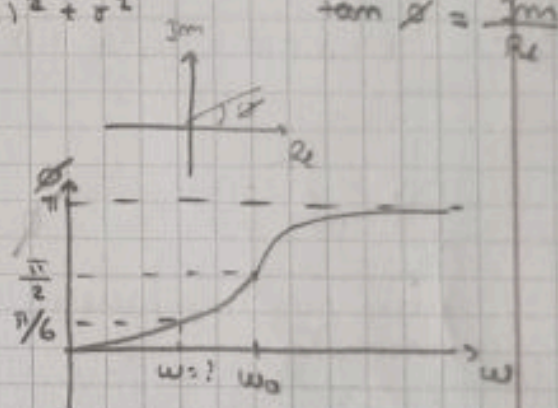
$$E_0 = 8680 \text{ V/m}$$

$$e = 1,6 \times 10^{-19} \text{ C}$$

$$m = 9,1 \times 10^{-31} \text{ Kg}$$

$$\text{Quando } \omega = \omega_0 \Rightarrow x(t) = -\frac{e E_0}{2m\omega} \frac{1}{i\gamma} e^{-i\omega t}$$

$$x_{max} = \left| \frac{-e E_0}{2m\omega \gamma} \right| = 0,24 \text{ pm}$$



$a_H \approx 53 \text{ pm}$

→ something atômica
boro

TP15/03/2022

Meio dielétrico é um meio transparente

luz a incidir num meio dielétrico o núcleo vai permanecer fixo mas os e^- mais distantes do meio oscilam gerando uma polarização

Modelo de Lorentz trata destes problemas

$$\vec{P} = \epsilon_0 \chi^{(1)} \vec{E}$$

Oscilador harmónico com atrito forçado e amplitude

$$m \frac{d^2 x}{dt^2} = -kx - b \frac{dx}{dt} - e E_0 e^{i\omega t}$$

$$\ddot{x} + 2\gamma \dot{x} + \omega_0^2 x = -e E_0 e^{i\omega t}$$

$$F_{\text{tot}} = -\frac{1}{4\pi\epsilon_0} \frac{qQ}{x^2} = -\frac{1}{4\pi\epsilon_0} \frac{qQ}{(x_0 + \delta x)^2}$$

$$= -\frac{1}{4\pi\epsilon_0} \frac{qQ}{x_0^2} \left(1 + \frac{\delta x}{x_0}\right)^{-2}$$

$$\approx \frac{1}{x_0^2} \left(1 - 2\frac{\delta x}{x_0}\right)$$

$$= F - k \delta x$$

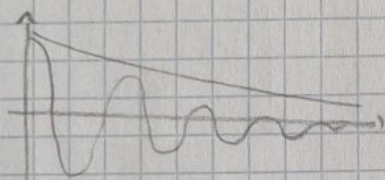
parece uma mola

se $x \ll 1$

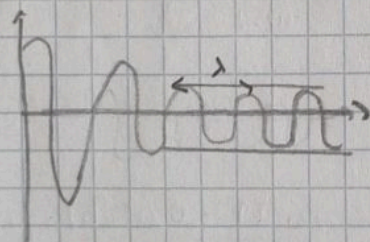
$$(1+x)^n \approx 1+nx$$

Solução Homogênea

$$\ddot{x} + 2\gamma \dot{x} + \omega_0^2 x = 0$$



Solução Não Homogênea



$$x(t) = A(\omega) e^{i\omega t}$$

substituindo na equação

$$- \omega^2 A(\omega) e^{i\omega t} - 2\gamma i\omega A(\omega) e^{i\omega t} + \omega_0^2 A(\omega) e^{i\omega t} = -e E_0 e^{i\omega t}$$

$$= -e E_0 e^{i\omega t}$$

$$A(\omega) = \frac{-e E_0}{(\omega_0^2 - \omega^2 - 2i\gamma\omega)}$$

$$\chi(\omega) = \frac{1}{\omega_0^2 - \omega^2 - 2i\gamma(\omega)}$$

$$\vec{P} = N e A(\omega) = \frac{N e}{\epsilon_0 \chi^{(1)}} \vec{E}$$

Eq. de onda

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2}$$

dielétrico

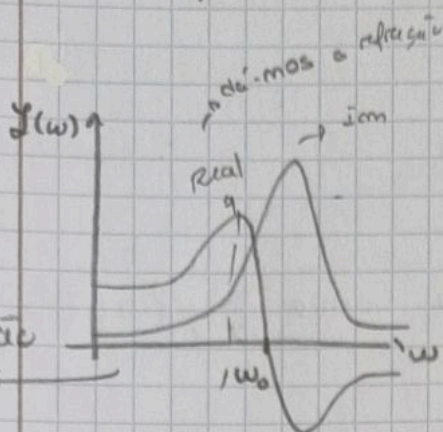
$$P = \epsilon_0 \chi^{(1)} E$$

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \epsilon_0 \chi^{(1)} \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$n^2 = 1 + \chi^{(1)} \Rightarrow \frac{1}{v^2} = \frac{1 + \chi^{(1)}}{c^2}$$

$$\nabla^2 \vec{E} - \frac{1 + \chi^{(1)}}{c^2} \frac{d^2 \vec{E}}{dt^2} = 0$$

$$\tilde{n} = \frac{c}{v} = \sqrt{1 + \tilde{\chi}^{(1)}}$$



Dispersão Normal

Dispersão anômala

Perfil de linha

$$\begin{aligned} \tilde{\omega}(\omega) &= \frac{\omega_0^2 - \omega^2 + 2i\delta\omega}{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2} \\ &= \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2} + \frac{2i\delta\omega}{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2} \end{aligned}$$

Quando $\omega = \omega_0$ $\tilde{\omega}(\omega)$ é só imaginário.

Onde plana a propagar-se

$$e^{-i\omega t + iKz}$$

se o tempo aumenta z tem de aumentar

$$e^{iKz}$$

$$k_0 = \frac{2\pi}{\lambda} \rightarrow \text{vazio}$$

$$\tilde{K} = \frac{2\pi}{\lambda} \tilde{n} \rightarrow \text{meio}$$

$$= \exp(i k_0 m z + i k_0 (1/\chi) z)$$

$$\tilde{n} = n + i\chi$$

$$= e^{i k_0 m z - k_0 \chi z}$$

campo elétrico

decaimento

1.

$$\text{Re}\{\tilde{\omega}(\omega)\} = \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\delta^2}$$

$$0 = \frac{dR(\omega)}{d\omega} = \frac{-2\omega[(\omega_0^2 - \omega^2)^2 + 4\omega^2\delta^2] + [4(\omega_0^2 - \omega^2)\omega - 8\delta^2\omega](\omega_0^2 - \omega^2)}{((\omega_0^2 - \omega^2)^2 + 4\omega^2\delta^2)^2}$$

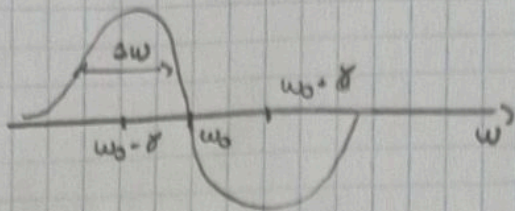
$$= \frac{-2\omega(\omega_0^2 - \omega^2)^2 - 8\omega\delta^2 + 4(\omega_0^2 - \omega^2)^2\omega - 8\delta^2\omega_0^2\omega + 8\delta^2\omega^3}{(\quad)^2}$$

$$= \frac{4\omega(\omega_0^2 - \omega^2)^2 - 8\omega\delta^2\omega_0^2}{(\quad)^2} = 0$$

$$\omega_0^2 - \omega^2 = \pm 2\delta\omega_0 \quad ; \quad \omega = \sqrt{\omega_0^2 \mp 2\delta\omega_0} = \omega_0 \left(1 \mp \frac{2\delta}{\omega_0}\right)^{1/2}$$

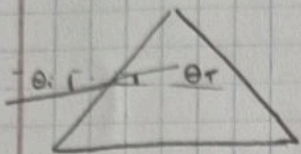
$$\approx \omega_0 \mp \delta$$

(5)

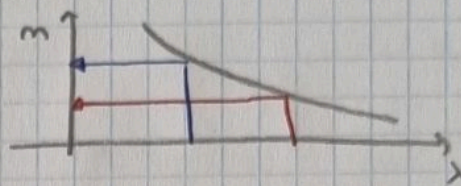


2. $n(\omega) = A + \frac{B}{\lambda^2}$

→ Quanto mais pequeno o λ maior o índice de refração



$$\sin \theta_i = n(\omega) \sin \theta_r$$



$$\left| \frac{dn}{d\lambda} \right| = \text{devide} = \left| \frac{2B}{\lambda^3} \right|$$

3. Modelo de Drude (frequência de plasma)

Ressonância de plasmom

Nos condutores os e^- andam a colidir pelo meio, não se conseguem propagar

a) $m_e^* = 0,67 m_e =$

→ Condensada

$$N = 10^{18} \text{ At/cm}^3 = 10^{18} \frac{\text{At}}{\text{cm}^3} = \frac{10^6 \text{ cm}^3}{1 \text{ m}^3} = 10^{24} \frac{\text{At}}{\text{m}^3}$$

$$\Delta t = 120$$

$$\tau = 120 \text{ fs} = 120 \times 10^{-15} \text{ s}$$

$$\sigma = \frac{Ne^2 \tau}{m_e^*} \rightarrow \frac{\frac{\text{At}}{\text{m}^3}}{\text{kg}} \frac{\text{s}^2}{\text{kg}} = \frac{\text{A}^2 \text{s}^2}{\text{kg m}^3}$$

TP-22/03/2022

4.

a) Para metais usamos o modelo de Drude

$$\epsilon(\omega) = m^2(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + 2i\delta)}$$

$$\omega_p^2 = \frac{e^2 N}{\epsilon_0 m}$$

frequência de plasma massa efetiva

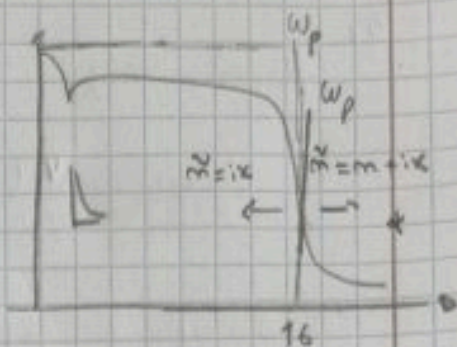
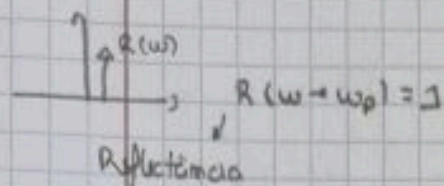
$$\omega \gg \delta : m(\omega) = \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

muito alto (oscila rapidamente)

como $\omega_p \gg \omega$ então o interior é imaginário que deverá descrever o decaimento

Se $\omega > \omega_p \Rightarrow m(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$

$R(\omega) = |r(\omega)|^2 = \left| \frac{m(\omega) - 1}{m(\omega) + 1} \right|^2 = \left| \frac{1 - \omega_p^2/\omega^2 - 1}{1 - \omega_p^2/\omega^2 + 1} \right|^2$



$E_p \approx 16 \text{ eV} = 16 \times (1,6 \times 10^{-19}) \text{ J}$

$E_p = h \omega_p \Rightarrow \nu_p = \frac{16 \times e}{h} = 3,87 \times 10^{15} \text{ Hz}$

b)

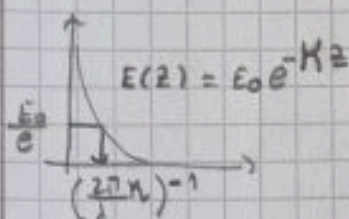
Para $\lambda = 500 \text{ nm}$

$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{500 \times 10^{-9}} = 6 \times 10^{14} \text{ Hz}$

Menor que a ν do visível

$\omega < \omega_p \Rightarrow \tilde{n} = i\kappa \quad m(\omega) \approx \sqrt{1 - \frac{\omega_p^2}{\omega^2}} = i \sqrt{\frac{\omega_p^2}{\omega^2} - 1}$

$\kappa \rightarrow$ profundidade de penetração

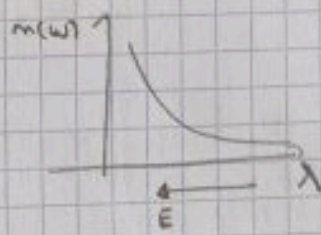


$\exp[-iRz] = \exp[-i\kappa m^2 z]$

$= \exp\left[-i \frac{2\pi}{\lambda} \frac{1}{2} (1 + i\kappa) z\right] \rightarrow \exp\left[-\frac{2\pi}{\lambda} \kappa z\right]$

c)

Quando $\nu \gg m(\omega)$ diminui então estaremos na dispersão normal.



4.

4.1.

O céu é azul por causa da expressão de Rayleigh. Sol emite luz branca com maior componente verde.

Quanto mais pequeno o comprimento de onda maior o efeito de Rayleigh

Se tivermos uma poeira, ela vai espalhar o campo de forma entrópica e circular/esférica.

$E_0 \propto \frac{E_{0i}}{r} \propto \frac{1}{r^2}$ (constante)

$[V] = m^3$

$[r] = m$

$[K] = m^{-2}$

$I_s \propto \frac{I_i}{\lambda^4}$



6

4.2.

Razão scattering

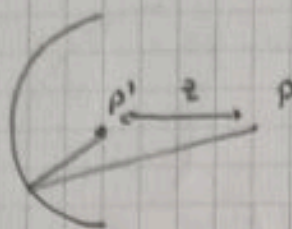
$$\frac{\sigma_{\text{violet}}}{\sigma_{\text{green}}} = \frac{\frac{1}{\lambda_0^4}}{\frac{1}{\lambda_0^4}} = \frac{\lambda_0^4}{\lambda_0^4} \approx \left(\frac{500}{300}\right)^4 = 7,6$$

O violeta é 8x mais espalhado do que o verde.

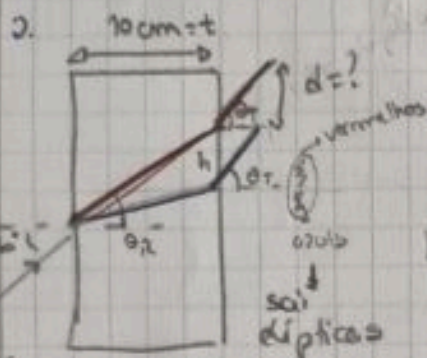
Protegemos os ultra-violeta

Folha 4

1. ?



Supostamente não pode voltar a P é impossível



$$n_{\text{Red}} = 1,505$$

$$n_{\text{violeta}} = 1,545$$

Queremos o diâmetro do feixe.

Lei de Snell-Descartes:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\theta_{r3} = \theta_{r2} = \theta_i$$

$$\sin \theta_i = n \sin \theta_r = \sin \theta_t$$

$$\sin \theta = \frac{\sin \theta_i}{n}$$

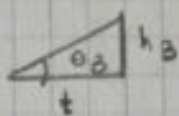
$$\Rightarrow \theta_{\text{Red}} > \theta_{\text{violeta}}$$



$$d = h \cos \theta_r = t (\tan \theta_r - \tan \theta_3) \cos \theta_r$$

$$h = h_2 - h_3$$

$$d = 0,10 (\tan(35^\circ) - \tan(34,09^\circ)) \cos(60^\circ) = 1,28 \text{ cm}$$



$$h_3 = t \tan \theta_3$$

$$h_2 = t \tan \theta_2$$

$$\sin \theta_2 = \frac{\sin \theta_i}{n_2} = \frac{\sin 60}{1,505} \Rightarrow \theta_2 = 35^\circ$$

$$\sin \theta_3 = \frac{\sin \theta_i}{n_3} = \frac{\sin 60}{1,545} \Rightarrow \theta_3 = 34,09^\circ$$

3. → Fazer em casa

$$d = \sqrt{t^2 + h^2}$$

4.

$$n = 1,33$$

$$\theta_i = 60,0^\circ$$

$$\vec{E}_{0,p} = 10 \text{ V/m}$$

$$\vec{E}_{0,s} = 10 \text{ V/m}$$

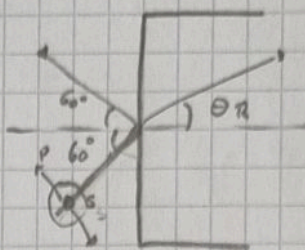
$$E_{0i} + E_{0r} = E_{0t}$$

$$TE \rightarrow \Delta$$

(Todos os campos estão alinhados)

$$r_{\perp} = r_s = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t}$$

$$r_{\parallel} = r_p = \frac{n_i \cos \theta_t - n_t \cos \theta_i}{n_i \cos \theta_t - n_t \cos \theta_i}$$

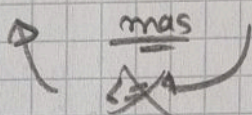


P → Paralelo
S → Perpendicular

Sem

5.

→ Luz não polarizada \Rightarrow componente de $S = P$

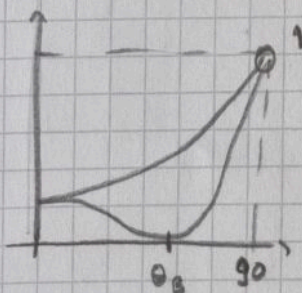


$$R = |r|^2$$

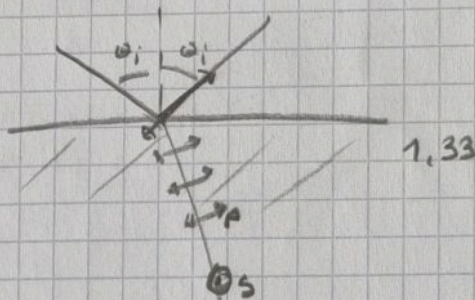
$$T \neq |t|^2 \Rightarrow T = |t|^2 \frac{n_1 \cos \theta_1}{n_2 \cos \theta_2}$$

(termos mais diferentes logo temos de considerar as diferentes velocidades)

$$\frac{R_p - R_s}{R_p + R_s}$$

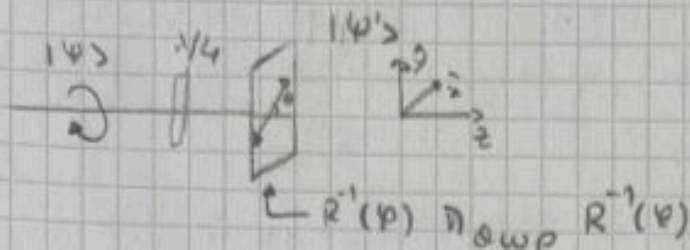


Brewster (polarização)



conjunto 3.
(5)

$$|\psi\rangle = \begin{bmatrix} A \\ B e^{i\delta} \end{bmatrix}$$



fast axis horizontal

$$H_{QWP} = \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} e^{i\frac{\pi}{4}}$$

$$R(\psi) = \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix}$$

$$R^{-1}(\psi) = \begin{bmatrix} \cos \psi & +\sin \psi \\ -\sin \psi & \cos \psi \end{bmatrix}$$

$$\begin{aligned} R(\psi) H_{QWP} R^{-1}(\psi) &= \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix} e^{i\frac{\pi}{4}} \begin{bmatrix} \cos \psi & +\sin \psi \\ -\sin \psi & \cos \psi \end{bmatrix} \\ &= \begin{bmatrix} \cos \psi & -\sin \psi \\ \sin \psi & \cos \psi \end{bmatrix} \begin{bmatrix} \cos \psi & \sin \psi \\ i\sin \psi & +i\cos \psi \end{bmatrix} e^{i\frac{\pi}{4}} \\ &= \begin{bmatrix} \cos^2 \psi + i\sin^2 \psi & 1/2 \sin(2\psi)(1-i) \\ 1/2 \sin(2\psi)(1-i) & \sin^2 \psi + i\cos^2 \psi \end{bmatrix} e^{i\frac{\pi}{4}} \end{aligned}$$

$$|\psi'\rangle = R(\psi) H_{QWP} R^{-1}(\psi) |\psi\rangle$$

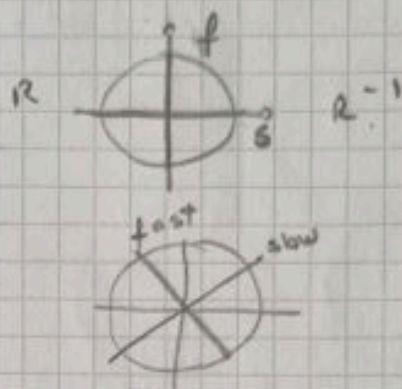
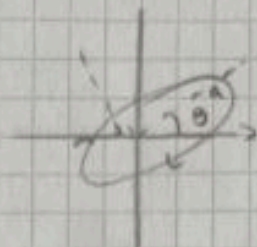
linear ~ elliptica

$$\begin{aligned} H &= \begin{bmatrix} \frac{1+\cos 2\psi}{2} & \frac{1-\cos 2\psi}{2} \\ \frac{1}{2} \sin(2\psi)(1-i) & \frac{1-\cos 2\psi}{2} + \frac{1+\cos 2\psi}{2} \end{bmatrix} \\ &= \frac{1}{2}(1-i) \begin{bmatrix} i + \cos 2\psi & \sin 2\psi \\ \sin 2\psi & 1 - \cos(2\psi) \end{bmatrix} \end{aligned}$$

$$|\psi\rangle = \begin{bmatrix} a \\ e^{i\delta} \end{bmatrix}$$

$$\hat{H} \begin{bmatrix} a \\ e^{i\delta} \end{bmatrix} \propto \begin{bmatrix} a(i + \cos 2\psi) + e^{i\delta} \sin(2\psi) \\ a \sin 2\psi + e^{i\delta} (1 - \cos(2\psi)) \end{bmatrix} = \begin{bmatrix} \psi'_x \\ \psi'_y \end{bmatrix}$$

$$\frac{\text{Im}(\psi'_x)}{\text{Re}(\psi'_x)} = \frac{\text{Im}(\psi'_y)}{\text{Re}(\psi'_y)}$$



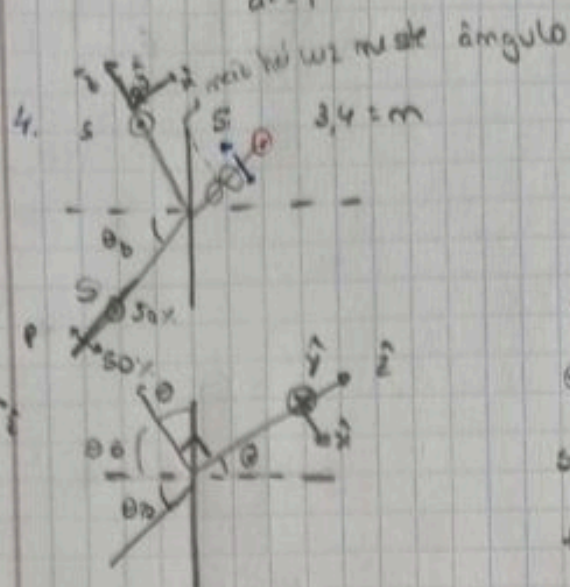
$$\cos^2 \theta = \frac{1 + \cos(2\theta)}{2}$$

$$\sin^2 \theta = \frac{1 - \cos(2\theta)}{2}$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\tan(2\theta) = \frac{2a}{a^2-1} \cos \delta$$

$$a = \frac{A}{B}$$



Só temos luz refletida com polarização S

Fronte \odot

Três \otimes

$$\sin \theta_B = m \sin \theta$$

$$\theta_B + \theta = 90$$

$$\sin \theta_B = m \sin(90 - \theta_B)$$

$$\tan \theta_B = m$$

vetores de Jones para a luz incidente:

$$|\psi_1\rangle = \frac{E_0}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|\psi_2\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \frac{E_0}{\sqrt{2}}$$

$$|\psi_3\rangle = \frac{E_0}{\sqrt{2}} \begin{bmatrix} t_p \\ t_s \end{bmatrix}$$

y é sempre o meu s

$$r_p = \frac{m_1 \cos \theta_i - m_2 \cos \theta_t}{m_2 \cos \theta_i + m_1 \cos \theta_t}$$

(pode ser nulo)

$$\frac{m \cos \theta_B - \cos \theta_t}{m \cos \theta_B + \cos \theta_t} = 0$$

$$r_s = \frac{m_1 \cos \theta_i - m_2 \cos \theta_t}{m_1 \cos \theta_i + m_2 \cos \theta_t}$$

(nunca é nulo)

$$* m \sin \theta_1 = m_2 \sin \theta_2$$

$$\sin \theta_B = m \sin \theta_t \quad \text{para que } r_p = 0 \Rightarrow m \cos \theta_B = \cos \theta_t$$

$$\frac{\sin^2 \theta_B}{m^2} + m^2 \cos^2 \theta_B = 1 \quad \Leftrightarrow (1 - \cos^2 \theta_B) + m^4 \cos^2 \theta_B = m^2$$

$$\cos^2 \theta_B (m^4 - 1) = m^2 - 1$$

$$\cos^2 \theta_B (m^2 + 1)(m^2 - 1) = m^2 - 1$$

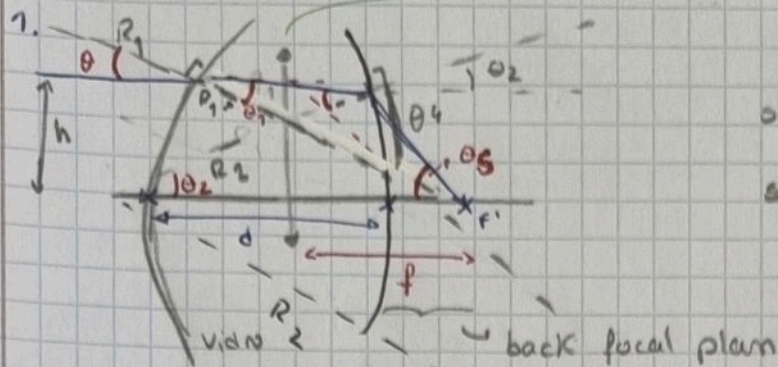
$$\cos^2 \theta_B = \frac{1}{1+m^2}$$

$$\Leftrightarrow m^2 + 1 = 1 + \tan^2 \theta_B \Rightarrow \tan \theta = m$$

$$r_s = \frac{\cos \theta_B - m \cos \theta_t}{\cos \theta_B + m \cos \theta_t}$$

conjunto 6 - TP 29/03 plano principal anterior da lente

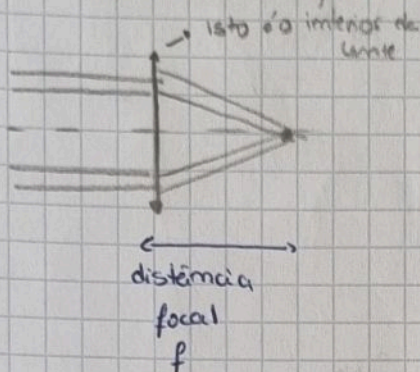
Trat
a ótica
geométrica



$$\sin \theta \approx \tan \theta \approx \theta$$

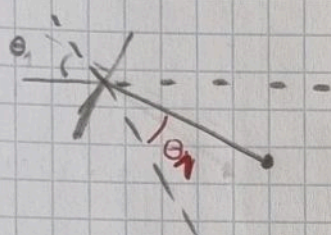
$$\sin \theta = \theta - \frac{\theta^3}{3!}$$

Feixe colimado



$$\tan \theta_4 = \frac{h}{f} \approx \theta_3 \Rightarrow f = \frac{\theta_3}{h}$$

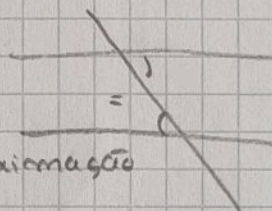
d = espessura da lente



Lei de Snell

Na aproximação

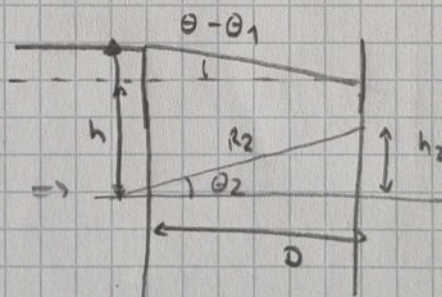
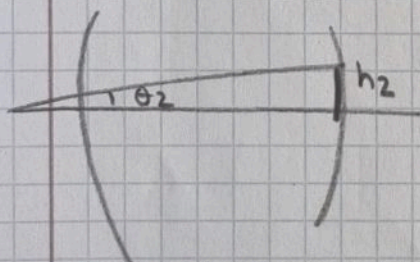
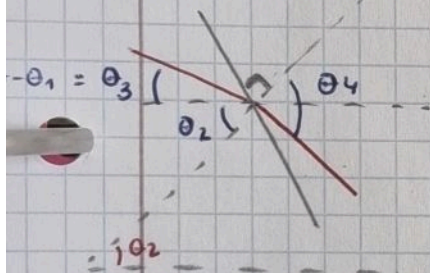
$$\theta = n \theta_1$$



Lei de Snell + aproximação

$$n(\theta_3 + \theta_2) = \theta_4$$

*



$$\theta_2 = \frac{h_2}{R_2}$$

$$h_2 = h - d(\theta - \theta_1)$$

$$* n \left[\theta - \theta_1 + \frac{h - d(\theta - \theta_1)}{R_2} \right] = \theta_4$$

$$\frac{n h}{R_2} + (\theta - \theta_1) n \left(1 - \frac{d}{R_2} \right) = \theta_4$$

$$\theta_3 = \theta_4 - \theta_2$$

$$\theta = \frac{h}{R_1}$$

$$= m \left[\theta_3 + \theta_2 \right] - \theta_2$$

$$= m \theta_3 + \theta_2 (m-1)$$

$$= m (\theta - \theta_1) + \theta_2 (m-1)$$

$$= (\theta + \theta_2) (m-1)$$

$$= \left[\theta + \frac{h}{R_2} - \frac{D}{R_2} \left(\theta - \frac{\theta}{m} \right) \right] \cdot \theta \frac{m-1}{m}$$

$$\theta_5 = \left[\theta + \frac{h}{R_2} - \frac{D}{R_2 m} \theta (m-1) \right] (m-1)$$

R_2 é negativo

$$\frac{\theta_5}{h} = \left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{D}{R_2 m R_1} (m-1) \right] (m-1)$$

Na forma matricial

$$C_1 \begin{pmatrix} \end{pmatrix} C_2 \quad \begin{bmatrix} C_2 \end{bmatrix} \begin{bmatrix} P(\theta) \end{bmatrix} \begin{bmatrix} C_1 \end{bmatrix} = \begin{bmatrix} 1/e \end{bmatrix}$$