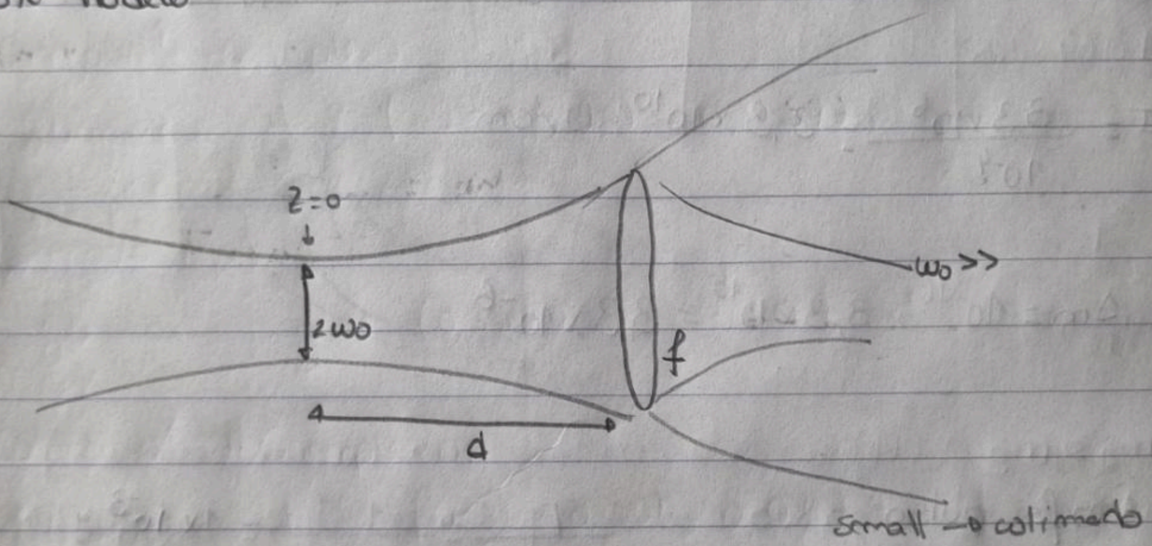
Teste Modelo

1.



$$\frac{1}{q_1} = \frac{1}{R_1} + \frac{i\lambda}{\pi w^2(z)}$$

Matrizes necessária

Lente:

$$\begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \qquad \frac{1}{q_2} = \frac{-1/f + 1/q_1}{1 + 0}$$

$$\frac{1}{f} = \frac{1}{R_1} - \frac{1}{R_2}$$

$$\frac{1}{q_2} = -\frac{1}{f} + \frac{1}{q_1} = \underbrace{-\frac{1}{f} + \frac{1}{R_1}}_{\frac{1}{R_2}} + \frac{i\lambda}{\pi w^2(z)}$$

$$= \frac{1}{R_2} + \frac{i\lambda}{\pi w^2(z)}$$

Como $R_2 \to \infty \Rightarrow \frac{1}{R_2} = 0$: $\frac{1}{f} = \frac{1}{R_1}$; $f = R_1$

$$\frac{1}{q_2} = \frac{i\lambda}{\pi w^2(z)} \quad , \quad q_1 = -iz_R + z = \frac{-i\lambda}{\pi w_0^2} + d$$

$$\frac{1}{q_1} = \frac{1}{\frac{-i\lambda}{\pi w_0^2} + d} = \frac{d + i\lambda/\pi w_0^2}{d^2 + \frac{\lambda^2}{\pi^2 w_0^4}} = \underbrace{\frac{1}{R_1}}_{\text{Parte real}} + \frac{i\lambda}{\pi w^2(z)}$$

$$\Rightarrow \frac{1}{R_1} = \frac{d}{d^2 + \frac{\lambda^2}{\pi^2 w_0^4}} = \frac{1}{f} \quad \Leftrightarrow \quad d = \frac{1}{f}\left(d^2 + \frac{\lambda^2}{\pi^2 w_0^4}\right)$$

$$df = d^2 + \frac{\lambda^2}{\pi^2 w_0^4} \quad \Leftrightarrow \quad w_0^4 = \frac{\lambda^2}{\pi^2(df - d^2)}$$

$$w_0 = \sqrt[4]{\frac{\lambda^2}{\pi^2(df - d^2)}}$$

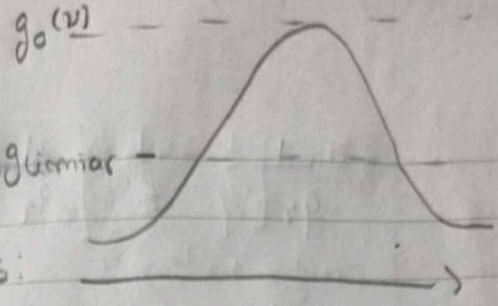
2.

$\lambda = 514{,}5\,nm$

$g_{\text{sinais peq}} = g_0(\nu_0) = 0{,}04\,m^{-1}$; $n = 1$; $L = 1\,m$; i)

ii)

$1 < L < 2$ (m)



a)

Para escolher os espelhos tem de se ter em conta 2 condições:

A → condição de estabilidade

$$0 \le g_1 g_2 \le 1$$

B → O ganho limiar da cavidade tem de ser menor que o ganho produzido para pequenos sinais.

$$g_{limiar} < g_0(\nu)$$

$$g_{limiar} = -\frac{1}{2l} \ln(r_1 r_2)$$

C → $1 \le L_{cav} \le 2$

B

1) Concavo¹ + Concavo²

$$g_{limiar} = -\frac{1}{2} \ln(0{,}99 \times 0{,}97) = 0{,}020 \ m^{-1}$$

2) Concavo¹ + complexo

$$g_{limiar} = -\frac{1}{2} \ln(0{,}99 \times 0{,}90) = 0{,}0577 \ m^{-1} \times$$

3) Concavo¹ + plano

$$g_{limiar} = -\frac{1}{2} \ln(0{,}99 \times 0{,}55) = 0{,}307 \ m^{-1}$$

4) Concavo² + Complexo

$$g_{limiar} = -\frac{1}{2} \ln(0{,}97 \times 0{,}90) = 0{,}0679 \ m^{-1}$$

5) Concavo² + Plano

$$g_{limiar} = -\frac{1}{2} \ln(0{,}97 \times 0{,}97) = 0{,}0409 \ m^{-1}$$

A/

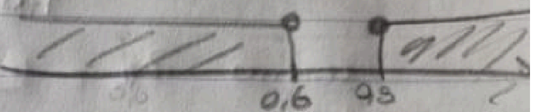
1) Concavo¹ + Concavo²

$$g_1 = 1 - \frac{L}{R_1} = 1 - \frac{L}{0{,}9} = \qquad g_2 = 1 - \frac{L}{0{,}6}$$

$$0 \le \left(1-\frac{L}{0,9}\right)\left(1-\frac{L}{0,6}\right) \le 1$$

$$0 \le \left(1-\frac{L}{0,9}\right)\left(1-\frac{L}{0,6}\right)$$

		0,6		0,9	
$\left(1-\frac{L}{0,9}\right)$	+	+	+	0	-
$\left(1-\frac{L}{0,6}\right)$	+	0	-	-	-
	+	0	-	0	+



$$1 - \frac{L}{0,6} - \frac{L}{0,9} + \frac{L^2}{0,54} = 1$$

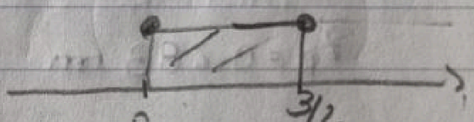
$$1 - \frac{25}{9}L + \frac{L^2}{0,54} = 0$$

		0		1,5	
L	-	0	+	+	+
$\frac{L}{0,54} - \frac{25}{9}$	-	-	-	0	+
	+	0	-	0	+

$$L=0 \vee -\frac{25}{9} = -\frac{L}{0,54} \Leftrightarrow \frac{L}{0,54} - \frac{25}{9}$$

$$L=0 \vee L=\frac{3}{2}$$

$$0 \le L \le 0,6 \wedge 0,9 \le L \le 1,5$$



3) Côncavo + plano

$$g_2 = 1 - \frac{L}{R_2} = 1 - \frac{L}{\infty} = 1$$

$$g_1 = 1 - \frac{L}{0,90} \qquad 0 \le \left(1-\frac{L}{0,90}\right) \le 1$$

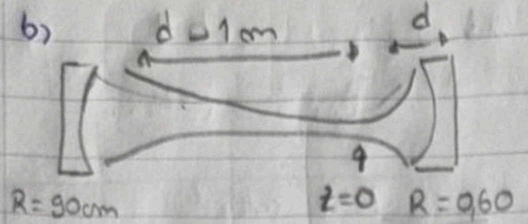
		0,90	
$\left(1-\frac{L}{0,90}\right)$	+	0	-

$0 \le L \le 90$ cm X

Então a única representação possível seria pela cavidade com os espelhos côncavos onde:

$$1 \le L \le 1,5$$

b)



$$R(z) = z + \frac{z_R^2}{z}$$

$$R(d) = d + \frac{z_R^2}{d} = 0,60 \Rightarrow z_R^2 = (0,60 - d)\, d$$

$$R(1-d) = 1-d + \frac{z_R^2}{1-d} = 0,50 \Rightarrow z_R^2 = (-0,10 + d)\,(1-d)$$

Como $z_R = \frac{\pi w_0^2}{\lambda}$ então

$$(0,60d - d^2) = -0,10 + 0,1d + d - d^2$$

$$0,60\,d - 0,1d - d = -0,10$$

$$d = 0,20 \text{ cm}$$

$$z_R^2 = 0,20\,(0,60 - 0,20) = 0,08 \text{ cm}$$

$$z_R = 0,283 \text{ cm}$$

$$w_0 = \sqrt{\frac{z_r \lambda}{\pi}} = 0,215 \text{ mm}$$

c)

3700 K

$A \sim 40$ g/mol

$$\delta\nu_D \sim 2,15 \times 10^5 \left(\frac{1}{514,5} \sqrt{\frac{3700}{40}} \right) \sim 4019 \text{ MHz}$$

$$* \quad g_0(\nu) = g_0(\nu_0) \exp\left[\frac{-2,77\,(\nu - \nu_0)^2}{\delta\nu_0} \right]$$

Se $g_{\text{limiar}} = \frac{g_0(\nu_0)}{2}$ então $\Delta\nu_{\text{Laser}} = \delta\nu_D$

Caso contrário calcula-se $(\nu - \nu_0)$ pela fórmula *

$$\Delta\nu_{cav} = \frac{c}{2L} = \frac{3\times10^{8}}{2\times1} = 150\,MHz$$

N^{o} de modos da cavidade $= \frac{\Delta\nu_{laser}}{\Delta\nu_{cav}} = \frac{4019\times10^{6}}{150\times10^{6}} \approx 27$ modos

d)

$I^{sat} = 14\,W/cm^2$, Sistema de 4 níveis $\Rightarrow g(\nu_0) = \frac{g_0(\nu_0)}{\sqrt{1+I/I^{sat}_{\nu_0}}}$

Modo da cavidade = transição Laser

$P = \frac{I}{A}$ $\quad g(\nu_0) = \frac{g_0(\nu_0)}{\sqrt{1+I/I_\nu^{sat}}}$

$$1+\frac{I}{I_\nu^{sat}} = \left(\frac{g_0(\nu_0)}{g(\nu_0)}\right)^2 \quad \therefore I = \left(\left(\frac{g_0(\nu_0)}{g(\nu_0)}\right)^2 - 1\right) I_\nu^{sat}$$

$$I = 3\times14 = 42\,\frac{W}{cm^2}$$

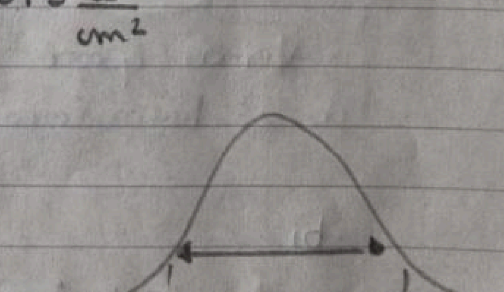
$I^{+} + I^{-} \sim I^{cav}$ $\qquad I^{out} \sim t\,I^{+}$

$\Rightarrow I^{+} = \frac{1}{2} I^{cav}$ $\qquad \sim 0{,}03\times 42\,\frac{W}{cm}\times\frac{1}{2}$

$1 = r + t + \overset{0?}{s}$ $\qquad = 0{,}615\,\frac{W}{cm^2}$

$t = 1 - 0{,}97 = 0{,}03$

$$P_{out} = \frac{I^{out}}{A} = 0{,}615\cdot A\ W$$



e)

Acordo de fase $\Rightarrow \Delta K = 0$

$\Delta\nu_{cav} = \frac{c}{2L} = \frac{1}{T}$

$\Delta t_{pulso} = \frac{T}{N} = \frac{\cancel{T}}{\Delta\nu\cancel{T}} = \frac{1}{\Delta\nu} = \frac{1}{\delta\nu_D}$ $\qquad N = \frac{\Delta\nu}{\Delta\nu_{cav}} = \Delta\nu\cdot T$

$= 2{,}48\times10^{-10}\,s$

$$t_{rep} = \frac{2\pi}{\Delta\omega_{cav}} = \frac{1}{\Delta\nu_{cav}} = \frac{2L}{c} = \frac{2\times1}{3\times10^{8}} = 6,66\times10^{-5}\,s$$

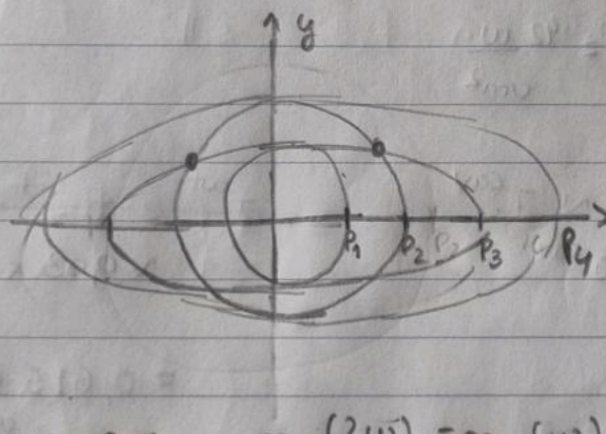
$$Pot_{pico} = \frac{E_{pulso}}{\tau_{pulso}} = \frac{1}{2,48\times10^{-10}} = 4\times10^{9}$$

3.

		m_o	m_e
ω	1550 nm	2,664	2,937
2ω	775 nm	2,780	3,094

a)

Como $m_o < m_e$ estamos perante um cristal positivo!
↳ elipse fora!



$P_1 \rightarrow m_{o\omega}$
$P_2 \rightarrow m_{o2\omega}$
$P_3 \rightarrow m_{e\omega}$
$P_4 \rightarrow m_{e2\omega}$

Vamos querer que $m_o(2\omega) = m_e(\omega)$, ou seja, o feixe do 2º harmónico deverá ter uma polarização ordinária

b)

Acordo de fase => $\Delta K = 0$
Neste caso teremos que:

$$\frac{1}{m_e^2(\omega,\theta)} = \frac{\cos^2\theta}{m_o^2(\omega)} + \frac{\overbrace{\sin^2\theta}^{1-\cos^2\theta}}{m_e^2(\omega)} = \frac{1}{m_o^2(2\omega)}$$

$$\cos^2\theta\left[\frac{1}{n_o^2(\omega)}-\frac{1}{n_e^2(\omega)}\right]=\frac{1}{n_o^2(2\omega)}-\frac{1}{n_e^2(\omega)}$$

$$\cos^2\theta = 0{,}539 \Rightarrow \theta = \underline{\underline{57{,}4^\circ}}$$

c)

$d_{eff} = 40{,}9$ pm/V

$I = 1\,\frac{MW}{cm^2} = \frac{1\times10^{+6}}{10^{-4}}$ $\quad I_{out} = 0{,}01 \times I$

Acordo de fase

$$I_{2\omega}(z) = \frac{(2\omega)^2}{2\varepsilon_0 c^3 n_{2\omega} n^2_\omega}(d_{eff}\, z\, I_\omega)^2 \underbrace{\left[\frac{\sin(\Delta k z/2)}{\Delta k z/2}\right]^2}_{1}$$

$\Delta k = 0$

$$0{,}01\, I = \left(\frac{4\pi^2}{(775\times10^{-9})^2}\right)\frac{(40{,}9\times10^{-12}\times z\times I)^2}{2\times(8{,}85\times10^{-12})\times(2{,}780)^3}$$

$$\left(\frac{2\omega}{c}\right)^2 = \left(\frac{2\pi}{\lambda_{2\omega}}\right)^2$$

$n_{2\omega} = n_\omega = 2{,}780$

$z = 1{,}86$ cm

d)

→ Já não haveria acordo de fase. Pelo que:

$$\frac{1}{n_e^2(\omega,\theta+1)} = \frac{\cos^2(58{,}4)}{2{,}664^2} + \frac{\sin^2(58{,}4)}{2{,}937^2} = 0{,}1227$$

$$\frac{2\pi/\cancel{2}}{\lambda\cancel{2}} = \frac{2\pi}{\lambda}$$

$n_e(\omega,\theta+1) = 2{,}85$

$n_o(2\omega) = 2{,}780$

$$\Delta k = \frac{2\omega}{c}(n_\omega - n_{2\omega}) = \frac{\pi}{\lambda_{2\omega}}\left[2{,}85-2{,}780\right]$$

$$= 28{,}37\times10^{5}\ \text{m}^{-1} \Rightarrow \left[\frac{\sin(\Delta k z/2)}{\Delta k z/2}\right] = 3{,}67\times10^{-5}$$

$$I_{2\omega} = 0{,}01\times3{,}67\times10^{-5}\times1\times10^{+6} = 367\ \frac{mW}{cm^2}$$