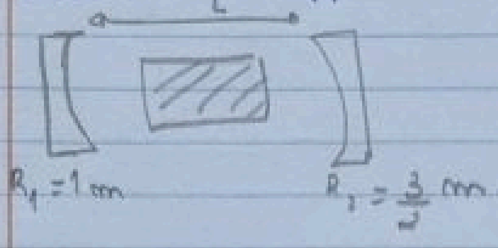
<u>Extra</u> (slide 4 pp. aula 25 de mai)



se $L \geq 0$:

$$g_1 = 1 - \frac{L}{1m} = 1 - L \ (m)$$

$$g_2 = 1 - \frac{L\,2}{3}m = 1 - \frac{2}{3}L$$

condição de estabilidade

$$0 \leq (1-L)(1 - 2/3\,L) \leq 1$$

$$0 \leq (1-L)\left(1 - \frac{2}{3}L\right)$$

$\Downarrow$ $L \leq 1m$

ou

$L \geq \frac{3}{2}m$

$$(1-L)\left(1 - \frac{2}{3}L\right) \leq 1$$

$$\cancel{1} - L - \frac{2}{3}L + \frac{2}{3}L^2 \leq \cancel{1}$$

$$\frac{2}{3}L^2 - \frac{5}{3}L \leq 0$$

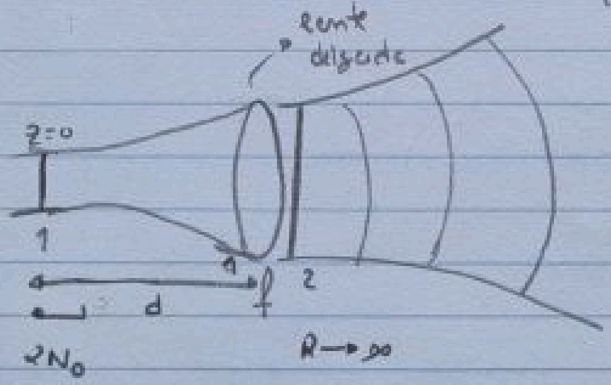
$$L\left(\frac{2}{3}L - \frac{5}{3}\right) \leq 0$$

$$L \leq \frac{5}{2}$$

Se $L \geq 0$: $0 \leq L \leq 1\,m$

$\frac{3}{2}m \leq L \leq \frac{5}{2}m$

3. (Slide 33)



Lente delgada $\to \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$

1: antes da lente

2: onda plana

$$\frac{1}{q} = \frac{i}{z_{R_1}} = \frac{i\lambda}{\pi w_0^2}$$

Sabemos que

$$\frac{1}{q_2} = \frac{C + \cancel{D}/q_1}{A + \cancel{B}/q_1} = -\frac{1}{f} + \frac{1}{q_1}$$

$$= -\frac{1}{f} + \underbrace{\frac{1}{R_1}}_{1/R_2} + \frac{i\lambda}{\pi w_1^2}$$

como $\frac{1}{R_2} = 0 \Rightarrow f = R_1$

$$q_1 = -\frac{i\pi w_0^2}{\lambda} + d$$

$$\frac{1}{q_1} = \frac{1}{d - \frac{i\pi w_0^2}{\lambda}}\left(\frac{d + i\pi w_0^2/\lambda}{d - i\pi w_0^2/\lambda}\right) = \frac{d + i\pi w_0^2/\lambda}{d^2 + \left(\frac{\pi w_0^2}{\lambda}\right)^2}$$

$1/f = 1/R_1$

Valores típicos (Dado)

$r_1 \sim 1$

$r_2 = 0,95$ $\Rightarrow 2\delta\nu_{cav} \sim 4,8\ MHz$

$L = 25cm$ Como $\Delta\nu_{Dop} \sim 1-2\ GHz$, o calculado é 2x maior do que a cavidade vazia.

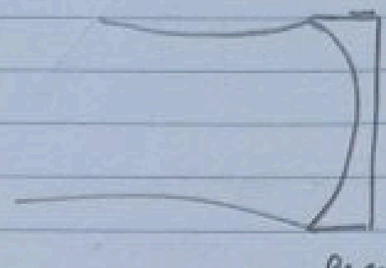
$$f = \frac{d^2 + \left(\frac{\pi w_0^2}{\lambda}\right)^2}{d} = \frac{d^2 + z_R^2}{d} = d + \frac{z_R^2}{d}$$

2: Parte

$$\frac{df}{dd} = 1 - \frac{z_R^2}{d^2} = 0 \iff d^2 = z_R^2 \quad \text{ou} \quad d = z_R$$

7.4.

$$M = \begin{bmatrix} 1 & 0 \\ -2/R_{esp} & 1 \end{bmatrix}$$



R_{esp}

$$\frac{1}{q_1} = \underbrace{\frac{1}{R_{G_1}}}_{\text{feixe}} + \frac{i\lambda}{\pi w_{G_1}^2}$$

$$\frac{1}{q_2} = \frac{C + D/q_1}{\underset{1}{A} + \cancel{B/q_1}} = -\frac{2}{R_{esp.}} + \frac{1}{R_{G_1}} + \frac{i\lambda}{\pi w_{G_1}^2} = \frac{1}{R_{G_2}} + \frac{i\lambda}{\pi w_{G_2}^2}$$

$$\left|\frac{1}{R_{G_1}}\right| = \frac{-2}{R_{esp}} + \frac{1}{R_{G_2}} \quad \text{ou}$$

Temos um espelho plano $\Rightarrow R_{esp_{plano}} \to \infty$

$\Rightarrow R_{esp} = R_{G_1}$

curvatura da frente da onda é igual à curvatura do espelho

7.5.

$$k = \frac{2\pi\nu}{c} = \frac{2\pi}{\lambda} \quad ; \quad z_2 - z_1 = L$$

$$\frac{2\pi \nu_m}{c} L - \tan^{-1}\left(\frac{z_2}{z_R}\right) + \tan^{-1}\left(\frac{z_1}{z_R}\right) = m\pi$$

Multiplicar e dividir por π

$$\nu_m = \frac{c}{2L}\left[m + \underbrace{\frac{1}{\pi}\left(\tan^{-1}\left(\frac{z_2}{z_R}\right) + \tan^{-1}\left(\frac{z_1}{z_R}\right)\right)}_{\text{Gouy}}\right]$$

$$z_R = \frac{\pi w_0^2}{\lambda} = L\,\frac{\sqrt{g_1 g_2 (1 - g_1 g_2)}}{g_1 + g_2 - 2 g_1 g_2}$$

$$\frac{z_2}{z_R} = \frac{g_1(1-g_1)}{\sqrt{g_1 g_2 (1 - g_1 g_2)}} \quad ; \quad \frac{z_1}{z_R} = \frac{-g_2(1-g_1)}{\sqrt{g_1 g_2(1-g_1 g_2)}}$$

4.8.

$$I_\nu^{sat} = \frac{h\nu}{2\sigma(\nu)}(P + \Gamma_{21})$$

Taxa de emissão estimulada $\quad$ Taxa de decaimento

$$I_\nu = I_\nu^{sat}$$

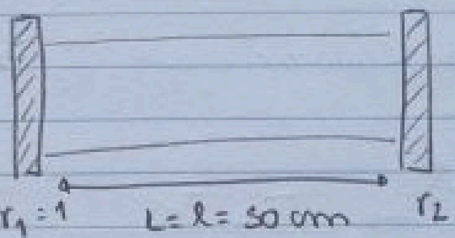
$$\underbrace{\frac{\sigma I_\nu}{h\nu}}_{\Phi_\nu} \longrightarrow \frac{\sigma}{h\nu}\frac{h\nu}{2\sigma}(P + \Gamma_{21})$$

$$\frac{P + \Gamma_{21}}{2}$$

TP conjunto 4 $\quad$ 11/11/2022

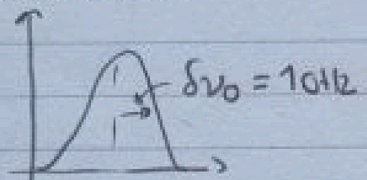
5.2. Teórica 11/11/2022

5.3.



$g_0(\nu_0) \sim 0{,}005\ cm^{-1}$

Lorentziano



$S = 0{,}02$

a)

$$t_{opt} = \sqrt{2 g_0(\nu_0)\, l\, S} - S$$

$$= \sqrt{2 \times (0{,}05\ cm^{-1}) \times 50\ cm \times 0{,}02} - 0{,}02$$

$$= 0{,}1 - 0{,}02 = 0{,}08 \rightarrow 8\%$$

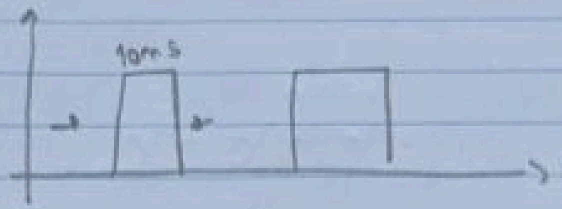
b)

$$(I_\nu^{out})_{max} = I_\nu^{sat}\left[\sqrt{g_0(0)\, l} - \sqrt{\frac{S}{2}}\right]^2$$

$$= 100\ \frac{kW}{cm^2}\left[\sqrt{0{,}005 \times 50} - \sqrt{\frac{0{,}02}{2}}\right]^2$$

$$= 16\ \frac{kW}{cm^2}$$ → é muito elevado comparativamente com $I_\nu^{sat} = 1\ \frac{kW}{cm^2}$

b) 30 pulsos/segundo $\Delta t = 10\,ms$ 30Hz → 30 pulsos/segundos



Potência média = 1W

$$\frac{\text{Energia}}{\text{pulso}} = \frac{1J}{30} = 3{,}3\times10^{-2}\,J = 33\,mJ$$

$$Pot_{pico} = \frac{33\,mJ}{10\,ms} = \frac{3{,}3\times10^{-2}\,J}{10^{-8}} = 3{,}3\times10^{6}\,W$$

Ao fim de 30 pulsos → 1 J

$$I_{pico} = \frac{Pot_{pico}}{\text{área}} = \frac{3{,}3\times10^{6}}{10^{-4}} = 3{,}3\times10^{10}\,\frac{W}{cm^2}$$

$$\Delta n = n_2^{I}\, I = 3{,}3\times10^{-6}$$

c) pulsos de 100 fs
$f = 1\,KHz$
Laser Ti Safira

$$\frac{\text{Energia}}{\text{pulso}} = \frac{1J}{1000} = 1\,mJ$$

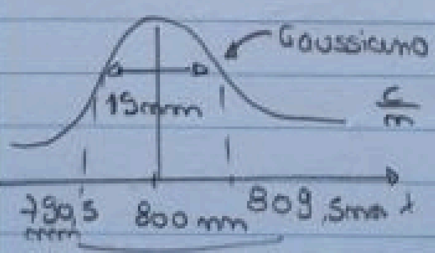
$$Pot = \frac{1\,mJ}{100\,fs} = \frac{10^{-3}}{10^{-13}} = 10^{10}\,W$$

$$I_{pico} = \frac{Pot}{\text{área}} = \frac{10^{10}}{10^{-4}} = 10^{14}\,W/cm^2$$

$$\Delta n = I\, n_2^{I} = 10^{-16}\times10^{14} = 10^{-2}\,W/cm^2 = 0{,}01$$

Dispersão numa fibra ótica

$$\Delta\nu\,\Delta T \approx \overset{0{,}44}{1}$$
↳ 50 fs



! varre !

$$t_{percurso} = \frac{z}{c}\,n$$

$$\Delta t = \frac{z}{c}\left(n_{790,5} - n_{809,5}\right) = 100\,fs$$

$$z = \frac{100\,fs\,(3\times10^{8})}{0{,}0003} = \frac{10^{-13}\,s\,(10^{8}\,m/s)}{10^{-4}} = 0{,}1\,m$$

c)

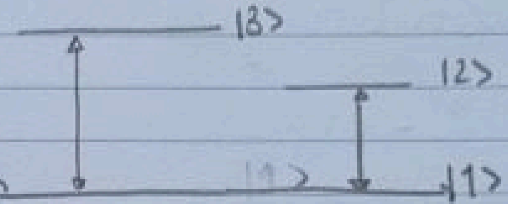
$$t_2 I_\nu^+ = I_\nu^{out} \qquad I_\nu^+ \simeq I_\nu^- \simeq \frac{1}{2} I_\nu^{cav}$$

$$I_\nu^+ = \frac{I_\nu^{out}}{t_2} \quad \Rightarrow \quad I_\nu^{cav} = \frac{2 I_\nu^{out}}{t_2} = \frac{2 \times 16\ kW/cm^2}{0{,}08}$$

$$= 400 \frac{kW}{cm^2}$$

→ Porquê é necessário arrefecar os espelhos?
↳ É muito calor dentro da cavidade, pelo que isto é necessário para não derreter o espelho

5.4.

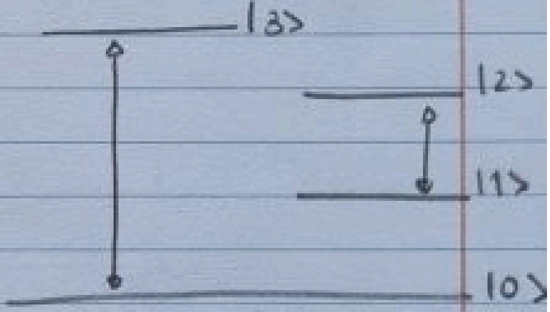


Rubi

Transição laser $\lambda_L = 694{,}3\ nm$
$\lambda_{excitação} = 550\ nm$

$$\epsilon_{quântica}^{Rubi} = \frac{550\ nm}{694\ nm} = 0{,}79$$ → não tem maior eficiência do que isto

Nd: YAG



Transição Laser $\lambda_L = 1064\ nm$
Excitação $2{,}38\ eV \sim 521\ nm$

$$E_{quântica}^{YAG} = \frac{521}{1064} = 0{,}49$$

+ de metade da energia fornecida é perdida

7.2. 25/11/2022

$$0 \leq g_1 g_2 \leq 1$$

Confocal $R_1 = R_2 = L$ | $R_1 = R_2 = -L$

$$g_1 = 1 - \frac{L}{R_1} = 0 \qquad g_1 = 1 + 1 = 2$$

$$\rightarrow g_1 g_2 = 0 \qquad g_1 g_2 = 4 \times$$

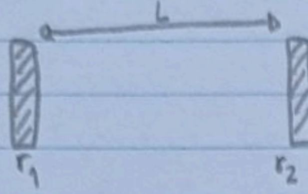
instável

$$g_2 = 1 - \frac{L}{R_2} = 0 \qquad g_2 = 1 + 1 = 2$$

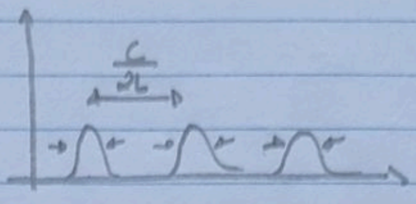
TP 18/11

5.7.

se tivermos uma cavidade vazia

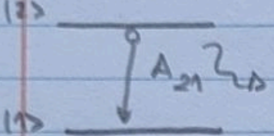


$$\nu_m = \frac{mc}{2L}$$

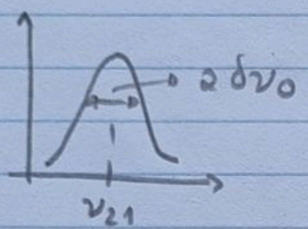


Como estimamos a largura dos picos?

Temos uma cavidade de Fabry-Perot (só espelhos)



$$\frac{dN_2}{dt} = -A_{21} N_2 \qquad N_2(t) = N(t=0)\, e^{-A_{21} t}$$



$$L(\nu) = \frac{\delta\nu_0/\pi}{(\nu-\nu_0)^2 + \delta\nu_0} \qquad 2\delta\nu_0 = \frac{A_{21}}{2\pi} = \frac{1}{2\pi\tau}$$

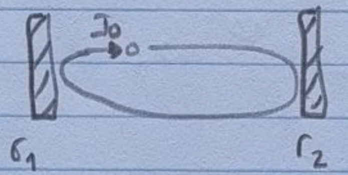
$\Delta E \, \Delta t \gtrsim \hbar$

$\cancel{\hbar} \Delta\omega \, \Delta t \gtrsim \cancel{\hbar}$

$\Delta\omega \, \Delta t \sim 1$

$2\pi \Delta\nu \, \Delta t \sim 1$

$\Delta\nu \sim \frac{1}{2\pi \Delta t}$



$$\Delta I = I_f - I_i = r_1 r_2 I_0 - I_0 = -(1 - r_1 r_2) I_0$$

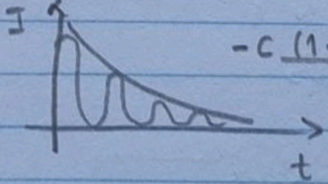
$$\Delta t = \frac{2L}{c} \Rightarrow \frac{\Delta I}{\Delta t} = \frac{-(1-r_1 r_2) I_0 \, c}{2L} \sim \frac{dI}{dt}$$

isto é equivalente a

$$\frac{dN_2}{dt} = -A_{21} N_2$$

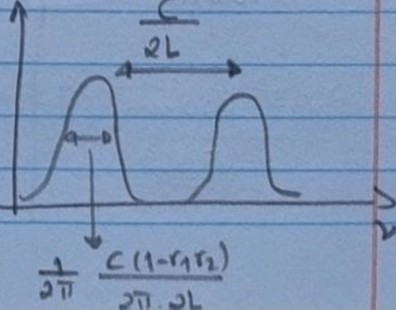
$\hookrightarrow A_{21} = \frac{1}{\tau}$

$$I(t) = I(0)\, e^{-\frac{c(1-r_1 r_2)}{2L} t}$$



$$\tau_{cav} = \frac{2L}{c(1-r_1 r_2)}$$

$$2\,\delta\nu_{cav} = \frac{1}{2\pi\,\tau_{cav}} = \frac{1}{2\pi} \quad \frac{c\,(1-r_1 r_2)}{2\pi \cdot 2L}$$



b)

! Falta-mos valores: r_1, r_2 e $\Delta\sigma$! É igual à alínea (a) então

c)

$$T = \frac{2L}{c} = \frac{0,8}{3\times10^8} = 2,7\ ms$$

$$\Delta\nu_{cav} = \frac{1}{T} = 375\ MHz$$

$$N = \frac{T}{\Delta t_{pulso}} = \frac{2,7\ ms}{670\ \mu s} = 4$$ modos que podem oscilar

d)

Não falamos sobre isso ainda.

Só depende da largura de banda da emissão.

TP - 9/12/2022

10.1. (b)

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x + a x^2 = \frac{eE_0}{m}\left[\frac{e^{-i\omega t} + e^{i\omega t}}{2}\right]$$

Método perturbativo

Desenvolver $x(t)$ numa série de Taylor à volta de $a=0$

$$\underset{(a,t)}{x(t)} = x^{(0)}(t) + a\,\underbrace{x^{(1)}(t)}_{\left.\frac{dx(a,t)}{da}\right|_{a=0}} + a^2 x^{(2)}(t) + \dots$$

$$x^{(2)}(t) \to \left.\frac{d^2x(a,t)}{2!\,da^2}\right|_{a=0}$$

A ordem 0:

$$\frac{d^2x^{(0)}}{dt^2} + 2\gamma\frac{dx^{(0)}}{dt} + \omega_0^2 x^{(0)} = \frac{eE}{m}e^{-i\omega t} + \dots$$

$$x^{(0)}(t) = x_0 e^{-i\omega t}$$

$$\to \left[-\omega^2 x_0 - i\omega 2\gamma x_0 + \omega_0^2 x_0 = \frac{eE}{m}\right] e^{i\omega t}$$

$$x_0 = \frac{eE_0}{m}\,\frac{1}{(\omega_0^2-\omega^2) - i\omega 2\gamma}$$

→ ordem 1 (lineares em a)

$$a\left[\frac{d^2x^{(1)}}{dt^2} + 2\gamma\frac{dx^{(1)}}{dt} + \omega_0^2 x^{(1)} + \underbrace{[x^{(0)}]^2}_{x_0^2\, e^{-2i\omega t}}\right] = 0$$

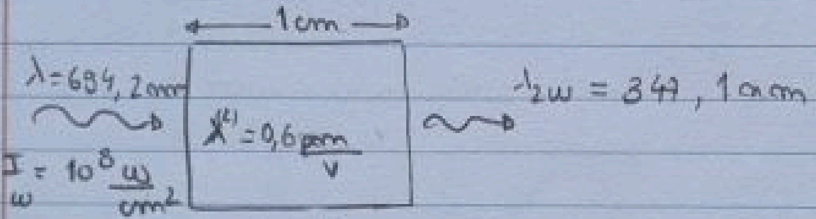
$$\frac{d^2x^{(1)}}{dt^2} + \frac{2\gamma\, dx^{(1)}}{dt} + \omega_0^2 x^{(1)} = -x_0^2\, e^{-i2\omega t} \qquad x^{(1)} = x_0^{(1)}\, e^{-2i\omega t}$$

$$(-4\omega^2 x_0^{(1)} - 4i\omega\gamma\, x_0^{(1)} + \omega_0^2 x_0^{(1)})\,e^{-i2\omega t} = -x_0^{(0)2}\, e^{-i2\omega t}$$

$$x_0^{(1)} = \frac{-x_0^{(0)2}}{(\omega_0^2 - 4\omega^2) - 4i\omega\gamma}$$

$$x(a,t) = \frac{eE_0}{m}\,\frac{e^{-i\omega t}}{(\omega_0^2-\omega^2)-i2\omega\gamma}\left[1 - \frac{x_a\, e^{-i\omega t}}{(\omega_0^2-4\omega^2)-4i\omega\gamma}\right] + \dots$$

10.3.



Quartzo

$$e_{SHG} = \frac{(2\omega)^2}{2\varepsilon_0 c^3 n_{2\omega} n_\omega^2}\,[\chi^{(2)}]^2 z^2 I_\omega(0,t)\left[\frac{\sin(\frac{1}{2}\Delta k z)}{1/2\,\Delta k z}\right]^2$$

Se área do foco ~ $10^{-3}\,cm^2$ e diâmetro ~ 530 μm

$Pot_\omega = I_\omega \times \text{Área} = 10^5\, W$

$\tau_{pulso} \simeq 100\, ns$

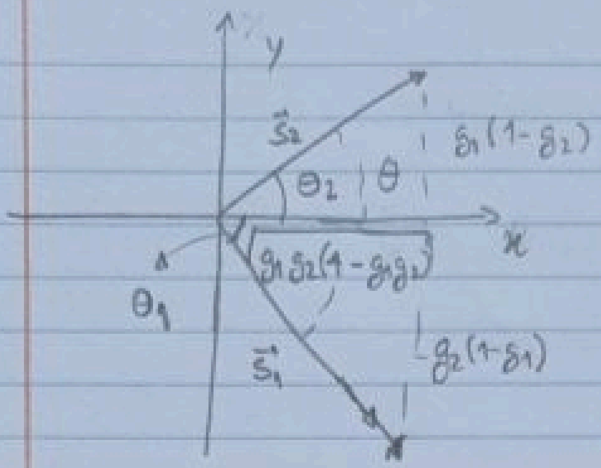
$\text{Energia}_\omega = (10^5\, W)(10^{-7}\, s) = 10\, mJ$

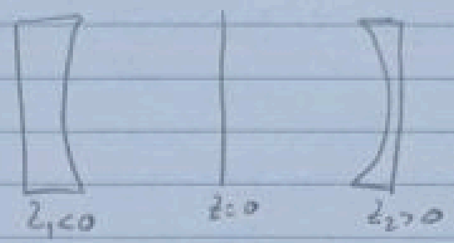
$N_{\text{fotões } \omega \text{ incidentes}} = \frac{\text{Energia}_\omega}{hc/\lambda_\omega} \sim 3{,}5\times10^{16}$

a) Assumir $\Delta k = 0$ e $n = 1{,}5$.

$\Rightarrow \frac{\Delta k z}{2} = 0$ | unidades $\left(\frac{2\omega}{c}\right)^2 = \frac{4\pi^2}{\lambda_{2\omega}^2}$

$I = \frac{1}{2}\varepsilon_0 c n |E|^2$ o $\frac{I}{\varepsilon_0 c} \sim |E|^2 \sim \frac{V^2}{m^2}$





$$\vec{S}_1 \cdot \vec{S}_2 = |\vec{S}_1|\,|\vec{S}_2| \cos\theta$$

$$\theta = \arccos\left(\underbrace{\frac{\vec{S}_1\cdot\vec{S}_2}{|\vec{S}_1|\,|\vec{S}_2|}}_{*}\right)$$

$$\theta = \theta_2 + \theta_1$$

$$\vec{S}_2 = \sqrt{\quad}\;\hat{x} + g_1(1-g_2)\,\hat{y}$$

$$\vec{S}_1 = \sqrt{\quad}\;\hat{x} - g_2(1-g_1)\,\hat{y}$$

$$\vec{S}_1 \cdot \vec{S}_2 = g_1g_2(1-g_1g_2) - g_1g_2(1-g_1)(1-g_2)$$

$* \sqrt{g_1g_2}$ ✓

$$g_1 = 1-\frac{L}{R_1}\;;\; g_2 = 1-\frac{L}{R_2}$$

$$\cos\theta = \sqrt{g_1g_2} \simeq 1-\frac{\theta^2}{2}$$

↓

é um ângulo mt pequeno

$$\sqrt{g_1g_2} = \sqrt{1-\left(\frac{L}{R_1}+\frac{L}{R_2}\right)}$$

Nota

$$\left(\sqrt{1-t}\right) \sim 1-\frac{t}{2}$$

$$\sim 1-\frac{1}{2}\left(\frac{L}{R_1}+\frac{L}{R_2}\right)$$

$$\theta \sim \sqrt{\frac{1}{R_1}+\frac{1}{R_2}}$$

02/12/2022

6.6.

a)

$$I(t) = I_0\,\frac{\sin^2\left[N\frac{\Delta\omega_{cav}}{2}t\right]}{\sin^2\left[\frac{\Delta\omega_{cav}}{2}t\right]}$$

No limite dos picos $t \to 0 \Rightarrow I(t) = I_0 \dfrac{\left(N\frac{\Delta\omega_{cav}t}{2}\right)^2}{\left(\frac{\Delta\omega_{cav}t}{2}\right)^2} = N^2 I_0$

$$= N\,(N I_0)$$

b)

$$\bar{I} = \frac{1}{T}\int_0^T I(t)\,dt = *$$

$$I(t) = \frac{1}{2}\varepsilon_0 c\,|E^N(t)|^2 = \frac{1}{2}\varepsilon_0 c\,|E_0|^2 \sum_{m=0}^{N-1} e^{-i(\omega_0 + m\Delta\omega_{cav})t} \times \sum_{m=0}^{N-1} e^{i(\omega_0+m\Delta\omega_{cav})t}$$

$$E^N(t) = E_0 \sum_{m=0}^{N-1} e^{-i(\omega_0 + m\Delta\omega_{cav})t}$$

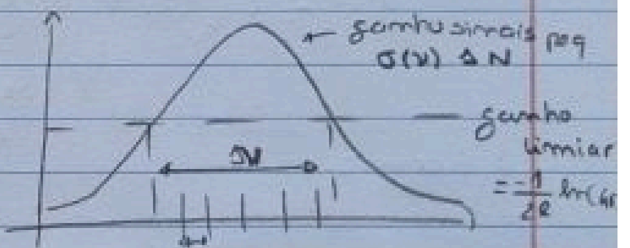
$$I(t) = I_0\left[\sum_{m=m} 1 + \sum_{m\neq m} e^{-i(m-m)\Delta\omega_{cav} t}\right]$$

$$* \quad \bar{I} = \frac{I_0}{T}\left[\underbrace{\int_0^T dt\,N}_{NT} + \underbrace{\int_0^T dt\, e^{-i(m-m)\Delta\omega_{cav}t}}_{\frac{1}{i(m-m)\Delta\omega_{cav}}\left[\underbrace{e^{-i(m-m)\Delta\omega_{cav}t}}_{e^{-i(m-m)2\pi}} - 1\right]}\right]$$

$$\Delta\omega_{cav} = 2\pi\,\Delta\nu_{cav} = 2\pi\frac{c}{2L} = \frac{2\pi}{T}$$

6.8.

a) He-Ne $\quad \lambda = 632{,}8\ \text{nm}$



$$\Delta\nu_{cav} = \frac{c}{2L} = \frac{1}{T}$$

$$\Delta t_{pulso} = \frac{T}{N}$$

tempo de 1 volta

$$N = \frac{\Delta\nu}{\Delta\nu_{cav}} = \Delta\nu\,T$$

Princípio de incerteza

$$\Rightarrow \Delta t_{pulso} = \frac{T}{\Delta\nu T} = \frac{1}{\Delta\nu}$$

menor tempo de pulso

$$\Delta\nu \sim \Delta\nu_{Doppler} \sim 2{,}15\times10^{11}\,\frac{1}{\lambda(nm)}\sqrt{\frac{T(K)}{M(g/mol)}} \sim 1{,}5\ \text{GHz}$$

$$\Delta t_{pulso} = \frac{1}{1{,}5\times10^{9}} = 670\ \text{ps}$$

$$\boxed{\frac{\partial I_{\omega_3}}{\partial z} = \frac{1}{2}\,\varepsilon_0\, c\, n_{\omega_3}\left[\mathcal{E}^*_{\omega_3}\,\frac{\partial \mathcal{E}_{\omega_3}}{\partial z} + \mathcal{E}_{\omega_3}\,\frac{\partial \mathcal{E}^*_{\omega_3}}{\partial z}\right]}$$

$$= \left\{ \begin{array}{l} \dfrac{1}{\omega_2}\dfrac{\partial I_{\omega_2}}{\partial z} = \left[i\,\dfrac{1}{2}\,\varepsilon_0\, d_{eff}\, \mathcal{E}_{\omega_1}\mathcal{E}^*_{\omega_2}\mathcal{E}^*_{\omega_3}\, e^{i(k_{\omega_1}-k_{\omega_2}-k_{\omega_3})z} + cc \right] \\[2ex] \dfrac{1}{\omega_3}\dfrac{\partial I_{\omega_3}}{\partial z} \left[i\,\dfrac{1}{2}\,\varepsilon_0\, d_{eff}\, \mathcal{E}_{\omega_1}\mathcal{E}^*_{\omega_2}\mathcal{E}^*_{\omega_3}\, e^{i(k_{\omega_1}-k_{\omega_2}-k_{\omega_3})z} + cc \right] \end{array} \right.$$

$$\frac{1}{\omega_1}\frac{\partial I_{\omega_1}}{\partial z} = \left[-i\,\frac{1}{2}\,\varepsilon_0\, d_{eff}\, \mathcal{E}_{\omega_1}\mathcal{E}^*_{\omega_2}\mathcal{E}^*_{\omega_3}\, e^{i(k_{\omega_1}-k_{\omega_2}-k_{\omega_3})z} + cc \right]$$

↳ com a parte do complexo conjugado

$$\boxed{\frac{1}{\omega_2}\frac{\partial I_{\omega_2}}{\partial z} = \frac{1}{\omega_3}\frac{\partial I_{\omega_3}}{\partial z} = \frac{1}{\omega_1}\frac{\partial \omega_1}{\partial z}} \times \frac{\Delta t\, \text{área}}{\hbar}$$

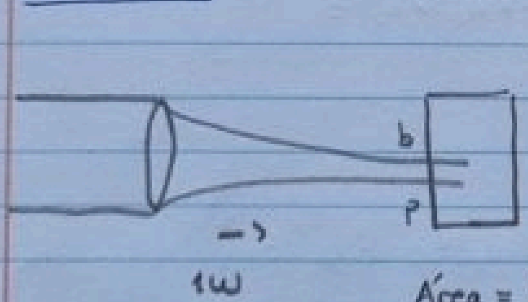
energia de cada fotão

$$I_\omega = \frac{N_{fotões}}{\Delta t}\,\frac{\hbar\omega}{\text{área}} \qquad \frac{I_\omega}{\omega} = N_{fotões}\,\frac{\hbar}{\Delta t\, \text{área}}$$

É equivalente a: $\dfrac{\partial N_{fotões}^{\omega_1}}{\partial z} = \dfrac{\partial N_{fotões}^{\omega_2}}{\partial z} = \dfrac{\partial N_{fotões}^{\omega_3}}{\partial z}$

→ Por cada fotão que perdemos ~~em~~ em ω_1, ganhamos em ω_2 e ω_3

Problema 2



$n_2^I = 10^{-16}\ cm^2/W$

Área $= 10^{-4}\ cm^2$

$d = 100\,\mu m$

a) Feixe CW

$$I = \frac{Pot}{\text{área}} = \frac{1W}{10^{-4} cm^2} = 10^4\ W/cm^4 \Rightarrow \Delta n = n_2^I\, I = 10^{-16}\times 10^4 = 10^{-12}$$

Desprezável

$$\frac{(\omega)^2}{c^3} \cdot I_\omega = \frac{1}{\lambda^2} |E|^2 z^2$$

$$e_{SHG} = \left(\frac{4\pi^2}{347{,}1\times10^{-9}\,m}\right)^2 \frac{1}{2\left(8{,}85\times10^{-12}\,\frac{C}{Vm}\right)\left(3\times10^{8}\,\frac{m}{s}\right)^3} \cdot \frac{1}{1{,}6^2}\underbrace{\left[6{,}0\times10^{-3}\,\frac{m}{V}\right]^2}_{(\chi^{(2)})^2} \times$$

$$\times\ 1\,cm^2\ 10^{8}\,\frac{W}{cm^2} = 0{,}66$$

b) Considere $n_{2\omega} = 1{,}57$ e $n_\omega = 1{,}54$

$$\Delta K = 2k_\omega - K_{2\omega} = 2\left(\frac{\omega}{c} n_\omega\right) - \frac{2\omega}{c} n_{2\omega}$$

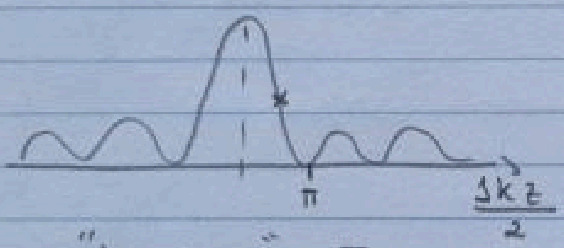
$$= \frac{2\omega}{c}\left[\overbrace{n_\omega - n_{2\omega}}^{-0{,}03}\right] = 5{,}4\times10^{5}\ cm^{-1}$$

$$\frac{2\pi}{\lambda_{2\omega}}$$

$$\left[\frac{\sin\left(\frac{\Delta kL}{2}\right)}{\frac{\Delta kL}{2}}\right]^2 \simeq 1{,}3\times10^{-7}$$

Nº de fotões gerados at 2ω $\sim (4{,}6\times10^{9})(0{,}66) \sim 3\times10^{8}$

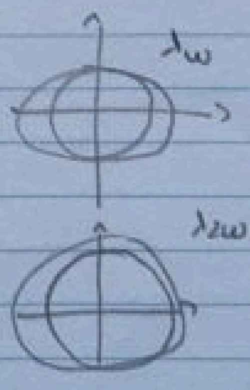
Energia$_{2\omega} \sim 2{,}6\,mJ$



$$\frac{\Delta k\,\text{"}L_{coerência}\text{"}}{2} = \frac{\pi}{2} \Leftrightarrow \text{"}L_{coerência}\text{"} = \frac{\pi}{\Delta k} \sim 6\,\mu m$$

10.4.

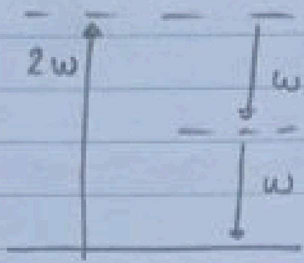
	n_o	n_e
$\lambda_\omega = 694{,}3$	1,5408	1,5498
$\lambda_{2\omega} = 347{,}15$	1,5664	1,5774



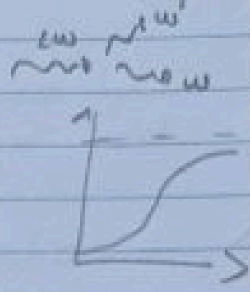
2 fatias o desfazer e fazer

Problema: o que acontece quando $\Delta k = 0$?

A 2ª harmónica pode ser desfeita em 2 fotões com $f = \omega$



$$I_{2\omega} = I_{\omega} \tanh^2\left(\sqrt{\frac{\ldots \omega m_\omega^2}{2\omega n_0{}^2 d_{ef}}}\right)$$



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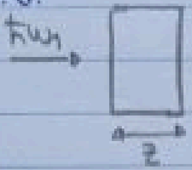
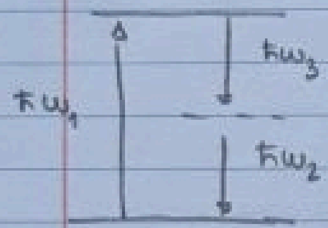
10.6.



$\omega_1 \to$ freq que queremos; $d = \frac{1}{2} \chi^{(2)}_{i,j,k}$

$$\frac{\partial \mathcal{E}_{\omega_1}}{\partial z} = \frac{i\omega_1}{n_{\omega_1} c} d_{eff}\, \mathcal{E}_{\omega_2} \mathcal{E}_{\omega_3}\, e^{-i(k_{\omega_1} - k_{\omega_2} - k_{\omega_3})z}$$

ver o que é



Em geral: $E_\omega = \frac{1}{2}\left[\mathcal{E}_\omega\, e^{i(kz-\omega t)} + \mathcal{E}_\omega^*\, e^{-i(kz-\omega t)}\right]$

vão cortando por causa da construção de energia

$\omega_1 = \omega_2 + \omega_3$

$$\frac{\partial \mathcal{E}_{\omega_2}}{\partial z} = \frac{i\,\omega_2}{c\, n_{\omega_2}} d_{eff}\, \mathcal{E}_{\omega_1} \mathcal{E}_{\omega_3}^*\, e^{i(k_{\omega_1} - k_{\omega_2} - k_{\omega_3})z}$$

é absorvido   é emitido

$\omega_2 = \omega_1 - \omega_3$

$$\frac{\partial \mathcal{E}_{\omega_3}}{\partial z} = \frac{i\,\omega_3}{c\, n_{\omega_3}} d_{eff}\, \mathcal{E}_{\omega_1} \mathcal{E}_{\omega_2}^*\, e^{i(k_{\omega_1} - k_{\omega_2} - k_{\omega_3})z}$$

$\omega_3 = \omega_1 - \omega_2$

Pelo vetor Poyting

$I_{\omega_1} = \frac{1}{2} \varepsilon_0 c\, n_{\omega_1} |\mathcal{E}_{\omega_1}|^2$ $\qquad |\mathcal{E}_{\omega_1}|^2 = \mathcal{E}_{\omega_1} \mathcal{E}_{\omega_1}^*$

$$\boxed{\frac{\partial I_{\omega_1}}{\partial z} = \frac{1}{2} \varepsilon_0 c\, n_{\omega_1} \left[\mathcal{E}_{\omega_1}^* \frac{\partial \mathcal{E}_{\omega_1}}{\partial z} + \mathcal{E}_{\omega_1} \frac{\partial \mathcal{E}_{\omega_1}}{\partial z}\right]}$$

já calculamos

$$\boxed{\frac{\partial I_{\omega_2}}{\partial z} = \frac{1}{2} \varepsilon_0 c\, n_{\omega_2} \left[\mathcal{E}_{\omega_2}^* \frac{\partial \mathcal{E}_{\omega_2}}{\partial z} + \mathcal{E}_{\omega_2} \frac{\partial \mathcal{E}_{\omega_2}^*}{\partial z}\right]}$$