Teste modelo:

A. Descreve dois exemplos de alargamento do perfil da linha que aumentam a largura do espetro da absorção acima do que é observado nas condições de alargamento natural.

1. Considere um gás de Ne (M = 20.2 g/mol) dentro uma laser He-Ne. O perfil da transição laser é dominado pelo efeito Doppler á uma temperatura de 400K. O meio ativo tem um diâmetro de 1 mm, um comprimento de 20 cm e preencha toda a cavidade laser. Estime o número dos modos eletromagnéticos que podem interagir com átomos Ne na transição laser (λ = 632.8nm) dentro da largura inteira a meia altura (FWHM) do perfil da transição. Pode assumir que o índice de refração de gás é 1. Quantos modos longitudinais se encontram nesta mesma gama das frequências?

2. Um conjunto de 10^{10} átomos de "dois níveis" foi arrefecido até o efeito Doppler seja desprezável. A transição ocorre num comprimento de onda igual á 600 nm e as degenerescências dos dois níveis são iguais.
Quando um laser de irradiância constante é sintonizado em ressonância com a transição, ao atingir um estado estacionário, 25% dos átomos se encontram no estado excitado. Ao dessintonizar a radiação por um valor igual à $\Delta\nu = \nu - \nu_0 = 10MHz$ apenas 1/12 dos átomos se encontram no estado excitado quando um estado estacionário é atingido. Assumindo que apenas alargamento natural contribua para o perfil da linha determine:
(a) o tempo da vida do estado excitado;
(b) a potência da luz emitida espontaneamente pelos átomos quando o laser está sintonizado em ressonância com a transição;
(c) a irradiância (em W/m^2) do laser.

(d) Se a irradiância da saturação é dada pela expressão $I_\nu^{sat} = h\nu_{21}A_{21} / \sigma(\nu_{21})$ (sistema de 4 níveis assumindo que $P \ll A_{21}$) a presunção que apenas alargamento natural contribua para o perfil da transição é valida?

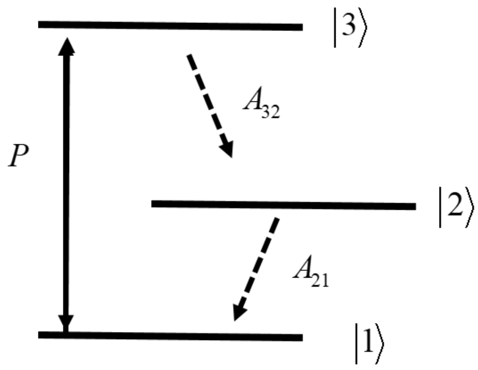
(e) Qual é temperatura duma fonte da radiação térmica que excitava 25% dos átomos no estado estacionário?

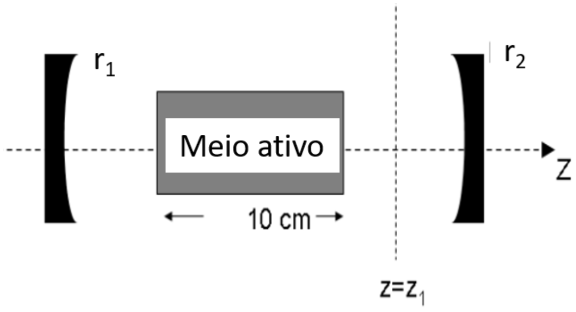
3. Considere um sistema laser de 3 níveis com alargamento homogéneo como ilustrado em baixo. Assuma que uma das frequências próprias da cavidade está em ressonância com a transição entre níveis 1 e 2 e despreze os efeitos de degenerescência.

(a) Escrever as equações dinâmicas para a variação das populações em ordem do tempo. No limite dos sinais pequenos com $A_{32} \gg P, A_{21}$ derive uma expressão para a inversão da população entre níveis 2 e 1, $\Delta N_{21} = N_2 - N_1$ no estado estacionário, em termos da taxa de excitação P, a taxa de emissão espontânea, A_{21} e a população total $N_T = N_1 + N_2 + N_3$.

(b) Dado os valores, $\lambda_{21} = 694nm$ $\sigma(\nu_{21}) = 10^{-12}cm^2$; $N_T = 3x10^{15}cm^{-3}$; $A_{21} = 10^8 s^{-1}$; $P = 2A_{21}$; $r_1 = 1$, qual é o valor mínimo de r_2 que permite que o ganho atinge o seu valor limiar?

(c) No caso em que $R_2 = 0.99$ qual seria a irradiância dentro da cavidade laser que faça que o ganho na frequência central baixa para o valor do ganho limiar?





13. (15 pts) Consider a stable 2-mirror laser cavity containing a 3-level homogeneous gain medium (see figure below). Assume that a resonant mode of the optical cavity (ν_q) is centered on the atomic transition frequency ν_0 between levels 1 and 2, so that $\Delta = 0$ (one can therefore ignore the frequency dependence of the gain coefficient for this problem).

(a) For the 3-level system shown in the figure below, set up the rate equations for the population densities N_1 and N_2. Then, derive the expression for the small signal steady-state population density inversion $\delta N = N_2 - N_1$ between levels 1 and 2 in terms of only the pumping rate P, the spontaneous emission rate (Γ_{21}), and total number density in the 3 level system, $N_t = N_1 + N_2 + N_3$. Assume the population decays instantaneously from level 3 to 2 (i.e. $N_3 \sim 0$).

9. (6 pts) Consider a linear laser cavity of length L= 1 meter with an inhomogeneouly-broadened gain medium. The gain medium has an upper state lifetime of τ_2 =100 μs. Suppose the small-signal gain coefficient $\gamma_o(\omega)$ has a peak value of $\gamma_o(\omega_o)$ that is twice the value of the threshold gain coefficient γ_T.
(a) If $\gamma_o(\omega)$ has a FWHM of 100 GHz, how many $TEM_{0,0}$ modes can simultaneously lase?

3. (2 pts) Give two examples of line broadening mechanisms that would increase the measured absorption profile of a transition beyond it's natural line width.

1. (4 pts) Consider an isolated 2-level atom with ground state $|\Psi_g\rangle$ and excited state $|\Psi_e\rangle$ and population densities in each state given by N_g and N_e, respectively. If all the atoms are initially in the excited state, write an expression for $N_e(t)$ in terms of the Einstein A coefficient.

6. (4 pts) Is a laser with a homogeneous or inhomogeneous gain medium more likely to operate on a single longitudinal mode? Describe why in 1-2 sentences.

$\lambda = 1 \mu m$ $\quad$ $l = 10$ cm

7. (4 pts) Consider a laser gain medium consisting a solid cylindrical rod of diameter 5 mm doped with an atomic number density of $N = 5 \times 10^{18}/\text{cm}^3$. If the laser is Q-switched and emits a single pulse of the form $P(t) = P_o e^{-t^2/\tau^2}$, where $\tau = 100$ ns, what is the maximum *peak* power, P_o, that could possibly be generated? For this calculation, assume all the atoms are initially in the excited state and the laser cavity mode fills the entire volume of the gain medium. (Hint: first calculate the total maximum energy that could be extracted from the laser gain medium to form the pulse assuming every inverted atom in the rod emits a photon.)

You will need the following integral relation to help calculate the peak power: $\int_{-\infty}^{\infty} e^{-(t/\tau)^2} dt = \tau\sqrt{\pi}$

$$E = N \times V \times \hbar\omega$$

$$V = \pi r^2 l = \pi (.25\,cm)^2\, 10\,cm = 1.96\ cm^3$$

$$\hbar\omega = h\nu_0 \; ; \; \nu = c/\lambda = 300 \cdot 10^{12}\ Hz \rightarrow \hbar\omega = 2 \cdot 10^{-19}$$

$$E = \int P(t)\, dt = P_o \tau \sqrt{\pi} \qquad E = (5 \cdot 10^{18}/cm^3)(1.96\ cm^3)\ \hbar\omega$$

$$= 2\ J \qquad E = 2 \text{ Joule}$$

$$\rightarrow P_o = \frac{2\ J}{100\,ns\ \sqrt{\pi}} = 11 \cdot 10^6 \text{ Watts}$$

7. (4 pts) Consider a laser gain medium operating at $\lambda = 1\ \mu$m, consisting of a solid cylindrical rod of diameter 5 mm and length 10 cm, doped with an atomic number density of $N = 5 \times 10^{18}/\text{cm}^3$. If the laser is Q-switched and emits a single pulse of the form $P(t) = P_o e^{-t^2/\tau^2}$, where $\tau = 100$ ns, what is the maximum *peak* power, P_o, that could possibly be generated? For this calculation, assume all the atoms are initially in the excited state and the laser cavity mode fills the entire volume of the gain medium. (Hint: first calculate the total maximum energy that could be extracted from the laser gain medium to form the pulse assuming every inverted atom in the rod emits a photon.)

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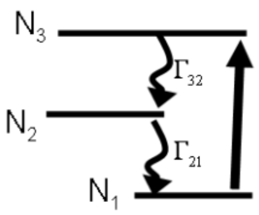
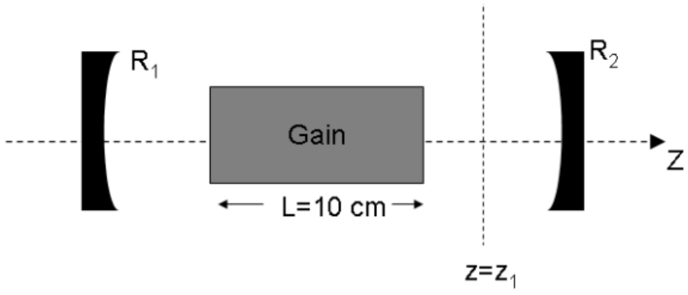
(b) Determine whether or not the laser will lase given the parameters below. Be sure to show your work.
$\sigma_o = 10^{-12} cm^2, N_t = 3 \times 10^9/cm^3, P = 2 \times \Gamma_{21}, R_1 = 1, R_2 = .99$





2. Excitação atómica [7.5 valores]

Luz laser com um comprimento de onda 589.2 nm e uma irradiância de 100mW/cm^2 excita um conjunto de átomos do sódio, com velocidades desprezáveis (i.e. existe apenas alargamento natural), na transição $3^2S_{1/2} \rightarrow 3^2P_{3/2}$. O tempo de vida média dos átomos no estado $3^2P_{3/2}$ é 16. 25ns. Considere o caso em que a polarização do feixe laser é linear e consequentemente as degenerescências do níveis fundamental e excitado que participam na transição são iguais.

(a) Estime a secção eficaz na ressonância nesta transição. Qual será a secção eficaz se a luz laser estiver dessintonizada da ressonância por uma largura inteira a meio altura do perfil da absorção?

(b) Ache as taxas de absorção, de emissão espontânea e de emissão estimulada que cada átomo é sujeito quando a luz laser está em ressonância com a transição.

(c) Estime a fração dos átomos que se encontrar no estado excitado quando o conjunto atinge um estado estacionário nestas condições.

(d) Imagine que pretende usar radiação do corpo negro em vezes radiação laser para obter a mesma razão entre átomos no estado excitado e no estado fundamental no estado estacionário. Qual terá ser a temperatura associada com a radiação?

13. (15 pts) Consider a stable 2-mirror laser cavity containing a 3-level homogeneous gain medium (see figure below). Assume that a resonant mode of the optical cavity (ν_q) is centered on the atomic transition frequency ν_0 between levels 1 and 2, so that $\Delta = 0$ (one can therefore ignore the frequency dependence of the gain coefficient for this problem).

(a) For the 3-level system shown in the figure below, set up the rate equations for the population densities N_1 and N_2. Then, derive the expression for the small signal steady-state population density inversion $\delta N = N_2 - N_1$ between levels 1 and 2 in terms of only the pumping rate P, the spontaneous emission rate (Γ_{21}), and total number density in the 3 level system, $N_t = N_1 + N_2 + N_3$. Assume the population decays instantaneously from level 3 to 2 (i.e. $N_3 \sim 0$).

$$dN_2/dt = -\Gamma_{21} N_2 + P N_1 = 0 \; ; \quad \frac{dN_1}{dt} = +\Gamma_{21} N_2 - P N_1 = 0 \quad \leftarrow \text{Steady state}$$

$$N_1 = \frac{\Gamma_{21}}{P} N_2 = \frac{\Gamma_{21}}{P}(N_t - N_1) \qquad \text{since } N_t = N_1 + N_2$$

$$N_1\left(1 + \frac{\Gamma_{21}}{P}\right) = \frac{\Gamma_{21}}{P} N_t \longrightarrow N_1 = N_T / (1 + P/\Gamma_{21})$$

likewise $N_2 = N_T / (1 + \Gamma_{21}/P)$

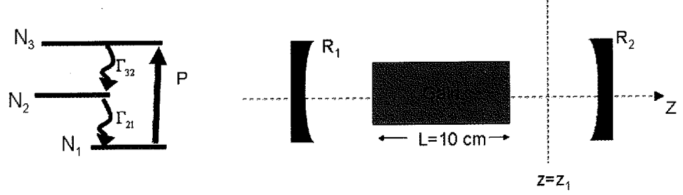
$$\rightarrow \delta N = N_t\left(\frac{1}{1+\Gamma_{21}/P} - \frac{1}{1+P/\Gamma_{21}}\right) = N_T\,\frac{(P-\Gamma_{21})}{P+\Gamma_{21}}$$

(b) Determine whether or not the laser will lase given the parameters below. Be sure to show your work.
$\sigma_o = 10^{-12} cm^2, N_t = 3 \times 10^9/cm^3, P = 2 \times \Gamma_{21}, R_1 = 1, R_2 = .99$

$\gamma_0 = \sigma_0\, \delta N$ w/ $P = 2 \cdot \Gamma_{21}$, $\delta N = N_t\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{N_t}{3}$

$\gamma_0 = .001$ /cm $\qquad \gamma_T = \beta = \frac{1}{20cm} \ln\left(\frac{1}{.99}\right)$

Yes, it will lase! $\qquad \gamma_T = .0005\ cm^{-1}$



14. (10 pts) A linearly polarized laser beam is sent through a 5 cm long cell containing ground state hydrogen atoms with a number density of $N = 10^{14}/cm^3$. For this problem, neglect Doppler broadening (i.e. assume the atomic ensemble is not moving) and assume that the atomic cross section for this transition is equal to its maximum possible value. The frequency of the laser is resonant with the $n = 1$ to $n = 2$ transition and is absorbed as it passes through the cell. Assume an effective (average) circular cross-sectional area of A_{eff} =1 cm^2 for the laser beam that is constant in the cell.

(a) Assuming the initial intensity of the laser is much less than the saturation intensity, calculate the percentage of power transmitted after propagating through the gas cell.

(b) Calculate the power of the laser that would saturate the absorption coefficient to half of its small signal value.

16. (15 pts) Ring laser cavity.
A 3-level homogeneous gain medium is placed within a stable 4-mirror ring optical cavity. The mirror power reflectivities are R_1=0.950, $R_2 = R_3 = R_4$=0.990. The length of the gain cell is l = 10 cm. Assume that the resonant mode of the optical cavity (ω_q) closest to the peak of the gain profile is detuned from the atomic transition frequency ω_0 by $\Delta = \Gamma/2$, where Γ is the full-width at half maximum (FWHM) of the gain profile. Assume the laser only operates in a single direction (e.g. clockwise).

a.) Calculate the numeric value of the threshold gain coefficient, γ_T.

b. Under steady-state lasing conditions, plot the gain coefficient $\gamma(\omega)$ vs. ω. On the axes, indicate γ_T, ω_q, and ω_o.

c.) On a single graph, plot both the output intensity I and the gain coefficient γ versus the pumping rate P. The pumping rate is the rate at which atoms from the ground state are excited to the upper state of the 3 level system. Be sure to indicate the position of the threshold gain coefficient γ_T in your graph, as well as the sign of γ at P=0 (ie, when P=0, is γ positive, negative, or equal to zero?).

(5.5) More steady-state amplifiers
The two amplifiers considered in the problem both operate on homogeneously broadened transitions and under steady-state conditions.

(a) When a signal of intensity $3.0\,\mathrm{kW\,m^{-2}}$ is applied at the input of an amplifier of length 10 m, the output signal intensity is $36\,\mathrm{kW\,m^{-2}}$. When the input signal is reduced to $1.0\,\mathrm{kW\,m^{-2}}$ the output is $20\,\mathrm{kW\,m^{-2}}$. Calculate the value of the saturation intensity of the amplifier and the small-signal gain coefficient.

(b) Another laser amplifier gives an output of $5I_0$ (
when it receives input intensity I_0 tuned to ω_0, the centre frequency of the laser transition. When the input radiation is tuned to a nearby angular frequency ω_1, an input intensity of $3I_0$ is amplified to an output intensity of $7I_0$. What is the ratio of the gain cross-sections at the two frequencies?

In the CO_2 laser the upper laser level is 2350 cm^{-1} above the ground state and the lower laser level is 1406 cm^{-1} above the ground state. Estimate how much more difficult it would be to obtain laser oscillation if the gas in the laser heats up from 400K to 1200K. (4 pts.) Assume that $g_2 = g_1$, and that only the ground state, upper laser level and lower laser level are populated. Take $N = 4 \times 10^{22}\mathrm{m}^{-3}$, $N_t = 10^{15}\mathrm{m}^{-3}$. At 300K kT=208.6 cm^{-1}.

A 4-level laser is pumped into its pump band at a rate 10^{24} m^{-3} s^{-1}, the transfer efficiency to the upper laser level is 0.5. The lifetime of the upper laser level is 7×10^{-4}s. For the laser transition $A_{21}=10^3$ s^{-1}, $\lambda_0=1\mu$m. The laser is homogeneously broadened with $\Delta\nu=$ 1GHz. Assume n=1.6. The amplifying medium is 20mm long. Neglect lower laser level population.
(a) What is the gain at line center? (6 pts.)
(b) What minimum value of R_2 is needed to get oscillation if R_1=1?
Assume α=0. (4 pts.)

Hint:

$$\gamma_t = \alpha - \frac{1}{l}\ln(r_1 r_2).$$

(3) Derive an expression for the threshold gain in a laser with a resonant cavity of length l and an amplifying medium of length L when both mirrors have identical reflectance R. There is a distributed loss α in the part of the cavity outside the amplifying medium. (4 pts.) Can you think of a laser system in which this type of behavior might occur? (1 pt.) In a laser of the sort described above, l=100mm, L=1mm, $R=0.95$ and $\alpha=0$. The cavity and amplifying medium both have refractive index 3.5. The small signal gain at line center is 1 mm^{-1}. The laser is inhomogeneously broadened with a Gaussian FWHM of 10GHz. How many modes will oscillate? (5 pts.)

Hint:

$$\gamma(\nu) = \gamma(\nu_0)\exp - \left[\frac{2(\nu-\nu_0)}{\Delta\nu_D}\right]^2 \ln 2.$$

Look at InstQunt in fotonics