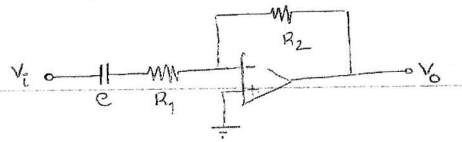
Filtro passa - alto de 1ª Ordem:

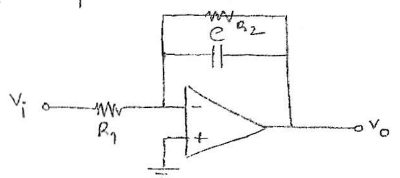


$$H(jf) = \frac{V_o}{V_i} = -\frac{Z_{R_2}}{Z_C + Z_{R_1}} = -\frac{R_2}{\frac{1}{j\omega C} + R_1} = -\frac{R_2}{\frac{1+j\omega R_1 C}{j\omega C}}$$

$$= -\frac{j\omega C R_2}{1+j\omega C R_1} = -\frac{R_2 \; j\omega C}{R_1\left(\frac{1}{R_1} + j\omega C\right)} = -\frac{R_2}{R_1}\,\frac{j\omega R_1 C}{1+j\omega R_1 C} \Leftrightarrow$$

$$\Leftrightarrow H(jf) = -\frac{R_2}{R_1}\,\frac{j(f/f_0)}{1+j(f/f_0)} \;,\; f_0 = \frac{1}{2\pi R_1 C}$$

Filtro passo baixo de 1ª Ordem:



$$H(jf) = \frac{-\, Z_C // Z_{R_2}}{R_1}$$

$Z_C // Z_{R_2} = ?$

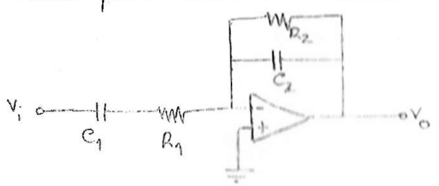
$$\frac{1}{Z_T} = \frac{1}{Z_C} + \frac{1}{R_2} \Leftrightarrow$$

$$\Leftrightarrow \frac{1}{Z_T} = j\omega C + \frac{1}{R_2} \Leftrightarrow$$

$$\Leftrightarrow Z_T = \frac{R_2}{j\omega R_2 C + 1}$$

$$H(jf) = -\frac{R_2}{R_1} \cdot \frac{1}{1+j\omega R_2 C} = -\frac{R_2}{R_1}\,\frac{1}{1+j(f/f_0)} \;,\; f_0 = \frac{1}{2\pi R_2 C}$$

Filtro passa banda de 1º ordem



$$H(jf) = -\frac{Z_2}{Z_1}$$

$$Z_2 = Z_{R_2} // Z_{C_2} = \frac{R_2}{j\omega R_2 C_2 + 1}$$

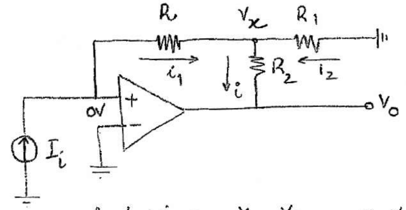
$$Z_1 = \frac{1}{j\omega C_1} + R_1 = \frac{1+j\omega R_1 C_1}{j\omega C_1}$$

$$H(jf) = -\frac{\frac{R_2}{1+j\omega R_2 C_2}}{\frac{1+j\omega R_1 C_1}{j\omega C_1}} = -\frac{R_2 \; j\omega C_1}{(1+j\omega R_2 C_2)(1+j\omega R_1 C_1)} =$$

$$= -\frac{R_2}{R_1}\,\frac{j\omega R_1 C_1}{(1+j\omega R_1 C_1)(1+j\omega R_2 C_2)} = -\frac{R_2}{R_1}\,\frac{j(f/f_1)}{[1+j(f/f_1)][1+j(f/f_2)]}$$

$$f_1 = \frac{1}{2\pi R_1 C_1} = f_{ic} \;;\; f_2 = \frac{1}{2\pi R_2 C_2} = f_{sc}$$

Conversor I-V de elevada sensibilidade



$$i = i_1 + i_2 \Leftrightarrow \frac{V_x - V_o}{R_2} = \frac{0 - V_x}{R} + \frac{0 - V_x}{R_1}$$

$$0 - i_1 R = V_x \Leftrightarrow V_x = -I_i R$$

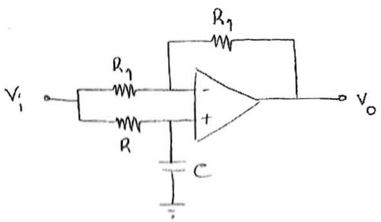
$$\frac{-I_i R - V_o}{R_2} = \frac{I_i \cancel{R}}{\cancel{R}} + \frac{I_i R}{R_1} \Leftrightarrow -I_i R - V_o = I_i R_2 + I_i R\,\frac{R_2}{R_1} \Leftrightarrow$$

$$\Leftrightarrow V_o = -I_i R - I_i R_2 - I_i R\,\frac{R_2}{R_1} \Leftrightarrow$$

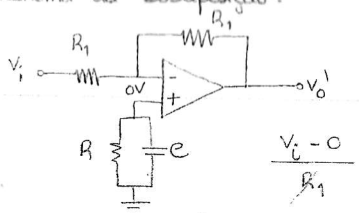
$$\Leftrightarrow V_o = -I_i R\left(1 + \frac{R_2}{R} + \frac{R_2}{R_1}\right)$$

Deslocadores de Fase:

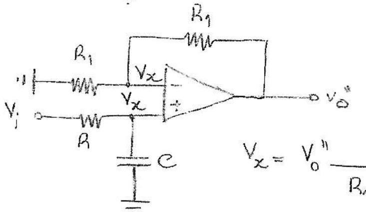
1



Teorema da Sobreposição:



$$\frac{V_i - 0}{\cancel{R_1}} = \frac{0 - V_o'}{\cancel{R_1}} \Leftrightarrow V_o' = -V_i$$



$$V_x = V_o'' \frac{R_1}{R_1+R_1} \Leftrightarrow V_x = \frac{1}{2} V_o''$$

$$V_x = V_C = V_i \frac{Z_C}{Z_C+R} = V_i \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C}+R} \Leftrightarrow$$

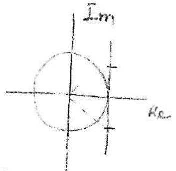
$$\Leftrightarrow V_x = V_i \frac{\frac{1}{j\omega C}}{\frac{1+j\omega RC}{j\omega C}} \Leftrightarrow V_x = V_i \frac{1}{1+j\omega RC}$$

$$\frac{1}{2} V_o'' = V_i \frac{1}{1+j\omega RC} \Leftrightarrow V_o'' = V_i \frac{2}{1+j\omega RC}$$

$$V_o = V_o' + V_o'' = -V_i + V_i \frac{2}{1+j\omega RC} \Leftrightarrow V_o = V_i \left(-1 + \frac{2}{1+j\omega RC}\right) \Leftrightarrow$$

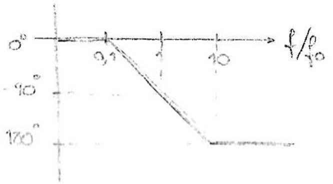
$$\Leftrightarrow \frac{V_o}{V_i} = H(jf) = \frac{-1-j\omega RC+2}{1+j\omega RC} \Leftrightarrow H(jf) = \frac{1-j\omega RC}{1+j\omega RC} \Leftrightarrow$$

$$\Leftrightarrow H(jf) = \frac{1-j(f/f_o)}{1+j(f/f_o)}$$

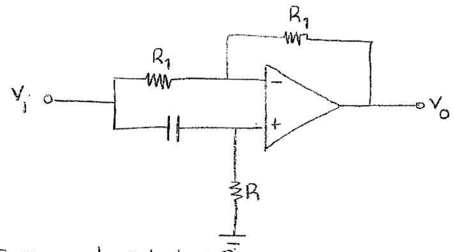


$$\angle H(jf) = \text{arctg} \frac{-(f/f_o)}{1} - \text{arctg} \frac{(f/f_o)}{1} =$$

$$= -\text{arctg}(f/f_o) - \text{arctg}(f/f_o) = -2\,\text{arctg}(f/f_o)$$



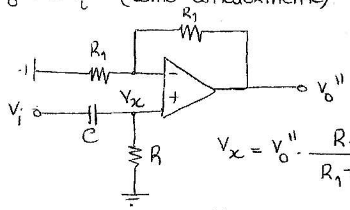
②



Teorema da Sobreposição:

1) $V_o' = -V_i$ (como anteriormente)

2)



$$V_x = V_o'' \cdot \frac{R_1}{R_1+R_1} \Leftrightarrow V_o = \frac{1}{2} V_o''$$

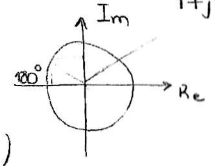
$$V_x = V_R = V_i \frac{R}{\frac{1}{j\omega C}+R} \Leftrightarrow V_x = V_i \frac{j\omega RC}{1+j\omega RC}$$

$$\frac{1}{2} V_o'' = V_i \frac{j\omega RC}{1+j\omega RC} \Leftrightarrow V_o'' = V_i \frac{2j\omega RC}{1+j\omega RC}$$

$$V_o = V_o' + V_o'' = -V_i + V_i \frac{2j\omega RC}{1+j\omega RC} \Leftrightarrow V_o = V_i \left(-1 + \frac{2j\omega RC}{1+j\omega RC}\right) \Leftrightarrow$$

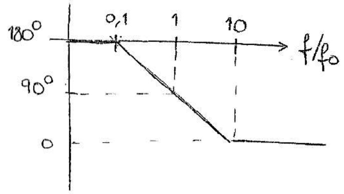
$$\Leftrightarrow \frac{V_o}{V_i} = H(jf) = \frac{-1-j\omega RC+2j\omega RC}{1+j\omega RC} \Leftrightarrow H(jf) = \frac{j\omega RC-1}{1+j\omega RC} \Leftrightarrow$$

$$\Leftrightarrow H(jf) = \frac{j(f/f_o)-1}{1+j(f/f_o)}$$



$$\angle H = \text{arctg} \frac{(f/f_o)}{-1} - \text{arctg} \frac{(f/f_o)}{1} =$$

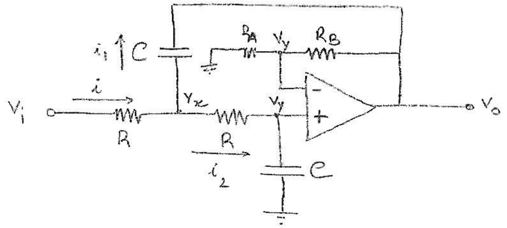
$$= 180° - \text{arctg} \frac{(f/f_o)}{1} - \text{arctg} \frac{(f/f_o)}{1} = 180° - 2\,\text{arctg}(f/f_o)$$



Filtros de 2ª Ordem:

$$H(jf) = \frac{N\, j(f/f_0)}{1-(f/f_0)^2 + (j/Q)(f/f_0)} \Bigg\} = \text{em todos os filtros!!}$$

1) Filtro passa baixo Sallen-Key KRC:



$$i = i_1 + i_2 \Leftrightarrow \frac{V_i - V_x}{R} = \frac{V_x - V_o}{Z_C} + \frac{V_x - V_y}{R}$$

$$V_y = V_o \frac{R_A}{R_A+R_B}$$

$$V_y = V_C = V_x \frac{Z_C}{R+Z_C} \Leftrightarrow V_y = V_x \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} \Leftrightarrow$$

$$\Leftrightarrow V_y = V_x \frac{1}{1+j\omega RC} = V_o \frac{R_A}{R_A+R_B} \Leftrightarrow$$

$$\Leftrightarrow V_x = V_o \frac{R_A}{R_A+R_B}(1+j\omega RC)$$

$$\frac{V_i - V_o \frac{R_A}{R_A+R_B}(1+j\omega RC)}{R} = \frac{V_o\frac{R_A}{R_A+R_B}(1+j\omega RC) - V_o}{\frac{1}{j\omega C}} + \frac{V_o \frac{R_A}{R_A+R_B}(1+j\omega RC) - V_o\frac{R_A}{R_A+R_B}}{R} \Leftrightarrow$$

$$\Leftrightarrow \frac{V_i(R_A+R_B) - V_oR_A(1+j\omega RC)}{\cancel{R}(R_A+R_B)} = \frac{[V_oR_A(1+j\omega RC) - V_o(R_A+R_B)]\,j\omega CR}{(R_A+R_B)\cancel{R}} +$$

$$+ \frac{V_oR_A(1+j\omega RC) - V_oR_A}{\cancel{R}(R_A+R_B)} \Leftrightarrow V_i(R_A+R_B) = V_oR_A + V_oR_A\, j\omega RC +$$

$$+ V_oR_A\, j\omega RC + V_oR_A(j\omega RC)^2 - \cancel{V_oR_A\, j\omega RC} - V_oR_B\, j\omega RC + \cancel{V_oR_A} + \cancel{V_oR_A\, j\omega RC}$$

$$-\cancel{V_oR_A} \Leftrightarrow$$

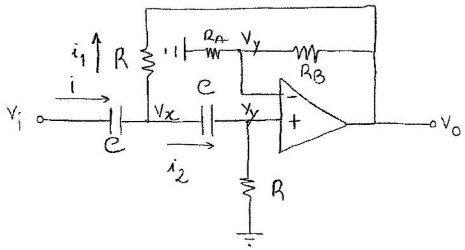
$$\Leftrightarrow V_i(R_A+R_B) = V_o\left(R_A + R_A(j\omega RC)^2 + (2R_A - R_B)\,j\omega RC\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{V_o}{V_i} = H(jf) = \frac{R_A+R_B}{R_A\left(1 - \left(\frac{f}{f_0}\right)^2 + \left(2-\frac{R_B}{R_A}\right)j\left(\frac{f}{f_0}\right)\right)} \Leftrightarrow$$

$$\Leftrightarrow H(jf) = \frac{1+\frac{R_B}{R_A} = K}{1-\left(\frac{f}{f_0}\right)^2 + \left(2-\frac{R_B}{R_A}\right)j\left(\frac{f}{f_0}\right)}$$

$$\frac{\cancel{j}}{Q} = \left(2-\frac{R_B}{R_A}\right)\cancel{j} \Leftrightarrow Q = \frac{1}{2-\frac{R_B}{R_A}} = \frac{1}{3-K}$$

2) Filtro passa-alto Sallen-Key KRC



$$i = i_1+i_2 \Leftrightarrow \frac{V_i - V_x}{Z_C} = \frac{V_x - V_o}{R} + \frac{V_x - V_y}{Z_C}$$

$$V_y = V_o\frac{R_A}{R_A+R_B}$$

$$V_y = V_R = V_x \frac{R}{R+Z_C} \Leftrightarrow V_y = V_x\frac{R}{R+\frac{1}{j\omega C}} \Leftrightarrow$$

$$\Leftrightarrow V_y = V_x\frac{j\omega RC}{1+j\omega RC} = V_o\frac{R_A}{R_A+R_B} \Leftrightarrow V_x = V_o \frac{R_A}{R_A+R_B}\frac{(1+j\omega RC)}{j\omega RC}$$

$$\frac{V_i - V_o\frac{R_A}{R_A+R_B}\frac{(1+j\omega RC)}{j\omega RC}}{\frac{1}{j\omega C}} = \frac{V_o\frac{R_A}{R_A+R_B}\frac{(1+j\omega RC)}{j\omega RC} - V_o}{R} +$$

$$+\frac{V_o\frac{R_A}{R_A+R_B}\frac{(1+j\omega RC)}{j\omega RC} - V_o\frac{R_A}{R_A+R_B}}{\frac{1}{j\omega C}} \Leftrightarrow$$

$$\Leftrightarrow \frac{\left[V_i (R_A+R_B)\, j\omega RC - V_o R_A (1+j\omega RC)\right] j\omega C R}{\cancel{R(R_A+R_B)\, j\omega RC}} =$$

$$= \frac{V_o R_A (1+j\omega RC) - V_o (R_A+R_B)\, j\omega RC}{\cancel{R(R_A+R_B)\, j\omega RC}} + \frac{\left[V_o R_A (1+j\omega RC) - V_o R_A (j\omega RC)\right] j\omega C R}{\cancel{R(R_A+R_B)}\, j\omega RC} \Leftrightarrow$$

$$\Leftrightarrow V_i (R_A+R_B)\, j^2 (\omega RC)^2 = V_o R_A\, j\omega C R + V_o R_A (j\omega RC)^2 + V_o R_A + \cancel{V_o R_A\, j\omega RC} -$$
$$- \cancel{V_o R_A\, j\omega RC} - V_o R_B\, j\omega RC + V_o R_A\, j\omega RC + \cancel{V_o R_A (j\omega RC)^2} - \cancel{V_o R_A (j\omega RC)^2} \Leftrightarrow$$

$$\Leftrightarrow -V_i (R_A+R_B) \left(\frac{f}{f_0}\right)^2 = V_o R_A + V_o R_A (j\omega RC)^2 + 2R_A\, j\omega RC - V_o R_B\, j\omega RC \Leftrightarrow$$

$$\Leftrightarrow -V_i (R_A+R_B) \left(\frac{f}{f_0}\right)^2 = V_o R_A \left[1 - \left(\frac{f}{f_0}\right)^2 + \left(2 - \frac{R_B}{R_A}\right) j \left(\frac{f}{f_0}\right)\right] \Leftrightarrow$$

$$\Leftrightarrow \frac{V_o}{V_i} = H(jf) = \frac{-\frac{(R_A+R_B)}{R_A} \left(\frac{f}{f_0}\right)^2}{1 - \left(\frac{f}{f_0}\right)^2 + \left(2 - \frac{R_B}{R_A}\right) j \left(\frac{f}{f_0}\right)} =$$

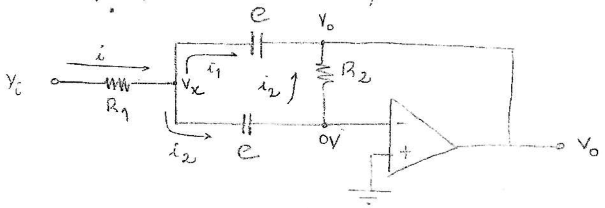
$$= \frac{-\left(1 + \frac{R_B}{R_A}\right)\left(\frac{f}{f_0}\right)^2}{1 - \left(\frac{f}{f_0}\right)^2 + \left(2 - \frac{R_B}{R_A}\right) j \left(\frac{f}{f_0}\right)}$$

$$Q = \frac{1}{3-K}$$

$$K = 1 + \frac{R_B}{R_A}$$

$$f_0 = \frac{1}{2\pi RC}$$

3) Filtro passo banda Sallen-Key RRC



$i = i_1 + i_2$

$$\frac{V_i - V_x}{R_1} = \frac{V_x - V_o}{Z_C} + \frac{V_x - 0}{Z_C}$$

$$\frac{V_x - 0}{Z_C} = \frac{0 - V_o}{R_2} \Leftrightarrow V_x = -\frac{V_o}{R_2} \cdot Z_C \Leftrightarrow V_x = -V_o \cdot \frac{1}{j\omega R_2 C}$$

$$\frac{V_i + V_o \frac{1}{j\omega R_2 C}}{R_1} = \frac{-V_o \frac{1}{j\omega R_2 C} - V_o}{1/j\omega C} - \frac{V_o \frac{1}{j\omega R_2 C}}{1/j\omega C} \Leftrightarrow$$

$$\Leftrightarrow \frac{V_i\, j\omega R_2 C + V_o}{\cancel{j\omega R_1 R_2 C}} = \frac{\left[-V_o - V_o j\omega R_2 C\right] j\omega C \cdot R_1}{\cancel{j\omega R_2 C \cdot R_1}} - \frac{V_o\, j\omega C R_1}{\cancel{j\omega R_2 C R_1}} \Leftrightarrow$$

$$\Leftrightarrow V_i\, j\omega R_2 C = -V_o - V_o j\omega R_1 C - V_o\, j^2 \omega^2 R_1 \cdot R_2 C^2 - V_o j\omega R_1 C \Leftrightarrow$$

$$\Leftrightarrow V_i\, j\omega R_2 C = V_o \left(-1 - 2j\omega R_1 C - j^2 \left(\omega \sqrt{R_1 R_2}\, C\right)^2\right) \Leftrightarrow$$

$$\Leftrightarrow \frac{V_o}{V_i} = H(jf) = \frac{j\omega R_2 C}{-\left(1 - \left(\frac{f}{f_0}\right)^2 + 2j\omega R_1 C\right)} \Leftrightarrow H(jf) = \frac{-j\omega R_2 C}{1 - \left(\frac{f}{f_0}\right)^2 + 2j\omega R_1 C}$$

$$f_0 = \frac{1}{2\pi \sqrt{R_1 R_2}\, C}$$

$$2\omega R_1 C \cancel{j} = \frac{\cancel{j}}{Q} \cdot \frac{f}{f_0} \Leftrightarrow \frac{1}{Q} \frac{f}{f_0} = 2\, 2\pi f R_1^{1/2} \cdot R_2^{1/2} C \times \frac{R_1^{1/2}}{R_2^{1/2}} = \frac{1}{Q}\left(\frac{f}{f_0}\right) \Leftrightarrow$$

$$\Leftrightarrow 2 \cancel{\left(\frac{f}{f_0}\right)} \sqrt{\frac{R_1}{R_2}} = \frac{1}{Q} \cancel{\left(\frac{f}{f_0}\right)} \Leftrightarrow Q = \frac{1}{2} \sqrt{\frac{R_2}{R_1}}$$

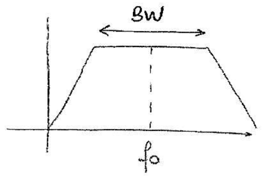
$$-j\omega R_2 C = -j\omega R_2^{1/2} \cdot R_1^{1/2} C \cdot \frac{R_2^{1/2}}{R_1^{1/2}} = -j\left(\frac{f}{f_0}\right) \sqrt{\frac{R_2}{R_1}} \cdot \frac{1}{2} \times 2 =$$

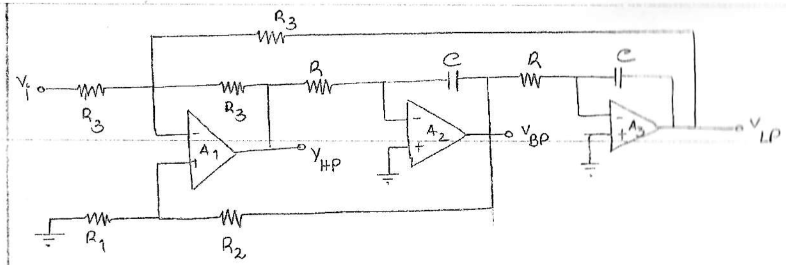
$$= -2jQ \left(\frac{f}{f_0}\right) = -2Q^2 \left(\frac{j}{Q}\right)\left(\frac{f}{f_0}\right)$$

$$H(jf) = \frac{-2Q^2 (j/Q)(f/f_0)}{1 - \left(\frac{f}{f_0}\right)^2 + \left(\frac{j}{Q}\right)\left(\frac{f}{f_0}\right)} \longrightarrow Q = \frac{1}{2}\sqrt{\frac{R_2}{R_1}}$$

$$f_0 = \frac{1}{2\pi \sqrt{R_1 R_2}\, C}$$

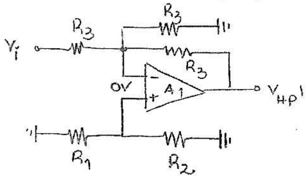
$$Q = \frac{f_0}{BW}$$

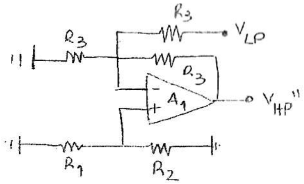




$H_{HP}(jf) = ?$

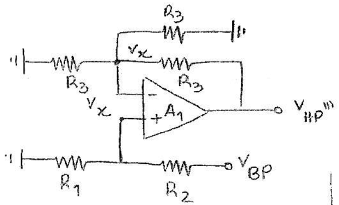
Teorema da sobreposição:





$\frac{V_i - 0}{R_3} = \frac{0 - V_{HP}'}{R_3} \Leftrightarrow V_{HP}' = -V_i$

$V_{HP}'' = -V_{LP}$



$V_x = V_{BP}\frac{R_1}{R_1+R_2}$

$V_x = V_{HP}''' \frac{R_3//R_3}{R_3 + R_3//R_3} \Leftrightarrow$

$\frac{1}{R_T} = \frac{1}{R_3} + \frac{1}{R_3} = \frac{2}{R_3} \Leftrightarrow$

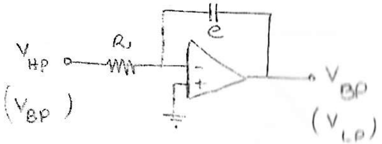
$\Leftrightarrow R_T = \frac{1}{2} R_3$

$\Leftrightarrow V_x = V_{HP}''' \frac{\frac{1}{2} R_3}{\frac{3}{2} R_3} \Leftrightarrow$

$\Leftrightarrow V_x = \frac{1}{3} V_{HP}'''$

$\frac{1}{3} V_{HP}''' = V_{BP}\frac{R_1}{R_1+R_2} \Leftrightarrow V_{HP}''' = V_{BP}\frac{3R_1}{R_1+R_2}$

$V_{HP} = -V_i - V_{LP} + V_{BP}\frac{3R_1}{R_1+R_2}$



$V_{BP} = -\frac{Z_C}{R} V_{HP} \Leftrightarrow$

$\Leftrightarrow V_{BP} = -\frac{1}{j\omega RC} V_{HP} \Leftrightarrow$

$\Leftrightarrow V_{BP} = -\frac{1}{j(f/f_0)} V_{HP} \; ; \; f_0 = \frac{1}{2\pi RC}$

$V_{LP} = -\frac{1}{j\omega RC} V_{BP} = +\frac{1}{(j\omega RC)^2} V_{HP} = -\frac{1}{(f/f_0)^2} V_{HP}$

$V_{HP} = -V_i + \frac{1}{(f/f_0)^2} V_{HP} - \frac{1}{j(f/f_0)} V_{HP} \frac{3R_1}{R_1+R_2} \Leftrightarrow$

$\Leftrightarrow V_i = V_{HP}\left(-1 + \frac{1}{(f/f_0)^2} - \frac{1}{j(f/f_0)}\frac{3R_1}{R_1+R_2}\right) \Leftrightarrow$

$\Leftrightarrow \frac{V_{HP}}{V_i} = H_{HP}(jf) = \frac{-1}{\left(1 - \frac{1}{(f/f_0)^2} + \frac{1}{j(f/f_0)}\frac{3R_1}{R_1+R_2}\right)} =$

$= \frac{-(f/f_0)^2 j(f/f_0)(R_1+R_2)}{(f/f_0)^2 j(f/f_0)(R_1+R_2) - j(f/f_0)(R_1+R_2) + 3R_1 (f/f_0)^2} =$

$= \frac{-(f/f_0)^2 \cancel{j(f/f_0)}\cancel{(R_1+R_2)}}{\cancel{j(f/f_0)(R_1+R_2)}\left[(f/f_0)^2 - 1 + \frac{3R_1}{j\cancel{(f/f_0)}(R_1+R_2)}(f/f_0)^{\cancel{2}}\right]} =$

$= \frac{(f/f_0)^2}{1 - (f/f_0)^2 - \frac{3R_1}{j(R_1+R_2)}(f/f_0)}$

$j/Q = -\frac{3R_1}{j(R_1+R_2)} \Leftrightarrow \frac{1}{Q} = \frac{3R_1}{(R_1+R_2)} \Leftrightarrow Q = \frac{1}{3}\left(1 + \frac{R_2}{R_1}\right)$

$H_{HP}(jf) = \frac{(f/f_0)^2}{1 - (f/f_0)^2 - (j/Q)(f/f_0)}$, $Q = \frac{1}{3}\left(1 + \frac{R_2}{R_1}\right)$

$f_0 = \frac{1}{2\pi RC}$