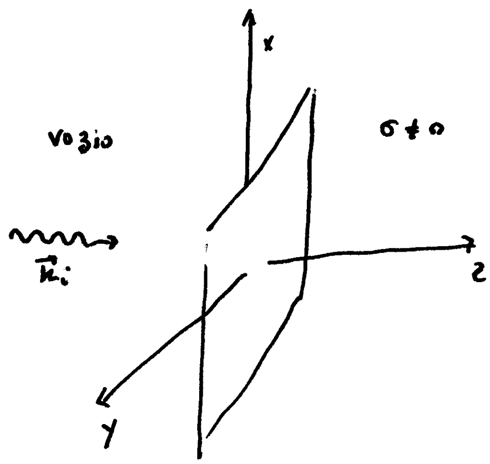
Reflectividade de uma superfície metálica

(vazio – metal ; fronteira $z=0$)

Condições de fronteira:

i) $\varepsilon_1 E_1^{\perp} - \varepsilon_2 E_2^{\perp} = \sigma_f$ ii) $E_1^{\parallel} = E_2^{\parallel}$

iii) $B_1^{\perp} = B_2^{\perp}$ iv) $\frac{1}{\mu_1} B_1^{\parallel} - \frac{1}{\mu_2} B_2^{\parallel} = \vec{K}_f \wedge \hat{n}$



$$\tilde{\vec{E}}_I(z,t) = \tilde{E}_{I0}\, \hat{x}\, e^{i(k_i z - \omega t)}$$

$$\tilde{\vec{B}}_I(z,t) = \tilde{B}_{0i}\, \hat{y}\, e^{i(k_i z - \omega t)} = \frac{\tilde{E}_{0I}}{v_1}\, \hat{y}\, e^{i(k_i z - \omega t)}$$

$$\tilde{\vec{E}}_R(z,t) = \tilde{E}_{0R}\, \hat{x}\, e^{i(-kz - \omega t)}$$

$$\tilde{\vec{B}}_R(z,t) = -\frac{\tilde{E}_{0R}}{v_1}\, \hat{y}\, e^{i(-kz - \omega t)}$$

$$\tilde{\vec{E}}_T = \tilde{E}_{0T}\, \hat{x}\, e^{i(\tilde{k}_T z - \omega t)}$$

$$\tilde{\vec{B}}_T = \frac{\tilde{k}_T}{\omega}\, \tilde{E}_{0T}\, \hat{y}\, e^{i(\tilde{k}_T z - \omega t)}$$

i) $\Rightarrow$ $E_1^{\perp} = E_2^{\perp} = 0$ se $\sigma_f = 0$ iii) $\Rightarrow$ $\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}$ $(z=0)$

ii) $\Rightarrow$ $B_1^{\perp} = B_2^{\perp} = 0$ iv) $\frac{1}{\mu_1 v_1}\left(\tilde{E}_{0I} - \tilde{E}_{0R}\right) = \frac{\tilde{k}_T}{\omega \mu_2}\, \tilde{E}_{0T}$

(se $\vec{K}_f = 0$)

(sup. metálica sem carga nem correntes superficiais)

Temos então:

iii) $$\tilde{E}_{0i} + \tilde{E}_{0R} = \tilde{E}_{0T}$$

iv) $$\tilde{E}_{0i} - \tilde{E}_{0R} = \underbrace{\frac{\mu_1 v_1 \tilde{k}_T}{\omega \mu_2}}_{\tilde{\beta}} \tilde{E}_{0T}$$

Resolvendo em ordem a $\tilde{E}_{0R}$ e $\tilde{E}_{0T}$:

$$\tilde{E}_{0R} = \left(\frac{1 - \tilde{\beta}}{1 + \tilde{\beta}} \right) \tilde{E}_{0i}$$

$$\tilde{E}_{0T} = \frac{2}{1 + \bar{\beta}} \tilde{E}_{0I}$$

<u>Observação</u>: Se tivermos um condutor perfeito, $\sigma = \infty$, $\Rightarrow$

$\Rightarrow |\tilde{k}_T| \to \infty \Rightarrow \tilde{\beta} \to \infty$; Logo $\tilde{E}_{0T} = 0$ e

$$\boxed{\tilde{E}_{0R} = - \tilde{E}_{0i}}$$

Toda a radiação é reflectida com uma mudança de fase de π !

Problema 9.21 (Griffiths)

Calcule o coeficiente de reflexão entre o ar e a prata para $\omega = 4 \times 10^{15}$ rad s^{-1} $[\mu_1 \sim \mu_2 \sim \mu_0 , \ \epsilon_1 = \epsilon_0 \ ; \ \sigma = 6 \times 10^{7} (\Omega \cdot m)^{-1}]$

$$R = \left| \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} \right|^2 = \left| \frac{1-\tilde{\beta}}{1+\tilde{\beta}} \right|^2 = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}} \right) \left(\frac{1-\tilde{\beta}^*}{1+\tilde{\beta}^*} \right)$$

$$\tilde{\beta} = \frac{\mu_1 v_1 \tilde{k}_T}{\omega \mu_2} = \frac{\mu_1 v_1}{\omega \mu_2} (k_T + i \eta_T)$$

$$\sigma >> 1 \Rightarrow k_z \sim \eta_z \sim \sqrt{\frac{\sigma \omega \mu_2}{2}}$$

$$\tilde{\beta} = \frac{\mu_1 v_1}{\mu_2 \omega} \sqrt{\frac{\sigma \omega \mu_2}{2}} (1+i) = \gamma (1+i)$$

$$\gamma = \mu_0 c \sqrt{\frac{\sigma}{\omega 2 \mu_0}} \simeq 29$$

$$R = \left(\frac{1-\gamma - i\gamma}{1+\gamma + i\gamma} \right) \left(\frac{1-\gamma+i\gamma}{1+\gamma-i\gamma} \right) = \frac{(1-\gamma)^2 + \gamma^2}{(1+\gamma)^2 + \gamma^2} \sim 0{,}93 \, !$$

Problema 9.19 : Mostre que o comprimento de penetração de um mau condutor ($\sigma << \mu \epsilon$) é $\frac{2}{\sigma}\sqrt{\frac{\epsilon}{\mu}}$ e $\therefore$ independente da frequência.

Calcule o comprimento de penetração para a água

Soluçaõ:

$$\eta = \omega \sqrt{\frac{\varepsilon\mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1 \right]^{1/2}$$

$$\frac{\sigma}{\varepsilon\omega} << 1 \Rightarrow \eta \simeq \omega \sqrt{\frac{\varepsilon\mu}{2}} \left[1 + \frac{1}{2}\left(\frac{\sigma}{\varepsilon\omega}\right)^2 - 1 \right]^{1/2} =$$

$$= \omega \sqrt{\frac{\varepsilon\mu}{2}} \cdot \frac{1}{\sqrt{2}} \frac{\sigma}{\varepsilon\omega} = \frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}}$$

$$d = \frac{1}{\eta} = \frac{2}{\sigma} \sqrt{\frac{\varepsilon}{\mu}}$$ (indep. da frequência)

Para a água:

$$\left. \begin{array}{l} \varepsilon \sim 80\, \varepsilon_0 \\ \mu \sim \mu_0 \\ \sigma \sim \frac{1}{2,5} \times 10^{-5} \end{array} \right\} \quad d \sim 1,19 \times 10^4 \text{ m} \quad !!!$$

Problema 9.20 (Griffiths)

(a) Calcule a densidade de energia (média temporal) de uma onda plana electromagnética num meio condutor. Mostre que a contribuição magnética é sempre dominante.

(b) Mostre que a intensidade decai como $\left(\frac{k}{2\mu\omega}\right) E_0^2 \, e^{-2\eta z}$

Soluçaõ:

(a) $$u = \frac{1}{2}\left(\varepsilon E^2 + \frac{1}{\mu} B^2\right)$$

$$E = E_0\, e^{-\eta z} \cos(\kappa z - \omega t + \delta_E)$$

$$B = B_0\, e^{-\eta z} \cos(\kappa z - \omega t + \delta_E + \phi) \qquad ; \quad \phi = \operatorname{arctg}\left(\frac{\eta}{\kappa}\right)$$

$$u = \frac{1}{2} e^{-2\eta z}\left[\varepsilon E_0^2 \cos^2(\kappa z - \omega t + \delta_E) + \frac{1}{\mu} B_0^2 \cos^2(\kappa z - \omega t + \delta_E + \phi)\right]$$

$$\langle u\rangle_T = \frac{1}{2} e^{-2\eta z}\left[\frac{1}{2}\varepsilon E_0^2 + \frac{1}{2\mu} B_0^2\right]$$

$$\frac{B_0}{E_0} = K/\omega \quad ; \text{ onde } \quad K = \sqrt{\kappa^2 + \eta^2} = \omega \sqrt{\varepsilon\mu \sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2}}$$

$$B_0 = \sqrt{\varepsilon\mu\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2}}\; E_0$$

$$\langle u\rangle_T = \frac{1}{2} e^{-2\eta z}\left[\frac{1}{2}\varepsilon E_0^2 + \frac{1}{2\mu}\varepsilon\mu\sqrt{1 + \left(\frac{\sigma}{\varepsilon\mu}\right)^2}\cdot E_0^2\right]$$

$$= \frac{1}{4} e^{-2\eta z}\varepsilon E_0^2\left[1 + \sqrt{1 + \left(\frac{\sigma}{\varepsilon\mu}\right)^2}\right]$$

Mas $$\kappa = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\mu}\right)^2} + 1\right]^{1/2} \Rightarrow \frac{\kappa^2}{\omega^2}\frac{2}{\varepsilon\mu} = \left[1 + \sqrt{1 + \left(\frac{\sigma}{\varepsilon\mu}\right)^2}\right]$$

Logo:

$$\langle u\rangle_T = \frac{1}{4} e^{-2\eta z}\varepsilon E_0^2 \cdot \frac{2}{\varepsilon\mu}\frac{\kappa^2}{\omega^2} = \frac{\kappa^2}{2\mu\omega^2} E_0^2\, e^{-2\eta z}$$

(a)

$$\frac{\langle u_{magt} \rangle}{\langle u_{elec} \rangle} = \frac{B_0^2/\mu}{E_0^2\,\varepsilon} = \frac{B_0^2}{E_0^2}\frac{1}{\mu\varepsilon} = \frac{1}{\mu\varepsilon}\,\varepsilon\mu\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}$$

$$= \sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} > 1$$

(A contribuição magnética para a densidade de energia é dominante)

(b) $$\vec{S} = \frac{1}{\mu}\left(\vec{E}\wedge\vec{B}\right) = \frac{\hat{z}}{\mu}\,E_0 B_0\, e^{-2\eta z}\cos\left(Kz-\omega t+\delta_E\right)\cdot\cos\left(Kz-\omega t+\delta_E+\phi\right)$$

$$\langle\vec{S}\rangle = \frac{\hat{z}}{\mu}\,E_0 B_0\, e^{-2\eta z}\,\underbrace{\frac{1}{T}\int_0^{T}\cos\left(Kz-\omega t+\delta_E\right)\cdot\cos\left(Kz-\omega t+\delta+\phi\right)dt}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\cos\theta\cdot\cos\left(\theta+\phi\right)d\theta$$

$$\frac{1}{2\pi}\int_0^{2\pi}\cos\theta\left[\cos\theta\cos\phi-\sin\theta\sin\phi\right]d\theta$$

$$\frac{1}{2}\cos\phi$$

$$\langle\vec{S}\rangle = \frac{\hat{z}}{2\mu}\,E_0B_0\,e^{-2\eta z}\cos\phi \Rightarrow I = \frac{1}{2\mu}\,E_0 B_0\, e^{-2\eta z}\cos\phi$$

$$= \frac{1}{2\mu}\,E_0^2\left(\frac{K}{\omega}\cos\phi\right)e^{-2\eta z}$$

$$K = |\tilde{k}| = \omega\sqrt{\varepsilon\mu\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}} \qquad \left(\phi = \arctan\left(\frac{\eta}{K}\right)\right)$$

$$K\cos\phi = k = \mathbb{R}(\tilde{k}) \Rightarrow \underline{\underline{I = \frac{1}{2\mu\omega}\,E_0^2\, e^{-2\eta z}}}$$

□