the popular CD40007 CMOS transistor array. For higher current levels, a power FET is better suited, for example, the VP1008B (p-channel, for current sourcing) or the VN1206B (n-channel, for current sinking) of the MOSPOWER FET series (Siliconix). Of course, a power FET can be used also for low-level currents. For high-current applications, the devices must be mounted on heat sinks.

2.3 CURRENT-TO-VOLTAGE CONVERTERS

The *current-to-voltage converter* (*I-V* converter), also called a *current-controlled voltage source* (CCVS), accepts an input current I_i and yields an output voltage V_o such that $V_o = AI_i$, where A is the gain of the circuit. Since A is measured in ohms, it is more appropriate to denote gain by the symbol R. Because of this, *I-V* converters are also called *transresistance amplifiers*.

Although the op amp is, strictly speaking, a voltage amplifier, it can be configured as a transresistance amplifier by connecting it as in Fig. 2.7a. By the current constraint, $I_R = I_i$; that is, $(V_n - V_o)/R = I_i$. Since the op amp keeps $V_n = 0$, we obtain

$$V_o = (-R)I_i \tag{2.6}$$

The gain, $-R$, is negative because of the way we have specified I_i's direction. Inverting I_i's polarity will, of course, also invert that of V_o. If desired, the gain magnitude can be varied by implementing R with a potentiometer, this being perhaps the reason the circuit is sometimes called a *potentiometric amplifier*. For that matter, the feedback element need not be purely resistive; it can be an otherwise arbitrary impedance Z, in which case Eq. (2.6) becomes $V_o = (-Z)I_i$ and the circuit is called a *transimpedance amplifier*.

The purpose of the op amp is to eliminate possible loading both at the input and at the output. In fact, should the input source exhibit some finite parallel resistance R_s, the op amp will eliminate any current loss through R_s by ensuring $V_n = 0$. Likewise, by delivering V_o with virtually zero output resistance, it eliminates any voltage loss due to the output load.

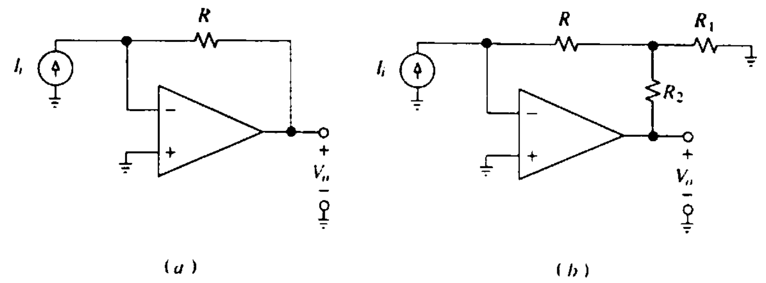

FIGURE 2.7 (*a*) Basic *I-V* converter. (*b*) High-sensitivity *I-V* converter.

Exercise 2. Design a circuit that accepts two *nonideal* current sinks I_{s1} and I_{s2} having parallel resistances R_{s1} and R_{s2}, and yields $V_o = (100\ \text{k}\Omega)(I_{s2} - I_{s1})$. The circuit must be insensitive to loading both at the input and at the output. Minimize the number of components in your circuit. ■

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• Note, incidentally, that the familiar inverting amplifier is itself a special case of the *I-V* converter since it accepts the current coming from the input source, $I_{V_i} = V_i/R_1$, and converts it to an output voltage, $V_o = -R_2 I_{V_i} = -(R_2/R_1)V_i$. This is a familiar result, expressed in terms of the *I-V* converter function. Stated in terms of negative feedback, the inverting amplifier is of the *shunt-shunt* type because of this.

High-Sensitivity *I-V* Converters

Gain R is also called the *sensitivity* of the *I-V* converter since it determines the amount of output voltage change for a given input current change. For instance, for a sensitivity of 1 V/mA we need $R = 1\ \text{k}\Omega$, for a sensitivity of 1 V/μA we need $R = 1\ \text{M}\Omega$, and so on. It is obvious that high-sensitivity applications tend to require unrealistically large resistances. Unless the circuit is properly sealed from moisture, the resistance of the surrounding medium, being in parallel with R, will decrease the net feedback resistance and degrade the accuracy of the circuit, particularly in high-humidity and high-salinity ambients. Figure 2.7b shows a widely used technique to avoid this limitation. The circuit utilizes a T-network to achieve high sensitivity without requiring unrealistically high resistances.

Denoting the voltage at the junction of the three resistors as V_x, we have $V_x = -RI_i$. By KCL at this node, $(0 - V_x)/R + (0 - V_x)/R_1 = (V_x - V_o)R_2$. Eliminating V_x, we obtain

$$V_o = (-R_{eq})I_i \tag{2.7a}$$

where

$$R_{eq} = \left(1 + \frac{R_2}{R} + \frac{R_2}{R_1}\right)R \tag{2.7b}$$

indicating that the circuit is in effect multiplying R by the amount within parentheses. We can therefore achieve high R_{eq} values by starting out with a reasonable R value and multiplying it by the needed amount.

Example 2.1. (*a*) In the circuit of Fig. 2.7b specify suitable component values to achieve a sensitivity of (1 V)/(10 nA). (*b*) If $a = 200{,}000$, what is the percentage deviation of R_{eq} from the ideal?

Solution. $R_{eq} = 1/(10 \times 10^{-9}) = 100\ \text{M}\Omega$, a fairly large value. Start out with $R = 1\ \text{M}\Omega$ and multiply it by 100 to meet the specifications. Thus, $(1 + R_2/10^6 + R_2/R_1) = 100$. Since we have two unknowns, we fix one; for example, let $R_1 = 1\ \text{k}\Omega$. Then $(1 + R_2/10^6 + R_2/10^3) = 100$. This yields $R_2 \simeq 99\ \text{k}\Omega$ (use $100\ \text{k}\Omega$). Summarizing,

where

$$H_0 = -\frac{R_2}{R_1} \tag{3.10b}$$

and

$$f_0 = \frac{1}{2\pi R_1 C} \tag{3.10c}$$

(*b*) Show that its magnitude response is as in Fig. 3.7b and justify it in terms of $|Z_C|$. (For obvious reasons H_0 is called the *high-frequency gain* and f_0 is called the *cutoff frequency*.) (*c*) Specify suitable component values for $f_0 = 100$ Hz, $H_0 = 40$ dB, and an input impedance of at least 10 kΩ. ■

Wide-Band Band-Pass Filter

The circuits of Figs. 3.6 and 3.7 can be combined as in Fig. 3.8 to yield a band-pass response. The input impedance $Z_1 = R_1 + 1/(j\omega C_1)$ forms a high-pass section with corner frequency $f_1 = 1/(2\pi R_1 C_1)$, while the feedback impedance $Z_2 = R_2 \,//\, [1/(j\omega C_2)]$ forms a low-pass section with corner frequency $f_2 = 1/(2\pi R_2 C_2)$. If we specify f_1 and f_2 so that $f_1 < f_2$, then input frequencies within the band $f_1 \le f \le f_2$ will succeed in making it through the circuit while those falling outside will be rejected by one section or the other.

The transfer function is easily determined as $H = -Z_2/Z_1$. Expanding Z_1 and Z_2 we obtain, after routine algebra,

$$H = H_0 \frac{j(f/f_1)}{[1 + j(f/f_1)][1 + j(f/f_2)]} \tag{3.11a}$$

where

$$H_0 = -\frac{R_2}{R_1}, \qquad f_1 = \frac{1}{2\pi R_1 C_1}, \qquad f_2 = \frac{1}{2\pi R_2 C_2} \tag{3.11b}$$

The magnitude plot is easily constructed as in Fig. 3.8b.

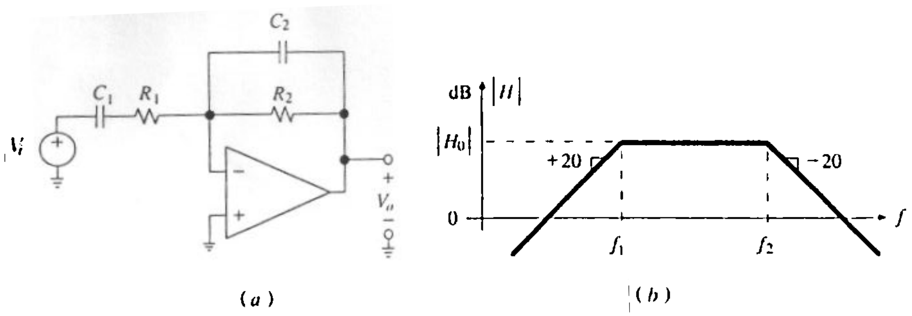


FIGURE 3.8 Wide-band band-pass filter.

Example 3.2. In the circuit of Fig. 3.8 specify suitable component values to achieve a band-pass response with a gain of 20 dB over the audio range, 20 Hz $\le f \le$ 20 kHz.

Solution. For a gain of 20 dB we need $R_2/R_1 = 10$. Try $R_1 = 20$ kΩ and $R_2 = 200$ kΩ, which are reasonable values. For $f_1 = 20$ Hz we need $C_1 = 1/(2\pi \times 20 \times 10^3 \times 20) = 0.39$ μF. For $f_2 = 20$ kHz we need $C_2 = 1/(2\pi \times 200 \times 10^3 \times 20 \times 10^3) =$ 39 pF. ■

This filter is especially suited to wide-band applications such as audio transmission, where it is desired to amplify signals within the audio range while blocking out subaudio components (e.g., dc) as well as noise above the audio range. If so desired, the filter can be made more selective by moving f_1 and f_2 closer together; however, there is a limit to the degree of selectivity achievable with this band-pass topology (more on this will be discussed in Exercise 3.10), and this is the reason for the designation wide-band. Band-pass filters having a ratio of upper cutoff frequency to lower cutoff frequency of 2 or less are classified as narrow-band, while those having a ratio of 2 or more are classified as wide-band. (Similar designations hold for the notch response.) The design of narrow-band filters will be covered in subsequent sections.

Phase Shifters

As implied by the name, the role of a phase shifter is to introduce a phase shift between output and input while leaving amplitude unchanged. The filters discussed so far do introduce phase shift; however, they also affect amplitude and therefore cannot be classified as pure phase shifters. Figure 3.9 shows a popular phase shifter realization.

Exercise 3.4. (*a*) Show that for the circuit of Fig. 3.9

$$H = \frac{1 - j(f/f_0)}{1 + j(f/f_0)} \tag{3.12a}$$

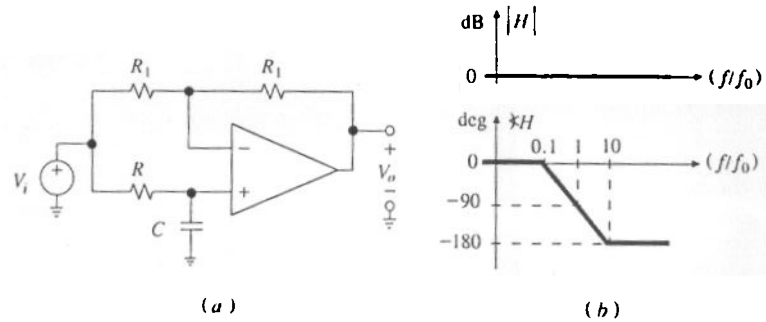


FIGURE 3.9 Phase shifter.

where

$$f_0 = \frac{1}{2\pi RC} \tag{3.12b}$$

Hint: First show that $V_p = V_i/(1 + jf/f_0), f_0 = 1/(2\pi RC)$. ■

S... the magnitude and argument of a ratio equal, respectively, the rati... of the magnitudes and the difference of the arguments, we obtain $|H| = \sqrt{[1 + (f/f_0)^2]}/\sqrt{[1 + (f/f_0)^2]} = 1$ and $\measuredangle H = \tan^{-1}[-(f/f_0)] - \tan^{-1}(f/f_0)$ ... (f/f_0). The frequency plots are therefore as shown in Fig. 3.9b. Wh... the ...magnitude response is flat (phase shifters are also called *all-pass filters* becau... of this), the ph...

Exercise 3.5. Specify suitable component values so that the frequency f_0 at which the circuit of Fig. 3.9 introduces a phase lag of 90° can be varied over the range $100\ \text{Hz} \le f_0 \le 2\ \text{kHz}$ by means of a 100 kΩ potentiometer. Don't forget to put a suitable resistor in series with the potentiometer. Show your final circuit. ■

Exercise 3.6. Redraw the circuit of Fig. 3.9, but with R and C interchanged. Derive the transfer function of this circuit and sketch its Bode plots. What is the main difference between the phase responses of the two circuits? ■

3.4 AUDIO FILTER APPLICATIONS

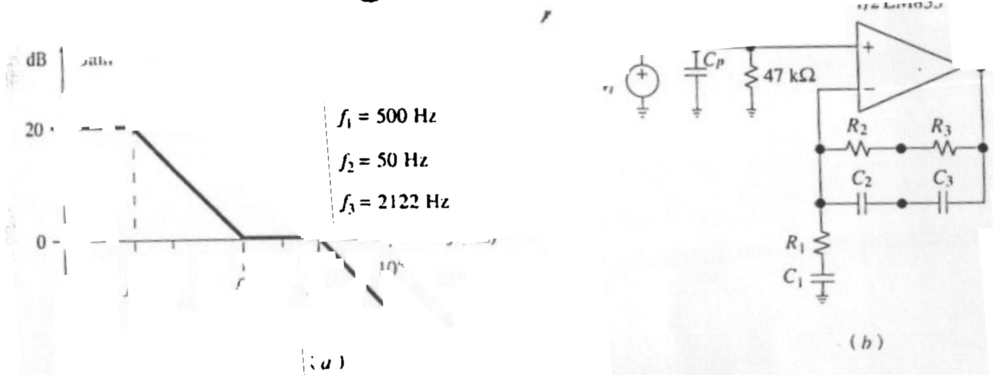
Audio signal processing provides a multitude of uses for active filters. Through simple filter functions, op amps can perform the precise response shaping and adjustments required in high-quality audio systems. Components utilizing these functions include equalized preamplifiers, active tone control, and graphic equalizers, to name a few.[1,2,3,4]

Equalized preamplifiers are used to compensate for the varying levels at which different parts of the audio spectrum are commercially recorded. Tone control and graphic equalization refer to response adjustments that the listener can effect to compensate for nonideal loudspeaker response, to match apparent room acoustics, or simply to suit one's taste.

Phono Preamplifier

The function of a phono preamplifier is to provide gain, as well as amplitude and phase equalization, for the signal from a moving magnet or a moving coil cartridge. The response must conform to the standard RIAA (Record Industry Association of America) curve of Fig. 3.10a.

Preamplifier gains are usually specified at 1 kHz. The required amount of gain is typically 30 to 40 dB for moving magnet cartridges, 50 to 60 dB for moving coil types. Since the RIAA curve is normalized for unity gain, the actual preamp response will be shifted upward by an amount equal to its gain.



...phono preamplifier.

Figure 3.10b shows one of several topologies commonly used to approxi... the RIAA response. The input shunting network provides impedance matching for the source, while C_1 provides a low-frequency breakpoint (usually below 20 Hz) to block out dc and subaudio frequencies. Since $X_{C_1} \ll R_1$ over the frequency range of interest, the transfer function can be determined as $H \simeq 1 + Z_F/R_1$, where Z_F is the impedance of the feedback network.

Exercise 3.7. Show that as long as $X_{C_1} \ll R_1$ the response of the circuit of Fig. 3.10b is

$$H \simeq 1 + \frac{R_2 + R_3}{R_1}\,\frac{1 + j(f/f_1)}{[1 + j(f/f_2)][1 + j(f/f_3)]} \tag{3.13a}$$

where

$$f_1 = \frac{1}{2\pi(R_2 /\!/ R_3)(C_2 + C_3)} \tag{3.13b}$$

$$f_2 = \frac{1}{2\pi R_2 C_2} \tag{3.13c}$$

$$f_3 = \frac{1}{2\pi R_3 C_3} \tag{3.13d}$$

■

As long as the circuit is configured for substantially high gain, the unity term in Eq. (3.13a) can be ignored. It is then straightforward to verify through linearized Bode plot reasoning that the response approximates the standard RIAA curve over the audio range.

Example 3.3. Design a 40 dB gain, RIAA phono amplifier.

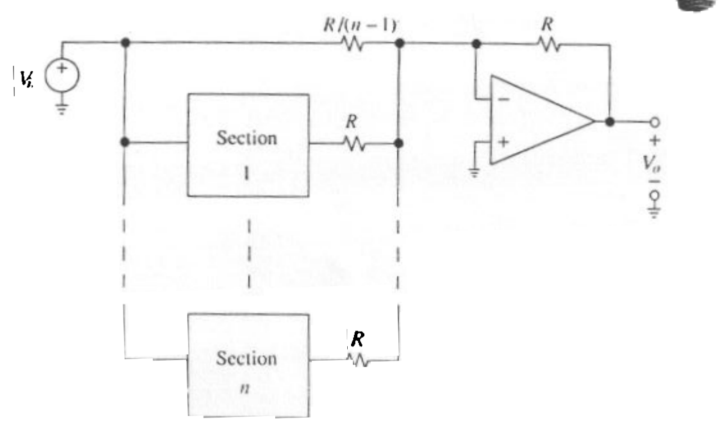


FIGURE 3.14 Graphic equalizer with n bands.

3.5 STANDARD SECOND-ORDER RESPONSES

Second-order responses are important in their own right and as building blocks of more complex responses. We shall therefore investigate them in some detail before turning to circuit realizations. When we studied first-order filters, we noticed that all responses have the same denominator, $1 + j(f/f_0)$, and that it is the *numerator* that determines the nature of the particular response: A numerator of unity yields the low-pass response, a numerator of $j(f/f_0)$ yields the high-pass, and a numerator of $1 - j(f/f_0)$ yields the all-pass. Moreover, multiplying a response by gain constant H_0 does not change the response type; it only shifts the magnitude plot up or down, depending on whether $|H_0| > 1$ or $|H_0| < 1$.

Similar considerations hold for second-order responses. However, since the degree of the denominator is now 2, we have an additional filter parameter besides f_0. As we shall verify below, all second-order functions can be put in the standard form

$$H(jf/f_0) = \frac{N(jf/f_0)}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.19}$$

where Q, a pure number, is the other parameter in question, and $N(jf/f_0)$ is a suitable polynomial of degree not greater than 2 whose form determines the nature of the particular response.

Second-Order Low-Pass Response

All second-order low-pass functions can be put in the standard form $H(jf/f_0) = H_{0LP}H_{LP}(jf/f_0)$ where H_{0LP} is a suitable constant referred to as *dc gain* and

$$H_{LP} \triangleq \frac{1}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.20}$$

To construct the magnitude plot we use asymptotic techniques.

1. For $(f/f_0) \ll 1$ the second and third denominator terms can be ignored in comparison with unity so that $H_{LP} \rightarrow 1$. The low-frequency asymptote is, therefore,

$$|H_{LP}|_{dB} = 0 \tag{3.21a}$$

2. For $(f/f_0) \gg 1$ the second denominator term dominates over the others so that $H_{LP} \rightarrow -1/(f/f_0)^2$. The high-frequency asymptote is, therefore, $|H_{LP}|_{dB} = 20 \log_{10}[1/(f/f_0)^2]$, that is,

$$|H_{LP}|_{dB} = -40 \log_{10}(f/f_0) \tag{3.21b}$$

This equation is of the type $y = -40x$, that is, a straight line with a slope of -40 dB/dec (decibels per decade). Compared to the first-order response, which had a slope of only -20 dB/dec, the second-order response is closer to the idealized brickwall model.

3. For $(f/f_0) = 1$ the two asymptotes meet, since setting $(f/f_0) = 1$ in Eq. (3.21b) yields Eq. (3.21a). Moreover, the first and second denominator terms cancel each other to yield $H_{LP} = 1/(j/Q) = -jQ$. Thus, for $(f/f_0) = 1$ we have

$$|H_{LP}|_{dB} = Q_{dB} \tag{3.21c}$$

indicating that in the frequency region near $(f/f_0) = 1$ we now have a *family of curves*, depending on the value of Q. Contrast this with the first-order case, where only one curve was possible. The second-order response, besides providing a high-frequency asymptotic slope *twice* as steep, offers an additional degree of freedom in specifying the filter's profile in the vicinity of $(f/f_0) = 1$.

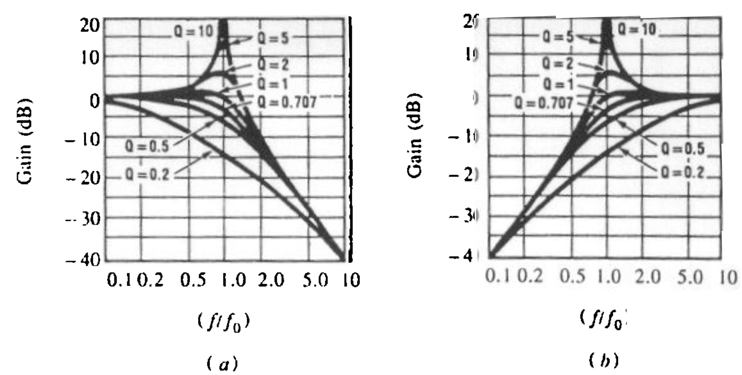


FIGURE 3.15 Standard second-order (a) low-pass and (b) high-pass responses

In actual applications the value of Q can range from 0.5 to 100, with values near unity being by far the most common. The magnitude plot is shown in Fig 3.15a for different values of Q. Note that for low Qs the transition from one asymptote to the other is very gradual, while for high Qs there is a range of frequencies in the neighborhood of $(f/f_0) = 1$ where $|H_{LP}|$ is greater than unity, indicating that the filter provides gain there. This phenomenon [illegible] ...ing, is similar to *resonance* in underdamped [illegible]

The borderline between gradual and peaked responses occurs for $Q = 1/\sqrt{2}$ 0.707, which one can prove to be the largest Q before peaking occurs. The corresponding curve is said to be *maximally flat* and the response is called the *Butterworth response*. Obviously, this curve is the closest to the brickwall model, hence its popularity. Note that the magnitude at $(f/f_0) = 1$ is, by Eq. (3.21c), $Q = 1/\sqrt{2} = -3$ dB. The meaning of f_0 for the Butterworth response i[illegible] [illegible] ...ency.

It can be proven that in th[illegible] $(Q > 1/\sqrt{2})$, the [illegible]

$$|H_{LP}|_{max} = \frac{Q}{\sqrt{[1 - 1/(4Q^2)]}} \tag{3.22a}$$

and the maximum is reached for

$$(f/f_0) = \sqrt{[1 - 1/(2Q^2)]} \tag{3.22b}$$

For sufficiently large Qs (in practice, for $Q \geq 5$) the values of the square roots can be taken as unity with negligible error. Then, $|H_{LP}|_{max} \simeq Q$ for $(f/f_0) \simeq 1$. Of course, in absence of peaking $(Q < 1/\sqrt{2})$ the maximum is reached at $(f/f_0) = 0$ that is, at dc.

Peaked responses are useful in the cascade synthesis of higher-order filters (see Chapter 4) but also in their own right. An example is the synthesis of formant filters for the simulation of the human vocal tract. This can be modeled with three second-order low-pass filters of the resonant type. Other applications include electronic and pop music effects such as voltage-controlled filters and *wah-wah* filters.

Second-Order High-Pass Response

All second-order high-pass functions can be put in the standard form $H(jf/f_0) = H_{0HP}H_{HP}(jf/f_0)$, where H_{0HP} is a suitable constant referred to as *high-frequency gain* and

$$H_{HP} \triangleq \frac{-(f/f_0)^2}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.23}$$

To construct the magnitude plot we can again use asymptotic reasoning; however, the procedure can be speeded considerably by noting that the expression for H_{HP} can be obtained from that of H_{LP} by making the substitution $j(f/f_0) \rightarrow [j(f/f_0)]^{-1}$, that is, $(f/f_0) \rightarrow -(f/f_0)$. Consequently, the magnitude plot of H_{HP} will be the *mirror image* of that of H_{LP} (see Fig. 3.15b).

Note that the low-frequency asymptotic slope is +40 dB/dec, indicating that a second-order high-pass is closer to ideal than a first-order high-pass. The maximally flat or Butterworth response is again the one corresponding to $Q = 1/\sqrt{2}$, where the magnitude is 3 dB below its high-frequency asymptotic value. For $Q > 1/\sqrt{2}$ the responses exhibit resonance. Equation (3.22) still holds provided (f/f_0) is replaced by (f_0/f).

Second-Order Band-Pass Response

All second-order band-pass functions can be put in the standard form $H(jf/f_0) = H_{0BP}H_{BP}(jf/f_0)$, where H_{0BP} is a suitable constant referred to as *resonant gain* and

$$H_{BP} \triangleq \frac{(j/Q)(f/f_0)}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.24}$$

To construct the magnitude plot we use asymptotic reasoning.

1. For $(f/f_0) \ll 1$ the second and third denominator terms can be ignored in comparison with the first, so that $H_{BP} \rightarrow (j/Q)(f/f_0)$. The low-frequency asymptote is, therefore, $|H_{BP}|_{dB} = 20 \log_{10}[(1/Q)(f/f_0)]$, that is,

$$|H_{BP}|_{dB} = 20 \log_{10}(f/f_0) - Q_{dB} \tag{3.25a}$$

This equation is of the type $y = 20x - Q_{dB}$, that is, a straight line with a slope of 20 dB/dec, but shifted by $-Q_{dB}$ with respect to the 0 dB axis at $(f/f_0) = 1$.

2. For $(f/f_0) \gg 1$ the second denominator term dominates over the others so that $H_{BP} \rightarrow -(j/Q)/(f/f_0)$. The high-frequency asymptote is, therefore,

$$|H_{BP}|_{dB} = -20 \log_{10}(f/f_0) - Q_{dB} \tag{3.25b}$$

This equation is of the type $y = -20x - Q_{dB}$, that is, a straight line with the same amount of downward shift as before, but with a slope of -20 dB/dec.

3. For $(f/f_0) = 1$ the first two denominator terms cancel each other to yield $H_{BP} = 1$. Thus, for $(f/f_0) = 1$

$$|H_{BP}|_{dB} = 0 \tag{3.25c}$$

One can prove that $|H_{BP}|$ peaks at $(f/f_0) = 1$ regardless of the value of Q and that all curves are symmetric with respect to this point. Because of this, f_0 is variously referred to as *resonant frequency*, *center frequency*, or *peak frequency*.

The magnitude plot is shown in Fig. 3.16a for different values of Q. Note that all curves peak at 0 dB and that the ones corresponding to low Qs

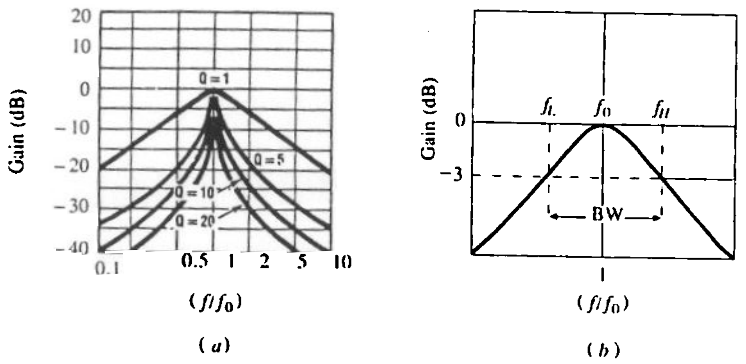


FIGURE 3.16 (a) Standard second-order band-pass response. (b) Bandwidth BW.

are broad and smooth, while those corresponding to high Qs are narrow and sharp, indicating a high degree of *selectivity*. In the vicinity of $(f/f_0) = 1$ the highly selective curves are much steeper than ± 20 dB/dec, although away from resonance they fall off at the same ultimate rate of ± 20 dB/dec

Looking at Fig. 3.16a, we note that the higher the Q term the more selective the filter. To express selectivity quantitatively we introduce the concept of *bandwidth*, defined as

$$BW \triangleq f_H - f_L \tag{3.26}$$

where f_L and f_H are the -3 dB frequencies, that is, the frequencies where the band-pass response drops 3 dB below its maximum (see Fig. 3.16b). One can prove that

$$\frac{f_L}{f_0} = \left(1 + \frac{1}{4Q^2}\right)^{1/2} - \frac{1}{2Q} \tag{3.27a}$$

and

$$\frac{f_H}{f_0} = \left(1 + \frac{1}{4Q^2}\right)^{1/2} + \frac{1}{2Q} \tag{3.27b}$$

From the above expressions it follows that

$$f_0 = \sqrt{(f_L f_H)} \tag{3.28}$$

Clearly, the narrower the bandwidth the more selective the filter. Selectivity also depends on the value of f_0 since a filter with BW = 10 Hz and f_0 = 1 kHz is certainly more selective than one with BW = 10 Hz and f_0 = 100 Hz. A proper measure of selectivity would therefore be the ratio f_0/BW. Subtracting Eq. (3.27a) from Eq. (3.27b) and taking the reciprocal yields

$$Q = \frac{f_0}{BW} \tag{3.29}$$

That is, Q *is* the selectivity. We now have a more concrete interpretation for this parameter.

Exercise 3.10. (*a*) By proper manipulation, put the wide-band band-pass function of Eq. (3.11a) in the standard form $H = H_{0BP}H_{BP}$, with H_{BP} as given in Eq. (3.24). (*b*) Verify that no matter how you select f_1 and f_2, Q in this filter can never exceed 0.5. This is why the filter is called wide-band. ■

The Second-Order Notch Response

The most common form for the notch function is $H(jf/f_0) = H_{0N}H_N(jf/f_0)$, where H_{0N} is an appropriate gain constant and

$$H_N \triangleq \frac{1 - (f/f_0)^2}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.30}$$

(Other notch functions are possible in which the value of f_0 in the numerator is not necessarily equal to the value of f_0 in the denominator; we shall see examples in Section 3.10.) By inspection we note that at sufficiently low and high frequencies H_N tends toward unity while for $(f/f_0) = 1$ it vanishes, thus leading to an infinitely deep magnitude notch there. For obvious reasons f_0 is called the *notch frequency*. The magnitude response of a notch filter is of the type of Fig. 3.17a, where we note that the higher the value of Q the narrower the width of the notch. In practical realizations, due to component nonidealities the notch will be deep, but not infinitely deep.

It is interesting to note that H_N can also be written as $H_N = H_{LP} + H_{HP}$ or as $H_N = 1 - H_{BP}$. These relationships indicate alternate ways of achieving a notch response when the other responses are available. We shall see examples in Section 3.10.

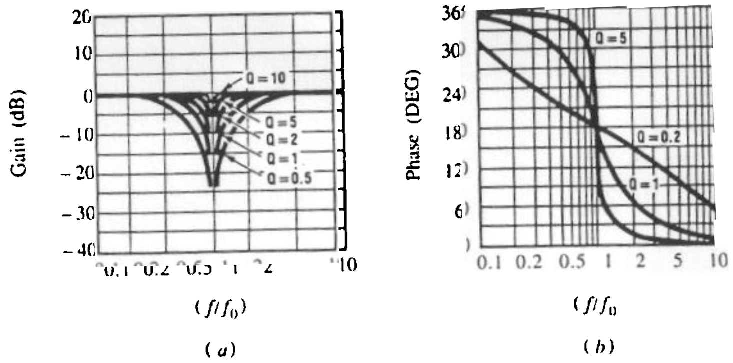


FIGURE 3.17 Standard second-order (*a*) notch and (*b*) all-pass responses

Second-Order All-Pass Response

The general form of a second-order all-pass response is $H_{AP}(jf/f_0) = H_{0AP}H_{AP}(jf/f_0)$, where H_{0AP} is the usual gain term and

$$H_{AP} \triangleq \frac{1 - (f/f_0)^2 - (j/Q)(f/f_0)}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.31}$$

The numerator and denominator are complex conjugates of each other, so their moduli are the same and $|H_{AP}|_{dB} = 0$ regardless of frequency. The argument is found as

$$\measuredangle H_{AP} = 360° - 2\tan^{-1}\left(\frac{(f/f_0)/Q}{1-(f/f_0)^2}\right) \tag{3.32a}$$

for $(f/f_0) < 1$, and

$$\measuredangle H_{AP} = 2\tan^{-1}\left(\frac{(f/f_0)/Q}{(f/f_0)^2 - 1}\right) \tag{3.32a}$$

for $(f/f_0) > 1$. Thus, as (f/f_0) is swept from 0 to ∞, $\measuredangle H_{AP}$ changes from 360° to 0°, (see Fig. 3.17b). The all-pass function can also be synthesized as $H_{AP} = 1 - 2H_{BP}$.

Exercise 3.11. Construct the phase plots of H_{LP}, H_{HP}, H_{BP}, and H_N for different values of Q. ■

3.6 SECOND-ORDER LOW-PASS FILTERS

Since an RC stage yields a first-order low-pass response, cascading two such stages as in Fig. 3.18a should yield a second-order response. Indeed, at low frequencies, where the capacitors act as effective open circuits, the input signal will pass through and $H = 1$ there. At high frequencies, where the capacitors act as effective shunts, the incoming signal will undergo a *two-step attenuation*, first by the R_1C_1 stage, then by the R_2C_2 stage, thus substantiating the designation *second-order*. Since the high-frequency response of an RC stage is $1/(f/f_0)$, the combined response of two stages will be $1/(f/f_0)^2$, indicating a high-frequency asymptotic slope of -40 dB/dec.

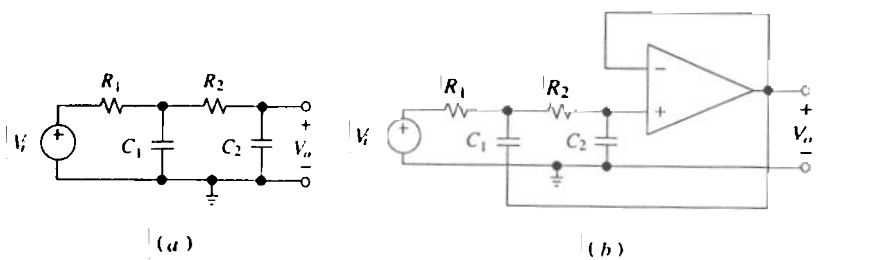


FIGURE 3.18 [illegible]

Although the network of Fig. 3.18a does meet the asymptotic criteria for a second-order low-pass response, it doesn't offer adequate flexibility for controlling the magnitude profile in the vicinity of $(f/f_0) = 1$. In fact, one can prove that the maximum Q achievable with this *all-passive* filter is a meager $Q = 0.5$. This limitation stems from energy loss in the resistors as well as loading of the first RC stage by the second.

One way of bolstering the magnitude profile in the vicinity of $(f/f_0) = 1$ is to replace the first RC stage with an inductor. Another is to use an active device such as an op amp to feed part of the output signal back into the circuit itself. As shown in Fig. 3.18b, this is achieved by lifting C_1 from ground and connecting it to the buffer amplifier output to provide a positive feedback path. This path must be effective only in the vicinity of $(f/f_0) = 1$, where bolstering is specifically needed. In fact, for $(f/f_0) \ll 1$, $|Z_{C_1}|$ is simply too large to feed back much of the signal, while for $(f/f_0) \gg 1$ the output signal is too small to do much good. With proper choice of the component values, the response in the vicinity of $(f/f_0) = 1$ can be bolstered until the desired amount of peaking is achieved. The op amp does what an inductor would, and a bit more since it also provides output buffering to make circuit behavior independent of any output load. It is precisely for these reasons that op amps play such an important role in filter synthesis. The circuit of Fig. 3.18b, referred to as the *Sallen-Key configuration* for its inventors, represents a cornerstone of active filters.

The Unity-Gain Sallen-Key Low-Pass Filter

To analyze our circuit we redraw it in the more conventional form of Fig. 3.19. To simplify the algebra, we express the values of the first RC pair in terms of those of the second pair by means of multipliers m and n, respectively. Denoting the voltage at the node where the two resistors meet as V_x, we note that V_p is related to V_x by the first-order low-pass function. Since $V_o = V_p$,

$$V_o = \frac{V_x}{1 + j\omega RC} \tag{3.33}$$

By KCL at node X,

$$\frac{V_i - V_x}{mR} = \frac{V_x - V_o}{R} + \frac{V_x - V_o}{1/(j\omega nC)} \tag{3.34}$$

Multiplying both sides by mR and collecting,

$$V_i = (1 + m + j\omega mnRC)V_x - (m + j\omega mnRC)V_o \tag{3.35}$$

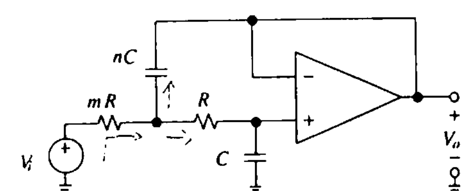


FIGURE 3.19 Unity-gain Sallen-Key low-pass filter.

Eliminating V_x and collecting once more,

$$V_i = [1 - \omega^2 mnR^2C^2 + j\omega(m + 1)RC]V_o \qquad (3.36)$$

Now we can write $\omega^2 mnR^2C^2 = [2\pi f\sqrt{(mn)}RC]^2 = (f/f_0)^2$, where we have set $f_0 = 1/[2\pi\sqrt{(mn)}RC]$. This in turn yields $j\omega(m + 1)RC = j(f/f_0)(m + 1)/\sqrt{(mn)} = (j/Q)(f/f_0)$, where we have set $Q = \sqrt{(mn)}/(m + 1)$. Thus, Eq. (3.36) yields

$$H = \frac{V_o}{V_i} = \frac{1}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \qquad (3.37a)$$

where

$$f_0 = \frac{1}{2\pi\sqrt{(mn)}RC} \qquad (3.37b)$$

and

$$Q = \frac{\sqrt{(mn)}}{m + 1} \qquad (3.37c)$$

Since $H = H_{LP}$ the Bode plot is as in Fig. 3.15a. Note that Q depends exclusively on resistance and capacitance ratios m and n, while f_0 depends on R and C as well as on ratios m and n.

Design Procedure

A common task is to find suitable component values to achieve given values of f_0 and Q. Standard resistance values offer more selection than do capacitances, so a reasonable approach is to use whatever capacitances are available and then fine-tune the resistance values. Here is a way to proceed:

1. Start with two equal ($m = 1$) resistances in the 10 kΩ to 100 kΩ range. Call their value R^*.
2. Calculate $C^* = 1/(4\pi Q f_0 R^*)$.
3. Calculate $n^* = 4Q^2$.
4. Find the closest standard capacitances C and nC, such that $C \simeq C^*$ and $n \geq n^*$.
5. Use the newly found value of n to compute the quantity $k = n/Q^2 - 2$. Hence, compute $m = [k + \sqrt{(k^2 - 4)}]/2$.
6. Find $R = 1/[2\pi\sqrt{(mn)}f_0C]$.
7. Select from the table of standard resistance values the resistances closest to R and mR.

The procedure looks much more complicated than it actually is. An example will substantiate this.

Example 3.5. For the circuit of Fig. 3.19 specify suitable component values to achieve $f_0 = 1$ kHz and $Q = 2$.

Solution. (1) $R^* = 33$ kΩ. (2) $C^* = 1/(4\pi \times 2 \times 10^3 \times 33 \times 10^3) = 1.2$ nF. (3) $n^* = 4 \times 2^2 = 16$. (4) Pick $C \doteq 1.2$ nF and $nC = 22$ nF so that $n = 22/1.2 = 18.33 > 16$. (5) $k = 18.33/2^2 - 2 = 2.583$; $m = [2.583 + \sqrt{(2.583^2 - 4)}]/2 = 2.11$. (6) $R = 1/[2\pi \times \sqrt{(2.11 \times 18.33)} \times 10^3 \times 1.2 \times 10^{-9}) = 21.33$ kΩ and $mR = 2.11 \times 21.33 = 45$ kΩ. Pick $R = 21.5$ kΩ and $mR = 45.3$ kΩ. Summarizing, $C = 1.2$ nF, $nC = 22$ nF, $R = 21.5$ kΩ, $mR = 45.3$ kΩ, all 1 percent. ■

Exercise 3.12. Design a second-order low-pass filter with $f_0 = 6$ kHz and $Q = 5$. Show your final design. ■

Second-Order Low-Pass Butterworth Filter

Thanks to its maximally flat characteristic, the Butterworth response is of special significance in filter applications. This response requires $Q = 1/\sqrt{2}$ and is achieved with $m = 1$ and $n = 2$, that is, with *equal resistances* and *2:1 capacitances*.

Exercise 3.13. Using standard component values, design a second-order Butterworth low-pass filter with $f_0 = 440$ Hz. ■

KRC Low-Pass Filter

Thanks to its simplicity, the filter of Fig. 3.19 is widely used in applications requiring fixed values of f_0 and Q, with Q in the vicinity of 1. If tuning is desired, for instance, to compensate for the effect of component tolerances, the circuit suffers from the drawback that adjusting one parameter also affects the other, as per Eq. (3.37). Moreover, high Qs result in excessive capacitance spread, as per Eq. (3.37c). These drawbacks can be alleviated by allowing the amplifier to have gain $K > 1$. The circuit of Fig. 3.20 is called a *KRC* filter (K to indicate gain, RC to indicate the type of passive elements being used). Circuit analysis proceeds as before, except that now $V_o = (1 + R_B/R_A)V_p$.

Exercise 3.14. Show that for the circuit of Fig. 3.20

$$H = \frac{K}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \qquad (3.38)$$

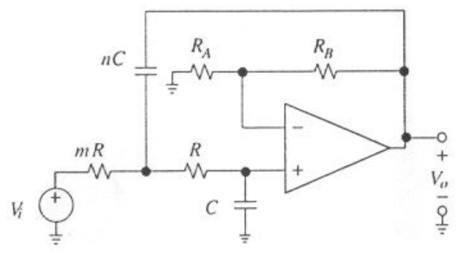


FIGURE 3.20 *KRC* low-pass filter.

where

$$K = 1 + \frac{R_B}{R_A} \quad (3.39a)$$

$$f_0 = \frac{1}{2\pi\sqrt{(mn)}RC} \quad (3.39b)$$

and

$$Q = \frac{\sqrt{(mn)}}{m + 1 + mn(1 - K)} \quad (3.39c)$$

■

Since $H = KH_{LP}$ the Bode plot is again as in Fig. 3.15a, but shifted upward by the amount K_{dB}. The expression for f_0 remains unchanged; however, Q now also depends on K so that it can be adjusted without affecting f_0. Thus, if both f_0 and Q require tuning, we can implement mR and R_B with two variable resistors. First we adjust mR to achieve the desired f_0 (this also affects Q), then we adjust R_B to achieve the desired Q (this adjustment does *not* affect f_0).

Equal-Component-Value *KRC* Filters

To simplify inventory, the *KRC* circuit is often implemented with equal capacitors ($n = 1$) and with $R_A = R_B$ so that $K = 2$ and

$$Q = \sqrt{m} \quad (3.40a)$$

$$f_0 = \frac{1}{2\pi QRC} \quad (3.40b)$$

Another frequently used implementation is $m = n = 1$, in which case the expressions for f_0 and Q become

$$f_0 = \frac{1}{2\pi RC} \quad (3.41a)$$

and

$$Q = \frac{1}{3 - K} \quad (3.41b)$$

The advantage of the second implementation is that it avoids excessive component spread, especially in designs with high Qs. This, however, is offset by the fact that the ratio R_B/R_A must be very precise since a small variation of K may lead to an unacceptably large variation of Q. Consider, for instance, the case $Q = 10$, which is achieved with $K = 2.9$, that is, $R_B/R_A = 1.9$. Now suppose that because of drift or aging, the ratio R_B/R_A changes by ± 1 percent, that is, $1.88 \le R_B/R_A \le 1.92$, so that $2.88 \le k \le 2.92$. This in turn implies $8.3 \le Q \le 12.5$, quite a variation compared with the change of the R_B/R_A ratio! Even worse, K may rise to 3 and make Q infinite, thus causing the circuit to oscillate. Because of

the high sensitivity of Q to component tolerances, the circuit is usually restricted to applications with $Q < 10$. In Section 3.10 we will study circuit configurations suited to high Qs.

Exercise 3.15. (*a*) Redesign the filter of Example 3.5, but using the equal-component-value *KRC* with $n = 1$ and $R_A = R_B$. Show the final design. (*b*) Repeat, but with $m = n = 1$. ■

Example 3.6. A *KRC* filter of the $m = n = 1$ type is implemented with $R = 88.7$ kΩ, $C = 18$ nF, $R_A = 100$ kΩ and $R_B = 59.0$ kΩ. If the input is a 1 kHz, 2 V_{rms} sine wave with a 5 V dc component, what comes out of the filter?

Solution. We can write $V_i = \{5 + 2\sqrt{2}\cos(2\pi 10^3 t)\}$ V, $V_o = \{|H(j0)|5 + |H(j10^3)|2\sqrt{2}\cos[2\pi 10^3 t + \measuredangle H(j10^3)]\}$ V. By Eqs. (3.39) and (3.41), $H = KH_{LP}$ where $K = 1.59$, $f_0 = 100$ Hz, and $Q = 1/\sqrt{2}$. Since $|H(j0)| = 1.59$, $|H(j10^3)| = 1.59 \times 10^{-2}$, and $\measuredangle H(j10^3) = -172°$, it follows that $V_o = \{7.95 + 4.5 \times 10^{-2} \times \cos(2\pi 10^3 t - 172°)\}$ V. As expected, the dc component goes through with a gain of 1.59 while the ac component undergoes significant attenuation. ■

3.7 SECOND-ORDER HIGH-PASS FILTERS

Interchanging resistors and capacitors in the Sallen-Key low-pass filter of Fig. 3.19 turns it into a high-pass filter, as in Fig. 3.21.

Unity-Gain Sallen-Key High-Pass Filter

Exercise 3.16. Show that for the circuit of of Fig. 3.21

$$H = \frac{-(f/f_0)^2}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \quad (3.42a)$$

where

$$f_0 = \frac{1}{2\pi\sqrt{(mn)}RC} \quad (3.42b)$$

and

$$Q = \frac{\sqrt{(mn)}}{n + 1} \quad (3.42c)$$

■

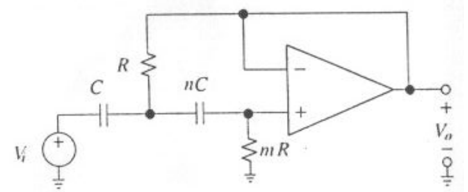


FIGURE 3.21 Unity-gain Sallen-Key high-pass filter.

Since $H = H_{HP}$ the Bode plot is as in Fig. 3.15b. As in the low-pass case, Q depends on resistor and capacitor ratios m and n while f_0 depends on R and C as well as on ratios m and n.

Design Procedure

Given f_0 and Q, the search for a suitable set of standard component values meeting the specifications proceeds as follows:

1. Use equal capacitors ($n = 1$) and determine the required value of m to achieve the given Q using Eq. (3.42c).
2. Select a reasonable pair of resistances (e.g., in the 1 kΩ to 1 MΩ range) satisfying the above ratio.
3. Using Eq. (3.42b) calculate C and select the closest standard capacitance.
4. Using Eq. (3.42b) recompute R and use the closest standard resistance values to R and mR.

Example 3.7. For the circuit of Fig. 3.21 specify suitable component values to achieve $f_0 = 200$ Hz and $Q = 1.5$.

Solution. Let $n = 1$ so that Eq. (3.42c) yields $1.5 = (\sqrt{m})/2$, that is, $m = 9$. Try $R = 30$ kΩ. Then by Eq. (3.42b), $C = 1/(2\pi \times \sqrt{9} \times 1 \times 30 \times 10^3 \times 200) = 8.8$ nF. Select $C = 10$ nF. Then $R = 20 \times 10/8.8 = 26.5$ kΩ (use 26.7 kΩ), and $mR = 9 \times 26.5 = 238.5$ kΩ (use 237 kΩ). Summarizing, $C = 10$ nF, $R = 26.7$ kΩ and $mR = 237$ kΩ, all 1 percent. ■

Second-Order High-Pass Butterworth Filter

This response corresponds to $Q = 1/\sqrt{2}$ and is achieved with $m = 2$ and $n = 1$, that is, with *equal capacitances* and *2:1 resistances*.

Exercise 3.17. Using standard component values, design a second-order high-pass Butterworth filter with $f_0 = 500$ Hz. ■

Equal-Component-Value *KRC* Filter

The tuning difficulties and component spread drawback of the unity-gain configuration of Fig. 3.21 are alleviated by allowing the amplifier to have gain $K > 1$.

The circuit is almost always used with equal component values, as in Fig. 3.22.

Exercise 3.18. Show that for the circuit of Fig. 3.22

$$H = \frac{-K(f/f_0)^2}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.43a}$$

where

$$K = 1 + \frac{R_B}{R_A} \tag{3.43b}$$

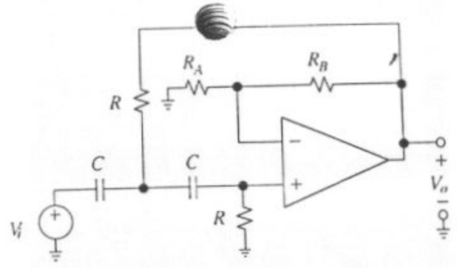


FIGURE 3.22 Equal-component-value *KRC* high-pass filter.

$$f_0 = \frac{1}{2\pi RC} \tag{3.43c}$$

$$Q = \frac{1}{3 - K} \tag{3.43d}$$

■

Since $H = KH_{HP}$ the Bode plot is again as in Fig. 3.15b, but shifted upward by the amount K_{dB}. We again note that f_0 can be tuned by adjusting one of the two resistors (e.g., the one going to ground), and Q is tuned by adjusting either R_A or R_B. For the reasons already discussed in the low-pass case, the *KRC* circuit is confined to applications requiring $Q < 10$ or so.

Exercise 3.19. Using the *KRC* circuit, design a high-pass filter with $f_0 = 100$ Hz and Q variable over the range $0.5 \le Q \le 5$ by means of a 100 kΩ potentiometer. Show your final circuit. ■

Exercise 3.20. If the input to the filter of Exercise 3.19 is a 60 Hz, 5 V_{rms} sine wave having a dc component of 3 V, what comes out of the filter? ■

3.8 SECOND-ORDER BAND-PASS FILTERS

Figure 3.23 shows what is probably the most popular single–op amp realization of the second-order band-pass response. Referred to as the *multiple-feedback configuration* (the designation stems from the presence of two separate feedback paths) or the *Delyiannis-Friend* circuit (from the names of the authors[5] who first reported it), this circuit is to band-pass filters what the Sallen-Key configurations are to low-pass and high-pass filters. Unlike the wide-band band-pass filter of Fig. 3.8, the response of the present circuit can be made much more selective. The circuit is also called *narrow-band-pass filter* because of this.

Although in general the two capacitors need not be equal, imposing equal values simplifies the algebra without diminishing the filter's versatility. To obtain the transfer function we designate the voltage at the node where R_1 meets the capacitors as V_x. Thanks to the input voltage and current constraints, we have $V_x/[1/(j\omega C)] = -V_o/R_2$, that is,

$$V_o = -(j\omega R_2 C)V_x \tag{3.44}$$

By KCL,

$$\frac{V_i - V_x}{R_1} = \frac{V_x}{1/(j\omega C)} + \frac{V_x - V_o}{1/(j\omega C)} = j\omega C(2V_x - V_o) \tag{3.45}$$

Eliminating V_x we obtain, after the usual algebra,

$$H = -2Q^2 \frac{(j/Q)(f/f_0)}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} \tag{3.46a}$$

where

$$f_0 = \frac{1}{2\pi\sqrt{(R_1R_2)}C} = \frac{1}{2\pi\sqrt{R_1R_2}\times C} \tag{3.46b}$$

$\sqrt{R_1R_2}\times C$

and

$$Q = \frac{1}{2}\left(\frac{R_2}{R_1}\right)^{1/2} \tag{3.46c}$$

Since $H = -2Q^2H_{BP}$ the magnitude plot can be obtained from that of Fig. 3.16a by shifting each curve *upward* by the amount $(2Q^2)_{dB}$. The result is shown in Fig. 3.23b for $Q = 1$ and $Q = 5$. Note in particular that the *resonant gain* is $H_{0BP} = -2Q^2$, so that even for moderate Qs this gain tends to be fairly high.

Design Procedure

To speed up design, the above equations can be turned around to yield

$$R_2 = \frac{Q}{\pi f_0 C} \tag{3.47a}$$

and

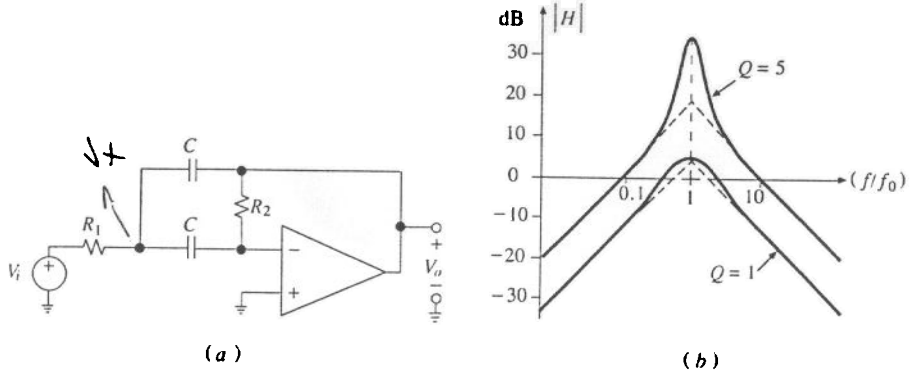


FIGURE 3.23 Multiple-feedback band-pass filter.

$$R_1 = \frac{R_2}{4Q^2} \tag{3.47b}$$

The specifications for a band-pass filter may be given directly in terms of f_0 and Q, or indirectly in terms of bandwidth and bandwidth position, or in terms of f_L and f_H. No matter what, one can always determine f_0 and Q by calculation using Eqs. (3.26) through (3.29). Then, starting out with a reasonable guess for C, one computes R_1 and R_2. If the resulting resistance values are out of range, scale all components accordingly.

Example 3.8. (*a*) For the circuit of Fig. 3.23 specify suitable component values to ensure a center frequency of 500 Hz and a bandwidth of 50 Hz. (*b*) What is the resonant gain?

Solution. (*a*) $Q = f_0/BW = 500/50 = 10$. Start out with $C = 10$ nF. Then by Eq. (3.47) $R_2 = 10/(\pi \times 500 \times 10^{-8}) = 636.7$ kΩ (use 634 kΩ, 1 percent) and $R_1 = (636.7 \times 10^3)/(4 \times 10^2) = 1.591$ kΩ (use 1.58 kΩ, 1 percent). Summarizing, $C = 10$ nF; $R_1 = 1.58$ kΩ; $R_2 = 634$ kΩ, all 1 percent. (*b*) The resonant gain is $-2Q^2 = -2 \times 10^2 = -200$. ■

Multiple Feedback with Input Attenuator

Since the resonant gain increases quadratically with Q, the op amp may easily end up in saturation even for moderate Qs unless the input signal level is kept suitably low. This can be done with an ordinary voltage divider at the input (see Fig. 3.24a).

To analyze the circuit we thevenize the input network and obtain the equivalent circuit of Fig. 3.24b, where $V_{eq} = V_iR_B/(R_A + R_B)$ and $R_{eq} = R_A /\!/ R_B$. Thus, the transfer function is as in Eq. (3.46), but with R_1 replaced by R_{eq}. In particular, we have $V_o = 2Q^2H_{BP}V_{eq} = 2Q^2H_{BP}V_iR_B/(R_A + R_B)$, that is,

$$H = H_{0BP}H_{BP} \tag{3.48a}$$

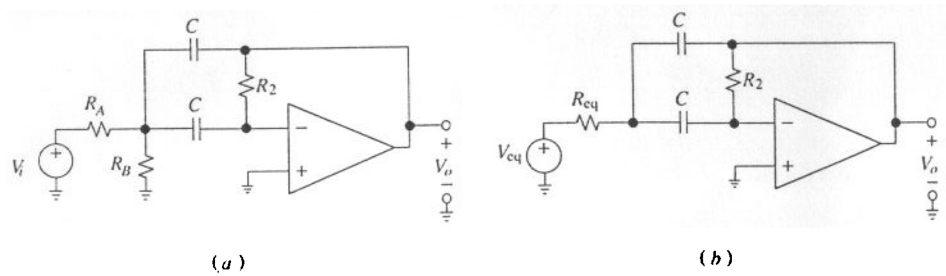


FIGURE 3.24 (*a*) Multiple-feedback configuration with input attenuator to reduce resonant gain. (*b*) Thevenin equivalent.

the band-passed output from the input signal itself. Design one such filter to achieve a 60 Hz notch with a Q of 10. ■

3.10 STATE-VARIABLE AND BIQUAD FILTERS

The second-order filters encountered so far are relatively simple circuits that achieve their goal with a minimum of components. However, simplicity doesn't come without a price, and these circuits, although enjoying wide popularity, are often awkward to tune and in some cases are quite sensitive to component nonidealities, particularly to the finite gain-bandwidth product of the op amp, which limits achievable Qs to 10 or so.

Component minimization, especially minimization of the number of op amps, was of concern when these devices were expensive. Now purchasable in volume quantities and in multiple–op amp packages, active devices are cost-competitive with precision passive components. The question then arises whether filter performance and versatility can be increased by shifting the burden from passive to active devices. The answer is provided by multiple–op amp filters of the *state-variable* and *biquad* types, which, while requiring more op amps than the circuits studied so far, can provide *more than one response simultaneously*, are *easier to tune*, and are *less prone to component nonidealities*. Although in a given application one may use only one of the simultaneously available responses, the same basic circuit can be utilized in different applications. These filters are also called *universal active filters* because of this and are available in hybrid form from various manufacturers. The most complete version consists of four op amps; however, this number can be reduced to three or even two in specific cases.

Inverting-Type State-Variable Filter

The state-variable configuration uses two op amp integrators and one op amp adder to provide simultaneous second-order low-pass, band-pass, and high-pass responses. A fourth op amp provides the notch response by combining the low-pass and high-pass outputs according to $H_N = H_{LP} + H_{HP}$. The circuit can also be viewed as the analog computer implementation of a second-order nonhomogeneous differential equation; hence its name.[6]

Figure 3.27 shows one of the most popular versions of the circuit. Although in general all component values would be different, imposing equal values as shown simplifies the algebra without diminishing versatility. However complicated the circuit may look, its analysis is less troublesome than those of the other filters discussed so far.

The leftmost op amp forms a linear combination of the type $V_{HP} = \alpha V_i + \beta V_{LP} + \gamma V_{BP}$, where α, β, and γ are found by means of the superposition principle. Suppressing two signals at a time yields

$$V_{HP} = -\frac{R_3}{R_3}V_i - \frac{R_3}{R_3}V_{LP} + \left(1 + \frac{R_3}{R_3 /\!/ R_3}\right)\frac{R_1}{R_1 + R_2}V_{BP}$$

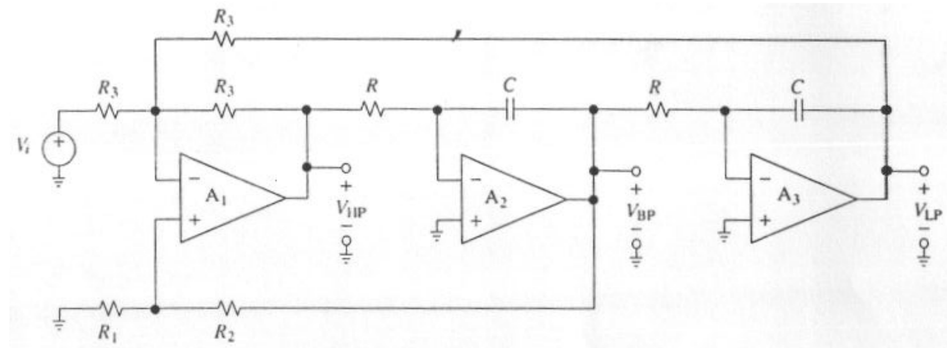


FIGURE 3.27 State-variable filter (inverting).

that is,

$$V_{HP} = -V_i - V_{LP} + \frac{3R_1}{R_1 + R_2}V_{BP} \tag{3.54}$$

By the integrator formula,

$$V_{BP} = -\frac{1}{j(f/f_0)}V_{HP} \tag{3.55}$$

and

$$V_{LP} = -\frac{1}{j(f/f_0)}V_{BP} = -\frac{1}{(f/f_0)^2}V_{HP} \tag{3.56}$$

where $f_0 = 1/(2\pi RC)$. Substituting Eqs. (3.55) and (3.56) into Eq. (3.54) and solving for the ratio V_{HP}/V_i, we obtain

$$\frac{V_{HP}}{V_i} = \frac{(f/f_0)^2}{1 - (f/f_0)^2 + (j/Q)(f/f_0)} = -H_{HP} \tag{3.57}$$

where

$$f_0 = \frac{1}{2\pi RC} \tag{3.58a}$$

and

$$Q = \frac{1}{3}\left(1 + \frac{R_2}{R_1}\right) \tag{3.58b}$$

Except for the negative sign, this is the standard second-order high-pass response, whose magnitude plot is shown in Fig. 3.15b.

Eliminating V_{HP} between Eqs. (3.55) and (3.57) yields