Aulas TP

23/09/2022

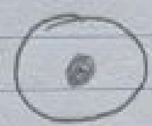
→ O que acontece quando se olha para um feixe laser?

Laser He-Ne 1 mW

$\lambda = 632,8 \text{ nm}$

- A pupila sobre iluminação durante 2 minutos

$* D^{ISR} = D^{\text{Imagem do sol na retina}}$



$d = 2 \text{ mm}$

$r = 1 \text{ mm}$

1. Vamos ver o que acontece primeiro com o sol.

1350 W/m²

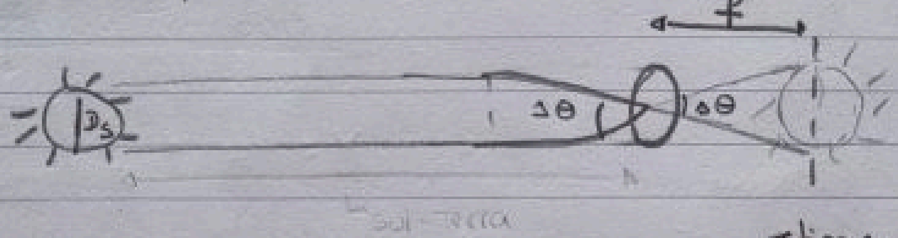
1000 W/m²

↳ atmosfera

$$I^{retina}_{sol} = \frac{P^{\to olho}_{sol}}{\pi (D^{ISR}/2)^2} = ?$$

$$P^{olho}_{sol} = 1000 \frac{W}{m^2} \times \left(\pi \times (10^{-3} m)^2\right)$$

$$= \pi \times 10^{-3} \text{ W}$$



$f = 2,5 \text{ mm}$

Como $\Delta\theta$ << então:

$\tan \Delta\theta \sim \Delta\theta$

$\frac{D^{ISR}}{f} \sim \Delta\theta$ $\wedge$ $\Delta\theta \sim \frac{D_{sol}}{L_{sol-Terra}}$

$$D^{ISR} \sim \frac{D_{sol}}{L_{sol-Terra}} \cdot f = (2,25 \times 10^{-3} \text{ m}) \frac{1,4 \times 10^9 \text{ m}}{1,5 \times 10^{11} \text{ m}}$$

$\sim 200 \ \mu m$

$$I_{sol}^{\to retina} = \frac{P_{sol}^{\to olho}}{\pi (R^{ISR})^2} \approx 10^5 \ W/m^2$$

2: olhar para o laser

$$P_{laser}^{\to olho} = 1 mW$$

$$\ell_{spot} = 1{,}22 \ f \frac{\lambda}{d} \Rightarrow D^{ISR} = \frac{4}{\pi} f \frac{\lambda}{d_{laser}} = \frac{4}{\pi}(2{,}23\times10^{-2}) \times \frac{632{,}8\times10^{-9}}{(2\times10^{-3})}$$

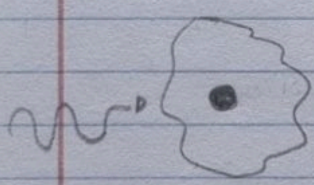
$$D^{ISR} \sim 9 \ \mu m$$

$$I_{laser}^{\to Retina} = \frac{P_{laser}^{\to olho}}{\pi (R^{ISR})^2} \sim \frac{1\times10^{-3}}{\pi(4{,}5\times10^{-6})^2} = 1{,}57\times10^7 \ W/m^2$$

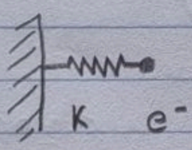
Isto é 150 vezes superior à $I_{sol}^{\to Retina}$!!

30/09/2022

3.3.



Lorentz

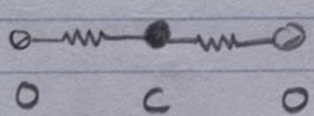


$$\nu_{átomo} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m_e}}$$

$$\nu_{átomo} \sim 5\times10^{14} \ Hz$$

$$\lambda = \frac{c}{\nu} = 600 nm$$

molécula



$$\nu_{molécula} = \frac{1}{2\pi}\sqrt{\frac{K}{m_{átomo}}}$$

$$m_{atómicos} \sim (1-200) \ m_{protão}$$

$$m_{protão} = 1{,}67\times10^{-27} \ kg$$

$$m_e = 9{,}1\times10^{-31} \ kg$$

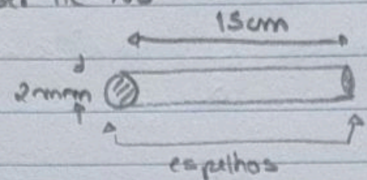
$$\frac{\nu_{molécula}}{\nu_{atómica}} = \sqrt{\frac{m_{e^-}}{m_{átomo}}}$$

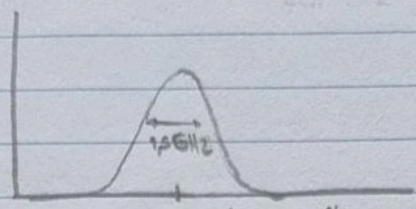
$$\nu_{molécula} \sim 8\times10^{11} - 10^{13} \ Hz$$

$$\lambda_{molécula} \sim 374 \mu m \quad 30 \mu m$$

1.

Laser He-Ne

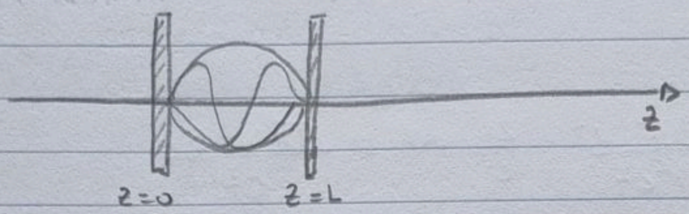




$\nu_0 = 4,79 \times 10^{14}$ Hz

$\lambda = 632,8$ nm

Caso 1: Cavidade de 1 Dimensão



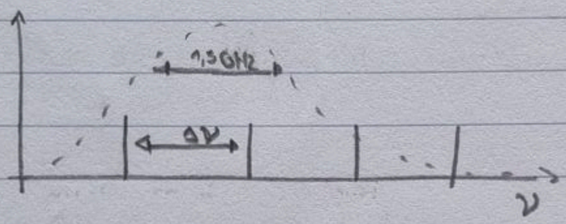
$$m \frac{\lambda_m}{2} = L$$

$$\pi m = \frac{2L}{\lambda_m} \pi = K_m L \quad (1)$$

$$C = \frac{\omega}{K} = \frac{2\pi \nu_m}{K} \rightarrow K = \frac{2\pi \nu_m}{C} \quad (2)$$

Tendo (1) e (2):

$$\pi m = \frac{2\pi \nu_m L}{C} \Leftrightarrow \nu_m = m \underbrace{\left[\frac{C}{2L}\right]}_{\Delta\nu}$$



$$\Delta\nu = \frac{C}{2L} = \frac{3 \times 10^8 \text{ m/s}}{0,3 \text{ m}}$$

$$= 10^9 \text{ Hz}$$

$$\frac{1,5 \text{ GHz}}{1 \text{ GHz}} = 1,5 \text{ modos}$$

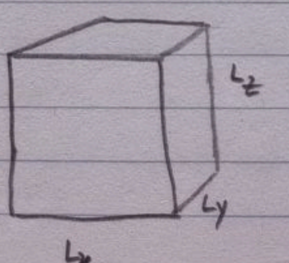
$\Downarrow$

1,2 (2 modos

$$N^{\underline{o}} \text{ modos} = \frac{\Delta\nu_{\text{He-Ne Laser}}}{\Delta\nu_{\text{cavidade}}}$$

Caso 2: Cavidade de 3D

Temos 3 cavidades de 1 dimensão



$$K_x L_x = m_x \pi \quad , \quad K_x = \frac{m_x \pi}{L_x}$$

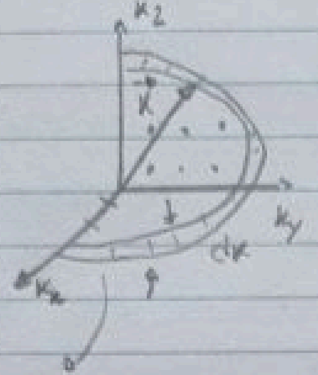
$$K_y L_y = m_y \pi$$

$$K_z L_z = m_z \pi$$

Quantos modos podem existir entre $|\vec{k}|$ e $|\vec{k}| + |d\vec{k}|$.

fator da polarização

$$N^{\underline{o}}\ \text{modos} = \frac{\left(\underbrace{\frac{1}{8}}_{k>0}\ \underbrace{4\pi k^2}_{\text{área}}\ dk\right) 2}{\underbrace{\left(\frac{\pi}{L_x}\right)\left(\frac{\pi}{L_y}\right)\left(\frac{\pi}{L_z}\right)}_{\text{volume tomado}}}$$



Volume de casca esférica

$$N^{\underline{o}}\ \text{modos} = \frac{k^2\, dk}{\pi^2} \cdot L_x L_y L_z$$

$$\frac{N^{\underline{o}}\ \text{modos}}{\text{volume}} = \frac{k^2\, dk}{\pi^2} = \frac{4\pi^2 \nu^2}{\pi^2 c^2} \cdot \frac{2\pi}{c}\, d\nu = \frac{8\pi \nu^2 d\nu}{c^3}$$

$$k = \frac{2\pi\nu}{c} \ ; \ dk = \frac{2\pi}{c}\, d\nu$$

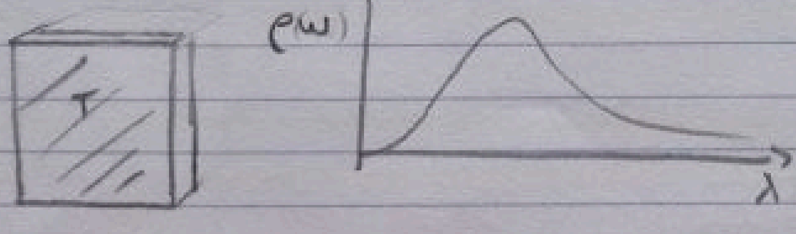
Área da cavidade

$$\text{Na cavidade 3D: } N^{\underline{o}}\ \text{modos} = \frac{8\pi\nu^2}{c^3}\, \underbrace{\Delta\nu_{laser}}_{1{,}5\ GHz} \cdot (0{,}15\ m) \cdot \pi 10^{-6}\ m$$
$$= 148 \times 10^6$$

Teoria

Lei de Planck

Radiação do corpo



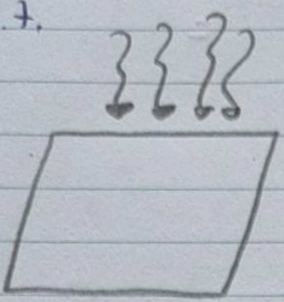
$$\text{Densidade espectral} \Rightarrow \rho(\nu) = \frac{\text{energia}}{(\text{volume})(\text{frequência})}$$

$$\rho(\nu)\, d\nu = \underbrace{\frac{8\pi\nu^2}{c^3}\, d\nu}_{\frac{N^{\underline{o}}\ \text{modos}}{\text{volume}}}\ \underbrace{h\nu}_{\text{Energia fotão}} \cdot \underbrace{\frac{1}{e^{h\nu/k_BT} - 1}}$$

↳ Nº médio dos fotões/modos no equilíbrio termodinâmico com temperatura T $\langle n(\nu) \rangle$

TP1 - Capítulo 3 07/10/2022

3.7.



$I = 1000 \, W/m^2$

→ Equilíbrio: a energia que absorve por segundo é igual à que emite.

$$I_{rad} = \frac{c}{4}\int_0^{\infty} \rho(\nu)\, d\nu \sim \sigma T^4$$

$$1000 \frac{W}{m^2} = 5{,}67 \times 10^{-8} \frac{W}{m^2} K^4 \cdot T^4 \Rightarrow T = 364\,K \sim 91°C$$

3.9.

$$\frac{dN_2}{dt} = \frac{1}{c}\left[B_{21} N_2 - B_{12} N_1\right] I_\nu S(\nu) - \underbrace{A_{21} N_2}$$

Taxa de emissão espontânea: $-fm = -A_{21} N_2$

$f = A_{21} \wedge m = N_2$

Taxa de emissão estimulada: $a = B_{21}$

Taxa de absorção: $\beta = B_{12}$

$m \equiv \frac{\text{Nº de átomos no estado excitado}}{\text{Volume}}$

$q \equiv$ Nº de fotões

3.10.

$1 \text{ atm} = 1{,}01325 \times 10^5 \, N/m^2$ $\frac{\text{Nº átomos}}{\text{Volume}}$

1 Torr = 1 mm Hg

760 Torr = 1 Atm

N: de átomos

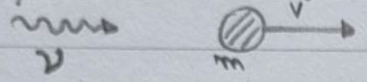
$$PV = N k_B T \quad , \quad k_B = 1,38 \times 10^{-23}\ J/K$$

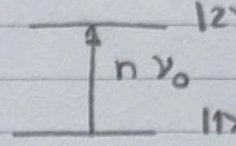
$$\frac{N}{V} = \frac{P}{k_B T} = \frac{1,01325 \times 10^5 \frac{N/m^2}{Atm} \cdot \frac{1 Atm}{760} Torr}{1,38 \times 10^{-23}\ J/K \ . \ T(K)} =$$

$$= 9,65 \times 10^{24} \frac{P(Torr)}{T(K)}$$

3.12.

fotão





$$p = \frac{h\nu}{c}$$

$$E = h\nu$$

$$E = pc$$

$\hookrightarrow E^2 = \underbrace{m^2c^4}_{0} + p^2c^2 \Rightarrow E = pc$

→ Nota que:

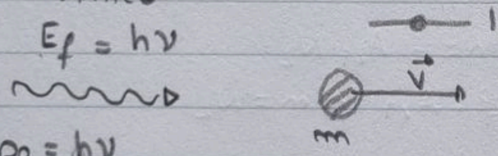
$v \ll c$

$\frac{h\nu}{c} \ll mv$

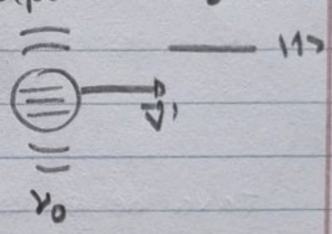
Antes

$E_f = h\nu$

$p_f = \frac{h\nu}{c}$



Depois



A energia é conservada: $h\nu + \frac{1}{2}mv^2 = h\nu_0 + \frac{1}{2}mv'^2$

O momento é conservado: $\frac{h\nu}{c} + mv = mv'$

$$\begin{cases} v' = \frac{h\nu}{mc} + v \\ h\nu + \frac{1}{2}mv^2 = h\nu_0 + \frac{1}{2}m\left(\frac{h\nu}{mc} + v\right)^2 \end{cases} \Leftrightarrow$$

$$\cancel{h}\nu + \cancel{\frac{1}{2}mv^2} = \cancel{h}\nu_0 + \frac{1}{2}\frac{h^{\cancel{2}}\nu^2}{mc^2} + \frac{\cancel{h}\nu v}{c} + \cancel{\frac{1}{2}mv^2}$$

$$\nu = \nu_0 + \frac{\nu v}{c} + \frac{1}{2}\frac{h\nu^2}{mc^2}$$

$$\nu = \nu_0 + \nu\left[\underbrace{\frac{v}{c}}_{10^{-6}} + \underbrace{\frac{1}{2}\cancel{\frac{h\nu}{mc^2}}}_{10^{-9}}\right]$$

podemos ignorar

$v \sim 3\times10^2$ m/s
$c \sim 3\times10^8$ m/s
$h\nu \sim 2$ eV
(e) $mc^2 \sim 0{,}5$ MeV
p $mc^2 \sim 10^9$ eV

$$\nu\left(1 - \frac{v}{c}\right) = \nu_0$$

$$\nu = \frac{\nu_0}{1 - v/c} \sim \underbrace{\nu_0}_{10^{14}\,Hz}\underbrace{\left(1 + \frac{v}{c}\right)}_{10^{-6}}$$

TP2 - Capítulo 3

21/10/2022

3.11.

CO_2 $\lambda = 10{,}6\ \mu m$

$\sigma(CO_2 \text{ e } N_2) = 1{,}22\ mm^2$
$\sigma(CO_2 \text{ e } O_2) = 0{,}95\ mm^2$

$$\gamma_c = \gamma_{CO_2-N_2} + \gamma_{CO_2+O_2} = N_y \bar{V}_{rel}\, \sigma_{x-y}$$

$$\bar{V}_{rel} = \left[\frac{8RT}{\pi}\left(\frac{1}{M_x} + \frac{1}{M_y}\right)\right]^{1/2} \quad ; \quad N = 9{,}65\times10^{24}\ \frac{P(Torr)}{T(K)}\ \frac{1}{m^3}$$

Da wikipedia:

$1\ atm = 760\,T$ $N_2 \simeq 78\% \simeq 593\,T$
$T = 300\,K$ $O_2 \simeq 21\% \simeq 160\,T$

$$N_{N_2} \simeq 9{,}65\times10^{24}\ \frac{593}{300}\ \frac{1}{m^3} \simeq 1{,}9\times10^{25}\ m^{-3}$$

$$N_{O_2} \simeq 5{,}2\times10^{24}\ m^{-3}$$

$M_{CO_2} = 44\,g/mol$
$M_{O_2} = 32\,g/mol$
$M_{N_2} = 28\,g/mol$

$$N_a k_B = R = 8{,}3\ \frac{J}{mol\,K}$$

$\overline{v}_{rel\,O_2-CO_2} = 585\ m/s$; $\overline{v}_{rel\,N_2-CO_2} \simeq 609\ m/s$

$\gamma^{colisão}_{CO_2-O_2} = N_{O_2}\ \overline{v}_{O_2-CO_2}\ \sigma_{CO_2-O_2} \simeq 2{,}86\times10^{9}\ rad/s$

$\gamma^{colisão}_{CO_2-N_2} = 1{,}39\times10^{10}\ rad/s = N_{N_2}\ \overline{v}_{N_2-CO_2}\ \sigma_{N_2-CO_2} \rightarrow ??$

$\gamma_T = \gamma_{CO_2-O_2} + \gamma_{CO_2-N_2} = 1{,}68\times10^{10}\ rad/s$

$\gamma^{colisões}_{CO_2} \simeq 1{,}68\times10^{10}\ rad/s$

$\Delta\nu^{colisão}_{CO_2} = 2{,}76\ Hz \rightarrow$ Lorentziano $\quad \Delta\nu = \frac{\gamma}{2\pi}$

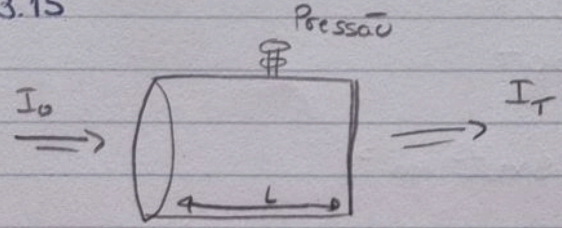
Largura de Dopler $\quad \Delta\nu_D = \frac{2\nu_0}{c}\sqrt{\frac{2k_BT\ln 2}{m(kg)}}$ $\quad \frac{1}{\lambda_0}$

$= 2{,}15\times10^{5}\ \frac{1}{\lambda_D(nm)}\sqrt{\frac{T(K)}{M(g/mol)}}\cdot MHz$

$= 2{,}15\times10^{5}\ \frac{1}{10600}\sqrt{\frac{300}{44}}\ MHz$

$\simeq 53\ MHz$

3.15



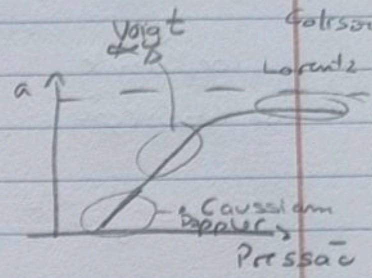
$\frac{I_T}{I_0} = e^{-\alpha L}$

$N_2 \sim 0$
$N_1 \sim N$

$\alpha(\nu) = \frac{\lambda^2}{8\pi}\ A_{21}\ \frac{g_2}{g_1}\ N\,S(\nu)$

Limite de Doppler dominante

$$S(\nu) = \frac{1}{\delta\nu_D}\left(\frac{4\ln 2}{\pi}\right)^{1/2} \exp\left[\frac{-4(\nu-\nu_0)^2 \ln 2}{\delta\nu_D^2}\right]$$



$$\delta\nu_D = \frac{2\nu_0}{c}\sqrt{\frac{2k_B T \ln 2}{m}}$$

independente N

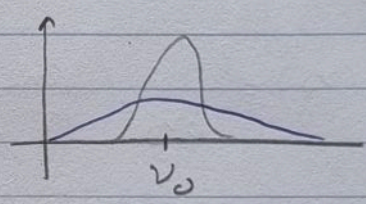
Limite onde as colisões são dominantes

$$S(\nu) = \frac{\delta\nu/\pi}{(\nu-\nu_0)^2+\delta\nu^2} \quad \text{Lorentz}$$

Quando N >>
1º Termo de $\delta\nu$ corta-se

$$\delta\nu = \cancel{\frac{A_{21}}{4\pi}} + \frac{\gamma_c}{2\pi} \quad ; \; \gamma_c = N\bar{v}_{rel} \cdot \sigma$$

Tempo de vida



$$S(\nu_0) = \frac{\delta\nu/\pi}{0+\delta\nu^2} = \frac{1}{\pi\,\delta\nu}$$

$$N \text{ elevado} \Rightarrow a(\nu) = \frac{\lambda^2}{8\pi} A_{21} \frac{g_2}{g_1} \cancel{N} \cdot \frac{1}{\pi \cancel{N} \bar{v}_{rel} \cdot \sigma} \quad *$$

3.19.

He $1s_0 \rightarrow 2p_1$ @ $\lambda = 58,4$ nm

$f = 0,28$ (p. 125)

a) $A_{21} = ?$

$2p_1$ $m_J=-1$ $m_J=0$ $m_J=1$ $g_2 = 3$

$1s_0$ $m_J=0$ $g_1 = 1$

$$A_{21} = \frac{g_1}{g_2}\frac{2\pi e^2}{\varepsilon_0 m c} f \frac{1}{\lambda_0^2}$$

$e = 1,6\times10^{-19}$ C
$m_e = 9,1\times10^{-31}$ kg
$c = 3\times10^{8}$ m/s
$\varepsilon_0 = 8,85\times10^{-15}\ \frac{Nm^2}{C^2}$

$\lambda = 58,4339$ nm
$A = 1,789\times10^{9}$ rad/s

b)

$a(\nu_0) = \lambda_0^2 \frac{A_{21}}{8\pi} \frac{g_2}{g_1} N\, S(\nu_0)$ $\qquad \delta\nu_D = 2{,}15\times10^5 \frac{1}{\lambda_0(nm)} \sqrt{\frac{T(K)}{M(g/mol)}}$ M_{He}

$\downarrow$
coeficiente
de absorção

$T = 300\,K \qquad M_{He} = 4\,g/mol$

$I(L) = I_0 e^{-aL}$ $\qquad \delta\nu_D^{He} = 31{,}9\,GHz$

A baixa pressão:

$$\delta\nu_D \sim \frac{A_{21}}{4\pi} + \frac{\delta_L}{2\pi} \sim \frac{1{,}8\times10^9}{4\pi} = 281\,MHz$$

Doppler $\qquad \nu = \nu_0$

$$S(\nu) = \frac{1}{\delta\nu_D}\left(\frac{4\ln 2}{\pi}\right)^{1/2} \exp\left[\cancel{\frac{-4(\nu-\nu_0)^2 \ln 2}{\delta\nu_D^2}}\right]$$

$$S(\nu_0) = \frac{1}{\delta\nu_D}\left(\frac{4\ln 2}{\pi}\right)^{1/2}$$

$$a(\nu_0) = \lambda_0^2 \frac{A_{21}}{8\pi} \frac{g_1}{g_2} N \frac{1}{\delta\nu_D}\left(\frac{4\ln 2}{\pi}\right)^{1/2}$$

$\lambda_0 = 58{,}4\,nm$
$g_2 = 3\ ;\ g_1 = 1$ $\qquad P = 1\,Torr \quad N = 9{,}65\times10^{24} \frac{P(Torr)}{T}$
$A_{21} = 1{,}8\times10^9\,s^{-1}$

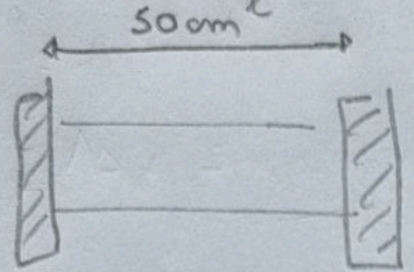
$T = 300\,K \qquad N \sim 3{,}2\times10^{22}\,m^{-3}$

$a(\nu_0) = 6{,}9\times10^{-5}\,nm^{-1}$

comprimento de absorção $\qquad \frac{1}{a} \sim 145\,\mu m$
muito absorvente

$\simeq 25\,\lambda_0$

Exemplo He-Ne $\lambda = 632,8$ nm — Exercício/Exemplo 5



1º Problema 4.1

$$g(\nu) = \frac{\lambda^2}{8\pi} A_{21} \underbrace{\left(N_2 - N_1 \frac{g_2}{g_1}\right)}_{\Delta N} S(\nu)$$

$r_1 = 0,998$ $r_2 = 0,98$

$t_1 \approx 0,002$ $t_2 = 0,02$

$$\Delta N_{limiar} = g_{limiar} \left(\frac{8\pi}{\lambda^2}\right) \frac{1}{A_{21}} \frac{1}{S(\nu)}$$

$\lambda = 632,8$ nm

$\tau \sim 715$ ns $= \frac{1}{A_{21}} \Rightarrow A_{21} = 1,4 \times 10^{6}$ rad/s

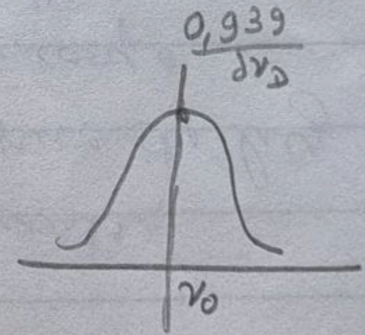
$$g_e = \frac{1}{2l} \ln(r_1 r_2) = 2,2 \times 10^{-4} \text{ cm}^{-1}$$

$$\Delta\nu_D = 2,15^{5} \frac{1}{\lambda(nm)} \left[\frac{T}{M}\right]^{1/2} \text{MHz} = 1,5 \text{ GHz}$$

Ne $\Rightarrow$ $M = 20$ g/mol $T = 400$ K

$$S(\nu)_{Doppler} = \frac{0,939}{\delta\nu_D} \exp\left[\frac{-2,77(\nu - \nu_0)^2}{\delta\nu_D^2}\right]$$

$$S(\nu)_{Doppler} = 6,3 \times 10^{-10} \text{ s}$$



$$\Delta N_{limiar} = 1,6 \times 10^{9} \frac{\text{atomos}}{\text{cm}^3}$$

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4.2

$$\frac{d}{dt}(m_2 + q_\nu) = -(\Gamma_2 + A_{21})\, m_2 + P - \frac{c}{2L}(1 - r_1 r_2)\, q_\nu$$

a)

$$\Gamma_2 + A_{21} = \frac{c}{2L}(1 - r_1 r_2) = \Gamma_{total}$$

$$\frac{d}{dt}(m_2 + q_\nu) = -\Gamma_{total}(m_2 + q_\nu) + P$$

solução particular $\left(\frac{d}{dt}(m_2 + q_\nu) = 0 \right)$

$$(m_2 + q_\nu) = \frac{P}{\Gamma_{total}}$$

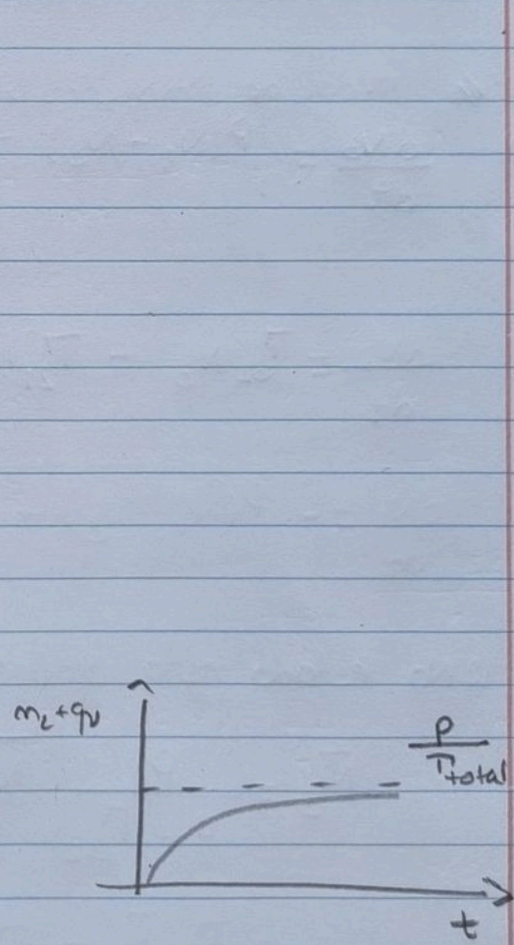
solução de equação homogénea

$$\frac{d}{dt}(m_2 + q_\nu) = -\Gamma_{total}(m_2 + q_\nu)$$

$$\frac{d}{dt}(m_2 + q_\nu) + \Gamma_{total}(m_2 + q_\nu) = 0$$

$$(m_2 + q_\nu) = A e^{-\Gamma_{total} t}$$

solução $(m_2 + q_\nu) = A e^{-\Gamma_{total} t} + \frac{P}{\Gamma_{total}}$



Condição

$t = 0 \Rightarrow m_2 + q_\nu = 0$

Então:

$$A + \frac{P}{\Gamma_{total}} = 0 \Leftrightarrow A = -\frac{P}{\Gamma_{total}}$$

$$\Rightarrow (m_2 + q_\nu) = \frac{P}{\Gamma_{total}}\left[1 - e^{-\Gamma_{total}}\right]$$

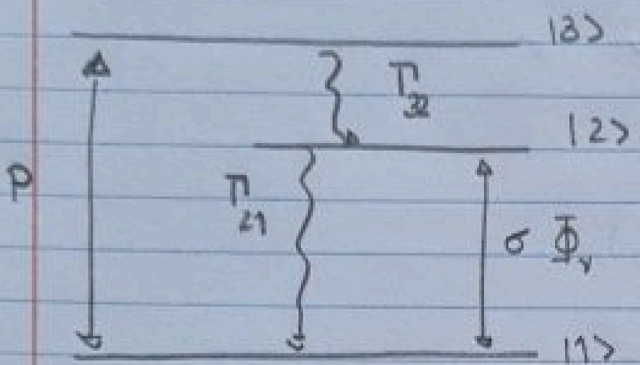
b)

caso geral e no estado estacionário

$$\frac{d}{dt}(\ ) = 0 \Leftrightarrow 0 = -(\Gamma_2 + A_{21})\overline{n_2} + P - \frac{c}{2L}(1 - r_1 r_2)\overline{q_\nu}$$

$$\overline{q_\nu} = \frac{P - (\Gamma_2 + A_{21})n_2}{\frac{c}{2L}}$$

4.3.



a) Perda de condições lineares

$$(3) \quad \frac{dN_3}{dt} = P(N_1 - N_3) - \Gamma_{32} N_3$$

$$\frac{d}{dt}(N_1 + N_2 + N_3) = 0$$

$$(2) \quad \frac{dN_2}{dt} = \Gamma_{32} N_3 - \Gamma_{21} N_2$$

$$N_T = N_1 + N_2 + N_3$$

$$(1) \quad \frac{dN_1}{dt} = -P(N_1 - N_3) + \Gamma_{21} N_2$$

No estado estacionário:

$$(3) \quad \overline{N}_3 = \frac{P}{P + \Gamma_{32}} \overline{N_1}$$

$$(3)+1) \quad 0 = -\Gamma_{32}\overline{N}_3 + \Gamma_{21} N_2 \Leftrightarrow \overline{N}_2 = \frac{\Gamma_{32}}{\Gamma_{21}} \overline{N}_3 = \frac{P\,\Gamma_{32}}{\Gamma_{21}(P + \Gamma_{32})} \overline{N}_1$$

$$N_T = \left(1 + \frac{P\,\Gamma_{32}}{\Gamma_{21}(P + \Gamma_{32})} + \frac{P}{P + \Gamma_{32}}\right) \overline{N_1}$$

$$N_T = \frac{T_{21}(P + T_{32}) + PT_{31} + PT_{21}}{T_{21}(P + T_{32})} \bar{N}_1$$

$$=$$

$$\bar{N}_1 = \frac{T_{21}\,(P + T_{32})}{T_{21}(2P + T_{32}) + PT_{31}} \bar{N}_T$$

$$\bar{N}_2 = \frac{T_{32}\,P}{T_{21}(2P + T_{32}) + T_{32}P} \bar{N}_T$$

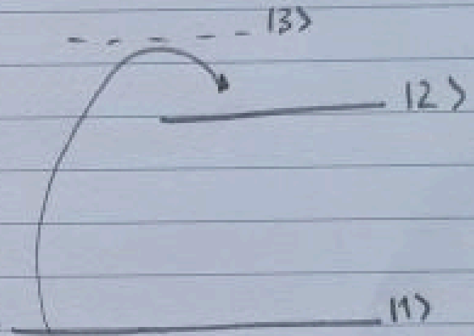
$$\bar{N}_3 = \frac{T_{21}\,P}{T_{21}(2P + T_{32}) + T_{32}P} \bar{N}_T$$

No limite favorável que $T_{32} \gg P, T_{21}$

$$\bar{N}_1 \rightarrow \frac{T_{21}\,\cancel{T_{32}}}{\cancel{T_{32}}(T_{21} + P)} \bar{N}_T$$

$$\bar{N}_2 \rightarrow \frac{P\,\cancel{T_{32}}}{\cancel{T_{32}}(T_{21} + P)} \bar{N}_T$$

$$\bar{N}_3 \rightarrow \frac{T_{21}\,P}{T_{32}(T_{21} + P)} \bar{N}_T \simeq 0$$



Eliminação adiabática da população N_3

$$\bar{N}_2 - \bar{N}_1 = \left(\frac{P - T_{21}}{P + T_{21}}\right) N_T$$

$P > T_{21}$

4.7.

Demora muito tempo, não vale a pena

4.8.

$$I_\nu^{sat} = \frac{h\nu}{2\sigma(\nu)} (P + \Gamma_{21})$$

Taxa de emissão estimulada

Taxa de decaimento

$$\underbrace{\frac{\sigma I_\nu}{h\nu}}_{\Phi_\nu} \xrightarrow{I_\nu = I_\nu^{sat}} \frac{\sigma}{\cancel{h\nu}} \frac{\cancel{h\nu}}{2\cancel{\sigma}} (P + \Gamma_{21})$$

$$\frac{P + \Gamma_{21}}{2}$$