Development of a single axis detumbling prototype for a 1U CubeSat

1. Introduction

CubeSat is a miniaturized version of a satellite that is composed of cube units (U) of 10 cm by 10 cm by 10 cm. Government Engineering College Barton Hill is developing this 1U CubeSat. Its mission is to serve as a platform for university students to learn to design and develop a 1U CubeSat bus capable of being reused in subsequent missions. A key system in this bus is the Attitude Determination and Control System (ADCS). The ADCS is responsible for determining and controlling a satellite's orientation in orbit. One of its first requirements is to reduce the rotation imparted by the CubeSat's deployer into a more stable motion, a pointing operation often referred to as detumble. This detumbling process can be managed either passively or actively. Passive control mechanisms are simple, often requiring no moving parts or power. For example, a passive control system could use permanent magnets or hysteresis rods to magnetically control orientation. This has several major drawbacks such as a limited attitude pointing accuracy of about +/- 10 degrees about Earth's magnetic field, constraining all other pointing requirements for power, communications and other sensors [1] . Another approach uses active control mechanisms like reaction wheels or magnetorquers to control the spacecraft's attitude and orientation. While somewhat more complicated, these systems allow the satellite to be more precisely controlled. This cubesat's planned ADCS will utilize an active control system using magnetorquers, also called torque rods, to manage detumble and some pointing requirements. This paper will lay out the design, assembly and testing of these torque rods.

2. Cubesat simulation

Before developing flight ready CubeSat, it is necessary to simulate the CubeSat in a virtual environment. Initially we had to develop an open source simulation script in Matlab for determining the global position, angular dynamics, detumbling effects etc which is catered to meet our requirements and specifications. The script was created based on "Space flight Mechanics" written by Dr. Carlos Jos´e Montalvo, University of South Alabama as well as his Matlab scripts from his official repository. The entire code is available on github at `https://github.com/NEONGASHMEN/arduinodemo1U`

2.0.1 Assumptions and reference values for the CubeSat

Orbital inclination = $98°$
Mass of CubSat = $2kg$
Mass moment of inertia, $M_{3\times3} = \begin{bmatrix} 0.006 & 0 & 0 \\ 0 & 0.006 & 0 \\ 0 & 0 & 0.006 \end{bmatrix}$

Maximum magnetic moment from Magnetorquer = $0.2Am^2$
Resolution of the Magnetorquer = 256 (8bit)

The magnetic field model chosen was IGRF. General gravitational model is formulated where the gravitational perturbations from celestial bodies other than earth is neglected.Atmospheric drag as well as solar pressure is neglected.

2.0.2 The control algorithm for the simulation

The first step in our workflow was to define an arbitary earth centered, non - rotating reference frame. CubeSat parameters shall be defined with respect to this reference frame. On this reference frame the Z-axis points towards the north pole and the X-axis points towards the equator through the Prime Meridian. The conversion between the aforementioned reference frame to the real world latitude and longitudes the equations given in 10.1 and 10.2 can be used.

The next step is to derive initial conditions for the CubeSat, referenced as initial state. Initial state is an array constituting of 13 elements.

$$S_i = [\ X_i \quad Y_i \quad Z_i \quad V_{xi} \quad V_{yi} \quad V_{zi} \quad q_{0i} \quad q_{1i} \quad q_{2i} \quad q_{3i} \quad w_{xi} \quad w_{yi} \quad w_{zi}\ ]$$

X_i, Y_i, Z_i are position coordinates given by (500,0,0) for 500km altitude at prime meridian

V_{xi}, V_{yi}, V_{zi} are velocity components with respect to body frame given by 10.3

$q_{0i}, q_{1i}, q_{2i}, q_{3i}$ are initial quarternions of CubeSat with respect to its body frame, initially taken as zero.

w_{xi}, w_{yi}, w_{zi} are initial angular velocities of CubeSat, initially taken as 10deg $/s$.

Runge Kutta 4 method of iteration 10.5 is used to find out consecutive states during the satellte's orbit. A partial differential eqution which yields the state parameters are fed to the RK-4 algorithm and iterated for consecutive timesteps to obtain the state parameters at consecutive timesteps. The PDE is of the form;

$$\frac{d[S]}{dt} = f([S], t) \quad \& \quad [S]_{t=0} = [S]_0$$

and $f()$ is given by,

$$\left(\frac{dX}{dt}, \frac{dY}{dt}, \frac{dZ}{dt}\right)_{t_i} = V_{t_{i-1}}$$

$$\left(\frac{d^2X}{dt^2}, \frac{d^2Y}{dt^2}, \frac{d^2Z}{dt^2}\right)_{t_i} = -\left(\frac{GM}{r^2}\right)\hat{r}$$

$$\left(\frac{dQ_0}{dt}, \frac{dQ_1}{dt}, \frac{dQ_2}{dt}, \frac{dQ_3}{dt}\right)_{t_i} = \omega_{t_{i-1}}$$

(Obtained from Euler angles to quarternions conversion 10.6)

$$\left(\frac{d^2\omega}{dt^2}\right) = I^{-1}(T_m)$$

(From rotational inertial equation 10.4)

Here the torque from magnetorquers T_m is computed from B-dot control algorithm.The algorithm takes in the current angular velocity of the CubeSat and the magnetic field around the satellite to determine the required magnetic moment that is to be produced by the Magnetorquer, inorder to achieve efficient detumbling. The B-dot algorithm states that the magnetic moment that is to be produced should be mututally perpendcular to the angular velocity vector ($\vec{\omega}$) and the Earth's magnetic field ($\vec{B}$). To emulate real world noissy sensors, a random noise generation is also hard coded in the script. This gives magnetic field (magnetometer sensor) with an error of few microteslas as well as angular velocity (gyro-error) with an error of few milli radians. This gave us the opportunity to implement a filtering algorithm which minimises the error. A modified Kalman filter is employed 10.7 - combining the current sensor data and previous sensor value, thereby smoothening any steep variations in the measured value. This corrected data is fed to the B-dot algorithm.

$$\vec{\mu} = k.\left(\vec{B} \times \vec{\omega}\right)$$

Here the control variable k can be adjusted as per the mission's requirements to fine tune the rate of detumbling. In our case k is adjusted to generate magnetic moments between -0.2 to $0.2Am^2$. In the script, the IGRF model is used to obtain the magnetic field at current location. The torque produced, in the magnetometer is given by,

$$\vec{T_m} = \vec{\mu} \times \vec{B}$$

Each consecutive states are stored in each iteration of timestep. For post processing, the numerical values are exported as excel and for visualisation, plotted in a graph.

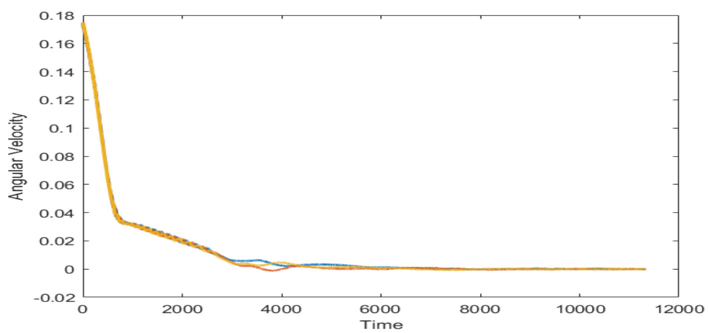
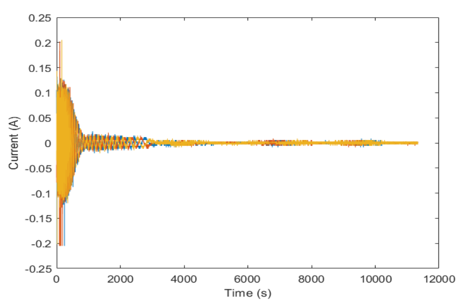
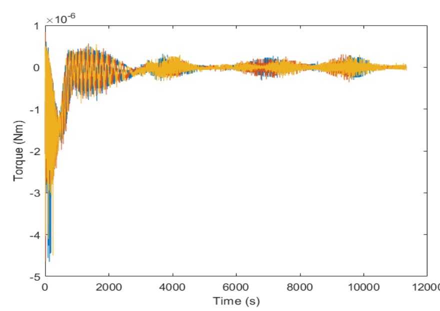


Figure 2.1: Decrease in Cubesat's angular velocity



(a) Current in magnetorquer coils



(b) Torque generated

From these results, the duration for detumbling from $\vec{\omega} = 10\hat{i} + 10\hat{j} + 10\hat{k}$ (in degrees) to $\vec{\omega} = 0.01\hat{i} + 0.013\hat{j} + 0.01\hat{k}$ was found to be 106.7667 mins (1.132 orbits) 4.3. Plot between Angular velocity of the CubeSat, Current in each magnetorquers and Torque produced from each torquers V/S time in seconds is given above 2.2a2.2b.

3. Designing and testing the magnetorquer using iron core

In this design we used iron as the core with diameter 5.5mm and number of turns 1088 and gauge 28SWG.

We tested it theoretically using the code written by ourself and practically using the magnetometer. Both the tests did not yield the desired result. So we decided to use Stainless steel FR430 instead of iron core and change the number of turns to 350.

3.0.1 Procedure

3.0.2 Inference

3.0.3 Result

4. Designing and testing of Helmholtz Coil

We constructed a Helmholtz coil inorder to test the detumbling system developed for our cubesat. Using Helmholtz coil a constant and uniform magnetic field is generated. Here we produced magnetic field greater than earth magnetic field to achieve detumbling at faster rate.

4.0.1 Objective

The magnetotorquer in the actual cubesat produces the opposite torque depending on the earth's magnetic field. Since earth's magnetic field is in the range of microteslas and hence it takes hours to stabilize the cubesat. But in the prototype of the cubesat we should demonstrate the detumbling process in a much lesser time. Inorder to achieve this task we provide a bigger magnetic field value in the order of milliteslas so the detumbling process occurs at a greater speed. The optimum value of magnetic moment was fixed as 1 millitesla keeping residual angular velocity low and achieving detumbling within few minutes. This is obtained by modifying the aforementioned CubeSat simulation script. We have to design a Helmholtz coil which generates the required magnetic field.

4.0.2 Theoretical Calculations

The formula used to calculate the theoretical value of magnetic field at the centre of two coils is given below

$$B = \frac{8}{5\sqrt{5}} \frac{\mu_0 N I}{R}$$

(Derived equation of magnetic field at centre of Helmholtz **??**)

Where n = number of turns I = current(measured using multi meter) R = radius of the Helmholtz coil.

For accomodating the CubeSat along with the airbearing inside the Helmholtz cage, we've alotted a gap of 26cm between the Coils. Therefore the radius of the Helmoltz coils should also be made to be equal to 26cm for maintaining uniform magnetic field at the midpoint of the two helmholtz cols. The cage needs to be fabricated as per the dimensions given below 4.1.

The magnetic field produced by the Helmholtz coil also depends upon number of turns and the current through the coil. The resistance per unit length depends upon

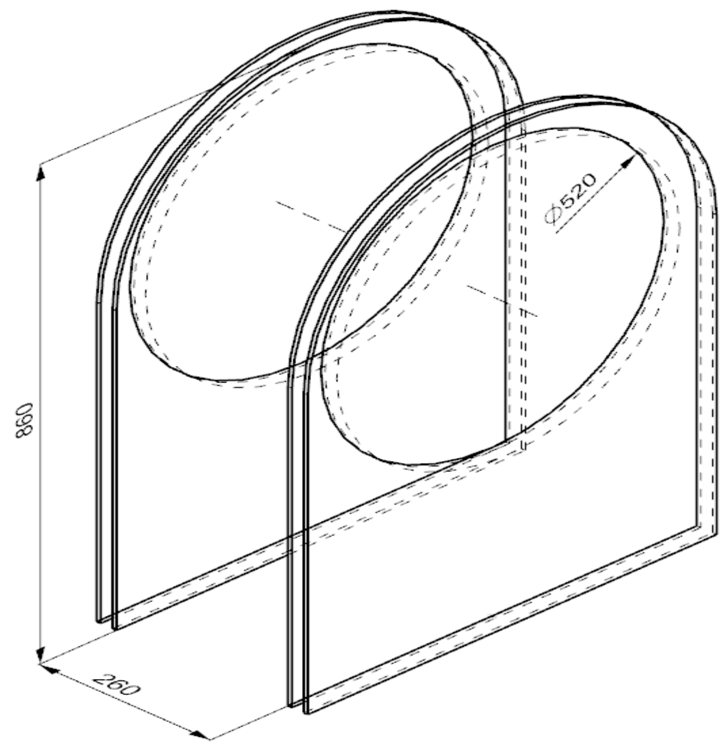


Figure 4.1: Model of the Helmholtz cage

the gauge of the wire used. So we have to fix the gauge of wire, value of current and number of turns in a perfect combination so as to achieve desired power draw. If we decrease the gauge of wire, thickness increases and we get less power draw but when thickness increases winding coil becomes difficult.We also limited the number of turns so that we are able to hand wind the coil.

To acheive this we wrote a script in Matlab that iterates between different guages of wire and different number of turns to find the current, power draw etc for each combination.

For 10SWG,
N = 50, I = 5.8A and P = 5.76W (obtained from 10.8)
N = 51, I = 5.7A and P = 5.66W
......
N = 200, I = 1.45A and P = 1.43W

For 11SWG,
N = 50, I = 5.8A and P = 7W
......
N = 200, I = 1.45A and P = 1.75W

For 12SWG,
N = 50, I = 5.8A and P = 8.7W
......
N = 200, I = 1.45A and P = 2.17W

and so on...

We found an optimum combination of 14SWG Copper wire with 150 turns and 1.966A 4.2 that produces the required magnetic field.

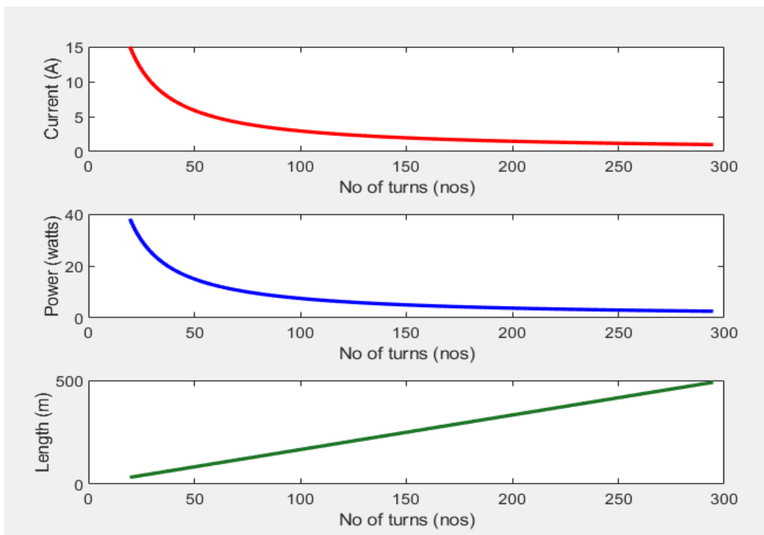


Figure 4.2: Current and Power draw for 14SWG wire

Matlabcode used for plots: github link- `https://github.com/NEONGASHMEN/arduinodemo1U/blob/main/helmholtz/hhz_vals.m`

4.0.3 Miniature model of Helmholtz coil

To check our theoretical assumptions before the fabrication of actual helmholtz coil we made a miniature model by 3D printing it with ABS(Acrylonitrile Butadiene Styrene) with diameter 10cm and winded with 28 AWG copper wire.

We tested it and proved that the theoretical value of the magnetic field at the centre of the two coils is equal to the practical value of magnetic field measured using EMF detector.

4.0.4 Fabrication of Helmholtz coil

A Plywood frame of aforementioned dimensions was fabricated using a woodcutter and adhesives. Plywood was choosed because of its relative permeability, which has the value close to that of air and also because it is easily available and is cheap. We wound the wire about 150 turns per coil around the plywood frameby hand. The Helmholtz coil is designed to produce 1mT magnetic field. A SMPS was used to control the voltage applied to the coil. Here we used a 12V SMPS according to our current requirement and for safety reasons.

We used a multimeter to measure the current flowing through the coil and measured the magnetic field using a magnetometer. The current was limited to 1.96A using a variable rheostat and the magnetic field was found to be xmT. The margin of error is x. The power draw was found out to be less than xW.

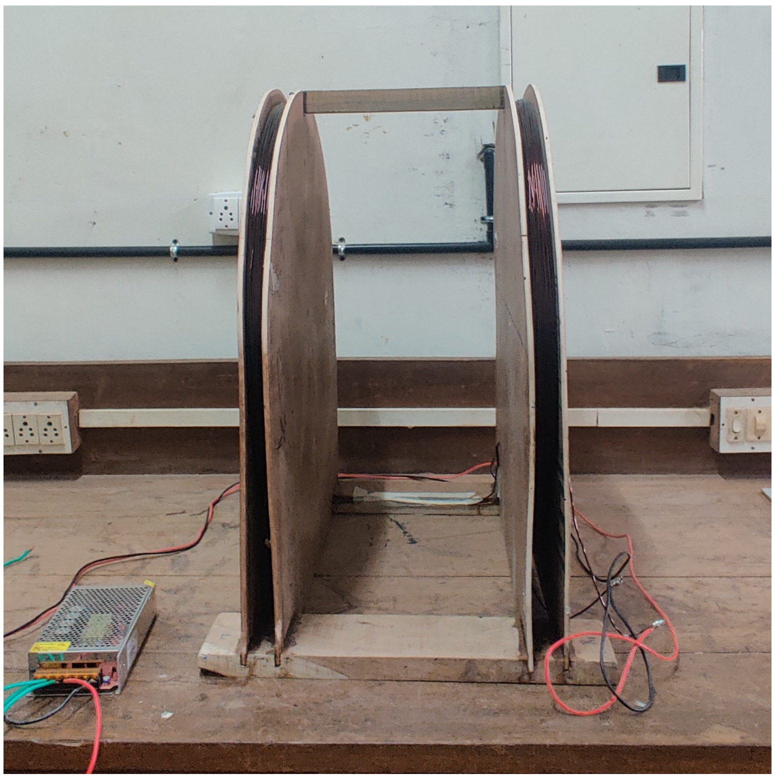


Figure 4.3: Helmholtz coil

5. Design and Development of Air-bearing

An airbearing is just a bearing in which moving surfaces are kept apart by a layer of air. Use of air as a medium creates a near friction less coupling between the mating surfaces. As the detumbling takes place at the orbit, we needed to have a frictionless environment to replicate the actual scenario. The objective was to create a frictionless free to rotate platform for the CubeSat to rest upon.

5.0.1 Design and Analysis

The main part of an airbearing is a porous material which is capable of producing uniform layer of air over which the corresponding mating part can be rested. The porous material selected for our purposes is Graphite (1.7g/cc density).

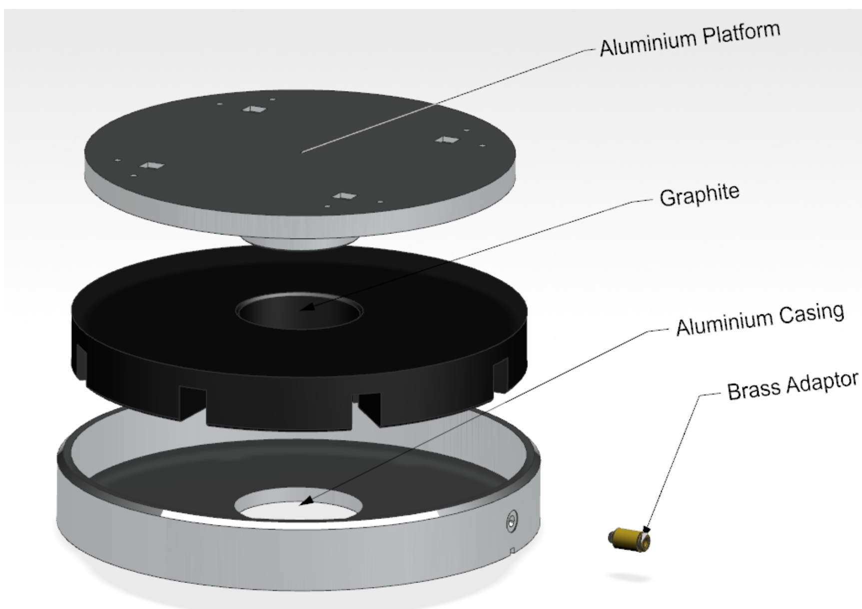


Figure 5.1: Air Bearing

The platform was made keeping in mind the base area of the CubeSat. A generous amount of 180mm diameter platform was designed for this case. The entire bearing was required to be low profile due to space constraints in the testing rig. A thickness of 25mm of graphite was first chosen to minimise the pressure drop through the graphite while maintaining sufficient stiffness. Air flow channels to allow the pumped air to

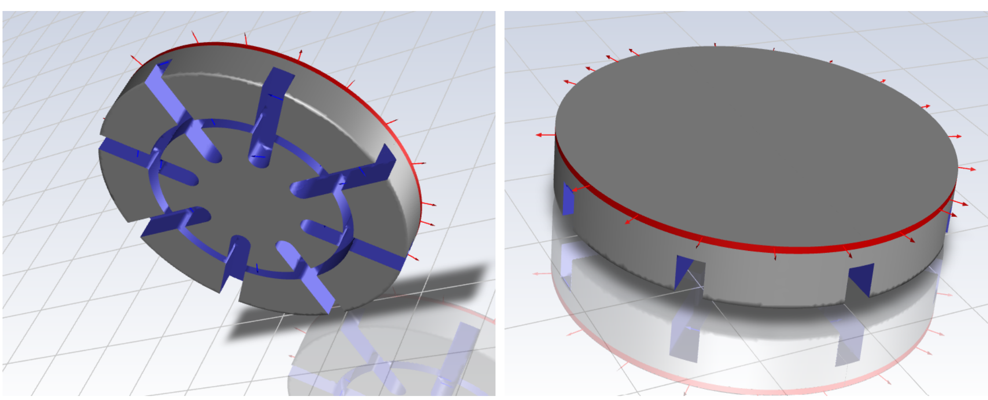
spread uniformly spread accross the entire graphite, is also designed. The performance of the designed graphite section was analysed on Ansys Fluent.
Fluid inlet is assigned as the air passages on the graphite. The air compressor that would be used alongside the air bearing was rated 24L capacity. Hence the flow rate from the compressor at 60psi was calculated as follows,

Cubic feet per metre, $CFM = \frac{V(P_f - P_i)}{14.7 t_{fill}}$

where, t_{fill} is the fill time to reach 60psi pressure and V is the tank capacity in cubic feet.

Hence the volume flow rate at compressor outlet was calculated to be $0.001088 m^3/s$
From the flow rate the velocity of flow was found and is set as the velocity inlet boundary condition (shown in blue color in 5.2a). The outlet is set to be ambient pressure condition (shown in red color in 5.2b). The porous zone of the graphite is set with the following parameters,

Flow permeability, $k = 1.85 \times 10^{-15}$ [10.9]
Viscous resistance, $R = 5.405 \times 10^{14}$
Mean diameter of graphite particle, $D_p = 3.16069 \times 10^{-6}$
Porosity of graphite, $\epsilon = 0.25$
Inertial loss coefficient, $C = 5.314 \times 10^{7}$ [10.10]



(a) Velocity inlet (b) Pressure outlet

A density based solver is employed and satidfactory convergence was obtained after about 1000 iterations. The static pressure contour plot of the airflow is shown below.

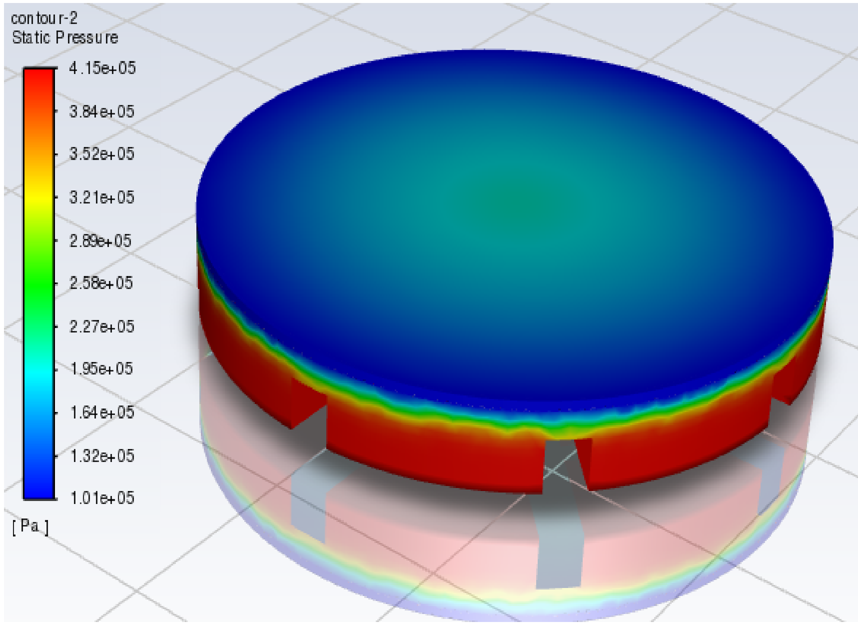


Figure 5.3: Pressure contour over the graphite layer

The results shows about 137684 Pa of static pressure on the bearing platform, which translates to 3924 N force. That is about 400 kg lift capacity. This is more than enough for our required application.

5.0.2 Fabrication

Casing and Platform

These were fabricated mainly by turning in lathe at 300RPM. A disk shaped Aluminium material of 210x50mm dimensions were turned using orthogonal cutting to yield the platform. A combination of orthogonal cutting, drilling and boring was done on a second piece of the same dimension to obtain the casing. The air inlet hole was machined using a vertical milling machine with a drill size of 8mm. The hole was later tapped with a thread of 1/8BSP, to fasten the tubing adaptor. The air vent groove was machined on the uderside of the casing using a 3mm end mill cutter.

Graphite

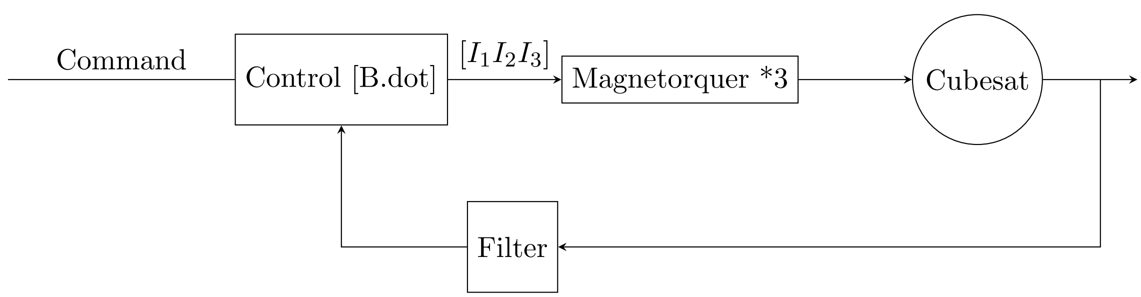
The air flow path on the graphite was machined using a CNC and later sanded manually to obtain better finishing.

The graphite was joined together with the casing using an airtight epoxy sealant. The platform is placed over the graphite to yield the finished bearing

6. Detumbling

The orbital injection of the cubesat is done with the help of a dispenser from the final stage of the rocket. The dispensing manoeuvre is net 100% inear and hence imparts angular momentum to the cubesat. For effective data transfer and data collection, the cubesat needs to be free of any angular momentum. Detumbling is the process of engaging attitude controllers on the cubesat to reduce its tumbling. The attitude control (actuation) system that we have is a magnetorquer board. The magnetorquer is a set of three solenoids that produces a sufficient magnetic field in mutually perpendicular directions, which interacts with the magnetic field of the earth to generate sufficient torque for the cubesat to detumble.

Effective detumbling can be done by energising the torquer rods in such a fashion as to maximise the torque generated at each instant. This is achieved through a control algorithm called "B.dot control". This algorithm computes the current that is required in each coil from the earth's magnetic field around the cubesat (measured from magnetometer) as well as the angular velocity components (measured by gyro of the cubesat.)



Here $[\omega_1, \omega_2, \omega_3]$ = Angular velocity about body axis of the cubesat
$[B_1, B_2, B_3]$ = Magnetic field vector about body axis of the cubesat
$[I_1, I_2, I_3]$ = Current in each magnetorquer coil

The sensor data ($[\omega]$ and [B]) obtained from the Gyro and Magnetometer will contain biases and hence is necessary to calibrate them. The filtering algorithm yields corrected values of the sensor data. The algorithm shall depend upon the sensors used in each one.

The b-dot algorithm relates the angular velocity and magnetic field to the magnetic moment (μ) as being orthogonal to each other.

ie,

$$\frac{\vec{\mu}}{|\mu|} = \frac{\vec{\omega} \times \vec{B}}{|\vec{\omega} \times \vec{B}|}$$

Using a control parameter k

$$\vec{\mu} = k \cdot [\ \frac{\vec{\omega} \times \vec{B}}{|\vec{\omega} \times \vec{B}|}]$$

$$\vec{\mu} = [mu_1\mu_2\mu_3] = [N_1I_1A_1N_2I_2A_2N_3I_3A_3]$$

where N_1, N_2, N_3 are number of turns on each coil
A_1, A_2, A_3 are area of cross section of each coil

Hence we will obtain $[I_1I_2I_3]$.Three PWM pinouts from the magnetorquer control IC generate sufficient voltage to energise the corresponding coils with I_1, I_2, I_3

7. Attitude Determination

This is the process of computing the relative orientation of the satellite (body frame) with respect to an inertial reference frame(Earth centred Inertial reference frame).This is done by analysing the position of celestial entities like sun star constellations. Attitude determination can be computed using several algorithms -among this ,TRIAD method is the most common and easiest algorithm.

For the proposed cubesat design we use sun sensors as well as magnetometers. From the sun sensor, we get the direction vector of the sun with respect to the body frame of cubesat ($\hat{S_B}$). And from the magnetorquer we get direction vector of the earth's magnetic field with respect to the body frame of the cubesat($\hat{\mu_B}$)

The relative position of the sun and the direction vector of the magnetic field with respect to the Earth centred Inertial frame is known to us a($\hat{S_B}\&\hat{\mu_B}$) using geographical models like the IGRF model.

The body vectors are related to the inertial vectors as follows

$$\hat{S_B} = [BN]\hat{S_B}$$

$$\hat{\mu_B} = [BN]\hat{\mu_B}$$

[BN] is the transformation matrix that needs to be computed which relates the body frame with the inertial frame. The TRIAD method uses a third frame $(\hat{t_1}, \hat{t_2}, \hat{t_3})$ where

$$t_{1B} = \hat{S_B}$$

$$t_{1N} = \hat{S_N}$$

$$\hat{t_{2B}} = \frac{\hat{S_B} \times \hat{\mu_B}}{|\hat{S_B} \times \hat{\mu_B}|}$$

$$\hat{t_{2N}} = \frac{\hat{S_N} \times \hat{\mu_N}}{|\hat{S_N} \times \hat{\mu_N}|}$$

$$\hat{t_{3B}} = \hat{t_{1B}} \times \hat{t_{2B}}$$

$$\hat{t_{3N}} = \hat{t_{1N}} \times \hat{t_{2N}}$$

$$[BT] = [\hat{t_{1B}}\hat{t_{2B}}\hat{t_{3B}}]$$

$$[NT] = [\hat{t_{1N}}\hat{t_{2N}}\hat{t_{3N}}]$$

$$[BN] = [BN][NT]^T$$

[BN] is the required transformation matrix for conversion between inertial frame to body frame.

8. Attitude Control

Currently the team is working on development of an attitude control system using magnetorquer. The magnetorquer rods are activated in such a way that we can produce the required torque.

Torque generated by a single magnetorquer rod is given by:

$$\vec{T} = \vec{\mu} \times \vec{B}$$

Where μ is the magnetic moment of the torquer given by

$$\mu =$$

And B is the magnetic field of earth in body frame given by the magnetorquer

If $\hat{r_N}$ is the required direction vector that the cubesat needs to be pointed in the inertial frame,

$$\hat{r_B} = [BN]\hat{r_N}$$

$$\hat{T} = k(\hat{r_B} - \hat{p_B})$$

where $\hat{p_B}$ is the direction vector of the cubesat with resopect to the body frame and k is a control parameter

9. Sensors for Attitude Determination

The attitude of the satellite is measured and estimated in order to control it and a sufficient amount of sensors has to be implemented to get the necessary accuracy on the estimated attitude. The main sensors used for attitude determination are star trackers, horizon sensors, sun sensors, GPS antenna arrays, magnetometers and angular rate gyroscopes.We use Magnetometer, Sun sensors and Gyroscopes for attitude determination.

9.1 Magnetometer

A magnetometer is a scientific instrument used to measure the strength and direction of the magnetic field in the vicinity of the instrument. The magnetometer, also known as a magnetic sensor, is an important sensor component in all types of aircraft and spacecraft. In the field of aeronautics, the magnetometer can be used to measure the geomagnetic field vector information of the position of the aircraft body, such as airplanes and satellites. And, according to the reference model for the Earth's magnetic field and local magnetic field, the angle information of a certain precision can be obtained through an algorithm, therefore, the magnetometer is widely used in aircraft attitude determination systems, especially in microsatellites, such as nanosatellites and picosatellites, etc.

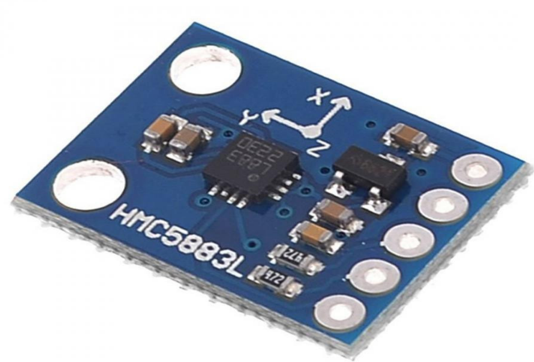


Figure 9.1: Magnetometer

9.1.1 Working Principle

Earth's magnetic field is present in space which points towards the magnetic north as shown in below image. Current carrying conductor also generates a magnetic field around itself. Hence, whenever a current carrying conductor is placed in space, it experiences the effect of the earth's magnetic field affecting the flow of the electrons through that conductor. These changes in the flow of the electrons are used for identifying the heading or direction of the magnetic field. This is the basic working principle of the magnetometer.

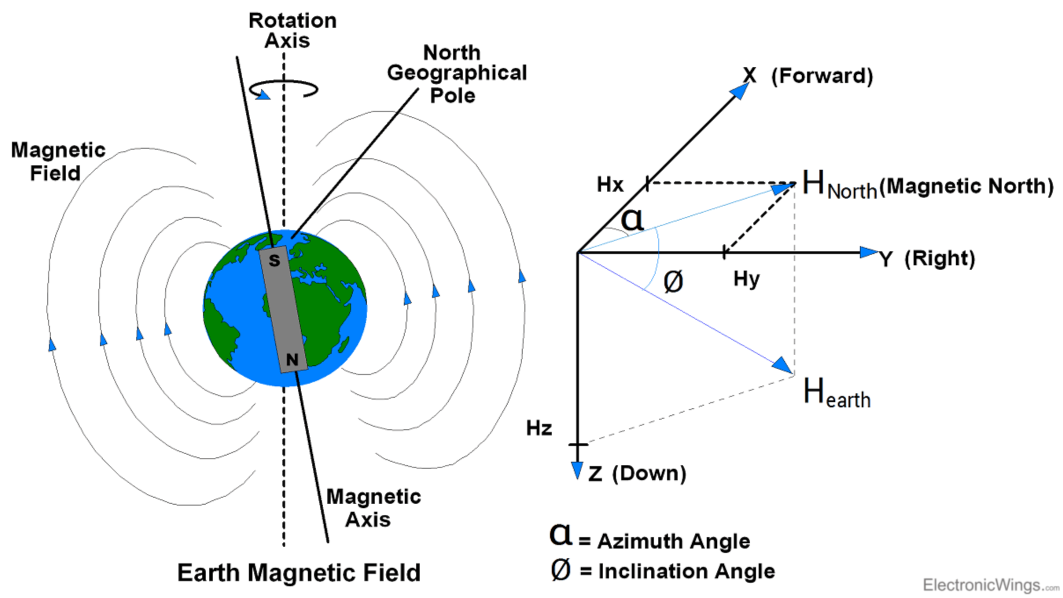


Figure 9.2: Earth's Magnetic field

9.2 Sun sensor

A sun sensor is a navigational instrument used by spacecraft to detect the position of the sun. Sun sensors are used for attitude control, solar array pointing, gyro updating, and fail-safe recovery. Sun sensors are widely used in spacecraft attitude determination systems for measuring the sun vector in spacecraft coordinates. Sun sensor determines a spacecraft's orientation with respect to the sun.

9.2.1 Working Principle

The sun sensor is operated based on the entry of light into a thin slit on top of a rectangular chamber whose bottom part is lined with a group of light-sensitive cells. The chamber casts an image of a thin line on the chamber bottom. The cells at the

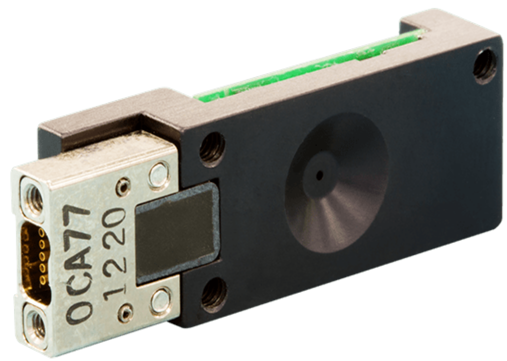


Figure 9.3: Sun sensor

bottom measure the distance of the image from a centerline and determine the refraction angle by using the chamber height. The cells are operated based on the photoelectric effect. They convert the incoming photons into electrons and hence voltages which are in turn converted into a digital signal. When two sensors are placed perpendicular to each other, the direction of the sun with reference to the sensor axes can be calculated.

The solar cells provide a current output depending on the angle made by the sensor normal with the sun vector based on cosine law,

$$I = I_0 cos(\theta)$$

where I_0 is the intensity at zero angle.

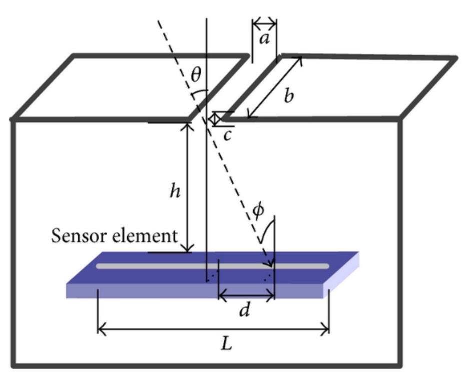
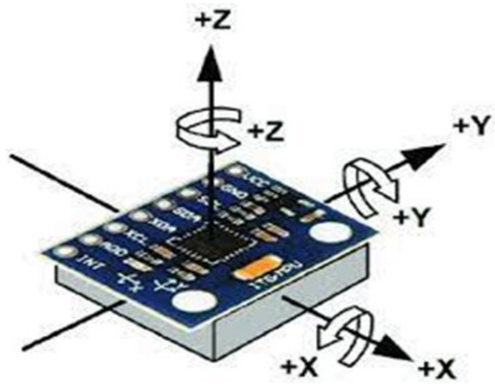


Figure 9.4: Sun sensor working principle

9.3 Gyroscope

A gyroscope is a device used for measuring or maintaining orientation and angular velocity. It is a spinning wheel or disc in which the axis of rotation (spin axis) is free to assume any orientation by itself. When rotating, the orientation of this axis is unaffected by tilting or rotation of the mounting, according to the conservation of angular momentum.

(a) Gyroscope



(b) MPU 6050

9.3.1 Working Principle

Depending on the direction there are three types of angular rate measurements. Yaw- the horizontal rotation on a flat surface when seen the object from above, Pitch- Vertical rotation as seen the object from front, Roll- the horizontal rotation when seen the object from front. The concept of Coriolis force is used in Gyroscope sensors. In this sensor to measure the angular rate, the rotation rate of the sensor is converted into an electrical signal. This sensor consists of an internal vibrating element made up of crystal material in the shape of a double – T- structure. This structure comprises a stationary part in the centre with 'Sensing Arm' attached to it and 'Drive Arm' on both sides. This double-T-structure is symmetrical. When an alternating vibration electrical field is applied to the drive arms, continuous lateral vibrations are produced. As Drive arms are symmetrical, when one arm moves to left the other moves to the right, thus cancelling out the leaking vibrations. This keeps the stationary part at the centre and sensing arm remains static. When the external rotational force is applied to the sensor vertical vibrations are caused on Drive arms. This leads to the vibration of the Drive arms in the upward and downward directions due to which a rotational force acts on the stationary part in the centre. Rotation of the stationary part leads to the vertical vibrations in sensing arms. These vibrations caused in the sensing arm are measured as a change in electrical charge. This change is used to measure the external rotational force applied to the sensor as Angular rotation. Besides sensing the angular velocity, Gyroscope sensors can also measure the motion of the object.

10. Equations

$$\left.\begin{array}{l}\rho = \sqrt{x^2+y^2+z^2}\\ \theta_E = \cos^{-1}\left(\frac{z}{\rho}\right)\\ \psi_E = \tan^{-1}\left(\frac{y}{x}\right)\end{array}\right\}\text{Cartesian to spherical coordinate conversion} \tag{10.1}$$

$$\left.\begin{array}{l}\lambda_{LAT} = 90-\theta_E\frac{180}{\pi}\\ \lambda_{LON} = \psi_E\frac{180}{\pi}\\ h = \rho - R_E\end{array}\right\}\text{Latitude, Longitude and height} \tag{10.2}$$

$$v = \sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)}, \qquad \text{Vis-viva equation, NB: for the simulation } a \approx r \tag{10.3}$$

$$\dot{\vec{\omega}} = I^{-1}\left(T_{Propulsion} + T_{Magnetorquer} + T_{Reactionwheel} - \dot{\vec{\omega}} \times \vec{H} - \dot{I}\vec{\omega}\right) \tag{10.4}$$

$$\left.\begin{array}{l}\text{If, } \frac{dy}{dx} = f\left(t_n, y_n\right) \text{ and } y\left(t_n\right) = y_n\\ k_1 = f\left(t_n, y_n\right)\\ k_2 = f\left(t_n+\frac{h}{2}, y_n + h\frac{k_1}{2}\right)\\ k_3 = f\left(t_n+\frac{h}{2}, y_n + h\frac{k_2}{2}\right)\\ k_4 = f\left(t_n+h, y_n+hk_3\right)\\ k = \frac{1}{6}\left(k_1+2k_2+2k_3+k_4\right)\\ y\left(t_n+h\right) = y_n + hk\end{array}\right\}\quad\text{RK4} \tag{10.5}$$

$$\left.\begin{bmatrix}q_0\\q_1\\q_2\\q_3\end{bmatrix} = \begin{bmatrix}\cos\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right)+\sin\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right)\\ \sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right)-\cos\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right)\\ \cos\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right)+\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right)\\ \cos\left(\frac{\phi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\psi}{2}\right)-\sin\left(\frac{\phi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\psi}{2}\right)\end{bmatrix}\right\}\text{Euler to Quarternion conversion} \tag{10.6}$$

$$X_{filtered} = S.X_{current} + (1-S)\,X_{previous} \quad ,0 < S < 1,\text{S being the sensor reliabilty} \tag{10.7}$$

$$\left.\begin{aligned}
&\left(\tfrac{R}{2}\right) = 2B_1\left(\tfrac{R}{2}\right)\\
&= \frac{2\mu_0 nIR^2}{2\left(R^2+\left(\frac{R}{2}\right)^2\right)^{\frac{3}{2}}}\\
&= \frac{\mu_0 nIR^2}{\left(R^2+\left(\frac{R}{2}\right)^2\right)^{\frac{3}{2}}}\\
&= \frac{\mu_0 nIR^2}{\left(R^2+\frac{1}{4}R^2\right)^{\frac{3}{2}}} = \frac{\mu_0 nIR^2}{\left(\frac{5}{4}R^2\right)^{\frac{3}{2}}}\\
&= \left(\tfrac{4}{5}\right)^{\frac{3}{2}}\frac{\mu_0 NI}{R}\\
&= \frac{8}{5\sqrt{5}}\frac{\mu_0 NI}{R}
\end{aligned}\right\}\quad \text{magnetic field at centre of Helmholtz} \tag{10.8}$$

$$k = \left(\frac{D_p^2}{150}\right)\left(\frac{\epsilon^3}{(1-\epsilon)^2}\right), \qquad \text{Permeability - Porosity relation} \tag{10.9}$$

$$C = \left(\frac{3.5}{D_p}\right)\frac{1-\epsilon}{\epsilon^3}, \qquad \text{Medium porosity - Inertial loss coefficient relation} \tag{10.10}$$