KEB-45250 Numerical Techniques for Process Modeling
Exercise 4 - 1D Heat Conduction
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Introduction

In today's exercise we will study one-dimensional heat conduction. First steady state and then unsteady. You may program your solutions in any language you like, but the examples are in Python.

Problem 1

Part a

Consider a source-free heat conduction in a insulated rod. The ends are maintained at constant temperatures of 100 °C and 500 °C, respectively. See problem sketch in Fig. 1.

Thermal conductivity $k = 1000\,\mathrm{W/mK}$, cross-sectional area $A = 10 \times 10^{-3}\,\mathrm{m}^2$.

Solve with one-dimensional Finite Volume Method (FVM) and compare results with analytical solution. Use 5 control volumes. Feel free to use Example 4.1 from Versteeg and Malalasekera or course notes for verification.

Part b

After verifying your code for heat conduction and boundary conditions in part a, add uniform heat generation $q = 5 \times 10^6\,\mathrm{W/m}^3$.

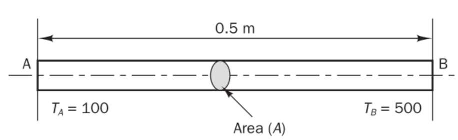

Figure 1: Problem sketch

Validate your code with analytical solution

$$T = \left[\frac{T_B - T_A}{L} + \frac{q}{2k}(L - x)\right] x + T_A \tag{1}$$

It may also be a good idea to change your input values to match those in Example 4.2 from Versteeg and Malalasekera to be able to check the matrix coefficients as you write them.

Part c *(plan only)*

How to would you add spatially varying heat conductivity

	node 1	node 2	node 3	node 4	node 5
k W/mK	10	300	500	200	1000

to you solver. How to verify/validate your code now?

There is probably no time to program your changes, just plan them.

Part d *(plan only)*

Let our medium be air at $p = 1$bar. Use temperature dependent fluid properties. Use $q = 100\,\mathrm{W/m^3}$.

There is probably no time to program your changes, just plan them.

Problem 2

One dimensional unsteady conduction. Save your code from problem 1 and make a copy to modify here.

Initial temperature $T_0 = 200\,°\mathrm{C}$.

East side temperature $T_B = 0\,°\mathrm{C}$.

Plate thickness $L = 2\,\mathrm{cm}$.

Thermal conductivity $k = 10\,\mathrm{W/mK}$.

Product of density and heat capasity $\rho c = 10 \times 10^6\ \mathrm{J/m^3K}$.

Use fully implicit scheme. Feel free to use Example 8.1 from Versteeg and Malalasekera.

Problem 3

What are the advantages/disadvantages of a numerical solution to one-dimensional heat conduction?

Suggested hobby projects

Expand your code to 2D. The process is the same as for 1D but requires more programming.