KEB-45250 Numerical Techniques for Process Modeling
Exercise 3 - CFD case study and time stepping
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Introduction

Today we will have two exercises. First one is a case study for a boiler super heater. The second one is a numerical solution of water freezing in a pipe during winter.

1 Super heater

The purpose of this exercise is to initiate discussion and thinking. It does not have a single right answer. It's recommended to form small groups and ask a lot of questions. We will use 20-30min for the total discussion.

Super heaters are used in many boilers. The super heater is located in the red area in Fig. 1. The assignment is to first calculate the heat transfer for a super heater and compare the results to measured data. The second part is to optimize the super heater geometry.

The dimensions of the super heater are $L_x = 2.5m$, $L_y = 15m$, $L_z = 20m$. The pipe diameter is $d = 5cm$. In flow direction there are $n_x = 30$ pipes in a row and in cross-flow direction there are $n_z = 150$ pipes. The pipes are arranged "in-line", see Fig. 2. There are five super heaters in a series and we are calculating the second one. The inlet velocity for the super heater is $V = 7m/s$ and temperature is $T_{in} = 900C$. The chemical composition of the gas is known.

There exists correlations for a tube bundle similar to that used in a super heater, See Fig. 3.

Discuss where the correlation based approach would work in our case and where CFD would be better. How would you combine the two approaches? What kind of CFD simulation would you do? The whole boiler or maybe just a couple of pipes? Consider the computational cost and the feasibility of optimization.

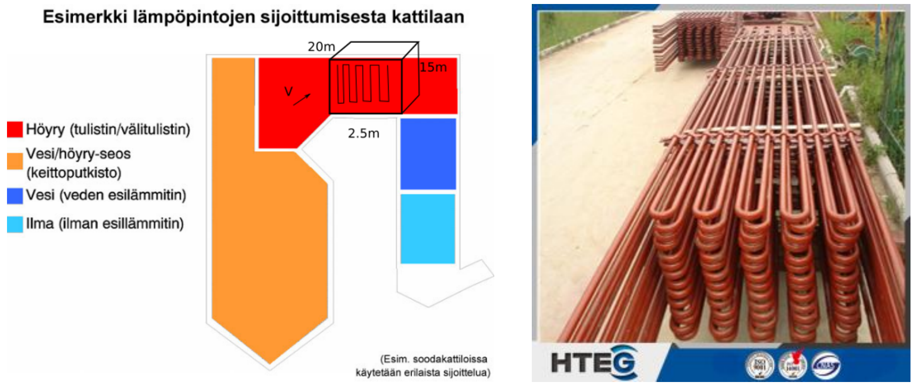

Figure 1: Boiler (kattila) and super heater (tulistin)

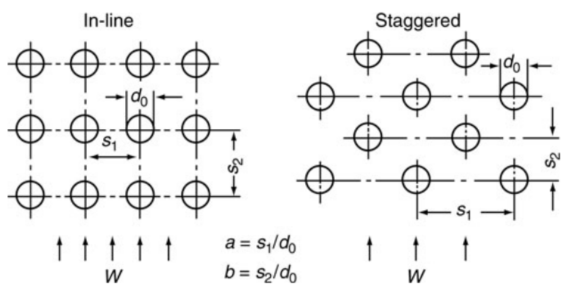

Figure 2: Pipe arrangements

$$\mathrm{Nu}_{0,\mathrm{bundle}} = f_\mathrm{A} \mathrm{Nu}_{l,0} \tag{8}$$

The void fraction and the arrangement factor f_A depend on the transverse pitch ratio $a = s_1/d_\mathrm{o}$ and the longitudinal pitch ratio $b = s_2/d_\mathrm{o}$ in the tube bundle.

The determination of a and b for various arrangements of the tube bundle is illustrated in Fig. 2. The void fraction is given by

$$\psi = 1 - \frac{\pi}{4\,a} \qquad \text{for } b \geq 1 \tag{9}$$

$$\psi = 1 - \frac{\pi}{4\,a\,b} \qquad \text{for } b < 1 \tag{10}$$

According to Eq. (3) in Chap. G6, $\mathrm{Nu}_{l,0}$ is given by

$$\mathrm{Nu}_{l,0} = 0.3 + \sqrt{\mathrm{Nu}^2_{l,\mathrm{lam}} + \mathrm{Nu}^2_{l,\mathrm{turb}}} \tag{11}$$

where

$$\mathrm{Nu}_{l,\mathrm{lam}} = 0.664\,\sqrt{\mathrm{Re}_{\psi,l}}\,\sqrt[3]{\mathrm{Pr}} \tag{12}$$

$$\mathrm{Nu}_{l,\mathrm{turb}} = \frac{0.037\ \mathrm{Re}_{\psi,l}^{0.8}\ \mathrm{Pr}}{1 + 2.443\ \mathrm{Re}_{\psi,l}^{-0.1}\ (\mathrm{Pr}^{2/3} - 1)} \tag{13}$$

$$\mathrm{Nu}_{0,\mathrm{bundle}} = \frac{\alpha\ l}{\lambda} \tag{14}$$

$$\mathrm{Re}_{\psi,l} = \frac{w\ l}{\psi\ \nu} \qquad 10 < \mathrm{Re}_{\psi,l} < 10^6 \tag{15}$$

$$\mathrm{Pr} = \frac{\nu}{a} \qquad 0.6 < \mathrm{Pr} < 10^3 \tag{16}$$

$l = (\pi/2)d_0$ is the streamed length of a single tube, w is the velocity of the flowing medium in the free cross section outside the bundle, and $T_\mathrm{m} = (T_\mathrm{i} + T_\mathrm{o})/2$ is the mean temperature of the flowing medium at which the physical properties are evaluated.

The factor f_A for in-line tube arrangement is given by

$$f_{\mathrm{A,in\text{-}line}} = 1 + \frac{0.7\ (b/a - 0.3)}{\psi^{1.5}(b/a + 0.7)^2}, \tag{17}$$

where ψ is given by Eq. (9).

Figure 3: Correlations from VDI Heat atlas p.725

2 Time stepping

In this exercise we implement explicit Euler time integration method. Our example case studies the time it takes for a horizontal water pipe to reach temperature $T = 0C$ if exposed to subzero environment.

Use explicit time stepping

$$f(t_0 + \Delta t) = f(t_0) + \Delta t f'(t_0) \tag{1}$$

For horizontal pipe natural convection

$$\overline{Nu} = 0.36 + \frac{0.518\mathrm{Ra}_\mathrm{D}^{1/4}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{4/9}}$$

$$\mathrm{Ra}_\mathrm{D} = \frac{\beta \Delta T g D^3}{\nu^2}\mathrm{Pr} \tag{2}$$

$$\beta = \frac{1}{T} \tag{3}$$

Ignore the heat conduction in pipe and convection inside the pipe. Assume a bulk temperature for water.

Take pipe diameter $D = 3cm$, water initial temperature $T_{init} = 4C$, outside temperature $T_\infty = -10C$.

For a control volume we have

$$\rho A_{pipe} \mathrm{d}x c_p \frac{\mathrm{d}T}{\mathrm{dt}} = hPdx(T_w - T_\infty) \tag{4}$$

$$\Leftrightarrow \rho A_{pipe} c_p \frac{\mathrm{d}T}{\mathrm{dt}} = hP(T_w - T_\infty) \tag{5}$$

Derive numerical solution using explicit time stepping. Program you own solver.

If you have time, try implementing a implicit solver.

Figures from

- http://www.knowenergy.net/suomi/monipoltt_kattilat/7_lammonsiirtimet/frame.htm
- Korkealaatuinen lämmönvaihdin Superheater for Power Plant Boiler, HTEG, http://fi.hengtaointernational.com/superheater/superior-quality-heat-exchanger-superheater.html