KEB-45250 Numerical Techniques for Process Modeling
Exercise 4 - 1D Heat Conduction
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Introduction

Today we will study one-dimensional heat conduction. All the problems are very similar and much of the same code and equations can be used.

Progress at your own speed and try to finish problems 1-3.

The extra problems add more complex properties to the solutions. If you have time, try these also.

The hobby project is time consuming but very educational.

Problem 1

Consider a source-free heat conduction in a insulated rod. The ends are maintained at constant temperatures of 100 °C and 500 °C, respectively. See Fig. 1.

Thermal conductivity is $k = 1000\,\mathrm{W/mK}$, cross-sectional area is $A = 10 \times 10^{-3}\,\mathrm{m}^2$.

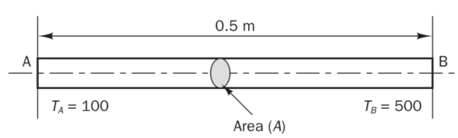

Figure 1: Problem sketch

Using Finite Volume Method with 5 control volumes

- derive the equations
- solve the problem
- compare the results with analytical solution

Feel free to use Example 4.1 from Versteeg and Malalasekera or course notes for reference.

Problem 2

After verifying your code in Problem 1, add uniform heat generation $q = 5 \times 10^6\,\mathrm{W/m^3}$.

Try to reuse you previous equations and code as much as possible.

Validate your code with analytical solution

$$T = \left[\frac{T_B - T_A}{L} + \frac{q}{2k}(L - x)\right] x + T_A \tag{1}$$

It may also be a good idea to change your input values to match those in Example 4.2 from Versteeg and Malalasekera to be able to check the matrix coefficients as you write the code.

Problem 3

Similarly to Problem 1, we have an insulated 1D rod. This time the left boundary is insulated and right boundary has a constant temperature. We want to solve the transient heat conduction.

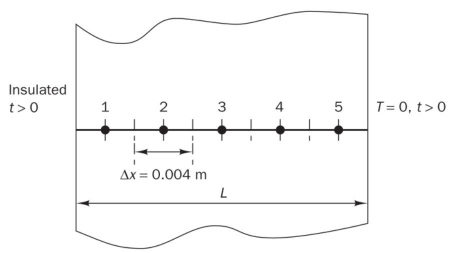

Figure 2: Problem sketch

Using a fully implicit scheme, derive the equations and solve the problem using FVM with 5 cells.

- Initial temperature $T_0 = 200\,°\mathrm{C}$.
- Right boundary temperature $T_{|x=L} = 0\,°\mathrm{C}$.
- Left boundary is insulated, $\frac{\mathrm{dT}}{\mathrm{d}x}_{|x=0} = 0$
- Rod length $L = 2\,\mathrm{cm}$.
- Thermal conductivity $k = 10\,\mathrm{W/mK}$.

- Product of density and heat capacity $\rho c = 10 \times 10^6$ J/m^{3}K
- Plot the results at times 40s, 80s, and 120s.

Feel free to use examples 8.1 and 8.2 from Versteeg and Malalasekera. Note that example 8.1 is for explicit time stepping.

Extra 1

How to would you add spatially varying heat conductivity

	node 1	node 2	node 3	node 4	node 5
k W/mK	10	300	500	200	1000

to you solver in Problem 2? How to verify/validate your code?

Extra 2

How would you add temperature dependent fluid properties in your code in Problem 2?

Let our medium be air at $p = 1$bar. Use $q = 100\,\mathrm{W/m^3}$.

Extra 3

What are the advantages/disadvantages of a numerical solution to one-dimensional heat conduction?

Suggested hobby project

Expand your code to 2D. The process is the same as for 1D, but requires more programming.