KEB-45250 Numerical Techniques for Process Modeling
Exercise 5 - 1D Heat Conduction and Advection
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Introduction

Today we will study one-dimensional heat conduction and advection.

Problems 1 and 2 share most of the code. The template code for this exercise is essentially copy-pasted from the previous exercise. Now we just add the advection to conduction.

If you have extra time and did not finish the extra problems from exercise 1, try them now. Or you may try the extra problems from this exercise.

The hobby project is time consuming, but very educational.

Problem 1

Consider a one dimensional source-free heat conduction and advection. The ends are maintained at constant temperatures of 10 °C and 5 °C, respectively. See Fig. 1.

Thermal conductivity is $k = 0.1\,\mathrm{W/mK}$, product of density and heat capacity is $c\rho = 1\,\mathrm{J/m^3K}$, and length $L = 1\,\mathrm{m}$.

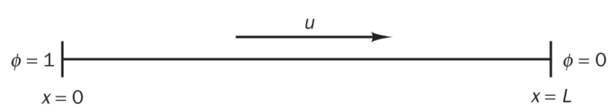

Figure 1: Problem sketch

Using Finite Volume Method and central differencing (linear interpolation) for advection term

- Grab a pencil and paper and derive the equations.
- Derive Peclet number boundedness criteria for the problem.

- solve the problem using

  a) $n = 5,\ u = 0.1\,\mathrm{m/s}$

  b) $n = 5,\ u = 1\,\mathrm{m/s}$

  c) $n = 5,\ u = 2.5\,\mathrm{m/s}$

  d) $n = 20,\ u = 2.5\,\mathrm{m/s}$

  e) $n = 5,\ u = -0.1\,\mathrm{m/s}$

  f) $n = 5,\ u = -1\,\mathrm{m/s}$

  g) $n = 5,\ u = -2.5\,\mathrm{m/s}$

  h) $n = 20,\ u = -2.5\,\mathrm{m/s}$

  where n is the number of cells and u is velocity.

- Calculate Peclet number for each case. How does Peclet number and your results correlate? Why?

- compare the results with analytical solution

$$\frac{T - T_0}{T_L - T_0} = \frac{\exp(c\rho u x/k) - 1}{\exp(c\rho u L/k) - 1} \tag{1}$$

Feel free to use Examples 5.1 and 5.2 from Versteeg and Malalasekera or course notes for reference. Note that we use non-zero boundary values here. Zero boundary values may easily mask a error in code as the contribution often becomes zero.

Problem 2

Repeat problem 1 using upwind scheme for advection.

Extra 1

Add a source term to you solver.

Extra 2

How would you implement a zero gradient boundary condition?

Extra 3

Make a transient case solver.

Suggested hobby project

Expand your code to 2D. The process is the same as for 1D, but requires more programming.