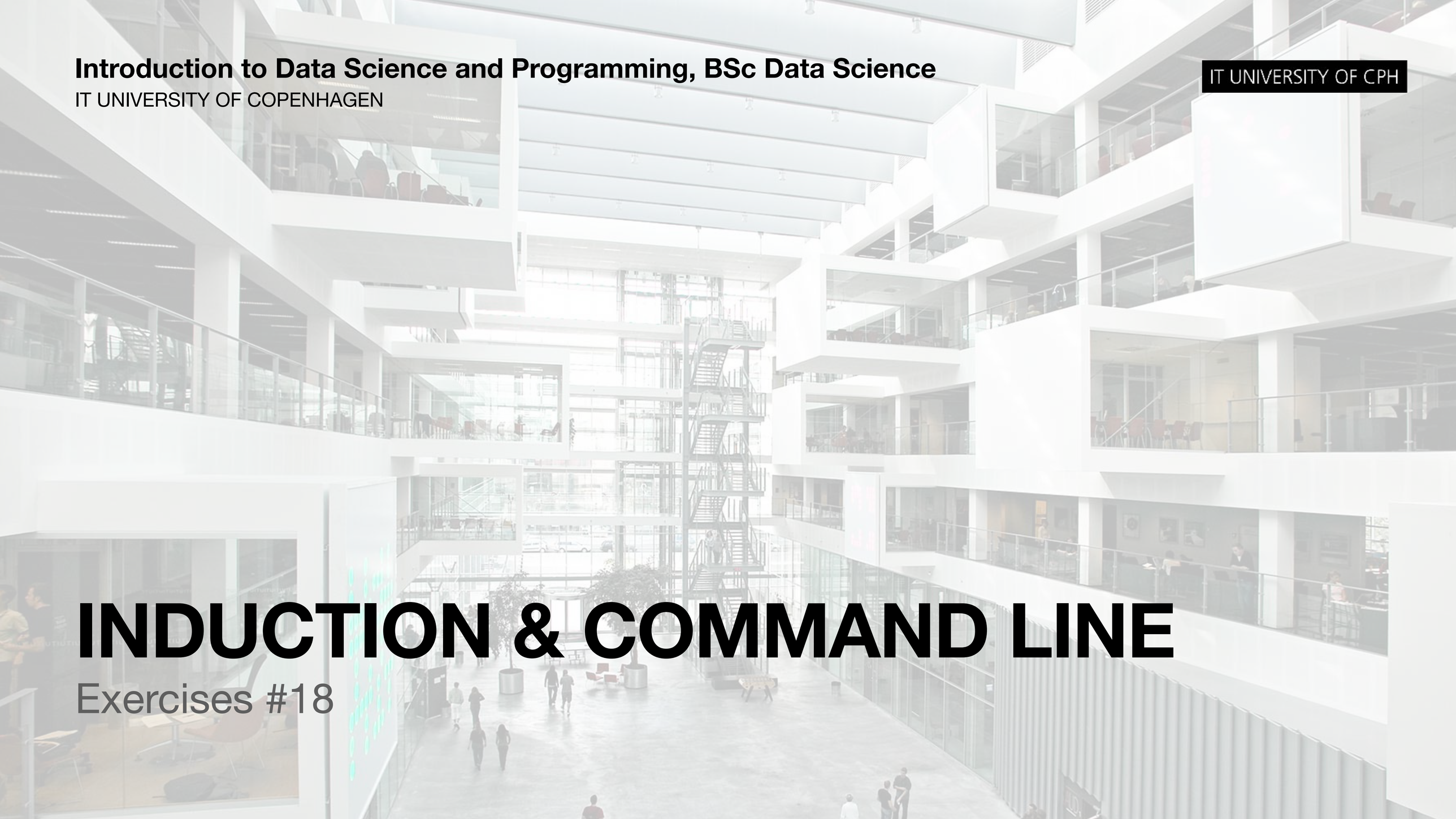

INDUCTION & COMMAND LINE

Exercises #18

What we will do today

In the first part of today's exercises, we are going to deal with the concept of mathematical induction, which is a mathematical proof technique.

In the second part, we will exercise our command line skills.

Mathematical Induction

Mathematical induction is a mathematical proof technique. It is used to prove that a statement $P(n)$ holds for every natural number $n = n \in \mathbb{N} = 0,1,2,...$; that is, the overall statement is a sequence of infinitely many cases $P(0), P(1), P(2), \ldots$

The method can be extended to prove statements about more general well-founded structures, such as trees; this generalisation, known as structural induction, is used in mathematical logic and computer science. Mathematical induction in this extended sense is closely related to recursion.

Structure of Proof

1. **Proof the initial/ base case:** Proof that the statement holds for the most basic example, usually P(0) or P(1()

2. **Perform the inductive step, also: induction step**: Assume that the statement holds for some arbitrary natural number k, and prove that the statement holds for k+1

1 Proof by Mathematical Induction

For each integer n with $n \geq 1$, prove the following statement using mathematical induction

$$2 + 4 + 6 + \ldots + 2n = n^2 + n$$

Hint:

1. First show, that the statement holds for the base-case, $P(1)$
2. Next, assume that the statement holds for any k, st. $P(k)$. Write this statement (inductive hypothesis)
3. Perform the inductive step, and write $P(k+1)$. Use basic tools of mathematics to show that the statements holds.

2 Fibonacci Sequence

Let the *Fibonacci Sequence* be defined by $F_0 = 0, F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Prove that $F_n < 2^n$ for all $n \in Z_+$ (the set of all positive integers).

3 Proof by Mathematical Induction

For each integer n with $n \geq 2$, prove the following statement using mathematical induction

$$\sum_{i=1}^{n-1} i(i+1) = \frac{n(n-1)(n+1)}{3}$$

Hint:

1. First show, that the statement holds for the base-case, $P(2)$
2. Next, assume that the statement holds for any k, st. $P(k)$. Write this statement (inductive hypothesis)
3. Perform the inductive step, and write $P(k+1)$. Use basic tools of mathematics to show, that the statements holds.

4 Proof by Mathematical Induction

For each integer n with $n \geq 0$, prove the following statement using mathematical induction

$$\sum_{i=1}^{n+1} i \cdot 2^i = n \cdot 2^{n+2} + 2$$

Hint:

1. First show, that the statement holds for the base-case, $P(0)$
2. Next, assume that the statement holds for any k, st. $P(k)$. Write this statement (inductive hypothesis)
3. Perform the inductive step, and write $P(k+1)$. Use basic tools of mathematics to show, that the statements holds.

Command line tools

Most tasks in this part can be solved fastest by searching the internet for solutions, or by looking at the man pages of commands, for example `man tail`

5 Exploring file properties

Use the command line to enter the folder that contains the files accidents.csv, dsalphabet.txt, and variable_lookup.xls from the lecture.

Then, write a command to list in the console the file properties and sizes of these 3 files in long format (not human-readable)

Solution: ls -l

Expected output (a few numbers or strings may be different):

```
total 32928
-rw-r--r--@ 1 mszell  staff  16021264 Aug 24 14:09 accidents.csv
-rw-r--r--@ 1 mszell  staff        83 Aug 24 14:29 dsalphabet.txt
-rw-r--r--@ 1 mszell  staff    829952 Aug 24 14:10 variable_lookup.xls
```

6 Saving to files

Save the previous console output without the first line in a new file called 'fileproperties.txt'

Hint: Use a pipe and the command "tail"

Solution: ls -l | tail -n +2 > fileproperties.txt

Expected content of fileproperties.txt (a few numbers or strings may be different):

```
-rw-r--r--@ 1 mszell  staff  16021264 Aug 24 14:09 accidents.csv
-rw-r--r--@ 1 mszell  staff        83 Aug 24 14:29 dsalphabet.txt
-rw-r--r--  1 mszell  staff         0 Oct  2 15:00 fileproperties.txt
-rw-r--r--@ 1 mszell  staff    829952 Aug 24 14:10 variable_lookup.xls
```

7 String replacements

Squeeze multiple occurrences of spaces in fileproperties.txt into one space, then replace the spaces by commas, and save the output in file properties.csv

Hint: Use pipes and the -s option of tr

Solution: cat fileproperties.txt | tr -s " " | tr " " "," > fileproperties.csv

Expected content of fileproperties.csv:

```
-rw-r--r--@,1,mszell,staff,16021264,Aug,24,14:09,accidents.csv
-rw-r--r--@,1,mszell,staff,83,Aug,24,14:29,dsalphabet.txt
-rw-r--r--,1,mszell,staff,0,Oct,2,15:00,fileproperties.txt
-rw-r--r--@,1,mszell,staff,829952,Aug,24,14:10,variable_lookup.xls
```

8 awk sum

Use awk to display the total size of files listed in fileproperties.csv

Expected output: 16851299

Solution: awk -F, '{s+=$5} END {print s}' fileproperties.csv

9 Sort

Display the contents of fileproperties.csv sorted by file size, largest files first

Hint: Search the internet for "sort file by field unix"

Solution: sort -t, -nk5 -r fileproperties.csv

Expected output:

```
-rw-r--r--@,1,mszell,staff,16021264,Aug,24,14:09,accidents.csv
-rw-r--r--@,1,mszell,staff,829952,Aug,24,14:10,variable_lookup.xls
-rw-r--r--@,1,mszell,staff,83,Aug,24,14:29,dsalphabet.txt
-rw-r--r--,1,mszell,staff,0,Oct,2,15:00,fileproperties.txt
```

10 Sort and extract data

Display the filenames in fileproperties.csv sorted by file size, largest files first, in one line separated by commas.

Solution: sort -t, -nk5 -r fileproperties.csv | awk -F, '{print $NF}' | tr "\n" ","

Expected output:

```
accidents.csv,variable_lookup.xls,dsalphabet.txt,fileproperties.txt,%
```

DONE FOR TODAY!