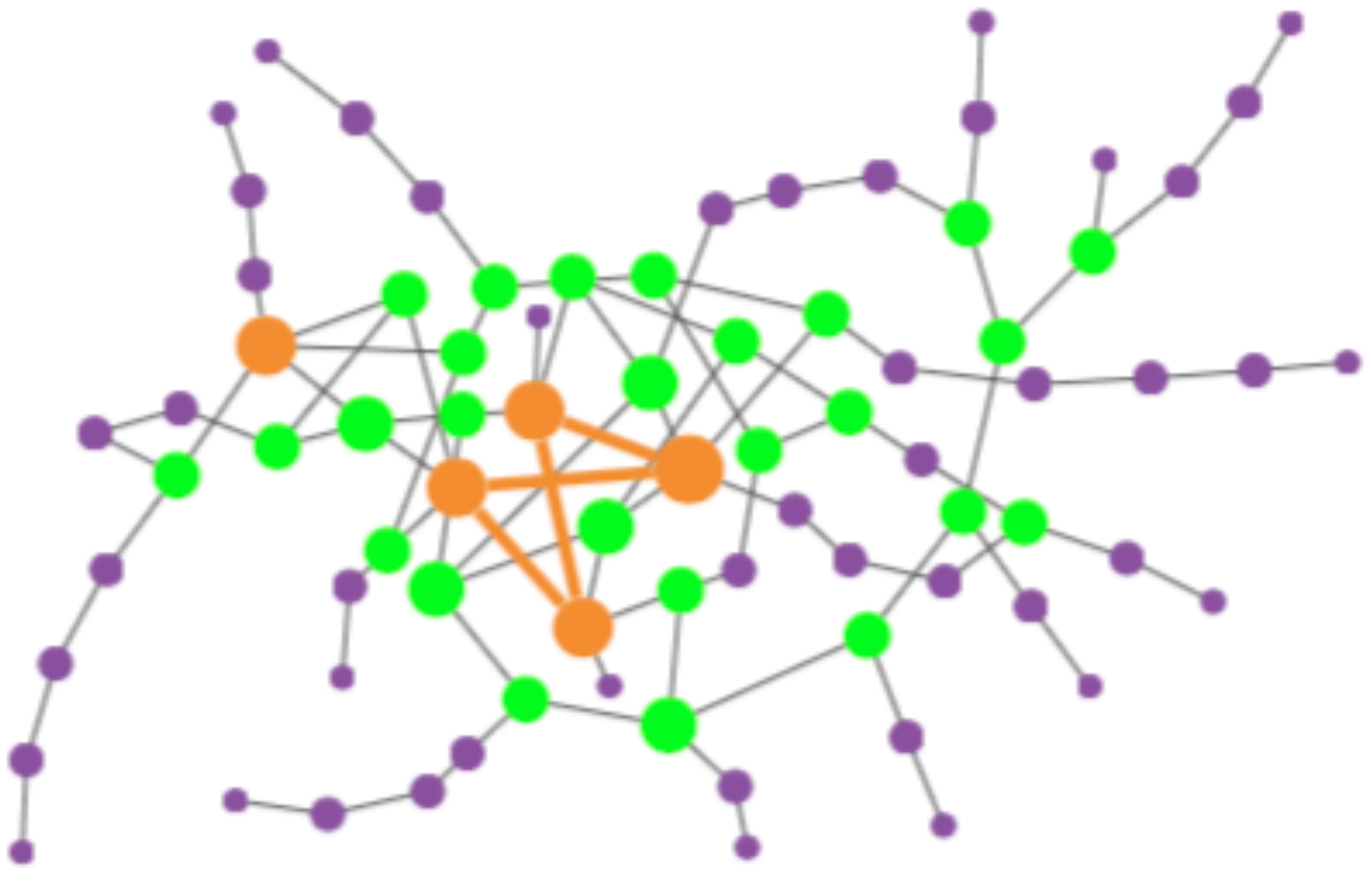
Class 23: Graph properties

Instructor: Michael Szell

Nov 20, 2019



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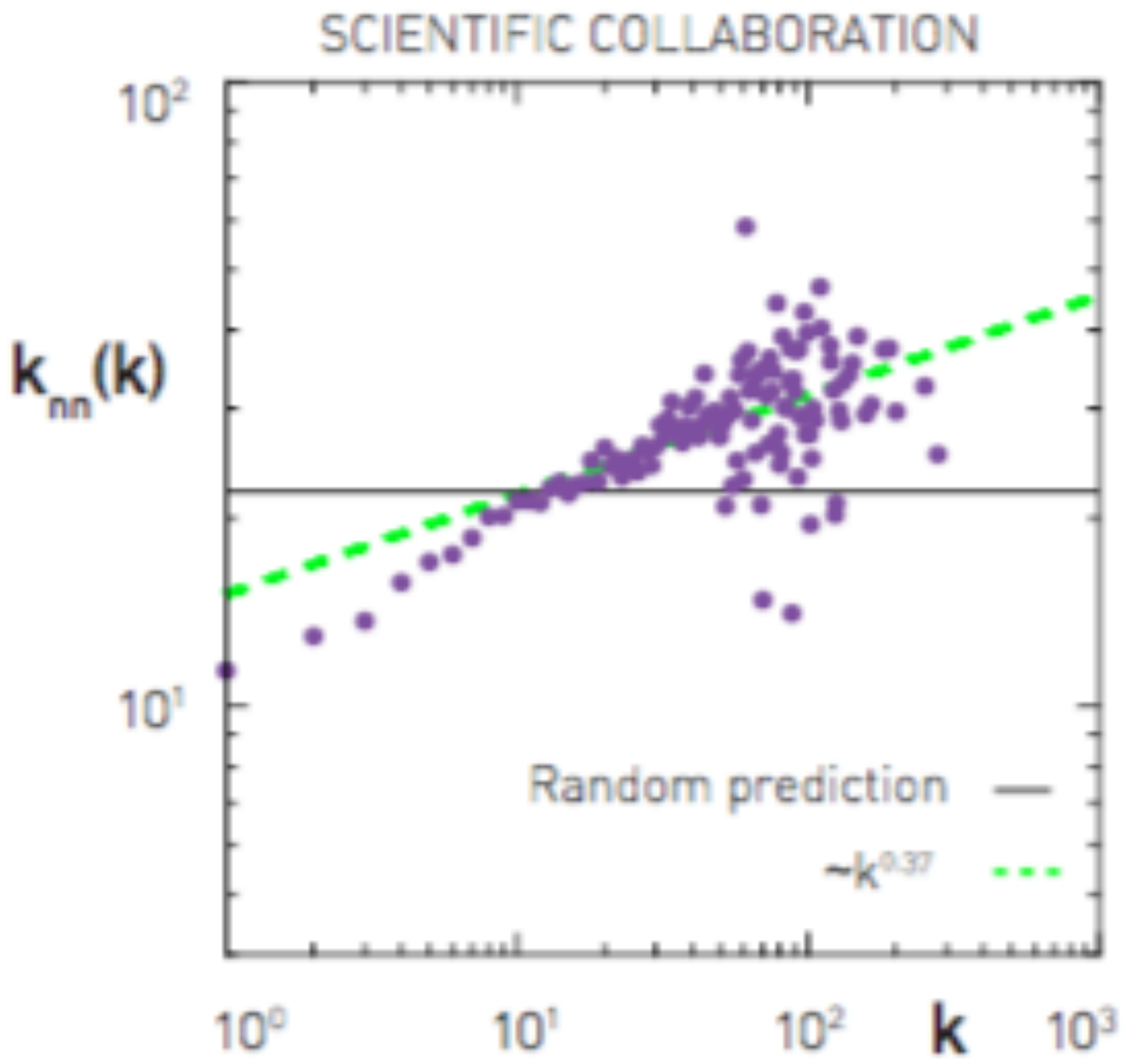
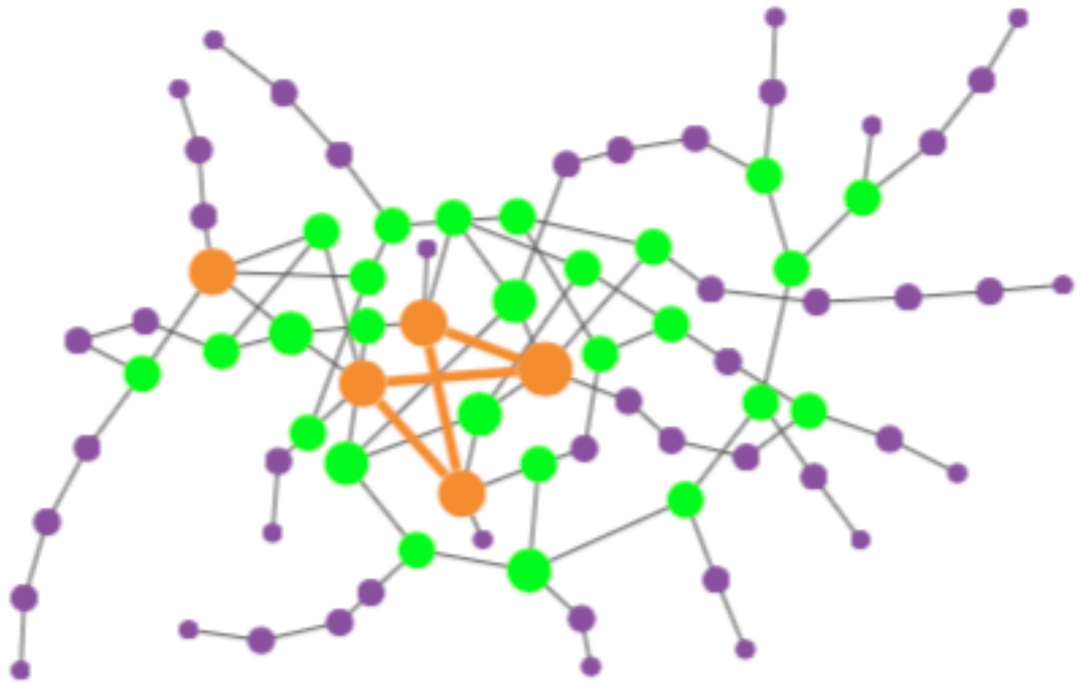
Today you will learn about degree-based graph properties

Assortativity

Nearest neighbors



Building our own Graph python class



What are the chances that celebrities marry each other?

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Picking another American at random: 1 in 100,000,000

Picking one of the 1000 similar celebrities by chance: 1 in 100,000

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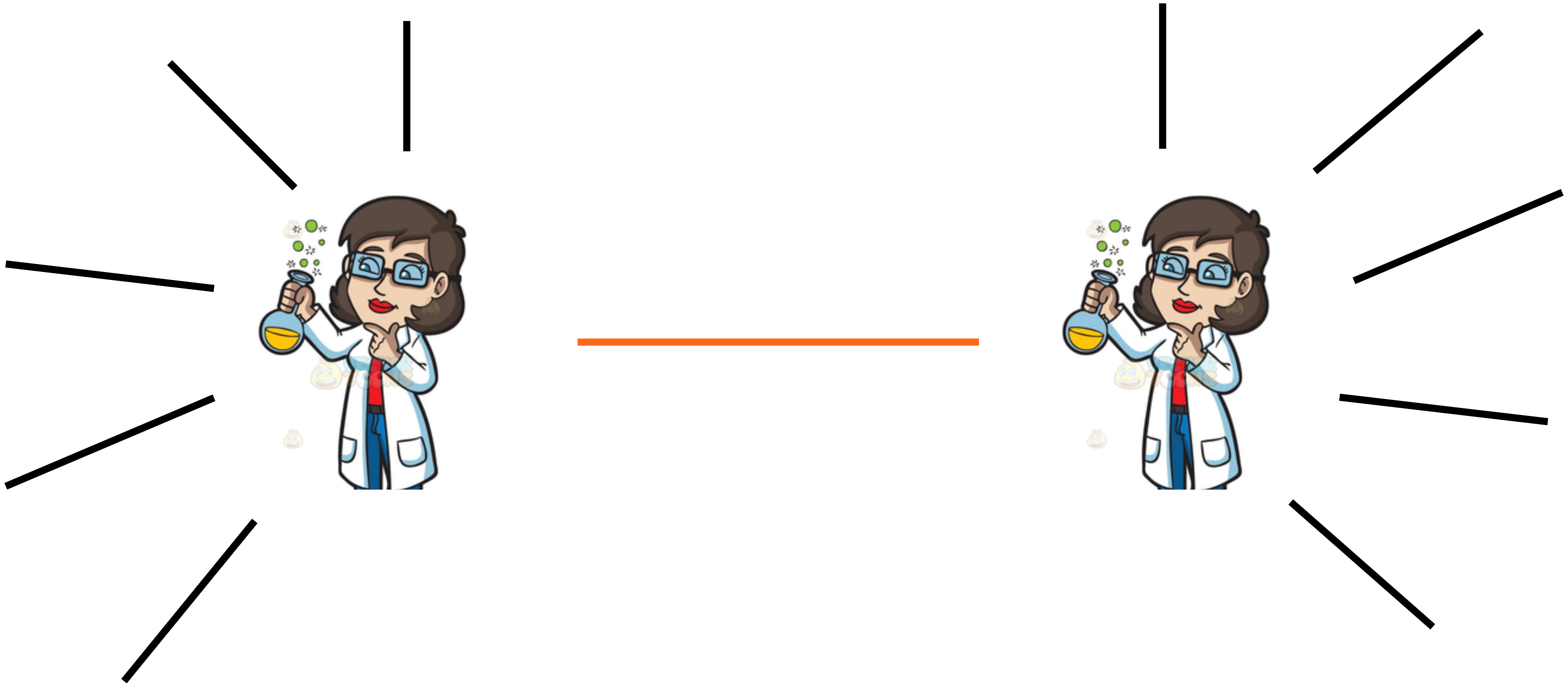
Picking another American at random: 1 in 100,000,000

Picking one of the 1000 similar celebrities by chance: 1 in 100,000

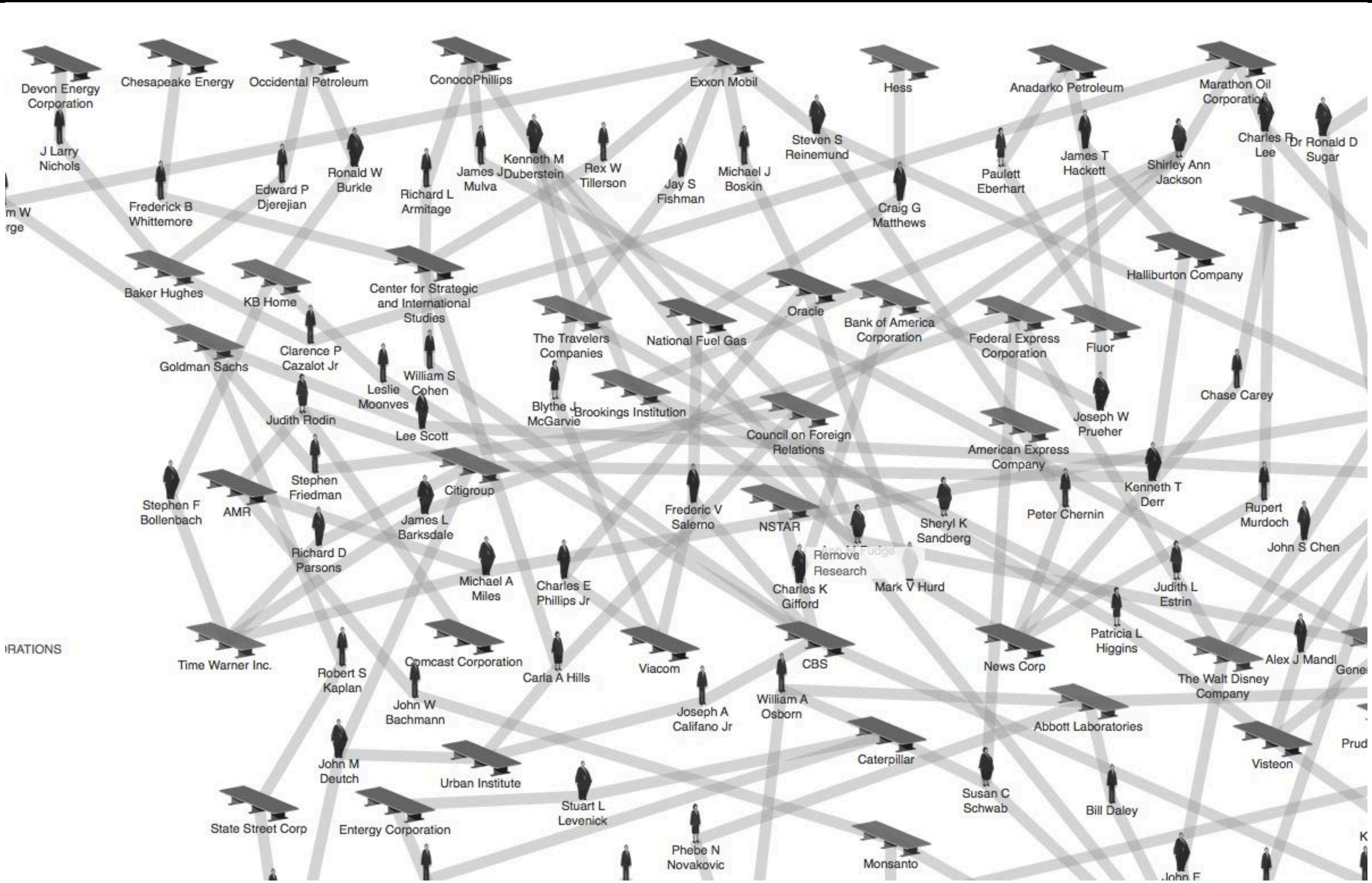
They do not pick at random, but choose each other more often

In social systems, hubs tend to connect to hubs



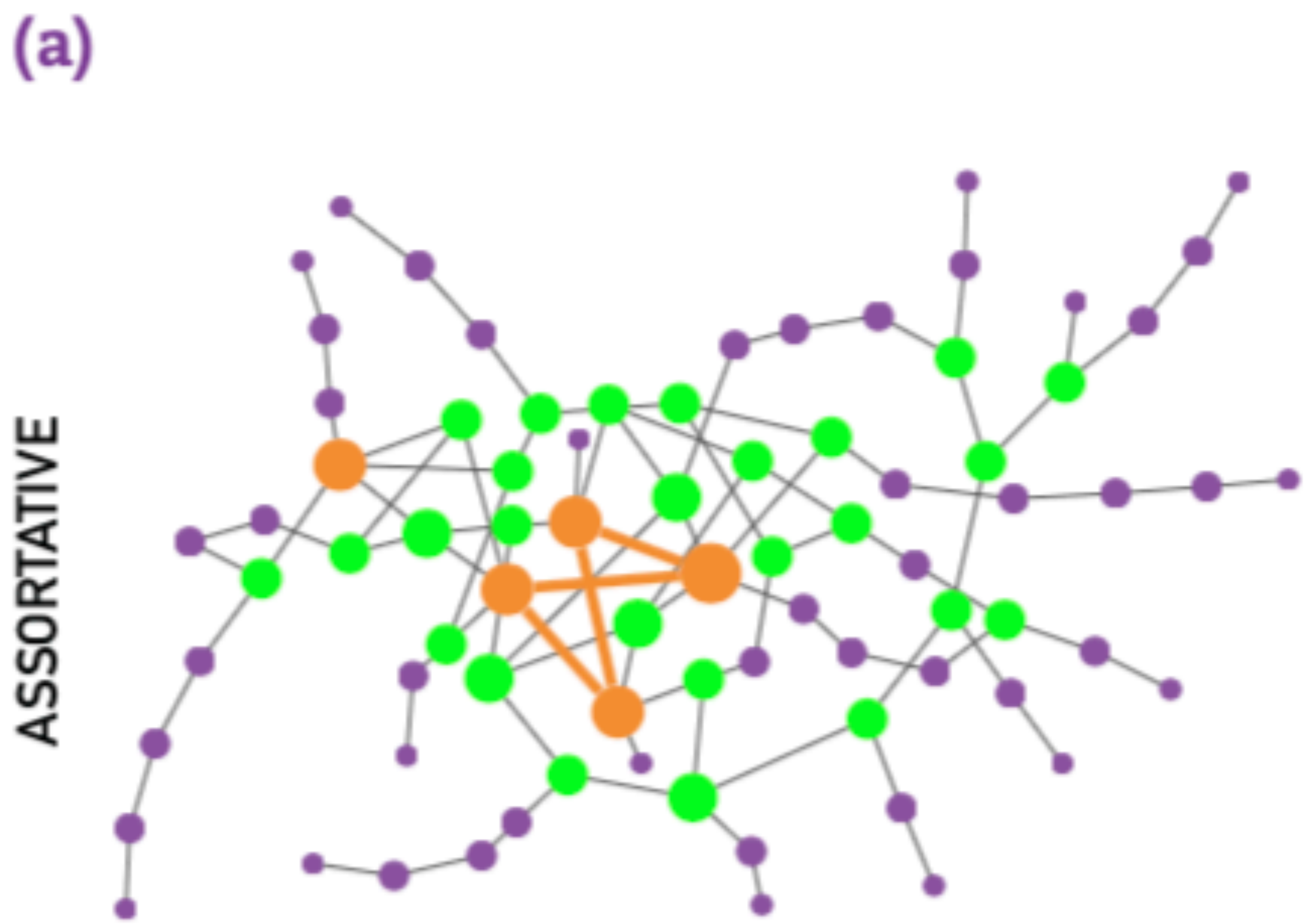
Researchers with many collaborators
tend to collaborate with each other

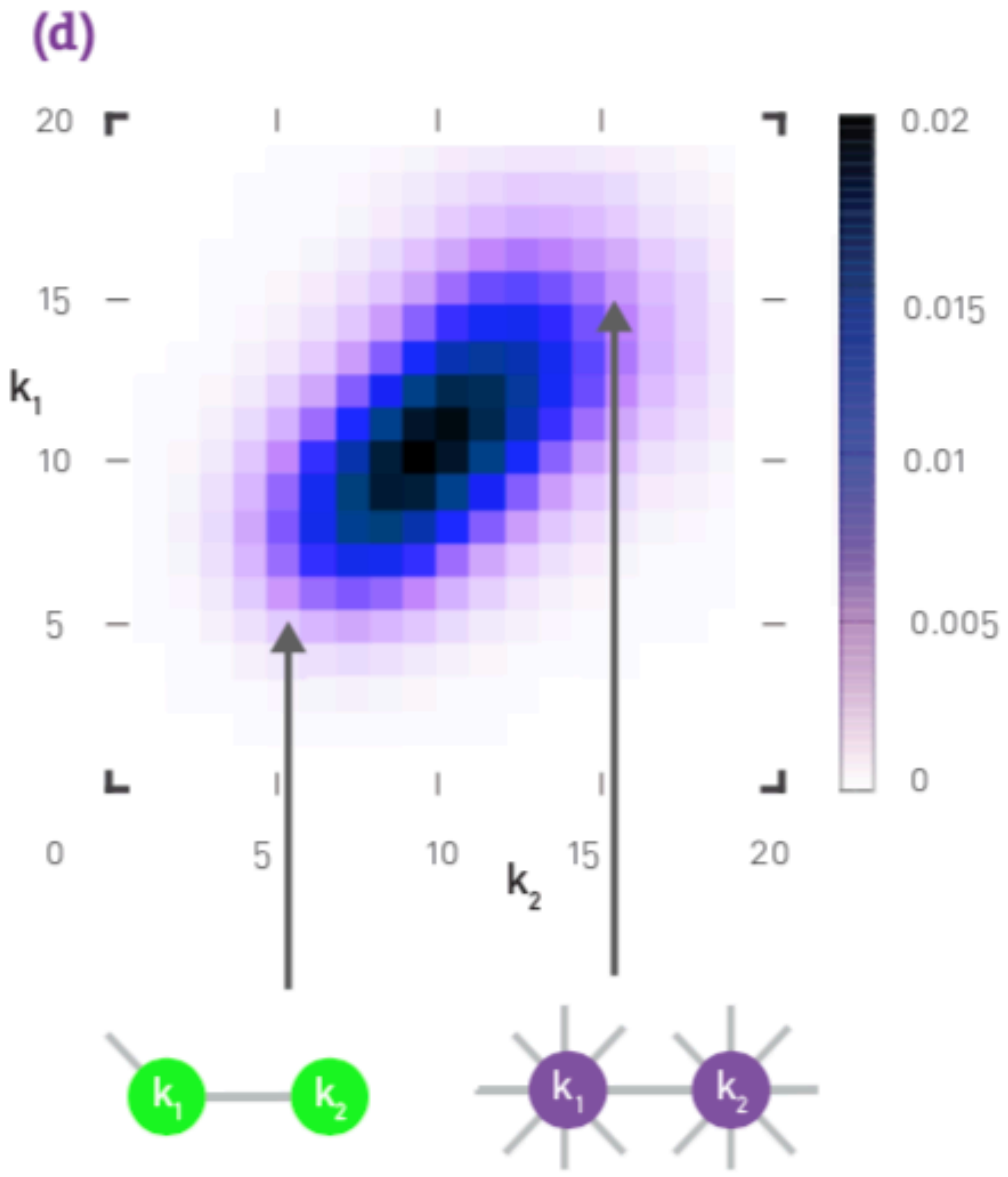
In social systems, hubs tend to connect to hubs



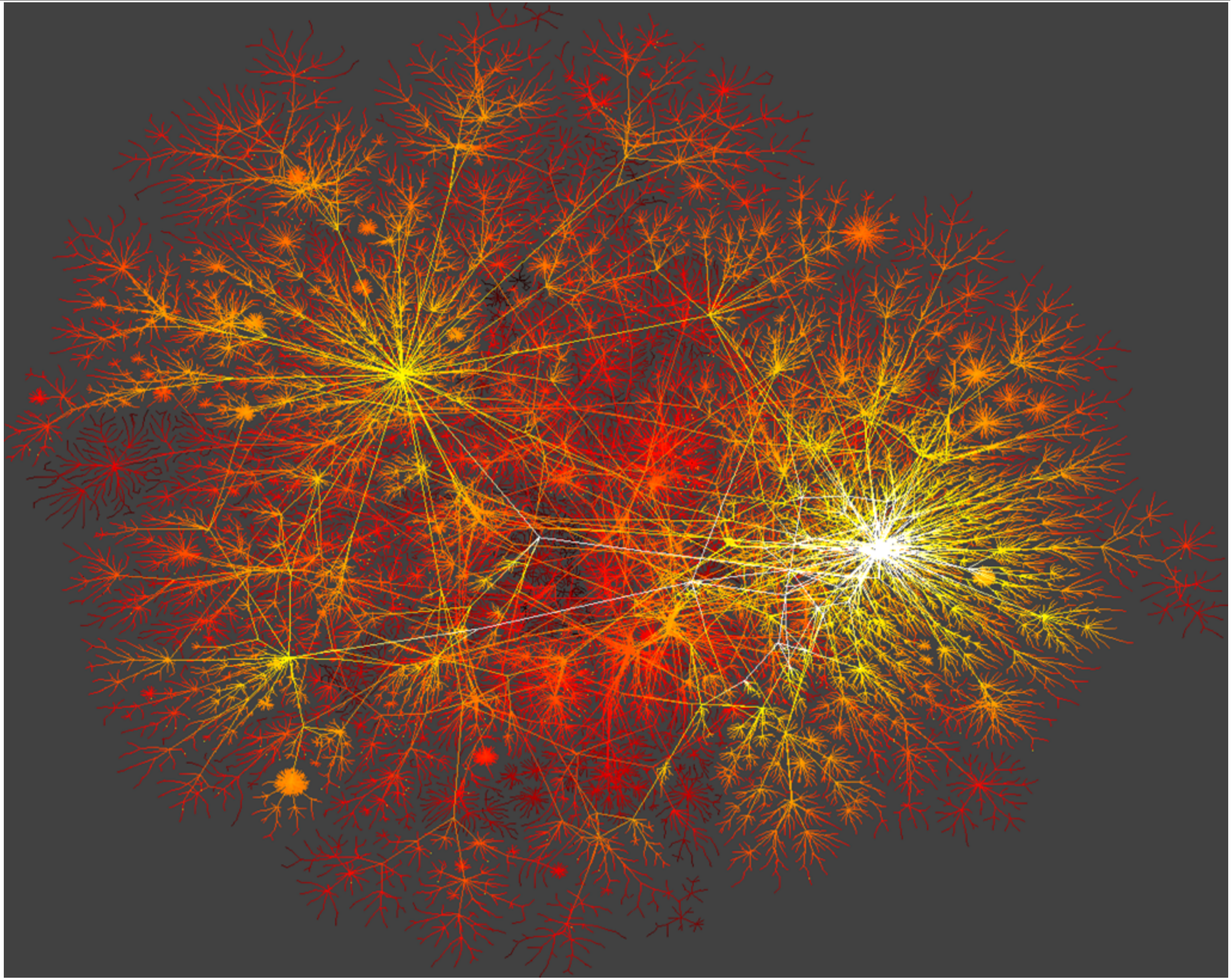
Company directors who sit on many boards tend to sit together with company directors who sit on many boards

A network is assortative when hubs tend to connect to hubs

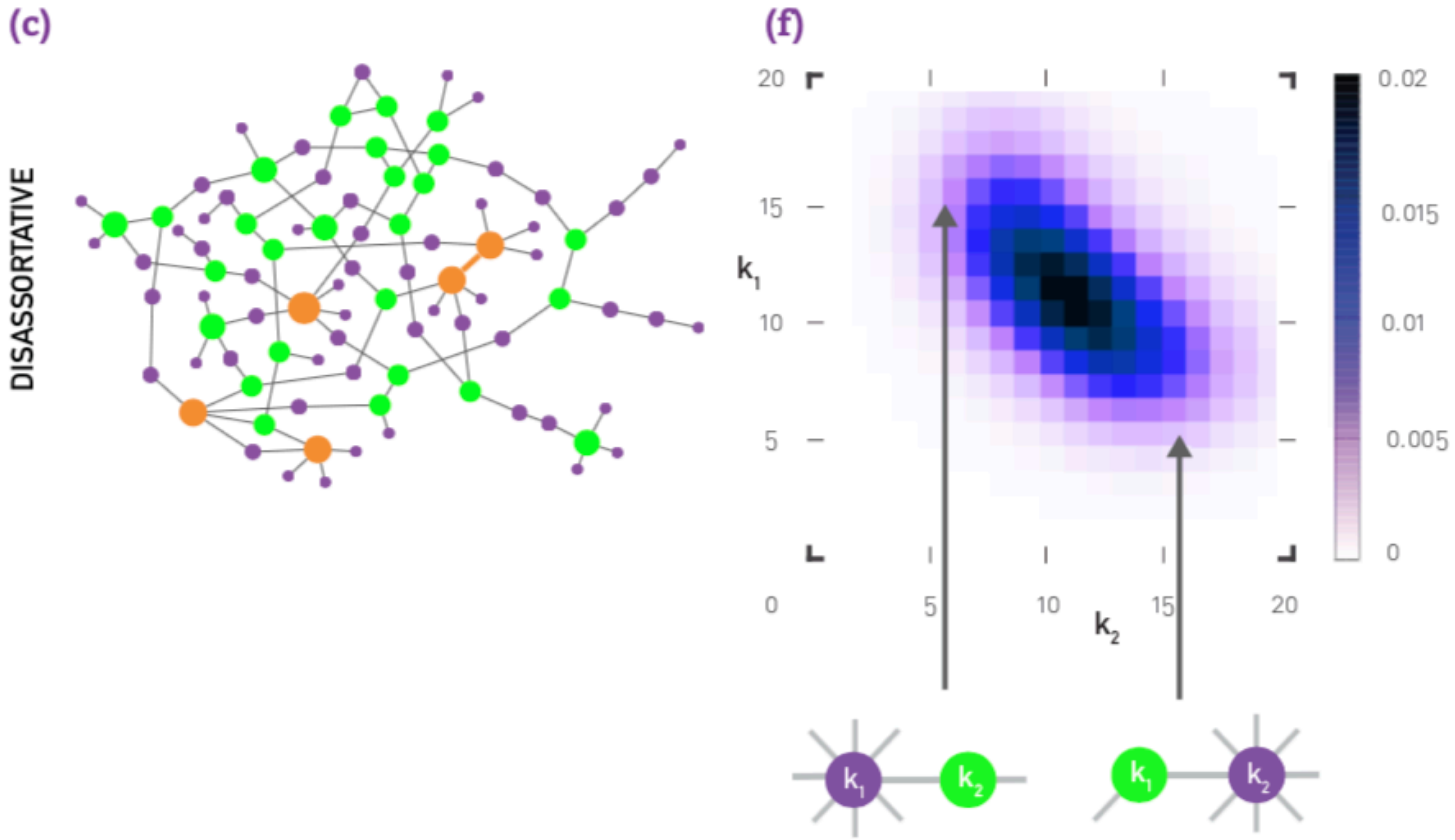




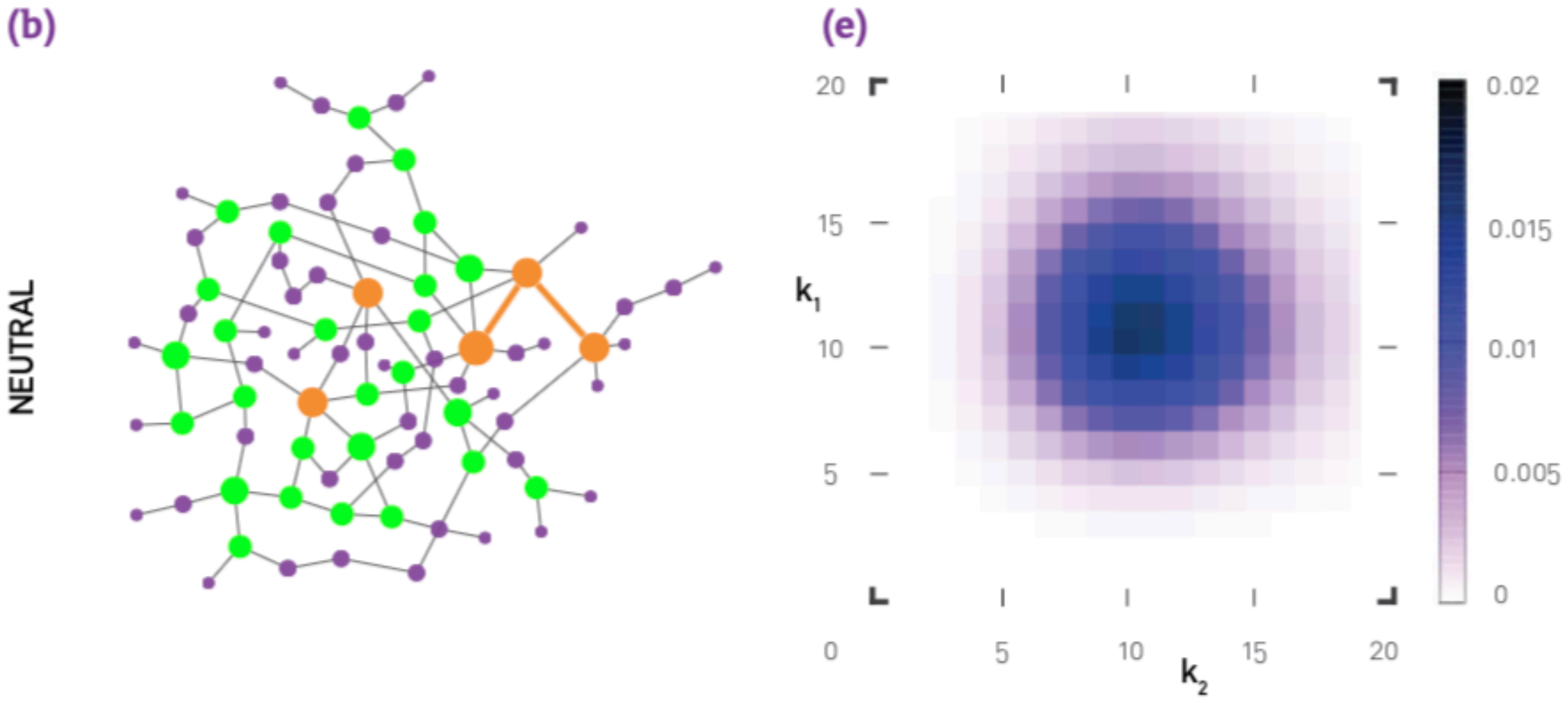
In technological networks, hubs tend to not connect to hubs



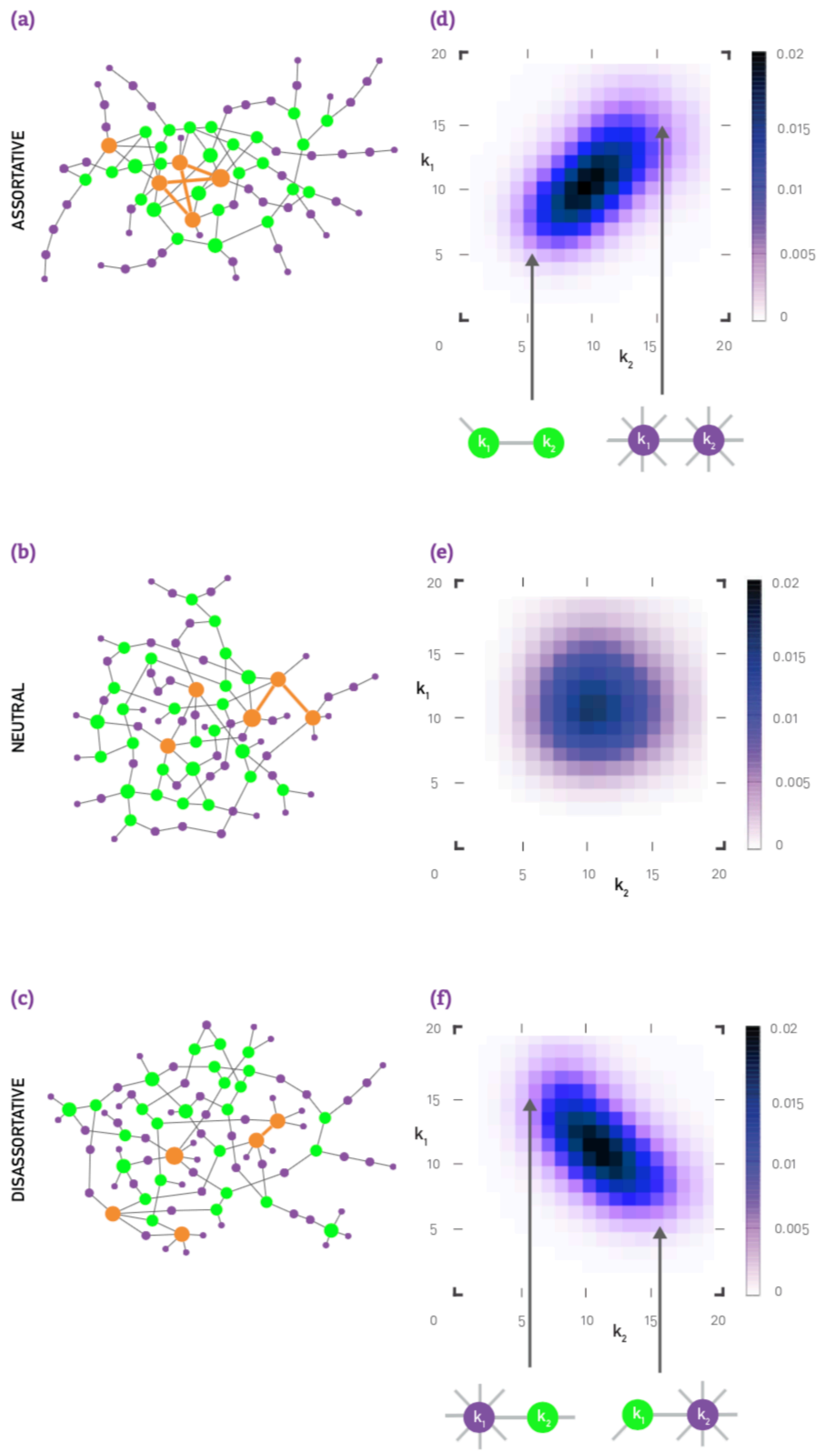
A network is disassortative when hubs tend to not connect to hubs



A network is neutral when wiring is independent of degrees



Assortative mixing can be quantified by the correlation coefficient r between degrees at two ends of all links



Assortative	$r > 0$
Neutral	$r = 0$
Disassortative	$r < 0$

	Group	Network	Type	Size n	Assortativity r
Social	a	Physics coauthorship	undirected	52 909	0.363
	a	Biology coauthorship	undirected	1 520 251	0.127
	b	Mathematics coauthorship	undirected	253 339	0.120
	c	Film actor collaborations	undirected	449 913	0.208
	d	Company directors	undirected	7 673	0.276
	e	Student relationships	undirected	573	−0.029
	f	Email address books	directed	16 881	0.092
Technological	g	Power grid	undirected	4 941	−0.003
	h	Internet	undirected	10 697	−0.189
	i	World Wide Web	directed	269 504	−0.067
	j	Software dependencies	directed	3 162	−0.016
Biological	k	Protein interactions	undirected	2 115	−0.156
	l	Metabolic network	undirected	765	−0.240
	m	Neural network	directed	307	−0.226
	n	Marine food web	directed	134	−0.263
	o	Freshwater food web	directed	92	−0.326

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Dublin

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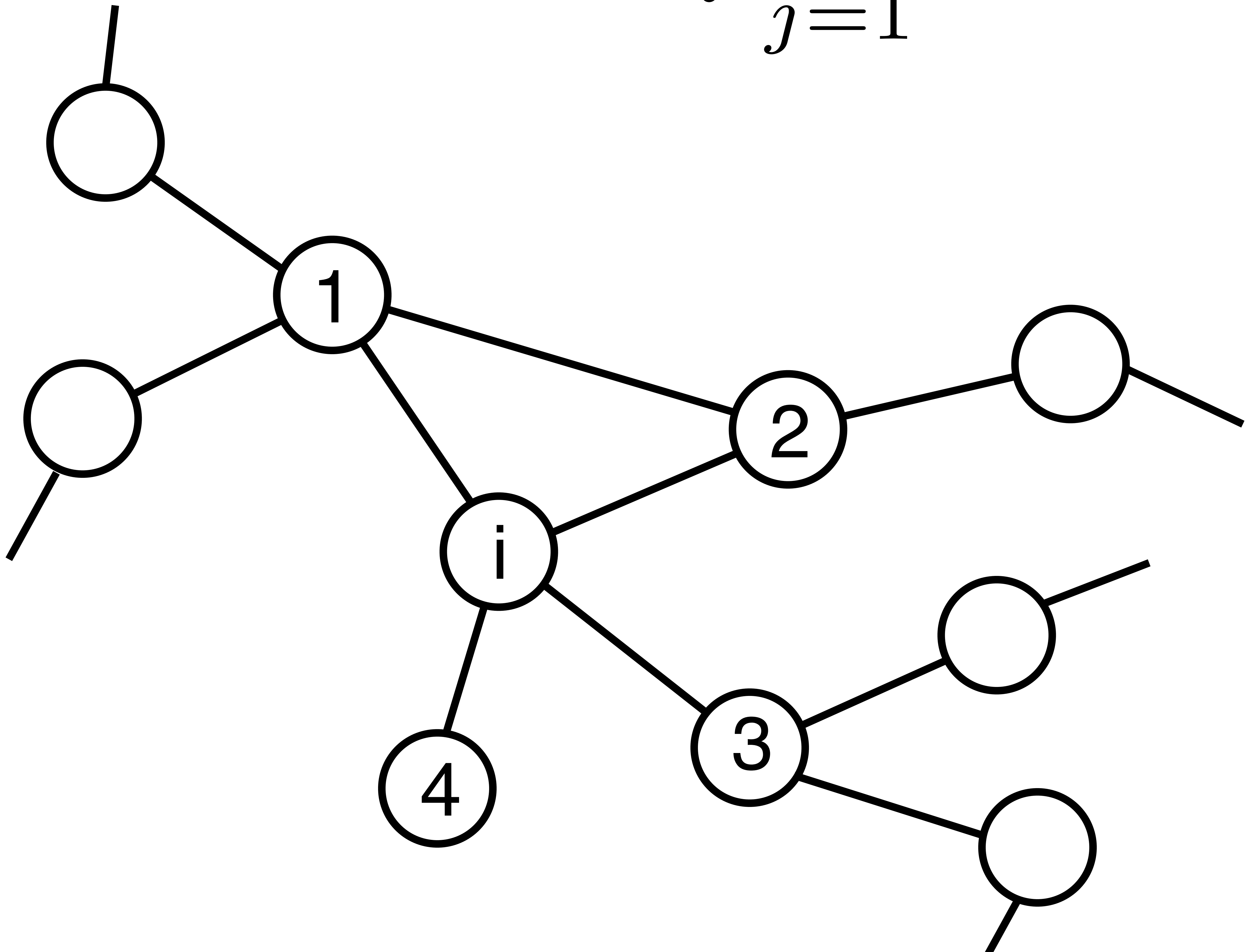
Tokyo Narita
Bangkok
Singapore
Beijing
Manila

Sydney
Brisbane
Auckland
Perth

Often there is a non-linear relation which we cannot capture with r

The nearest neighbor degree $k_{nn,i}$ of a node i is the average degree of its neighbors

$$k_{\mathrm{nn},i} = \frac{1}{k_i} \sum_{j=1}^{N} A_{ij} k_j$$



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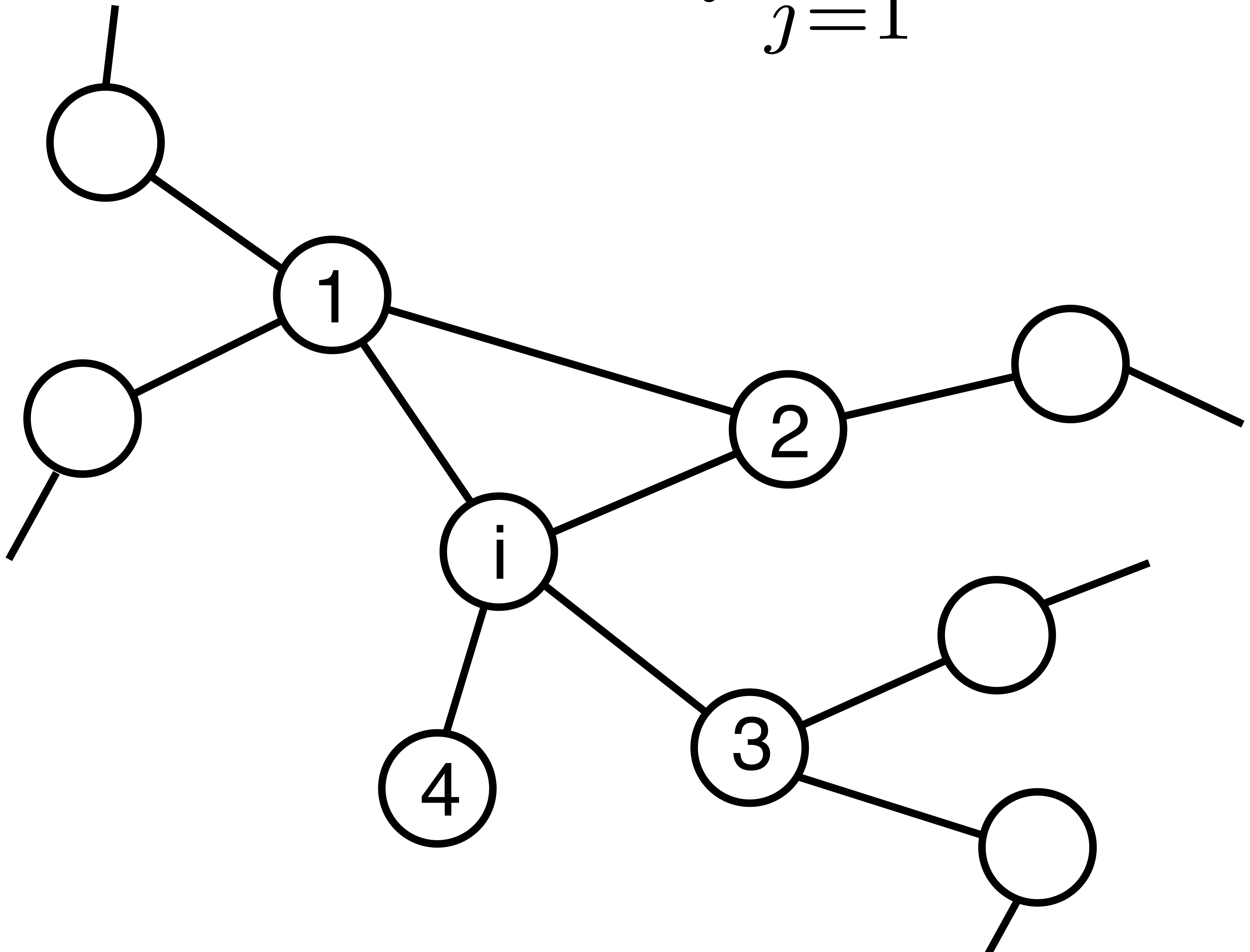
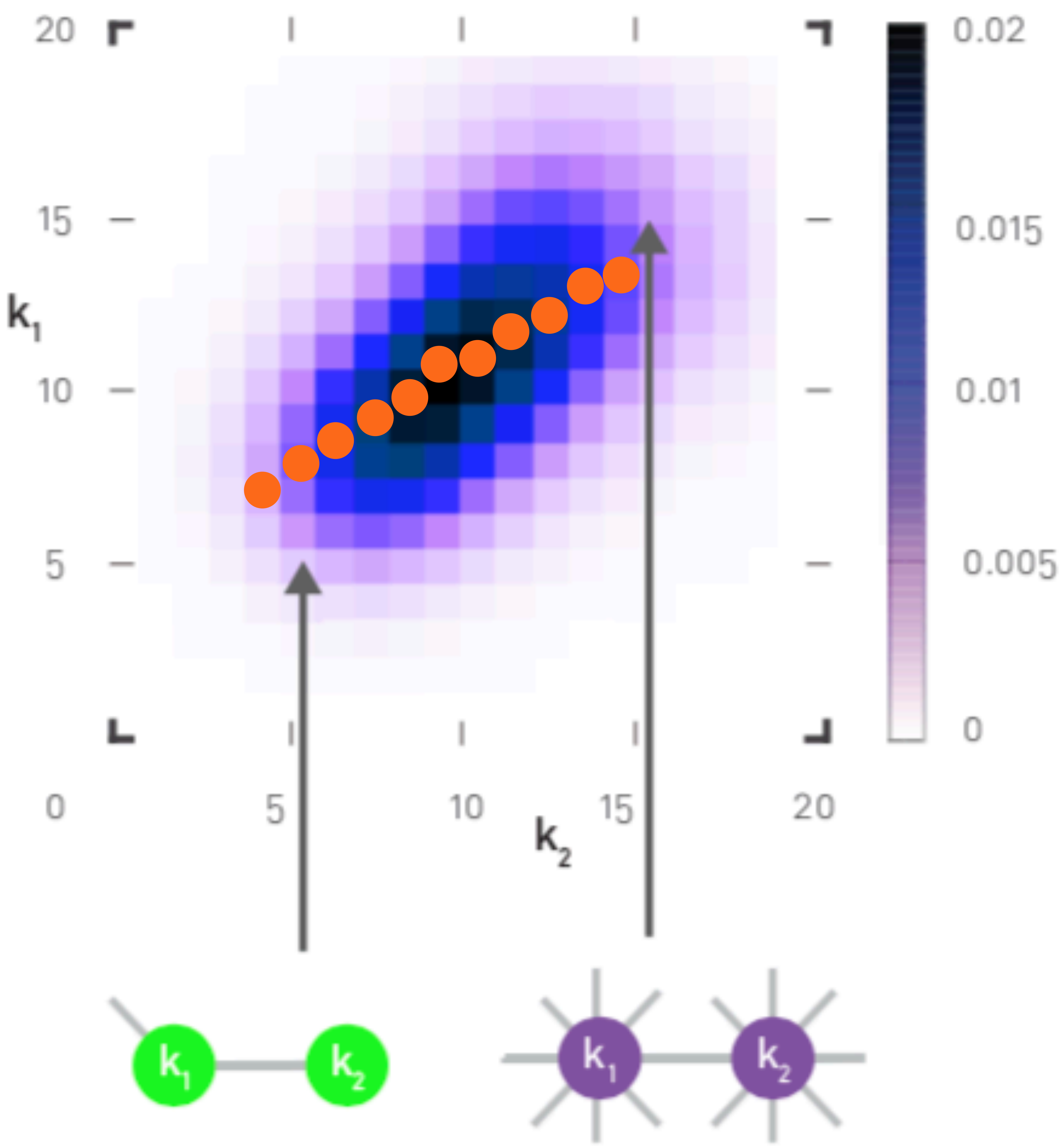


Example:

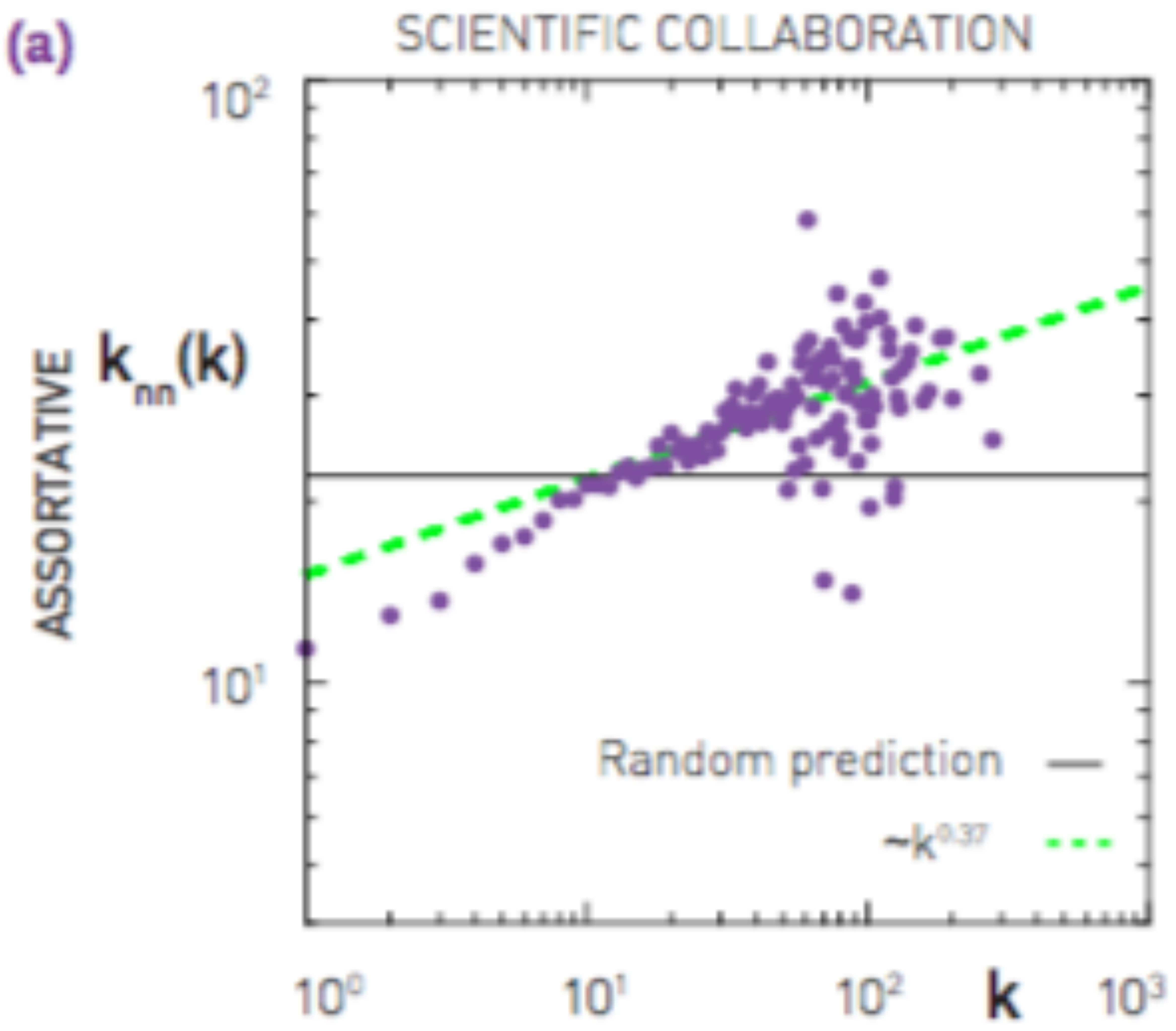
$$k_i = 4 \qquad k_1 = 4$$
$$k_2 = 3$$
$$k_3 = 3$$
$$k_4 = 1$$

$$k_{\mathrm{nn},i} = \frac{4+3+3+1}{4} = 2.75$$

We can calculate $k_{nn}(k)$, the average degree of the neighbors of all degree-k nodes



Often $k_{nn}(k)$ has the nonlinear relation k^{μ}, where μ is called the correlation exponent



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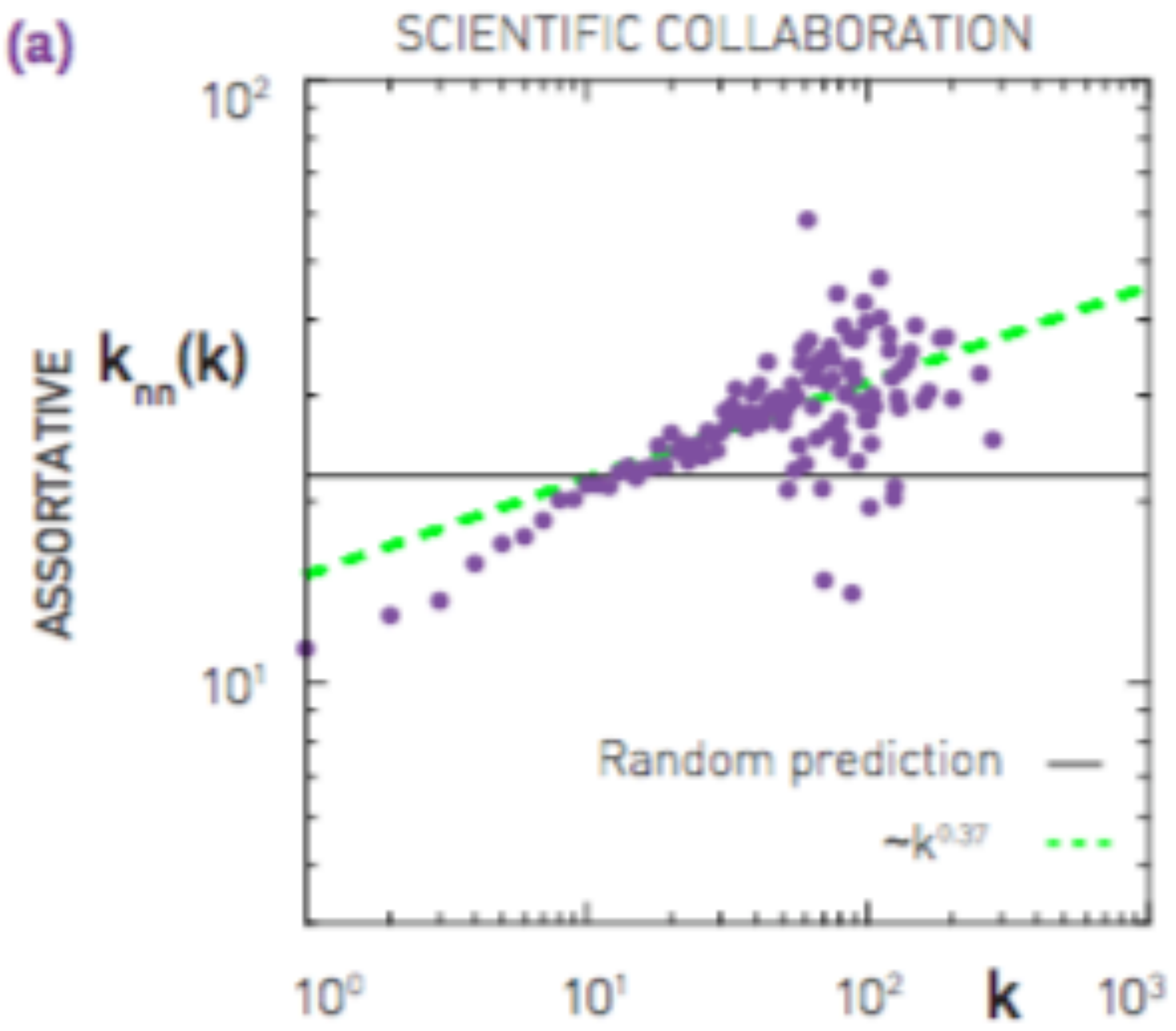

If $k_{nn}(k)$ is increasing, the network is assortative. $\mu > 0$

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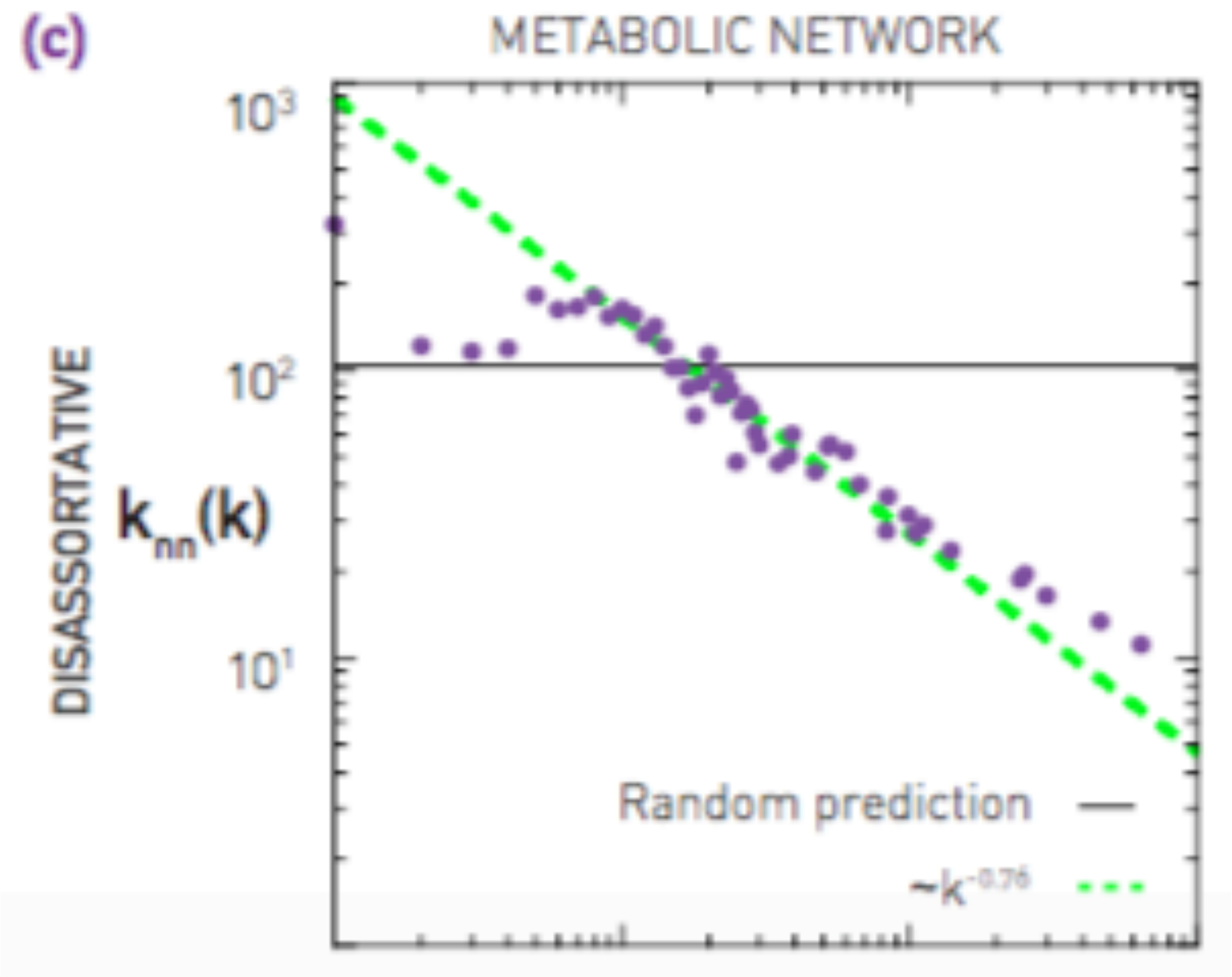
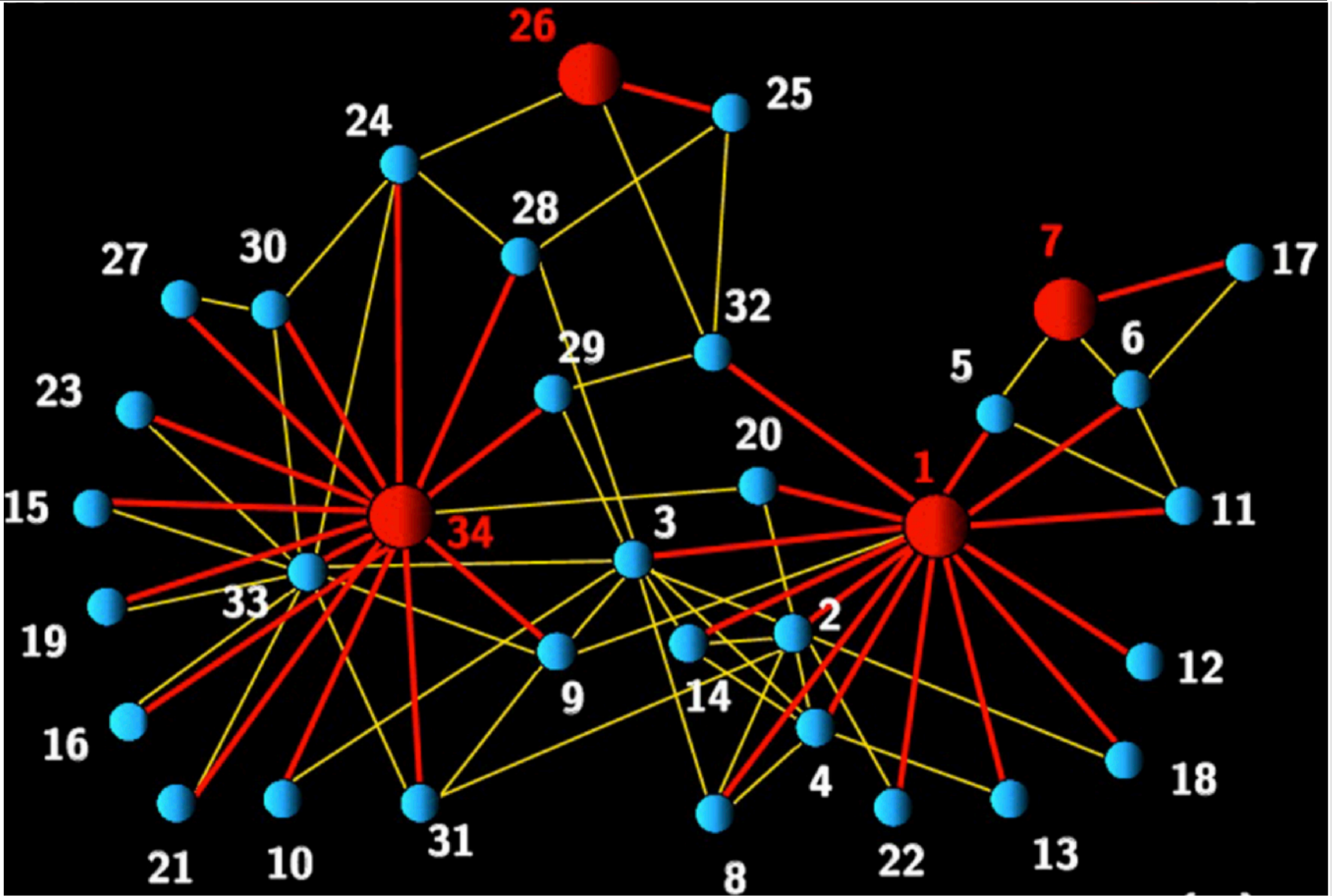
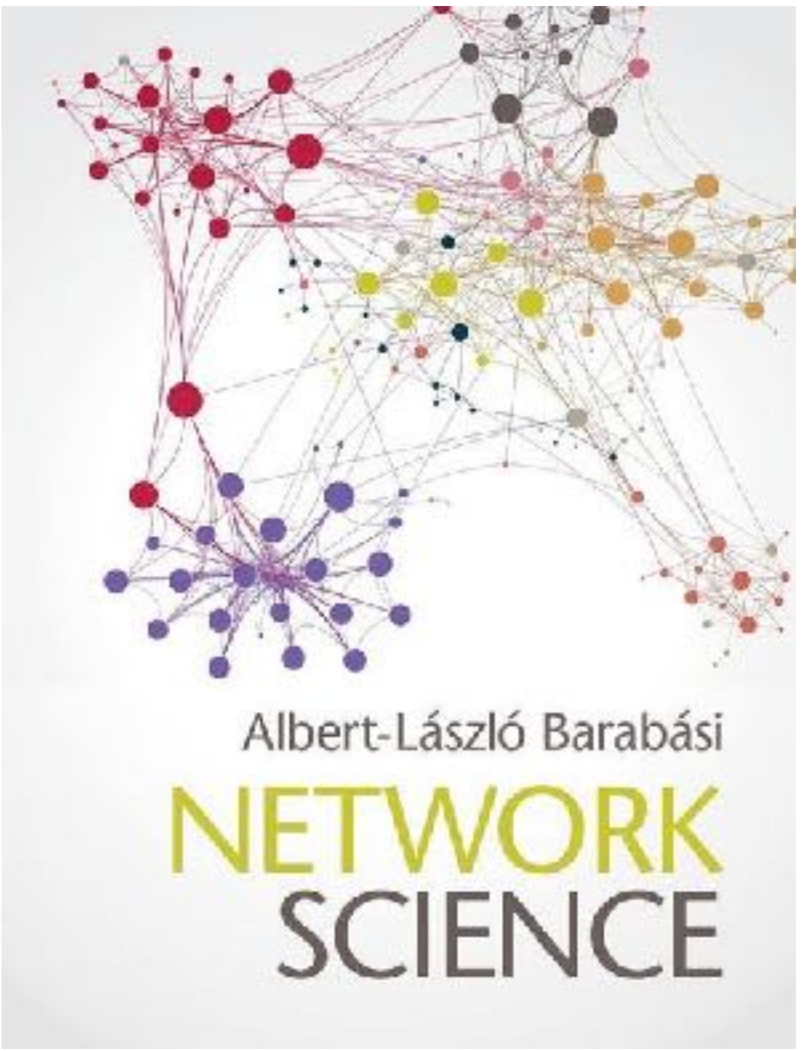


If $k_{nn}(k)$ is decreasing, the network is disassortative. $\mu < 0$

The Zachary Karate Club network shows a dispute between two instructors that led to a split into 2 groups



Sources and further materials for today's class



A.-L. Barabási.
Network Science.
Cambridge University Press (2016)

http://barabasi.com/networksciencebook/

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