

# Subsampling and image pyramids



# Course announcements

- Homework 0 and homework 1 will be posted tonight.
  - Homework 0 is not required and is not graded!
  - Homework 1 is due on February 7<sup>th</sup> at midnight.
- Course website updated.
  - Syllabus slightly updated.
  - Added homework schedule.
  - Note homework 4 that spans spring break.

# Overview of today's lecture

- Image downsampling.
- Aliasing.
- Gaussian image pyramid.
- Laplacian image pyramid.

# Slide credits

Most of these slides were adapted directly from:

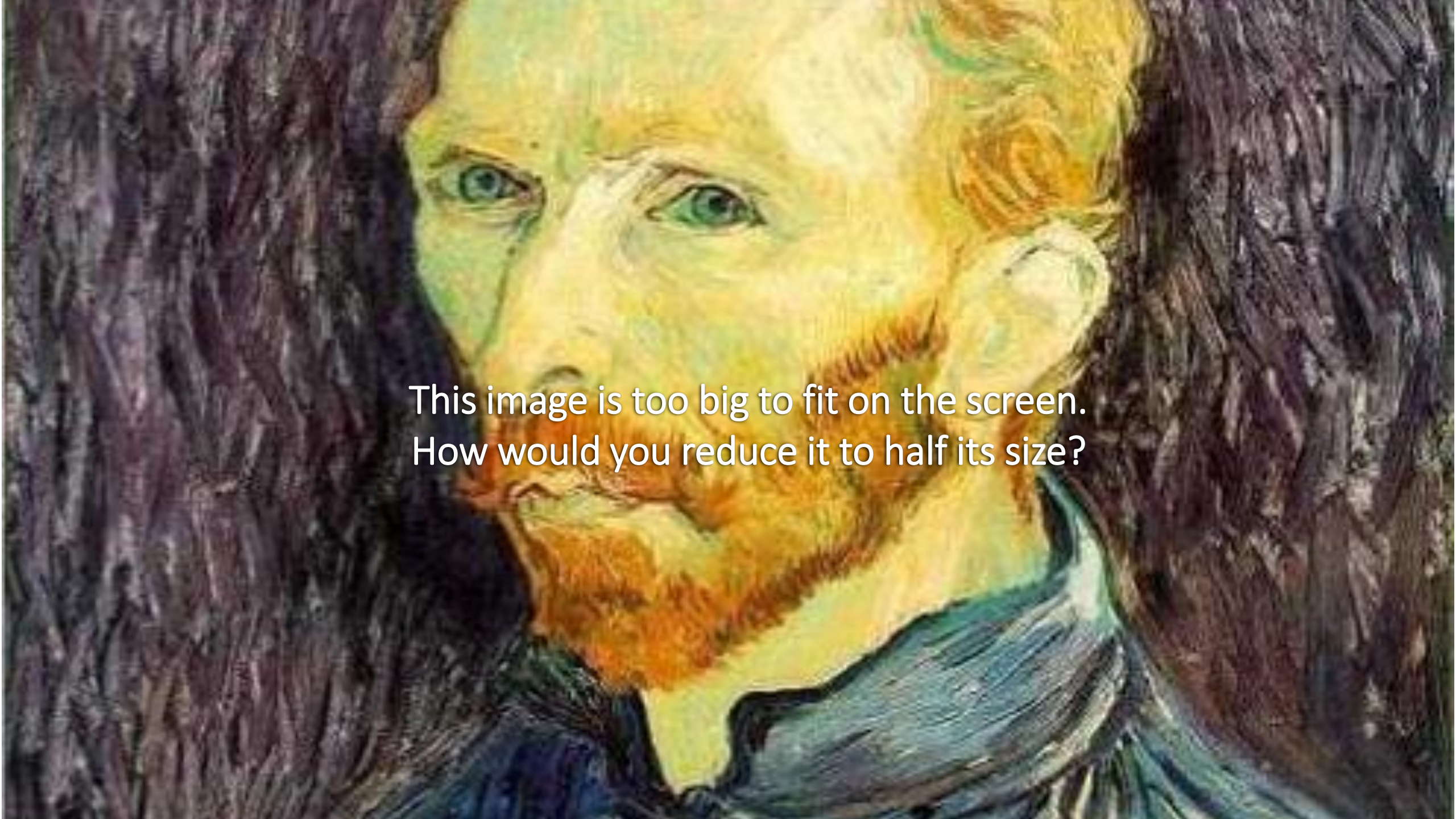
- Kris Kitani (15-463, Fall 2016).

Some slides were inspired or taken from:

- Fredo Durand (MIT).
- Bernd Girod (Stanford University).
- James Hays (Georgia Tech).
- Steve Marschner (Cornell University).
- Steve Seitz (University of Washington).

Image downsampling

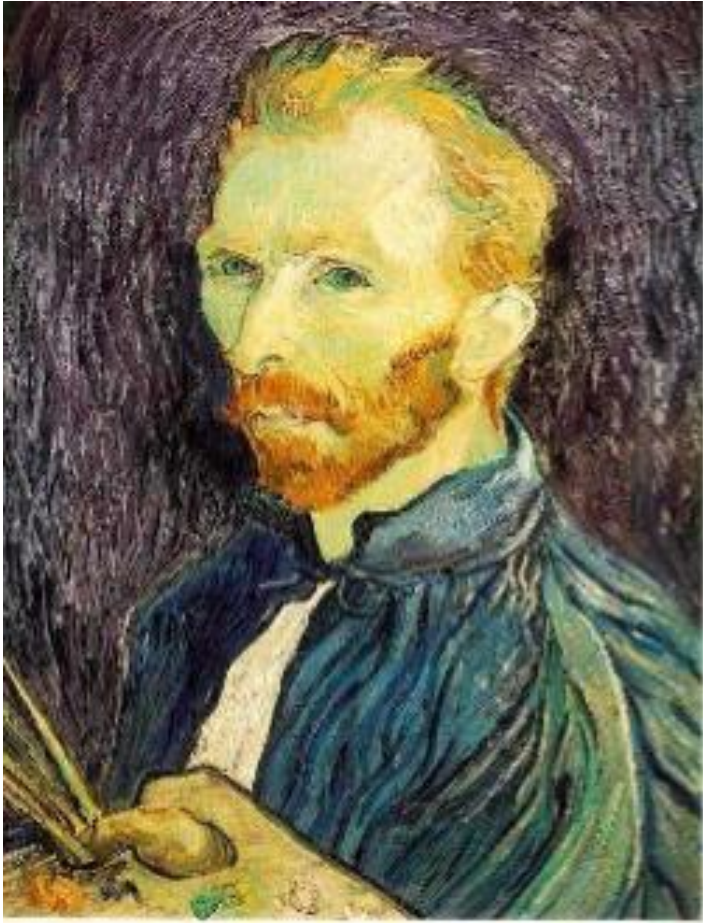


A close-up of Vincent van Gogh's 'Self-Portrait with Bandaged Ear'. The painting shows a man with a pale, yellowish-green face, looking slightly to the left. He has a thick, reddish-brown beard and mustache. His hair is a mix of yellow and green. He is wearing a dark blue jacket. The background is a dark, textured brown. The brushstrokes are visible and expressive.

This image is too big to fit on the screen.  
How would you reduce it to half its size?



# Naïve image downsampling



$1/2$

Throw away half the rows and columns

delete even rows  
delete even columns



$1/4$

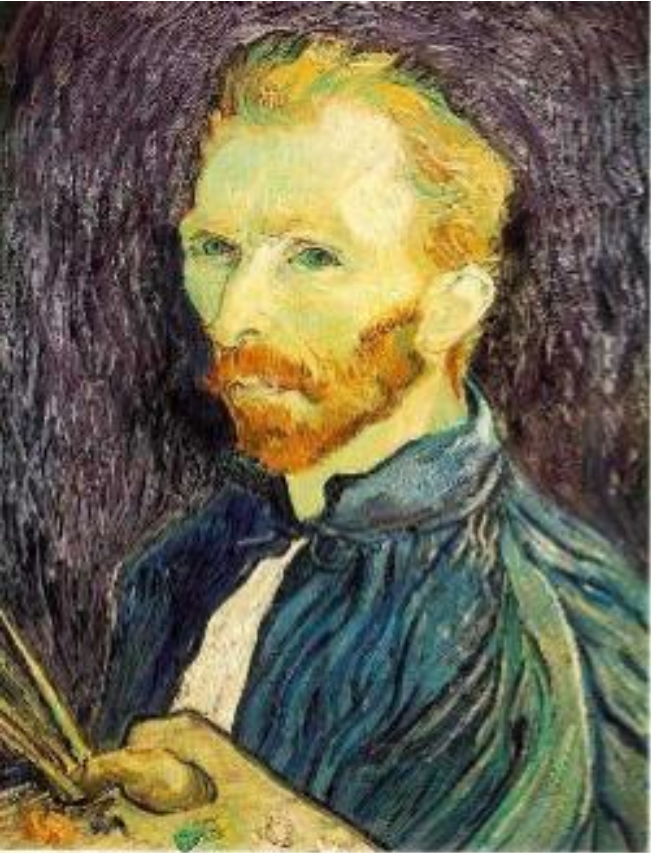
delete even rows  
delete even columns



$1/8$

What is the problem with this approach?

# Naïve image downsampling



1/2



1/4 (2x zoom)



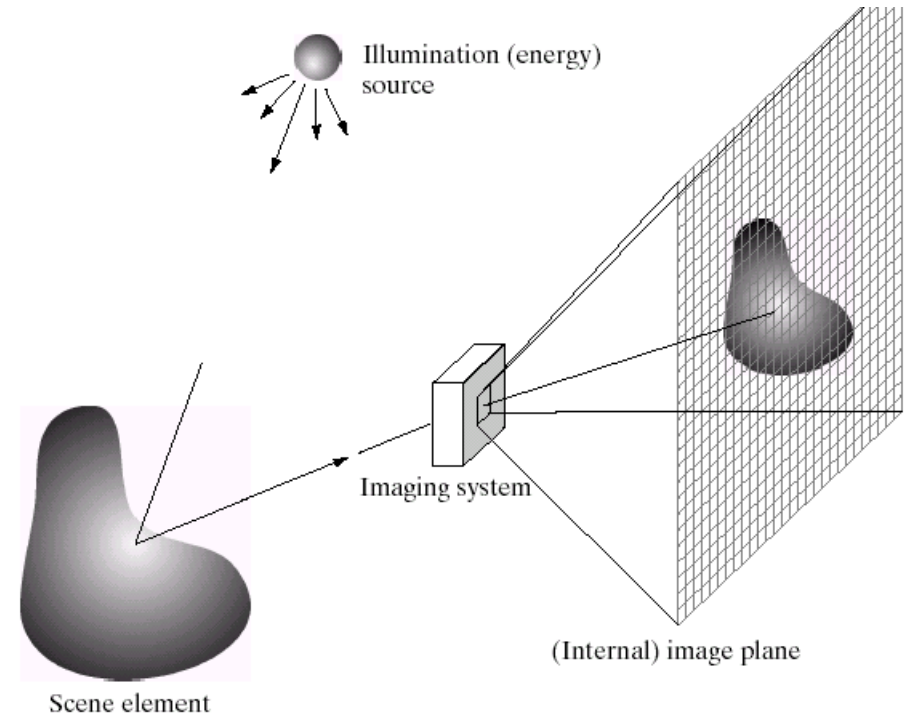
1/8 (4x zoom)

What is the 1/8 image so pixelated (and do you know what this effect is called)?



Aliasing

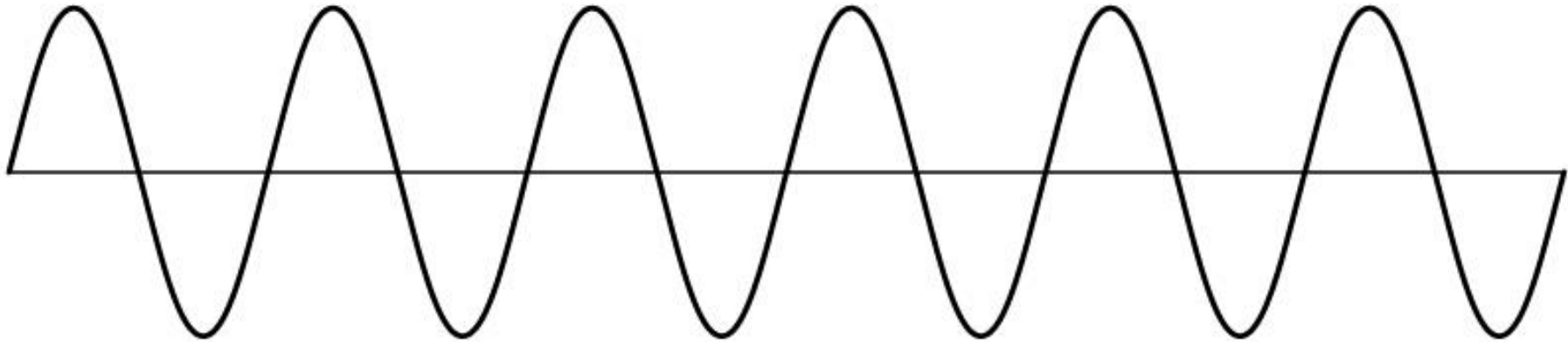
# Reminder



Images are a *discrete*, or *sampled*, representation of a *continuous* world

# Sampling

Very simple example: a sine wave

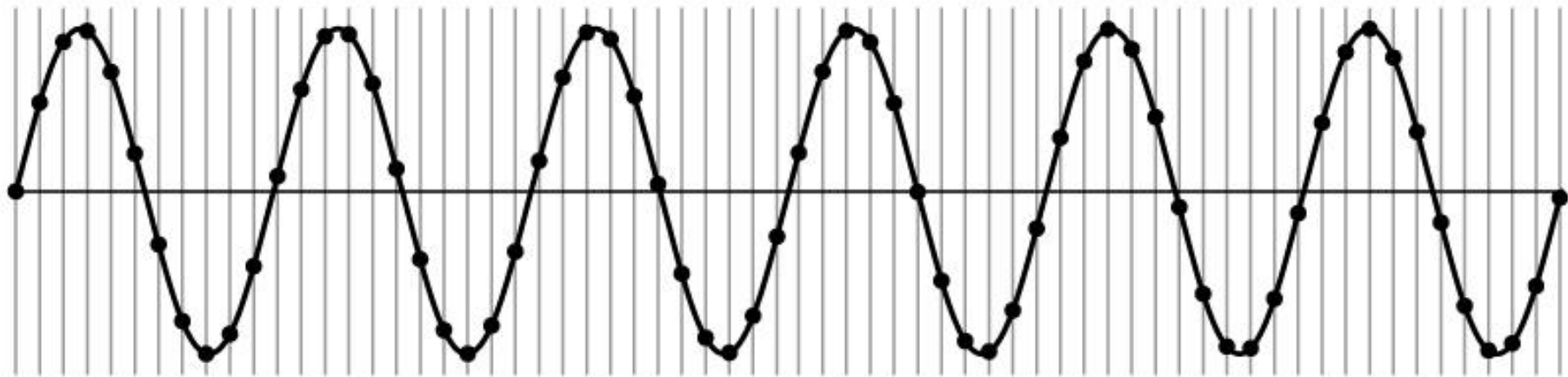


How would you discretize this signal?



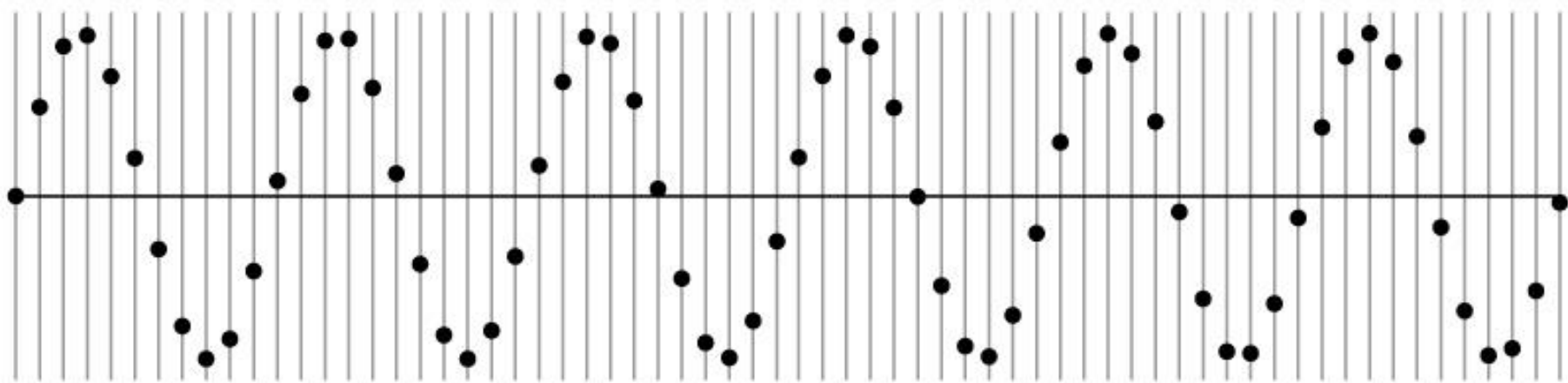
# Sampling

Very simple example: a sine wave



# Sampling

Very simple example: a sine wave

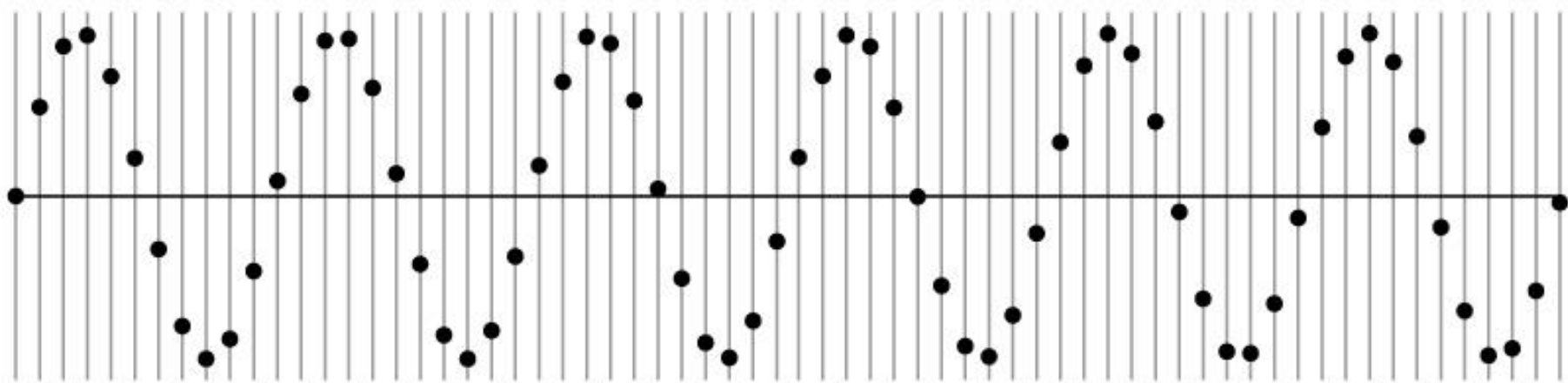


How many samples should I take?

Can I take as *many* samples as I want?

# Sampling

Very simple example: a sine wave



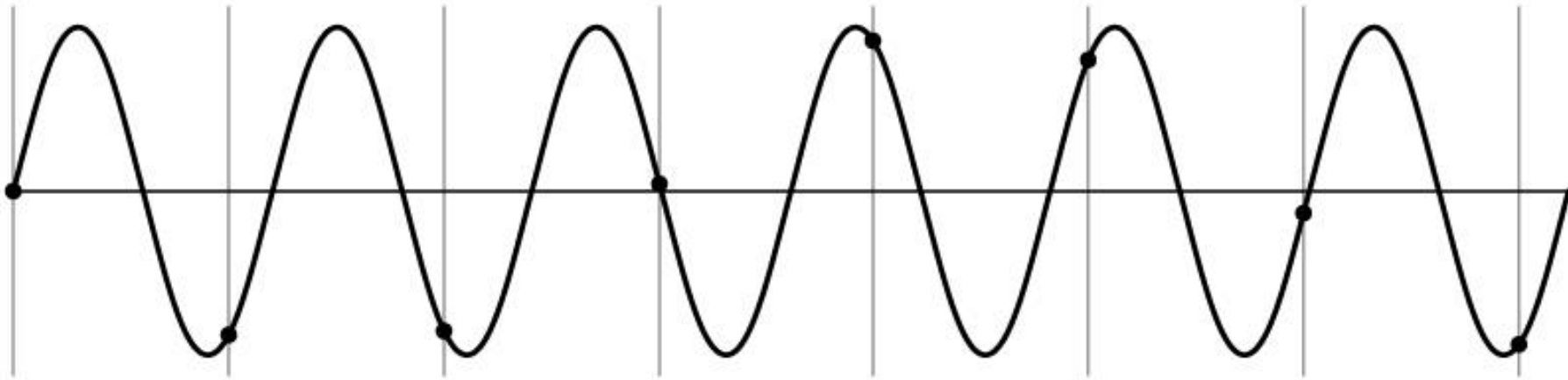
How many samples should I take?

Can I take as *few* samples as I want?



# Undersampling

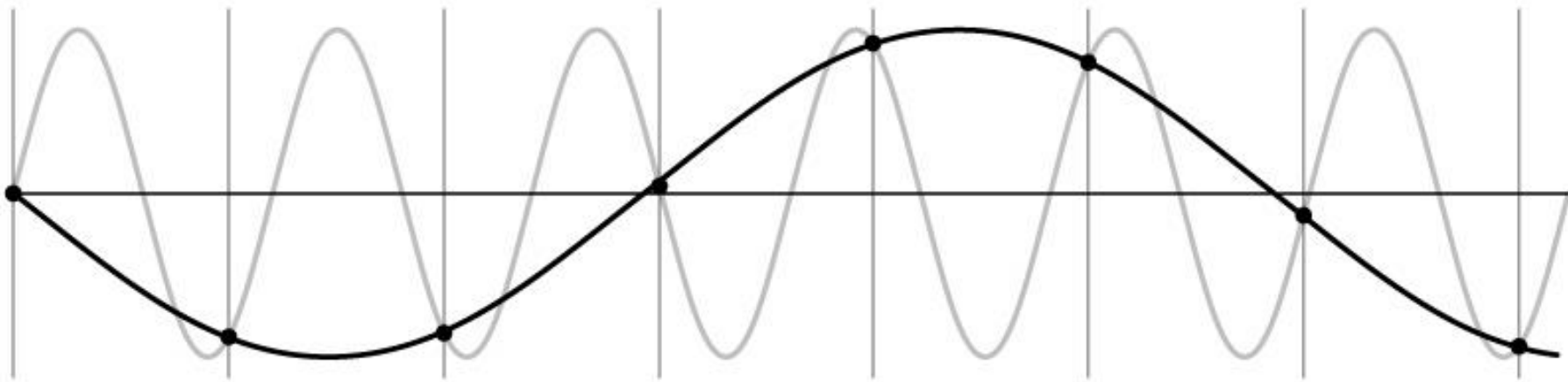
Very simple example: a sine wave



Unsurprising effect: information is lost.

# Undersampling

Very simple example: a sine wave

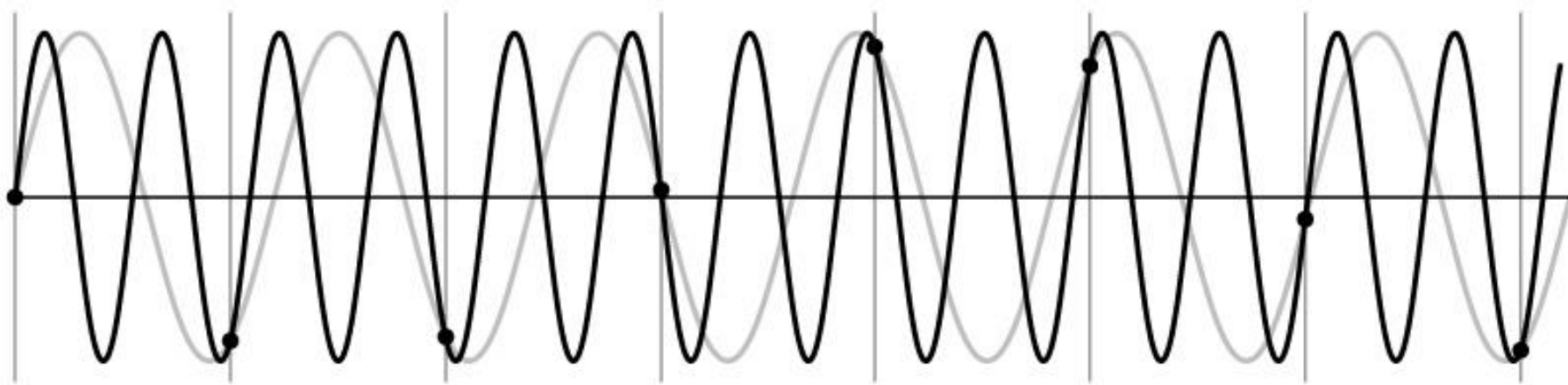


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

# Undersampling

Very simple example: a sine wave



Unsurprising effect: information is lost.

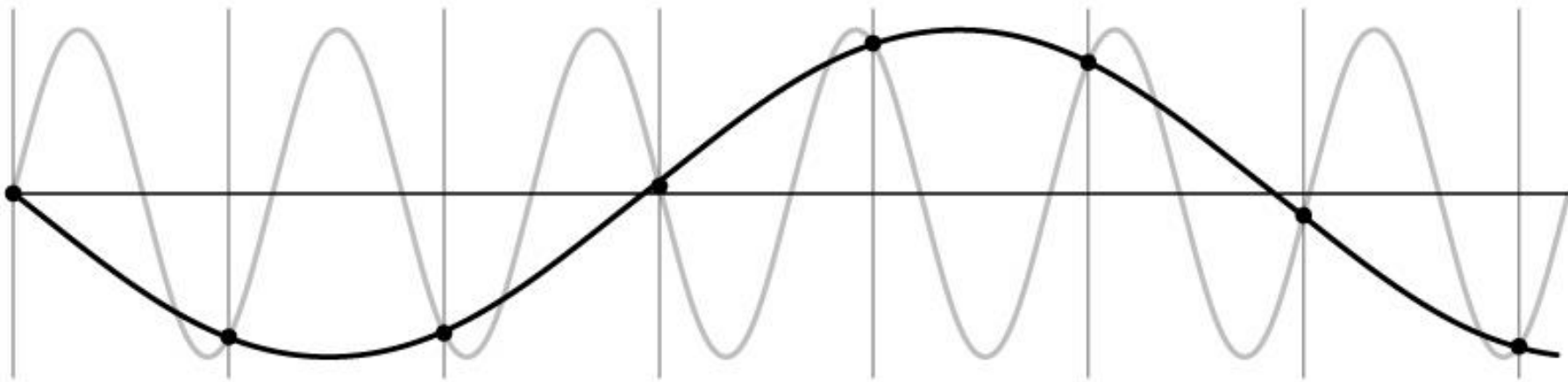
Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.



# Aliasing

Fancy term for: *Undersampling can disguise a signal as one of a lower frequency*

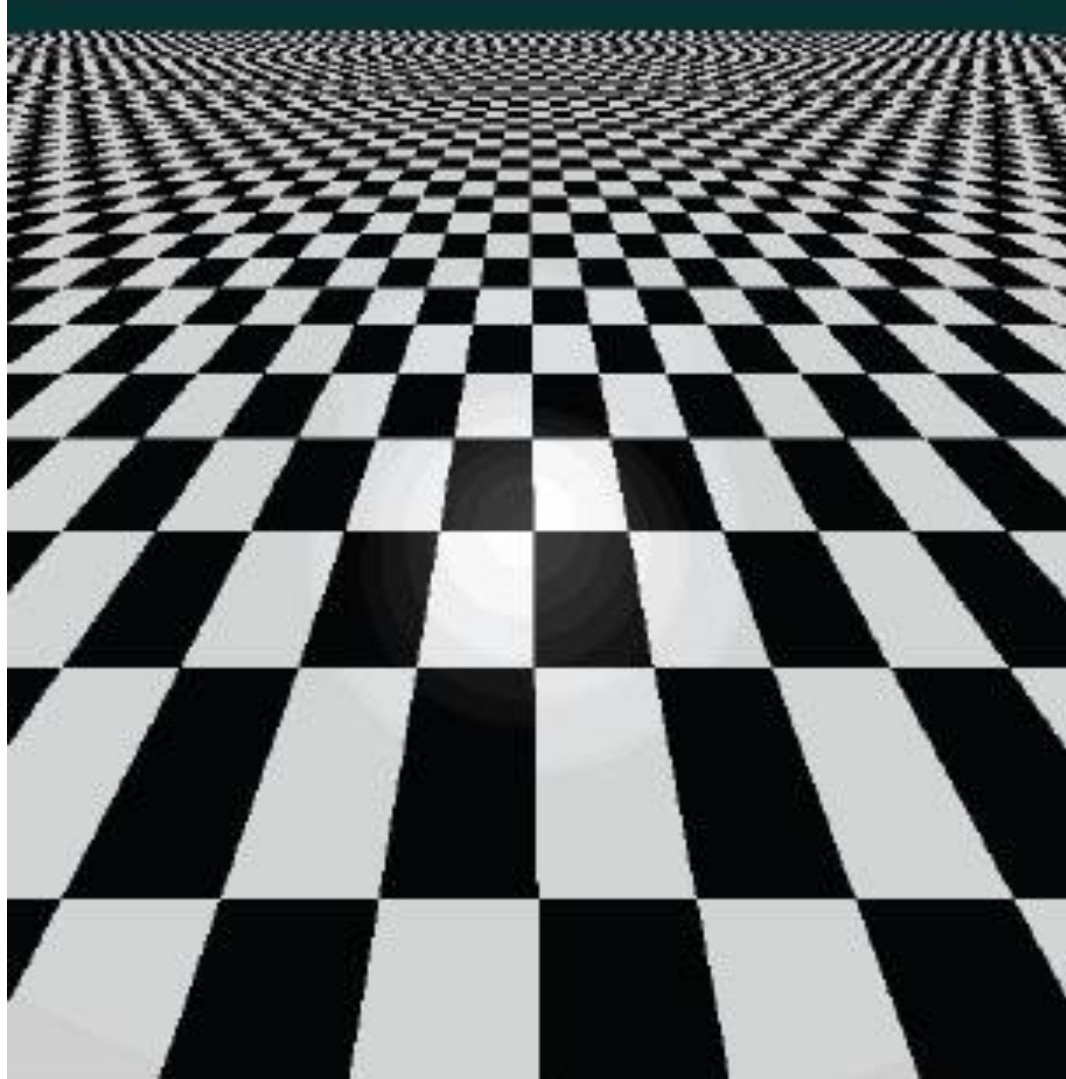


Unsurprising effect: information is lost.

Surprising effect: can confuse the signal with one of *lower* frequency.

Note: we could always confuse the signal with one of *higher* frequency.

# Aliasing in textures



# Aliasing in photographs

This is also known as “moire”

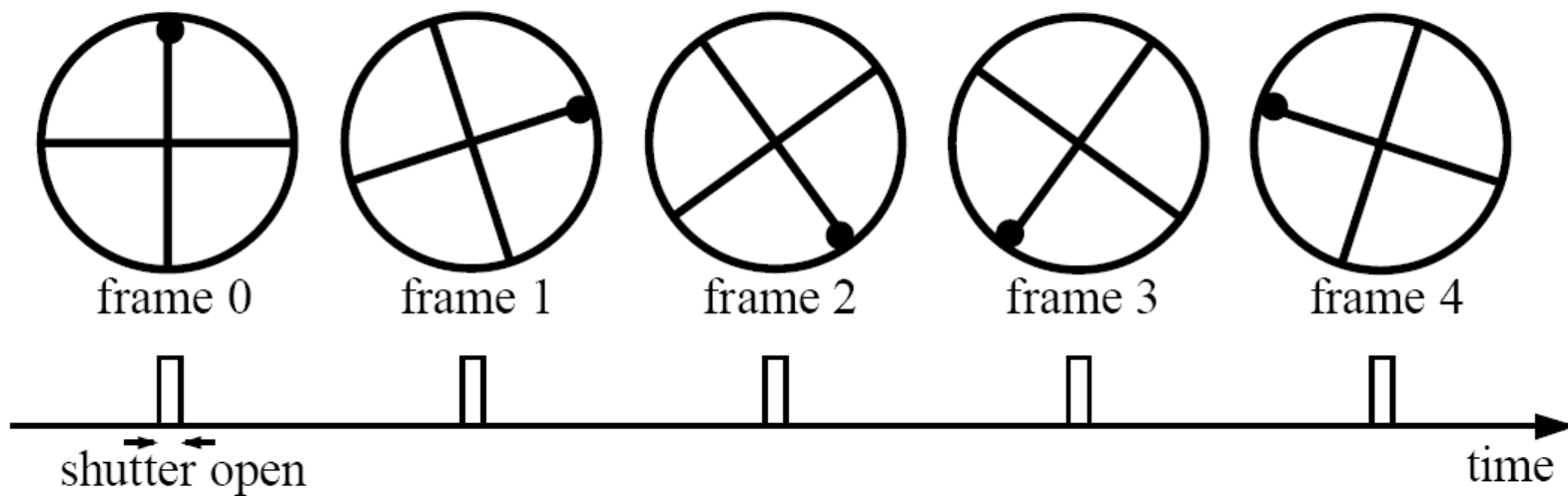


# Temporal aliasing

Imagine a spoked wheel moving to the right (rotating clockwise).

Mark wheel with dot so we can see what's happening.

If camera shutter is only open for a fraction of a frame time (frame time =  $1/30$  sec. for video,  $1/24$  sec. for film):



Without dot, wheel appears to be rotating slowly backwards!  
(counterclockwise)



# Wagon wheel effect





# Anti-aliasing

How would you deal with aliasing?

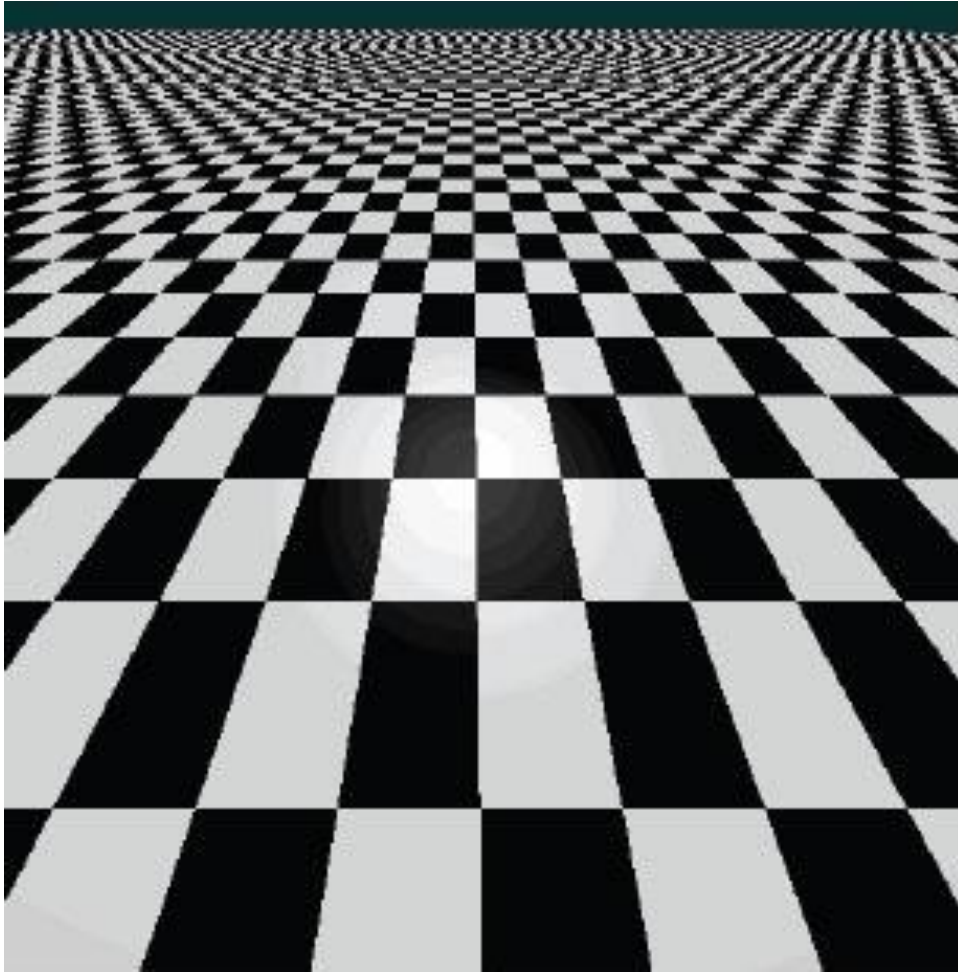
# Anti-aliasing

How would you deal with aliasing?

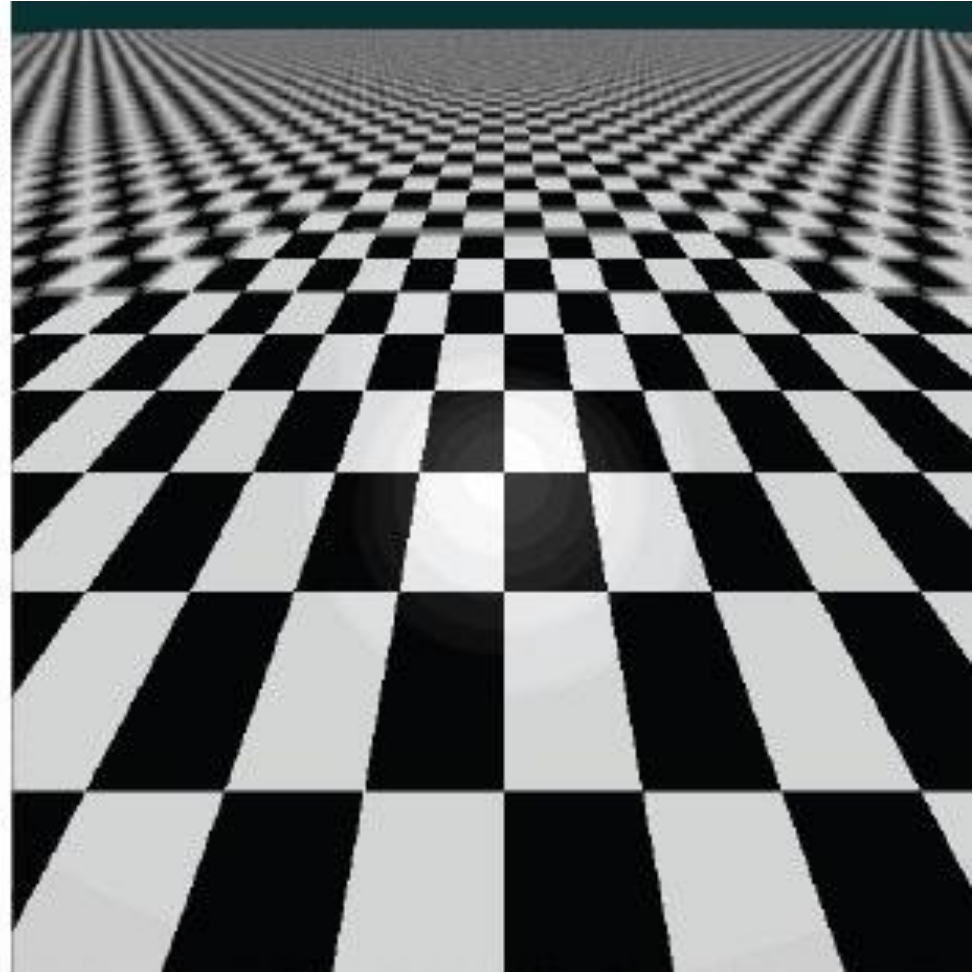
Approach 1: Oversample the signal



# Anti-aliasing in textures



aliasing artifacts



anti-aliasing by oversampling

# Anti-aliasing

How would you deal with aliasing?

Approach 1: Oversample the signal

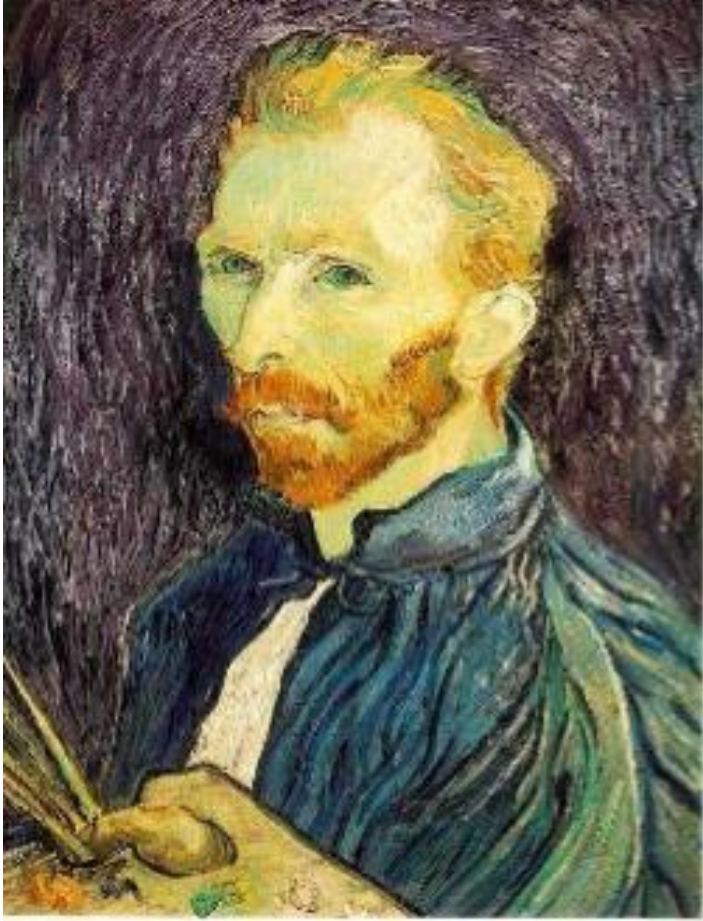
Approach 2: Smooth the signal

- Remove some of the detail effects that cause aliasing.
- Lose information, but better than aliasing artifacts.

How would you smooth a signal?

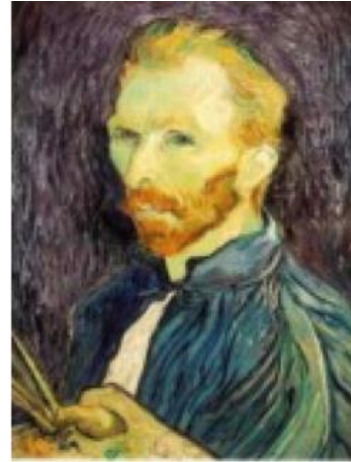
# Better image downsampling

Apply a smoothing filter first, then throw away half the rows and columns



1/2

Gaussian filter  
delete even rows  
delete even columns



1/4

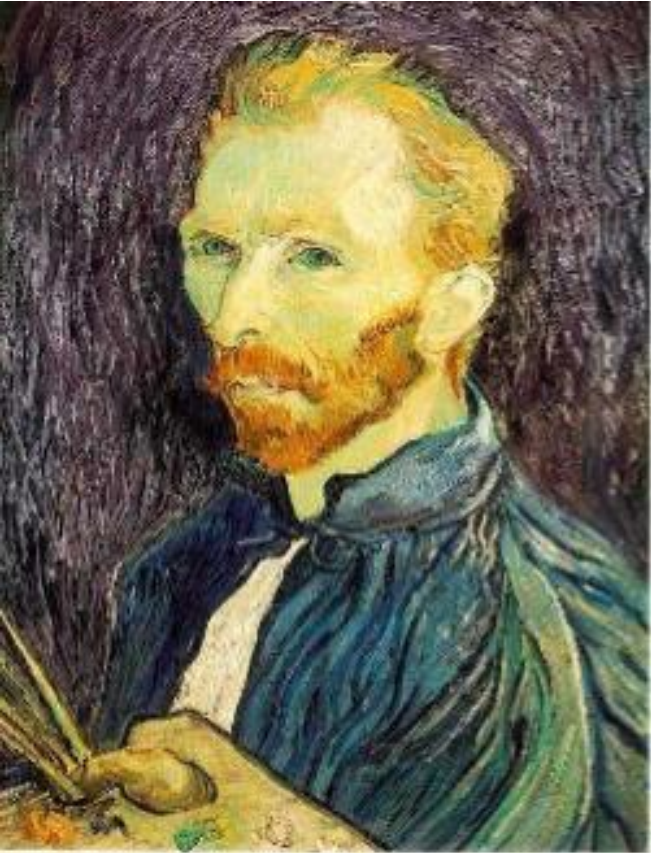
Gaussian filter  
delete even rows  
delete even columns



1/8



# Better image downsampling



$1/2$

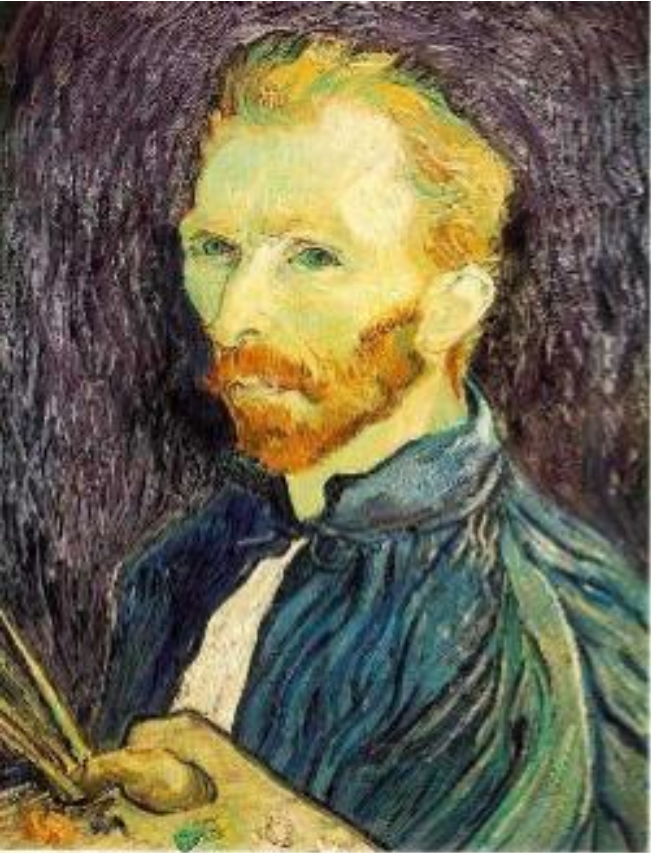


$1/4$  (2x zoom)



$1/8$  (4x zoom)

# Naïve image downsampling



$1/2$



$1/4$  (2x zoom)



$1/8$  (4x zoom)



# Anti-aliasing

Question 1: How much smoothing do I need to do to avoid aliasing?

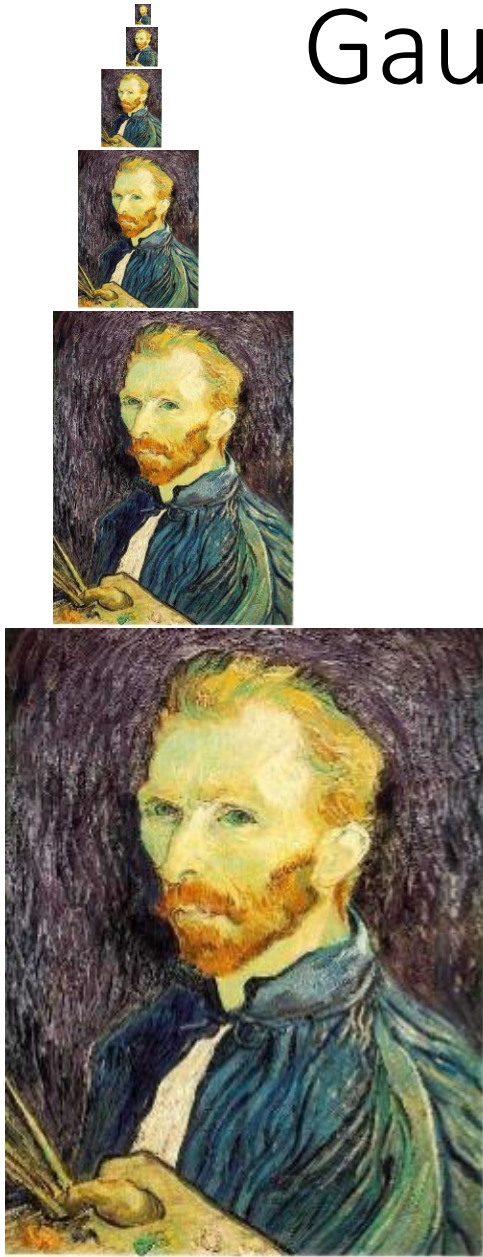
Question 2: How many samples do I need to take to avoid aliasing?

Answer to both: Enough to reach the Nyquist limit.

We'll see what this means soon.

Gaussian image pyramid

# Gaussian image pyramid

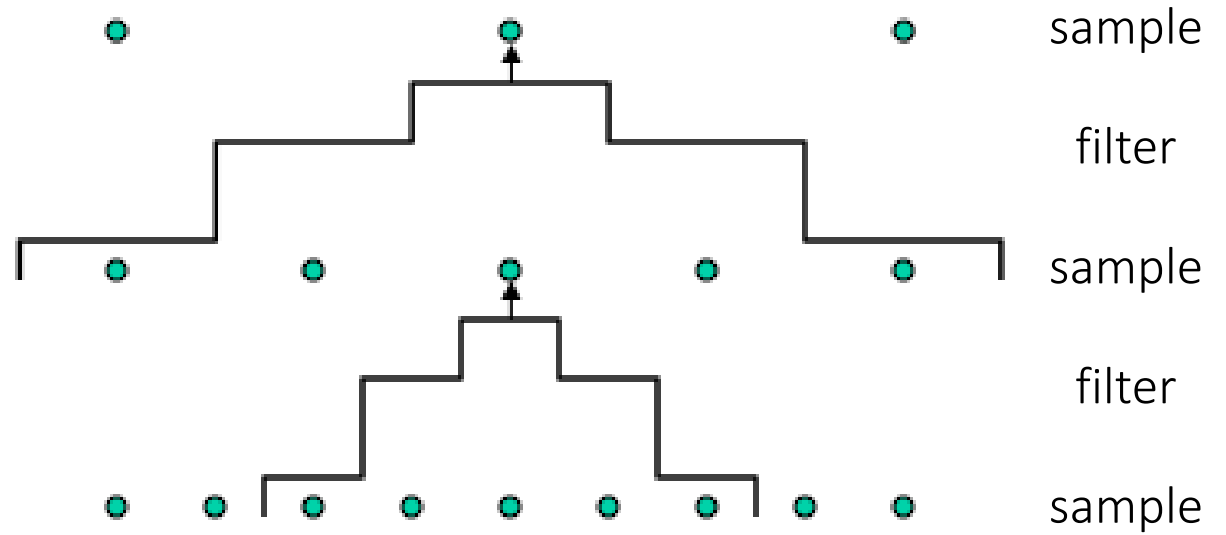


The name of this sequence of subsampled images

# Constructing a Gaussian pyramid

## Algorithm

```
repeat:  
    filter  
    subsample  
until min resolution reached
```

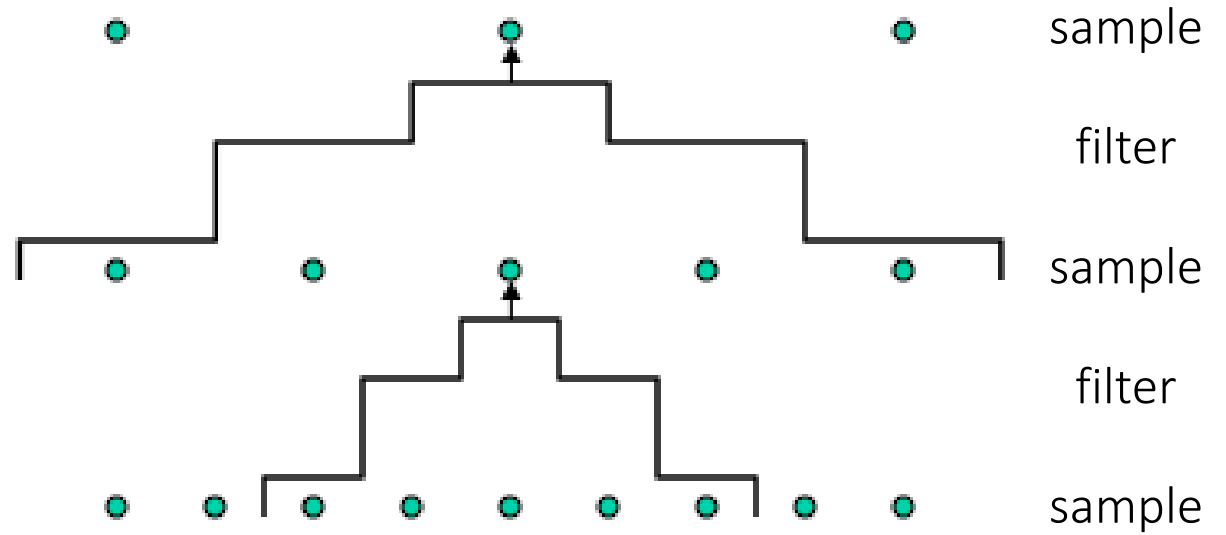


Question: How much bigger than the original image is the whole pyramid?

# Constructing a Gaussian pyramid

## Algorithm

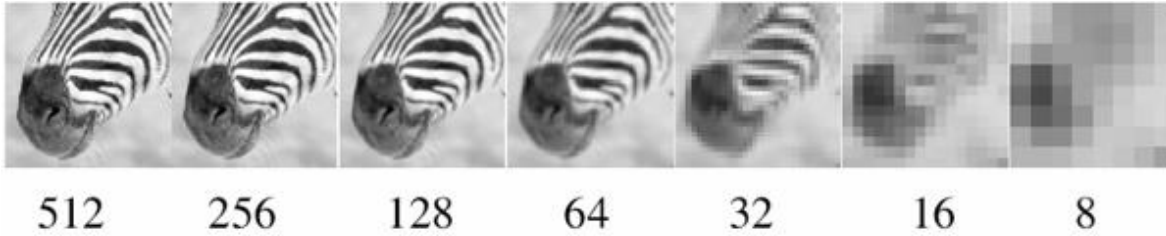
```
repeat:  
    filter  
    subsample  
until min resolution reached
```



Question: How much bigger than the original image is the whole pyramid?

Answer: Just  $\frac{4}{3}$  times the size of the original image! (How did I come up with this number?)

# Some properties of the Gaussian pyramid

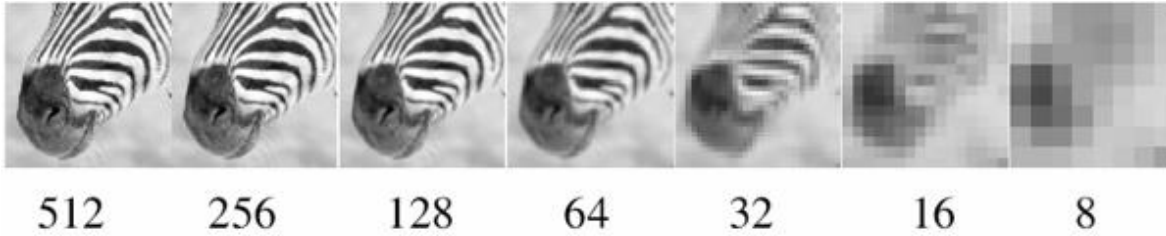


What happens to the details of the image?





# Some properties of the Gaussian pyramid



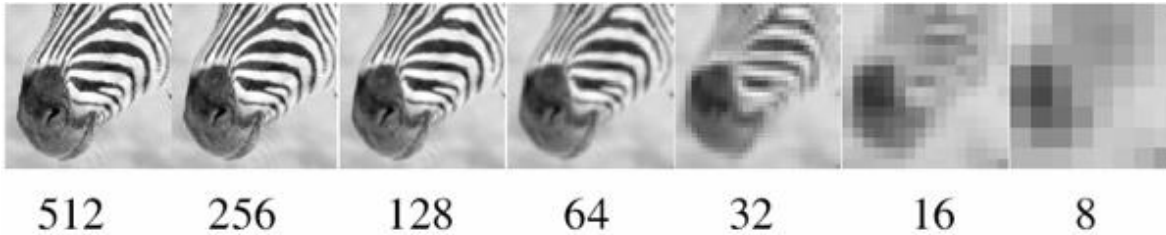
What happens to the details of the image?

- They get smoothed out as we move to higher levels.



What is preserved at the higher levels?

# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

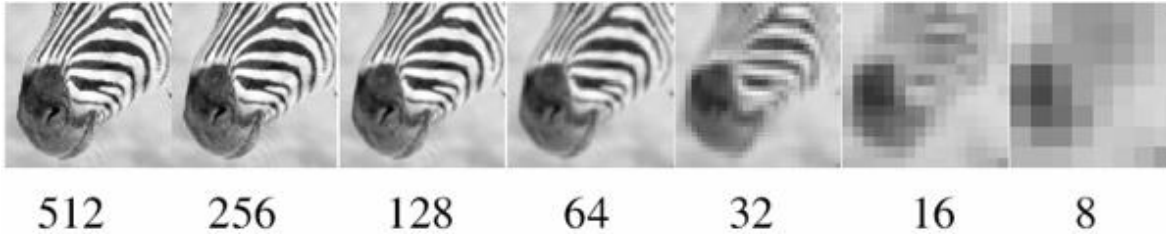
What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?



# Some properties of the Gaussian pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

- That's not possible.



# Blurring is lossy



level 0

-



level 1 (before downsampling)

=



residual

What does the residual look like?

# Blurring is lossy



level 0

-



level 1 (before downsampling)

=



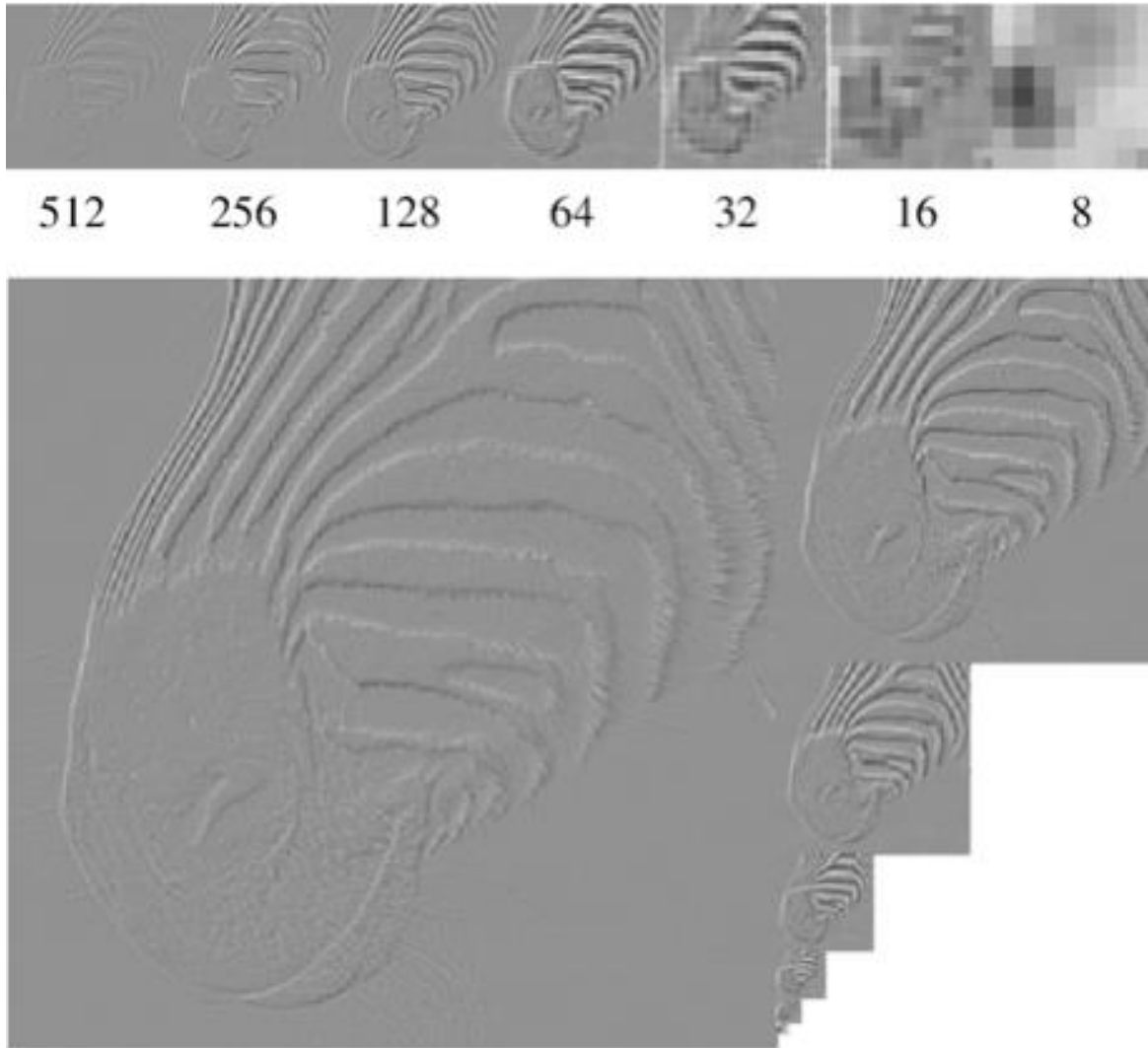
residual

Can we make a pyramid that is lossless?

Laplacian image pyramid



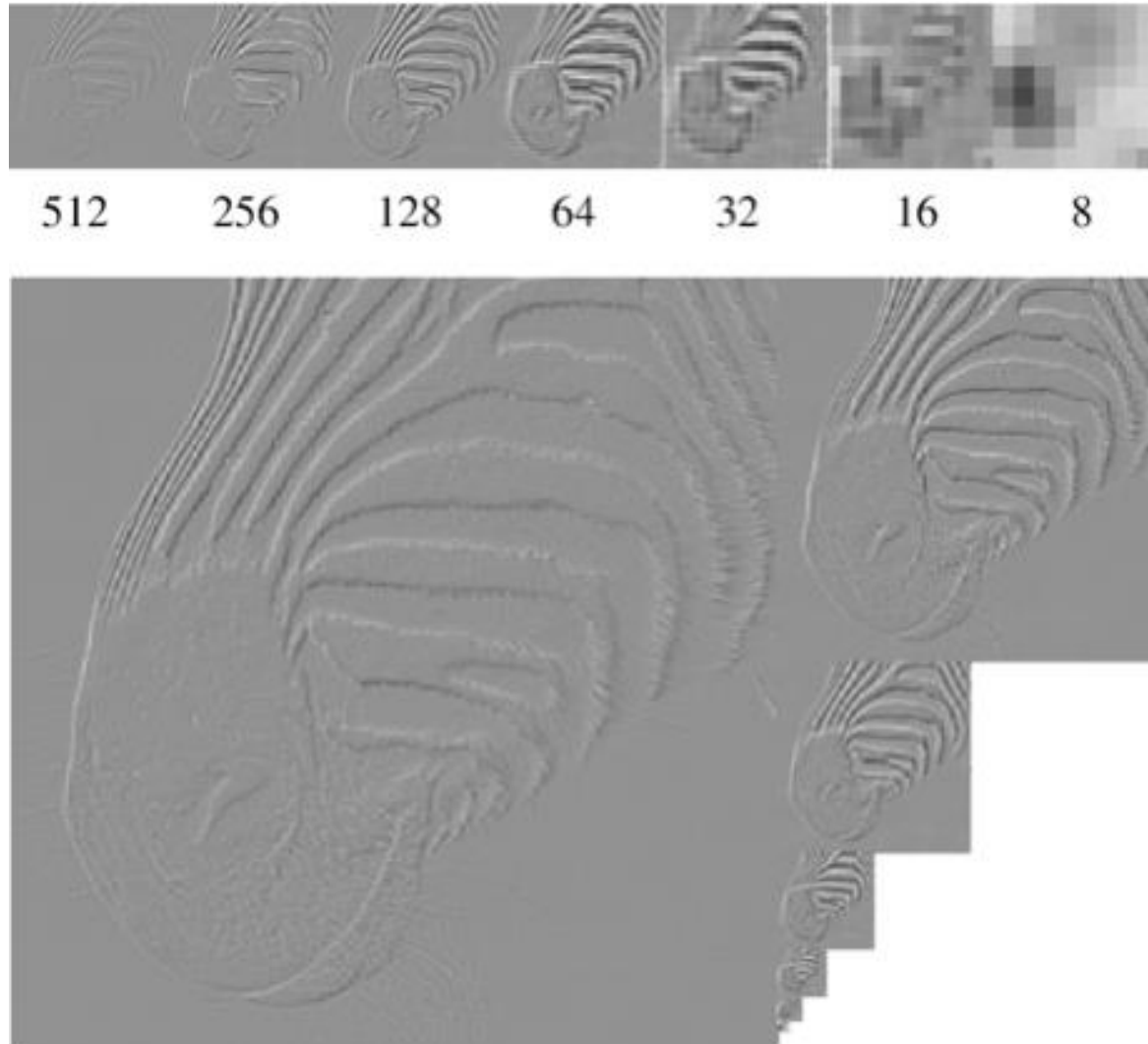
# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?

# Laplacian image pyramid



At each level, retain the residuals instead of the blurred images themselves.

Can we reconstruct the original image using the pyramid?

- Yes we can!



What do we need to store to be able to reconstruct the original image?

# Let's start by looking at just one level



level 0

=



level 1 (upsampled)

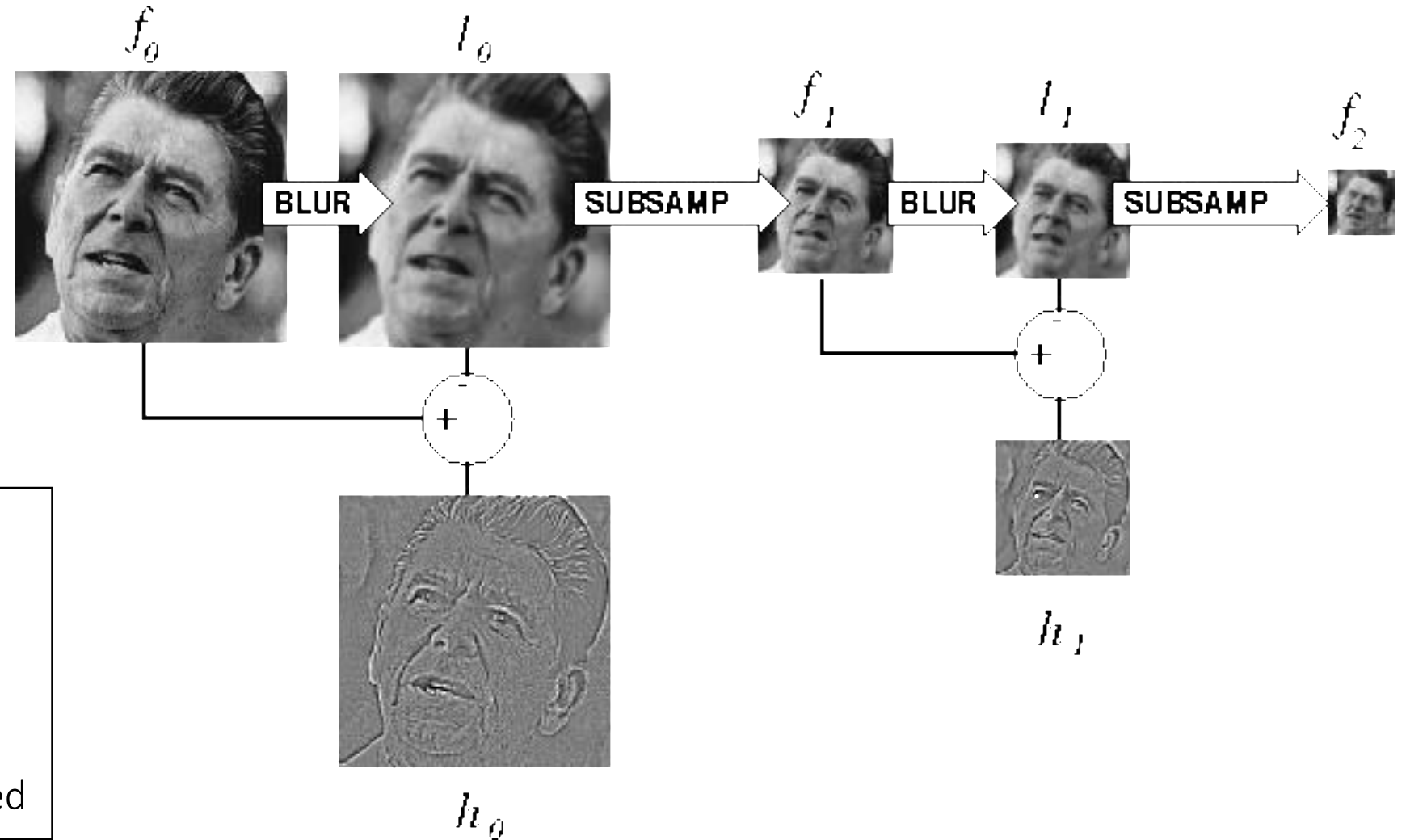
+



residual

Does this mean we need to store both residuals and the blurred copies of the original?

# Constructing a Laplacian pyramid



## Algorithm

repeat:

filter

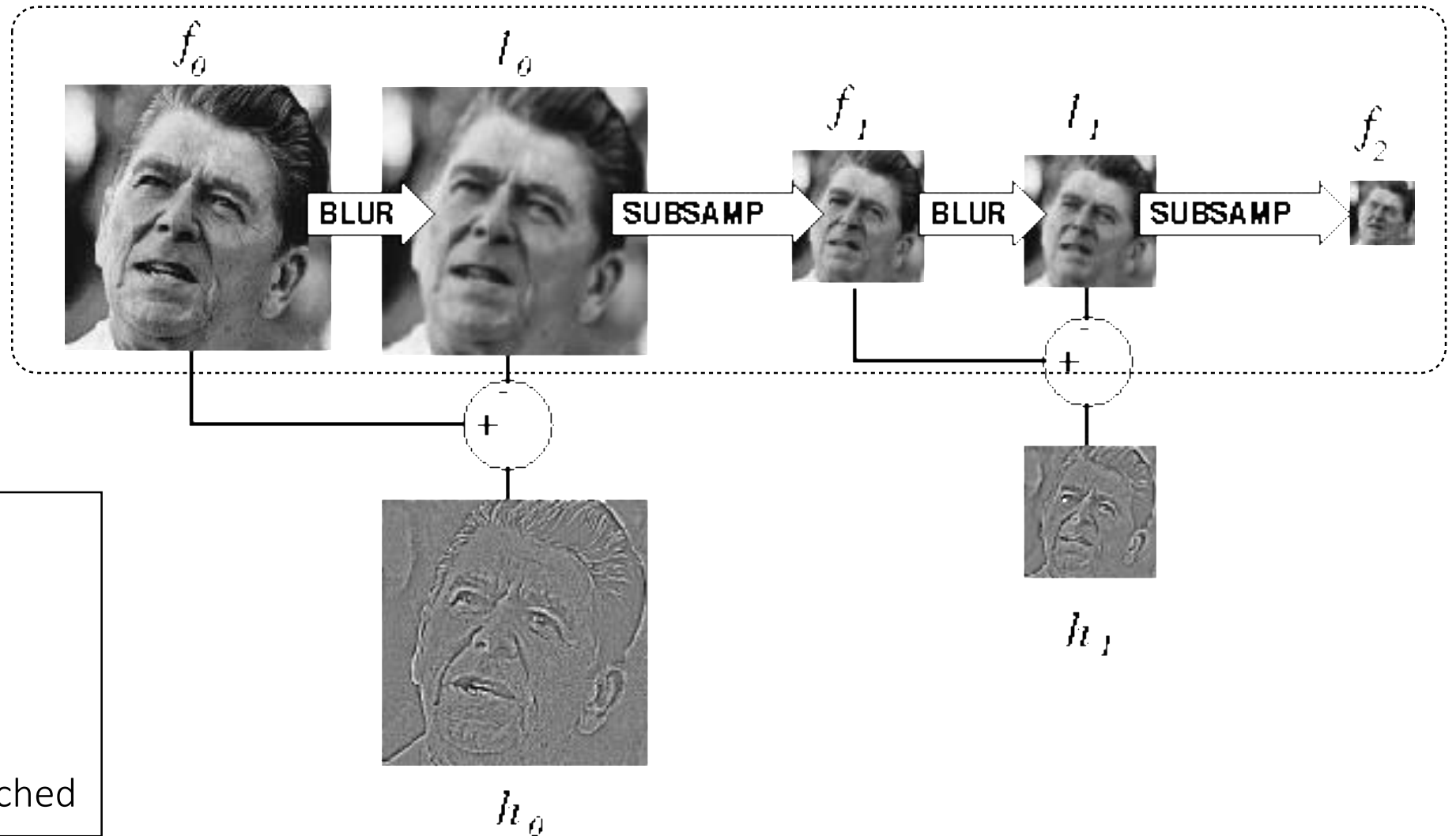
compute residual

subsample

until min resolution reached

# Constructing a Laplacian pyramid

What is this part?



## Algorithm

repeat:

- filter

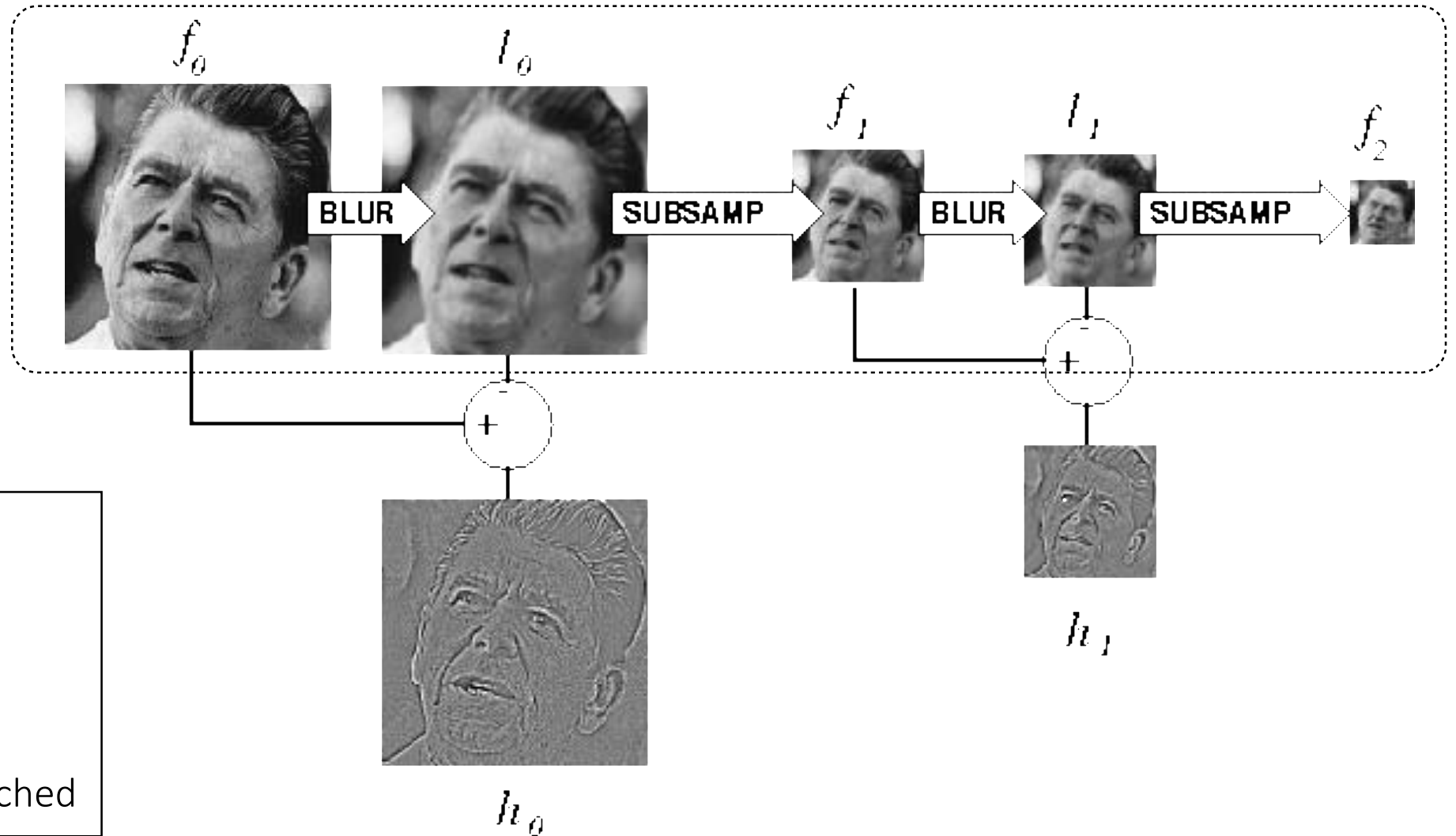
- compute residual

- subsample

until min resolution reached

# Constructing a Laplacian pyramid

It's a Gaussian pyramid.



## Algorithm

repeat:

filter

compute residual

subsample

until min resolution reached

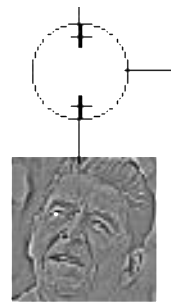


# What do we need to construct the original image?

$f_0$

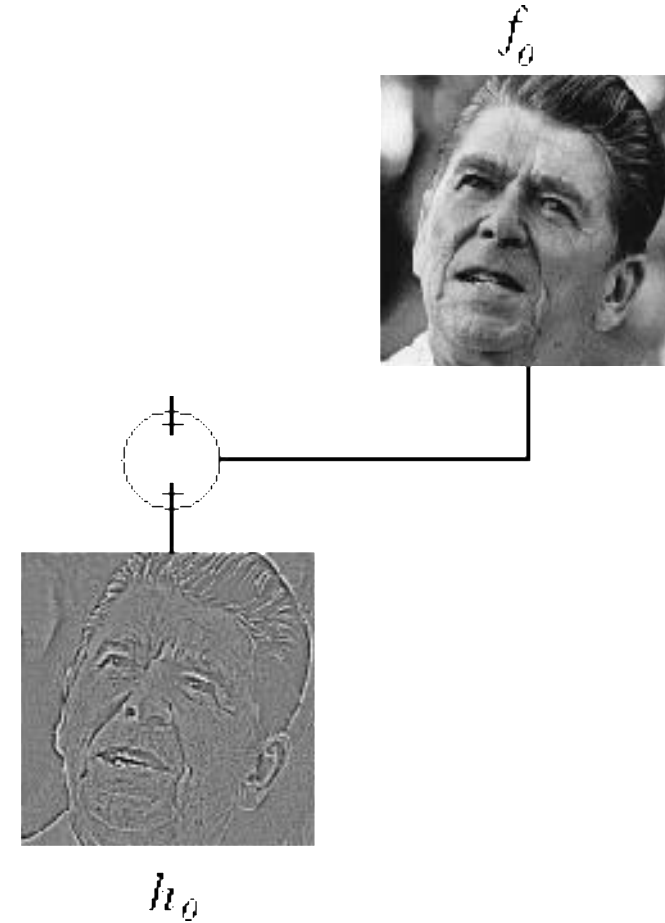


# What do we need to construct the original image?



$h_1$

(1) residuals



$h_0$

# What do we need to construct the original image?

(2) smallest  
image  $f_2$

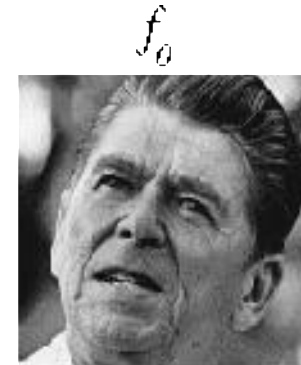


$h_1$

(1) residuals

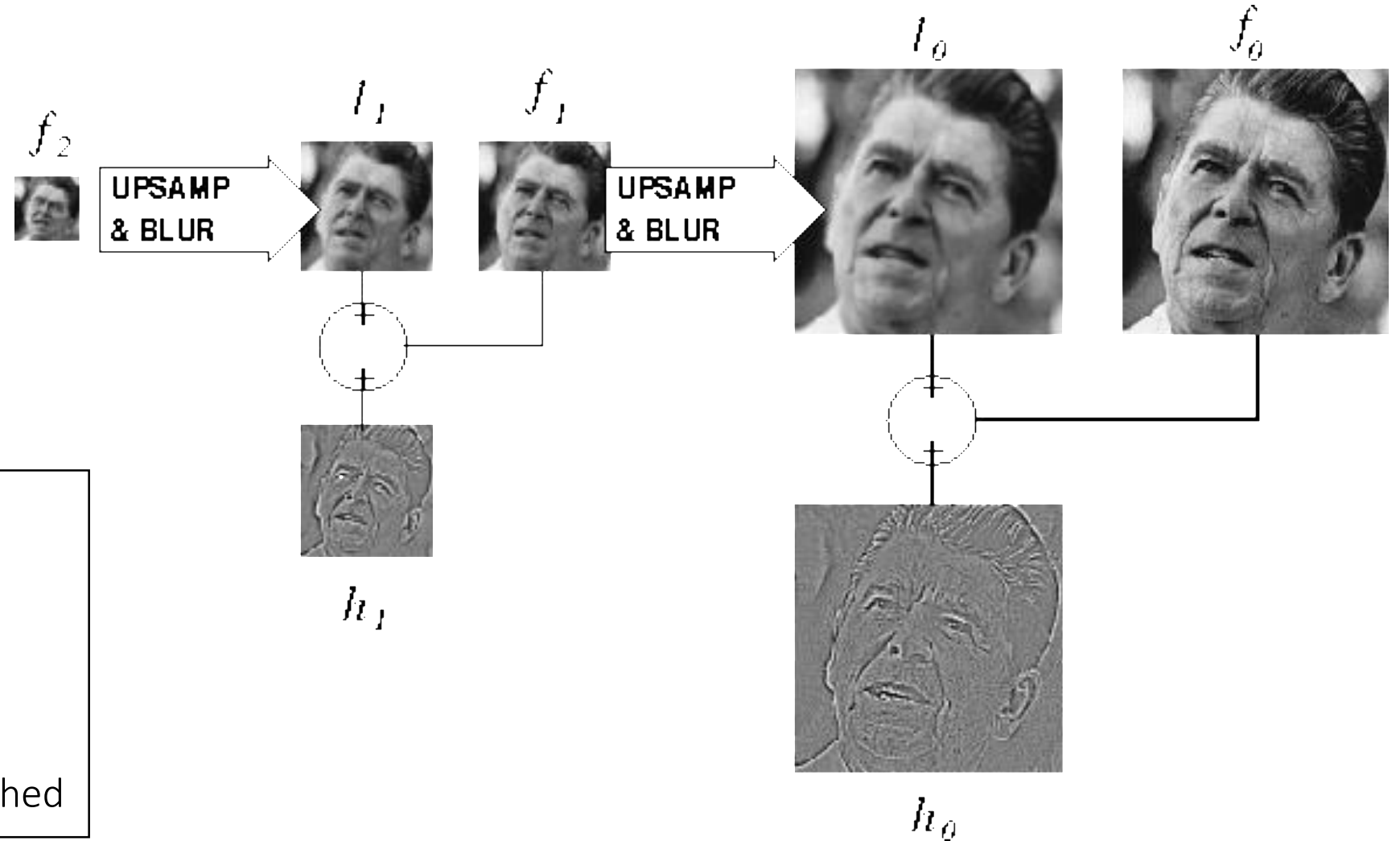


$h_0$



$f_0$

# Reconstructing the original image



## Algorithm

repeat:

    upsample

    sum with residual

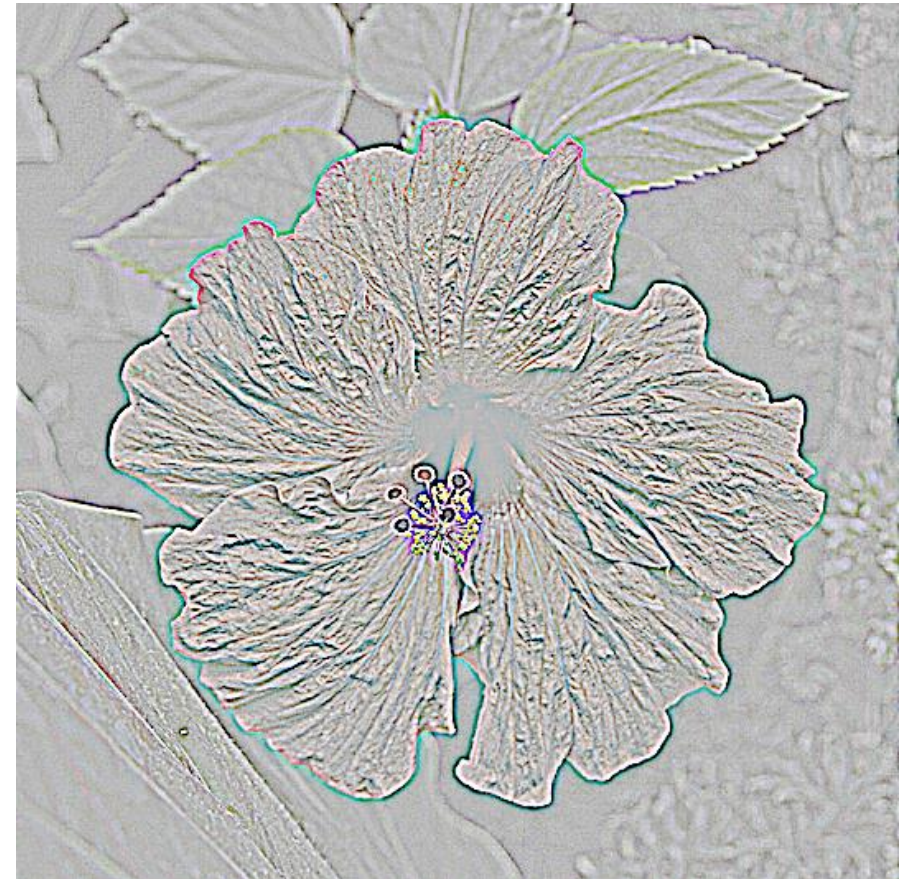
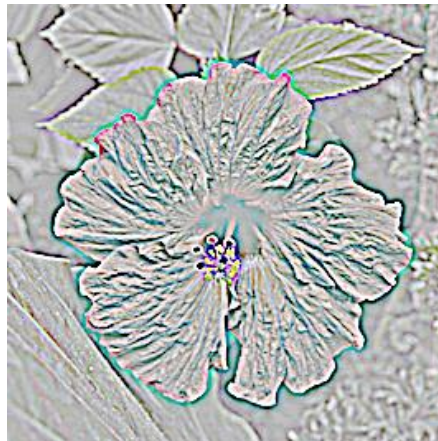
until orig resolution reached

# Gaussian vs Laplacian Pyramid



Shown in opposite  
order for space.

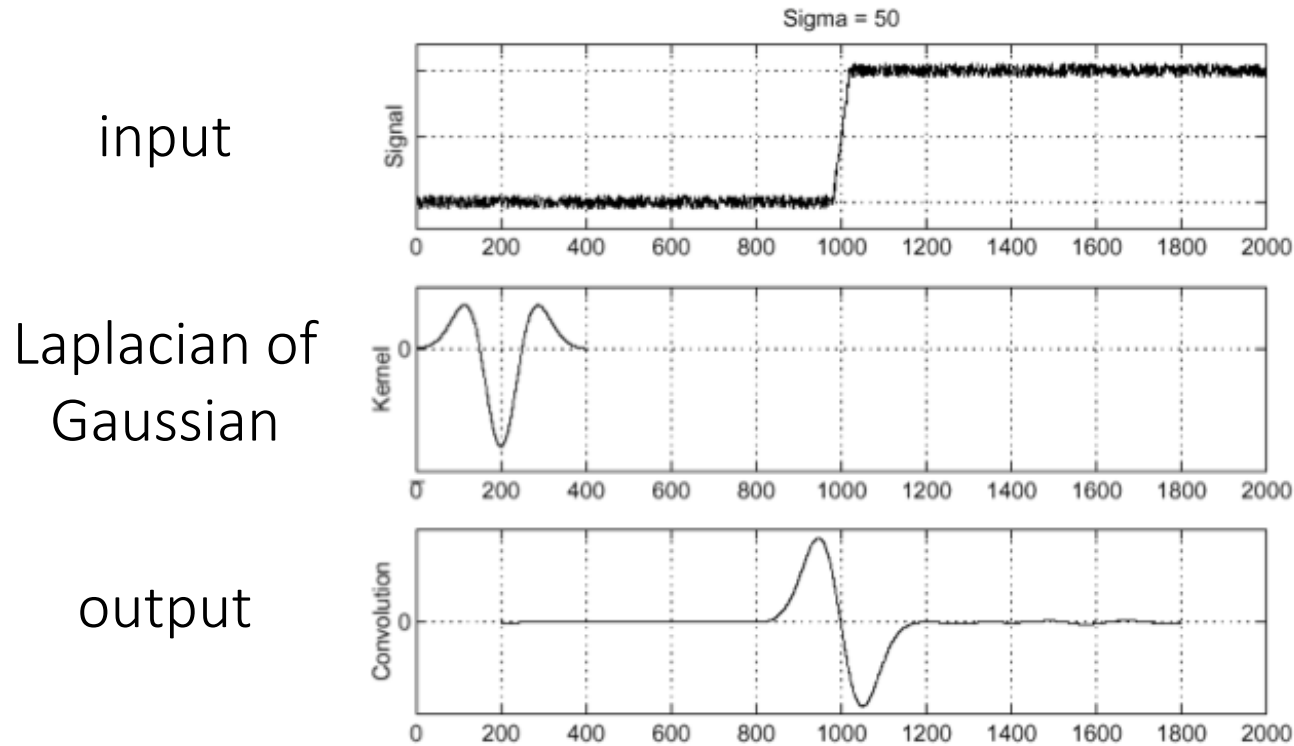
Which one takes  
more space to store?



Why is it called a Laplacian pyramid?

# Reminder: Laplacian of Gaussian (LoG) filter

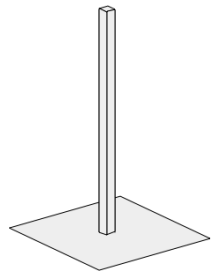
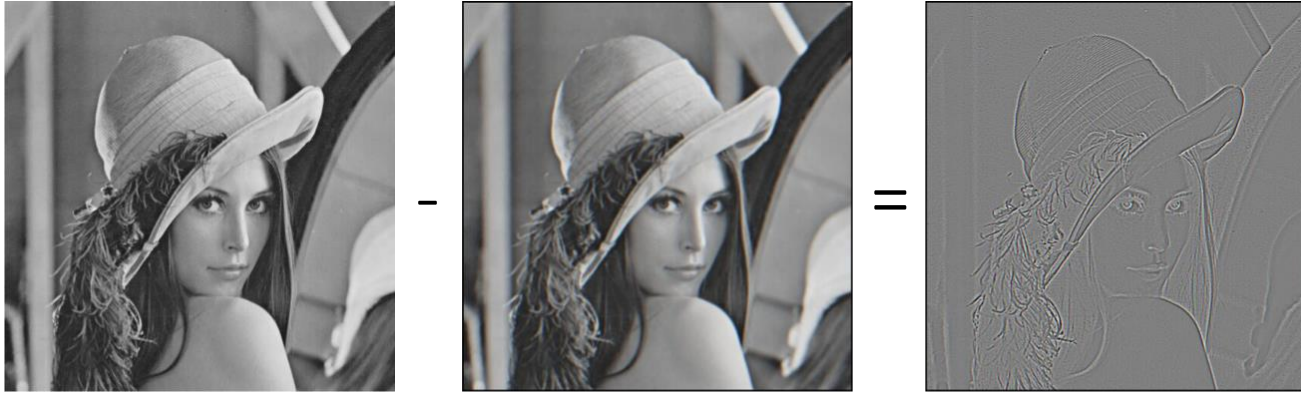
As with derivative, we can combine Laplace filtering with Gaussian filtering



“zero crossings” at edges

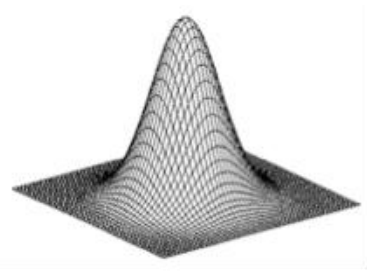


# Why is it called a Laplacian pyramid?



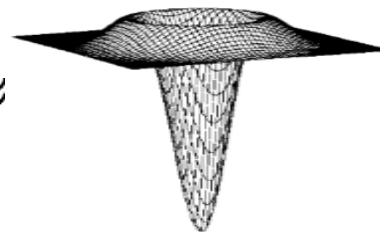
unit

-

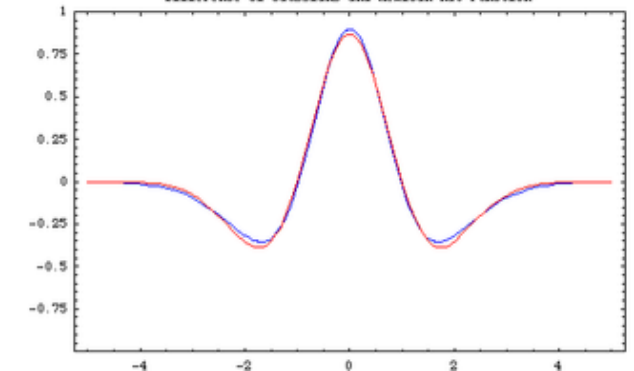
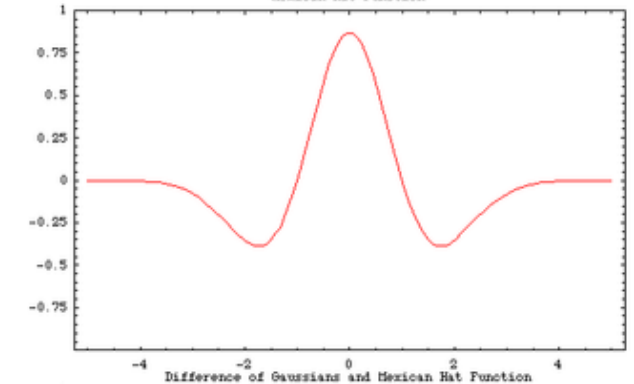
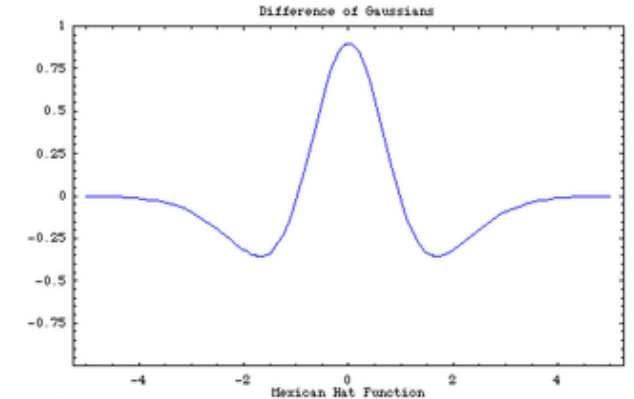


Gaussian

$\approx$

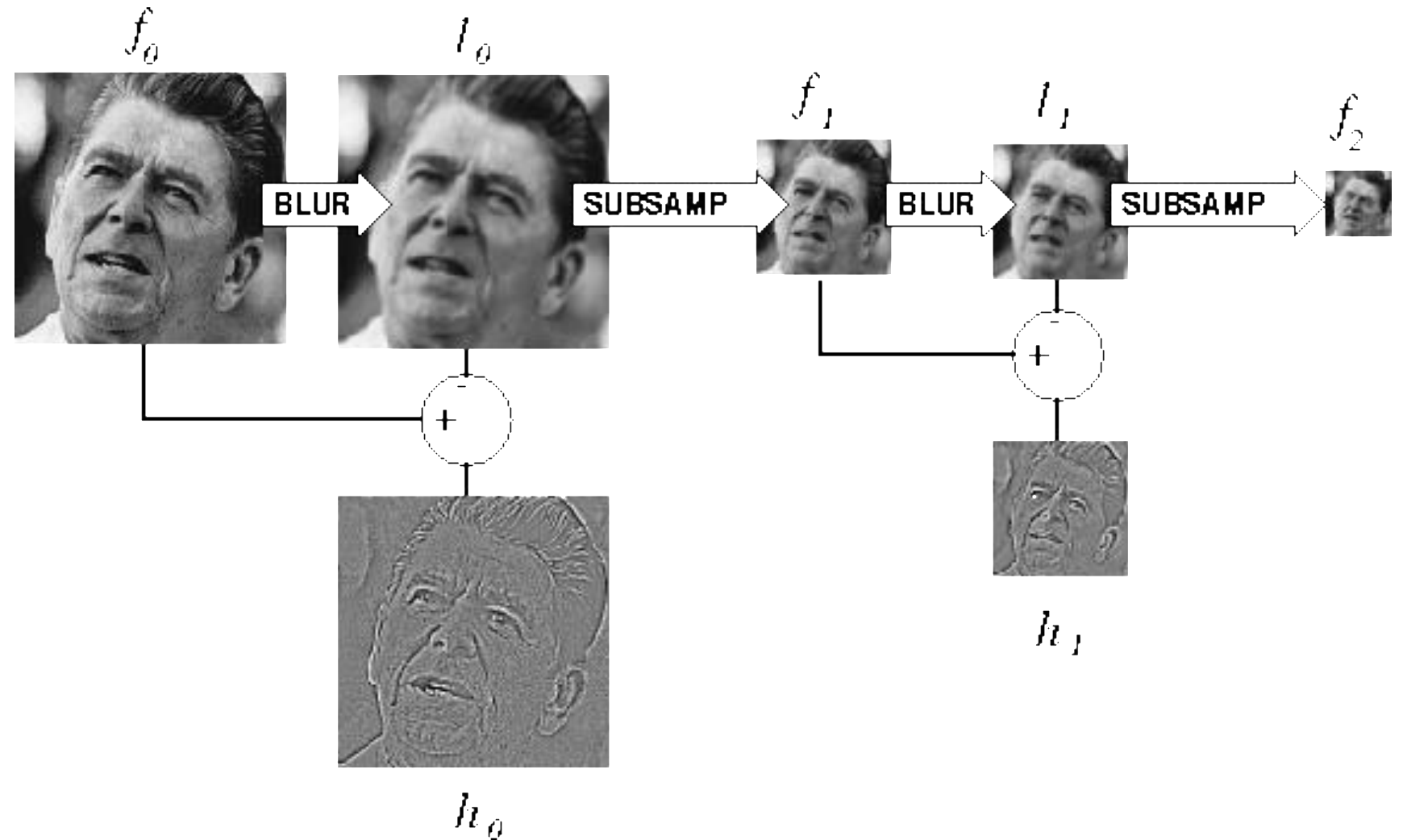


Laplacian



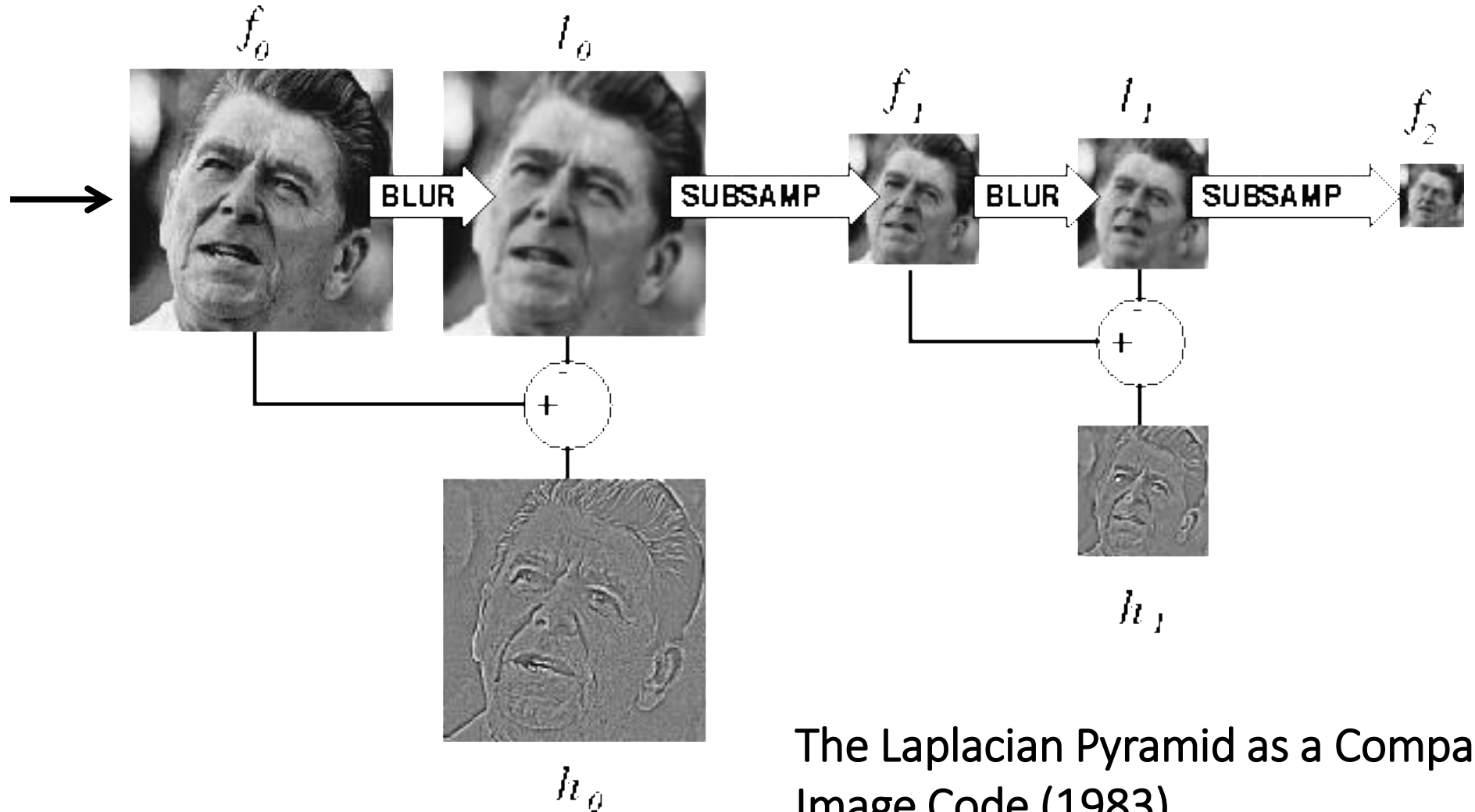
Difference of Gaussians approximates the Laplacian

# Why Reagan?



# Why Reagan?

Ronald Reagan was  
President when the  
Laplacian pyramid  
was invented



The Laplacian Pyramid as a Compact  
Image Code (1983)

Peter J. Burt , Edward H. Adelson



Still used extensively





# Still used extensively



foreground details enhanced, background details reduced



input image



user-provided mask

# Other types of pyramids

Steerable pyramid: At each level keep multiple versions, one for each direction.



Wavelets: Huge area in image processing (see 18-793).





# What are image pyramids used for?

image compression



multi-scale  
texture mapping

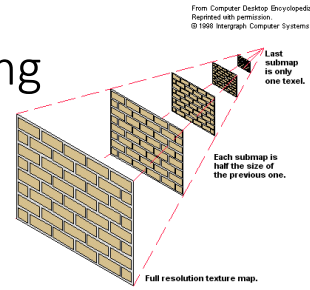
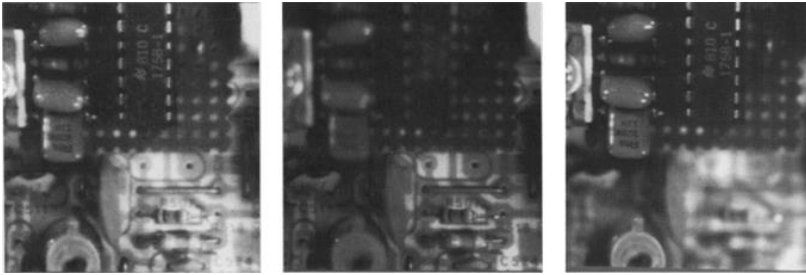


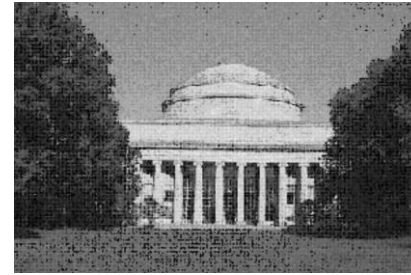
image blending



focal stack compositing



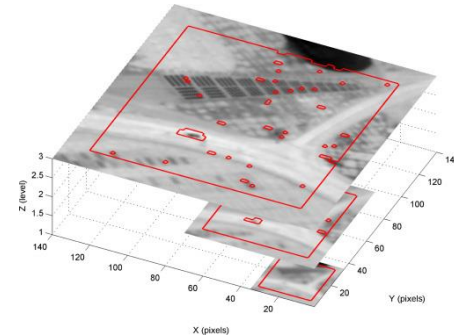
denoising



multi-scale detection



multi-scale registration



# References

Basic reading:

- Szeliski textbook, Sections 3.5

Additional reading:

- Burt and Adelson, “The Laplacian Pyramid as a Compact Image Code,” IEEE ToC 1983.  
the original Laplacian pyramid paper