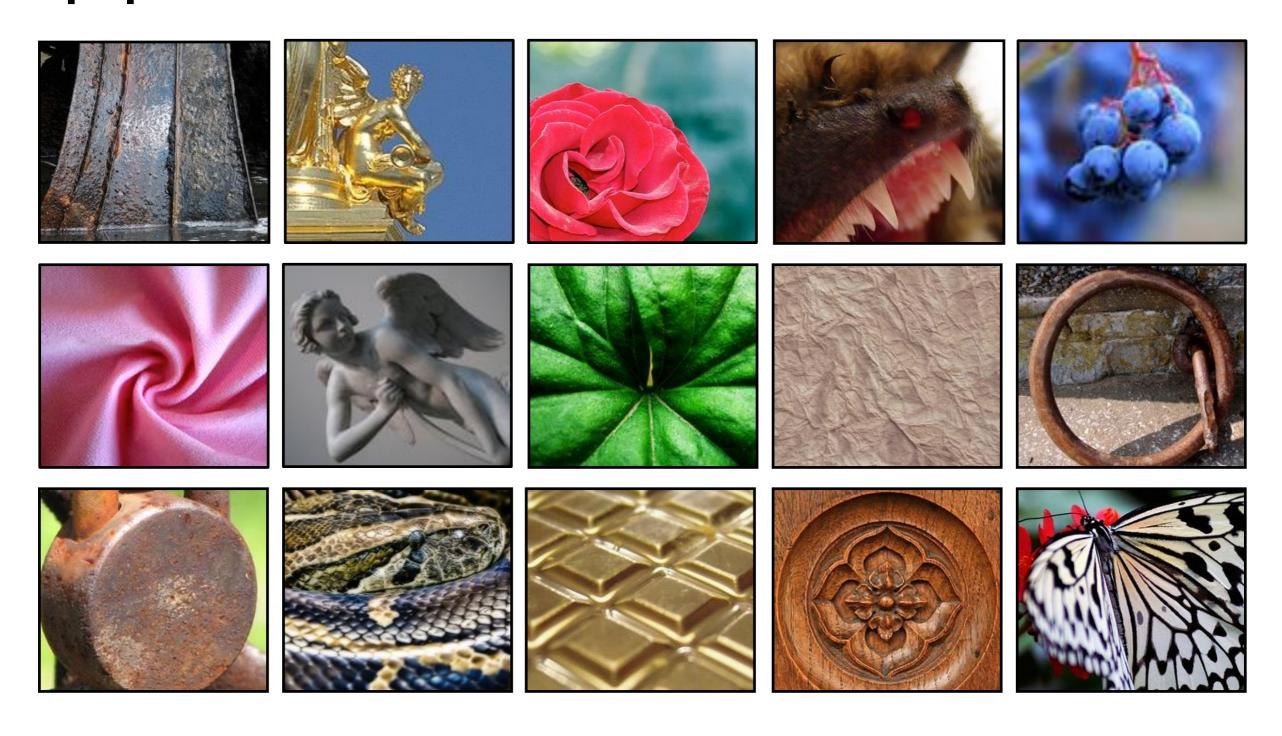
Appearance and reflectance



16-385 Computer Vision Spring 2018, Lecture 13

http://www.cs.cmu.edu/~16385/

Course announcements

- Apologies for cancelling last Wednesday's lecture.
- Homework 3 has been posted and is due on March 9th.
 - Any questions about the homework?
 - How many of you have looked at/started/finished homework 3?
- Office hours for Yannis' this week: Wednesday 3-5 pm.
- Results from poll for adjusting Yannis' regular office hours: They stay the same.
- Many talks this week:
- 1. Judy Hoffman, "Adaptive Adversarial Learning for a Diverse Visual World," Monday March 5th, 10:00 AM, NSH 3305.
- 2. Manolis Savva, "Human-centric Understanding of 3D Environments," Wednesday March 7, 2:00 PM, NSH 3305.
- 3. David Fouhey, "Recovering a Functional and Three Dimensional Understanding of Images," Thursday March 8, 4:00 PM, NSH 3305.
- How many of you went to Pulkit Agrawal's talk last week?

Overview of today's lecture

- Appearance phenomena.
- Measuring light and radiometry.
- Reflectance and BRDF.

Slide credits

Most of these slides were adapted from:

- Srinivasa Narasimhan (16-385, Spring 2014).
- Todd Zickler (Harvard University).
- Steven Gortler (Harvard University).

Course overview

1. Image processing.

Lectures 1-7See also 18-793: Image and Video Processing

2. Geometry-based vision.

←

Lectures 7 – 12 See also 16-822: Geometry-based Methods in Vision

3. Physics-based vision.

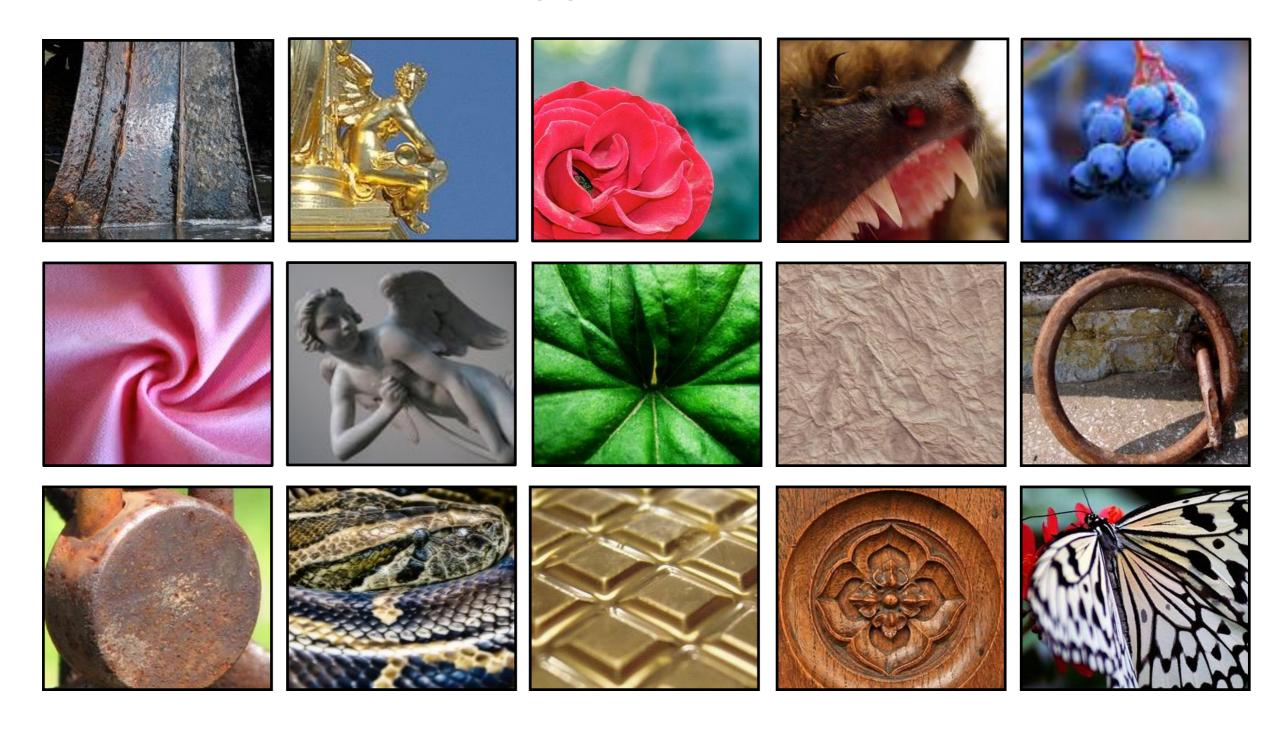
We are starting this part now

4. Learning-based vision.

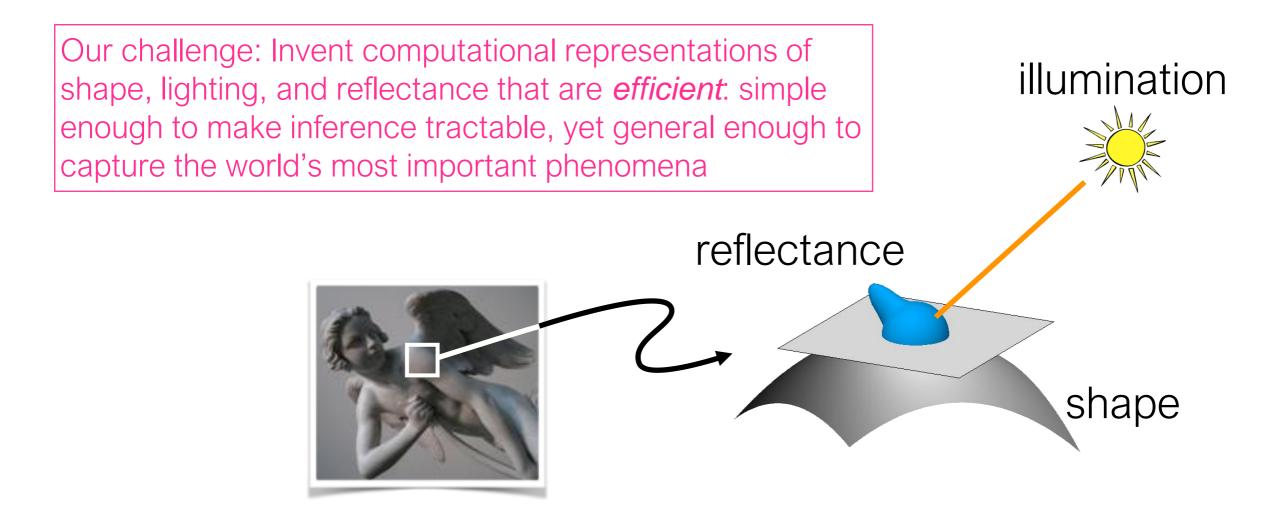
5. Dealing with motion.

Appearance

Appearance

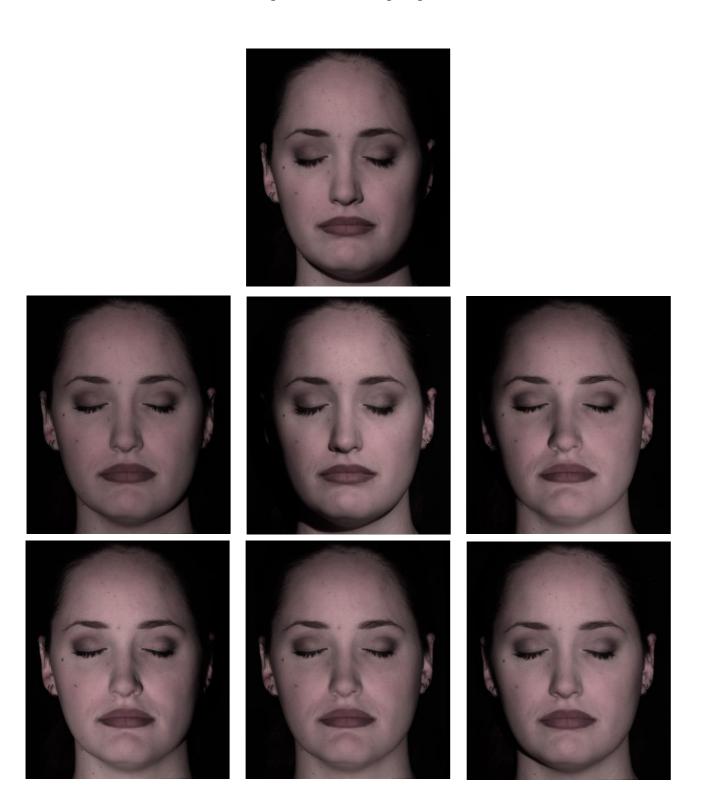


"Physics-based" computer vision (a.k.a "inverse optics")



I ⇒ shape, illumination, reflectance

Example application: Photometric Stereo

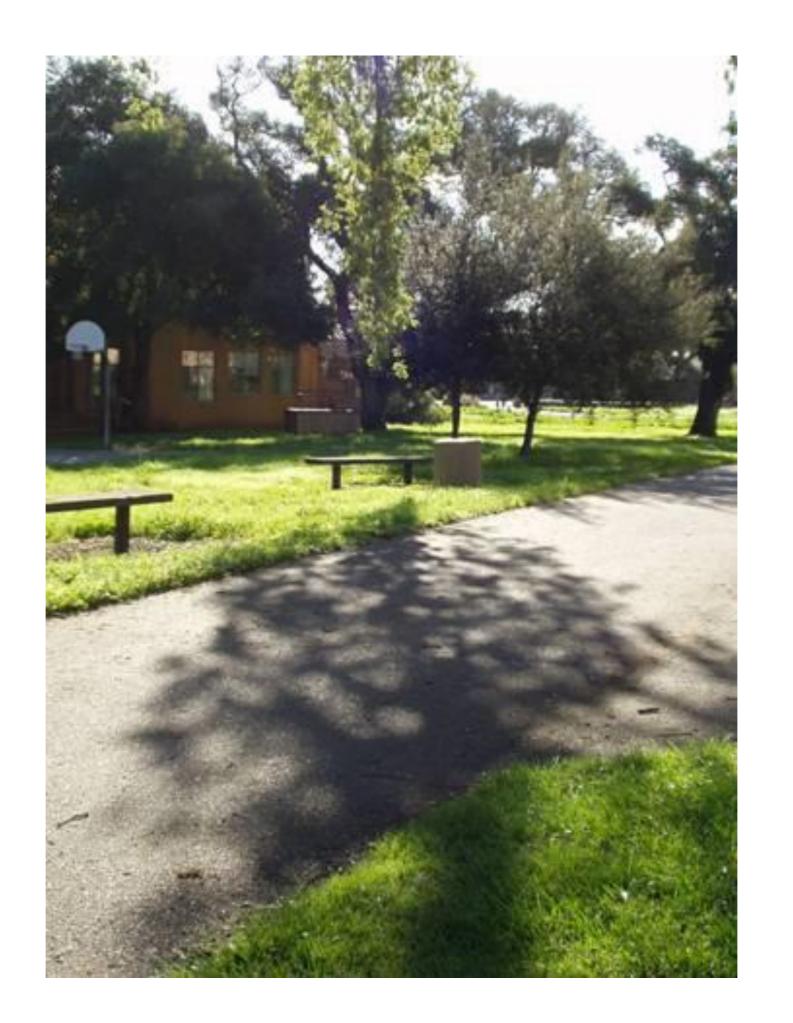




Why study the physics (optics) of the world?

Lets see some pictures!

Light and Shadows



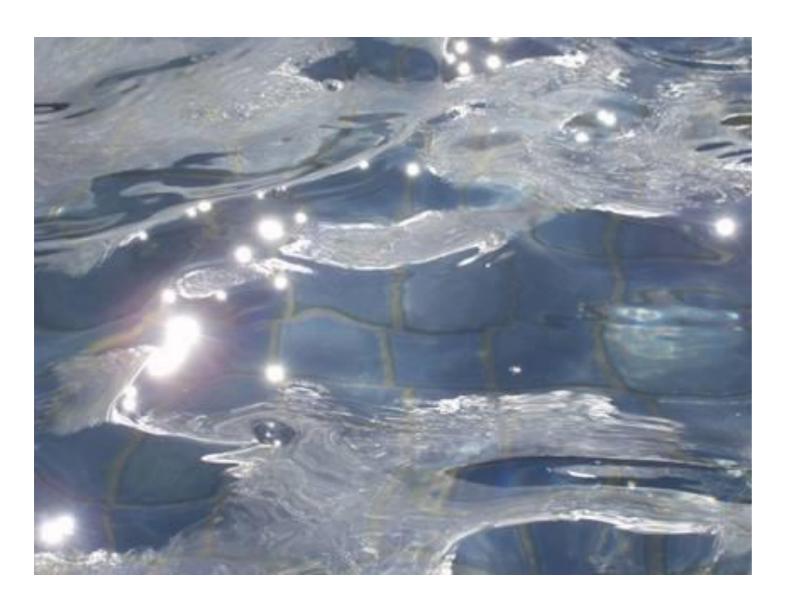


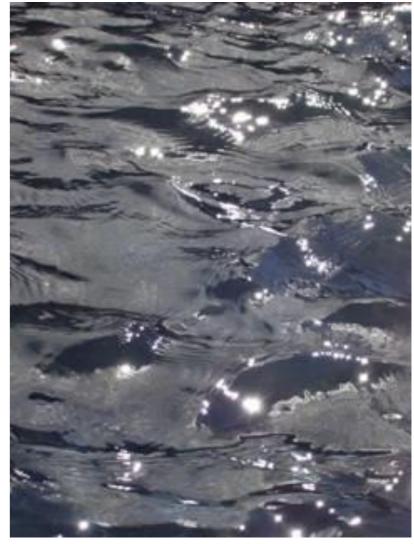


Reflections

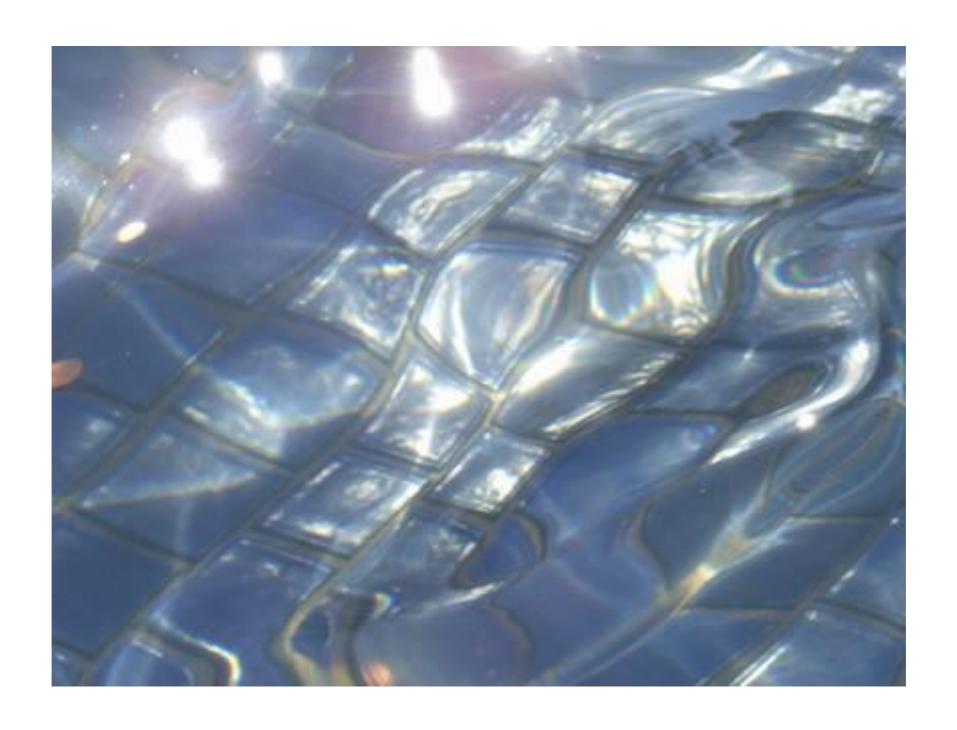




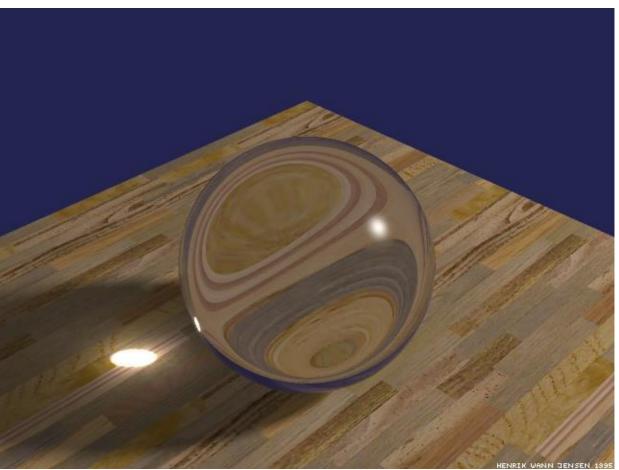




Refractions

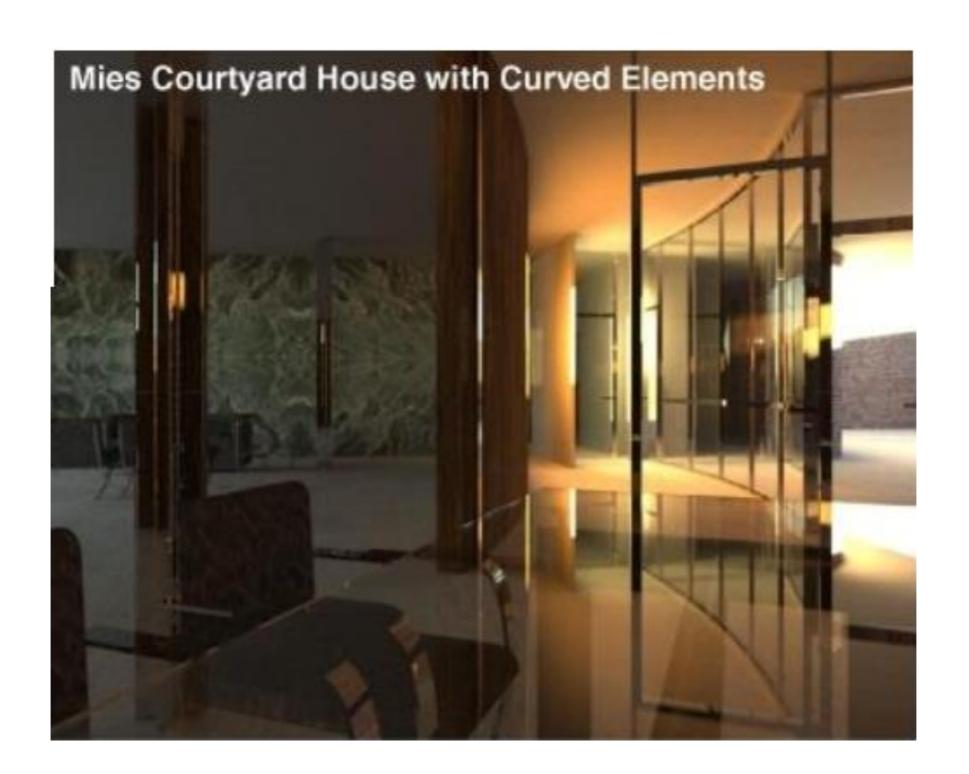




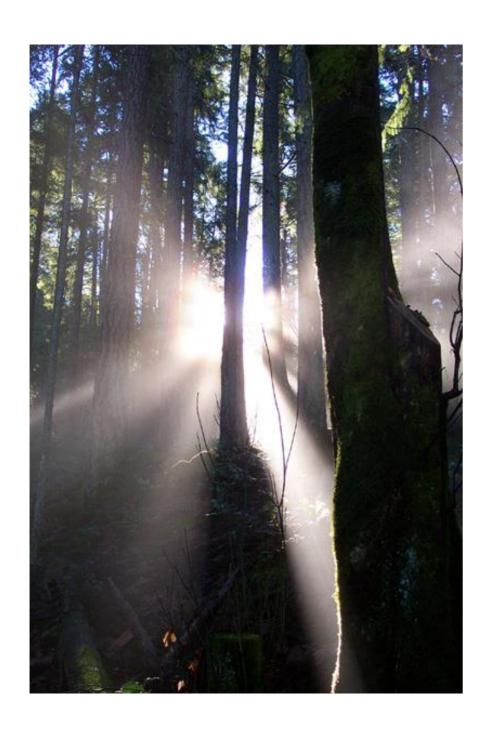


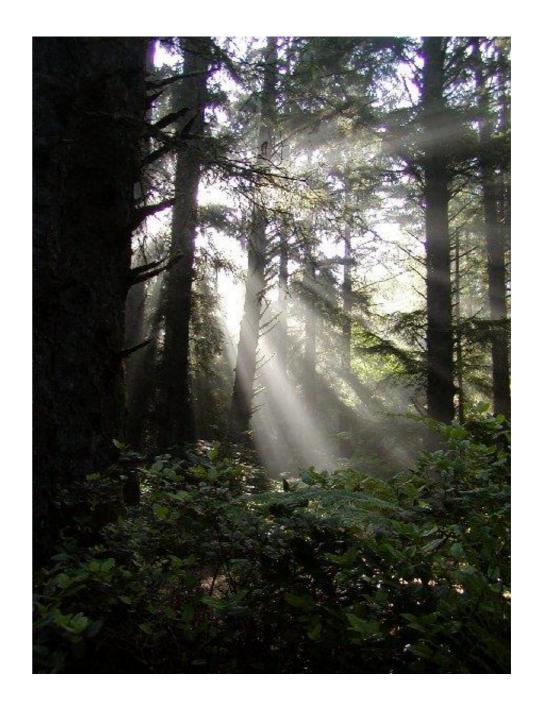


Interreflections

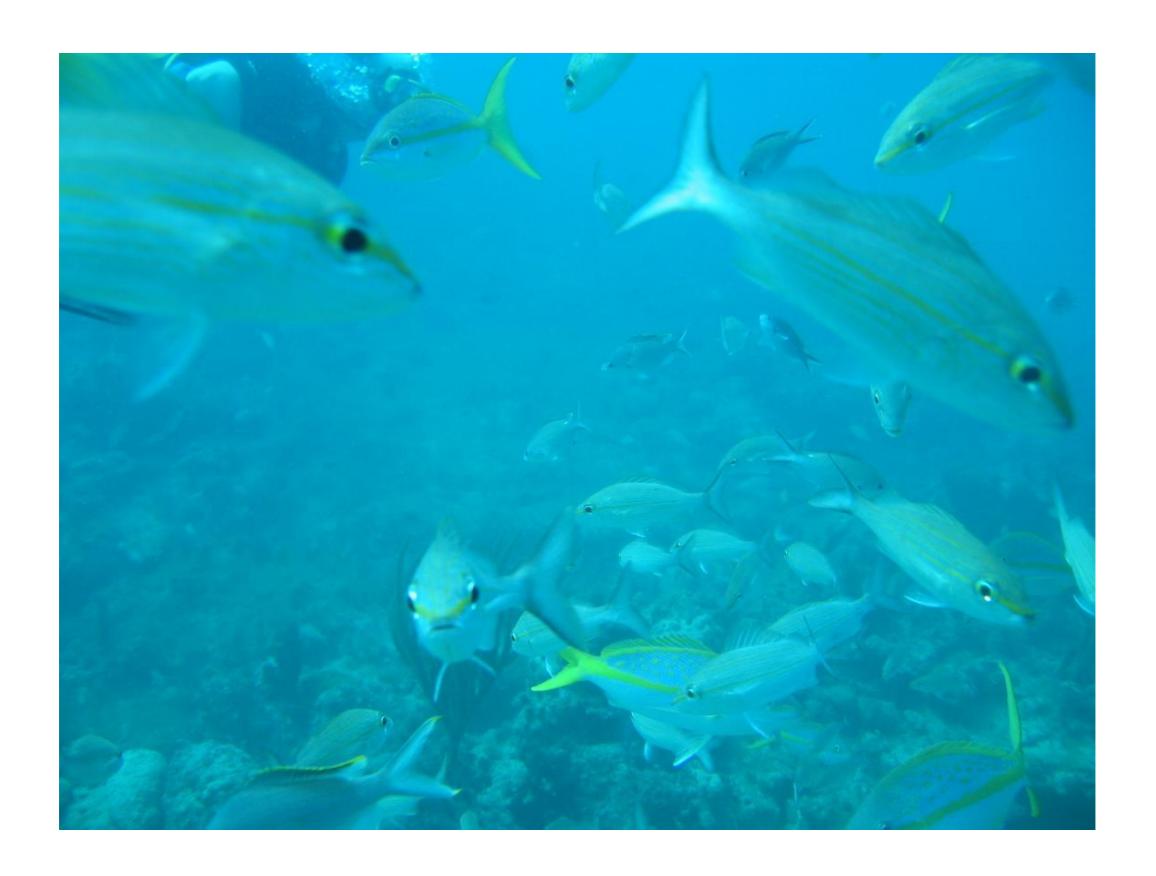


Scattering





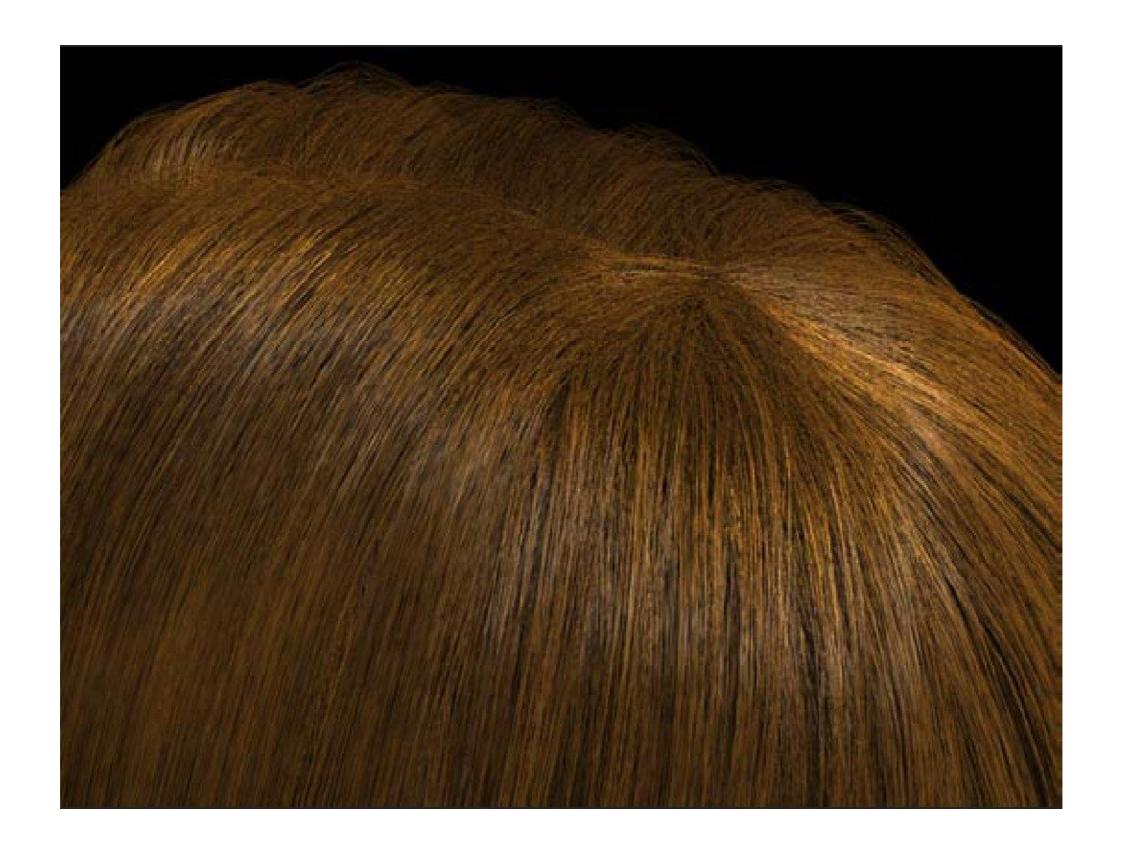




More Complex Appearances





























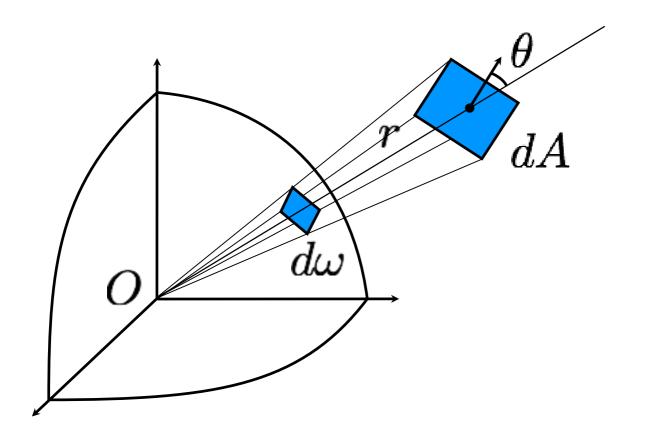




Measuring light and radiometry

Solid angle

The solid angle subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O

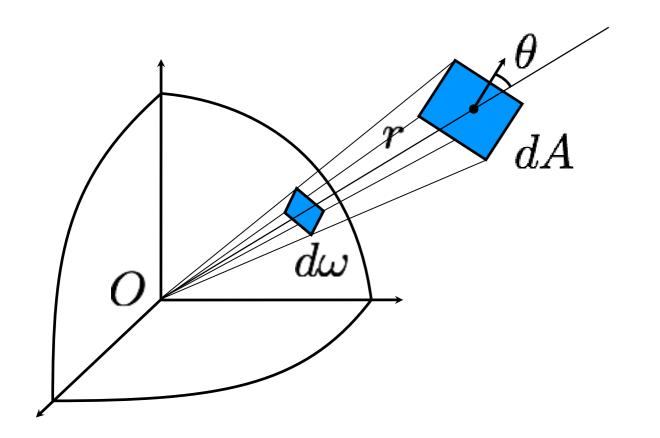


Depends on:

- orientation of patch
- distance of patch

Solid angle

The solid angle subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

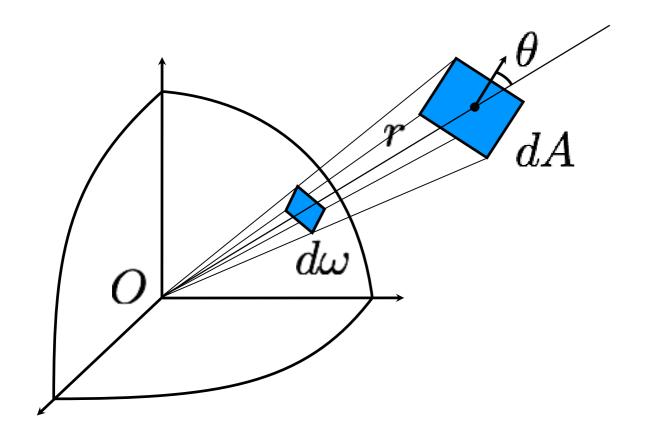
One can show:

$$d\omega = \frac{dA\cos\theta}{r^2}$$

Units: steradians [sr]

Solid angle

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Depends on:

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One can show:

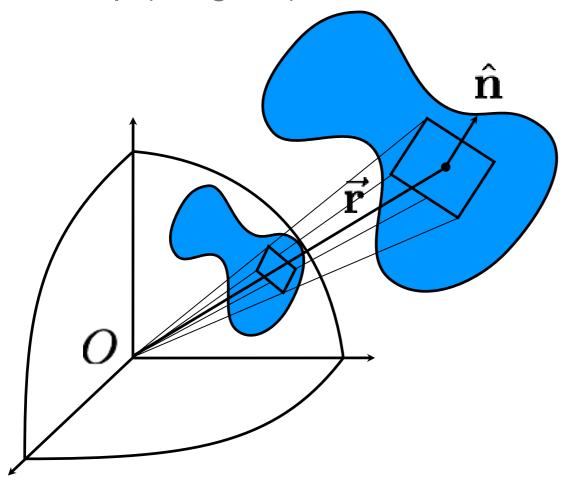
"surface foreshortening"

$$d\omega = \frac{dA\cos\theta}{r^2}$$

Units: steradians [sr]

Solid angle

 To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{\mathbf{r}} \cdot \hat{\mathbf{n}} \ dS}{|\vec{\mathbf{r}}|^3}$$

One can show:

"surface foreshortening"

$$d\omega = \frac{dA\cos\theta}{r^2}$$

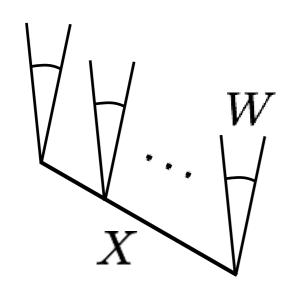
Units: steradians [sr]

• Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

• Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

 Answer: 2\pi (area of sphere is 4\pi*r^2; area of unit sphere is 4\pi; half of that is 2\pi)

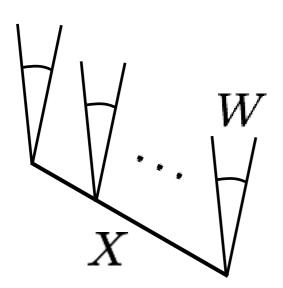
- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W
- It measures radiant flux [watts = joules/sec]
- Measurement depends on sensor area |X|



^{*} shown in 2D for clarity; imagine three dimensions

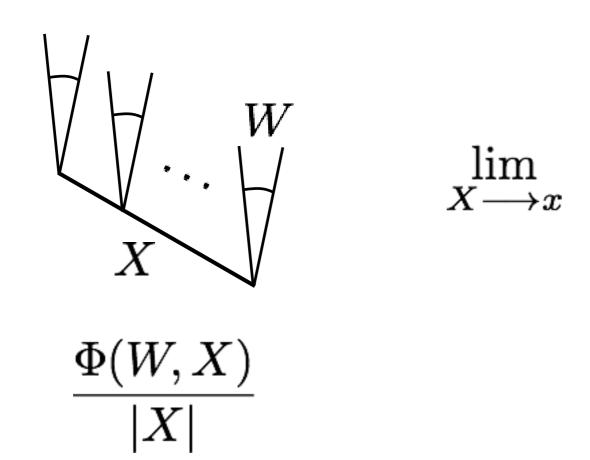
radiant flux $\Phi(W,X)$

- Irradiance:
 - A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m²]

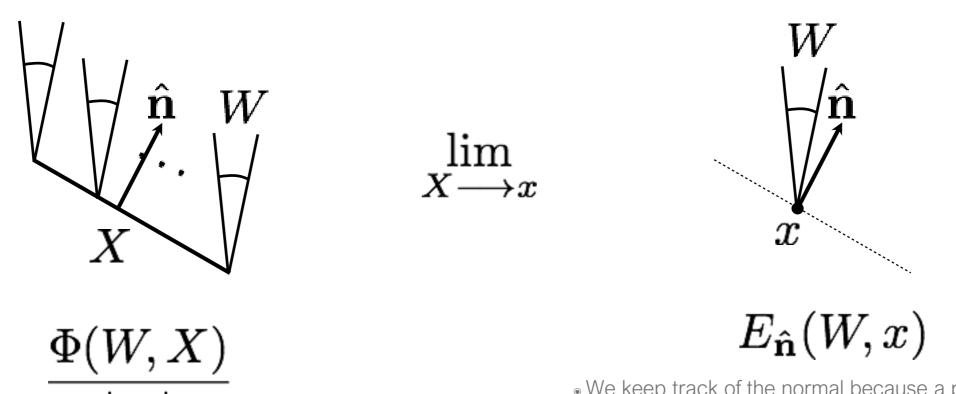


$$\frac{\Phi(W,X)}{|X|}$$

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- Irradiance:
 - A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m²]
- Depends on sensor direction normal.

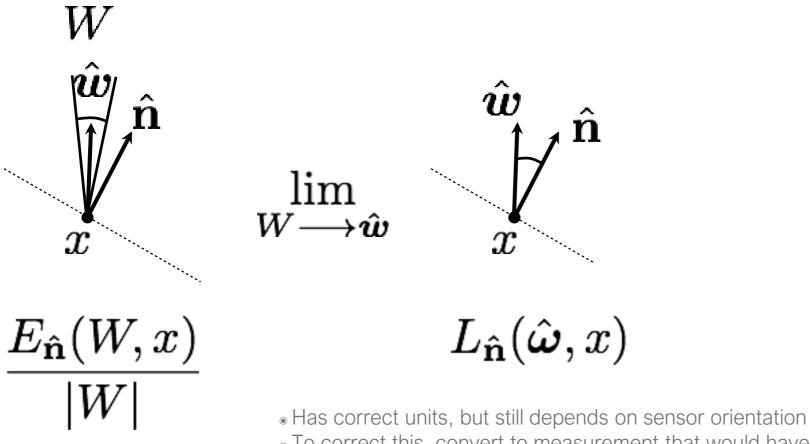


- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit
- In the literature, notations n and W are often omitted, and values are implied by context

• Radiance:

A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|

Units: watts per steradian per square meter [W/(m²-sr)]

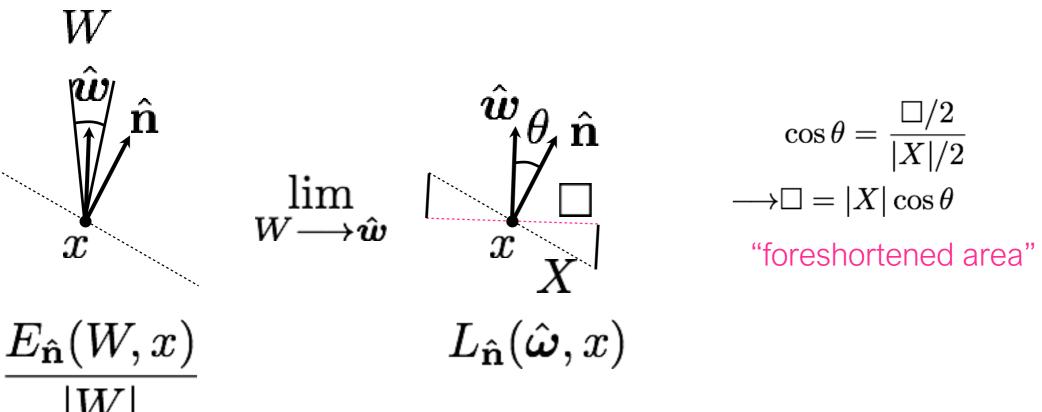


 $_{\odot}$ To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

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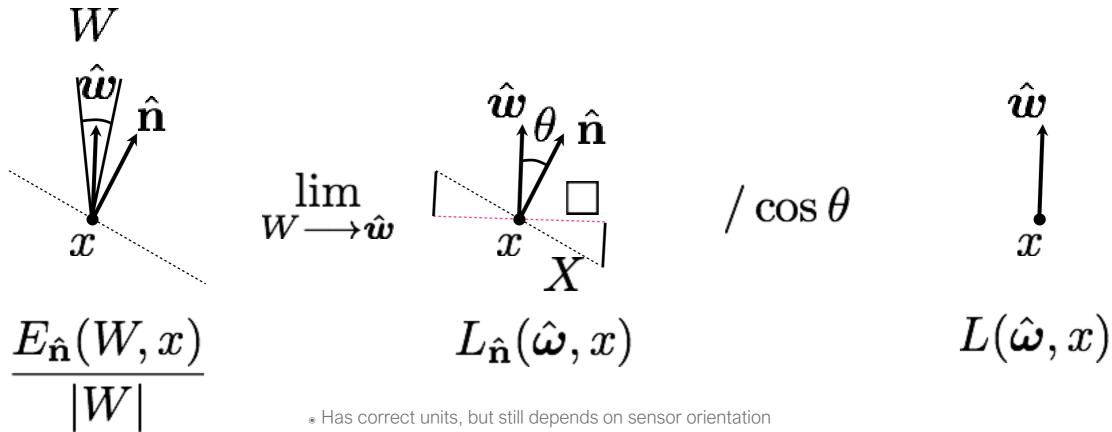


- Has correct units, but still depends on sensor orientation
- $_{\odot}$ To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

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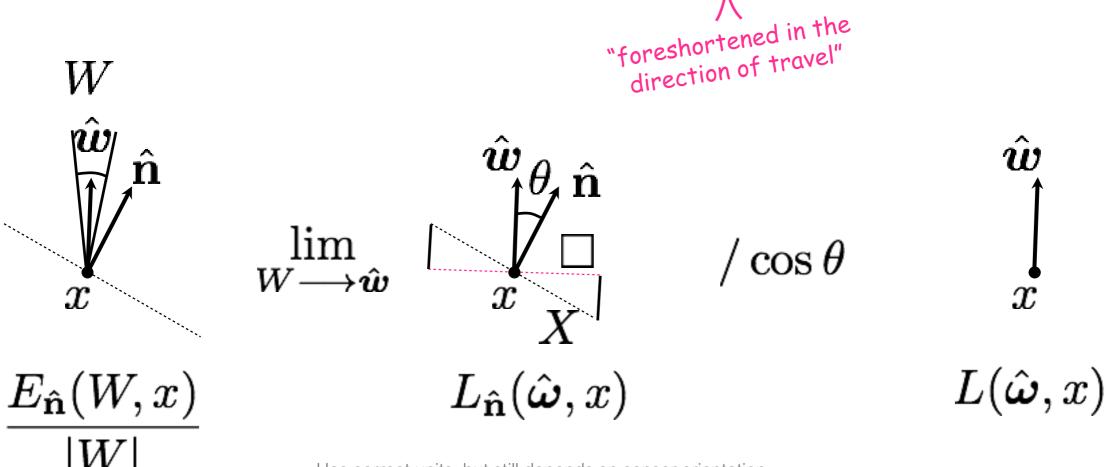


 $_{\text{o}}$ To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

• Radiance:

A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|

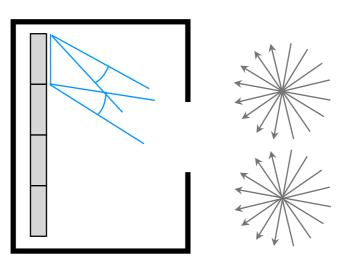
Units: watts per steradian per square meter [W/(m²-sr)]



Has correct units, but still depends on sensor orientation

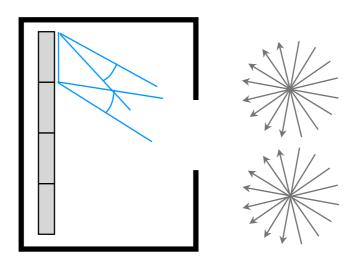
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- Attractive properties of radiance:
 - Allows computing the radiant flux measured by any finite sensor



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 - Allows computing the radiant flux measured by any finite sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos \theta d\omega dA$$

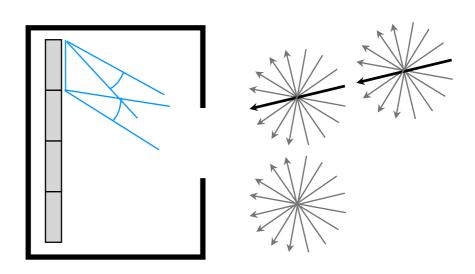


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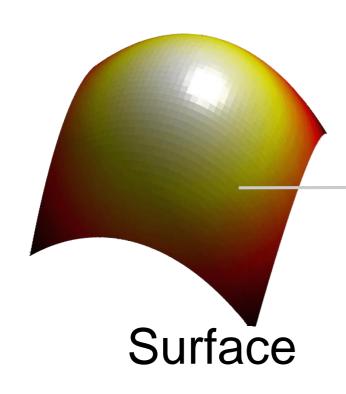
Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$



The Fundamental Assumption in Vision





No Change in

Surface Radiance



Camera

- Attractive properties of radiance:
 - Allows computing the radiant flux measured by any finite sensor

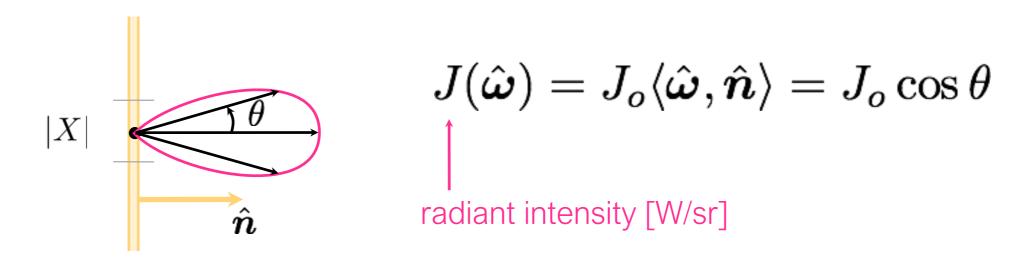
$$\Phi(W, X) = \int_X \int_W L(\hat{\boldsymbol{\omega}}, x) \cos \theta d\boldsymbol{\omega} dA$$

Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$

 A camera measures radiance (after a one-time radiometric calibration; more on this later). So RAW pixel values are proportional to radiance.

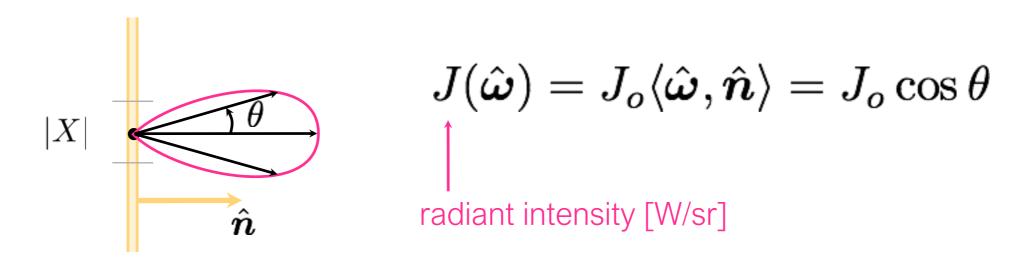
Most light sources, like a heated metal sheet, follow Lambert's Law



"Lambertian area source"

 \bullet What is the radiance $L(\hat{\omega}, x)$ of an infinitesimal patch [W/sr·m²]?

Most light sources, like a heated metal sheet, follow Lambert's Law

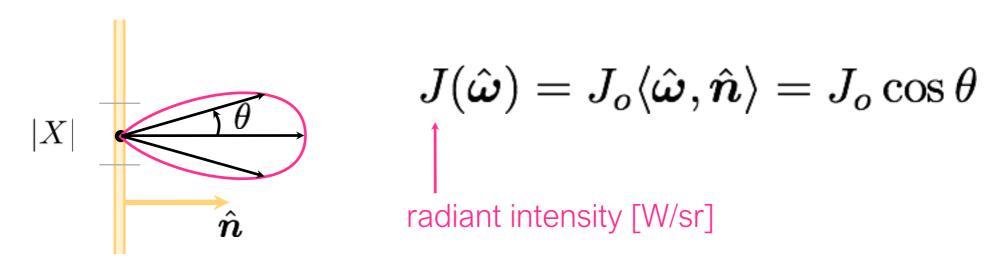


"Lambertian area source"

 $_{ullet}$ What is the radiance $L(\hat{oldsymbol{\omega}},oldsymbol{x})$ of an infinitesimal patch [W/sr·m²]?

Answer: $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x}) = J_o/|X|$ (independent of direction)

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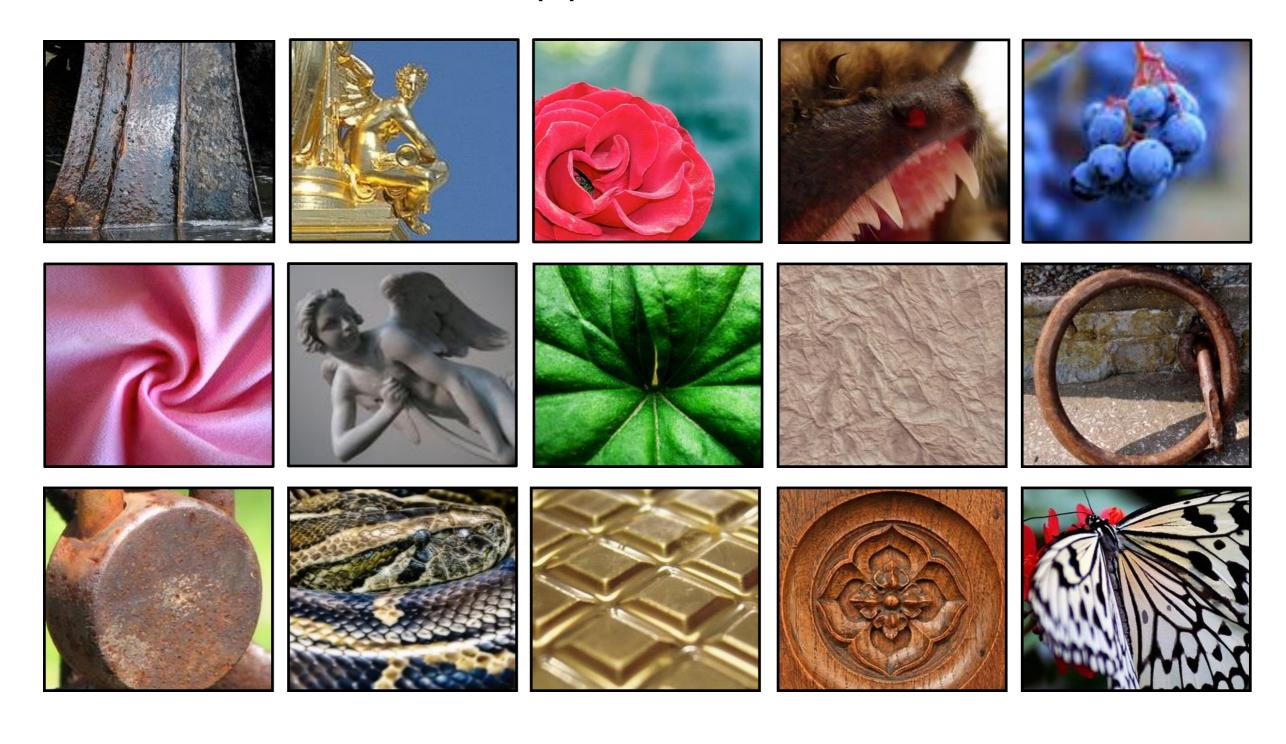
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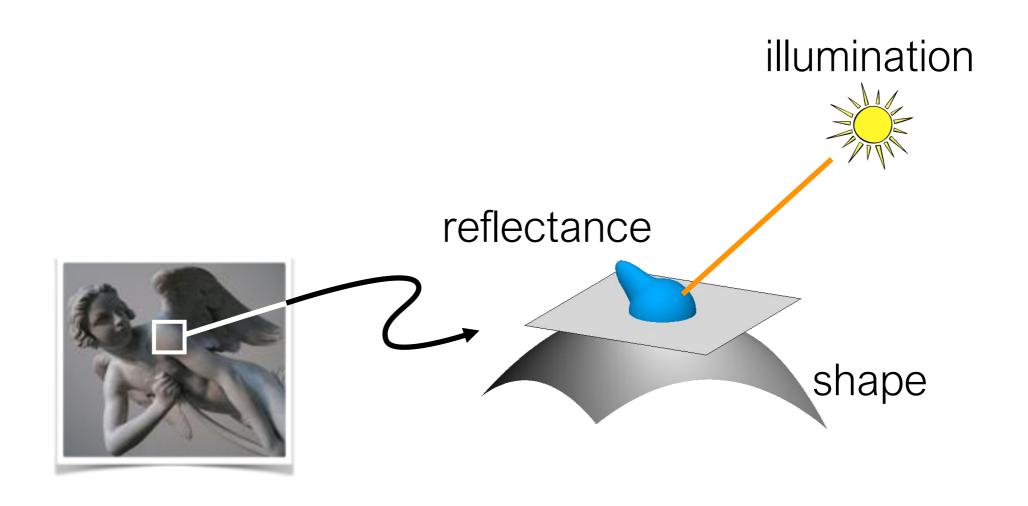
Answer: $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x}) = J_o/|X|$ (independent of direction)

"Looks equally bright when viewed from any direction"

Appearance



"Physics-based" computer vision (a.k.a "inverse optics")



I ⇒ shape, illumination, reflectance

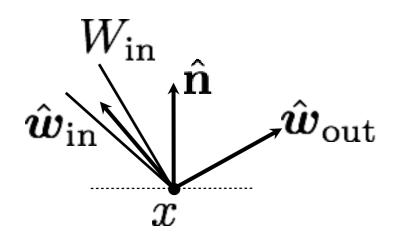
Reflectance and BRDF

Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
 - converges as we use smaller and smaller incoming and outgoing wedges
 - does not depend on the size of the wedges (i.e. is intrinsic to the material)

Reflectance

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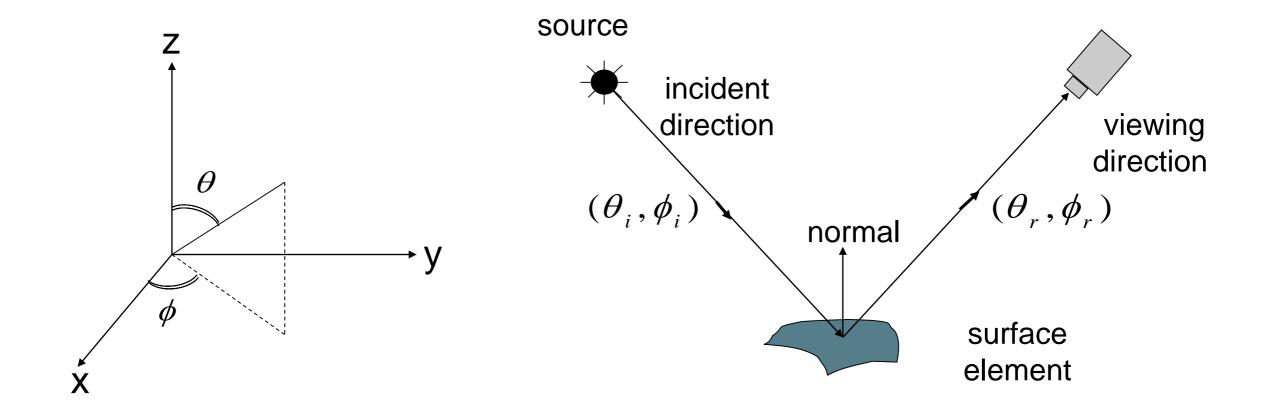
$$\lim_{W_{ ext{in}} o \hat{m{w}}_{ ext{in}}}$$

$$f_{x,\hat{\mathbf{n}}}(\hat{oldsymbol{\omega}}_{\mathrm{in}},\hat{oldsymbol{\omega}}_{\mathrm{out}})$$

$$f_{x,\hat{\mathbf{n}}}(W_{\mathrm{in}},\hat{\boldsymbol{\omega}}_{\mathrm{out}}) = rac{L^{\mathrm{out}}(x,\hat{\boldsymbol{\omega}}_{\mathrm{out}})}{E^{\mathrm{in}}_{\hat{\mathbf{n}}}(W_{\mathrm{in}},x)}$$

- Notations x and n often implied by context and omitted; directions \omega are expressed in local coordinate system defined by normal n (and some chosen tangent vector)
- Units: sr⁻¹
- Called Bidirectional Reflectance Distribution Function (BRDF)

BRDF: Bidirectional Reflectance Distribution Function



$$E^{surface}$$
 (θ_i, ϕ_i) Irradiance at Surface in direction (θ_i, ϕ_i)
 $L^{surface}$ (θ_r, ϕ_r) Radiance of Surface in direction (θ_r, ϕ_r)

$$\mathsf{BRDF}: f\left(\theta_{i}, \phi_{i}; \theta_{r}, \phi_{r}\right) = \frac{L^{\mathit{surface}}\left(\theta_{r}, \phi_{r}\right)}{E^{\mathit{surface}}\left(\theta_{i}, \phi_{i}\right)}$$

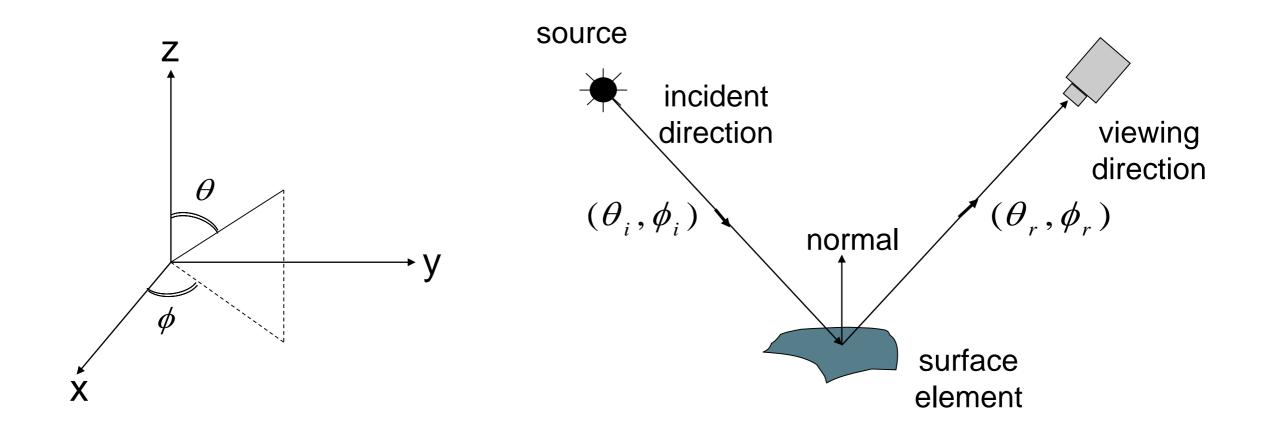
Reflectance: BRDF

- Units: sr⁻¹
- Real-valued function defined on the double-hemisphere
- Allows computing output radiance (and thus pixel value) for any configuration of lights and viewpoint

$$L^{
m out}(\hat{m{\omega}}) = \int_{\Omega_{
m in}} f(\hat{m{\omega}}_{
m in}, \hat{m{\omega}}_{
m out}) L^{
m in}(\hat{m{\omega}}_{
m in}) \cos heta_{
m in} d\hat{m{\omega}}_{
m in}$$

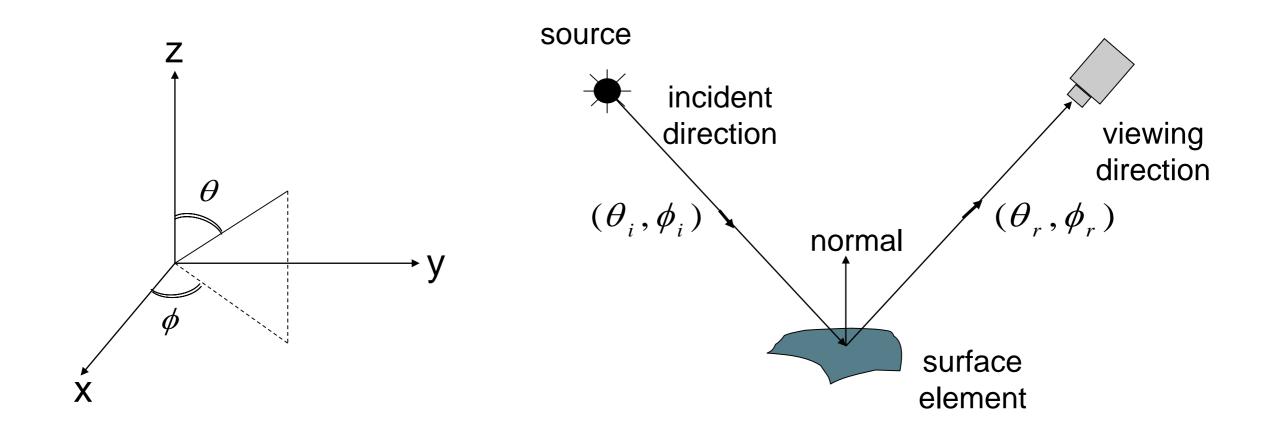
reflectance equation

Important Properties of BRDFs



Conservation of Energy:

Important Properties of BRDFs

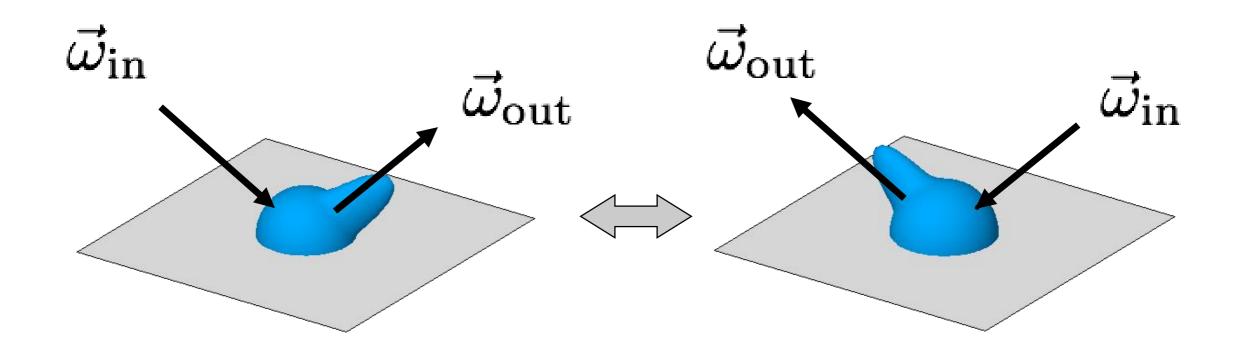


• Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

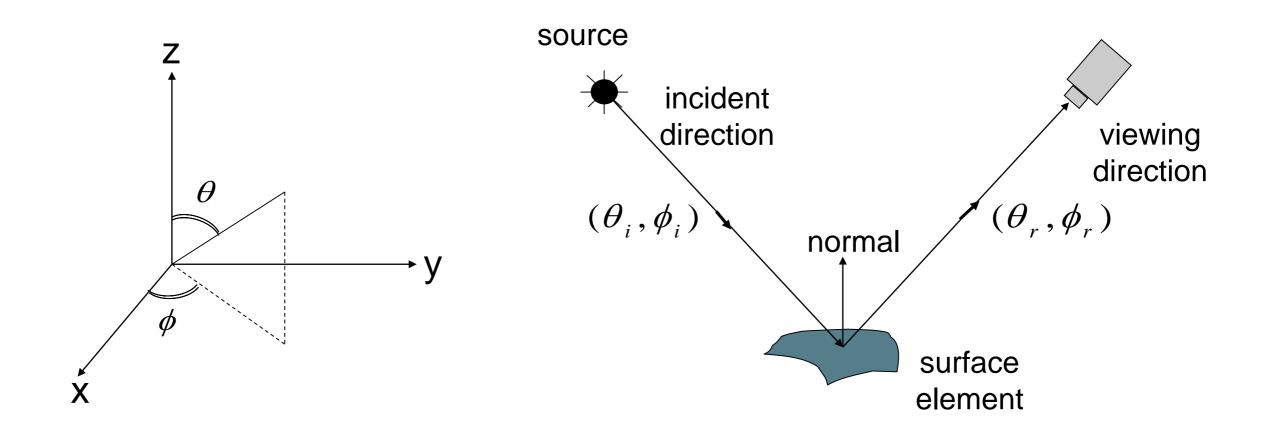
$$f(\theta_i, \phi_i; \theta_r, \phi_r) = f(\theta_r, \phi_r; \theta_i, \phi_i)$$

Property: "Helmholtz reciprocity"



$$f_r(\vec{\omega}_{\rm in}, \vec{\omega}_{\rm out}) = f_r(\vec{\omega}_{\rm out}, \vec{\omega}_{\rm in})$$

Important Properties of BRDFs

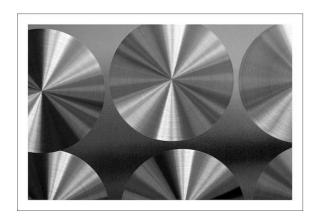


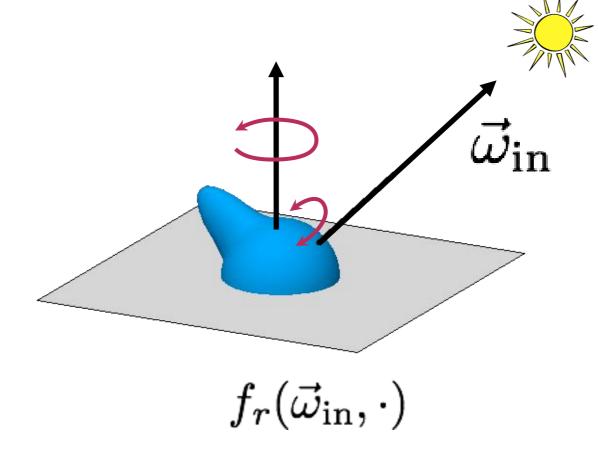
Rotational Symmetry (Isotropy):

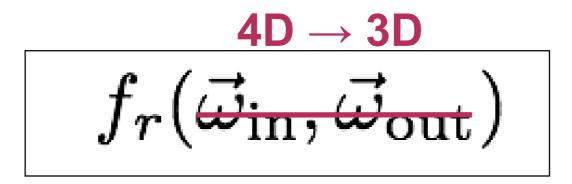
BRDF does not change when surface is rotated about the normal.

Can be written as a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

Common <u>assumption</u>: Isotropy





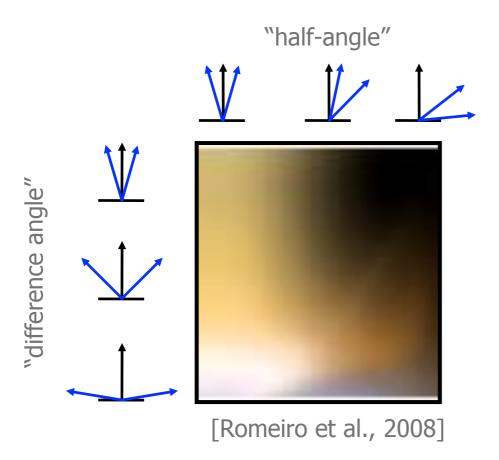


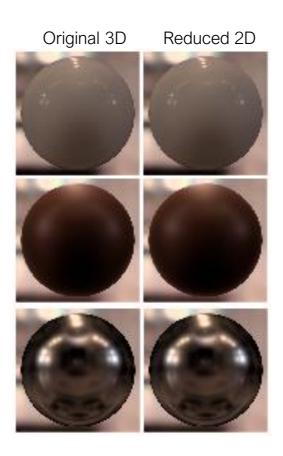


[Matusik et al., 2003]

Bi-directional Reflectance Distribution Function (BRDF)

Simplification: Bivariate





$$f_r(ec{arphi_{
m in},ec{arphi}_{
m out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Reflectance: BRDF

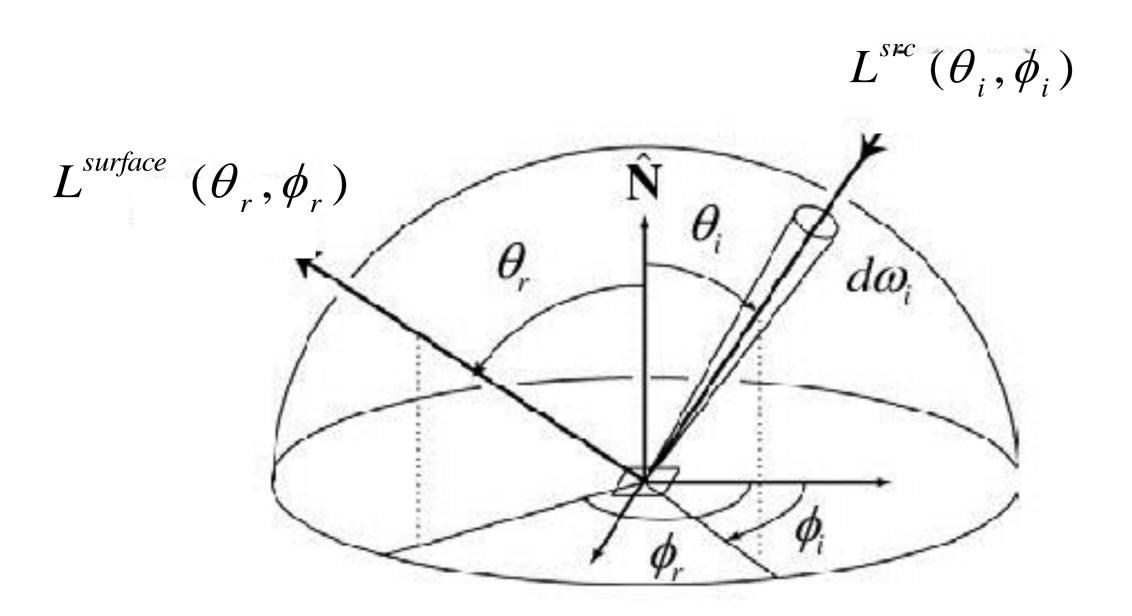
- Units: sr⁻¹
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- Allows computing output radiance (and thus pixel value) for any configuration of lights and viewpoint

$$L^{
m out}(\hat{m{\omega}}) = \int_{\Omega_{
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m in}, \hat{m{\omega}}_{
m out}) L^{
m in}(\hat{m{\omega}}_{
m in}) \cos heta_{
m in} d\hat{m{\omega}}_{
m in}$$

reflectance equation

Why is there a cosine in the reflectance equation?

Derivation of the Scene Radiance Equation



From the definition of BRDF:

$$L^{surface} (\theta_r, \phi_r) = E^{surface} (\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{\textit{surface}} (\theta_r, \phi_r) = E^{\textit{surface}} (\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface} (\theta_r, \phi_r) = L^{src} (\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i d\omega_i$$

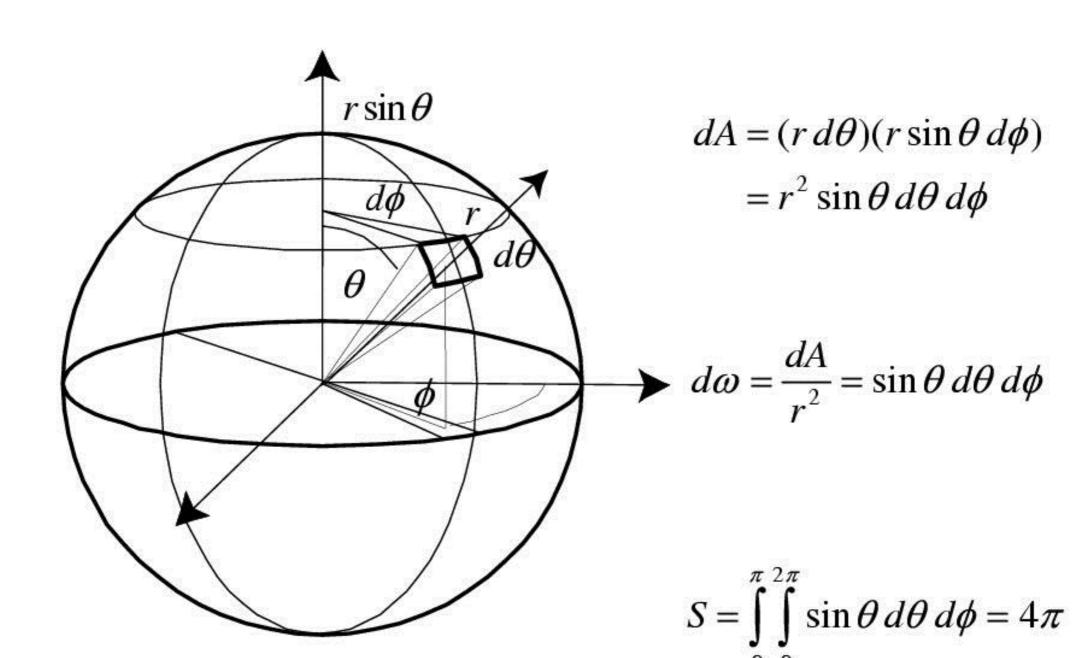
Integrate over entire hemisphere of possible source directions:

$$L^{surface} (\theta_r, \phi_r) = \int_{2\pi} L^{src} (\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \, d\omega_i$$

Convert from solid angle to theta-phi representation:

$$L^{surface} (\theta_r, \phi_r) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} L^{src} (\theta_i, \phi_i) f(\theta_i, \phi_i; \theta_r, \phi_r) \cos \theta_i \sin \theta_i d\theta_i d\phi_i$$

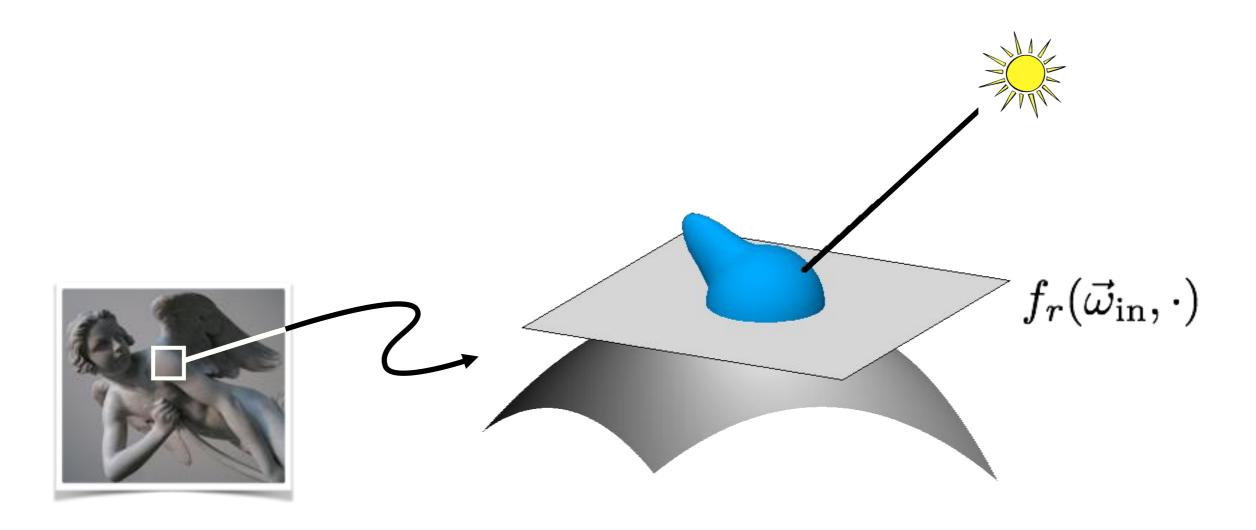
Differential Solid Angles



CS348B Lecture 4

Pat Hanrahan, Spring 2002

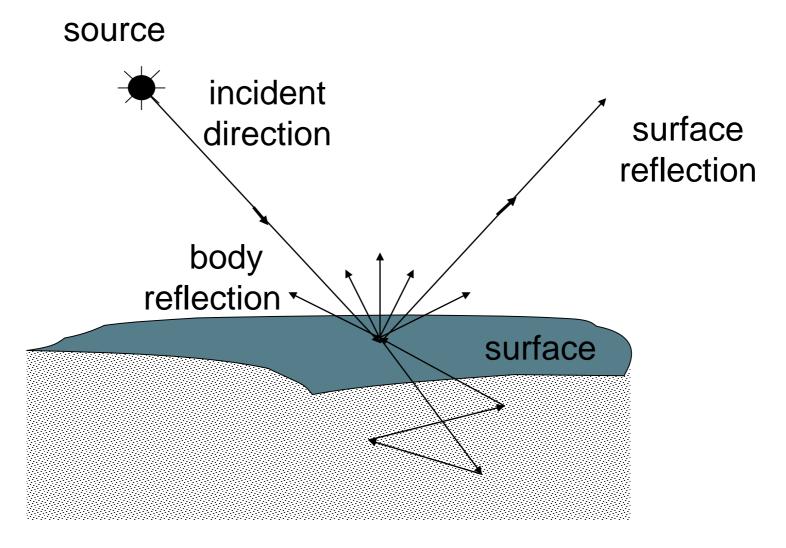
BRDF



$$f_r(\vec{\omega}_{\mathrm{in}}, \vec{\omega}_{\mathrm{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Mechanisms of Reflection



Body Reflection:

Diffuse Reflection

Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc

Surface Reflection:

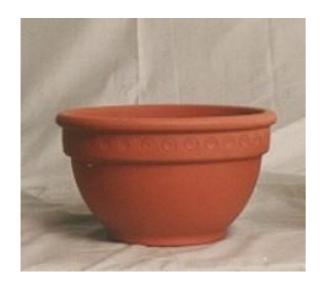
Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection

Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc



Many materials exhibit both Reflections:

Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

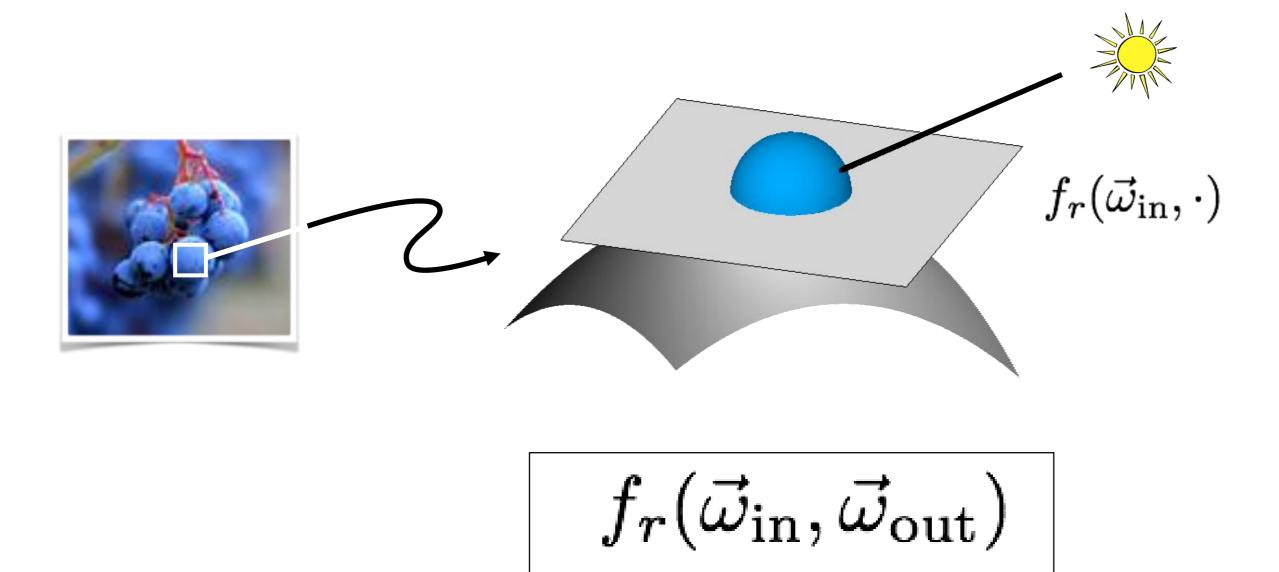






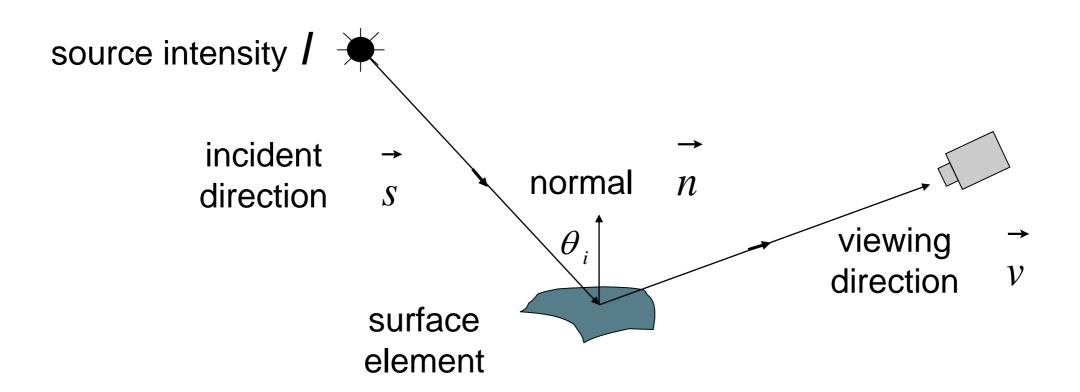
BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions



Bi-directional Reflectance Distribution Function (BRDF)

Diffuse Reflection and Lambertian BRDF



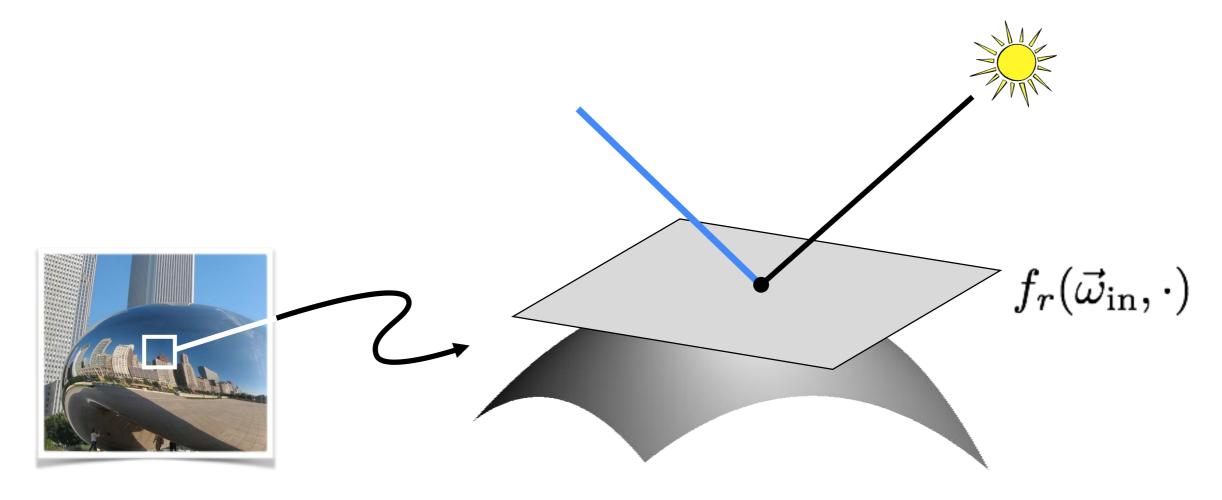
- Surface appears equally bright from ALL directions! (independent of $\,v\,$)
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ albedo

• Surface Radiance :
$$L = \frac{\rho_d}{\pi} I \cos \theta_i = \frac{\rho_d}{\pi} \underbrace{I \stackrel{\rightarrow}{n}.s}_{\text{source intensity}}$$

Commonly used in Vision and Graphics!

BRDF

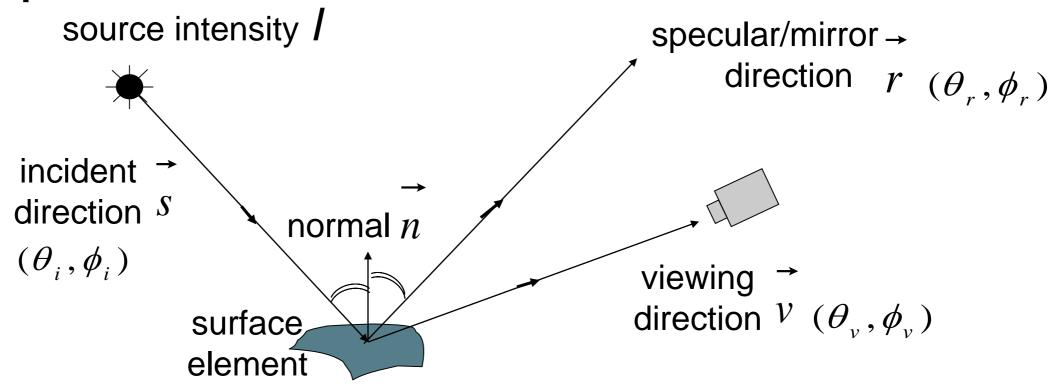
Specular BRDF: all energy concentrated in mirror direction



$$f_r(\vec{\omega}_{
m in}, \vec{\omega}_{
m out})$$

Bi-directional Reflectance Distribution Function (BRDF)

Specular Reflection and Mirror BRDF



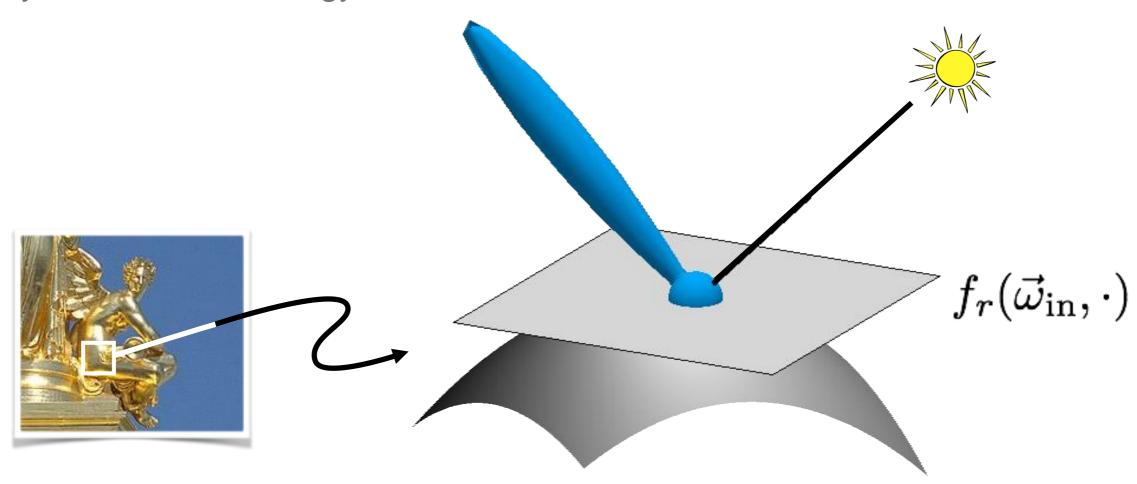
- · Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when v = r).
- Mirror BRDF is simply a double-delta function :

specular albedo
$$f(\theta_i,\phi_i;\theta_v,\phi_v)=\rho_s \ \delta(\theta_i-\theta_v) \ \delta(\phi_i+\pi-\phi_v)$$

• Surface Radiance: $L = I \rho_s \delta(\theta_i - \theta_v) \delta(\phi_i + \pi - \phi_v)$

BRDF

Glossy BRDF: more energy concentrated in mirror direction



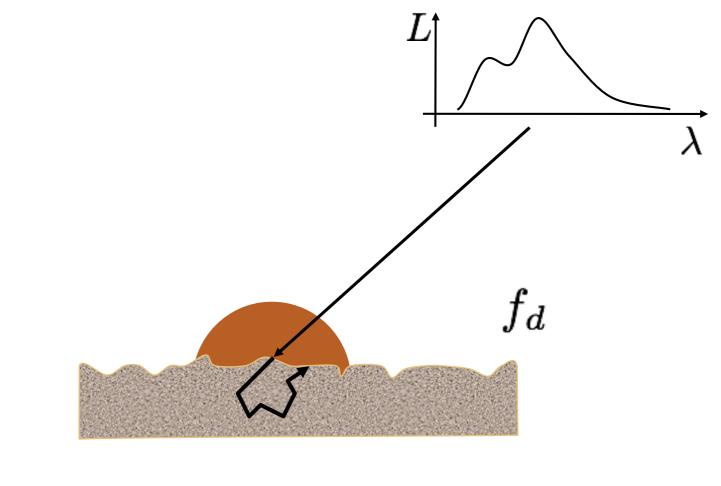
$$f_r(\vec{\omega}_{
m in}, \vec{\omega}_{
m out})$$

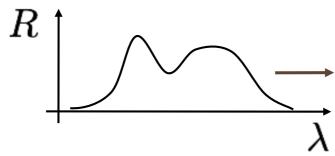
Bi-directional Reflectance Distribution Function (BRDF)

- BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components
- The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

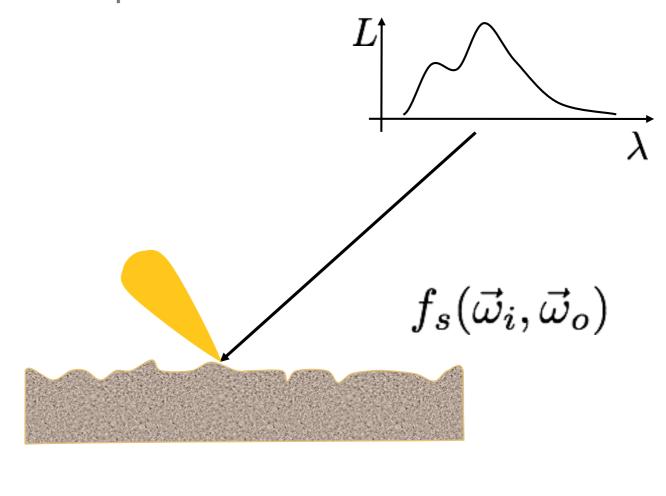
$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

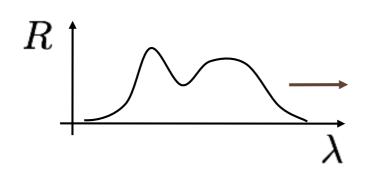
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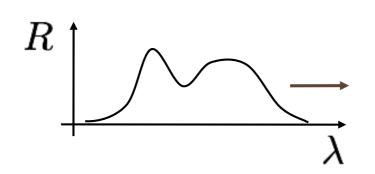
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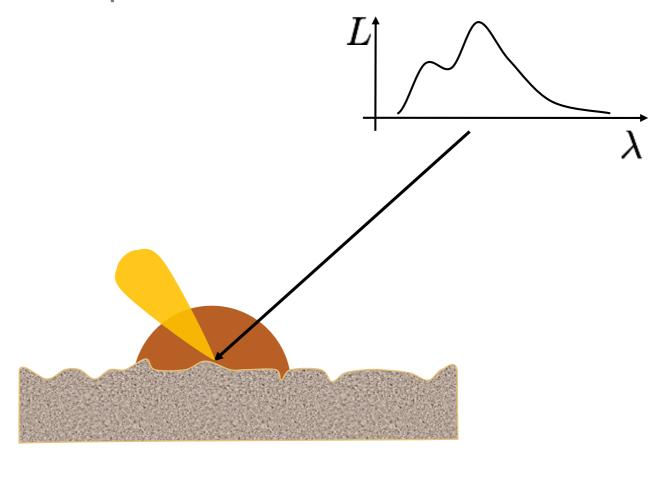




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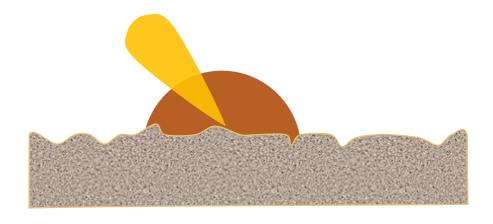


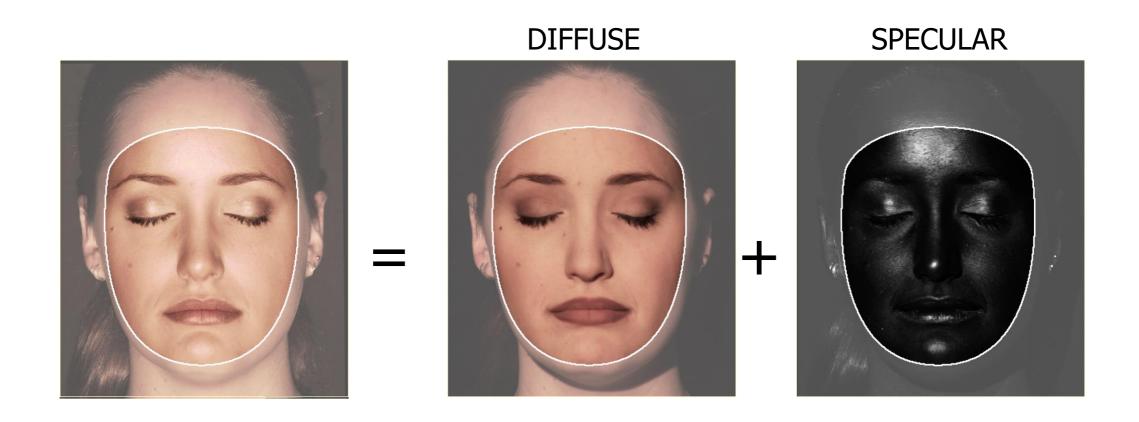
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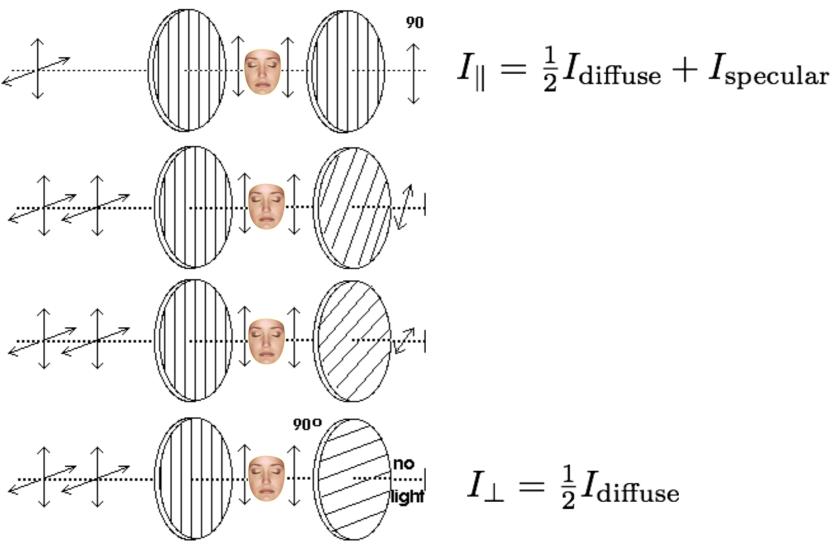
Often called the dichromatic BRDF:

- Diffuse term varies with wavelength, constant with polarization
- Specular term constant with wavelength, varies with polarization

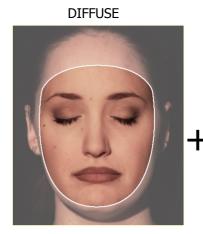




• In this example, the two components were separated using linear polarizing filters on the camera and light source.

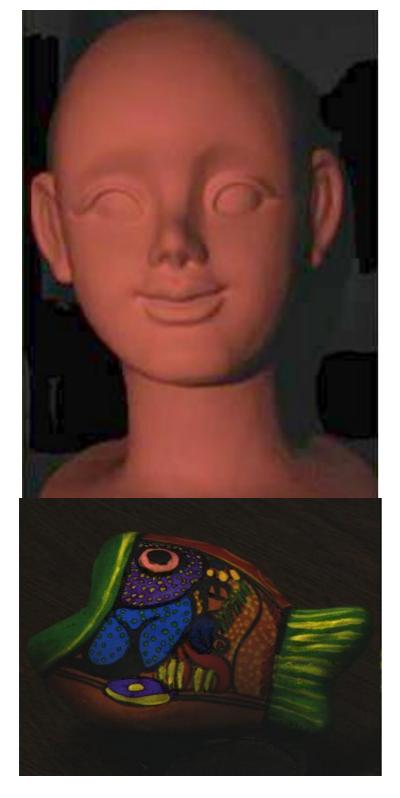




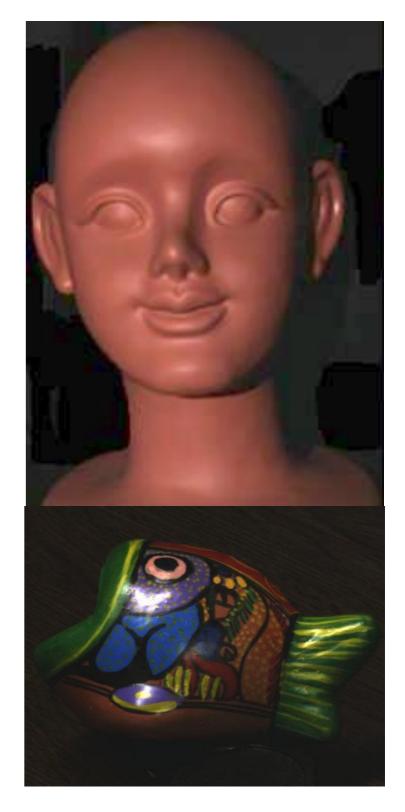


SPECULAR

Diffuse and Specular Reflection





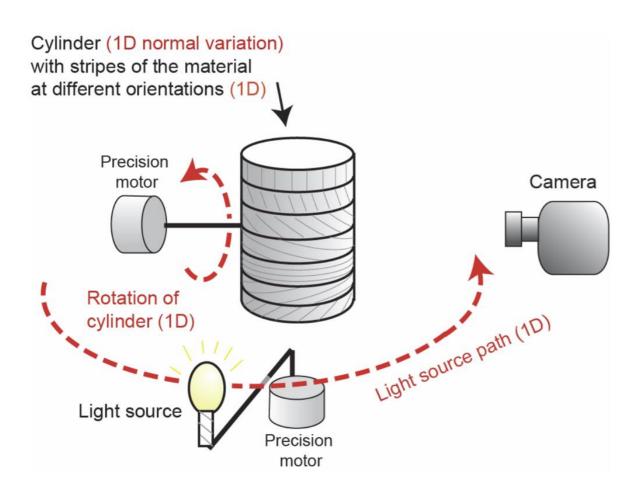


diffuse

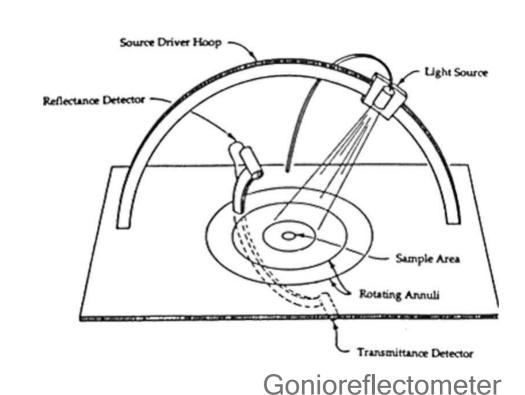
specular

diffuse+specular

Tabulated 4D BRDFs (hard to measure)







Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D,3D, or 4D) function
- Except for Lambertian, the BRDF is non-linear in these parameters
- Examples:

Lambertian:
$$f(\omega_i,\omega_o)=rac{a}{\pi}$$
 Where do these constants come from?

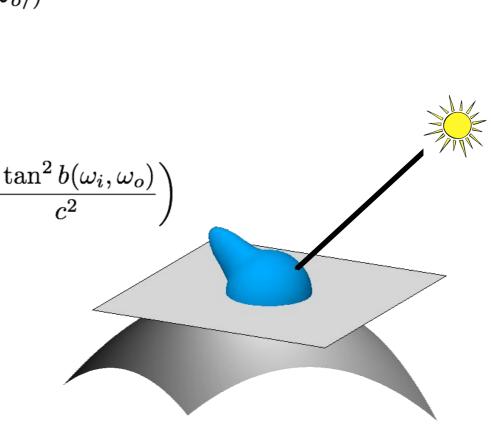
Phong:
$$f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c (2\langle \omega_i, n \rangle \langle \omega_o, n \rangle - \langle \omega_i, \omega_o \rangle)$$

Blinn:
$$f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c b(\omega_i, \omega_o)$$

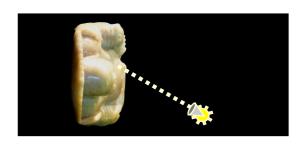
Lafortune:
$$f(\omega_i, \omega_o) = \frac{a}{\pi} + b(-\omega_i^\top A \omega_o)^k$$

$$\text{Ward:} \quad f(\omega_i, \omega_o) = \frac{a}{\pi} + \frac{b}{4\pi c^2 \sqrt{\langle n, \omega_i \rangle \langle n, \omega_o \rangle}} \exp\left(\frac{-\tan^2 b(\omega_i, \omega_o)}{c^2}\right)$$

α is called the *albedo*

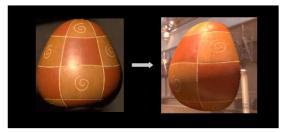


Recent progress: "Active appearance capture"



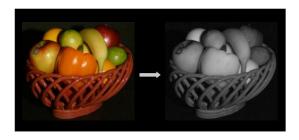
Reciprocity

[ICCV 2001; ECCV 2002; CVPR 2003; CVPR 2006; Sen et al., 2005; Hawkins et al., 2005, SIGGRAPH 2010]



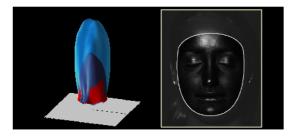
Isotropy

[Lu and Little 1999; CVPR 2007; Alldrin and Kriegman 2007; CVPR 2008; CVPR 2009]



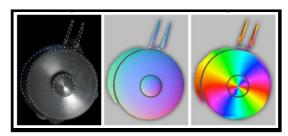
Separability

[Sato & Ikeuchi, 1994; Schluns and Wittig, 1993; ...; Barsky and Petrou, 2001; CVPR 2005; CVPR 2006; IJCV 2008; ...]



Spatial regularity

[Lensch et al., 2001; Hertzmann & Seitz, 2003; EGSR 2005; PAMI 2006; Lawrence et al., 2006; Weistroffer et al., 2007; CVPR 2008; Garg et al. 2009;...]

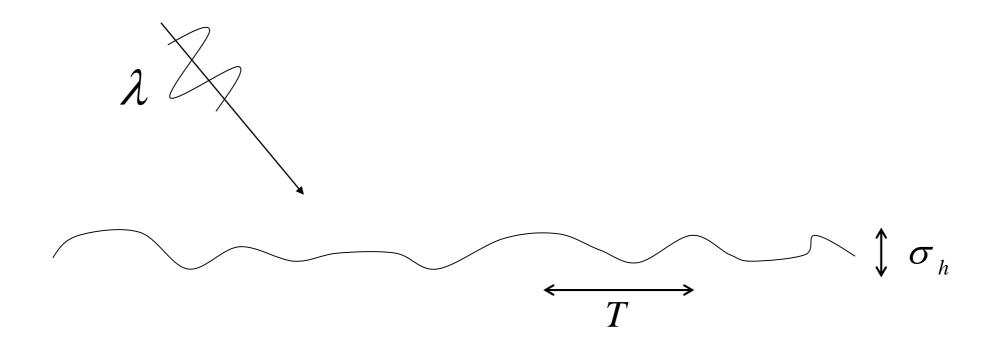


Tangent-plane symmetry

[SIGGRAPH Asia 2008]

Reflectance Models

Reflection: An Electromagnetic Phenomenon



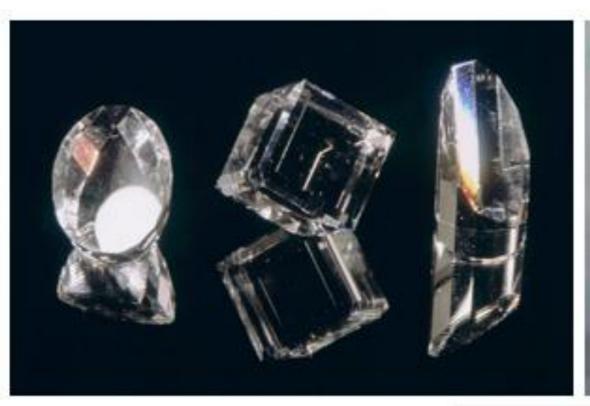
Two approaches to derive Reflectance Models:

- Physical Optics (Wave Optics)
- Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models

But they are easier to use!

Reflectance that Require Wave Optics













References

- Basic reading:Szeliski, Section 2.2.
- Gortler, Chapter 21.