K-Means

In [70]:
```
pip install -U numpy
```

```
Requirement already satisfied: numpy in /Users/haoranzhang/opt/anaconda3/
lib/python3.8/site-packages (1.24.2)
Note: you may need to restart the kernel to use updated packages.
```

In [71]:
```
import sklearn
```

In [72]:
```
from sklearn.cluster import KMeans
```

In [73]:
```
import sklearn.cluster as cluster
```

In [74]:
```
import matplotlib.pyplot as plt
import math
```

In [75]:
```
import pandas as pd
```

In [76]:
```
import numpy as np
```

Q1 (a)(b)

In [77]:
```
data = np.loadtxt("/Users/haoranzhang/CS6220/assignment-4-hrcheung/data/f15
```

In [78]:
```
data
```

Out[78]:
```
array([[-11.96999577,  -8.03962819],
       [-26.96141582,  -6.96210943],
       [-12.91584891,  -1.37894148],
       ...,
       [ 12.92550271,  -0.10319682],
       [-34.33677838,   4.4057531 ],
       [-26.81273855,  -1.63995599]])
```

In [79]:
```
data.shape
```

Out[79]:
```
(5000, 2)
```

In [80]:
```
#initialization vectors are:
initialization=np.array([[10,10],[-10,-10],[2,2],[3,3],[-3,-3]])
```

In [81]:
```
type(initialization)
```

Out[81]:
```
numpy.ndarray
```

In [82]:
```
initialization.shape
```
Out[82]:
```
(5, 2)
```

In [83]:
```
kmeans=cluster.KMeans(n_clusters=5,init=initialization,max_iter=100)
```

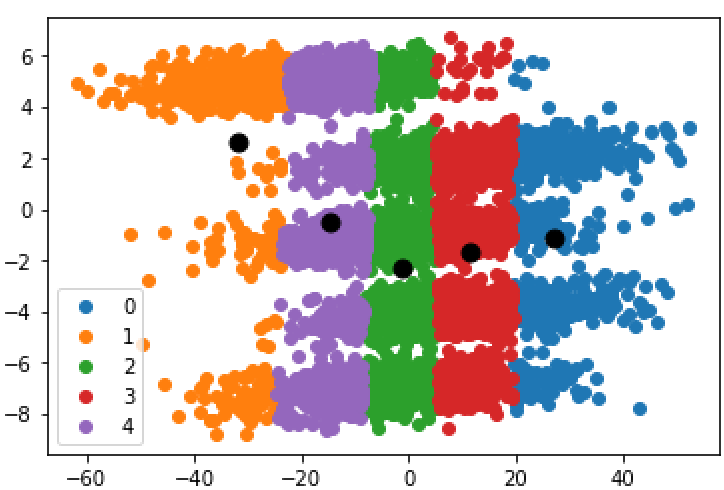
In [84]:
```
label=kmeans.fit_predict(data)
```

```
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:1362: RuntimeWarning: Explicit initial center position pas
sed: performing only one init in KMeans instead of n_init=10.
  super()._check_params_vs_input(X, default_n_init=10)
```

In [85]:
```
label
```
Out[85]:
```
array([4, 1, 4, ..., 3, 1, 1], dtype=int32)
```

In [86]:
```
centroids = kmeans.cluster_centers_
centroids
```
Out[86]:
```
array([[ 27.3664243 ,  -1.09580993],
       [-31.94805334,   2.59901959],
       [ -1.04061308,  -2.28300242],
       [ 11.68619774,  -1.68891573],
       [-14.50973012,  -0.54232205]])
```

In [87]:
```
u_labels=np.unique(label) #allocat label to different clusters
for i in u_labels:
    plt.scatter(data[label==i,0],data[label==i,1],label=i)
plt.scatter(centroids[:,0] , centroids[:,1] , s = 80, color = 'k')
plt.legend()
plt.show()
```



Q1(c)

Based on the cluster figure above, the cluster result does not look good.

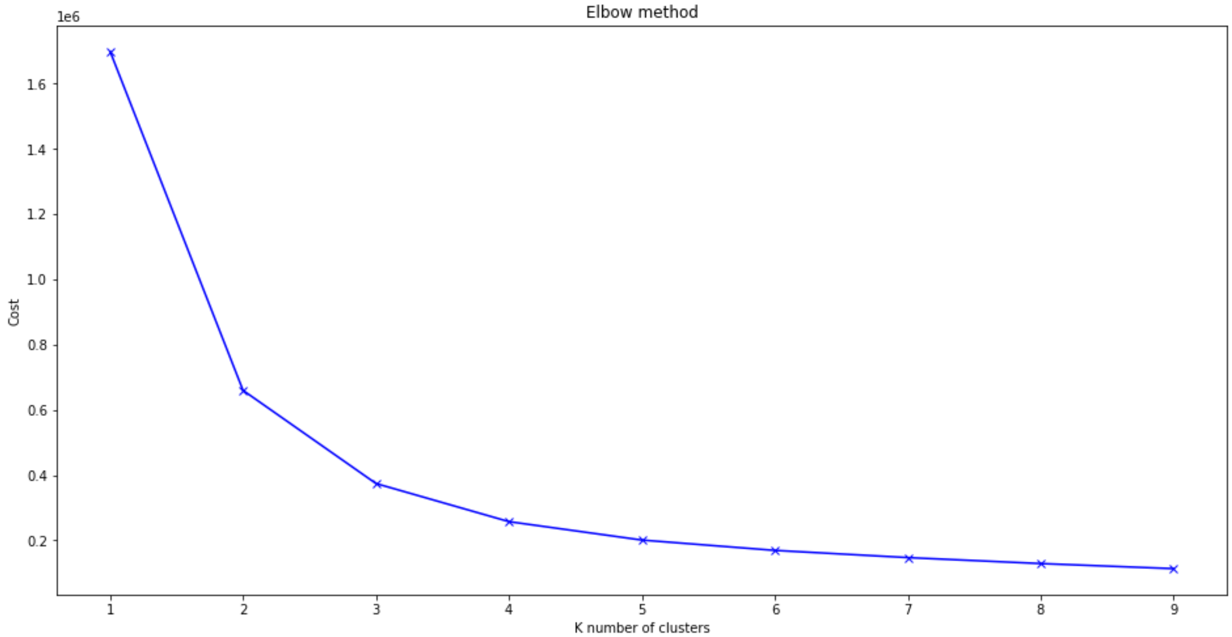
I am suspecting the initialization vectors and centroids are incorrect. They are all on the line of y=x, which disturbed the accurate clustering process.

Based on the picture, a better intialization would be y=c, where c in the constant, since apparently points can be clustered as blocks layered parallel to the x axis.

In [88]:

```
cost = []
K = range(1,10)
for k in K:
    kmeanModel = cluster.KMeans(n_clusters=k)
    kmeanModel.fit(data)
    cost.append(kmeanModel.inertia_) #sum of distances to cluster center

plt.figure(figsize=(16,8))
plt.plot(K, cost, 'bx-')
plt.xlabel('K number of clusters')
plt.ylabel('Cost')
plt.title('Elbow method')
plt.show()
```

```
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
/Users/haoranzhang/opt/anaconda3/lib/python3.8/site-packages/sklearn/clus
ter/_kmeans.py:870: FutureWarning: The default value of `n_init` will cha
nge from 10 to 'auto' in 1.4. Set the value of `n_init` explicitly to sup
press the warning
  warnings.warn(
```



k=3 is a relatively good choice from the perspective of cost, according to Elbow method.

Q1 a) b) write k-means by hand

In [89]:
```
df = pd.DataFrame(data, columns = ['x','y'])
df
```

Out[89]:

	x	y
0	-11.969996	-8.039628
1	-26.961416	-6.962109
2	-12.915849	-1.378941
3	22.476144	2.066612
4	-13.146631	4.835322
...	...	...
4995	6.852668	1.549076
4996	-2.340729	-7.343469
4997	12.925503	-0.103197
4998	-34.336778	4.405753
4999	-26.812739	-1.639956

5000 rows × 2 columns

In [90]:
```
def euclideanDistance(p1,p2):
    return math.sqrt(((p1[0]-p2[0])**2)+((p1[1]-p2[1])**2) )

def find_closest_centroid(point,centroids):
#     for i in range(len(centroids)):
#         clusters[i]=list()
    minV=float("inf")
    label=-1
    for i, center in enumerate(centroids):
        if euclideanDistance(center,point)<minV:
            label=i
            minV=euclideanDistance(center,point)
    return label

#compute new centroid after assign
def computeNewCentroid(X):
    new=list()
    tmp=X.groupby(by='label').mean()
    for row in range(len(tmp)):
        new.append([tmp.at[row,'x'],tmp.at[row,'y']])
    return new


#check if centroids change
def centroidsChange(old,new):
    if euclideanDistance(old,new)>0.000000000001:
        return True
    else:
        return False

def fit(X,initialization,max_iter,n_cluster):
    centroids=initialization
#     print("initial centroids =",centroids)

    X['label']=X.apply(lambda row: find_closest_centroid((row[0],row[1]),ce
    for i in range(max_iter):
        new_centroids=computeNewCentroid(X)
#         print("new centroids is",new_centroids)
        for i in range(len(centroids)):
            if not centroidsChange(centroids[i],new_centroids[i]):
                break
            else:
                centroids=new_centroids
                X['label']=X.apply(lambda row: find_closest_centroid((row[0
    return centroids

initialization=[[10,10],[-10,-10],[2,2],[3,3],[-3,-3]]
#         new_centroids=findNewCentroids(centroids)
clusters=fit(df,initialization,100,5)
clusters=np.asarray(clusters)
clusters
```

```
Out[90]: array([[ 28.09146831,  -1.0241717 ],
       [-30.93833893,   2.43436785],
       [  0.52511293,  -2.3158082 ],
       [ 12.86956124,  -1.68099097],
       [-12.88359021,  -0.80312447]])
```

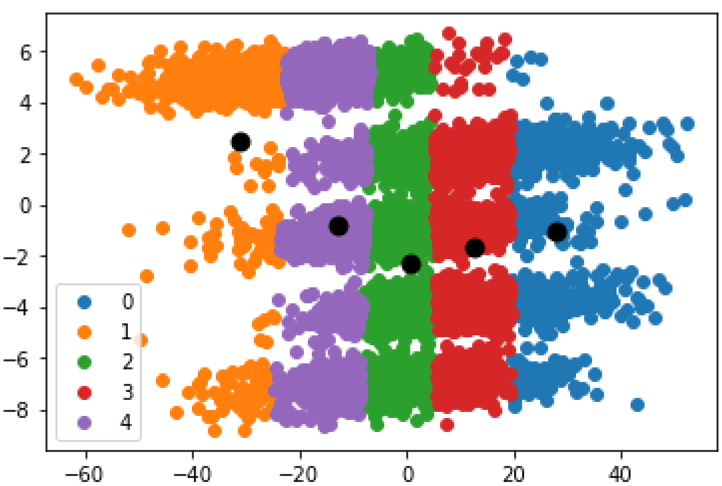
```
In [91]: df
```

Out[91]:

	x	y	label
0	-11.969996	-8.039628	4
1	-26.961416	-6.962109	1
2	-12.915849	-1.378941	4
3	22.476144	2.066612	0
4	-13.146631	4.835322	4
...	...	...	...
4995	6.852668	1.549076	3
4996	-2.340729	-7.343469	2
4997	12.925503	-0.103197	3
4998	-34.336778	4.405753	1
4999	-26.812739	-1.639956	1

5000 rows × 3 columns

```
In [92]: u_labels=np.unique(df['label']) #allocat label to different clusters
for i in u_labels:
    plt.scatter(data[label==i,0],data[label==i,1],label=i)
plt.scatter(clusters[:,0] , clusters[:,1] , s = 80, color = 'k')
plt.legend()
plt.show()
```



Q2

Q2 a)

In [93]:
```
#compute P-1 first
P_value=[[10,0.5],[-10,0.25]]
P=np.matrix(P_value)
R=(P.T @ P).I
```

In [94]:
```
R
```

Out[94]:
```
matrix([[ 0.00555556, -0.04444444],
        [-0.04444444,  3.55555556]])
```

In [95]:
```
def mahalaDistance(x,y):
    x=np.asarray(x)
    y=np.asarray(y)
    return (x-y).T @ R @ (x-y)

def find_closest_centroid(point,centroids):
#     for i in range(len(centroids)):
#         clusters[i]=list()
    minV=float("inf")
    label=-1
    for i, center in enumerate(centroids):
        if mahalaDistance(center,point)<minV:
            label=i
            minV=mahalaDistance(center,point)
    return label

#compute new centroid after assign
def computeNewCentroid(X):
    new=list()
    tmp=X.groupby(by='label').mean()
    for row in range(len(tmp)):
        new.append([tmp.at[row,'x'],tmp.at[row,'y']])
    return new


#check if centroids change
def centroidsChange(old,new):
    if mahalaDistance(old,new)>0.000000000001:
        return True
    else:
        return False

def fit(X,initialization,max_iter,n_cluster):
    centroids=initialization
#     print("initial centroids =",centroids)

    X['label']=X.apply(lambda row: find_closest_centroid((row[0],row[1]),ce
    for i in range(max_iter):
        new_centroids=computeNewCentroid(X)
#         print("new centroids is",new_centroids)
        for i in range(len(centroids)):
            if not centroidsChange(centroids[i],new_centroids[i]):
                break
            else:
                centroids=new_centroids
                X['label']=X.apply(lambda row: find_closest_centroid((row[0
    return centroids
```

In [96]:
```
df= pd.DataFrame(data, columns = ['x','y'])
```

In [97]:
```
df
```

Out[97]:

	x	y
0	-11.969996	-8.039628
1	-26.961416	-6.962109
2	-12.915849	-1.378941
3	22.476144	2.066612
4	-13.146631	4.835322
...	...	...
4995	6.852668	1.549076
4996	-2.340729	-7.343469
4997	12.925503	-0.103197
4998	-34.336778	4.405753
4999	-26.812739	-1.639956

5000 rows × 2 columns

In [98]:
```
initialization=[[10,10],[-10,-10],[2,2],[3,3],[-3,-3]]
#          new_centroids=findNewCentroids(centroids)
clusters=fit(df,initialization,100,5)
clusters=np.asarray(clusters)
clusters
```

Out[98]:
```
array([[-20.40310599,   5.00655283],
       [ -3.24764066,  -7.01766444],
       [ -1.00542627,  -0.97629543],
       [  9.96452001,   2.01588007],
       [  8.72391858,  -4.00459056]])
```

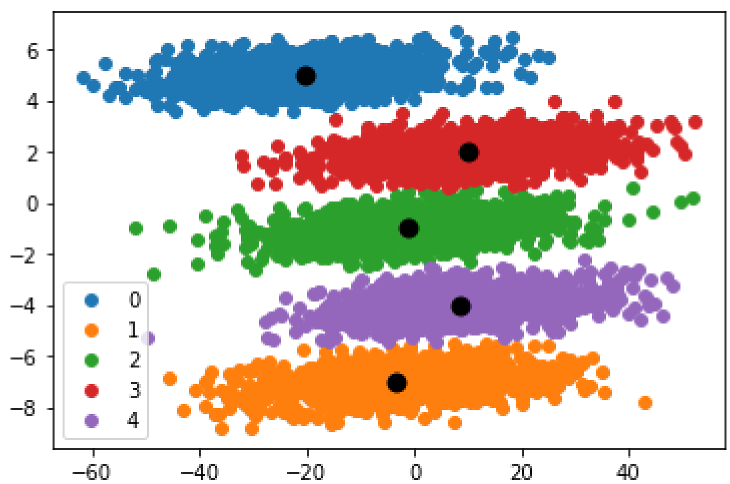
In [99]:
```
df
```

Out[99]:

	x	y	label
0	-11.969996	-8.039628	1
1	-26.961416	-6.962109	1
2	-12.915849	-1.378941	2
3	22.476144	2.066612	3
4	-13.146631	4.835322	0
...	...	...	...
4995	6.852668	1.549076	3
4996	-2.340729	-7.343469	1
4997	12.925503	-0.103197	2
4998	-34.336778	4.405753	0
4999	-26.812739	-1.639956	2

5000 rows × 3 columns

In [100]:
```
u_labels=np.unique(df['label']) #allocat label to different clusters
for i in u_labels:
    row=np.where(df.label==i)
    plt.scatter(df.iloc[row]['x'],df.iloc[row]['y'],label=i)
plt.scatter(clusters[:,0] , clusters[:,1] , s = 80, color = 'k')
plt.legend()
plt.show()
```



Points are clustered into a right way. Using Mahalanobis Distance allows a different way to calculate similarity between points.

Q2 b)

In [101]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
import sklearn
```

In [102]:
```
data_q2=df
data_q2.head()
```

Out[102]:

	x	y	label
0	-11.969996	-8.039628	1
1	-26.961416	-6.962109	1
2	-12.915849	-1.378941	2
3	22.476144	2.066612	3
4	-13.146631	4.835322	0

In [103]:
```
target=data_q2['label']
```

In [104]:
```
data=data_q2.loc[:,['x','y']]
```

In [105]:
```
data
```

Out[105]:

	x	y
0	-11.969996	-8.039628
1	-26.961416	-6.962109
2	-12.915849	-1.378941
3	22.476144	2.066612
4	-13.146631	4.835322
...	...	...
4995	6.852668	1.549076
4996	-2.340729	-7.343469
4997	12.925503	-0.103197
4998	-34.336778	4.405753
4999	-26.812739	-1.639956

5000 rows × 2 columns

In [106]:
```
from sklearn.preprocessing import StandardScaler
scaler = StandardScaler()
print(scaler.fit(data))
```

```
StandardScaler()
```

In [107]:
```
print(scaler.mean_)
```

```
[-1.22012285 -1.00360882]
```

In [108]:
```
data_z_score=scaler.transform(data)
```

In [109]:
```
print(data_z_score)
```

```
[[-0.59957277 -1.64159106]
 [-1.43571729 -1.39019249]
 [-0.65232761 -0.08756979]
 ...
 [ 0.78897044  0.21007735]
 [-1.84707718  1.26207159]
 [-1.42742483 -0.14846773]]
```

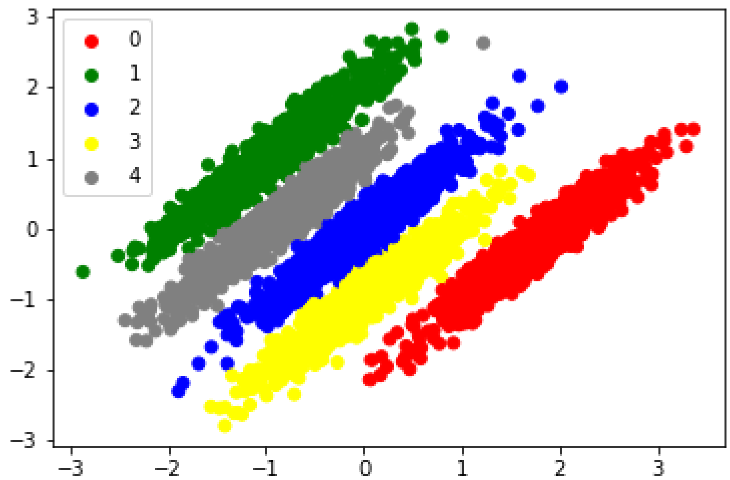
PCA

In [110]:
```
from sklearn.decomposition import PCA
# Two components of PCA
pca = PCA(2)

# Fit on data
pca.fit(data_z_score)

# Access values and vectors
print(pca.components_)
print(pca.explained_variance_)

# transform data
B = pca.transform(data_z_score)
print(B)
```

```
[[-0.70710678  0.70710678]
 [-0.70710678 -0.70710678]]
[1.22552735 0.77487273]
[[-0.7368182   1.58474214]
 [ 0.03219089  1.99821997]
 [ 0.39934408  0.52318647]
 ...
 [-0.40933923 -0.70643346]
 [ 2.19850018  0.41366142]
 [ 0.90435924  1.11432431]]
```

In [111]:
```
B.shape
```
Out[111]:
```
(5000, 2)
```

In [112]:
```
import matplotlib.pyplot as plt
import matplotlib
colors=['red','green','blue','yellow','grey']
scatter=plt.scatter(B[:, 0], B[:, 1],c=target,\
                    cmap=matplotlib.colors.ListedColormap(colors),marker='o
plt.legend(*scatter.legend_elements())
plt.show()
```



if not Z-score scalar

In [113]:
```
data
```
Out[113]:

	x	y
0	-11.969996	-8.039628
1	-26.961416	-6.962109
2	-12.915849	-1.378941
3	22.476144	2.066612
4	-13.146631	4.835322
...	...	...
4995	6.852668	1.549076
4996	-2.340729	-7.343469
4997	12.925503	-0.103197
4998	-34.336778	4.405753
4999	-26.812739	-1.639956

5000 rows × 2 columns

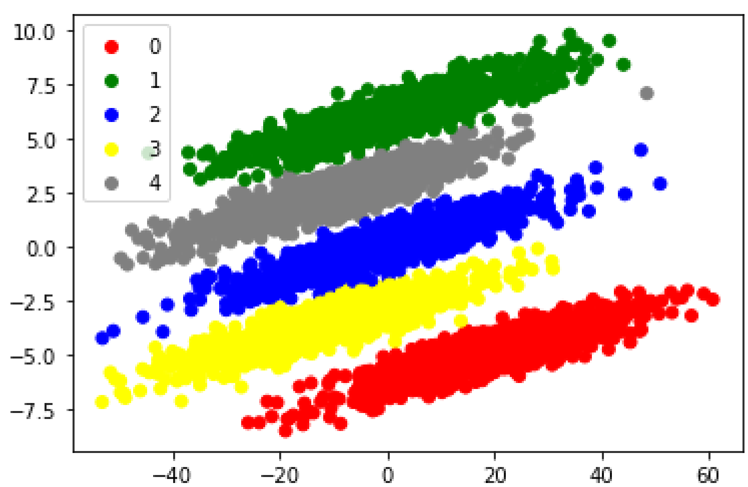
In [114]:

```
from sklearn.decomposition import PCA
# Two components of PCA
pca = PCA(2)

# Fit on data
pca.fit(data)

# Access values and vectors
print(pca.components_)
print(pca.explained_variance_)

# transform data
B = pca.transform(data)
print(B)
```

```
[[-0.99838317  0.05684225]
 [-0.05684225 -0.99838317]]
[322.50713273  17.38845582]
[[ 10.33254906   7.6356903 ]
 [ 25.36097916   7.41205973]
 [ 11.65548133   1.03953719]
 ...
 [-14.07157308  -1.70302537]
 [ 33.3705919   -3.51819072]
 [ 25.51506544   2.09006015]]
```

In [115]:

```
import matplotlib.pyplot as plt
import matplotlib
colors=['red','green','blue','yellow','grey']
scatter=plt.scatter(B[:, 0], B[:, 1],c=target,\
                    cmap=matplotlib.colors.ListedColormap(colors),marker='o
plt.legend(*scatter.legend_elements())
plt.show()
```



Q2 c)

In [116]:
```
data_q2
```

Out[116]:

	x	y	label
0	-11.969996	-8.039628	1
1	-26.961416	-6.962109	1
2	-12.915849	-1.378941	2
3	22.476144	2.066612	3
4	-13.146631	4.835322	0
...	...	...	...
4995	6.852668	1.549076	3
4996	-2.340729	-7.343469	1
4997	12.925503	-0.103197	2
4998	-34.336778	4.405753	0
4999	-26.812739	-1.639956	2

5000 rows × 3 columns

In [117]:
```
cluster_0=data_q2[data_q2['label'] == 0]
cluster_1=data_q2[data_q2['label'] == 1]
cluster_2=data_q2[data_q2['label'] == 2]
cluster_3=data_q2[data_q2['label'] == 3]
cluster_4=data_q2[data_q2['label'] == 4]
```

In [118]:
```
cluster_0
```
Out[118]:

	x	y	label
4	-13.146631	4.835322	0
12	-38.508282	4.112202	0
19	6.843345	4.550918	0
22	-36.768240	4.173526	0
25	-16.883207	4.664295	0
...	...	...	...
4985	-41.441343	5.164262	0
4987	-39.028493	5.153460	0
4991	-14.425117	5.286585	0
4994	-11.915634	5.320493	0
4998	-34.336778	4.405753	0

1000 rows × 3 columns

In [119]:
```
cluster_0=cluster_0.iloc[:,:2]
cluster_1=cluster_1.iloc[:,:2]
cluster_2=cluster_2.iloc[:,:2]
cluster_3=cluster_3.iloc[:,:2]
cluster_4=cluster_4.iloc[:,:2]
```

In [120]:
```
def pca_onto_cluster(x):
#     scaler = StandardScaler()
#     scaler.fit(data)
#     data_z_score=scaler.transform(data)
    # Two components of PCA
    pca = PCA(2)

    # Fit on data
    pca.fit(x)

    # Access values and vectors
    # transform data
    B = pca.transform(x)
    return B
```

In [121]:
```
res0=pca_onto_cluster(cluster_0)
res0
```

Out[121]:
```
array([[  7.25405752,   0.25378307],
       [-18.11417956,   0.68829451],
       [ 27.23950349,   0.76561297],
       ...,
       [  5.98078814,  -0.21199726],
       [  8.49049436,  -0.21735037],
       [-13.9396063 ,   0.44222502]])
```

In [122]:
```
res1=pca_onto_cluster(cluster_1)
res1
```

Out[122]:
```
array([[ -8.73419248,   0.91529791],
       [-23.71132555,  -0.34533961],
       [  1.94266087,  -0.75298475],
       ...,
       [  6.37240317,   0.70787872],
       [-15.07539706,  -0.44106933],
       [  0.90286284,   0.33686259]])
```

In [123]:
```
res2=pca_onto_cluster(cluster_2)
res2
```

Out[123]:
```
array([[-11.81898073,  -0.22222287],
       [ -7.96831058,  -0.07484901],
       [-21.787302  ,   0.26996112],
       ...,
       [-15.0302669 ,  -0.59978662],
       [ 14.03719841,   0.70566602],
       [-25.71812447,  -0.29620696]])
```

In [124]:
```
res3=pca_onto_cluster(cluster_3)
res3
```

Out[124]:
```
array([[ 12.50154109,  -0.080114  ],
       [ 26.88083768,   0.15701845],
       [-12.34056372,  -0.61900941],
       ...,
       [ 28.29556699,  -0.14442832],
       [ -8.76609494,   0.96638809],
       [ -3.12687729,  -0.41684377]])
```

In [125]:
```
res4=pca_onto_cluster(cluster_4)
res4
```

Out[125]:
```
array([[25.99078891, -0.97033838],
       [58.70744189, -0.41801869],
       [ 8.42336008, -0.14735909],
       ...,
       [-1.4299455 , -0.83051362],
       [24.30622751, -0.48802078],
       [ 0.58512901, -0.68303187]])
```

In [126]:
```python
res=list()
res.append(res0)
res.append(res1)
res.append(res2)
res.append(res3)
res.append(res4)
```

In [127]:
```python
res
```

Out[127]:
```
[array([[  7.25405752,   0.25378307],
        [-18.11417956,   0.68829451],
        [ 27.23950349,   0.76561297],
        ...,
        [  5.98078814,  -0.21199726],
        [  8.49049436,  -0.21735037],
        [-13.9396063 ,   0.44222502]]),
 array([[ -8.73419248,   0.91529791],
        [-23.71132555,  -0.34533961],
        [  1.94266087,  -0.75298475],
        ...,
        [  6.37240317,   0.70787872],
        [-15.07539706,  -0.44106933],
        [  0.90286284,   0.33686259]]),
 array([[-11.81898073,  -0.22222287],
        [ -7.96831058,  -0.07484901],
        [-21.787302  ,   0.26996112],
        ...,
        [-15.0302669 ,  -0.59978662],
        [ 14.03719841,   0.70566602],
        [-25.71812447,  -0.29620696]]),
 array([[ 12.50154109,  -0.080114  ],
        [ 26.88083768,   0.15701845],
        [-12.34056372,  -0.61900941],
        ...,
        [ 28.29556699,  -0.14442832],
        [ -8.76609494,   0.96638809],
        [ -3.12687729,  -0.41684377]]),
 array([[25.99078891, -0.97033838],
        [58.70744189, -0.41801869],
        [ 8.42336008, -0.14735909],
        ...,
        [-1.4299455 , -0.83051362],
        [24.30622751, -0.48802078],
        [ 0.58512901, -0.68303187]])]
```

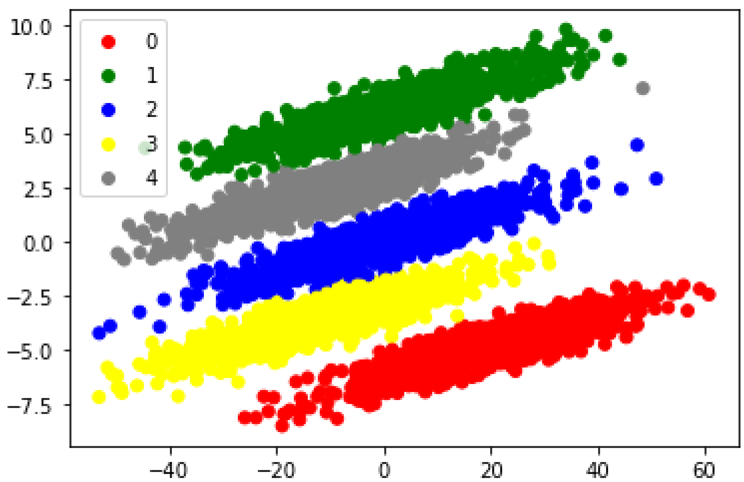
In [128]:
```python
res[0]
```

Out[128]:
```
array([[  7.25405752,   0.25378307],
       [-18.11417956,   0.68829451],
       [ 27.23950349,   0.76561297],
       ...,
       [  5.98078814,  -0.21199726],
       [  8.49049436,  -0.21735037],
       [-13.9396063 ,   0.44222502]])
```

In [129]:
```
res[1]
```

Out[129]:
```
array([[ -8.73419248,   0.91529791],
       [-23.71132555,  -0.34533961],
       [  1.94266087,  -0.75298475],
       ...,
       [  6.37240317,   0.70787872],
       [-15.07539706,  -0.44106933],
       [  0.90286284,   0.33686259]])
```

In [132]:
```
import matplotlib.pyplot as plt
import matplotlib
colors=['red','green','blue','yellow','grey']
scatter=plt.scatter(B[:, 0], B[:, 1],c=target,\
                    cmap=matplotlib.colors.ListedColormap(colors),marker='o
plt.legend(*scatter.legend_elements())
plt.show()
```



The result of 2b and 2c are the same.

Q2 d)

In [133]:
```
vector,mat=np.linalg.eig(P.T @P )
```

In [134]:
```
vector
```

Out[134]:
```
array([200.031294,   0.281206])
```

In [135]:
```
mat
```

Out[135]:
```
matrix([[ 0.99992166, -0.01251662],
        [ 0.01251662,  0.99992166]])
```

In [136]:
```
vector * mat
```

Out[136]:
```
matrix([[200.01914411,  -2.22253163]])
```

In [137]:
```
R.I
```

Out[137]:
```
matrix([[200.    ,   2.5   ],
        [  2.5   ,   0.3125]])
```

In [ ]:
```

```