

BST02: Using R for Statistics in Medical Research

Part D: Statistics with R

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In this Section

- ▶ Common statistical tests
 - ▶ for continuous data
 - ▶ for categorical data
- ▶ (Generalized) linear regression
- ▶ Useful functions for regression models
- ▶ Modeling non-linear effects

t-test: `t.test()`

One-sample t-test

- ▶ compares the **mean of one sample** with a fixed value μ

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Two sample / independent samples t-test

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Related samples t-test

- ▶ compares the **mean of the difference** between related observations with a fixed value μ (same as one-sample t-test)

Wilcoxon Test: `wilcox.test()`

Wilcoxon Signed Rank Test

- tests if **one sample** (or the differences between two paired samples) is/are **symmetric about** μ

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Wilcoxon Rank Sum Test / Mann-Whitney test

- ▶ test for a **location shift between the distributions** of two independent samples

See also BBR Sections 7.2 & 7.3 (<http://hbiostat.org/doc/bbr.pdf>)

Kruskal-Wallis Rank Sum Test: `kruskal.test()`

- ▶ **extension** of the Wilcoxon rank sum test for **more than two groups**
- ▶ test for a **difference in location** of a continuous variable between multiple groups
- ▶ the **Wilcoxon rank sum test is a special case** of the Kruskal-Wallis rank sum test

Other tests for continuous data

- ▶ **Kolmogoriv-Smirnov Test:** `ks.test()`
tests if two samples are drawn from the same continuous distribution
- ▶ **Shapiro-Wilk Normality Test:** `shapiro.test()`
- ▶ **Friedman Rank Sum Test:** `friedman.test()`
non-parametric test for two or more related samples
- ▶ ...

Tests for Continuous Data

Demo

- ▶ Tests for Continuous Data

R

html

Tests for Categorical Data / Proportions

One-sample Proportion Test

- ▶ tests if the **proportion in one sample** is equal to a fixed value p
- ▶ `prop.test()` and `binom.test()`

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Tests for Proportions in Multiple (independent) Groups

- ▶ tests if the **proportions in several samples** are equal
- ▶ `chisq.test()` and `fisher.test()` (when there are cells with 0)

See also BBR Sections 5.7 & 6 (<http://hbiostat.org/doc/bbr.pdf>)

Tests for Categorical Data / Proportions

Related Samples: McNemar Test

- ▶ Tests for **symmetry** in a 2×2 table
- ▶ `mcnemar.test()`

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3-Dimensional Contingency Table

- ▶ Cochran-Mantel-Haenszel Test
- ▶ χ^2 test for **independence** of two nominal variables **within each stratum**
- ▶ `mantelhaen.test()`

Tests for Categorical Data

Demo

- ▶ Tests for Categorical Data

R **html**

Practical

- ▶ Statistical Tests

html

Useful Functions: Statistical Tests

Continuous Outcomes

- ▶ `t.test()`
- ▶ `wilcox.test()`
- ▶ `kruskal.test()`
- ▶ `ks.test()`
- ▶ `friedman.test()`
- ▶ `shapiro.test()`

Categorical Outcomes

- ▶ `prop.test()`
- ▶ `binom.test()`
- ▶ `chisq.test()`
- ▶ `fisher.test()`
- ▶ `mcnemar.test()`
- ▶ `mantelhaen.test()`

Pairwise tests

- ▶ `pairwise.prop.test()`
- ▶ `pairwise.t.test()`
- ▶ `pairwise.wilcox.test()`

Variance and Correlation

- ▶ `cor.test()`
- ▶ `bartlett.test()`
- ▶ `var.test()`

Multiple Testing Adjustment

- ▶ `p.adjust()`

Linear Regression

A standard linear regression model has the form

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p + \varepsilon \quad \text{with} \quad \varepsilon \sim N(0, \sigma^2)$$

where

- ▶ y is the **outcome** variable ("dependent variable")
- ▶ x_1, \dots, x_p are the **covariates** ("independent variables")
- ▶ β_0, \dots, β_p are the **regression coefficients**
 - ▶ β_0 is the intercept
 - ▶ β_1, \dots, β_p estimate the effects of the covariates
- ▶ ε is a vector of **residuals**, which we assume to be (approximately) normally distributed.

Linear Regression

To fit a **linear regression** in R we use the function `lm()`.

The most important arguments are

- ▶ **formula:**
a formula object
- ▶ **data:**
a `data.frame` (optional, but usually needed)
- ▶ **subset:**
a vector specifying which observations should be used (optional)
(works like the `subset` argument of the function `subset()`)

Model Formula

A `formula` object has the form

```
outcome ~ linear predictor
```

for example

```
y ~ x1 + x2 + x3
```

- ▶ Variables are separated by "+" signs.
- ▶ An intercept is automatically included.
- ▶ One-sided formulas (omitting the outcome) are possible (used for random effects specification).

Model Formula: Interactions

Interaction terms are written using ":" or "*".

"*" includes the main effects and interaction terms, i.e.,

```
y ~ x1 * x2
```

is equivalent to

```
y ~ x1 + x2 + x1:x2
```

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is equivalent to

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y ~ x1 + x2 + x1:x2
```

Interactions between multiple variables can be written using “()”, i.e.,

```
y ~ x1 * (x2 + x3)
```

is equivalent to

```
y ~ x1 * x2 + x1 * x3
```

Model Formula: Interactions

To specify a **higher level interaction** “^” is used.

For example:

```
y ~ (x1 + x2 + x3)^3
```

will create all interactions up to 3-way and is equivalent to

```
y ~ x1 * x2 * x3
```

and equivalent to

```
y ~ x1 + x2 + x3 + x1:x2 + x1:x3 + x2:x3 + x1:x2:x3
```

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and equivalent to

```
y ~ x1 + x2 + x3 + x1:x2 + x1:x3 + x2:x3 + x1:x2:x3
```

and

```
y ~ (x1 + x2 + x3)^2
```

will create all two-way interactions and is equivalent to

```
y ~ x1 + x2 + x3 + x1:x2 + x1:x3 + x2:x3
```

Model Formula: Removing terms

The “-” sign can be used to remove terms from a model formula.

For example

```
y ~ x1 * x2 * x3 - x2 - x1:x3
```

is equivalent to

```
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```

The **intercept** can be removed from a formula by using “-1” or “+0”, i.e.

```
y ~ x1 + x2 - 1
```

```
y ~ x1 + x2 + 0
```

Generalized Linear Regression (GLM)

A **generalized linear regression** model has the form

$$g(\mathbb{E}(y)) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

where $g(\)$ is a link function and y is from the exponential family.

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where $g(\cdot)$ is a link function and y is from the exponential family.

For example **logistic regression** for binary y :

$$\log \left(\frac{P(y = 1)}{1 - P(y = 1)} \right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$$

$\log \left(\frac{p}{1-p} \right)$ is the **logit** link.

Generalized Linear Regression (GLM)

To fit a **GLM** in R we use the function `glm()`.

The most important arguments are

- ▶ **formula:**
a formula object
- ▶ **family:**
a family object or name of the family function, describing the error distribution and link function
- ▶ **data:**
a `data.frame` (optional, but usually needed)
- ▶ **subset:**
a vector specifying which observations should be used (optional)

Families and Link Functions

Common families & available links in R:

(see also ?family)

family	link
binomial	logit, probit, cauchit, log, cloglog
gaussian	identity, log, inverse
Gamma	inverse, identity, log
poisson	log, identity, sqrt

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The `family` argument in `glm()` can be specified in the following ways:

- ▶ `binomial(link = "logit")`
- ▶ `binomial()`
- ▶ `binomial`
- ▶ `"binomial"`

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(see also `?family`)

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The `family` argument in `glm()` can be specified in the following ways:

- ▶ `binomial(link = "logit")`
- ▶ `binomial()`
- ▶ `binomial`
- ▶ `"binomial"`

Note:

When the link is not explicitly specified (i.e. options 2-4), the default link is used.

Regression

Demo

- ▶ Regression Basics

R

html

Practical

- ▶ Linear Regression **html**

Model Evaluation

Linear model:

Evaluate the assumptions of a linear regression model visually, for example:

- ▶ Histogram
- ▶ Normal QQ-plot
- ▶ Plot residuals vs fitted values

Model Comparison

Nested models:

- ▶ model is a **special case** of the other, i.e.,
- ▶ model B is a special case of model A when B can be obtained by setting some regression coefficients in A to zero

Comparison using a **likelihood ratio (LR) test**, for example:

```
anova(modelA, modelB)
```

```
anova(modelA, modelB, test = "LRT") # for a glm
```

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Comparison using a **likelihood ratio (LR) test**, for example:

```
anova(modelA, modelB)  
anova(modelA, modelB, test = "LRT") # for a glm
```

Non-nested models:



Comparison using information criteria, e.g.

```
AIC(modelA, modelB)  
BIC(modelA, modelB)
```

The model with the **smaller** AIC (or BIC) has the **better** fit.

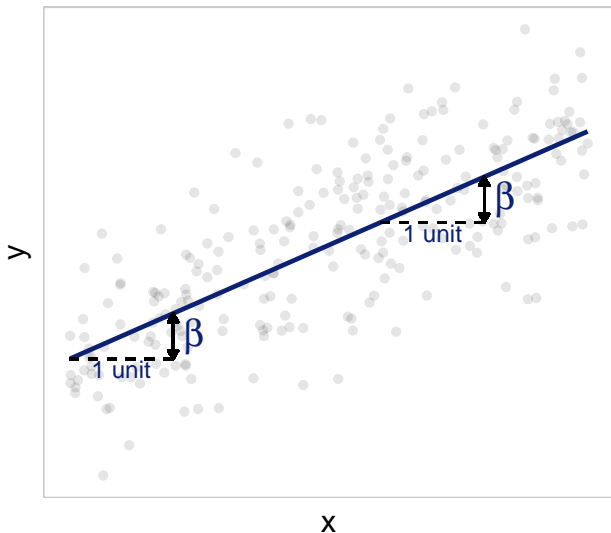
Model Evaluation

Demo

► Model Evaluation  

Non-linear Effects

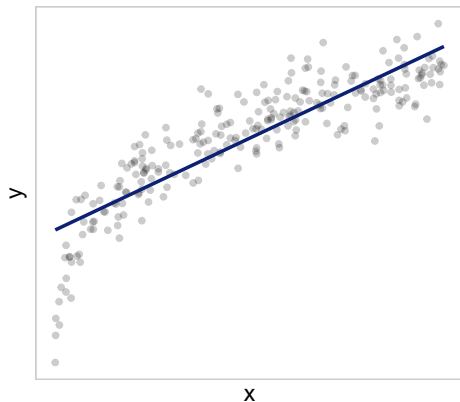
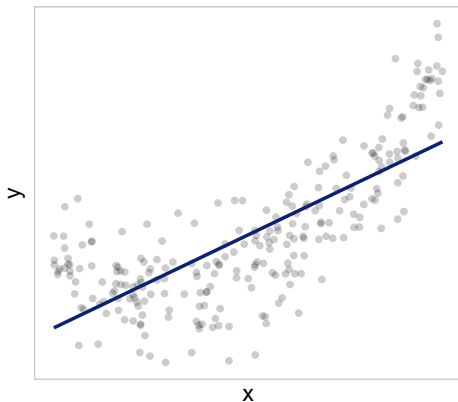
Default assumption: **linear effect**, i.e., $x \rightarrow y \rightarrow x + 1 \rightarrow y + \beta$, $\forall x$



Non-linear Effects

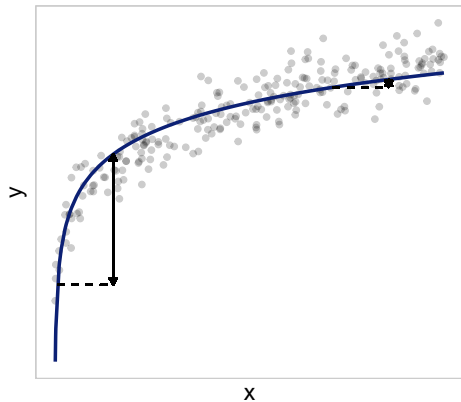
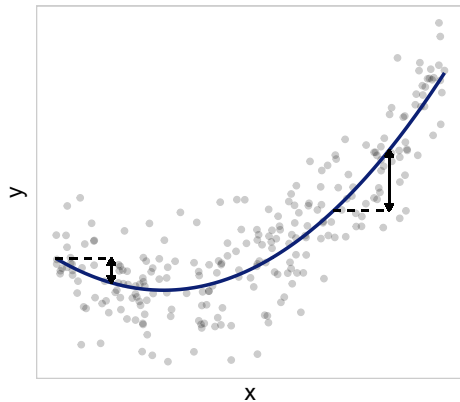
Default assumption: **linear effect**, i.e., $x \rightarrow y \Rightarrow x + 1 \rightarrow y + \beta, \quad \forall x$

This may not always be the case:



Non-linear Effects

Here, we would like to allow the **effect** of a one-unit increase **of x** to **change with the value of x** :



Non-linear Effects

Sometimes, we can use

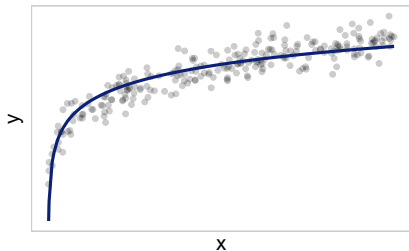
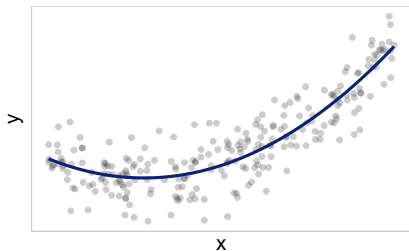
- ▶ a **transformation of x**, or
- ▶ x as well as a **polynomial** of x (or a transformation).

For example:

$$y \sim x + I(x^2)$$

or

$$y \sim \log(x)$$



Non-linear Effects: `I()`

The function `I()` is needed to distinguish between operators that need to be interpreted as

- ▶ arithmetic operators and
- ▶ formula operators

Example:

```
y ~ I(a + b)
```

would be the same as

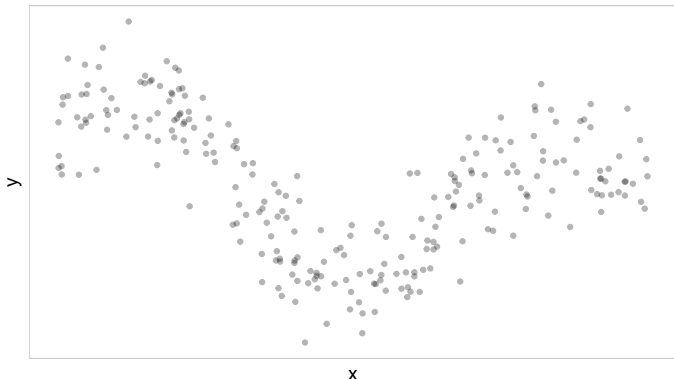
```
z <- a + b  
y ~ z
```

but not the same as

```
y ~ a + b
```

Complex Non-linear Effects

Non-linear effects may be **more complex** than can be modeled with a simple transformation or polynomial.



Also: the shape may depend on other covariates in the model

➔ we do not always know the shape in advance

➔ **Regression Splines / B-Splines**

B-Splines

A **B-Spline** is a linear combination of a set of **basis functions**.

These basis functions are defined so that they are

- ▶ a **polynomial functions** inside a given interval, and
- ▶ zero outside that interval.

The intervals are defined by a set of **knots**.

The polynomial function have a certain **degree** (i.e., constant, linear, quadratic, ...)

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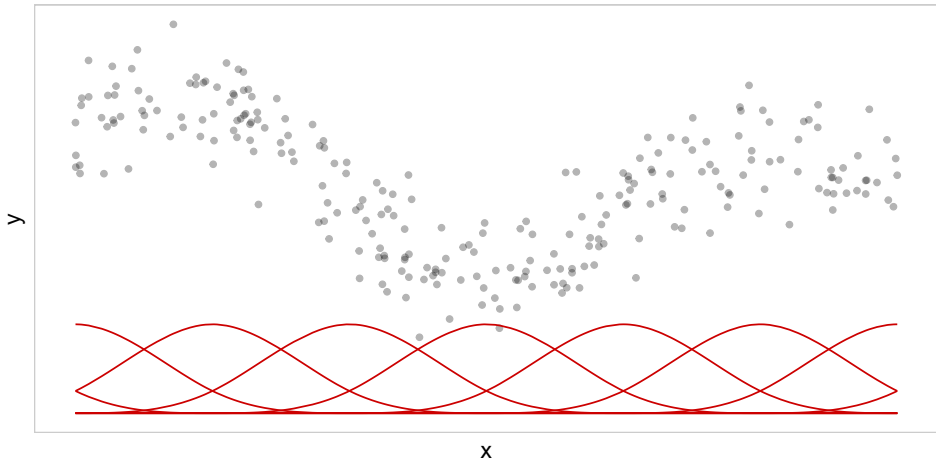
B-Splines in R

The R package **splines** provides the functions

- ▶ **bs()**: B-splines
- ▶ **ns()**: natural cubic (B-)splines

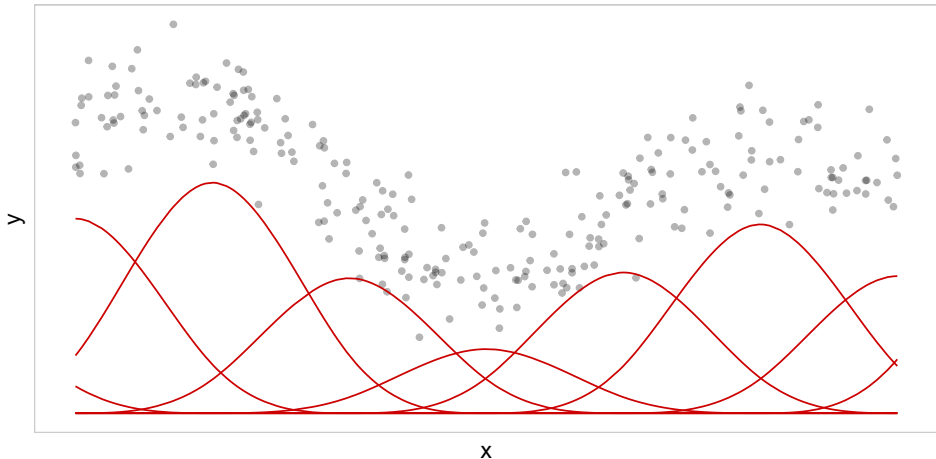
B-Splines

Instead of $y \sim \beta_0 + \beta_1 x + \dots$ we assume $y \sim \beta_0 + \sum_{\ell=1}^d \beta_{\ell} B_{\ell}(x) + \dots$



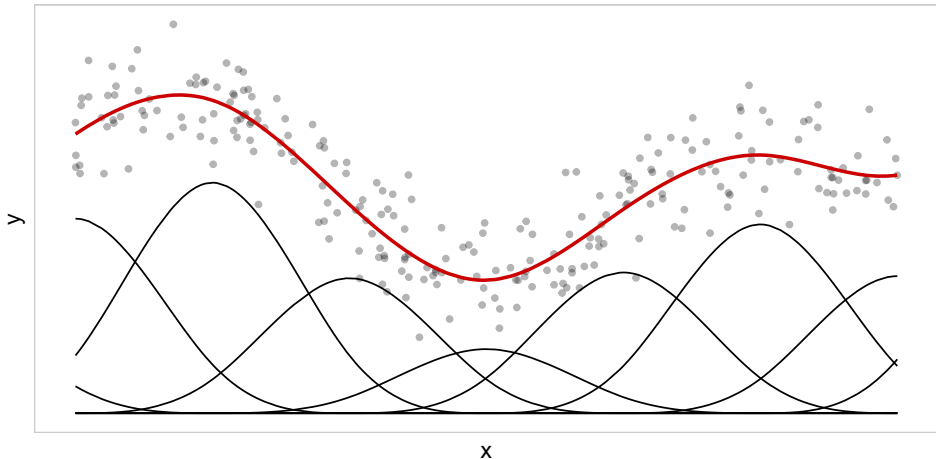
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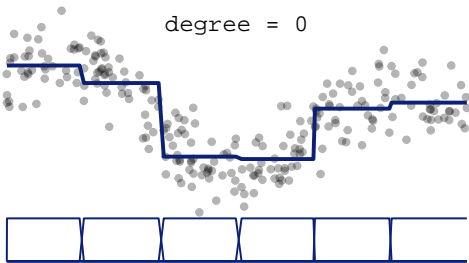
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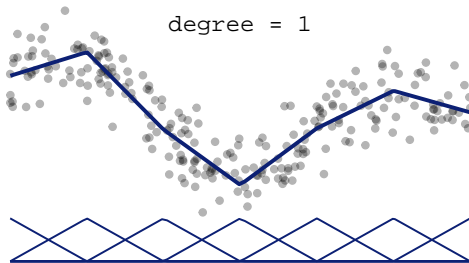


B-Splines: degree

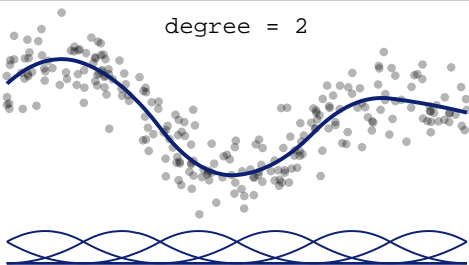
degree = 0



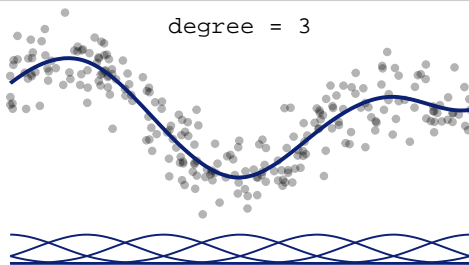
degree = 1



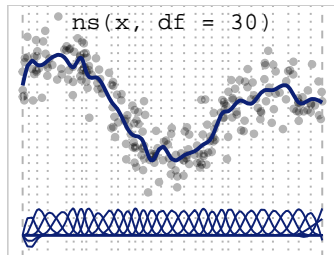
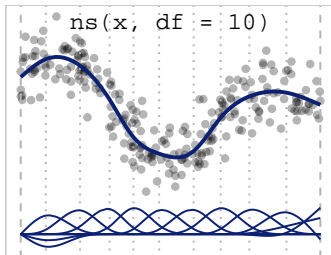
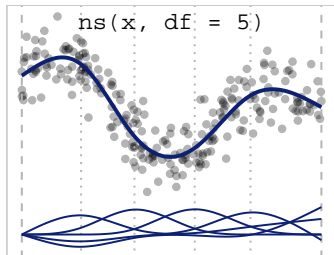
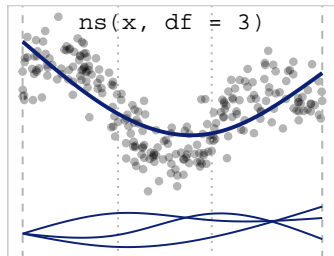
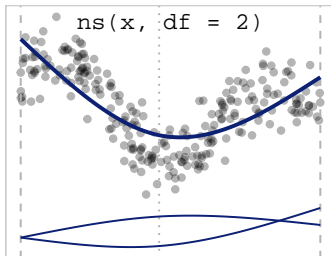
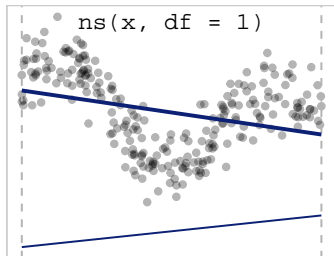
degree = 2



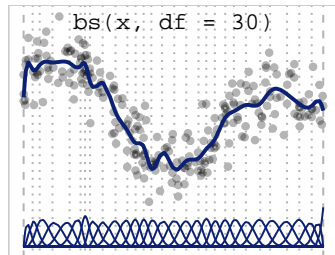
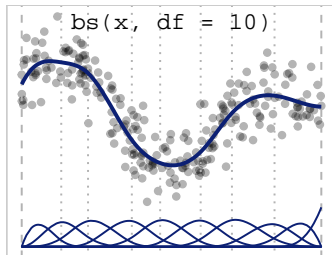
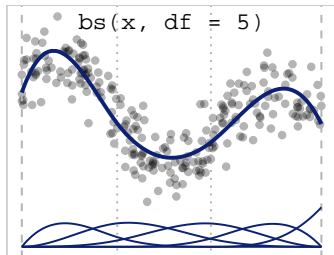
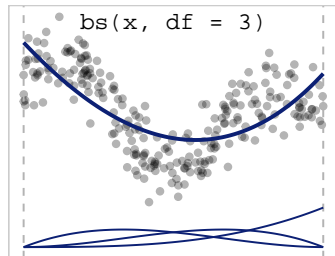
degree = 3



B-Splines: df



B-Splines: df



B-Splines in R: `bs()` & `ns()`

Important arguments of `ns()` and `bs()` are:

degree

- ▶ degree of the polynomial in each of the basis functions
- ▶ in `bs()`: default is 3
- ▶ in `ns()`: always 3 (“cubic”)

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df

- ▶ degrees of freedom, i.e., “number of regression coefficients” used
- ▶ for `bs()`: has to be \geq degree

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- ▶ in `bs()`: default is 3
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knots

- ▶ position of (inner) knots
- ▶ if unspecified:
 - ▶ `df`-degree knots are used
 - ▶ positioned at equally spaced quantiles

df

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- ▶ for `bs()`: has to be \geq degree

B-Splines in R: `bs()` & `ns()`

Important arguments of `ns()` and `bs()` are:

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 - ▶ df-degree knots are used
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Boundary.knots

- ▶ by default: `range(x)`
- ▶ outside the `Boundary.knots` the fit is extrapolated

Non-linear Effects

Demo

- ▶ Splines [R](#) [html](#)

Practical

- ▶ Logistic Regression & More [html](#)
- ▶ Logistic Regression II [html](#)

Regression

Regression Models

- ▶ `lm()`
- ▶ `glm()`

Regression Results

- ▶ `summary()`
- ▶ `coef(), confint()`
- ▶ `fitted(), residuals(),
rstandard()`
- ▶ `AIC(), BIC()`
- ▶ `anova()`

Plots

- ▶ `plot()`
- ▶ `qqnorm(), qqline(), qqplot()`

Formulas

- ▶ Formula operators: `+`, `-`, `*`, `:`, `^`
- ▶ `ns(), bs(), I()`
- ▶ `all.vars()`
- ▶ `update()`
- ▶ `as.formula()`