Formulario

Nicola Ferru

8 settembre 2022

1 Derivate

- $D(x^n) = n * x^{n-1}$
- $D(\log_a x) = \frac{1}{x} \log_a e$
- $D(a^x) = a^x \ln a$
- $D(\sin x) = \cos x$
- $D(\cos x) = -\sin x$
- $D(k) = 0$
- $D(\ln x) = \frac{1}{x}$
- $D(e^x) = e^x$
- $D(\tan x) = \frac{1}{\cos^2 x} = 1 + \tan^2 x$
- $D(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$
- $D(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$
- $D(\arctan x) = \frac{1}{1+x^2}$

Casi:

$$D[k * f(x)] = k * f'(x) \tag{1}$$

$$D[f(x) + g(x) + h(x)] = f' + g' + h' \tag{2}$$

$$D\left[\frac{f(x)}{g(x)}\right] = \frac{f' * g - f * g'}{[g]^2} \tag{3}$$

$$D\left[\frac{1}{f(x)}\right] = \frac{f'(x)}{[f(x)]^2} \tag{4}$$

$$D[f(g(x))] = f'[g(x)] * g' \tag{5}$$

1.1 Limiti Notevoli

1.1.1 esponenziali e logaritmici

$$\lim_{x \to \pm\infty} \left(1 + \frac{1}{x}\right)^x = e \tag{6}$$

$$\lim_{x \to +\infty} \left(1 + \frac{a}{x}\right)^x = e^a \tag{7}$$

$$\lim_{x \to +\infty} \left(1 + \frac{a}{x}\right)^{nx} = e^{na} \tag{8}$$

$$\lim_{x \to -\infty} \left(1 + \frac{a}{x}\right)^x = \frac{1}{e} \tag{9}$$

$$\lim_{x \to 0} (1 + ax)^{\frac{1}{x}} = e^a \tag{10}$$

$$\lim_{x \to 0} \lg_a (1 + x)^{\frac{1}{x}} = \frac{1}{\lg_e a} \tag{11}$$

$$\lim_{x \to 0} \frac{\lg_a (1 + x)}{x} = \lg_a e = \frac{1}{\ln a} \tag{12}$$

$$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a \tag{13}$$

$$\lim_{x \to 0} \frac{(1 + x)^a - 1}{x} = a \tag{14}$$

$$\lim_{x \to 0} \frac{(1 + x)^a - 1}{x} = 1 \tag{15}$$

$$\lim_{x\to 0} x^r \lg_a x = 0 \quad \forall \in R^+ - \{1\}, \quad \forall r \in R^+ \tag{16}$$

$$\lim_{x\to 0} \frac{\lg_a x}{x^r} = 0 \quad \forall \in R^+ - \{1\}, \quad \forall r \in R^+ \tag{17}$$

$$\lim_{x\to +\infty} x^r a^x = \lim_{x\to +\infty} a^x \tag{18}$$

$$\lim_{x\to -\infty} |x|^r a^x = \lim_{x\to \infty} a^x \tag{19}$$

$$\lim_{x\to +\infty} \frac{e^x}{x^r} = \lim_{x\to +\infty} a^x \quad \forall r \in R^+ \tag{20}$$

$$\lim_{x\to +\infty} \frac{x^x}{e^r} = \lim_{x\to +\infty} a^x \quad \forall r \in R^+ \tag{21}$$

$$\lim_{x\to -\infty} e^x * x^r = 0 \quad \forall r \in R^+ \tag{22}$$

1.2 Goniometrici

$$\lim_{x\to 0} \frac{\sin x}{x} = 1 \tag{23}$$

$$\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b} \tag{24}$$

$$\lim_{x\to 0} \frac{\tan x}{x} = 1 \tag{25}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0 \tag{26}$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \tag{27}$$

$$\lim_{x\to 0} \frac{\arcsin ax}{bx} = \frac{a}{b} \tag{28}$$

$$\lim_{x\to 0} \frac{arctanx}{x} = 1 \tag{29}$$

2 formula retta tangente

$$(y - y_0) = m(x - x_0) \tag{30}$$

$$m = f'(x_0) \tag{31}$$

formula per il massimo e minimo relativo

$$f(x_0) \tag{32}$$

3 Taylor-Mc Laurin

1. Esponenziali
$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{24} + \frac{x^4}{120} + \cdots + \frac{x^n}{n!} + o(x^n) \forall x \in \mathbb{R}$$

2. Logaritmi
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} + \cdots + \frac{(-1)^{n+1}}{n} x^n + o(x^2)$$

3. Binomio
$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{6}x^3 + \cdots + \binom{\alpha}{n} x^n + o(x^n)$$

4. radicali pari
$$\sqrt{1+x} = (1+x)^{\frac{1}{2}} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \frac{5}{128}x^4 + \frac{7}{256}x^5 - \frac{21}{1024}x^6 + o(x^6)$$

5. radicali dispari

$$\sqrt[3]{1+x} = (1+x)^{\frac{1}{3}} = 1 + \frac{x}{3} - \frac{x^2}{9} + \frac{5}{81}x^3 + \frac{10}{243}x^4 + \frac{22}{729}x^5 - \frac{154}{6561}x^6 + o(x^6)$$

6. funzioni goniometriche

$$\sin(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{40320} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + o(x^{2n})$$
$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^4}{720} + \frac{x^8}{40320} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + o(x^{2n})$$
$$\tan x = x + \frac{x^3}{3} + \frac{x^3}{3} + \frac{2}{15}x^5 + o(x^6)$$
$$\arcsin(x) = x + \frac{x^3}{6} + \frac{3}{40}x^5 + \frac{5}{112}x^7 + \frac{35}{1152}x^9 + o(x^9)$$
$$\arctan(x) = x - \frac{x^3}{6} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + o(x^9)$$

4 Integrali

1. Situazione base

$$\int f'(x)dx = f(x) + c$$

$$\int f'(g(x)) * g'(x)dx = f(g(x)) + c$$

2. Integrale di dx

$$\int a * dx = ax + c$$

3. Integrale di x^n

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

Integrale notevole in forma generale

$$\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

4. Integrale di $\frac{1}{x}$

$$\int \frac{1}{x}dx = \ln(|x|) + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln(|f(x)|) + c$$

5. Integrale di $\sin(x)$

$$\int \sin(x)dx = -\cos(x) + c$$

$$\int \sin(f(x)) * f'(x)dx = -\cos(f(x)) + c$$

6. Integrale di $\cos x$

$$\int \cos(x)dx = \sin(x) + c$$

$$\int \cos(f(x)) * f' dx = \sin(f(x)) + c$$

7. Integrale di $\frac{1}{\cos^2 x}$

$$\int \frac{1}{\cos^2 x} dx = \tan(x) + c$$

$$\int \frac{1}{\cos^2 x} * f' dx = \tan(f(x)) + c$$

8. integrale di e^x

$$\int e^x dx = \frac{a^x}{\ln(x)} + c$$

$$\int e^{f(x)} = e^{f(x)} + c$$

9. Integrale di a^x

$$\int a^x dx = \frac{a^x}{\ln(x)} + c$$

$$\int a^{f(x)} * f'(x) dx = \frac{a^{f(x)}}{\ln(a)} + c$$

10. Integrale di $\frac{1}{1+x^2}$

$$\int \frac{1}{1+x^2} dx = \arctan(x) + c$$

$$\int \frac{1}{1+[f(x)]^2} * f'(x) dx = \arcsin(f(x)) + c$$

11. Integrale di $\frac{1}{\sqrt{1-x^2}}$

$$\int \frac{1}{\sqrt{1-x^2}} = \arcsin(x) + c$$

$$\int \frac{1}{\sqrt{1-[f(x)]^2}} * f'(x) dx = \arcsin(f(x)) + c$$

12. Integrale di $\frac{-1}{\sqrt{1-x^2}}$

$$\int \frac{-1}{\sqrt{1-x^2}} = \arccos(x) + c$$

$$\int \frac{-1}{\sqrt{1-[f(x)]^2}} * f'(x) dx = \arccos(f(x)) + c$$

5 Integrazione delle funzioni razionali fratte

5.1 Caso generico

Consideriamo l'integrale (indefinito o definito)

$$\int \frac{N(x)}{D(x)} dx$$

ove N(x), D(x) <u>polinomi a coefficienti reali.</u>

<u>Supponiamo grado(N)$\geq$ grado(D).</u>

Dividiamo $N(x)$ per $D(x)$, cioè scriviamo

$$\int \frac{N(x)}{D(x)}dx = Q(x) + \frac{R(x)}{D(x)}, \text{con } Q(x) \text{ polinomio quoziente,}$$

$R(x)$ polinomio resto, grado(R)<grado (D).
Allora

$$\int \frac{N(x)}{D(x)}dx = \int Q(x)dx + \int \frac{R(x)}{D(x)}dx$$

e grado (R)<grado (D)!

Tre casi:

- grado(D)=1
- grado(D)=2
- grado(D)>2

5.2 Caso I: grado(D)=1

SI ha

$$\begin{aligned} grado(D) = 1 \quad &\Rightarrow \quad D(x) = ax + b, a \neq 0 \\ grado(R) < grado(D) \quad &\Rightarrow \quad grado(R) = 0 \\ &\Rightarrow \quad R(x) = k. \end{aligned}$$

Quindi

$$\begin{aligned} \int \frac{R(x)}{D(x)}dx \qquad &= \int \tfrac{k}{ax+b}dx \\ &= \tfrac{k}{a} \int \tfrac{a}{ax+b}dx \\ &= \tfrac{k}{a} \ln|ax + b| + c \end{aligned}$$

6 metodo per sostituzione

6.1 primo metodo

$$t = e^x$$
$$dt = D(e^x);\ dx = e^x dx$$
$$\int \tfrac{e^x}{e^{2x}+1} = \int \tfrac{1}{t^2+1}dt$$
$$\int \tfrac{1}{t^2+1} = \arctan t + c = \arctan e^x + c$$

6.2 secondo metodo

$$t = e^x$$
$$x = \ln t$$
$$dx = D[\ln t]dt = \frac{1}{t}dt$$
$$\int \frac{e^x}{e^{2x}+1} dx = \int \frac{t}{t^2+1}\frac{1}{t}$$
$$\int \frac{\cancel{t}}{t^2+1}\frac{1}{\cancel{t}}dt = \int \frac{1}{t^2+1}dt$$
$$\int \frac{1}{t^2+1} = \arctan t + c = \arctan e^x + c$$

7 equazioni differenziali

- $\Delta > 0$

$$\boxed{y(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}}$$

- $\Delta = 0$

$$\boxed{y(x) = e^{\lambda_1 x}\,(c_1 + c_2 x)}$$

- $\Delta < 0$

$$\boxed{y(x) = e^{ax}\,[c_1 \cos(\beta x) + c_2 \sin(\beta x)]}$$

$$y(x) = e^{A(x)} \left(y_0 + \int_{x_0}^{x} e^{-A(t)} b(t) dt \right)$$